Sewing Entanglement Wedges

Jan de Boer, Amsterdam

Based on work with Bartek Czech, Dongsheng Ge, and Lampros Lamprou, arXiv:1903.04493 + work in progress
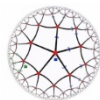(see also arXiv:1807.04276 and arXiv:1712.07123)

YIPQS long-term and Nishinomiya-Yukawa memorial workshop
Quantum Information and String Theory 2019
May 27 - June 28, 2019
Yukawa Institute for Theoretical Physics, Kyoto University
It from Qubit school/workshop
June 17 - June 28, 2019

Kyoto, June 20, 2019

Recent work has uncovered interesting relations between:

- Various notions of quantum information theory
- Algebraic QFT
- Chaos
- Shock waves
- Energy Conditions
- Bootstrap methods
- Bulk reconstruction
- Constraints on semiclassical gravitational backgrounds

Here, by thinking about a notion of parallel transport for entanglement wedges/modular Hamiltonians we will run into another set of such connections..
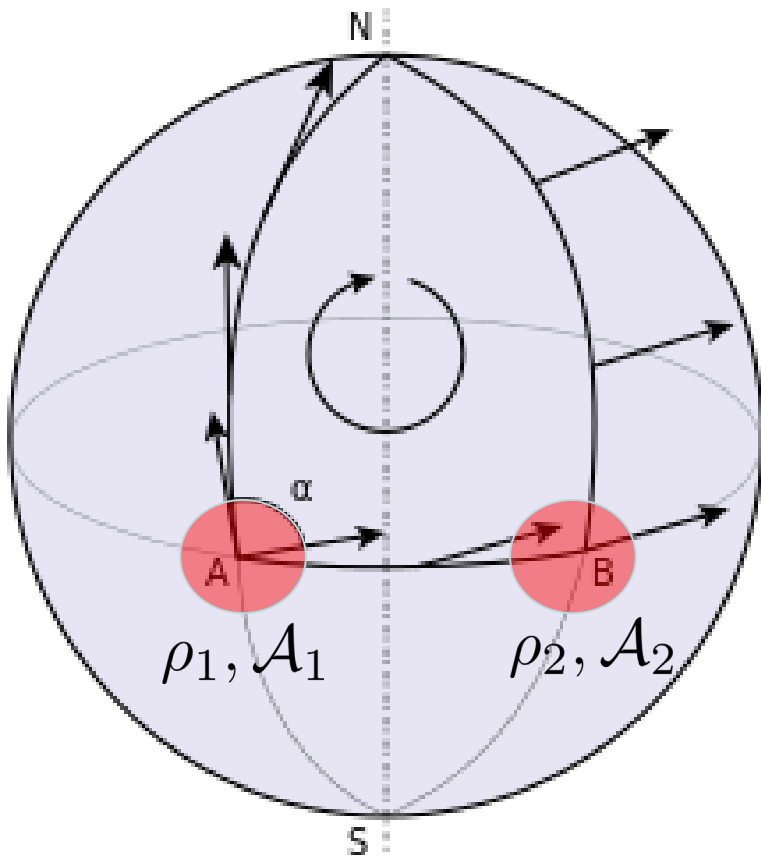
In general relativity, parallel transport provides a way to compare tangent vectors or frames at different points.

Freely falling observers accomplish parallel transport
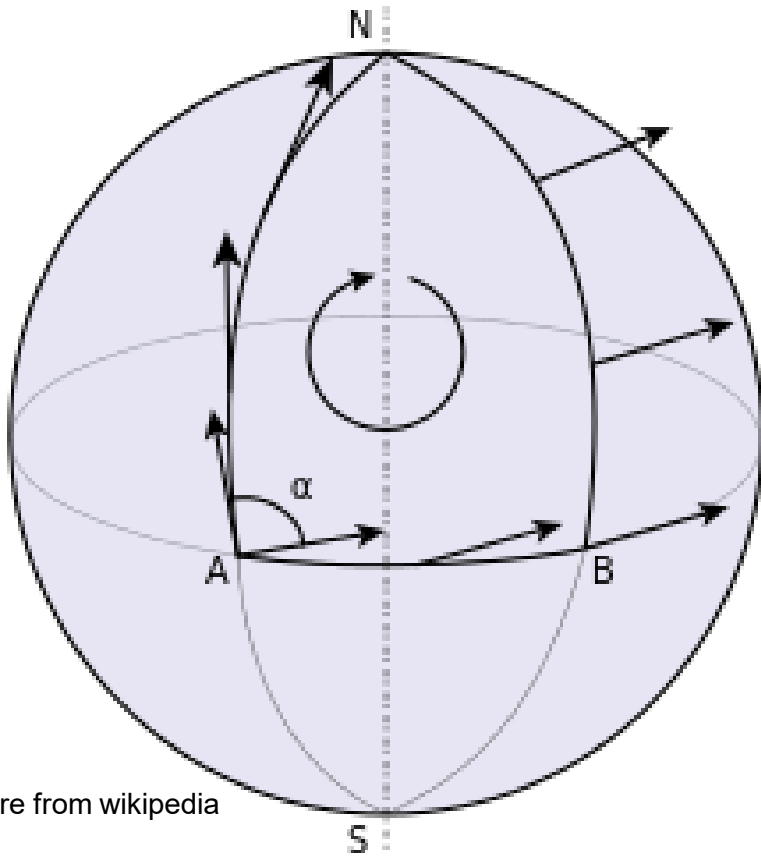
It would be nice to generalize this to (local) states and observables in order to compare different observers.

