A computational lens on Quantum Experiments

Dorit Aharonov
Hebrew University

Joint work with Jordan Cotler and Xiaoliang Qi

Some quantum revolutions…

90's: Quantum computation

Exponential Q algorithms,

cryptography

2000's:
Quantum Hamiltonian complexity

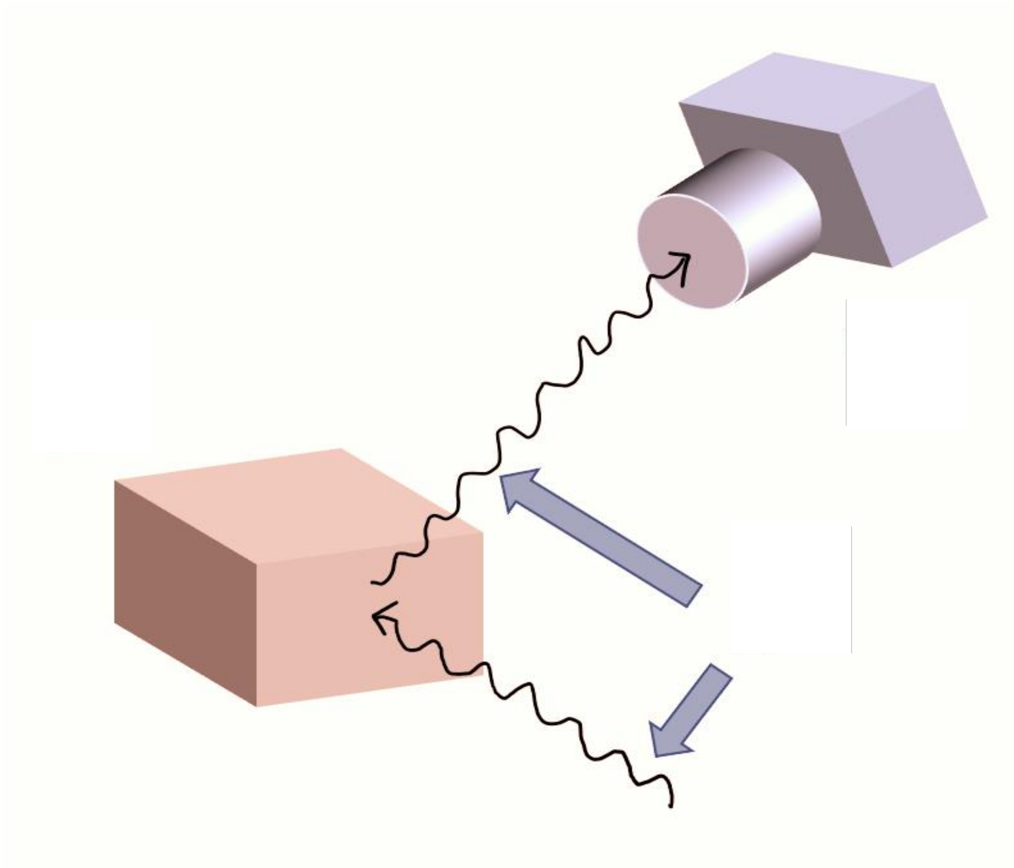
The computational lens on Q physics;
QMA hardness +
Complexity of tensor networks

2010's:
Quantum algorithmic Experiments

Introducing quantum algorithmic techniques into experiments

X-ray diffraction

Determining the atomic and molecular structure of a crystal



Adding Computational Ingredients: Sensing and Metrology

Example 1:
Enhancing resolution in metrology from standard quantum limit ($1/\sqrt{t}$) to Heisenberg limit ($1/t$) using entangled Noon states

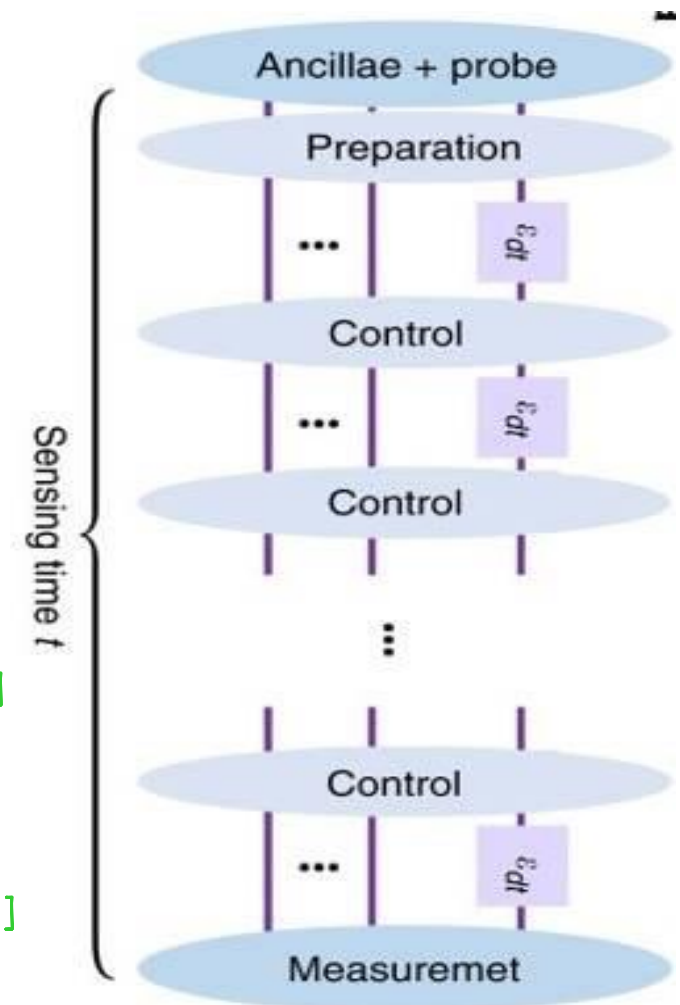
$$\frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}}$$

entanglement

[Jiovannetti, Lloyd, Maccone'11]

Example 2:
Increasing **sensing** resolution using quantum error correction, from standard quantum limit ($1/\sqrt{t}$) to Heisenberg limit ($1/t$)

Arrad Vincler Aharonov Retzker[PRL'14]
KesslerLovchinskySushkovLukin[PRL'14]
DurSkotiniotisFrowisKraus[PRL'14]
Ozeri[Preprint'13]
Unden et al [PRL'16]
ZhouZhangPreskillJiang[NatureComm'18]



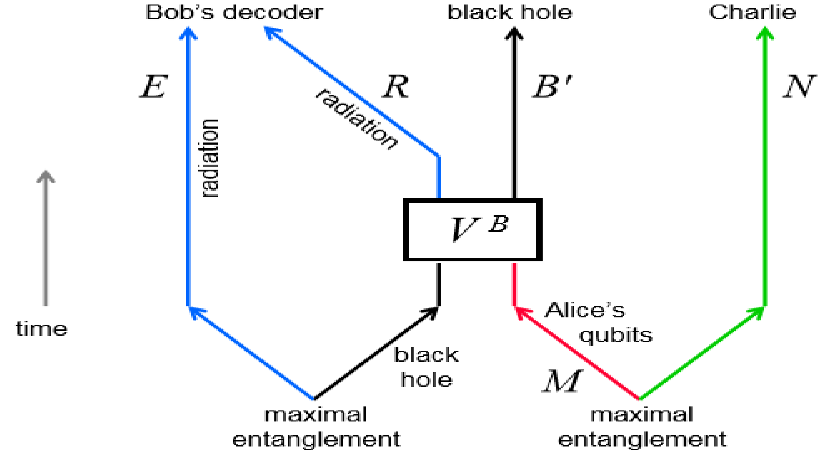
Quantum error correction

Adding Computational Ingredients to Experiments: Black holes

Example 3: Blackholes as mirrors

(An experiment that tests the hypothesis that Black holes reradiate Quantum information quickly, related to the information paradox)

