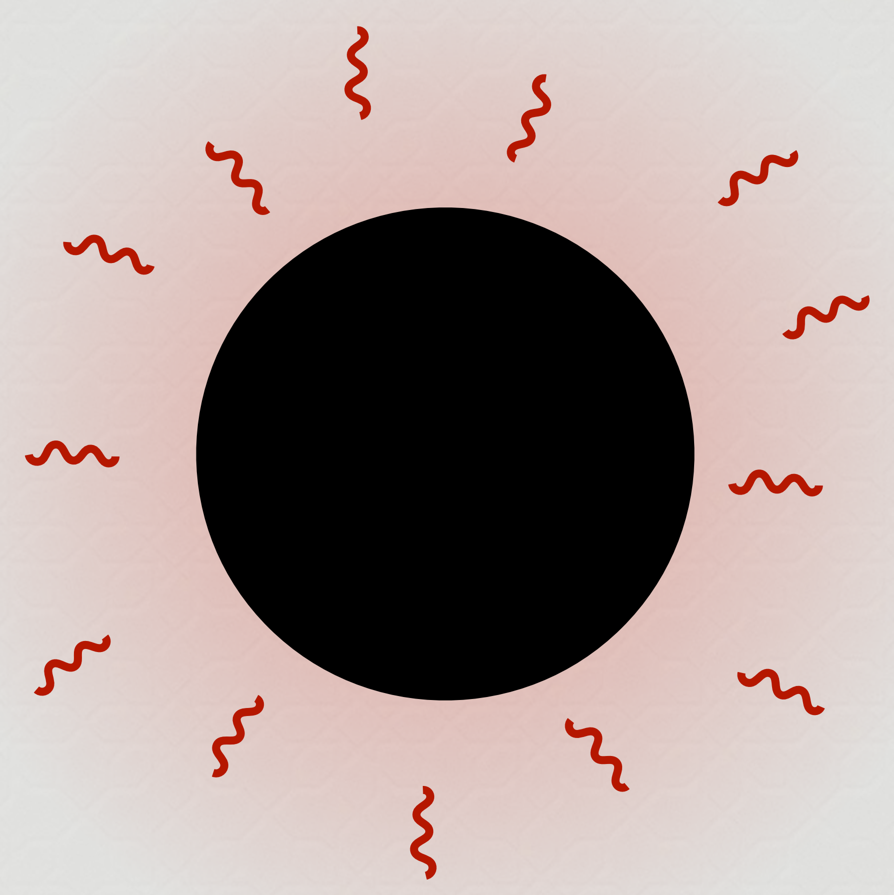
Replica Wormholes for an Evaporating 2D Black Hole

Kanato Goto (RIKEN)