Symmetries?

$$\rho \quad \to \quad U^{\dagger} \rho U \equiv \rho$$
$$\mathcal{O} \quad \to \quad U^{\dagger} \mathcal{O} U$$

Connection to
spacetime↔entanglement
ER=EPR
….?

Toy example in quantum mechanics:

$$|\psi\rangle_{AB} = \sum_{ij} W_{ij} |i\rangle_A |j\rangle_B$$

Assume for simplicity ρ_A, ρ_B are maximally mixed.

$$\langle U_A^\dagger \mathcal{O}_A U_A \rangle = \langle \mathcal{O}_A \rangle \qquad \langle U_B^\dagger \mathcal{O}_B U_B \rangle = \langle \mathcal{O}_B \rangle$$

But

$$\langle U_A^\dagger \mathcal{O}_A U_A U_B^\dagger \mathcal{O}_B U_B \rangle \neq \langle \mathcal{O}_A \mathcal{O}_B \rangle$$

unless we correlate the two unitaries in a precise way.

$$|\psi\rangle_{AB} = \sum_{ij} W_{ij} |i\rangle_A |j\rangle_B$$

So in this case:

- Entanglement connects the bases in A and B
- W transforms like an open Wilson loop under the local symmetries
- W yields an anti-unitary map from A to B. This maps operators in A to mirror operators in B (cf Papadodimas-Raju) and automatically relates bases in the right way.
- Using W to go from A to B and another W' to go back from B to A results in a non-trivial holonomy.

Plan: use these ideas to study parallel transport for a family of entanglement wedges in AdS/CFT.
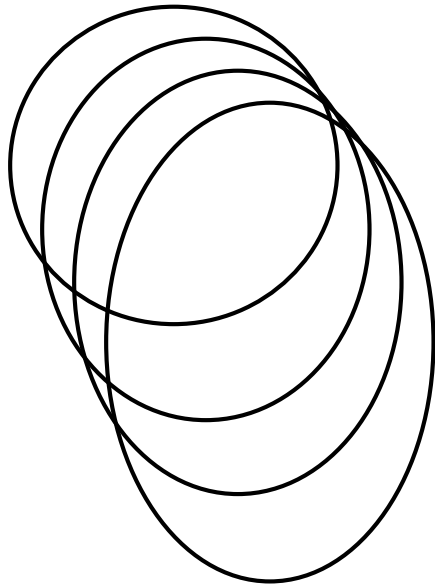
boundary
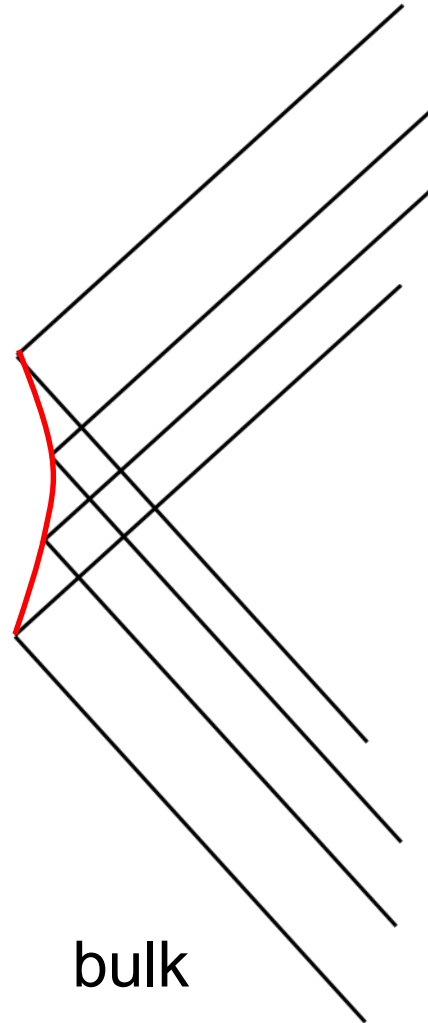
bulk

What are the symmetries?