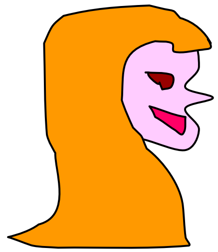
Hayden Preskill [JHEP'07]



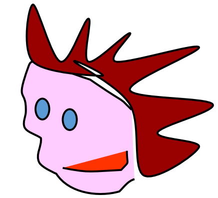
Howking radiation emitted from black hole is stored in quantum memory

Using a full-fledged quantum computer in a gedanken experiment

Example 4: Quantum Interactive experiments

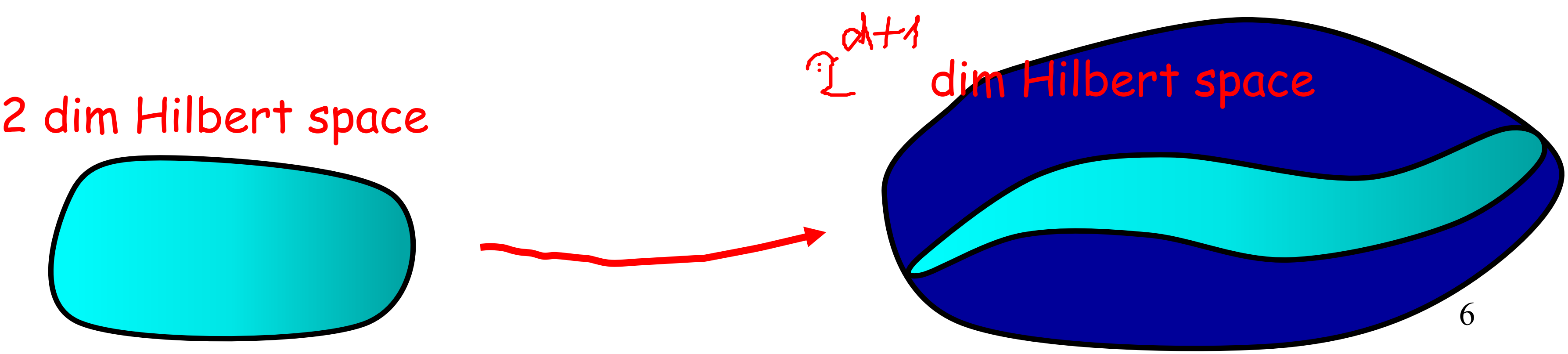


Verifier: BPP
+ O(1) qubits



Quantum
system

Theorem: The correctness of *any* quantum circuit can be tested by an interaction with a BPP+O(1) qubits verifier!



Aharonov Ben-Or Eban [2008], Aharonov BenOr Eban Mahadev [2017]
Broadbent Fitzsimons Kashefi [2008]

Interaction, adaptivity

Example 5: Exponentially precise energy measurements

factor N $\equiv$ find min r. s.t. $y^r = 1 \bmod N$

$n \simeq \log N$ bits

In poly(n) time

Shor's unitary: $U|x\rangle = |y \cdot x \bmod N\rangle$

U^t can be applied for exponential t!

$H = U + U^\dagger$

e^{iHt} can be applied by a QC for $t \simeq e^n$

Quantum computation

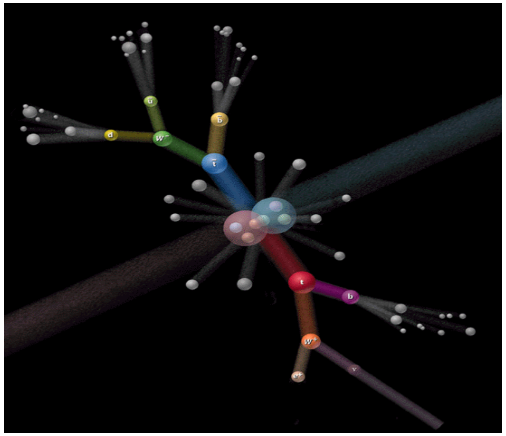
[Atia Aharonov NatureComm'17]

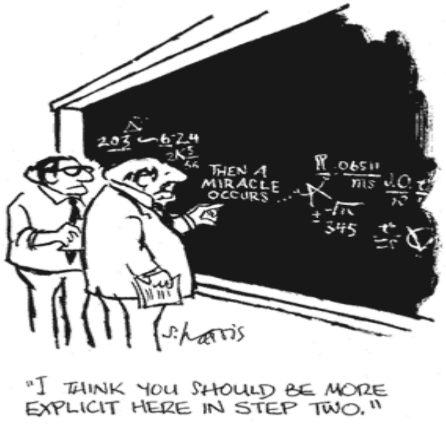
$$|\psi\rangle \frac{|0\rangle + |1\rangle}{\sqrt{2}} \longrightarrow \frac{1}{\sqrt{2}}\left(|\psi\rangle|0\rangle + e^{iHt}|\psi\rangle|1\rangle\right)$$

$|\psi\rangle$ is e.vec with e.v E $\Longrightarrow$ $|\psi\rangle\left(\frac{|0\rangle + e^{iEt}|1\rangle}{\sqrt{2}}\right)$

fast forwarding $\left(e^{iHt}\text{ in time }\log^c t\right)$ $\Longleftrightarrow$ Exponential violations of $\Delta E \Delta T \geq 1$

What is a Physical Experiment?