Based on a joint work with Tom Hartman & Amir Tajdini

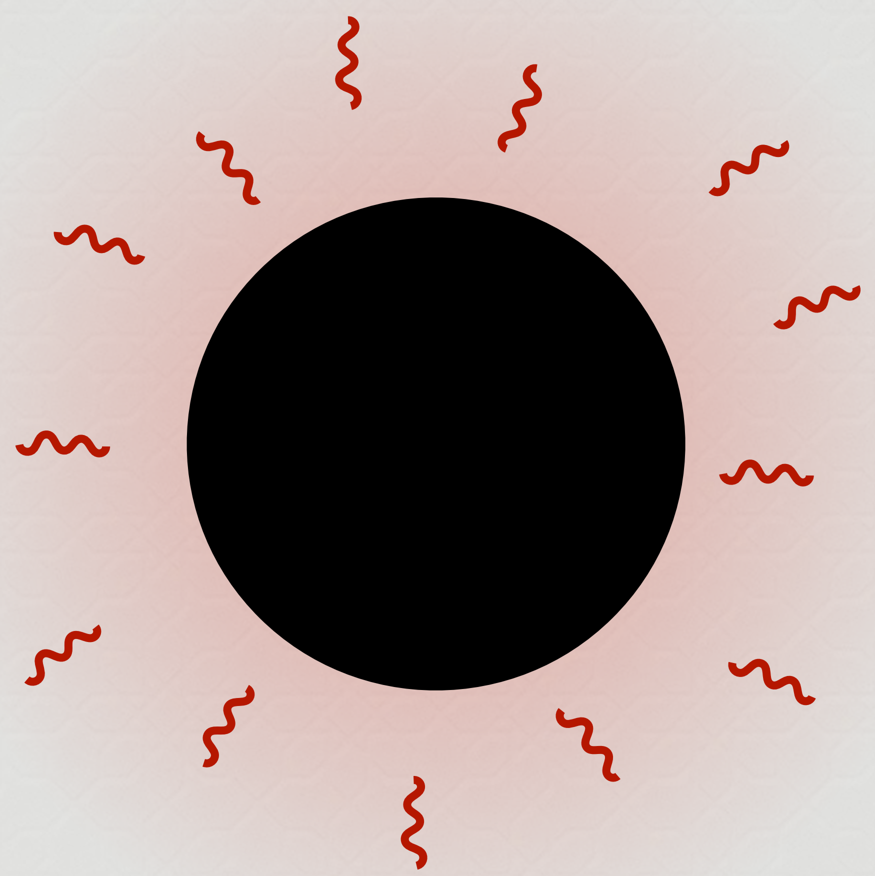
arXiv: 2011.09043

Black Hole Information Paradox



- *Temperature* T_{Hawking}
- *Entropy* $S_{\text{BH}} = \frac{\text{Area}}{4G}$

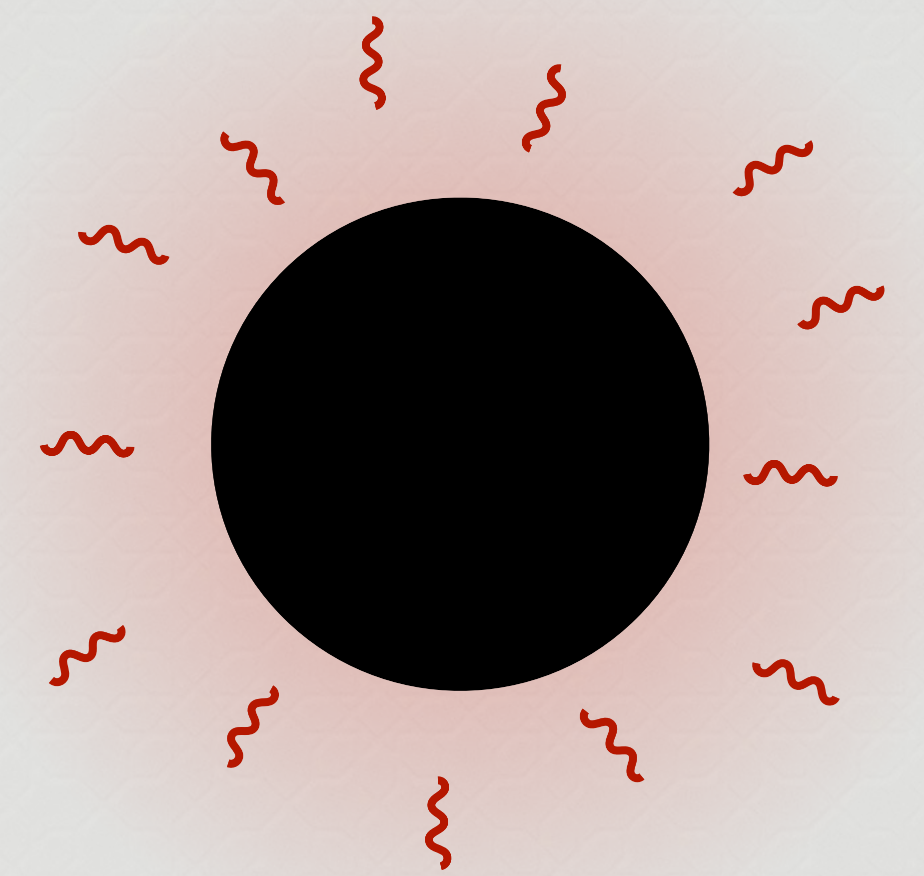
Black Hole Information Paradox

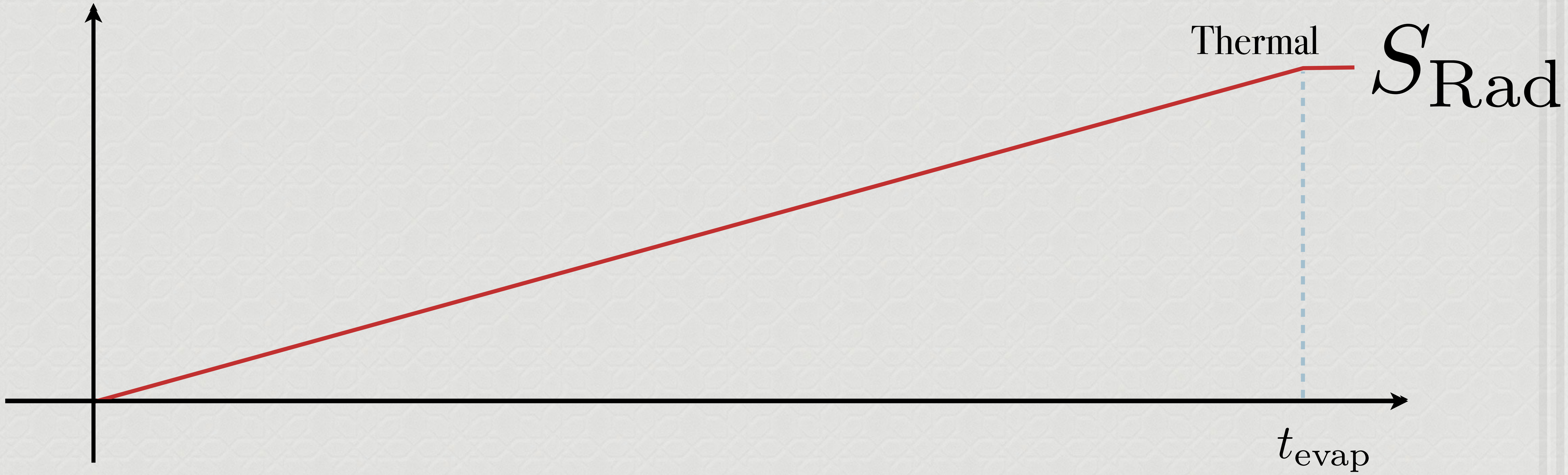


- *Temperature* T_{Hawking}
- *Entropy* $S_{\text{BH}} = \frac{\text{Area}}{4G}$

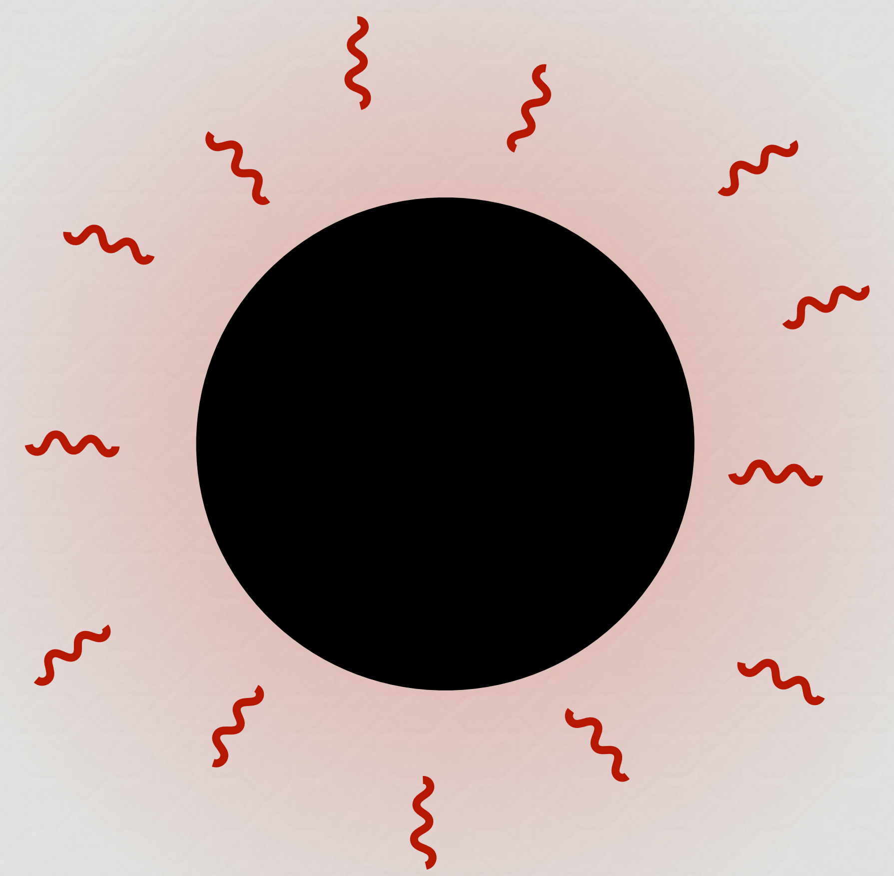
→ *Black Hole Thermally Radiates*

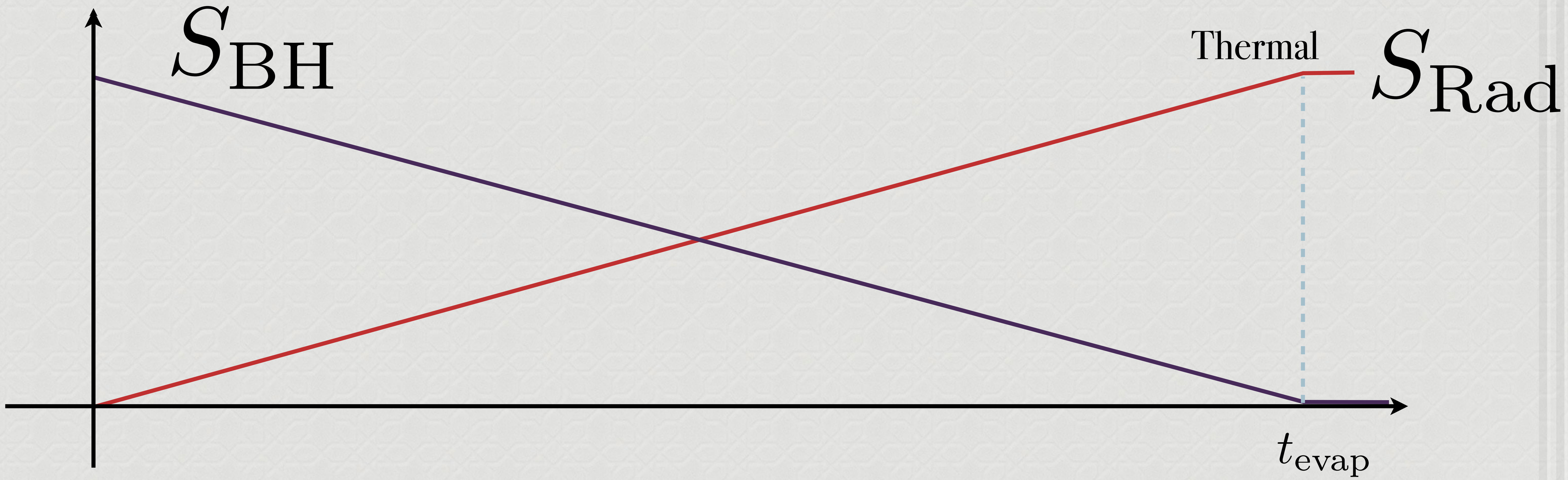
The entropy of Hawking radiation



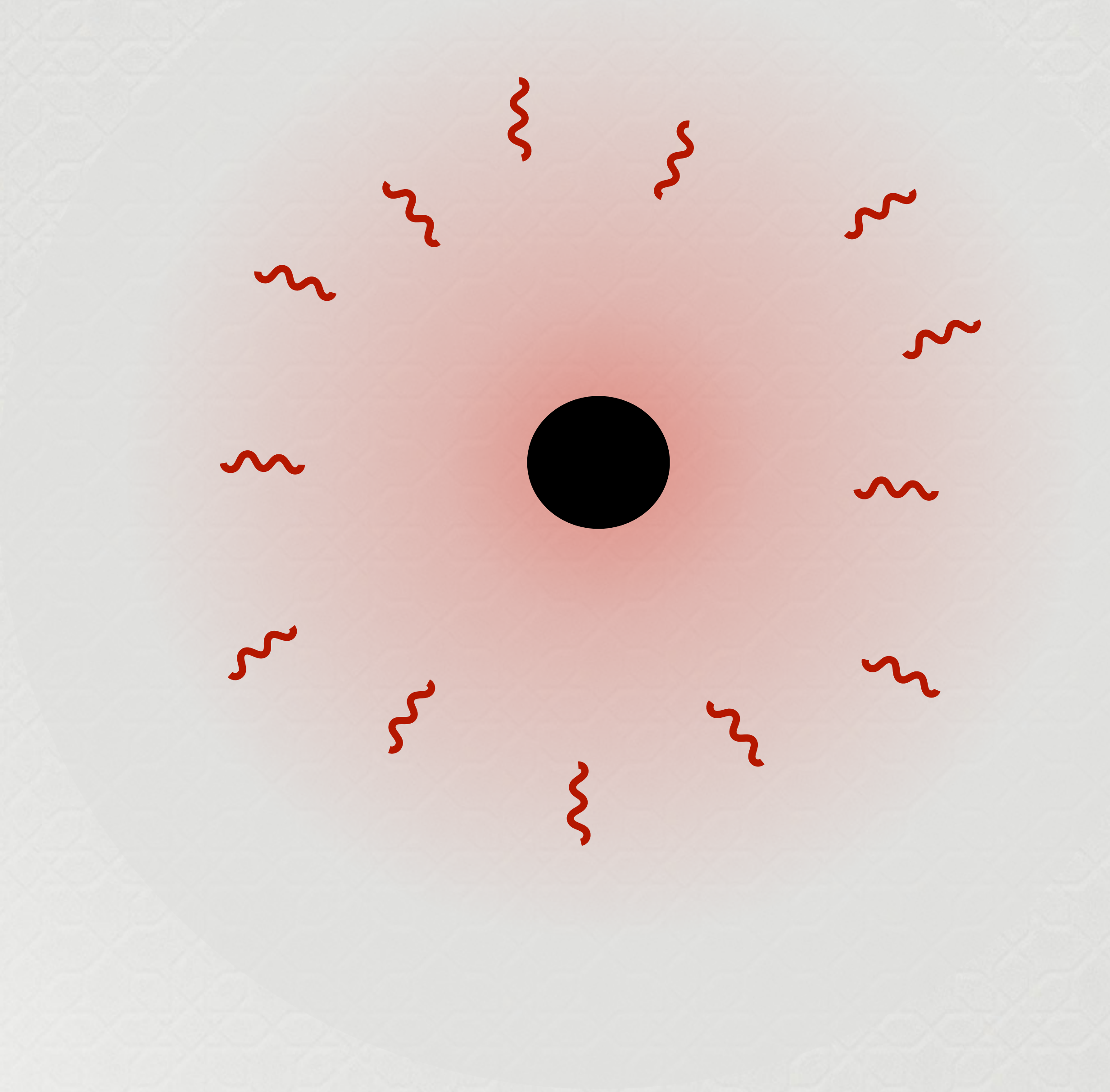


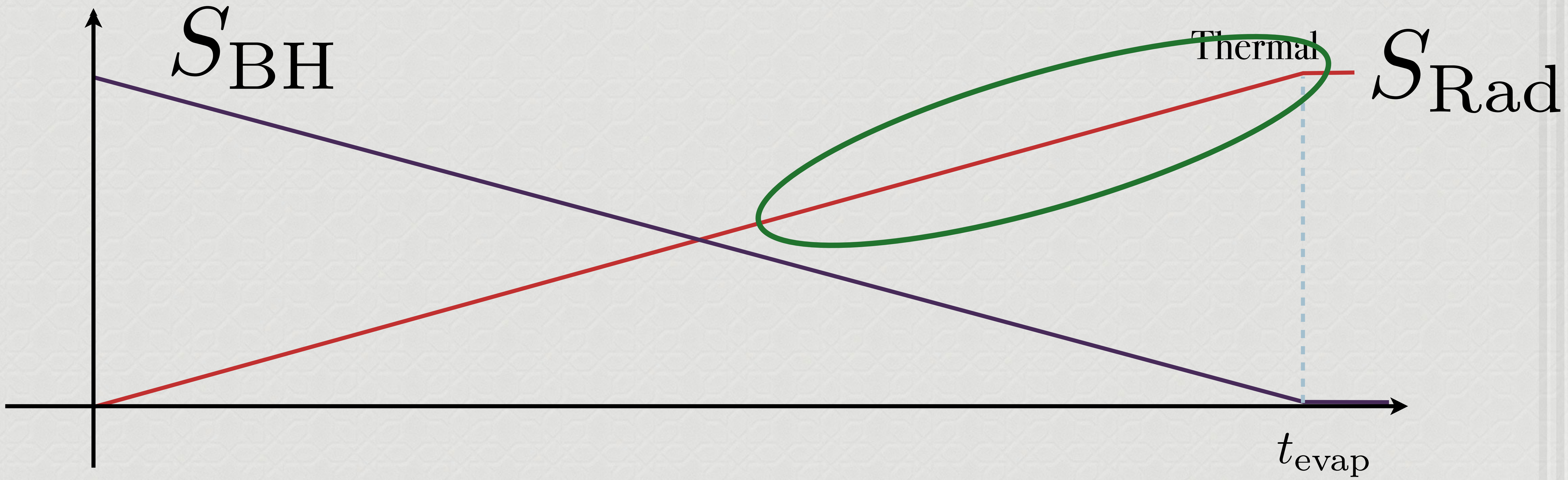
The entropy of Hawking radiation





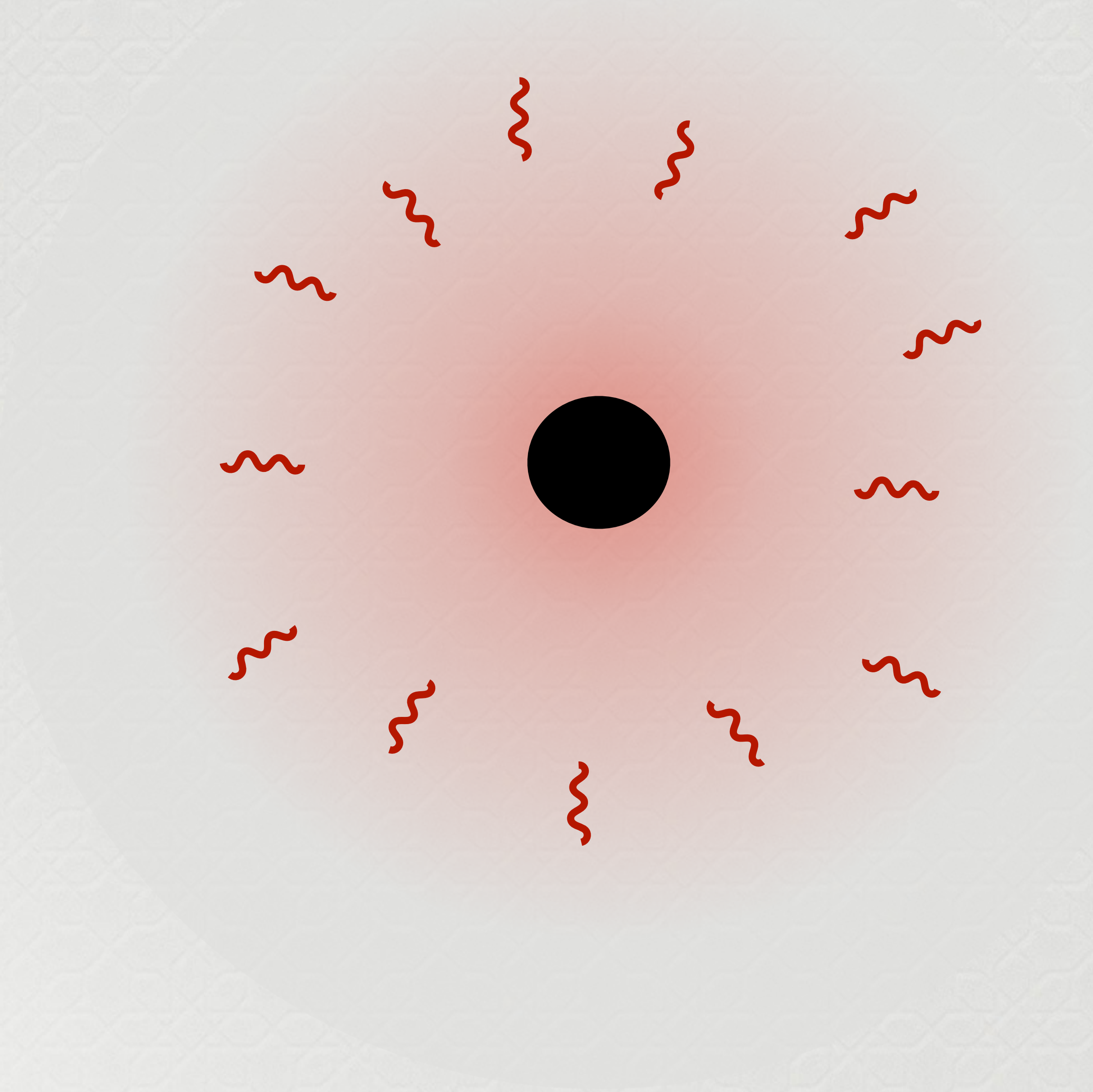
BH becomes too small to
be entangled with the radiation!

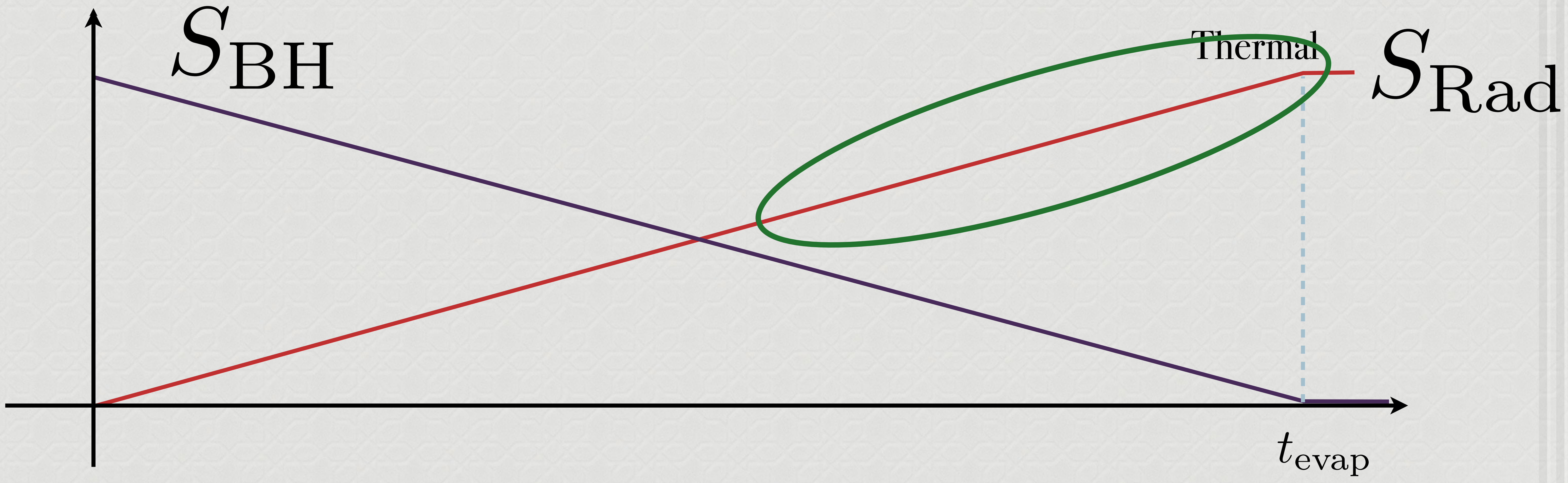




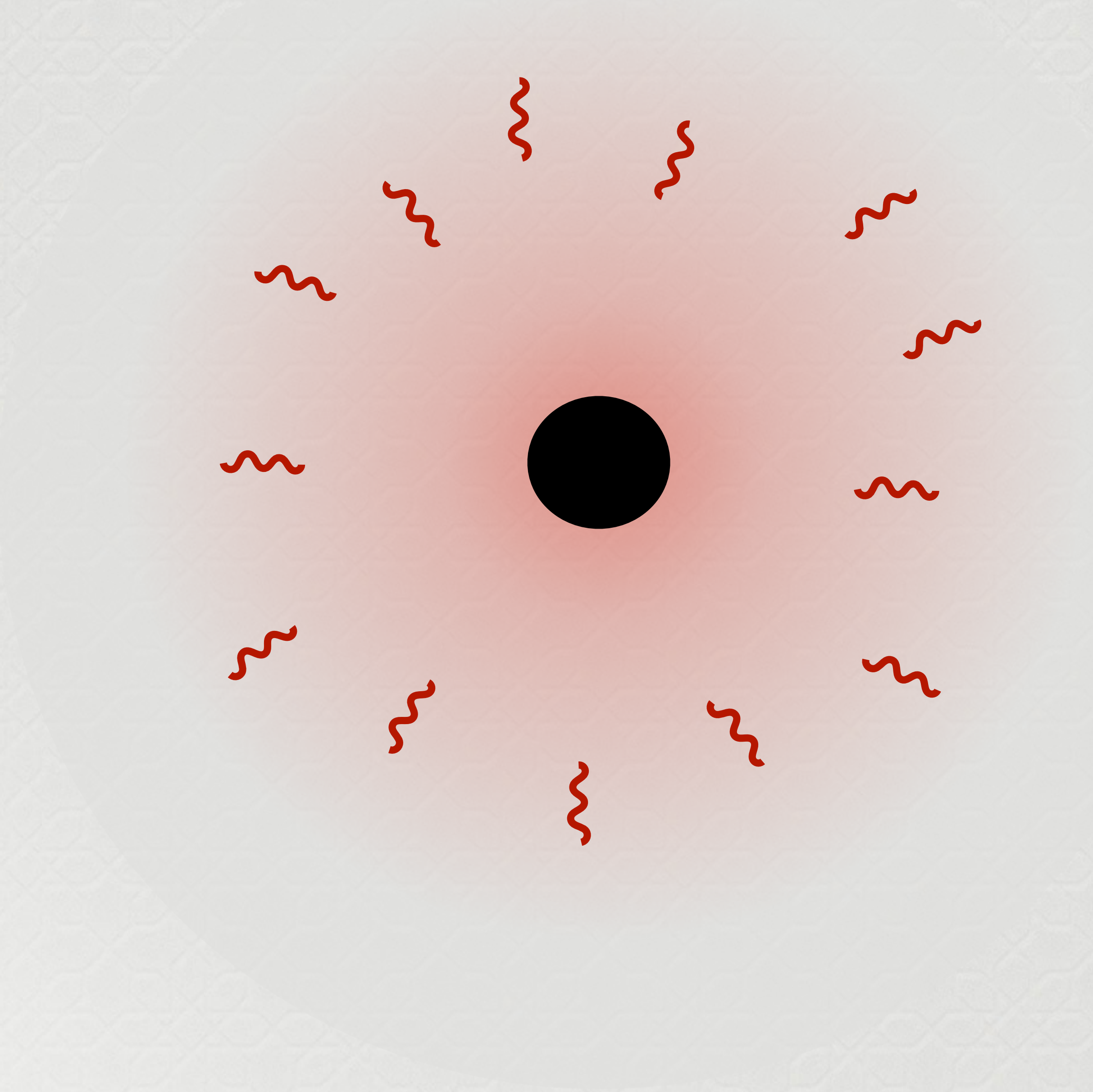
Pure states evolves
to the mixed state

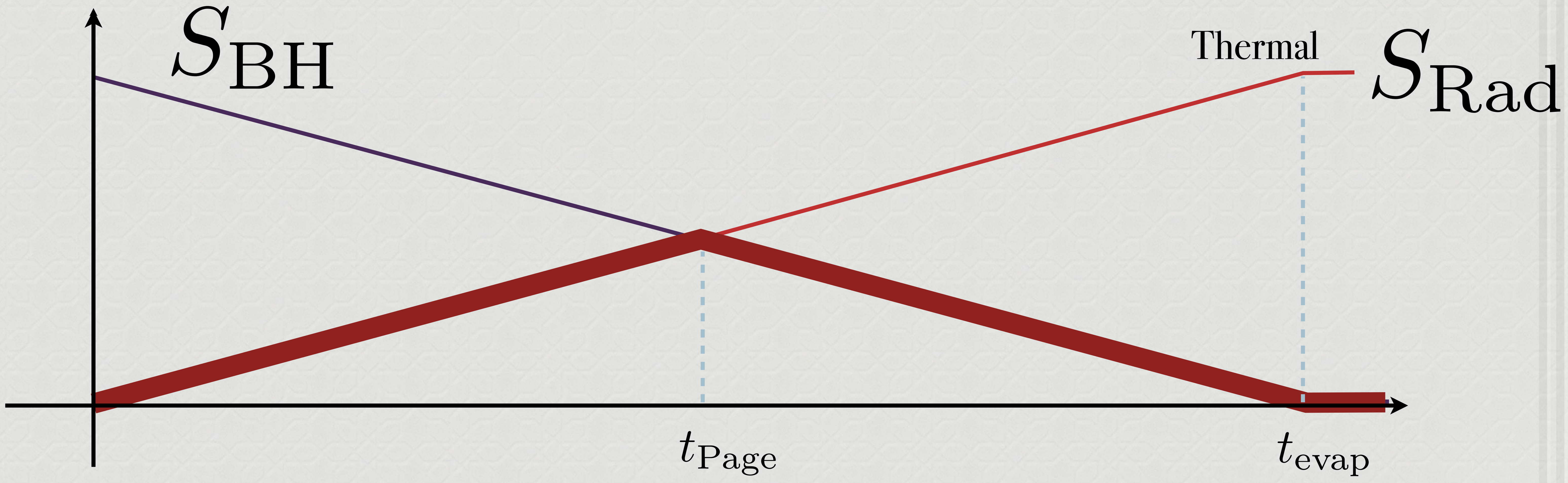
BH becomes too small to
be entangled with the radiation!





