A Generalized Entanglement Entropy and Holography

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Based on 1809.09109 (Phys. Rev. Lett. 122, 141601)

and work in progress with Yuya Kusuki (YITP)

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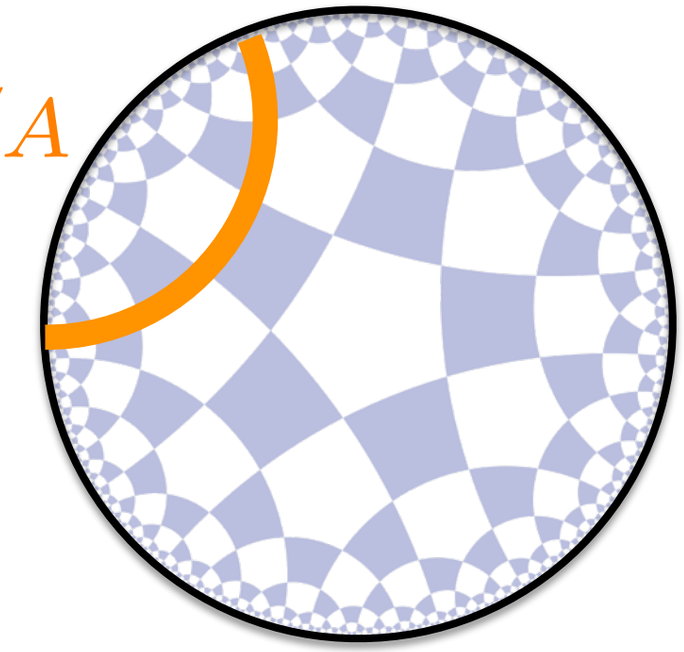
Motivation

Measure for Mixed states in AdS/CFT?

Ryu-Takayanagi formula

Ryu-Takayanagi '06,…

$$S(\rho_A) =$$



Entanglement entropy $S(\rho_A)$:
nice measure for pure states

How is the measure for mixed states?

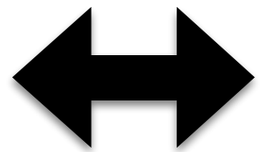
There are many candidates in the literature …
(Any nice ones in AdS/CFT ?)

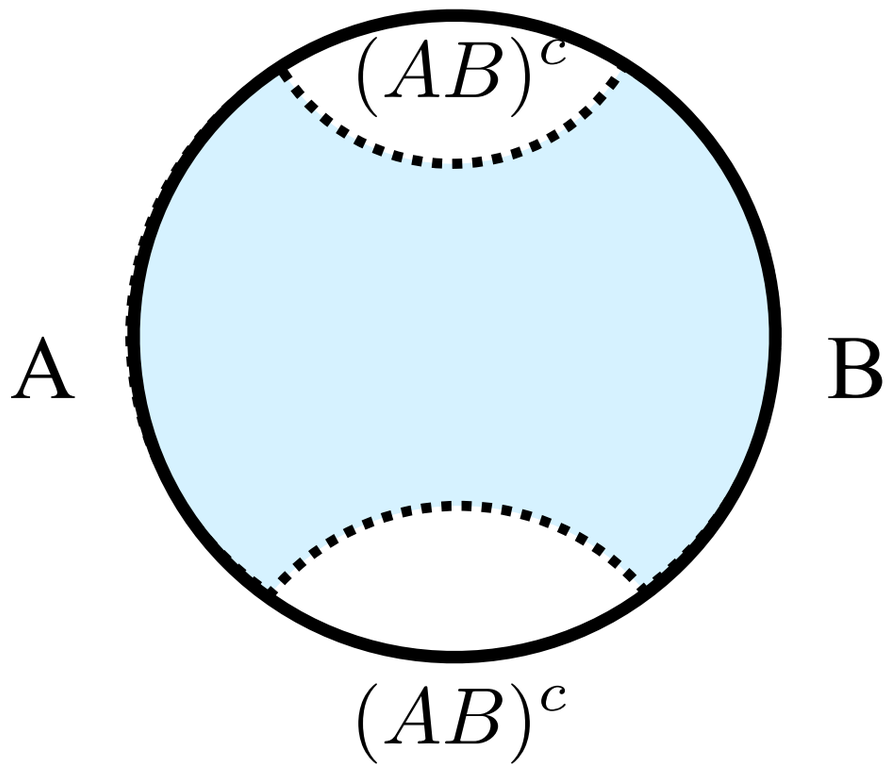
Measure for Mixed states in AdS/CFT?