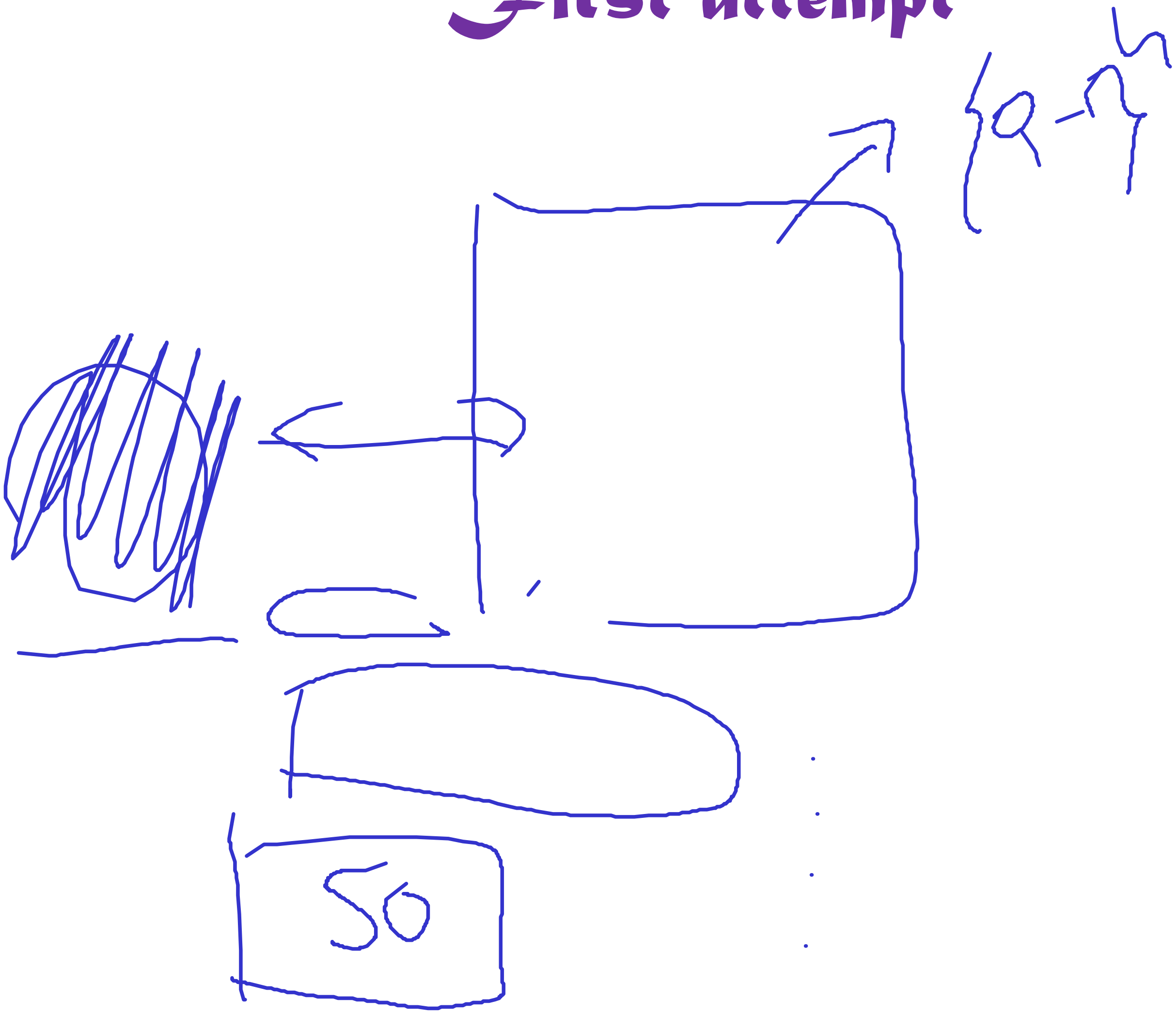


How to model the most general quantum experiment?

We want a model that will enable us to study Measurement processes and compare their resources without strong dependence on the physical implementations

A computational complexity model of measurements

First attempt

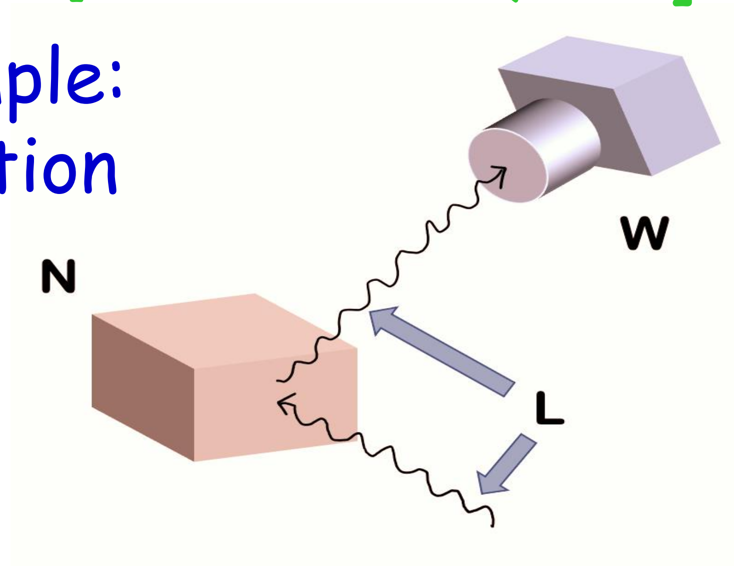


Quantum Algorithmic Measurements (QUALMs)

[AharonovCotlerQi'2021]

A useful example: X-ray diffraction

Crystal sample



Camera, Computer etc

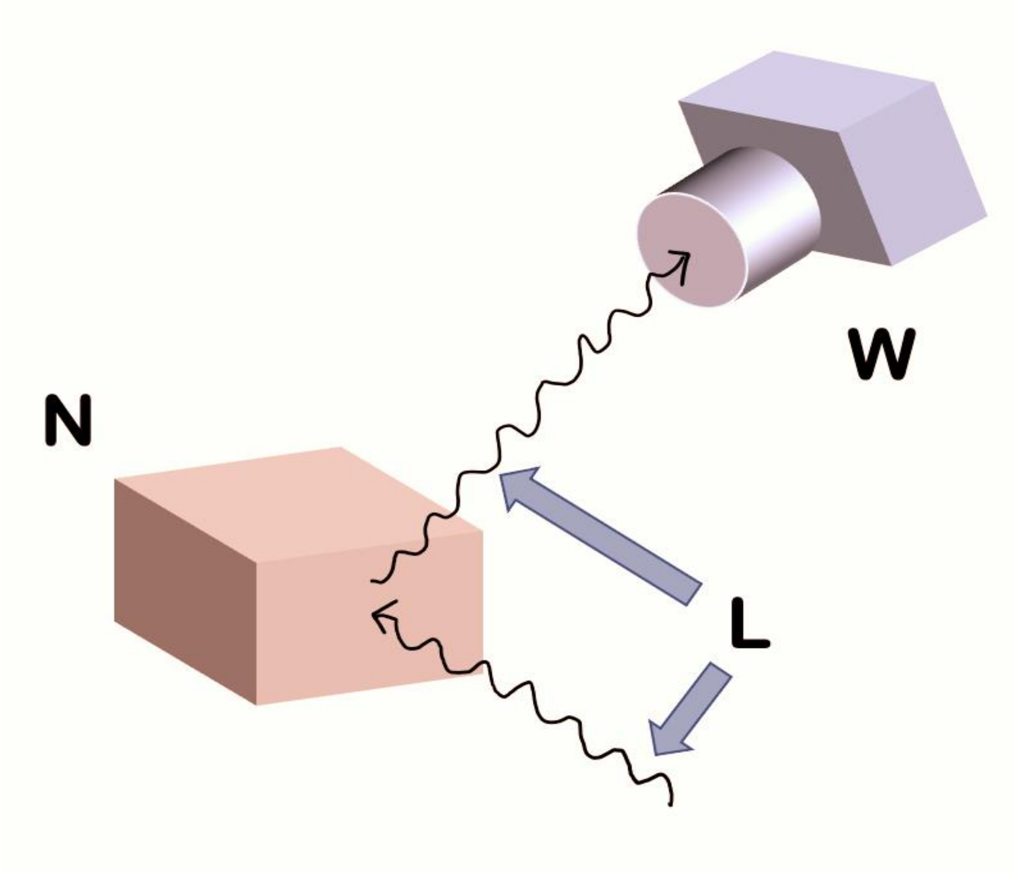
X-ray Photons

Important inrgedients

- Experiment computes a **function**: f: $\mathit{Physical\ system} \rightarrow \{0,1\}^n$
- We are not given full access to the **input** of this function, namely to the physical system; some DOF are **hidden**
- Quantum **interaction**s with other degrees of freedom.