Boundary: Unitaries that preserve the state, or equivalently, the modular Hamiltonian

$$U_A H_{\mathrm{mod},A} U_A^\dagger = H_{\mathrm{mod},A}$$

Using JLMS (Jafferis, Lewkowycz, Maldacena, Suh)

$$P_{\mathrm{code}} H_{\mathrm{mod}\,A} P_{\mathrm{code}} = \frac{A}{4G_N} + H_{\mathrm{bulk}}$$

Bulk: Diffeomorphisms that preserve the entanglement wedge, the area, and leave H_{bulk} invariant.

Boundary

$$H_{\mathrm{mod}}(\lambda) = U^\dagger(\lambda)\Delta(\lambda)U(\lambda)$$

Δ diagonal

U is ambiguous up to unitaries that leave Δ invariant.
Generically U(1)^d. Under infinitesimal change

$$\dot{H}_{\mathrm{mod}} = [\dot{U}^\dagger U, H_{\mathrm{mod}}] + U^\dagger \dot{\Delta} U$$

Parallel transport = particular choice of $U(\lambda)$
(flatness condition)

$$\dot{H}_{\mathrm{mod}} = [\dot{U}^\dagger U, H_{\mathrm{mod}}] + U^\dagger \dot{\Delta} U$$

Proposal: parallel transport is defined by requiring that $\dot{U}^\dagger U$ does not possess a *modular zero mode*

$$P_0^\lambda[V] \equiv \lim_{\Lambda \to \infty} \frac{1}{2\Lambda} \int_{-\Lambda}^{\Lambda} ds \, e^{iH_{\mathrm{mod}}(\lambda)s} V e^{-iH_{\mathrm{mod}}(\lambda)s}$$

$$P_0^\lambda[\dot{U}^\dagger U] = 0$$

$$\dot{H}_{\mathrm{mod}} = [\dot{U}^\dagger U, H_{\mathrm{mod}}] + U^\dagger \dot{\Delta} U$$

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zero mode

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$$\dot{H}_{\text{mod}} = [\dot{U}^\dagger U, H_{\text{mod}}] + U^\dagger \dot{\Delta} U$$

zero mode   zero mode

More generally, $P_0^\lambda[\dot{U}^\dagger U]$ is the connection for parallel transport:

If $\dot{V} - P_0^\lambda[\dot{U}^\dagger U]V = 0$ then $V^\dagger U^\dagger H U V$ obeys the parallel transport condition.

These equation may look peculiar, but are nothing but a generalization of the Berry phase:

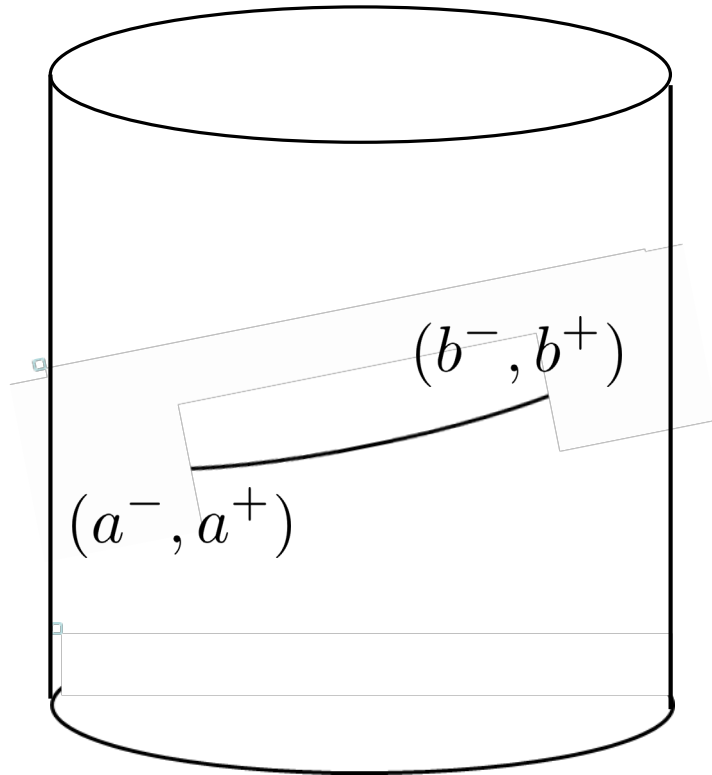Take $\rho(\lambda) = |\psi(\lambda)\rangle\langle\psi(\lambda)|$ then

$$|\dot{\psi}\rangle\langle\psi| + |\psi\rangle\langle\dot{\psi}| = [\,|\dot{\psi}\rangle\langle\psi| - |\psi\rangle\langle\dot{\psi}|\,,\,|\psi\rangle\langle\psi|\,]$$

project

$$\langle\psi\,|\dot{\psi}\rangle\langle\psi| - |\psi\rangle\langle\dot{\psi}|\,\psi\rangle = \langle\psi|\dot{\psi}\rangle - \langle\dot{\psi}|\psi\rangle$$

usual Berry connection

Example 1: 2d CFT with intervals

(b^-, b^+)