Subregion/subregion duality

Czech-Karczmarek-Nogueira-VanRaamsdonk, Wall '12, Headrick, Hubeny-Lawrence-Rangamani '14…

$$\rho_{AB} = \mathrm{Tr}_{(AB)^c} |0\rangle\langle 0|$$





reduced density matrix

(a mixed state)

entanglement wedge

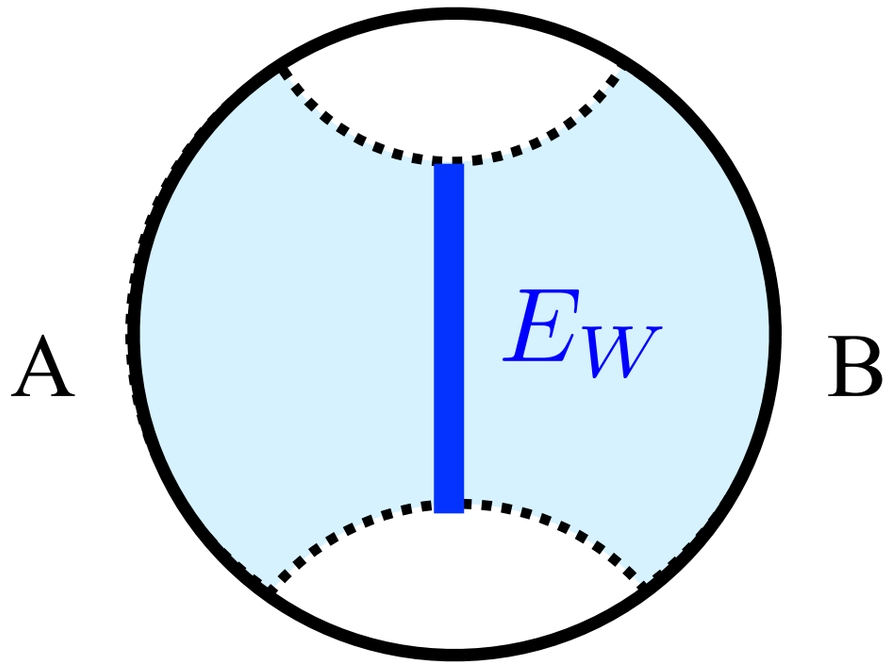
Measure for Mixed states in AdS/CFT?

Subregion/subregion duality

Czech-Karczmarek-Nogueira-VanRaamsdonk, Wall ’12, Headrick, Hubeny-Lawrence-Rangamani ‘14…

A natural object in the bulk :

Umemoto-Takayanagi ‘17,
Nguyen-Devakul-Halbasch-Zaletel-Swingle ‘17



entanglement wedge cross section

Measure for Mixed states in AdS/CFT?