Quantum Algorithmic Measurements (QUALMs)

The Hilbert spaces



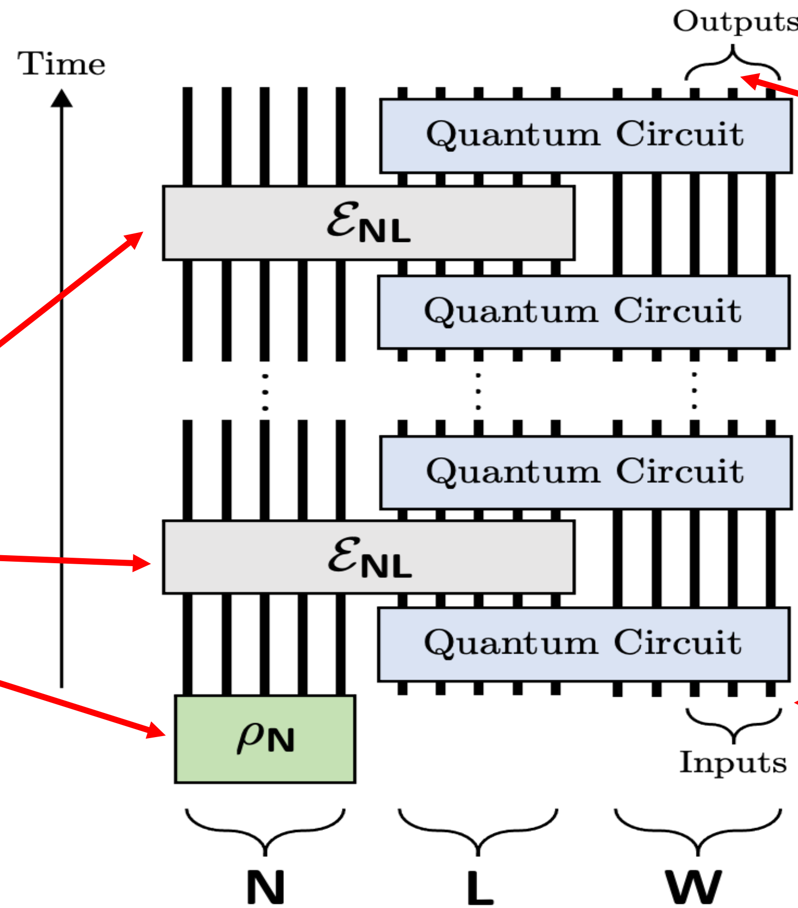
Crystal sample

Camera, Computer etc

X-ray Photons

Nature, **L**aboratory, **W**ork space

Quantum Algorithmic Measurements (QUALMs)



The output
Of the experiment
(or the QUALM)

The quantum
"input" system

Additional
Classical input
paramaters

A **hybrid** of
quantum interactive protocols & black box algorithms

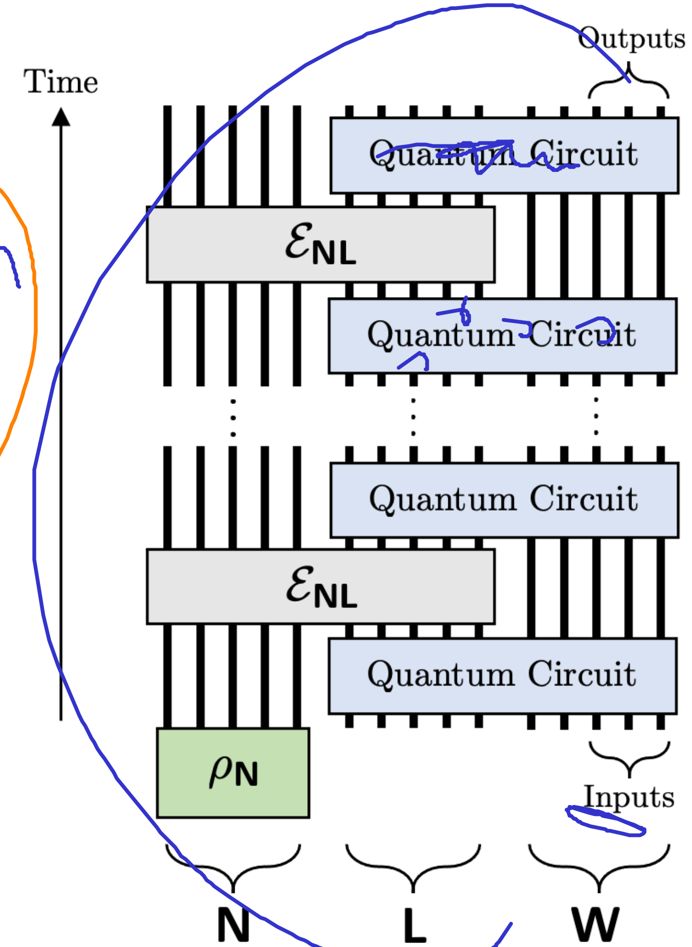
Quantum Algorithmic Measurements (QUALMs)

Definition

Lab oracle

Task

QUALM: A quantum circuit using gates from $\mathcal{G}$ interlaced with oracle Calls

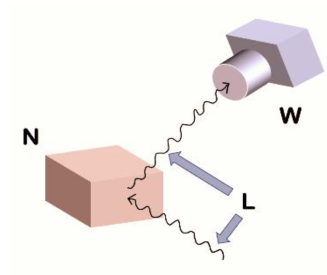


A hybrid of quantum interactive protocols & black box algorithms

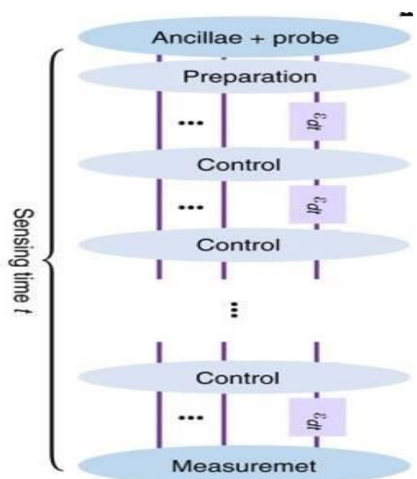
Back to examples

1) X ray diffraction:

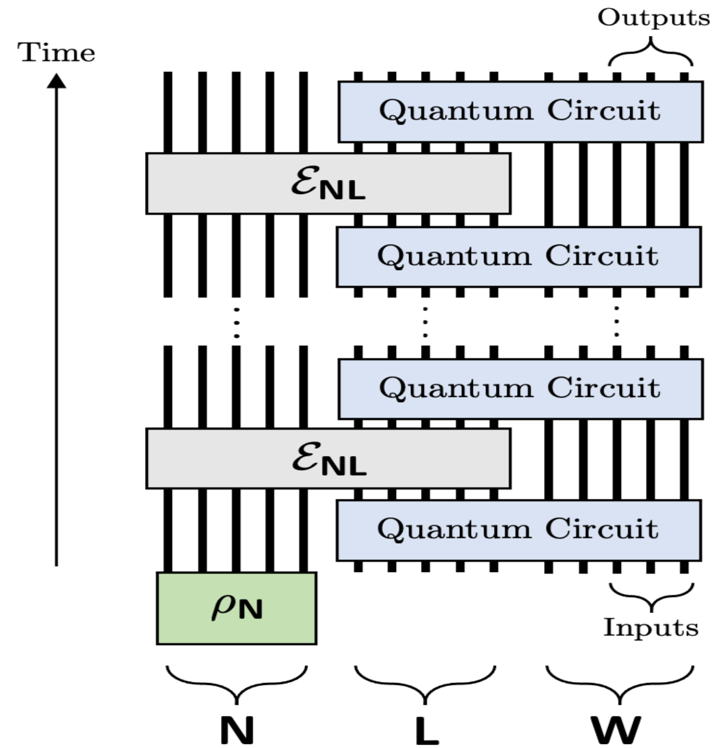
Nature: crystal
Lab:photons,
Work space: Camera