(a^-, a^+)

$$H_{\mathrm{mod}} = s_1 L_1 + s_0 L_0 + s_{-1} L_{-1} + t_1 \bar{L}_1 + t_0 \bar{L}_0 + t_{-1} \bar{L}_{-1}$$

$$s_1 = \frac{2\pi \cot(b^+ - a^+)/2}{e^{ia^+} + e^{ib^+}} \qquad s_0 = -2\pi \cot(b^+ - a^+)/2 \qquad \text{etc}$$

$$\frac{\partial H_{\mathrm{mod}}}{\partial \lambda} = \frac{\partial a^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^+} + \frac{\partial b^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^+} + \frac{\partial a^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^-} + \frac{\partial b^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^-}$$

$$2\pi i V = \frac{\partial a^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^+} - \frac{\partial b^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^+} - \frac{\partial a^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^-} + \frac{\partial b^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^-}$$

then

$$\partial_\lambda H_{\mathrm{mod}} = [V, H_{\mathrm{mod}}]$$

This reproduces the results of Czech, Lamprou, McCandlish and Sully. In particular, the holonomy is related to differential entropy.
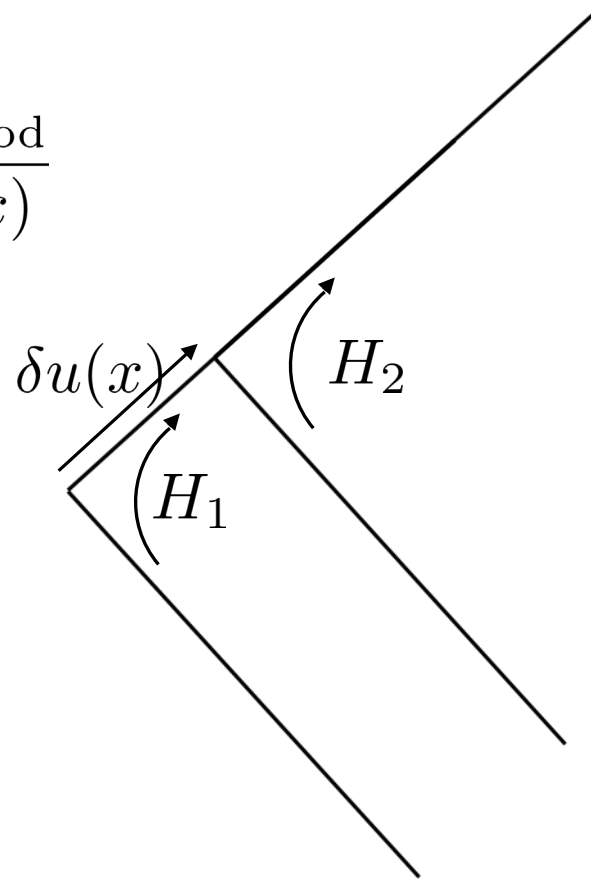
Example 2: modular inclusions (Casini, Teste, Torroba)

$$[H_2, H_1] = 2\pi i (H_2 - H_1)$$

$$\left[\frac{\delta H_{\mathrm{mod}}}{\delta u(x)}, H_{\mathrm{mod}}\right] = 2\pi i \frac{\delta H_{\mathrm{mod}}}{\delta u(x)}$$
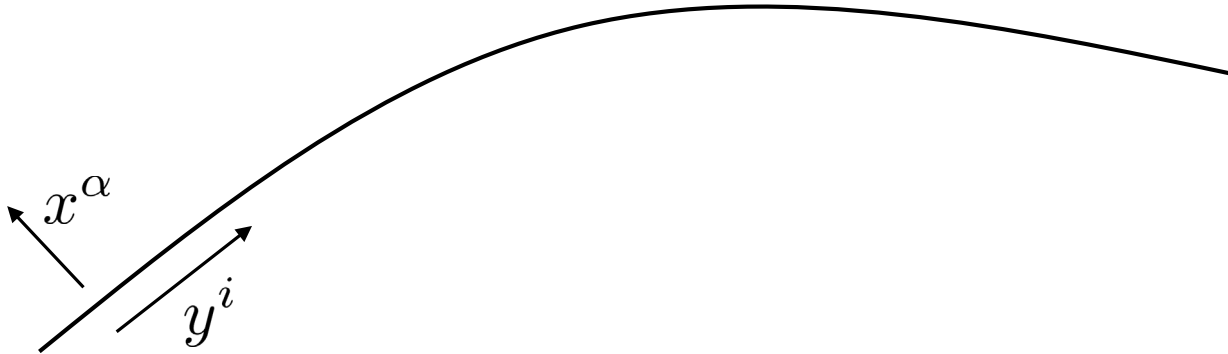
connection