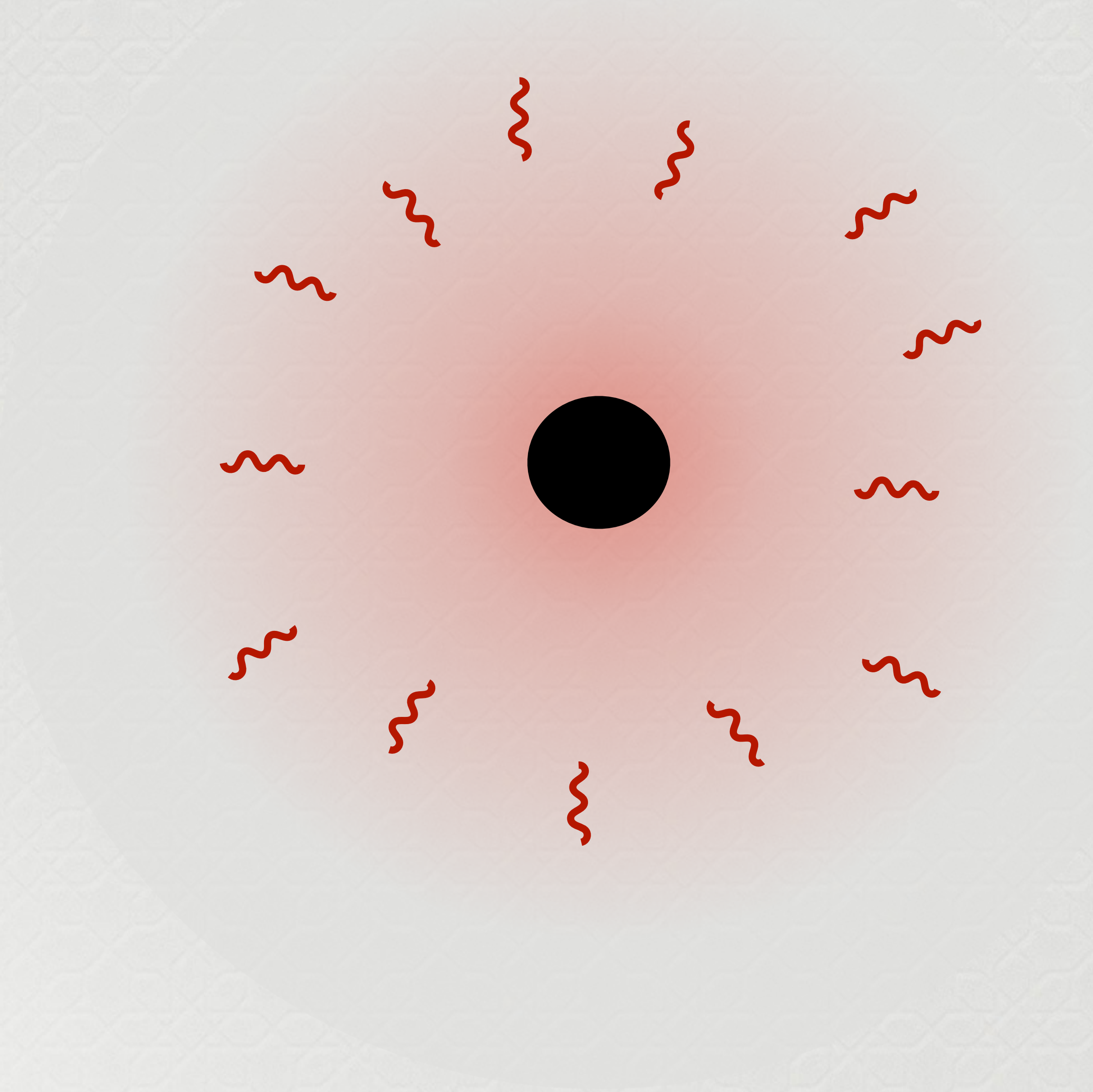
~~Pure states evolves to the mixed state~~

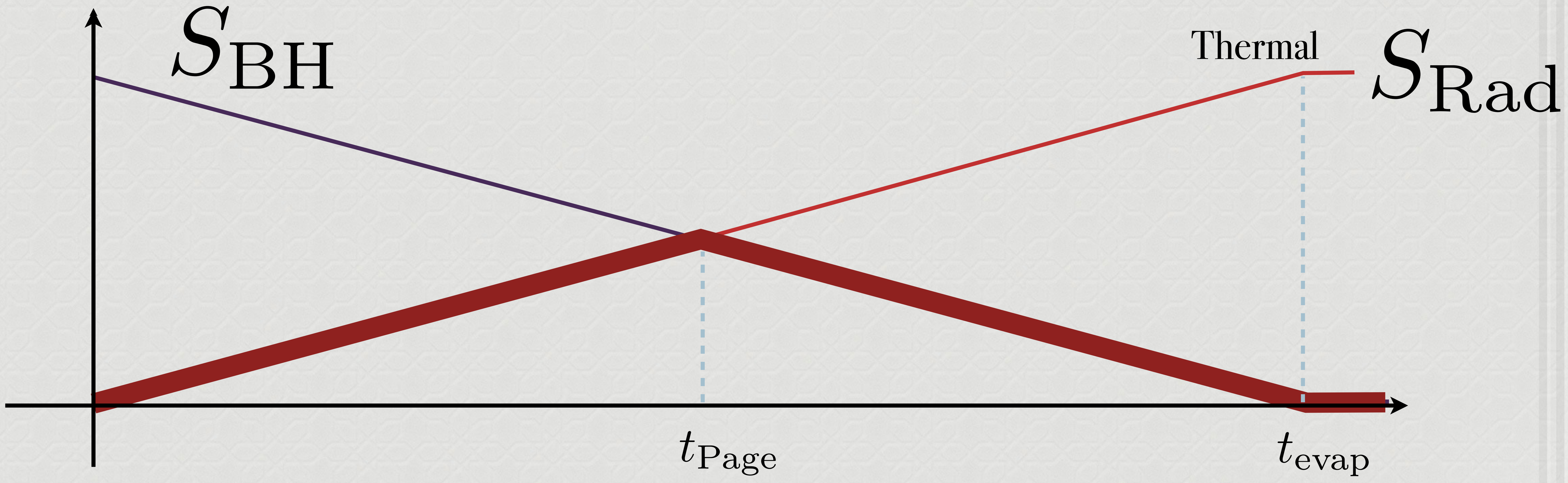




Unitary time-evolution

→ Page curve



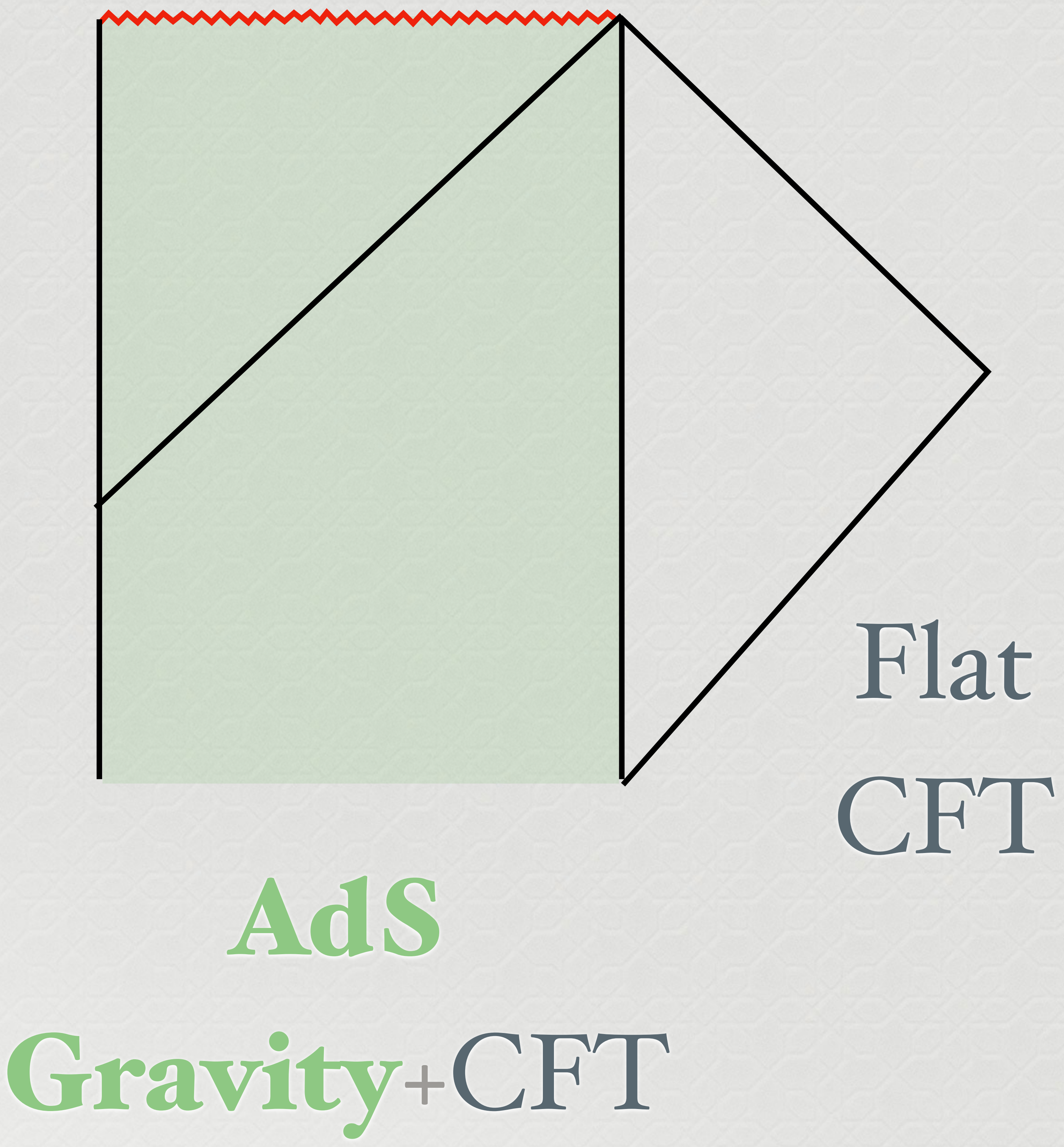


Unitary time-evolution
→ Page curve

How to get the unitary Page curve?

- Information paradox in an evaporating AdS BH coupled to Bath

Almheiri-Engelhardt-Marolf-Maxfield (AEMM) '19



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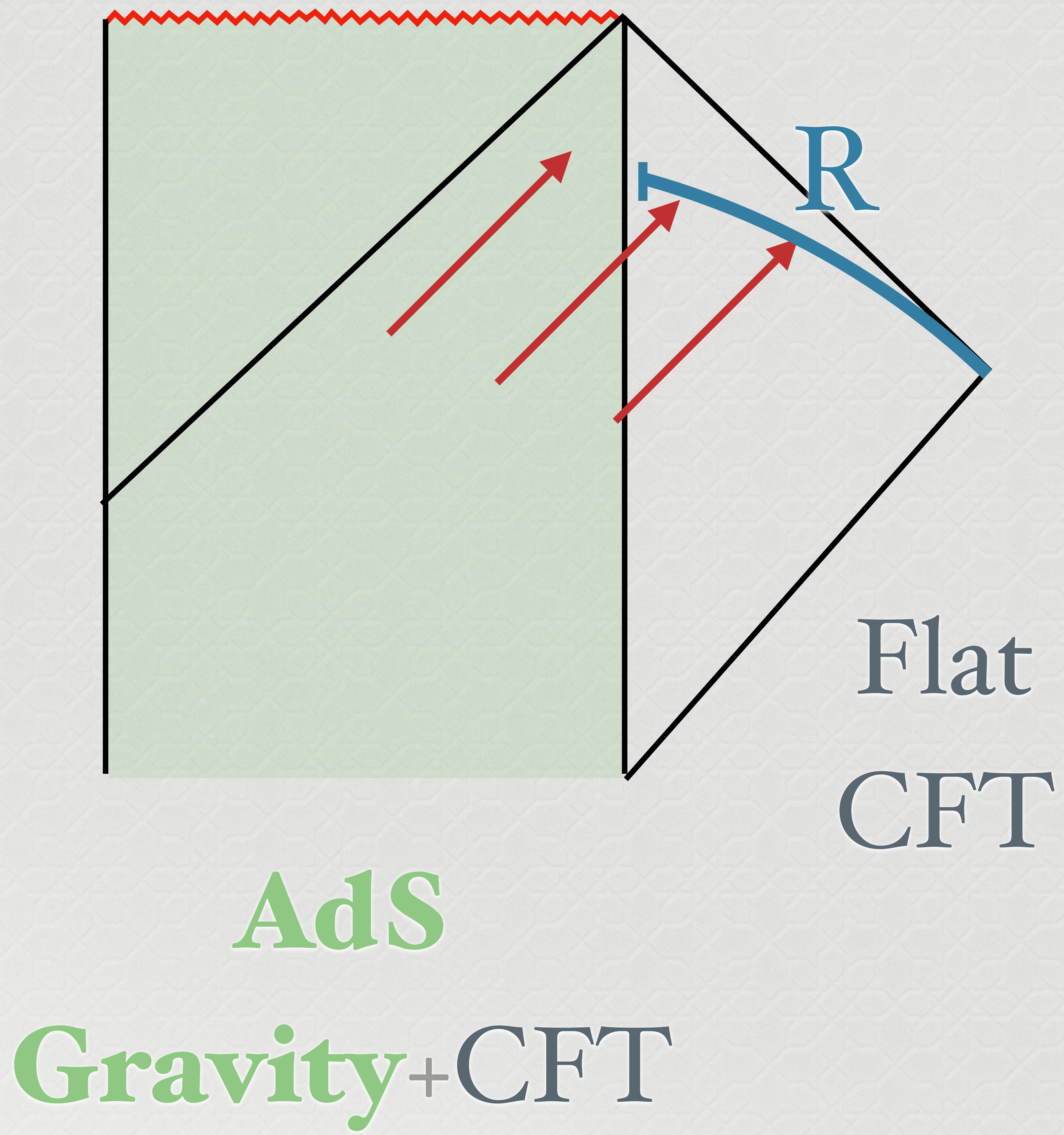