Subregion/subregion duality

Czech-Karczmarek-Nogueira-VanRaamsdonk, Wall '12, Headrick, Hubeny-Lawrence-Rangamani '14…

Purification conjectures:

$$E_P = E_W$$

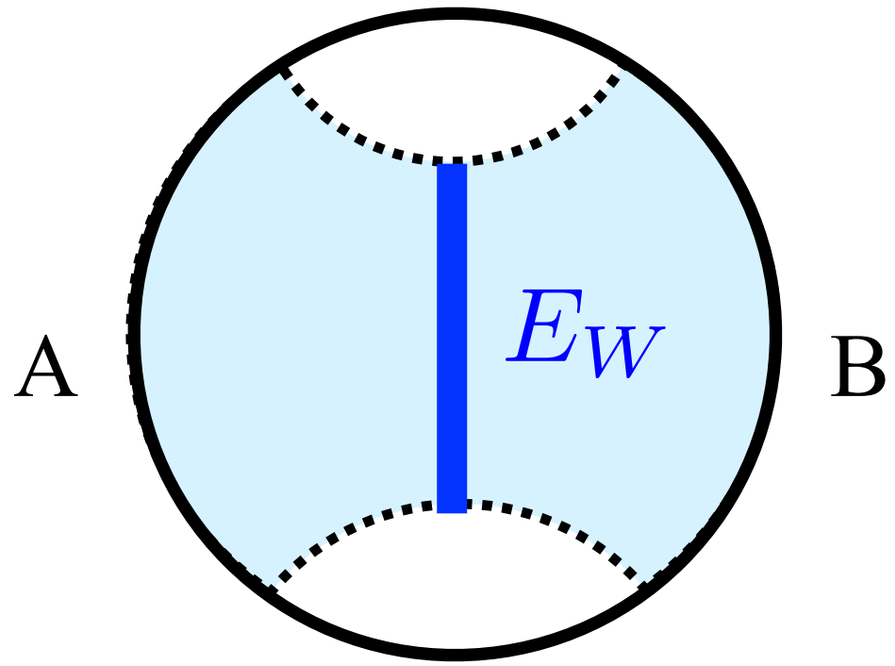
(min S(AA') for ∀ purification $|\Psi_{AA'BB'}\rangle$)

Umemoto-Takayanagi '17,
Nguyen-Devakul-Halbasch-Zaletel-Swingle '17

$$S_R = 2E_W$$

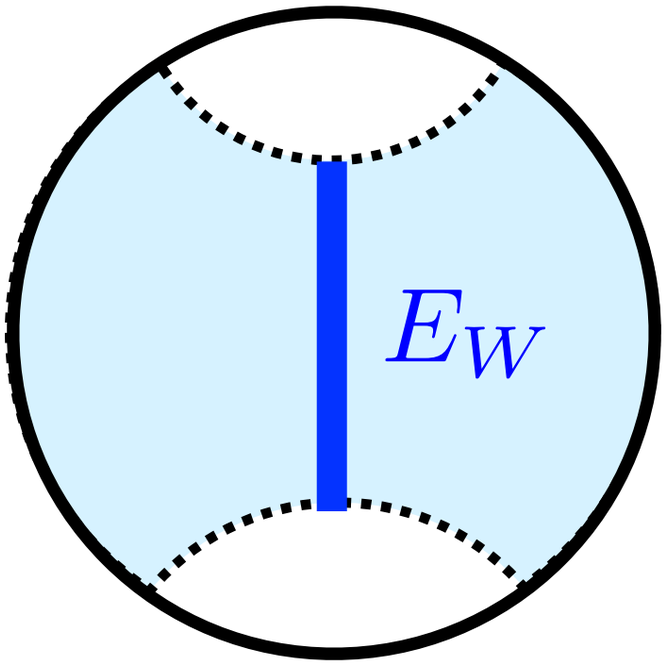
(S(AA') for TFD-like purification $|\Psi_{AA'BB'}\rangle$)

Dutta-Faulkner '19



entanglement wedge
cross section

This talk



from CFT without purification

the entanglement wedge should know
its cross section without introducing purified states!