Bulk picture

$$P_{\mathrm{code}} H_{\mathrm{mod}A} P_{\mathrm{code}} = \frac{A}{4G_N} + H_{\mathrm{bulk}}$$

What are the relevant symmetries? The bulk modular Hamiltonian becomes approximately a boost near the minimal surface.

Will take the point of view of "surface symmetries", "edge modes", "asymptotic symmetry group", etc (cf Donnelly, Freidel; Camps)

Consider a minimal surface

Choose coordinates

$$ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta + \gamma_{ij}dy^i dy^j + \mathcal{O}(x)$$

Modular Hamiltonian is boost in transversal plane

$$
\begin{aligned}
\zeta^\alpha_{\text{mod}} &= 2\pi\epsilon^{\alpha\beta}x_\beta + \mathcal{O}(x^2) \\
\zeta^i_{\text{mod}} &= \mathcal{O}(x^2)
\end{aligned}
$$

$$\zeta^{\alpha}_{\text{mod}} = 2\pi\epsilon^{\alpha\beta}x_{\beta} + \mathcal{O}(x^2)$$
$$\zeta^{i}_{\text{mod}} = \mathcal{O}(x^2)$$

Symmetries:

$$\zeta^{\alpha}_{\text{sym}} = \omega(y)\epsilon^{\alpha\beta}x_{\beta} + \mathcal{O}(x^2)$$
$$\zeta^{i}_{\text{sym}} = \zeta^{i}_0(y) + \mathcal{O}(x^2)$$

Preserve asymptotic form of metric, have non-trivial charges, and commute with modular Hamiltonian vector field.

Two components: diffeomorphisms along surface and location-dependent frame rotations.

Microscopic interpretation? Only visible in code subspace?

Parallel transport should provide:

- a relation between the coordinates y of nearby minimal surfaces
- a relation between choices of orthonormal bases for the normal plane
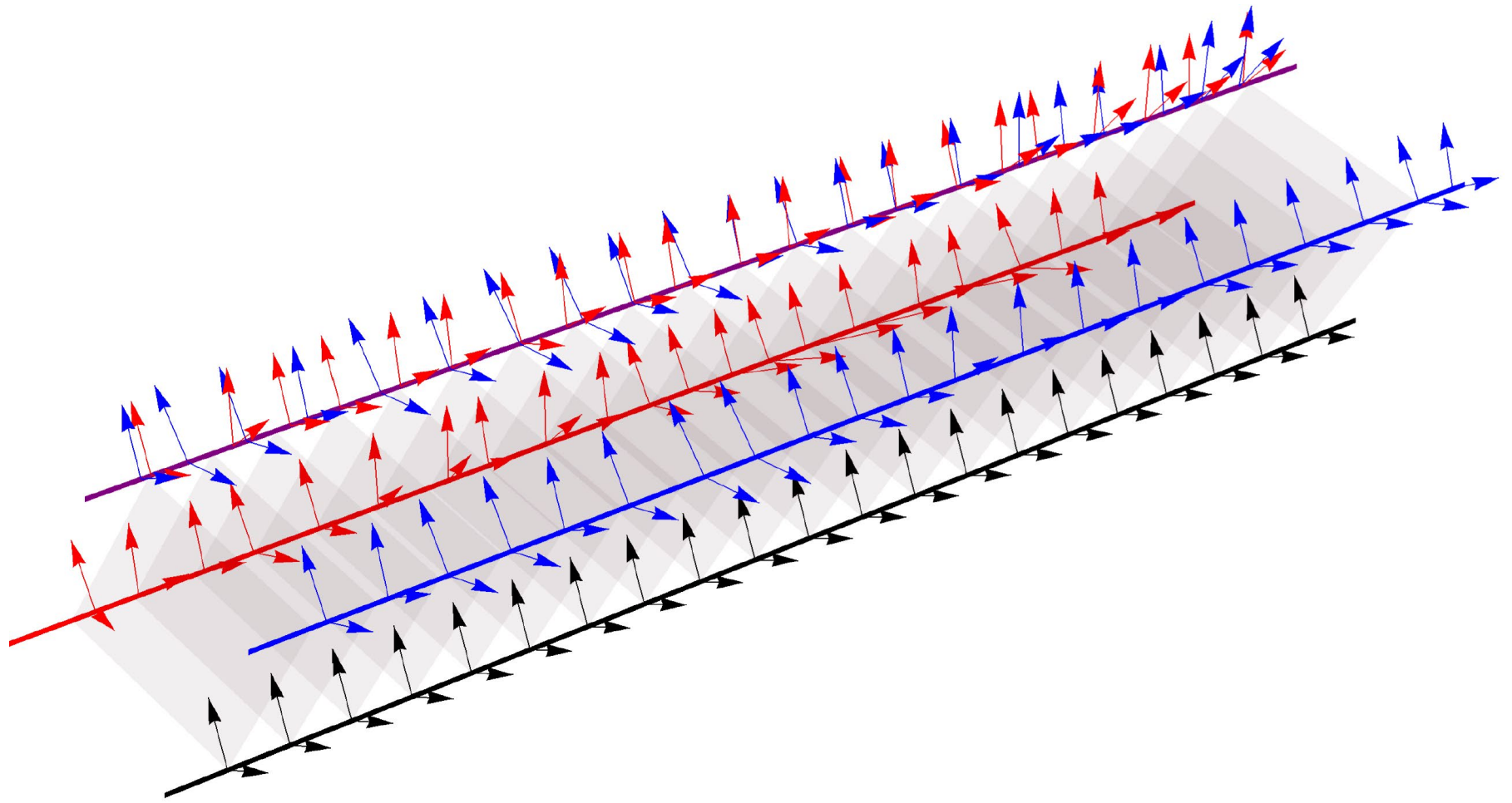
One can repeat analysis:

