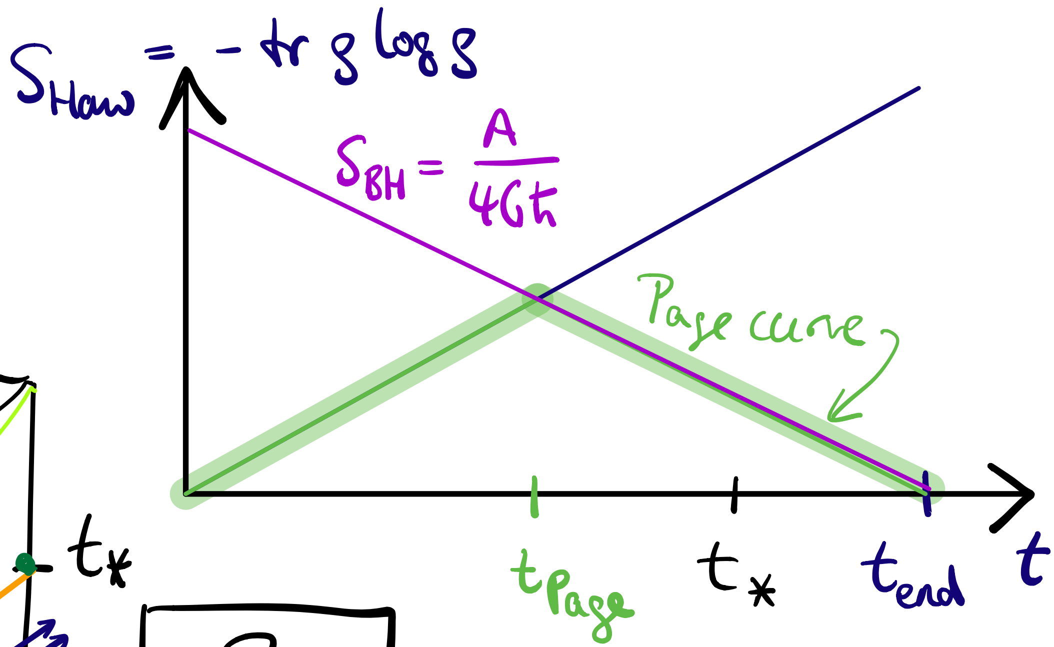
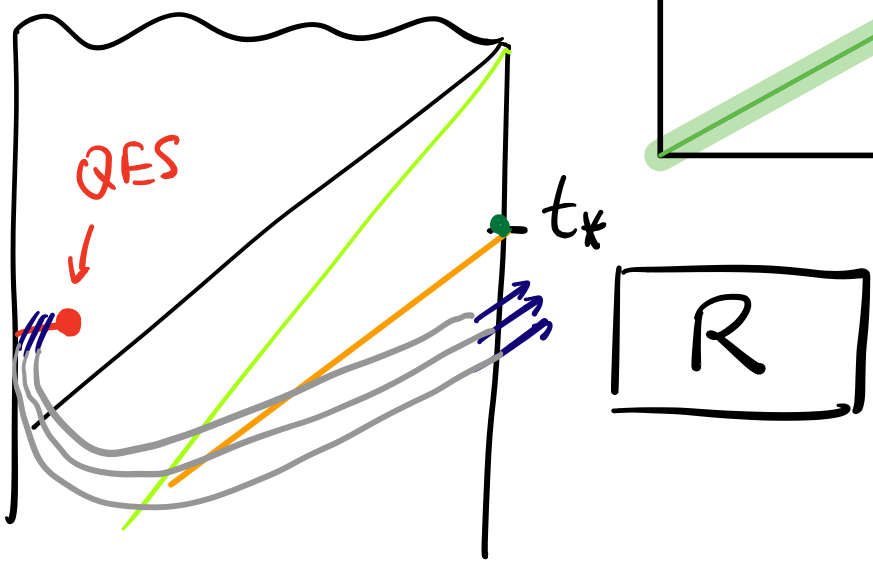
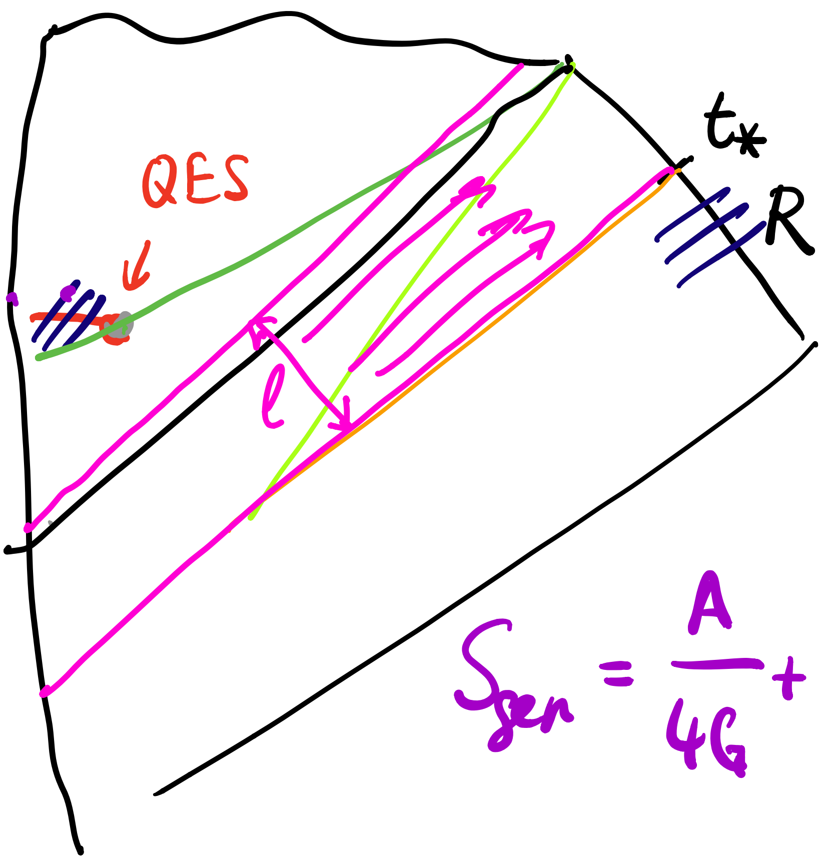
Island Finder