Information paradox in an evaporating AdS BH coupled to Bath

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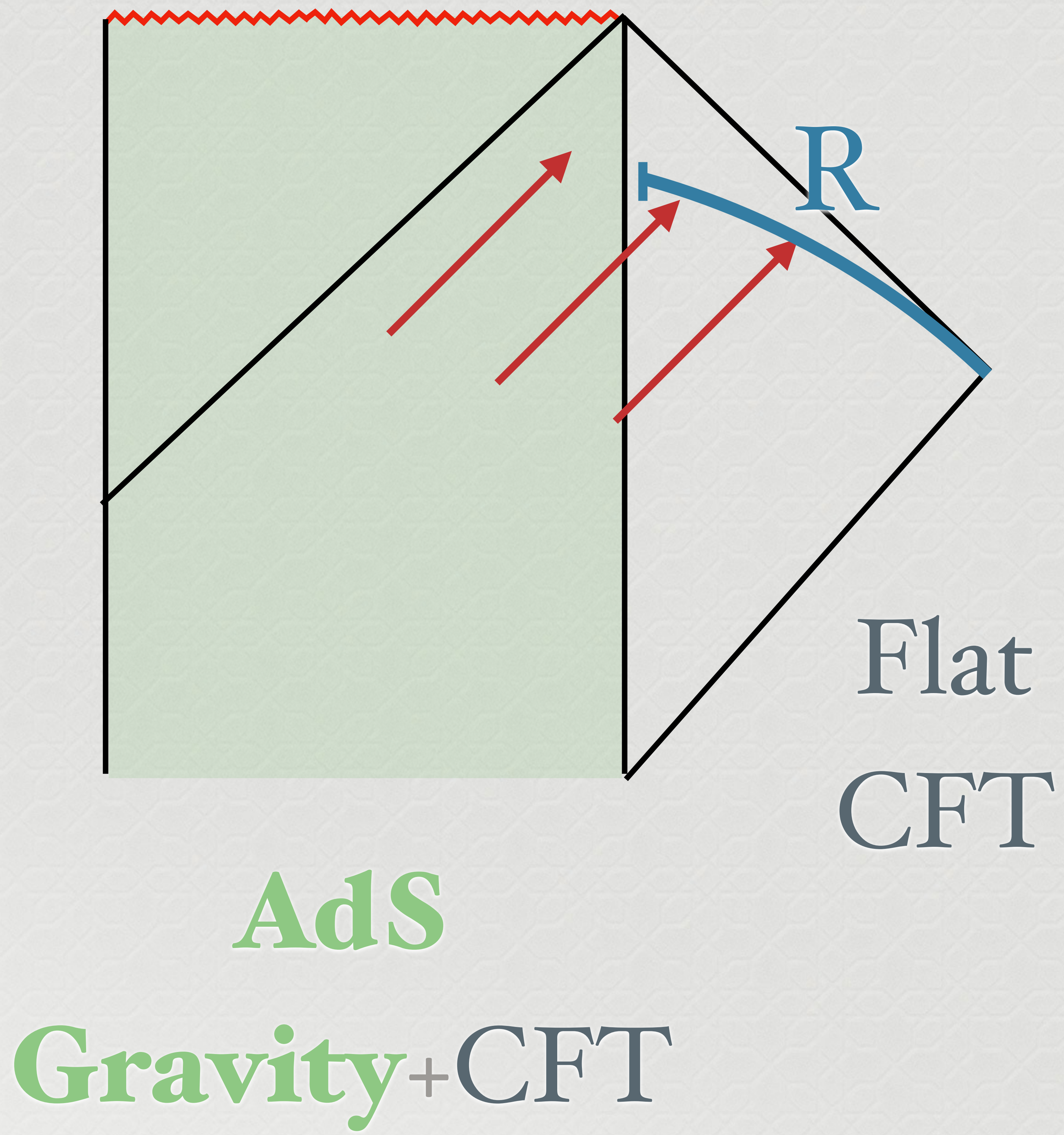
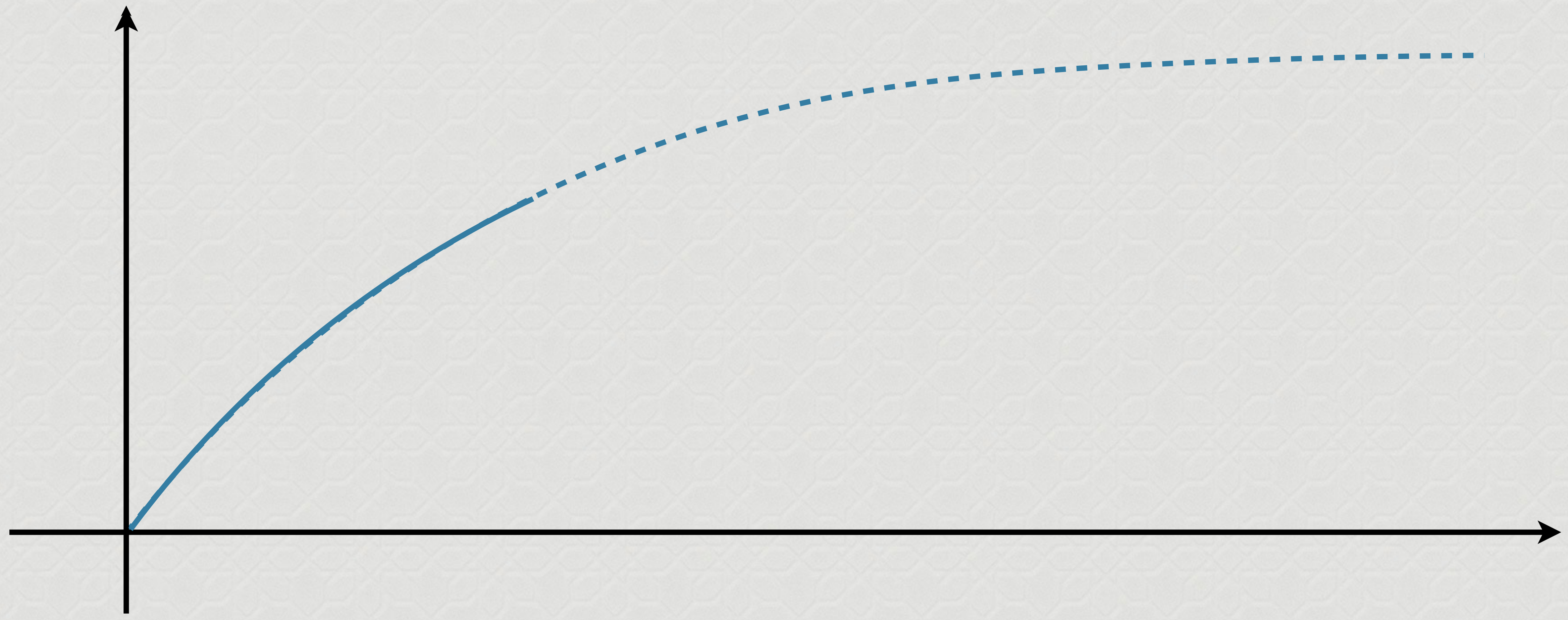


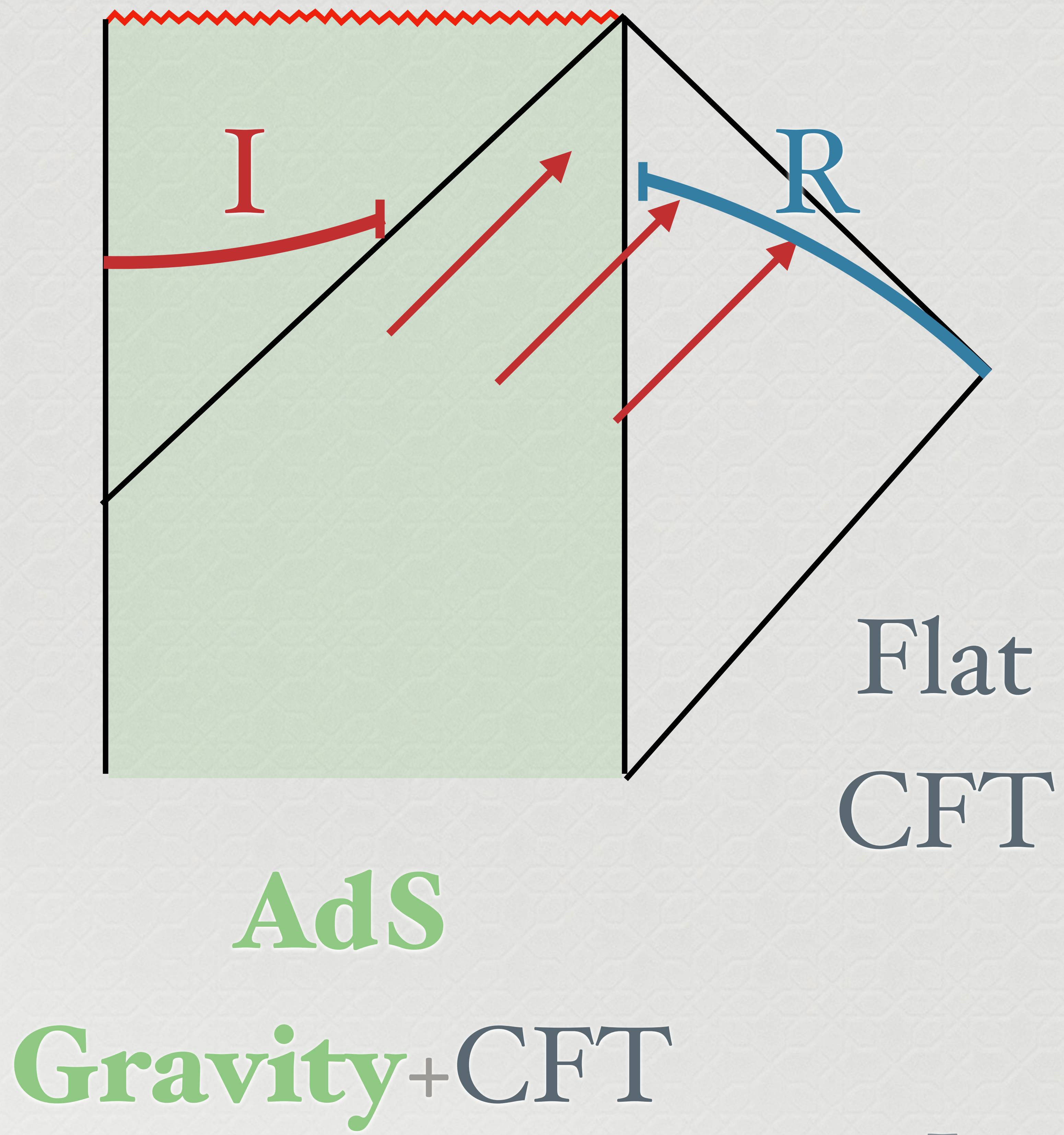


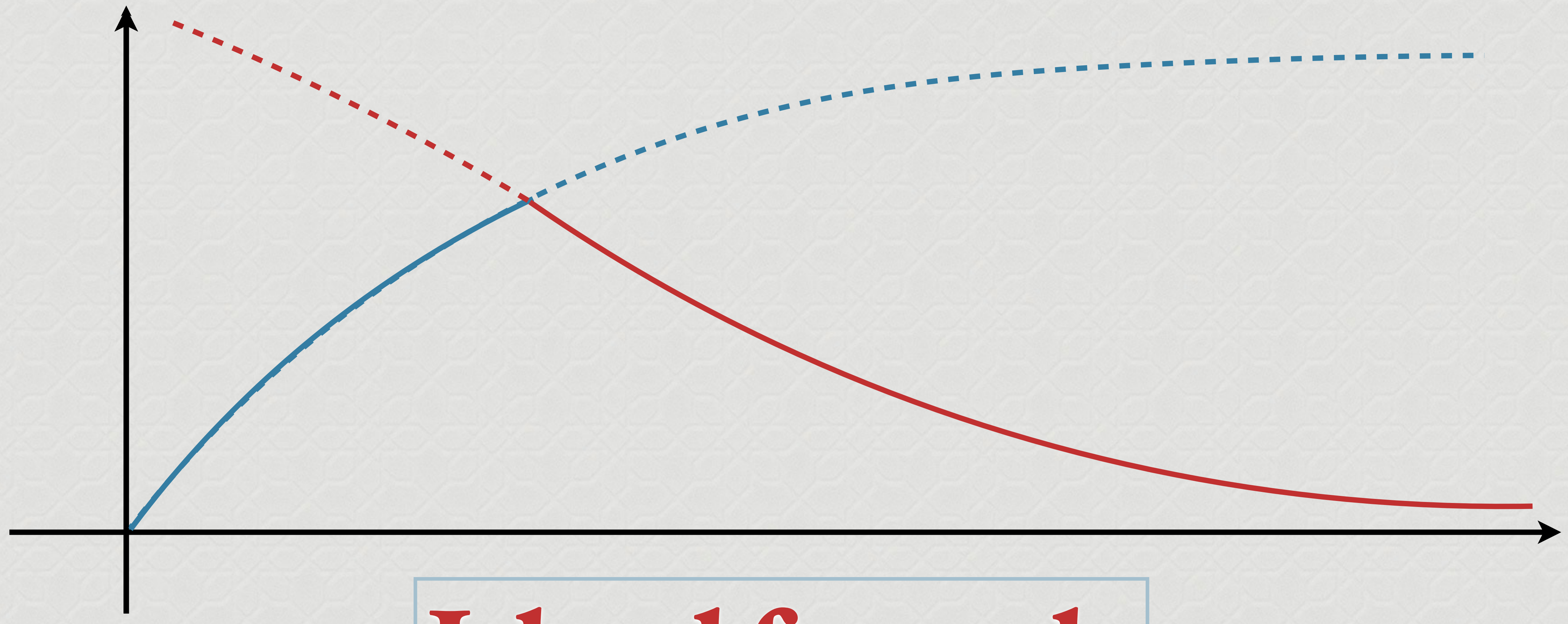
Hawking's calculation

$$S_{\text{Rad}}^{\text{Hawking}} = S_{\text{matter}}(R)$$

Information paradox in an evaporating AdS BH coupled to Bath

Penington, Almheiri-Engelhardt-Marolf-Maxfield (AEMM) '19





Island formula

$$S_{\text{Rad}} = \min \text{ext}_I \left[\frac{\text{Area}(\partial I)}{4G_N} + S_{\text{matter}}(I \cup R) \right]$$

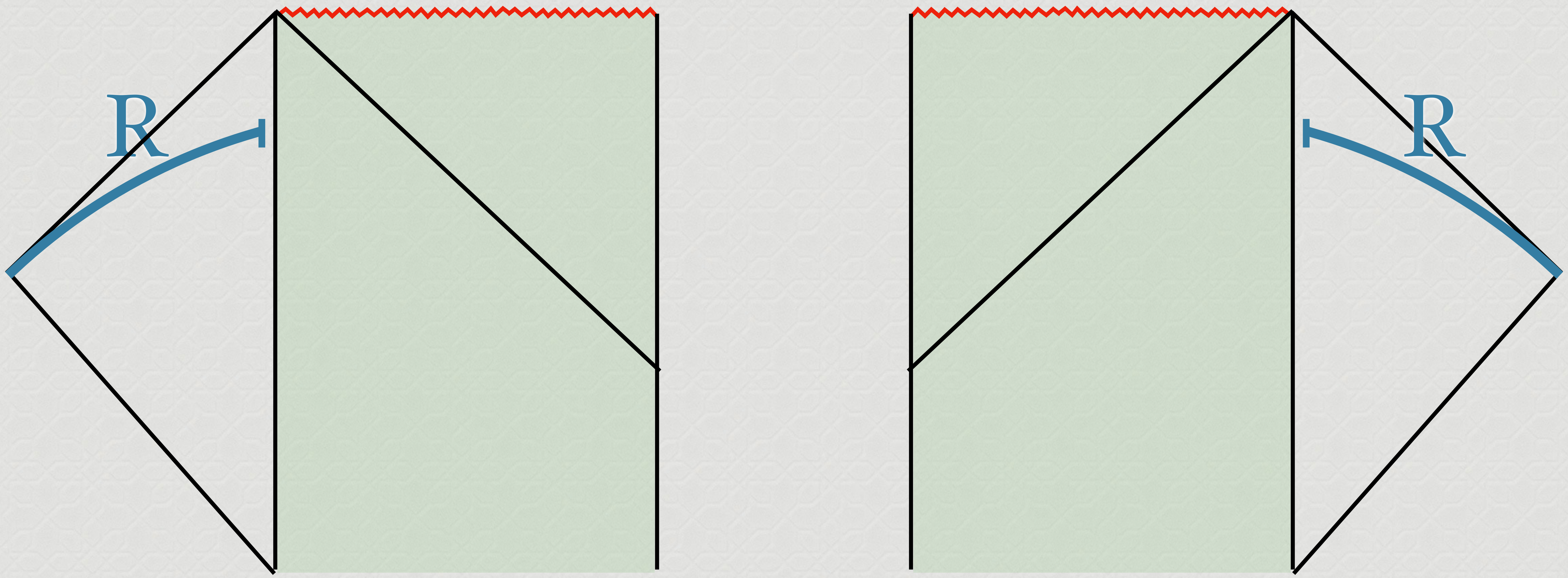
I: island = BH interior belong to Hawking radiation