It from an “odd” generalization of the entropy

Odd (Entanglement) Entropy

KT '18

$$S_o(\rho_{AB}) := \lim_{n_o:\mathrm{odd}\to 1} \frac{\mathrm{Tr}(\rho_{AB}^{T_B})^{n_o} - 1}{1 - n_o}$$

- partial transposition:

$$\langle i_A, j_B | \, \rho_{AB}^{T_B} \, | k_A, \ell_B \rangle \equiv \langle i_A, \ell_B | \, \rho_{AB} \, | k_A, j_B \rangle$$

- $\rho_{AB}^{T_B}$ can have negative eigenvalues

∃ negative eigenvalue

→ ∃ entanglement btw A&B Peres '96

Odd (Entanglement) Entropy

KT '18

$$S_o(\rho_{AB}) = -\sum_{\lambda_i>0} |\lambda_i| \log |\lambda_i| + \sum_{\lambda_i<0} |\lambda_i| \log |\lambda_i|$$

λ_i : eigenvalues for $\rho_{AB}^{T_B}$

For pure states, the same as usual EE!

$$S_o(\rho_{AB}) = S(\rho_A) \text{ (if } \rho_{AB} \text{ is a pure state)}$$

Note: it also counts classical correlations

$\leftrightarrow$ logarithmic negativity

$$\mathcal{E} = \lim_{n_e:\text{even}\to 1} \log \mathrm{Tr}(\rho_{AB}^{T_B})^{n_e}$$

Vidal-Werner '02, Calabrese-Cardy-Tonni '12

Kudler-Flam—Ryu '18

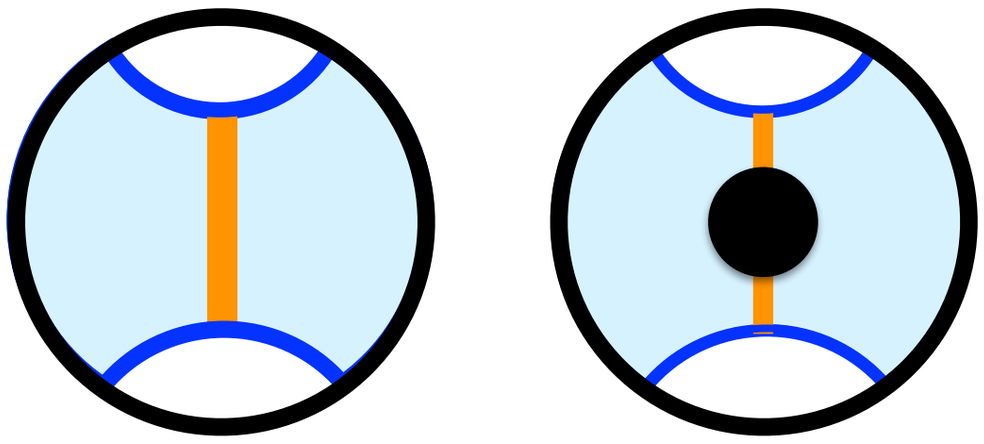
$\propto E_W$

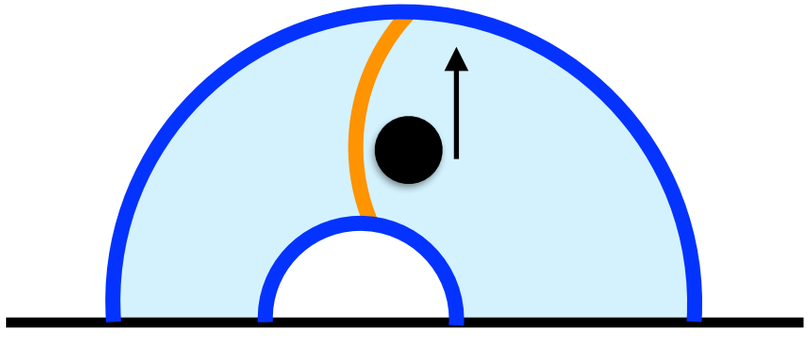
(with back-reaction)

Results in 2d holographic CFT

$$S_o(\rho_{AB}) = S(\rho_{AB}) + E_W(\rho_{AB})$$

minimal surfaces　　EWCS





- Vacuum and Thermal state

KT'18

- Heavy Excited states
- Quench by local heavy op.