with Arvin Shahbazi-Moghaddam



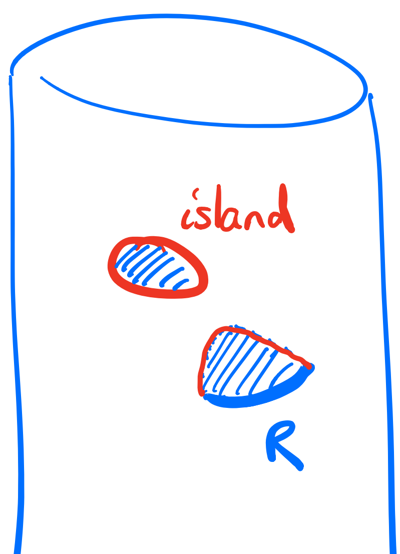
$$S_{gen} = \frac{A}{4G\hbar} + S_{vN}$$

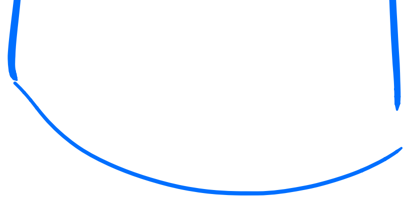




$$S_{gen} = \frac{A}{4G} + S$$

$\log \ell$





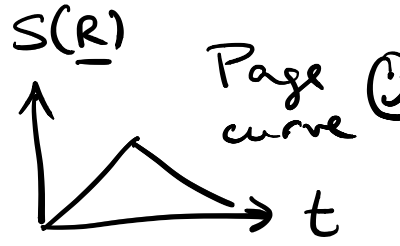
RT/HRT/FLM/EW:

$$S(\underline{R}) = S_{gen}(EW(R)) = S_{gen}(R \cup I)$$

$R \cup I$ is q-extremal and smallest S_{gen} among all such regions

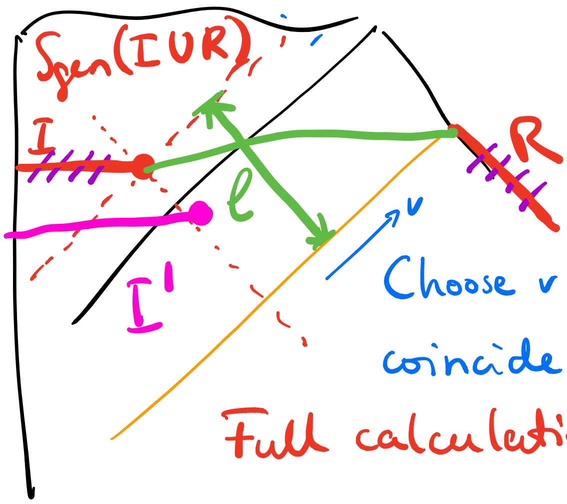
$t_* < t_{Page}$: $I = \emptyset$ has smallest S_{gen}

$t_* > t_{Page}$: I = red $\neq \emptyset$ wins. →



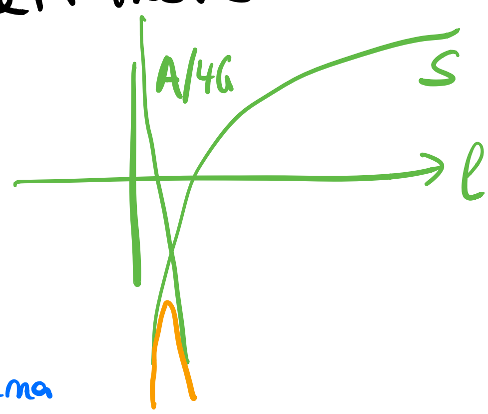
To show this rigorously is a bit of work: P, AEMM

Even at the handwaving level, it requires attention to detailed behavior of the entropy. E.g. suppose we restrict to 1+1 D, right and left movers.



$S \sim \log \ell/\varepsilon$

$A \sim r_1^2 - 2r_1 \cdot \ell$



Choose v so that the maxima coincide.

Full calculation, say in 3+1 D, is not viable for now.

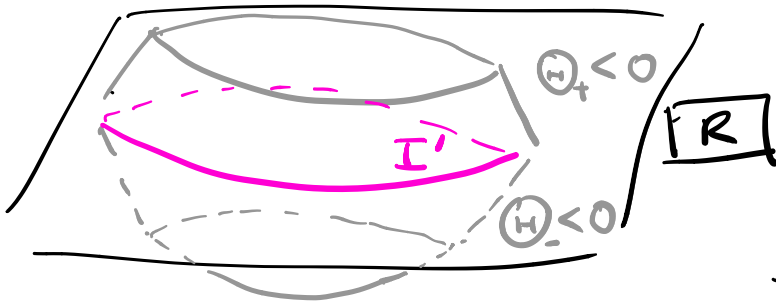
$\boxed{\text{Sufficient condition :}}$ • $I' \cup R$ is • q-(anti)-normal and • $S_{gen}(I' \cup R) < S(R)$