∂I: quantum extremal surface (QES)

- Replica wormhole "explains" the island formula

Almheiri-Maldacena-Hartman-Shaghoulian-Tajdini
Penington-Stanford-Shenker-Yang '19

$\mathrm{tr}\rho_R^2 \supset$



$$S(R) = -\mathrm{tr}\rho_R \log \rho_R = -\partial_n \mathrm{tr}\rho_R^n|_{n\to 1}$$

Replica wormhole: wormhole sol. of Einstein eq. that connects different replica sheets

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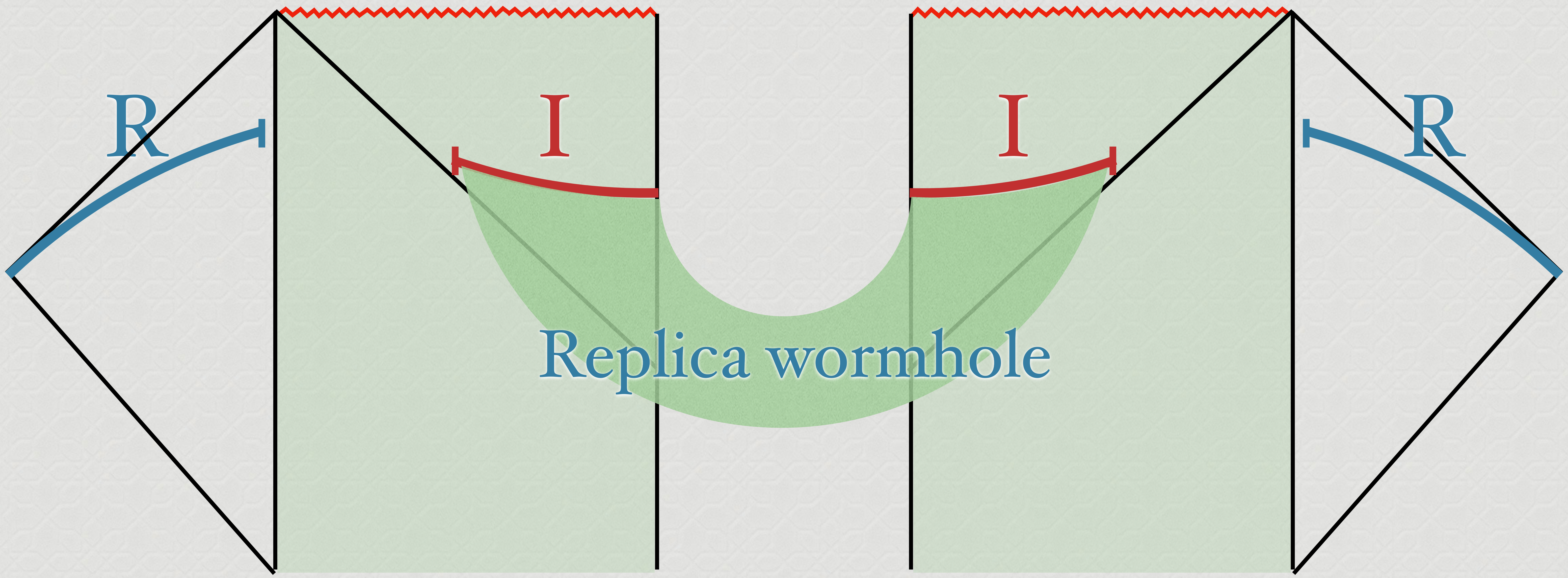

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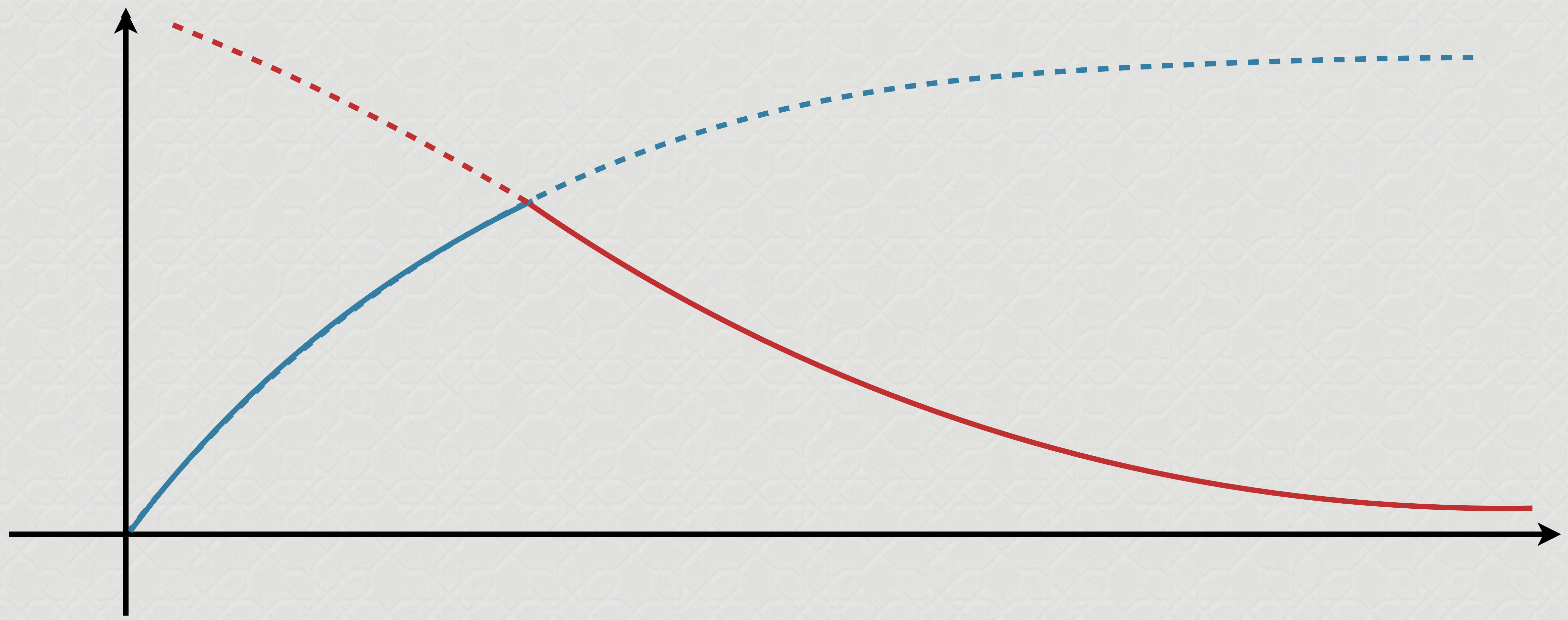
Replica wormhole: wormhole sol. of Einstein eq. that connects different replica sheets

- The derivation is formal: relies on **Euclidean** method

  not clear for evaporating BHs where there's actually information loss!



Our Goal

Derive the island formula and Page curve for BHs that form by collapse and then evaporate by using the gravitational path integral

PLAN

Part I. Background geometry (n=1)

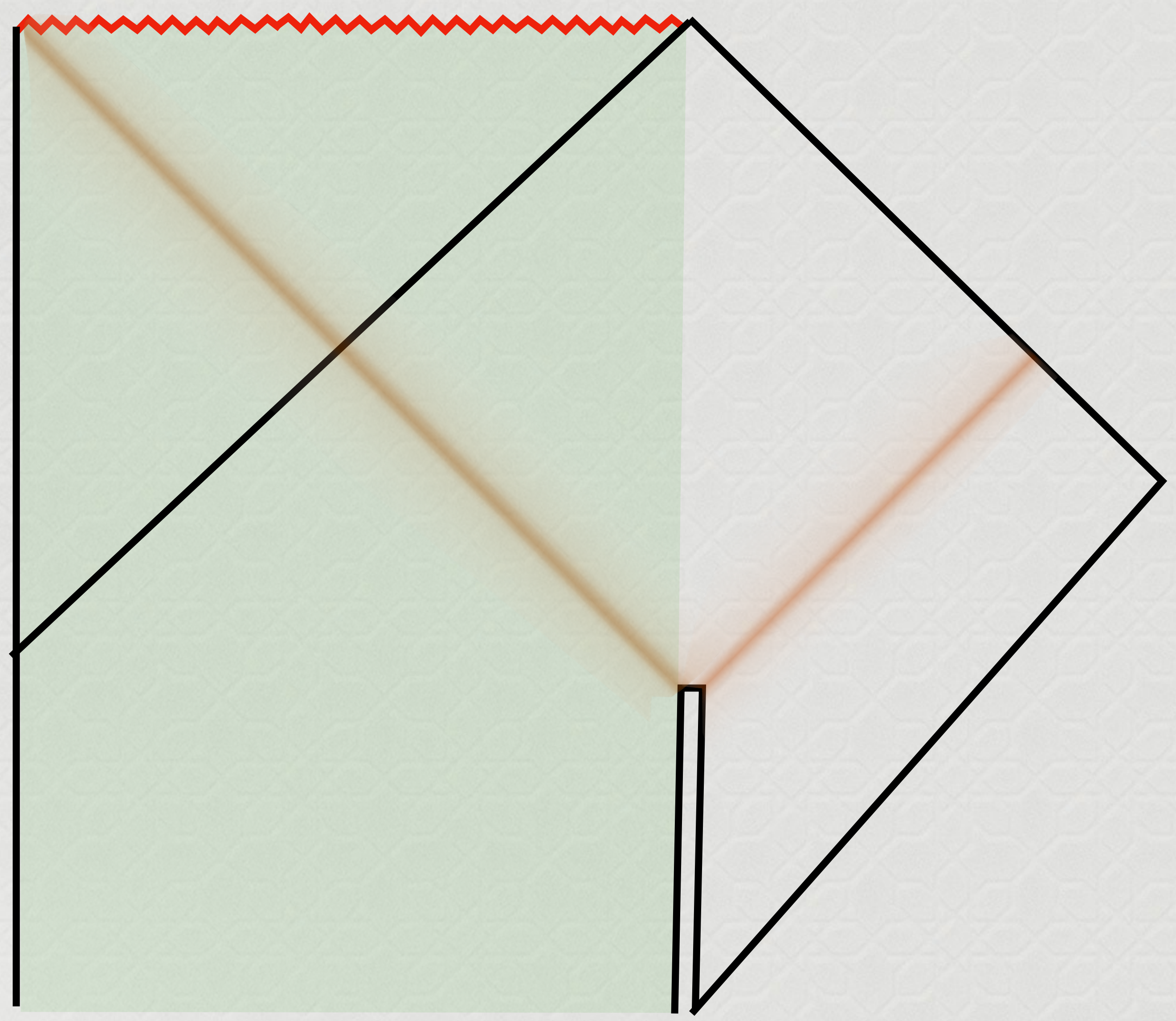
- Construction of evaporating BH from the Euclidean path-integral

Part II. Replica geometry ($n=1+\varepsilon$)

- Replica wormholes and derivation of the island (QES condition)

Part I. Background geometry (n=1)

- How to apply gravitational path integral techniques to evaporating black holes ?



AEMM: Joining quench

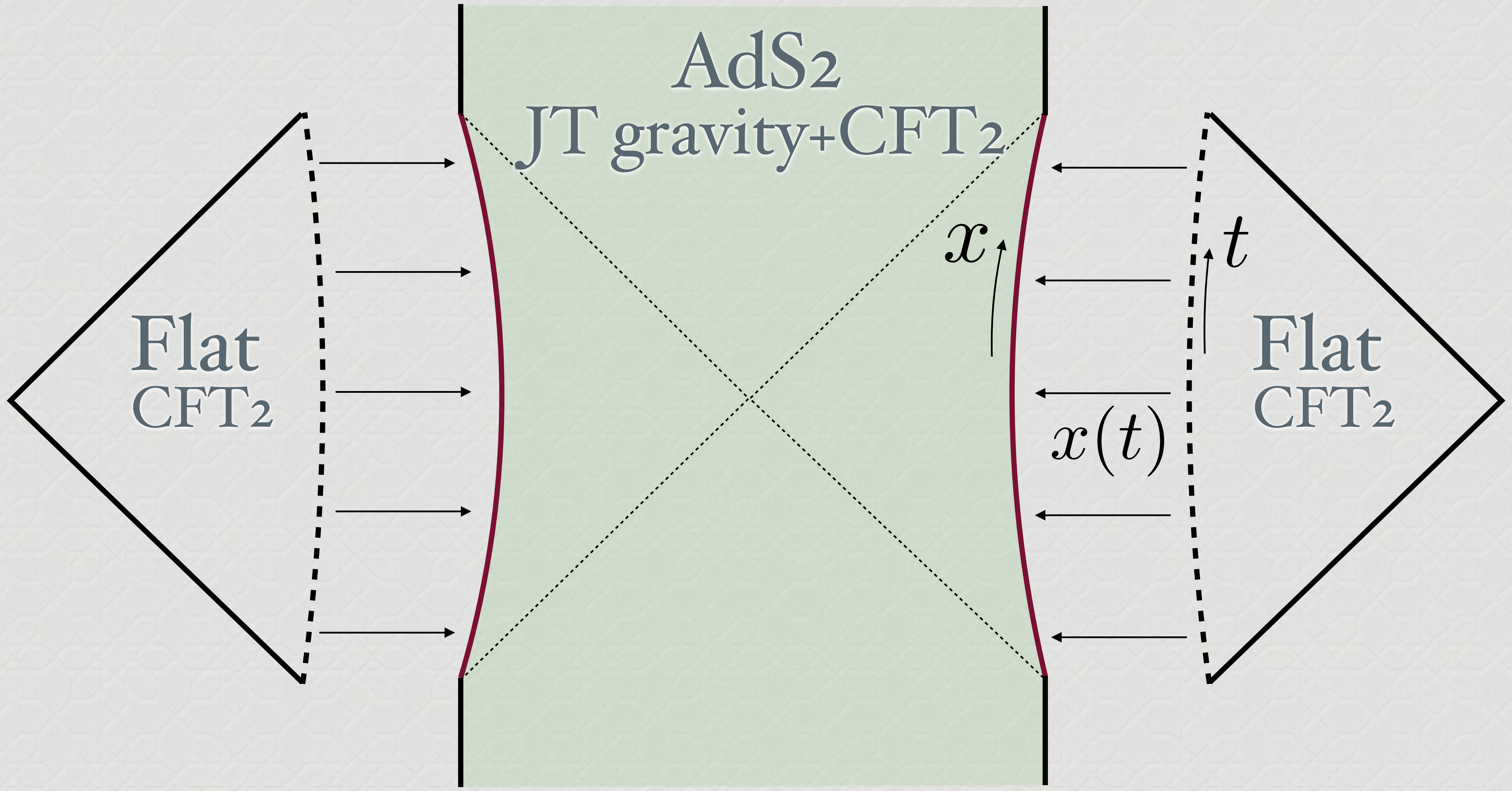
- No time-reflection symmetry

→ geometry has no real Euclidean continuation

- Gluing gravity to a flat region in Euclidean signature is highly non-local due to "the conformal welding problem"