2) Measurement
using a QECC



5) Verification…



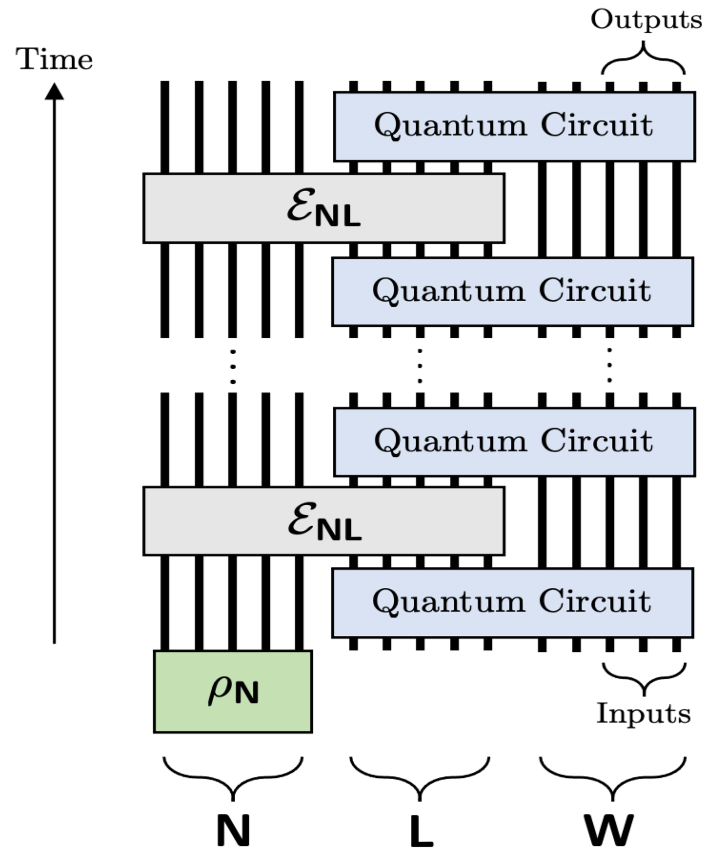
Lab oracle $(\rho_N, \mathcal{E}_{NL})$

Task $f : \{L\,0\} \times \{0,1\}^n \to \{0,1\}^m$

QUALM: A quantum circuit using gates from $\mathcal{G}$ interlaced with oracle Calls

Computational complexity of QUALMs

QUALM complexity:
Number of gates + oracle calls



Question of complexity of measurements becomes independent of exact physical Implementation!

We believe this is a universal model for quantum experiments

Study QUALMs from a computational complexity viewpoint

How much do the
New computational ingredients in
Quantum measurements help us?

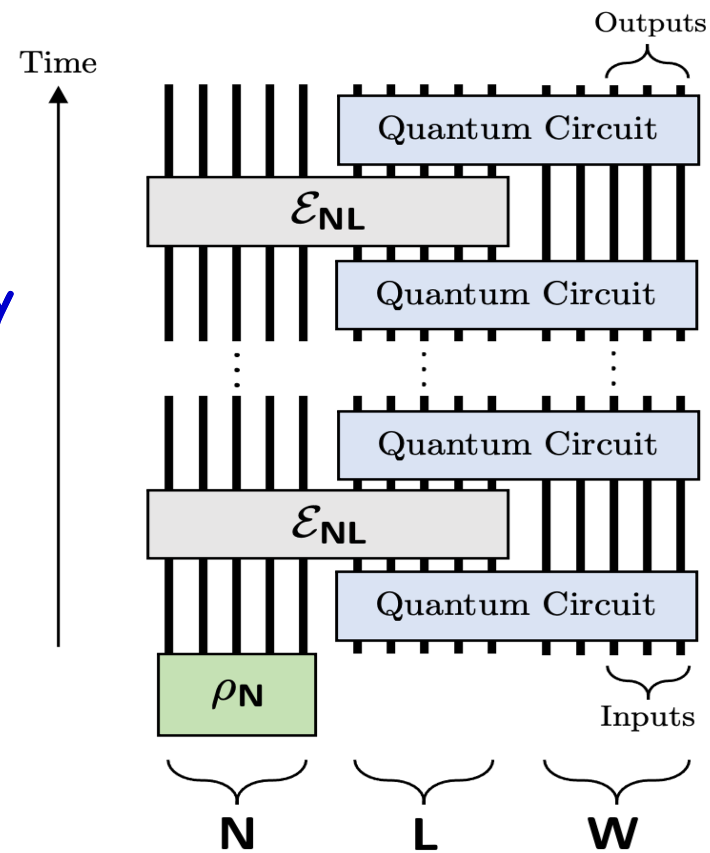
Possible interesting ingredients:

1. Entanglement & coherence

2. Sequentially versus parallelism, adaptivity

3. #of queries

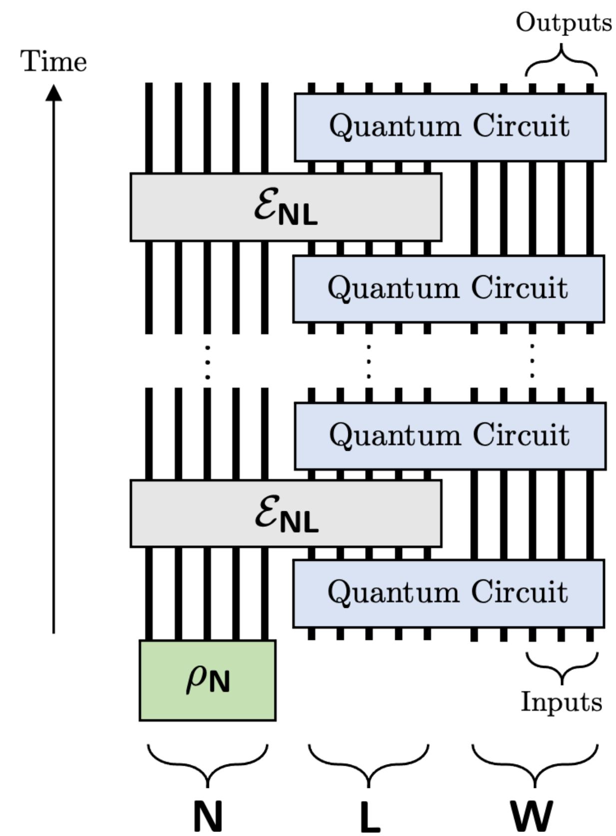
4. How Complex are quantum states and operations
   ….



How important is coherent access?

Heirarchy of Access-coherence

Coherent
Unentangled (classical L <-> W communication
Incoherent (no entanglement,no coherence in t)
Local-Local (product states prep. & meas.)
Computational basis (prep. & meas.)



Is there an advantage for coherent access?

Do we gain anything from a
Quantum computer interacting with the physical system
Being measured?

Connections to similar model and questions in quantum machine Learning [HuangKuengPreskill'2021]

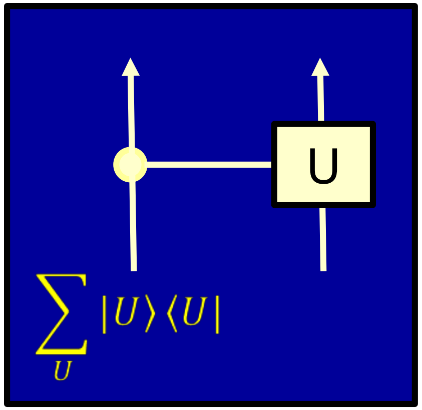
Two TASKs

Fixed unitary problem: Two lab oracles:

LO_1: Pick $U \in_{Haar} \mathbb{U}_n$, remember it and apply it each time the oracle is called

LO_2: Pick a new $U \in_{Haar} \mathbb{U}_n$ Every time

QUALM which distinguishes between them!