$$\frac{d\zeta_{\text{mod}}}{d\lambda} = [\xi, \zeta_{\text{mod}}]$$

where on minimal surface

$$
\begin{aligned}
\epsilon^{\alpha\beta}\partial_\alpha\xi_\beta &= 0 \\
\xi^i &= 0
\end{aligned}
$$

Qualitative picture:

➢ To transport a point on a minimal surface to a point on a nearby minimal surface: separation should be orthogonal to the minimal surfaces

➢ To transport normal frame: parallel transport in orthogonal direction and project into new normal plane
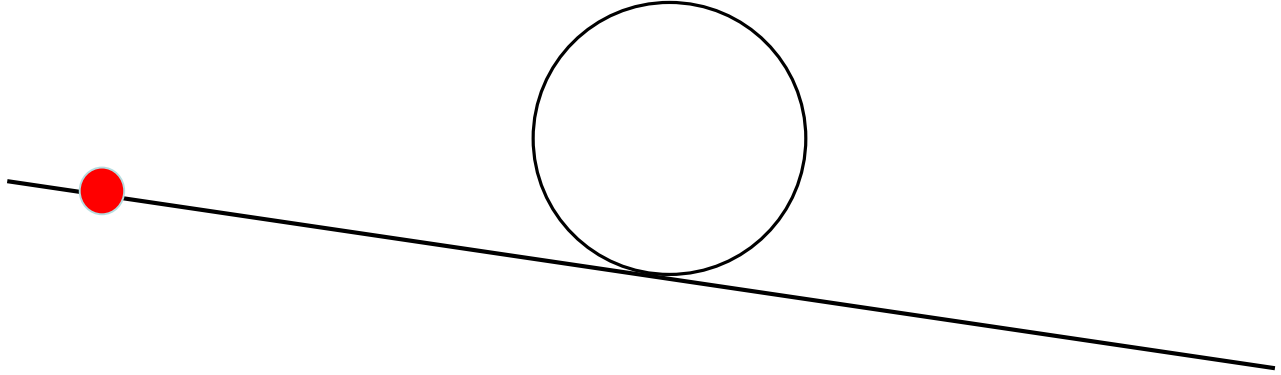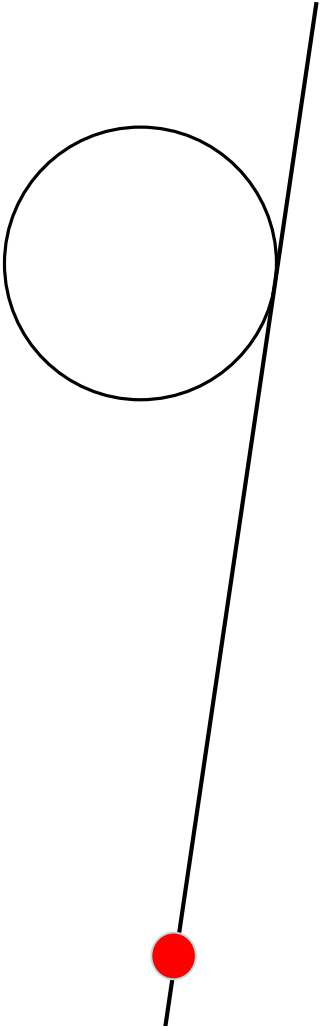
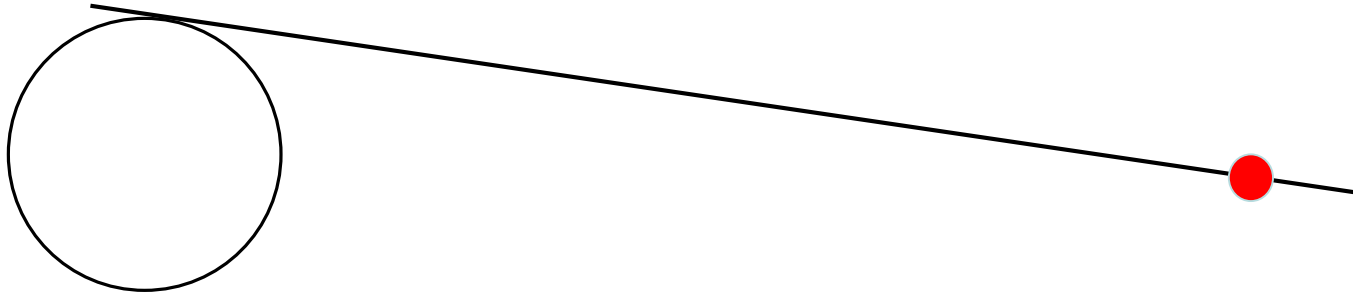Holonomy: surface diffeomorphism + frame rotation