$\Downarrow$

There exists an island

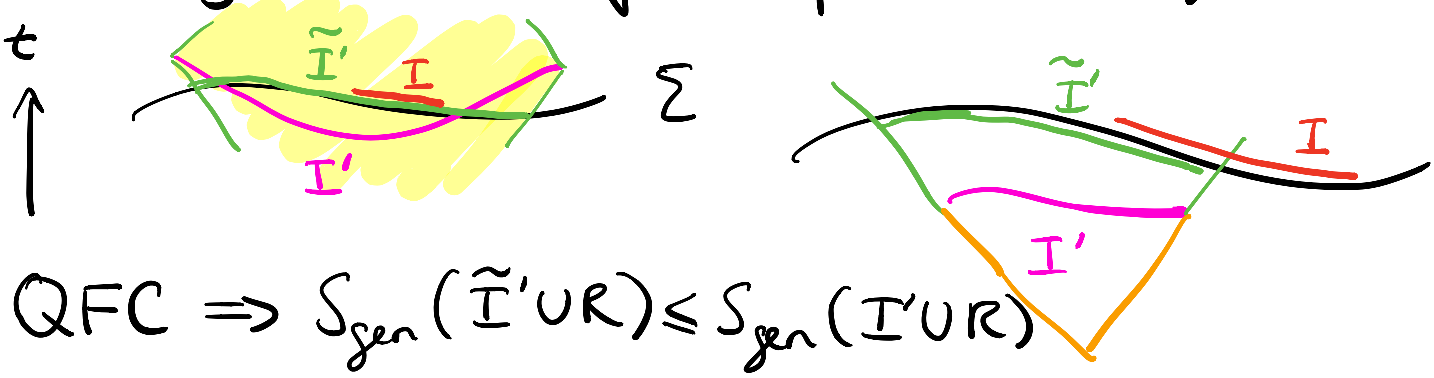
$I \neq \emptyset$; $S(\underset{=}{R}) = \underset{Q}{S}(I \cup R) < S(R)$

$\leq S_{gen}(I' \cup R)$



(Physics-) Proof : use maximin [Wall]: the qHRT region can be found by finding the region of minimal S_{gen} on every Cauchy surface, then maximizing over all Cauchy surfaces. Let Σ be the maximin Cauchy surface and let

$$\tilde{I}' \equiv \begin{cases} D(I') \cap \Sigma & (\text{if } I' \text{ is q-normal}), \text{ or} \\ J(I') \cap \Sigma & (\text{if } I' \text{ is q-anti-normal}). \end{cases}$$

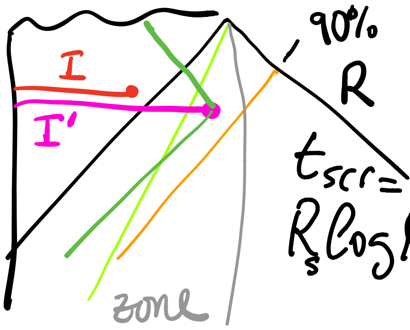


QFC $\Rightarrow$ $S_{gen}(\tilde{I}' \cup R) \leq S_{gen}(I' \cup R)$

Maximin $\Rightarrow$ $\underline{S_{gen}(I \cup R)} \leq S_{gen}(\tilde{I}' \cup R) \leq \underline{S_{gen}(I' \cup R)} < S(R)$

$I \neq \emptyset$

• Examples:

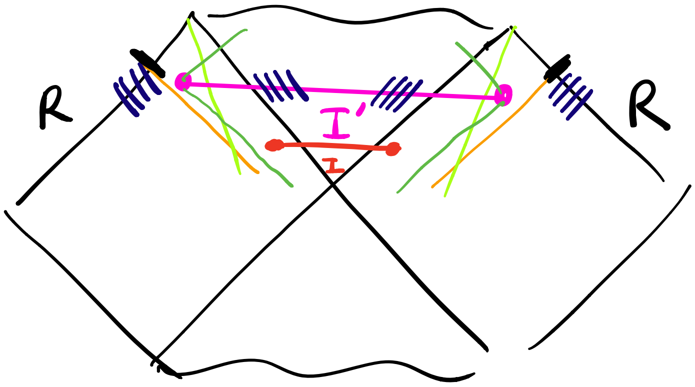


$t_{scr} = R_s \log R_s / \ell_p$

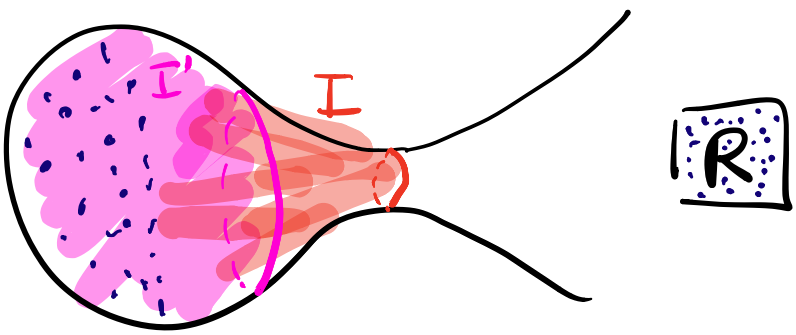
I' is classically and quantum normal, $S_{gen}(I' \cup R) \underset{\uparrow}{\approx} A_H/4G$