Almheiri-Engelhardt-Marolf-Maxfield (AEMM) '19

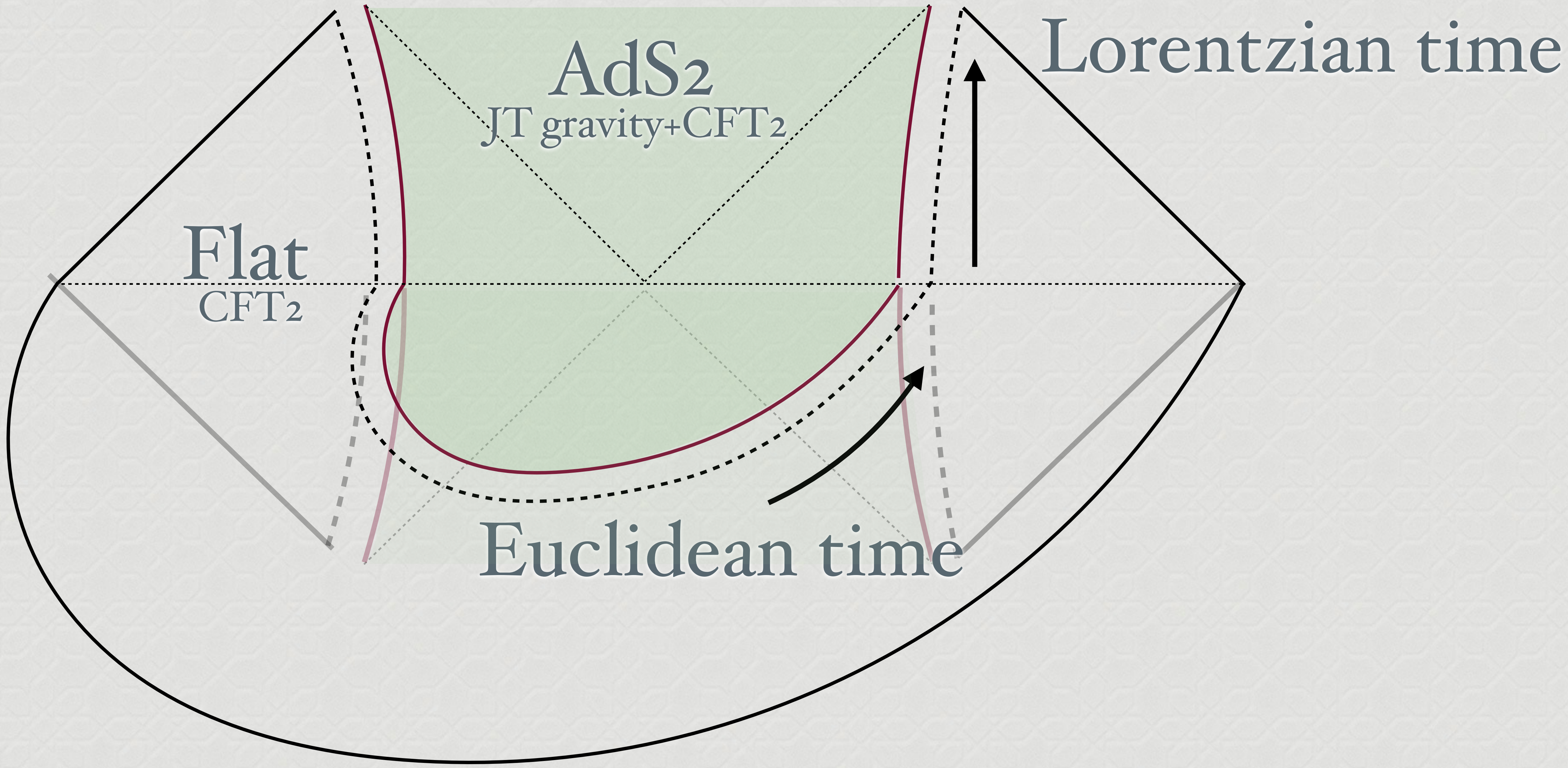
- Eternal AdS black hole coupled to bath



Eternal BH + External non-gravitational system at same temperature

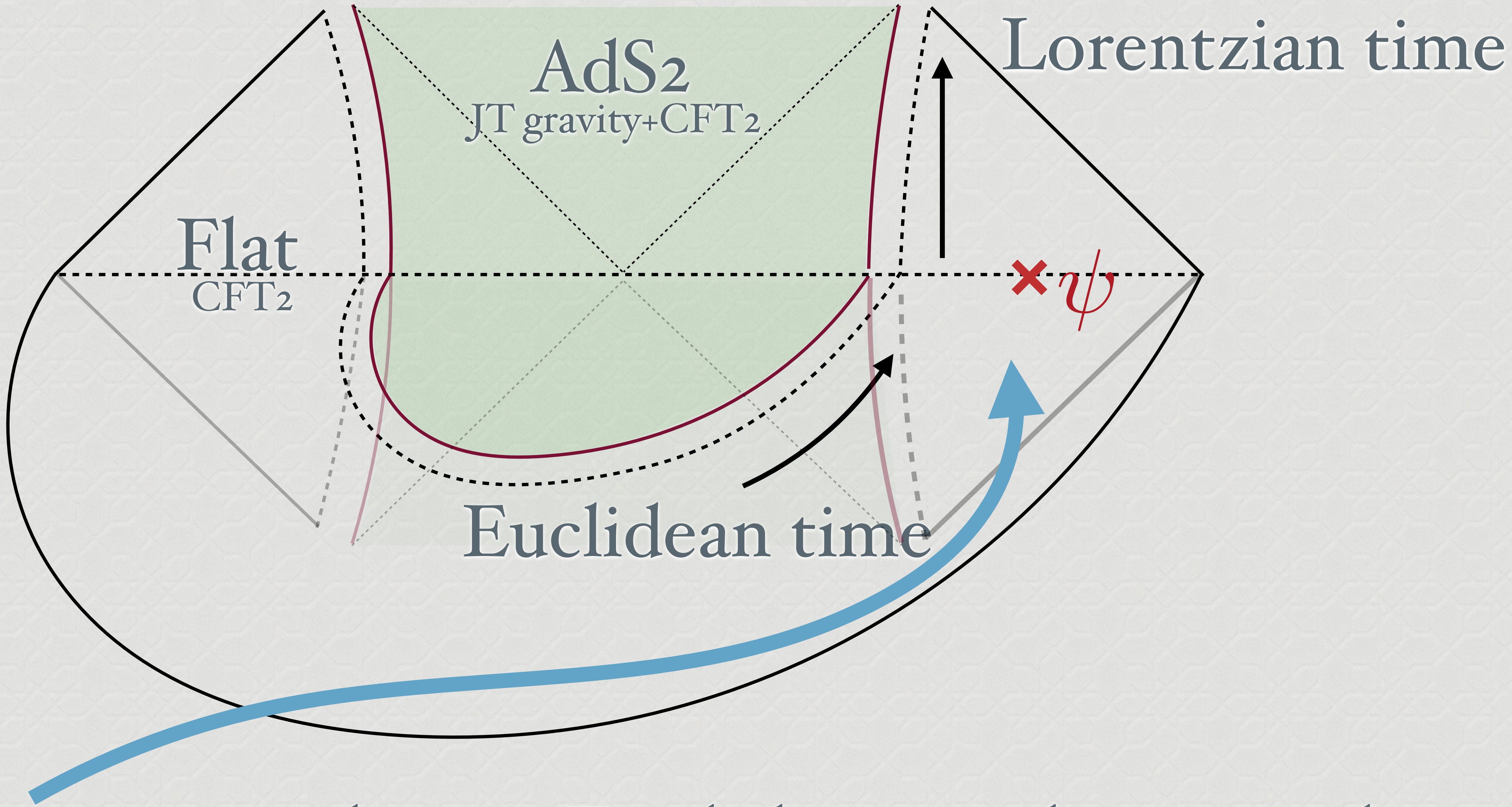
Dynamics of JT gravity→ Schwarzian action for gluing map: $x(t)$

- Eternal AdS black hole coupled to bath



Euclidean path-integral from $t_E = -\beta/2$ to $t_E = 0$

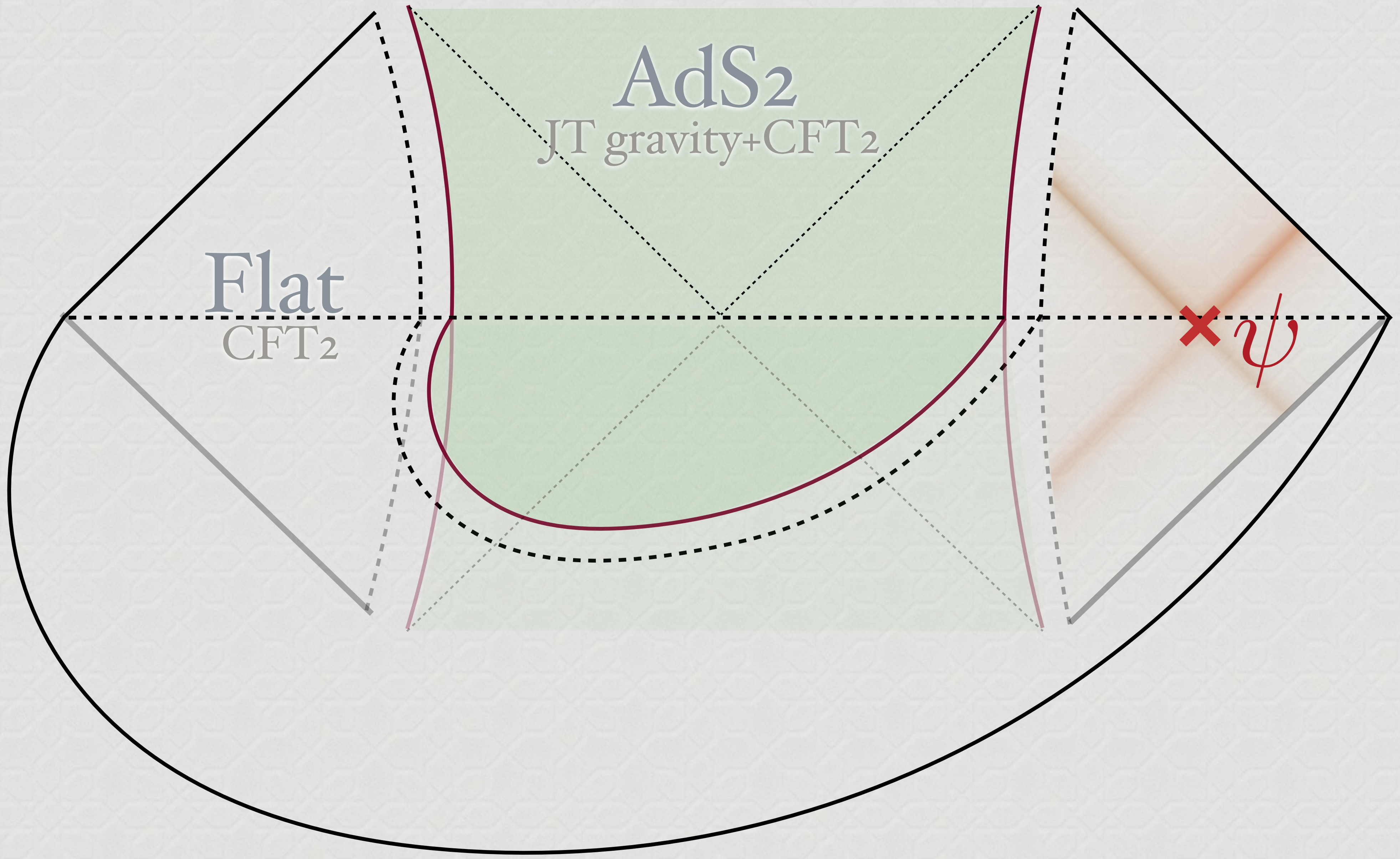
Evaporating one side of the eternal black hole



Insert a **primary operator** during Euclidean path-integral near $t_E = 0$

Conformal dim: $(h_\psi, \bar{h}_\psi)$

- Evaporating one side of the eternal black hole



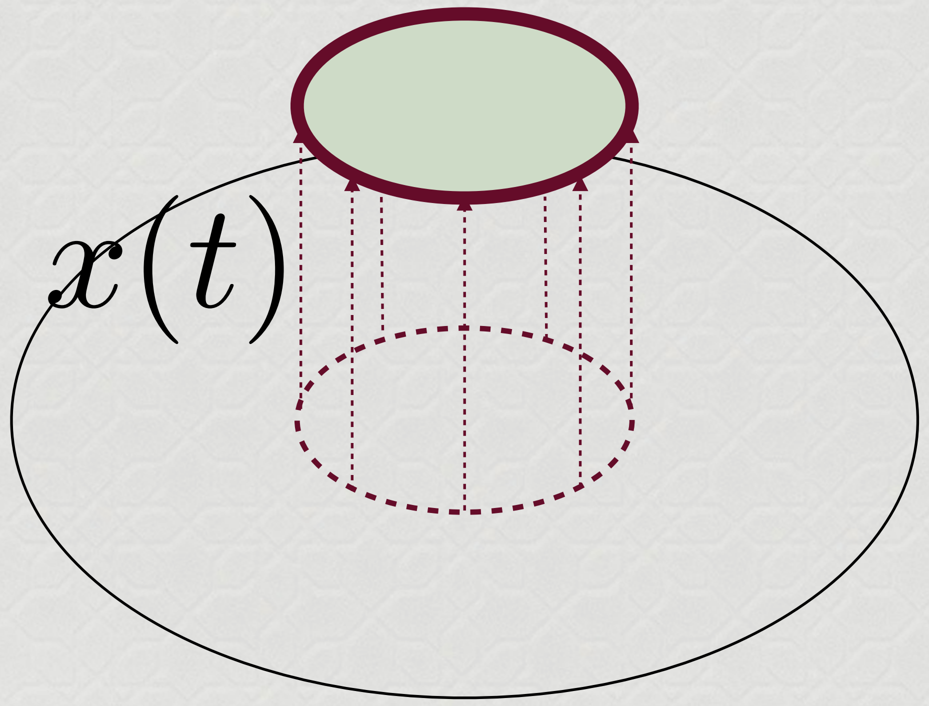
Shockwave with $E\psi \propto h_\psi$ injected from the flat region

Einstein equation for $x(t)$ $\to$ need to compute the matter stress tensor

Conformal welding problem

Gravity system

$$ds^2_{\text{inside}} = \frac{4dxd\bar{x}}{(1-x\bar{x})^2}$$



$$x = f(y, \bar{y})$$

Bath system

$$ds^2_{\text{outside}} = \frac{|dy|^2}{\epsilon^2}$$

Gravity+Bath system

$$ds^2_{\text{whole}} \propto (\partial f \bar{\partial}\bar{f} + \bar{\partial} f \partial \bar{f}) dy d\bar{y} + \partial f \partial \bar{f} dy dy + \bar{\partial} f \bar{\partial} \bar{f} d\bar{y} d\bar{y}$$

diagonal components

conformally flat part

The gravity and the bath region have different coordinates

$\rightarrow$We want to cover the whole region by a single coordinate

- Naive non-holomorphic extension $x = f(y, \bar{y})$ gives conformally non-flat metric

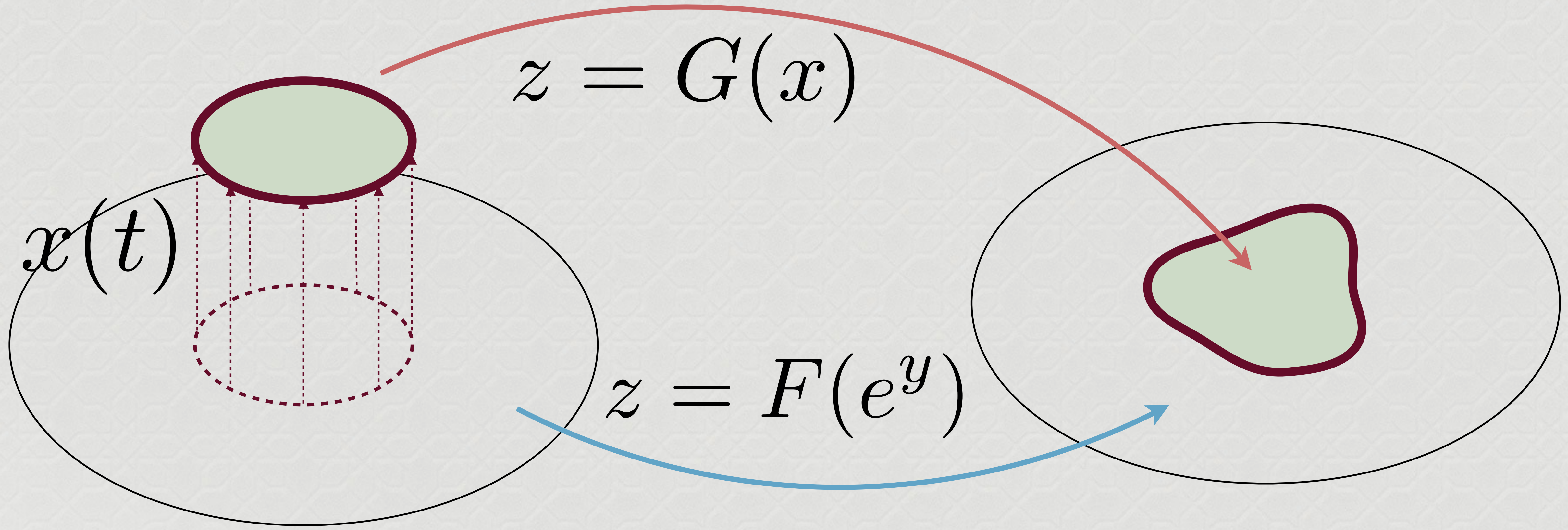
Conformal welding problem

For given $x(t)$, look for a new covering coordinate defined by holomorphic maps

$$\text{Bath:} \quad z = F(e^y)$$
$$\text{Gravity:} \quad z = G(x)$$

with matching condition at the interface $F(e^y)|_{\text{bdy}} = G(x)|_{\text{bdy}}$