Surface diffeomorphism illustrated
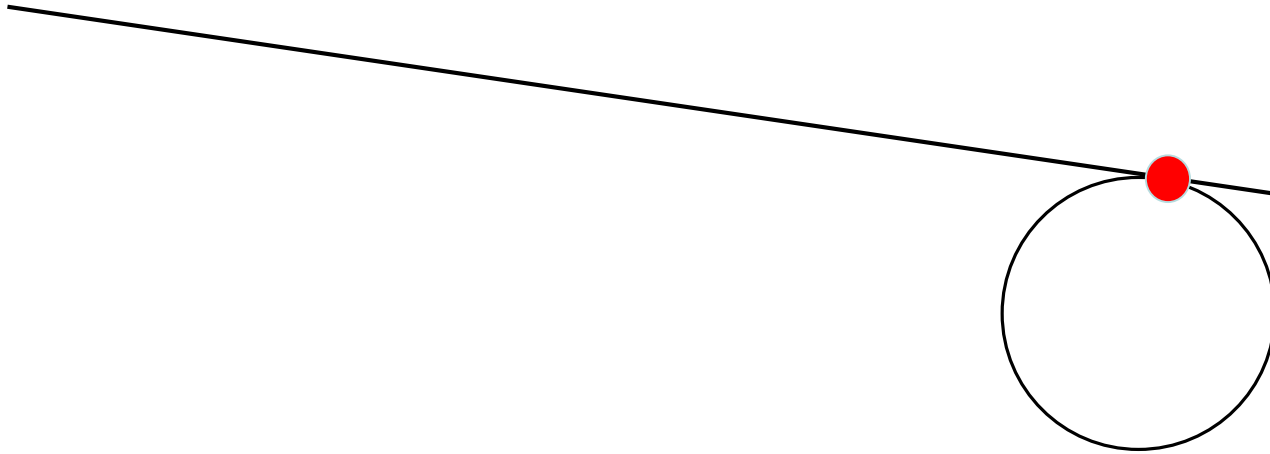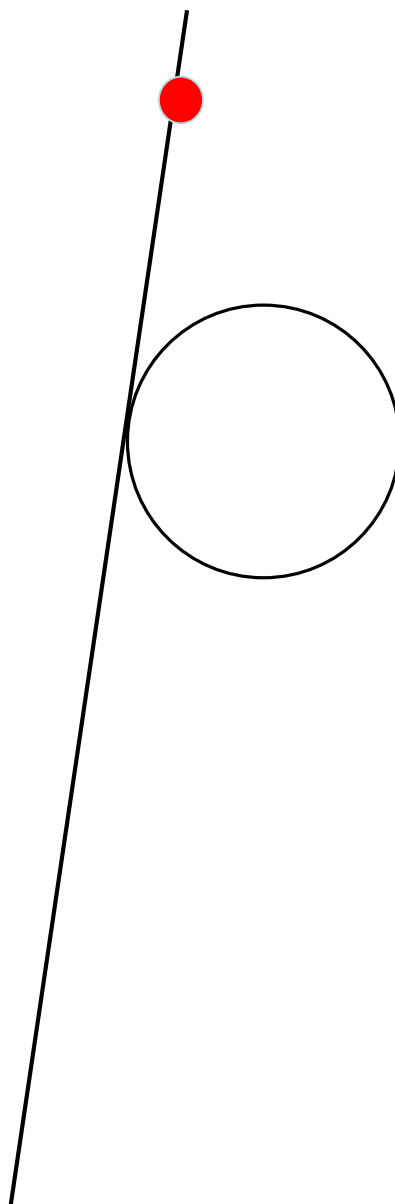
Red dot gets displaced by circumference of circle. Can use this to reconstruct length of curves.