Physically motivated by distinguishing time dependent vs. Time Indepenent Hamiltonians

Symmetry distinction problem:
Three lab oracles:

LO_1: Pick a fixed $U \in_{Haar} \mathbb{U}_n$

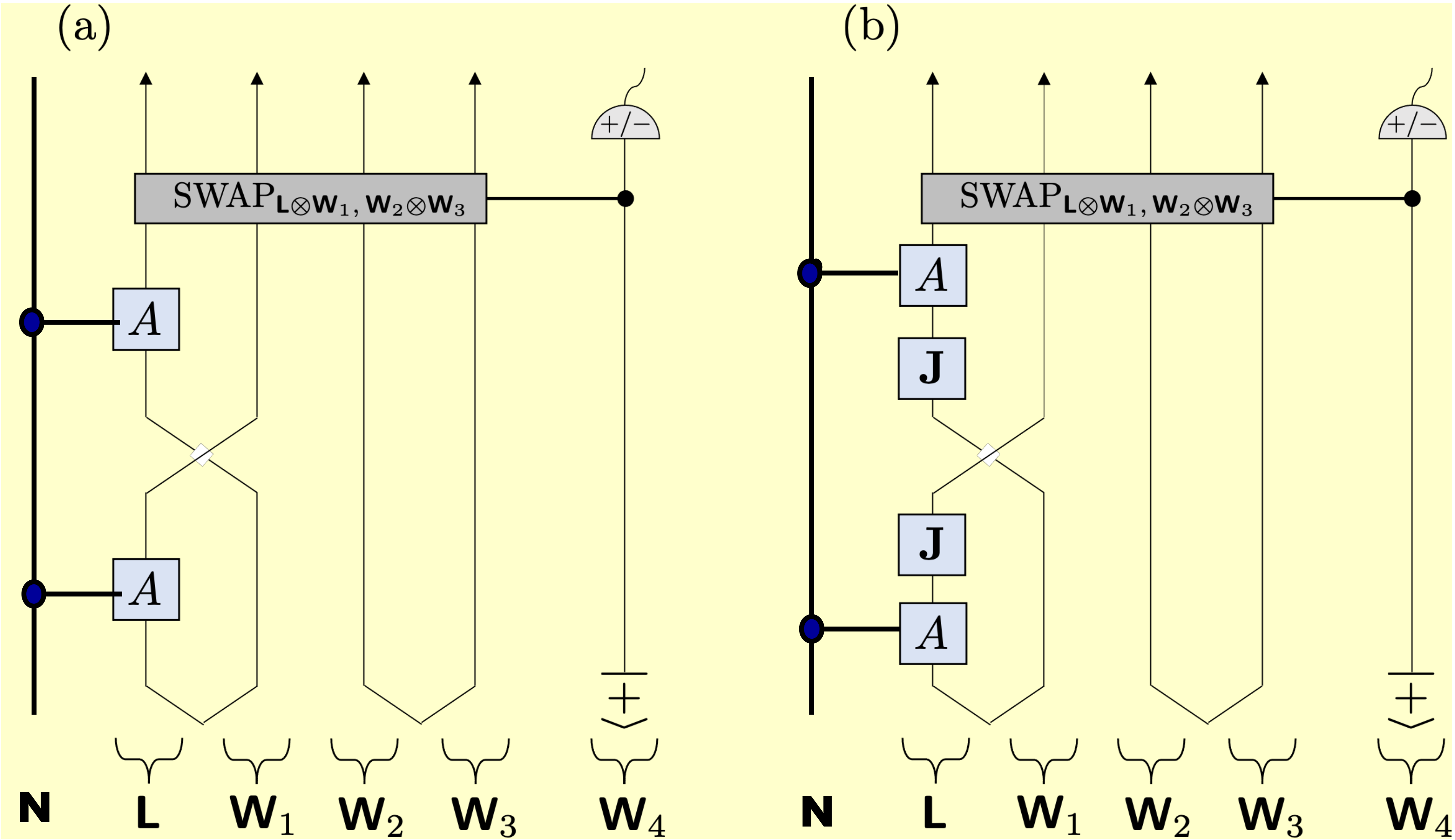
LO_2: Pick a fixed $U \in_{Haar} \mathbb{O}_n$

LO_3: Pick a fixed $U \in_{Haar} \mathbb{SP}_n$

Find a QUALM which distinguishes between the three (time) symmetry types!

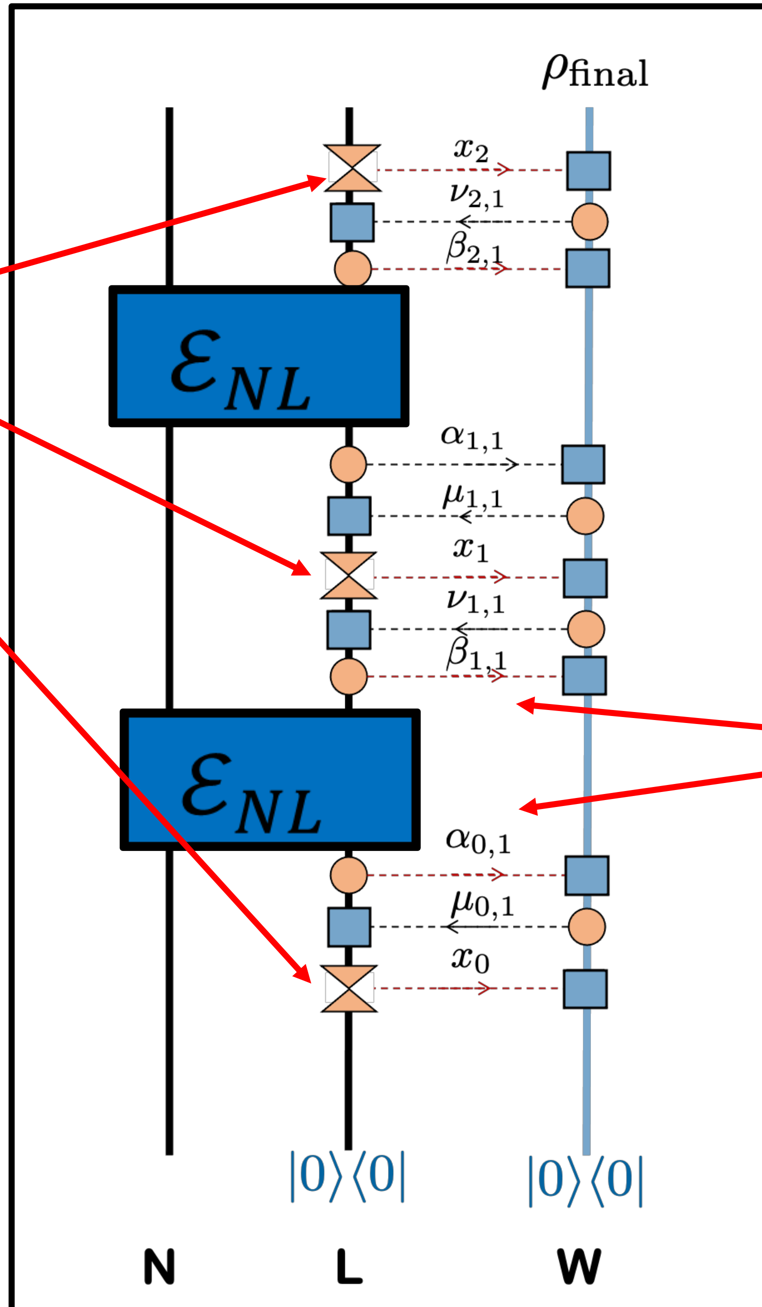
Physically motivated by distinguishing time reversal Symmetries of different types

QUALMs for the symmetry task:



(J is canonical symplectic form)

Incoherent QUALMs



Incoherent:
Both in space
And in time.