$\rightarrow F, G$ non-locally depends on $x(t)$



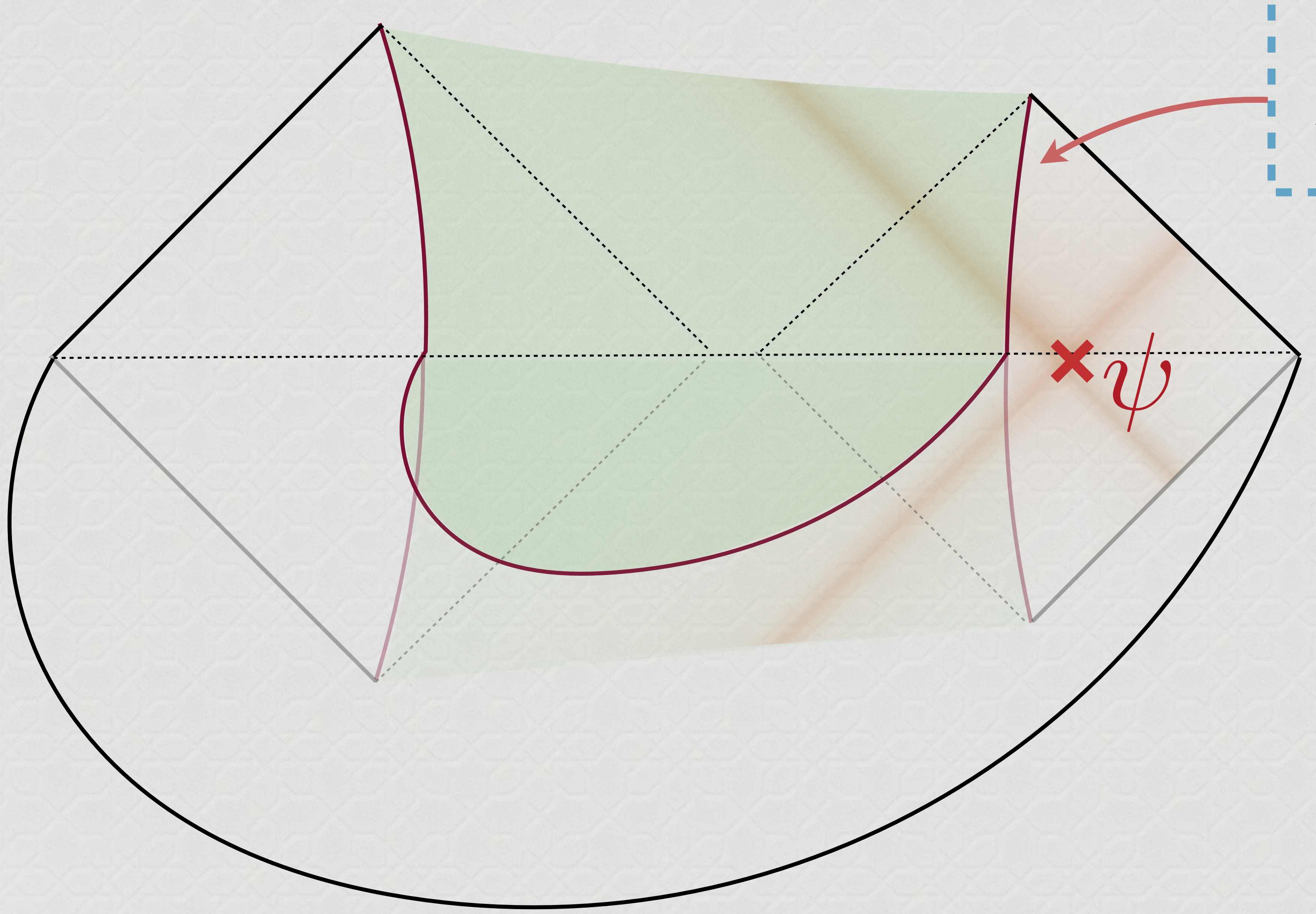
Gravity+Bath

$$ds^2 = \Omega^{-2}(z, \bar{z}) dz d\bar{z}$$

Conformally flat

$$T_{yy} = (F'(e^y))^2 T_{zz} - \frac{c}{24}\{F(e^y), y\}$$

Non-trivial solution on a Schwinger-Keldysh contour!



$$z^+ = e^{y^+}, \;\; z^- = x^-$$

$$T_{y^-y^-} = -\frac{c}{24\pi}\{x(y^-), y^-\}$$

Hawking radiation

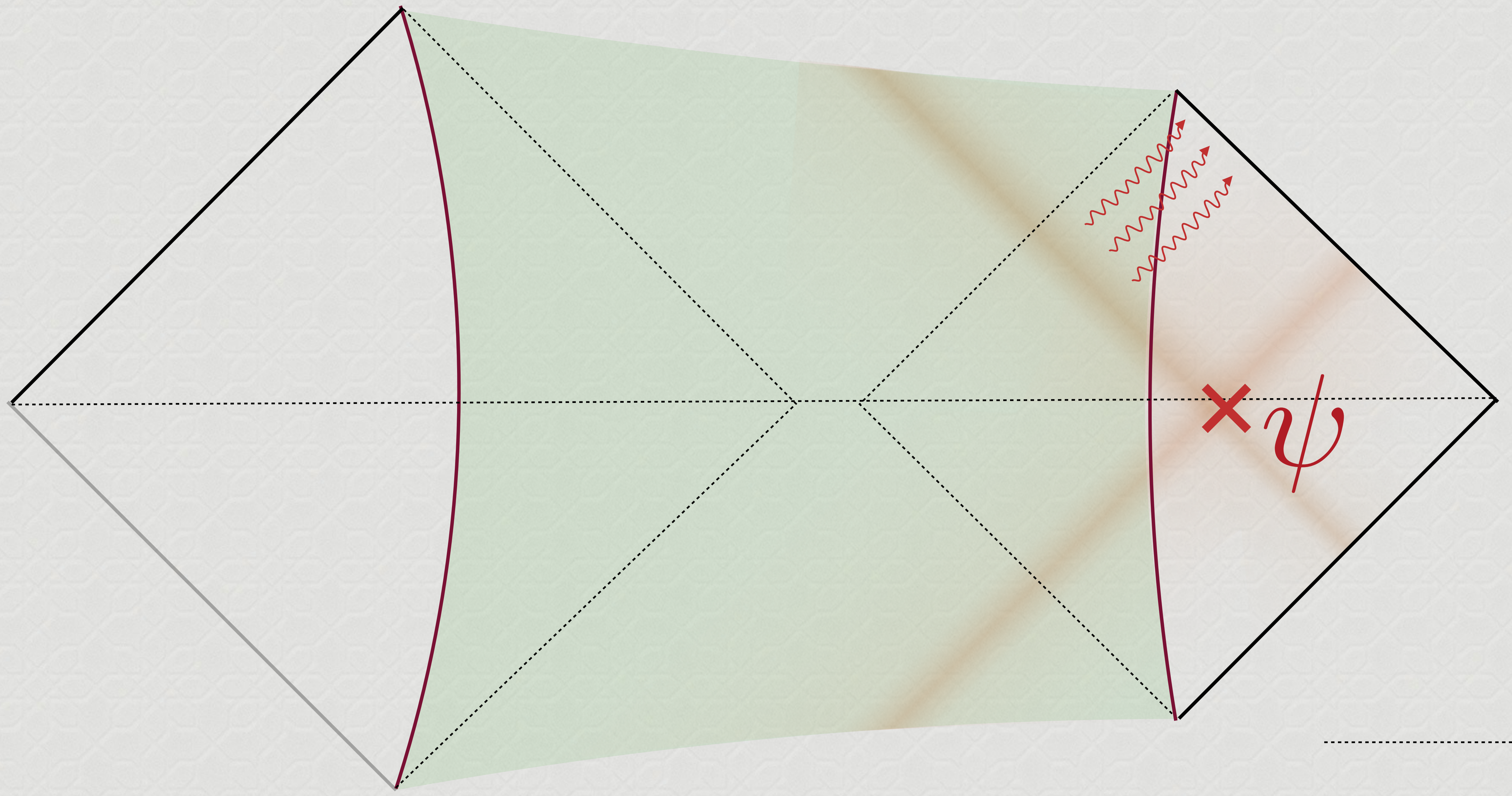
$$T_{y^+y^+} = E_\psi \delta(y^- - L) - \frac{c}{24\pi}\{e^{y^+}, y^+\}$$

Shockwave + Thermal flux

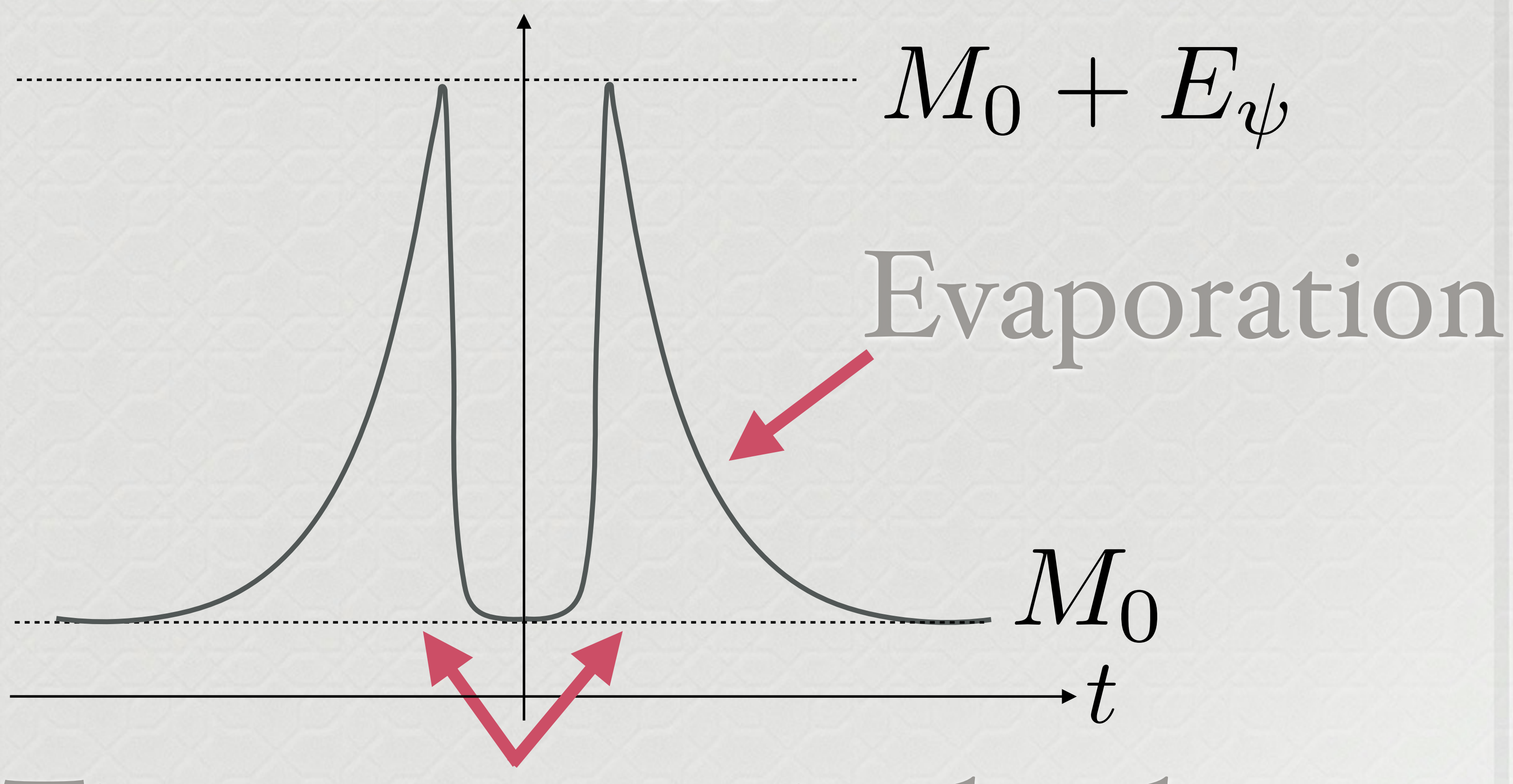
Equation of motion for $x(t)$: $\frac{d}{dt}\{x(t), t\} = T_{y^+y^+} - T_{y^-y^-}$

Local stress tensor & E.O.M!

- Evaporating one side of the eternal black hole



An evaporating black hole created by shockwave, decays into the original black hole



- Evaporation stops when it equilibrate with bath

PLAN

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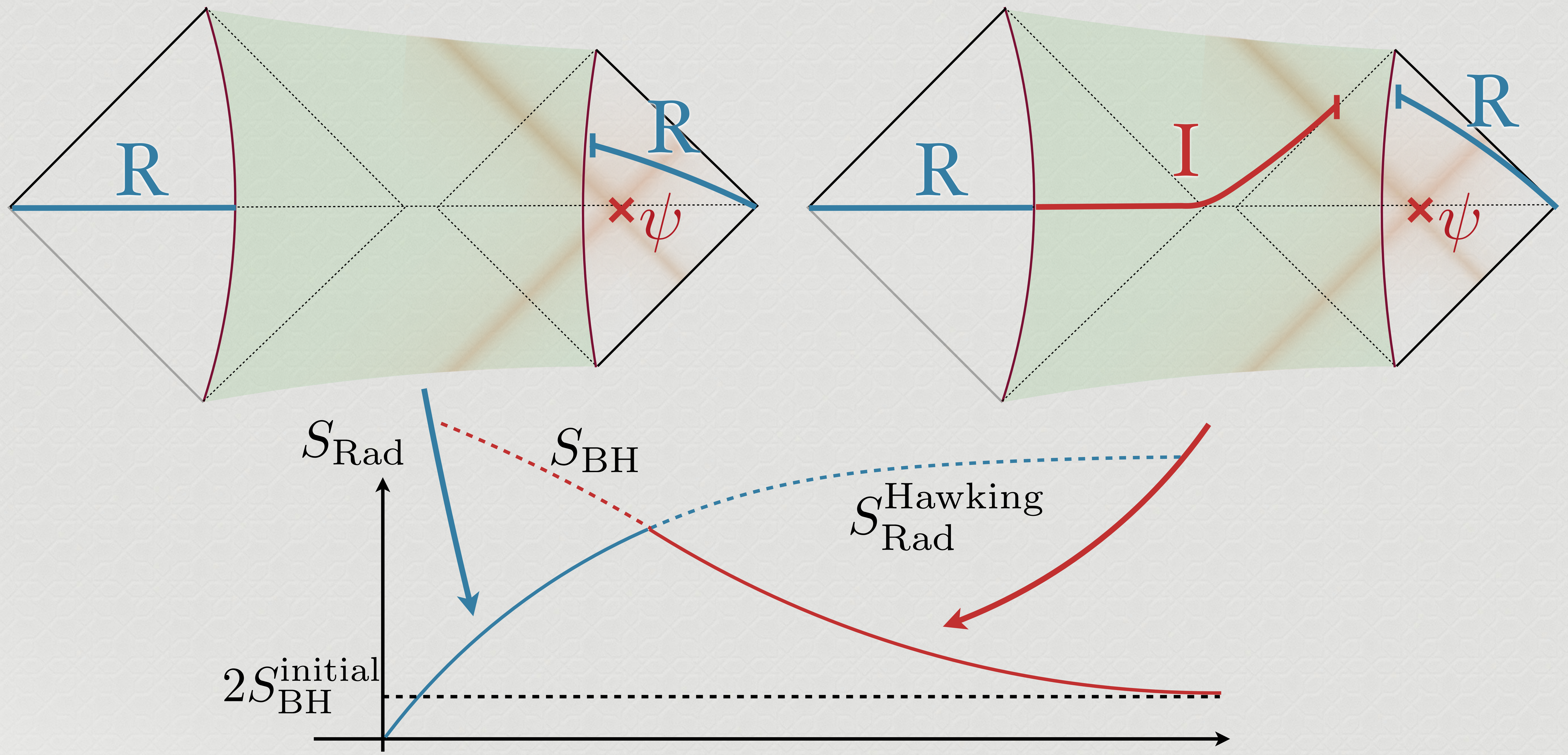
Island formula and Page curve