up to $O(1)$ factor

$< S(R)$ ✓



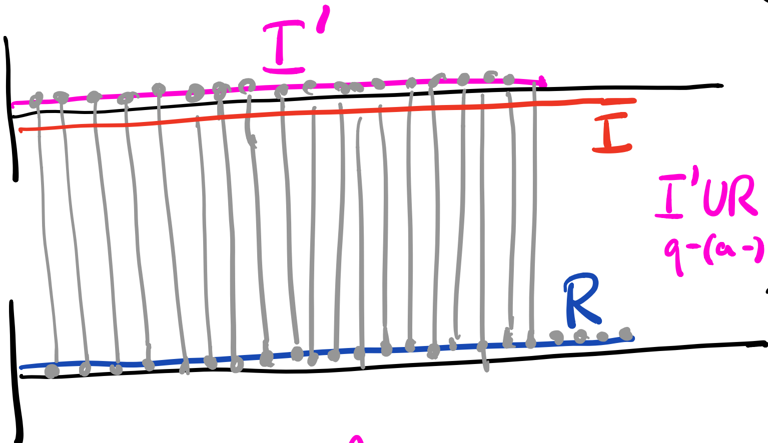
Eternal black hole [AMM]



Antinormal example:

Bag of gold

(time-symmetric slice)



Cosmology, time-symm. slice, $S = \sigma \cdot \frac{4\pi}{3} r^3$

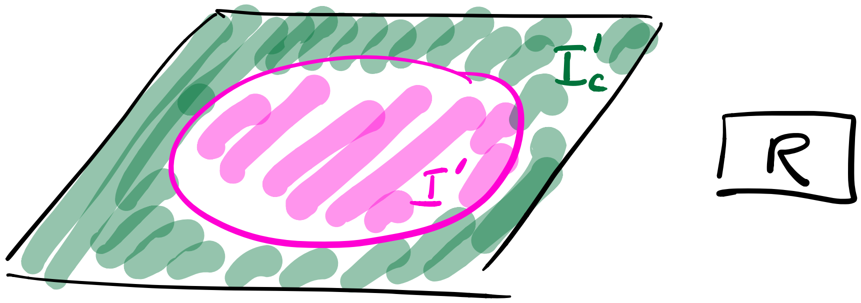
$I' \cup R$ is q-(a-) normal

$A = 4\pi r^2$,

TFD-like purification

r large $\Rightarrow S > \frac{A}{4G} \Rightarrow S_{gen}(I' \cup R) < S(R) \Rightarrow I \neq \emptyset$

Entropy bound



Let's drop the assumption that

$S_{gen}(I' \cup R) < S(R)$, but add the assumption that the global state (of $I' \cup I'_c \cup R$) is pure.

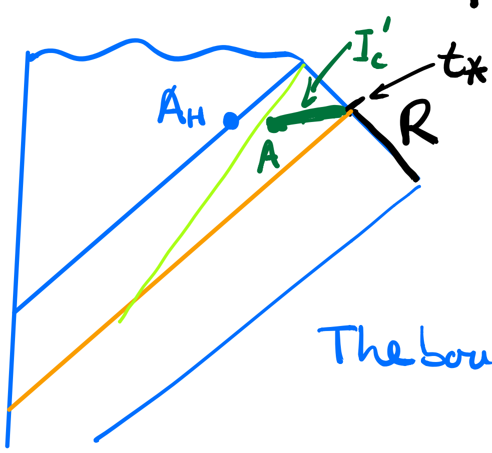
We still require that $I' \cup R$ (or equivalently, I'_c) is quantum normal or anti-normal. Then maximin implies $S(\underline{R}) = S_{gen}(I \cup R) \leq S_{gen}(I' \cup R) = S_{gen}(I'_c)$.

$$\boxed{S(\underline{R}) \leq S_{gen}(I'_c)}$$

For example, if I'_c is empty, we find

$$S(\underline{R}) \leq \frac{A}{4G}.$$

Example :