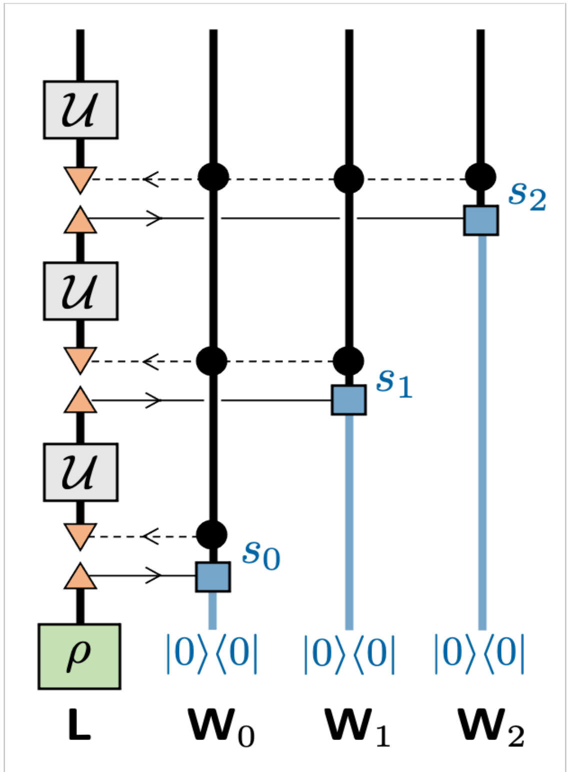
Complete measurements

LOCC Interactions between **L** and **W**

Adaptive!

Incoherent QUALMs are exponential

<u>Lowerbound:</u> Incoherent QUALMs need Exponentially many quries – hard due to adaptivity (proof based on Weingarten functions)



$$Q_k(s) = \int_{\text{Haar}} dU \, \mathrm{tr}\left(U^{\otimes k} A_s U^{\dagger \otimes k} B_s\right)$$

where

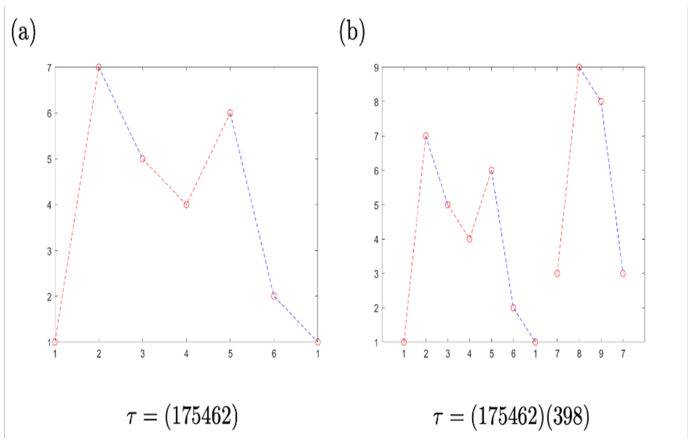
$$A_s = \bigotimes_{i=1}^{k} \sigma^{i-1}_{s_0 s_1 \ldots s_{i-1}}, \quad B_s = \bigotimes_{i=1}^{k} |y^i_{s_0 s_1 \ldots s_i}\rangle\langle y^i_{s_0 s_1 \ldots s_i}| \lambda^i_{s_0 s_1 \ldots s_i} .$$

$$\int_{U \in Haar} dU \, U_{i1,j1} U_{i2,j2} \ldots U_{ik,jk} U^*_{i1',j1'} U^*_{i2'j2'} \ldots U^*_{ik'jk'}$$

$$\int_{\text{Haar}} dU \left[U^{\otimes k}\right]_{IJ} \left[U^{*\otimes k}\right]_{KL} = \sum_{\sigma,\tau \in S^k} \tau_{KI} \sigma_{LJ} W(\tau\sigma^{-1}, D) .$$

I, J, K, L label an orthogonal basis in the k-copied Hilbert space,

S^k is the permutation group on k elements .



efficient coherent QUALMs
(Proof is very simple - variants on swap tests)

Incoherent QUALMs need exponentially many queries – hard (based on Weingarten functions)

↓

Theorem[AharonovCotlerQi2020]: Provable **Exponential** advantage for coherent over incoherent (even adaptive) experiments

Doesn't Simon's algorithm already give such an advantage?
No, Simon's advantage is incoherent!

Interestingly, other examples (though not related to physical notions) are implied by new results on **depth** of oracle models
[Chia Chung Lai'2020], [CoudronMenda'2020]

(New type of) exponential advantage
for quantum computers, in quantum experiments

Discussion & Open questions

We are at the midst of a new era of quantum experiments and quantum measurements, in which new computational ingredients will enter Experiments more and more.

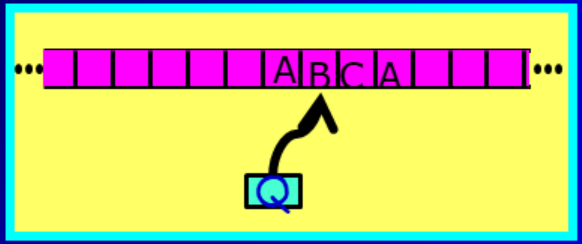
Despite the many examples, this is very far from understood theoretically and experimentally.

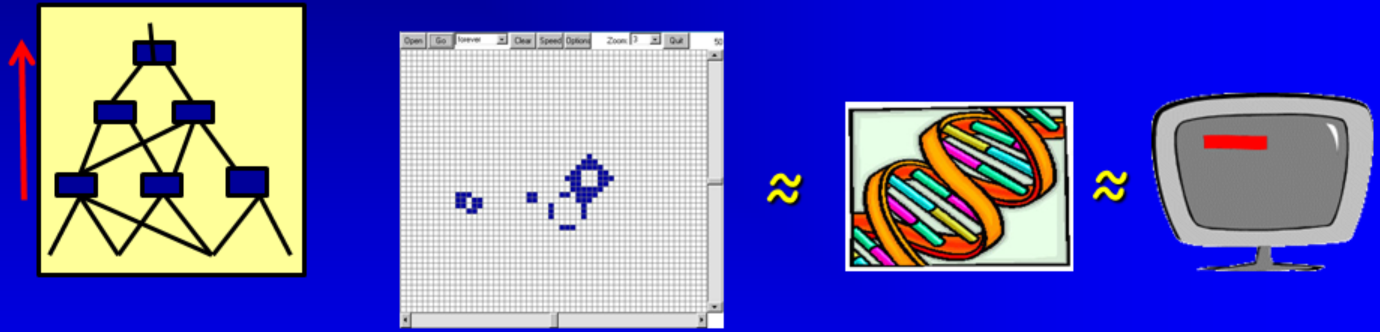
1. What new fancy quantum experiments can be done?
   E.g., can adpativeness be used in more sophisticated ways in experiments? Maybe complicated entangled initial states?

2. Exponential adv. for coherent QUALMs in the NISQ era?
   (local noise destroys our QUALM)
   And more generally, achieve advantage in more realistic settings…
   Also experimentally.