Yuya Kusuki & KT to appear

Can use Replica trick

Calabrese-Cardy '04 ,Calabrese-Cardy-Tonni '12,

Large-c + Sparse spectrum

Hartman '13,

Can use Fusion (Crossing) Kernel

Kusuki '18, Collier-Gobeil-Maxfield-Perlmutter '18, Kusuki-Miyaji '19

A Lesson from the odd entropy

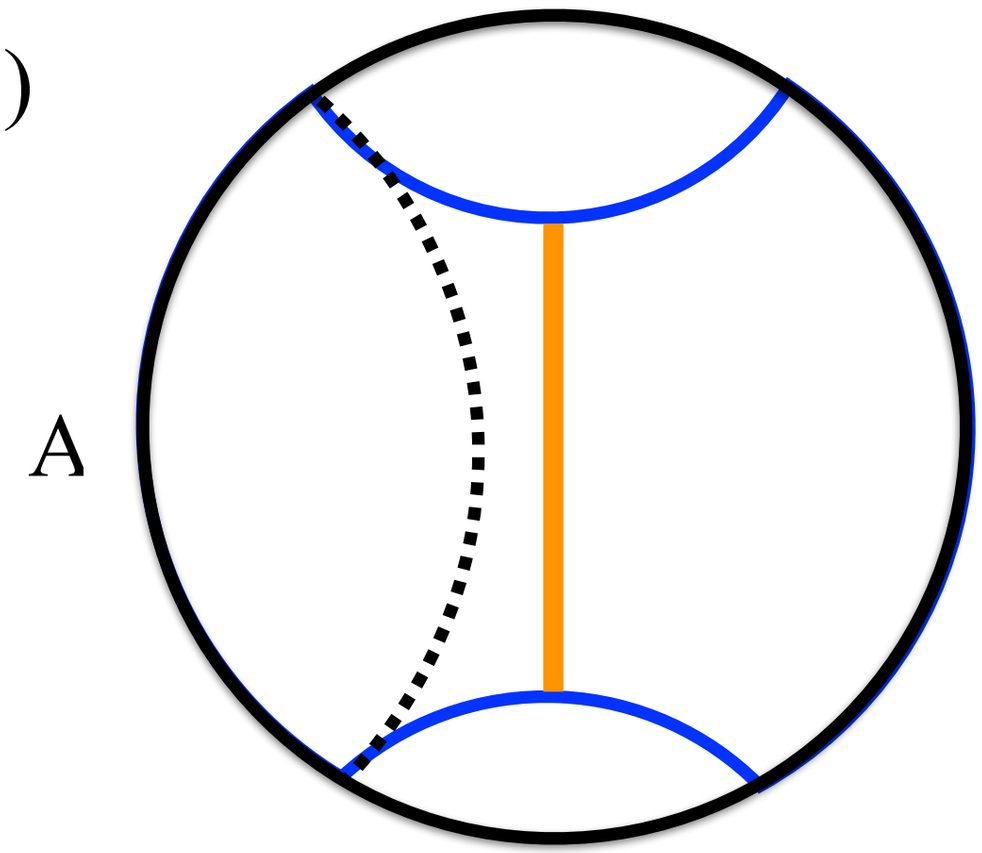
e.g.) an inequality from entanglement wedge

Freedman-Headrick'16, Umemoto-Takayanagi '17,
Nguyen-Devakul-Halbasch-Zaletel-Swingle '17