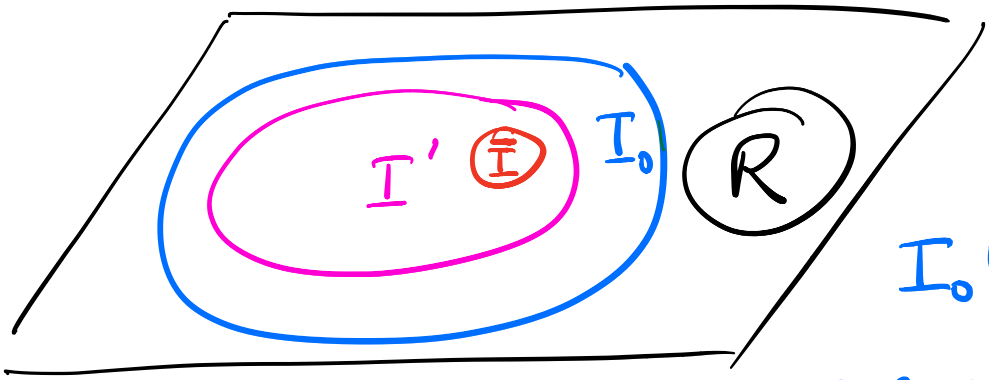


I_c is antinormal, q-antinormal

$A \simeq A_H(t_*)$

The bound is boring for $t_* < t_{Page}$
tight for $t_* > t_{Page}$

These results can be generalized to the case where R is inside the spacetime. The role of "infinity" is played by I_0, such that $I_0 \cup R$ is quantum normal (not antinormal).



With I' satisfying the same sufficient conditions as before (q-(anti)-normal) we can perform "wedge-restricted maximin" in $D(I_0)$. Quantum normalcy of I_0, plus the QFC, imply that this finds a (possibly non-minimal) quantum extremal region $\bar{I}$.

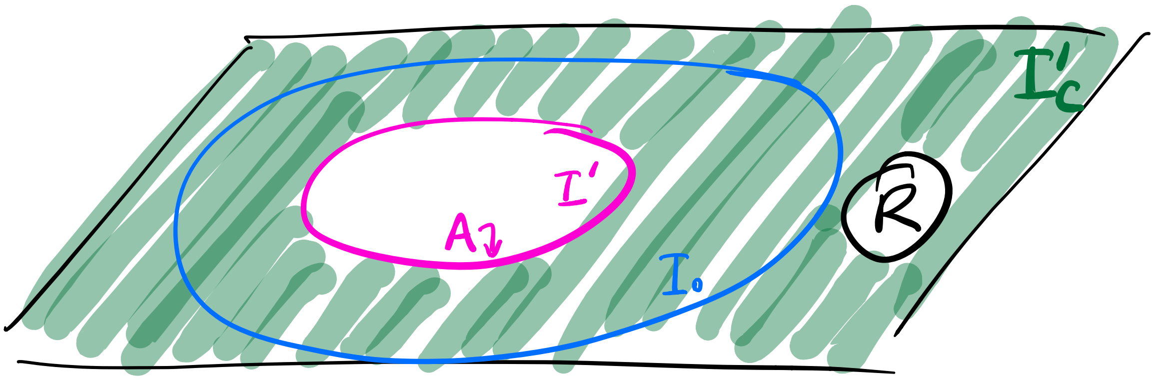
- Again, if $S_{gen}(I' \cup R) < S(R)$, then $\bar{I} \neq 0$ and $I \neq 0$.

- Again, if the global state is pure then

$$S_{gen}(\underline{R}) \leq S_{gen}(I'_c),$$

or, subtracting area(∂R) from both sides and assuming I'_c empty with inner boundary area A,

$$S(\underline{R}) \leq \frac{A}{4G}.$$



Area: $\theta_\pm = 0 \Leftarrow$ Focussing thm: $\theta' \leq 0$

S_{gen}: $\Theta_\pm = 0 \Leftarrow$ Quantum Focusing Conjecture

$$\Theta' \leq 0$$