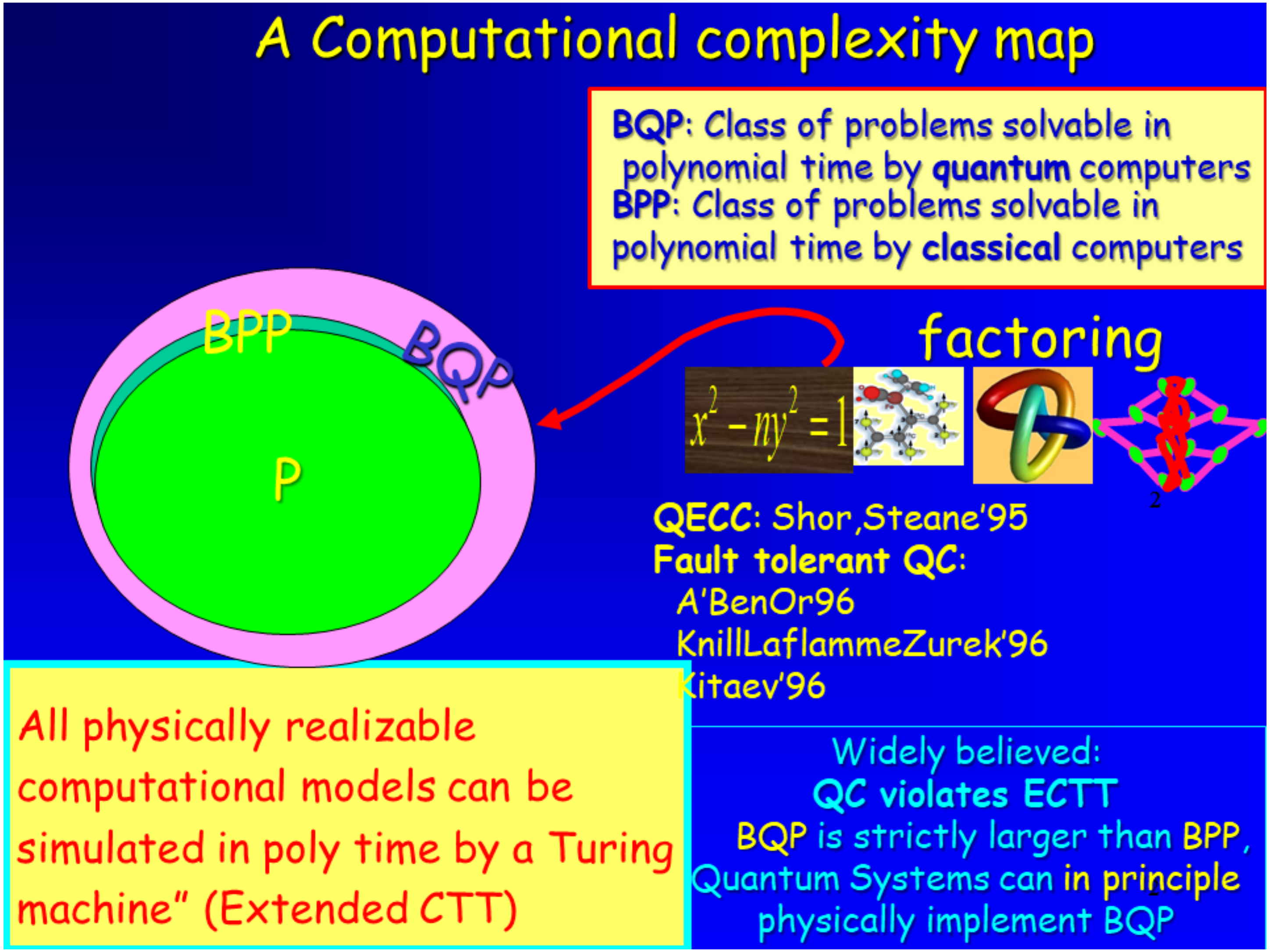

Thanks!

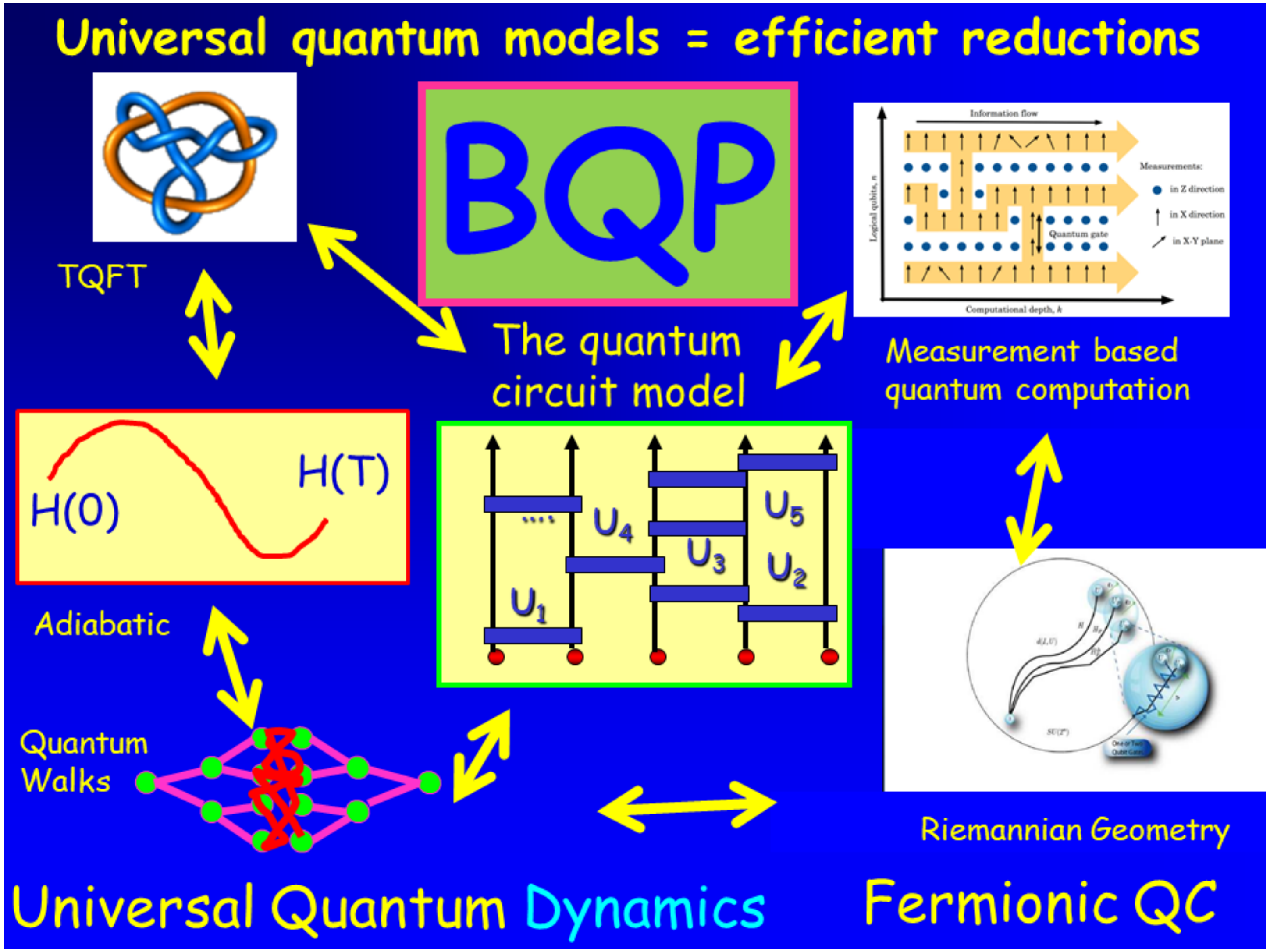
The Extended Church Turing Thesis





Extended Church Turing Thesis: "All physically reasonable (classical) computational models can be simulated with polynomial overhead by a Turing machine"


A Computational complexity map
BQP: Class of problems solvable in
polynomial time by quantum computers
BPP: Class of problems solvable in
polynomial time by classical computers
BPP
BQP
P
factoring
x^2 − ny^2 = 1
QECC: Shor,Steane'95
Fault tolerant QC:
A'BenOr96
KnillLaflammeZurek'96
Kitaev'96
All physically realizable
computational models can be
simulated in poly time by a Turing
machine" (Extended CTT)
Widely believed:
QC violates ECTT
BQP is strictly larger than BPP,
Quantum Systems can in principle
physically implement BQP



Universal quantum models = efficient reductions
BQP
TQFT
The quantum circuit model
Information flow
Measurements:
in Z direction
in X direction
in X-Y plane
Quantum gate
Logical qubits, n
Computational depth, k
Measurement based quantum computation
H(0)
H(T)
Adiabatic
U_1
U_2
U_3
U_4
U_5
Quantum Walks
One or Two Qubit Gates
Riemannian Geometry
Universal Quantum Dynamics
Fermionic QC