(Balasubramanian, Chowdhury, Czech, JdB, Heller)

Interpretation of frame rotation? Closely related to bulk curvature.

Consider e.g. 2d case where minimal surfaces are points.

Procedure reduces to ordinary parallel transport of tangent vectors, and curvature of connection = geometric curvature.
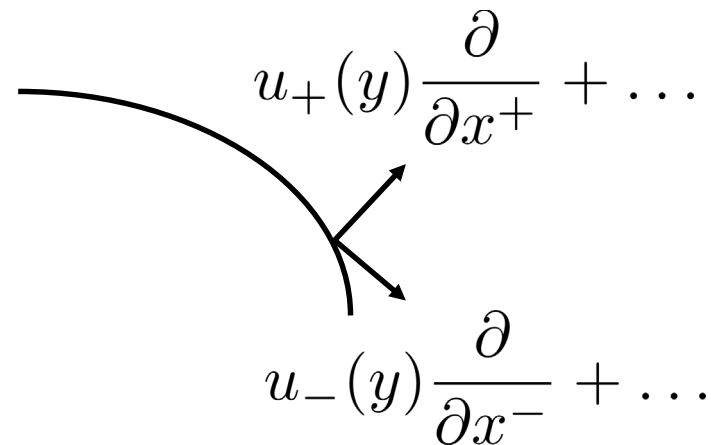
Connection to curvature only clear in case $K^2, \partial K \ll R$

For AdS3, boundary discussion can be directly translated into bulk (symmetries=Killing vectors) and agrees with above picture.

Examples show that there are special situations in which

$$\delta H_{\text{mod}} = \pm \left[\frac{\delta H_{\text{mod}}}{2\pi i}, H_{\text{mod}} \right]$$

In the bulk, these correspond to deformations in light cone directions. One can think of these vector fields as "shock waves"

$$u_+(y) \frac{\partial}{\partial x^+} + \dots$$

$$u_-(y) \frac{\partial}{\partial x^-} + \dots$$

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta + \gamma_{ij} dy^i dy^j + \mathcal{O}(x)$$

In the bulk, these modes always exist. But what about the boundary theory?

Conjecture:

If $[H_{\mathrm{mod}}, \delta H_{\mathrm{mod}}] = i\lambda \delta H_{\mathrm{mod}}$ then $|\lambda| \leq 2\pi$. This is closely related to the bound on chaos. If there is a semiclassical gravitational dual the bound is saturated.

cf modular chaos of
Faulkner, Li, Wang

- holds for Virasoro deformations of AdS3
- holds for inclusions of operator algebras
- does not hold in higher spin theories in AdS3 – maximal value of λ agrees with result of Perlmutter

The "shock wave" vector fields have an interesting commutator

$$[V_+, V_-] \sim (\gamma^{ij} \partial_i u_+ \partial_j u_- + R u_+ u_-) H_{\mathrm{mod}}$$

which is similar to the expression for Planckian scattering found by 't Hooft; Verlinde, Verlinde...

Key in this computation is to pick the right vector fields, i.e. where the modular zero mode has been projected out.

Many questions:

- General interpretation of the curvature/holonomy?
- (dis)prove conjecture
- Clarify the precise relation to chaos and shockwaves
- Connect to recent work on shock waves (Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov; Belin, Hofman, Mathys)
- Connection to other recent appearances of the Berry connection? (Belin, Lewkowycz, Sarosi)
- Connection to bulk reconstruction? (Faulkner, Li, Wang; Kabat, Lifschytz)
- Useful to get a handle on the code subspace?
- Connection to recent discussions of soft hair? (…)
- Include other fields (e.g. gauge fields) with non-trivial edge modes?
- Generalize to other spacetimes?
- Can we get dynamics in this framework?
- Applications to two-sided case?