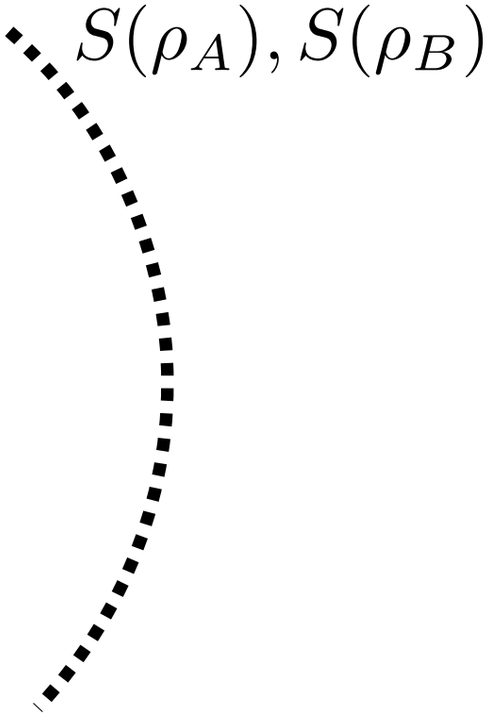
$$E_W(\rho_{AB}) \geq I(A:B)/2$$

$$\Leftrightarrow \quad 2E_W(\rho_{AB}) + S(\rho_{AB}) \geq S(\rho_A) + S(\rho_B)$$

$\because$)



$\geq$



A Lesson from the odd entropy

$$S_o(\rho_{AB}) - S(\rho_{AB}) \overset{?}{\geq} I(A:B)/2$$

It rather specializes the holographic CFT!

A counterexample:

$$\rho_{AB} = q\,|\Psi\rangle\langle\Psi| + (1-q)\sigma_A \otimes \sigma_B$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle\,|1_B\rangle - |1_A\rangle\,|0_B\rangle)$$

$$\sigma = \frac{1}{2}\,|0\rangle\langle 0| + \frac{1}{2}\,|1\rangle\langle 1|$$

Discussion

- More general many body systems

  General properties of odd EE ? / How holographic CFT is special?

- Relation to conditional & differential entropy

  Balasubramanian-Chowdhury-Czech-de Boer-Heller '13, ...

- From gravitational path integral

  c.f. Lewkowycz-Maldacena, Faulkner-Lewkowycz-Maldacena '13

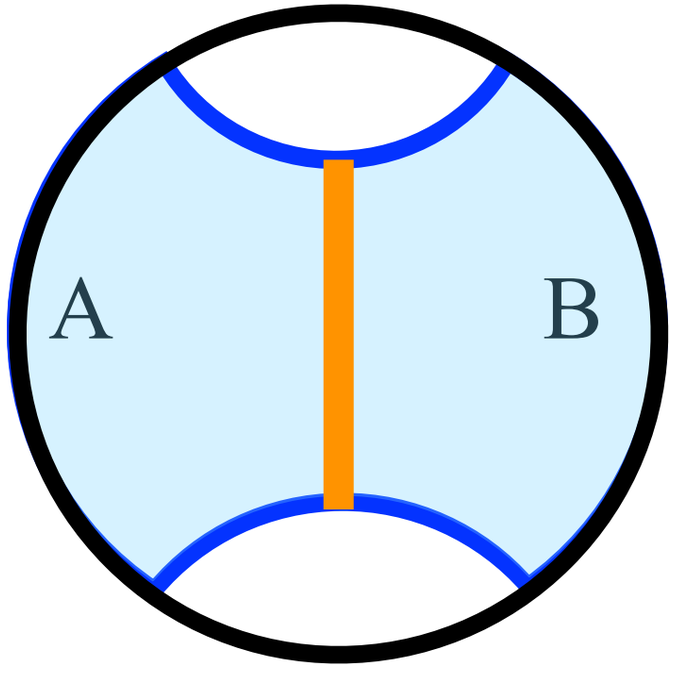
  ⋮

Summary: odd entropy and holography

$$S_o(\rho_{AB}) := \lim_{n_o:\text{odd}\to 1} \frac{\mathrm{Tr}(\rho_{AB}^{T_B})^{n_o} - 1}{1 - n_o}$$

$$S_o(\rho_{AB}) = S(\rho_{AB}) + E_W(\rho_{AB})$$

minimal surfaces　　EWCS



→ new constraints for holographic CFT