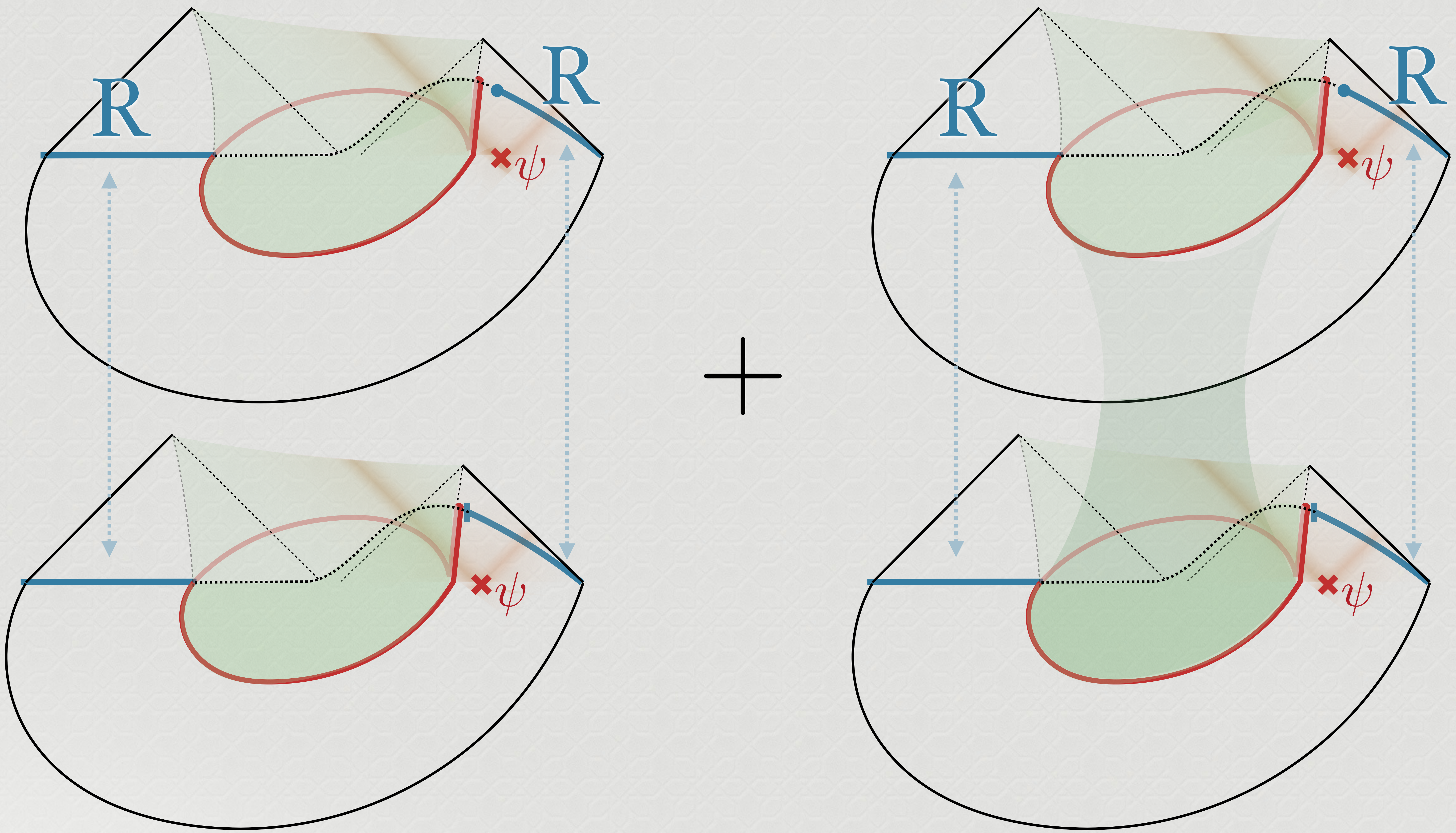
Island formula

$$S_{\rm Rad} = \min \mathrm{ext}_I \left[\frac{\mathrm{Area}(\partial I)}{4G_N} + S_{\rm matter}(I \cup R) \right]$$

- Replica wormhole for an evaporating black hole

$$S(R) = -\mathrm{tr}\rho_R \log \rho_R = -\partial_n \mathrm{tr}\rho_R^n|_{n\to 1}$$

$$\mathrm{tr}\rho_R^2 = \quad + \quad + \ldots$$

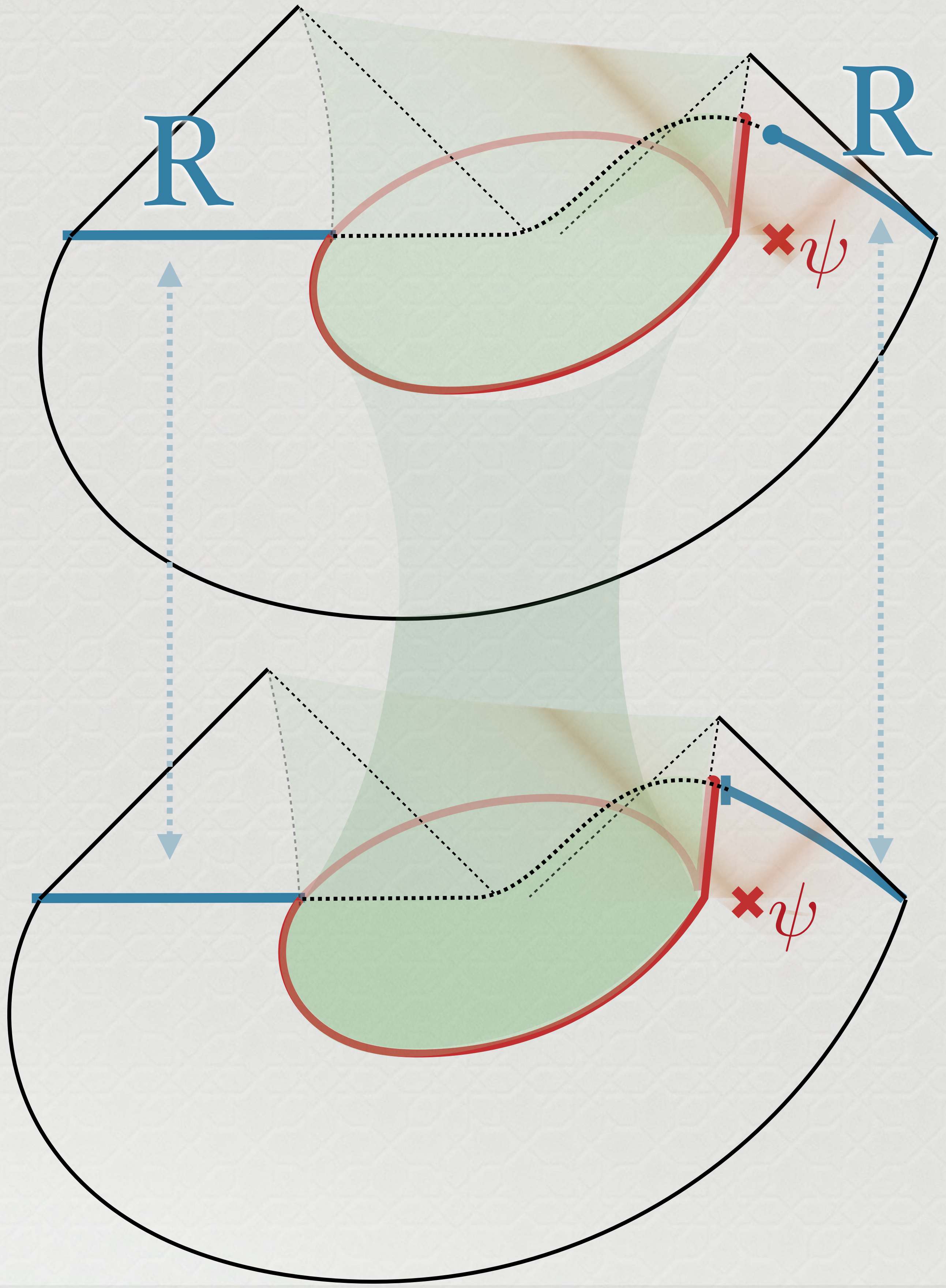


$S_{\mathrm{Rad}}^{\mathrm{Hawking}}$ $S_{\mathrm{Rad}}^{\mathrm{Wormhole}}$

Replica wormhole for an evaporating black hole

$$S(R) = -\mathrm{tr}\rho_R \log \rho_R = -\partial_n \mathrm{tr}\rho_R^n|_{n\to 1}$$

$\mathrm{tr}\rho_R^2 \supset$



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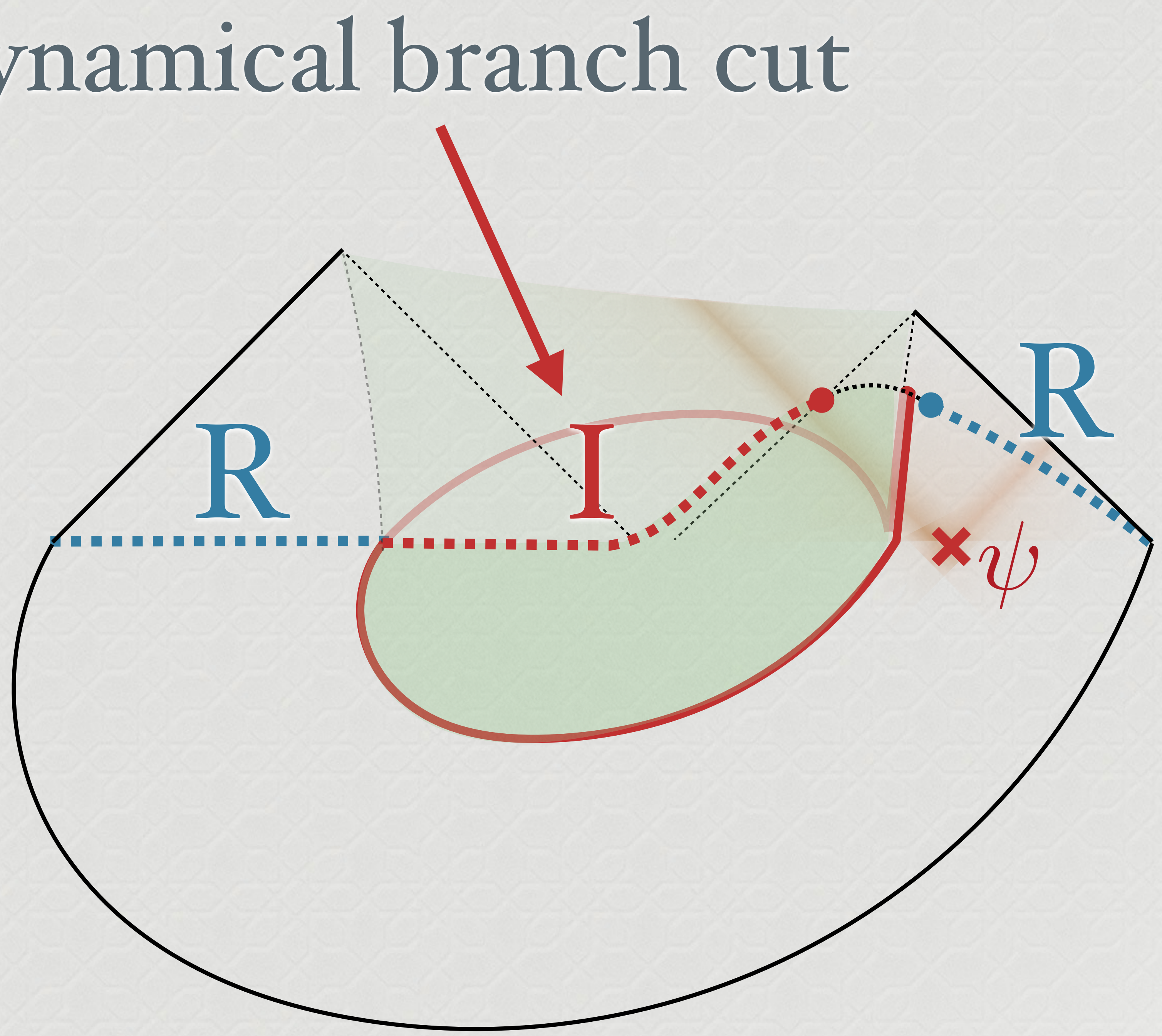
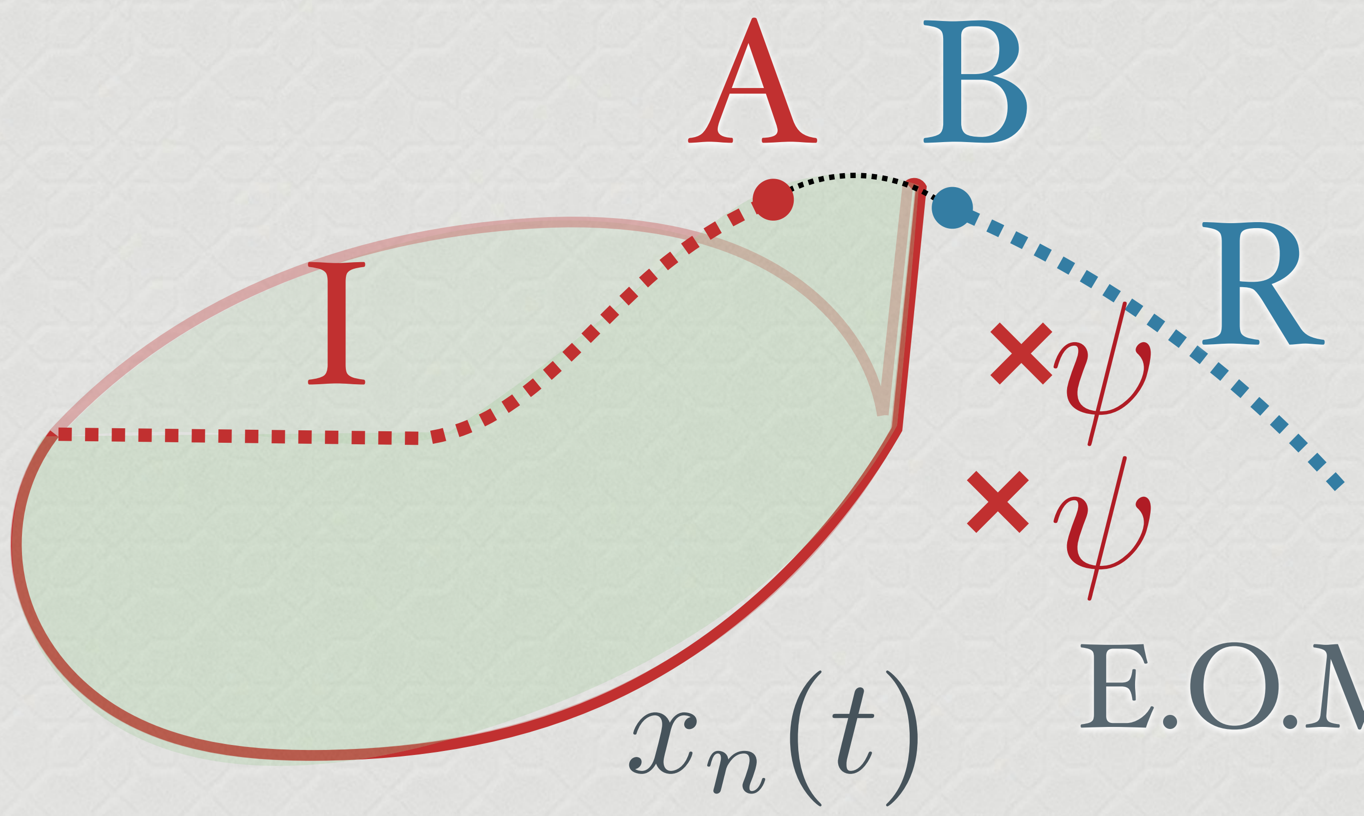


- Replica wormhole equation



- Replica wormhole :

On-shell solution of the gluing function $x_n(t)$ on a Schwinger-Keldysh contour

E.O.M: $\partial_\tau\{x_n(\tau),\tau\} + \left(1-\frac{1}{n^2}\right)\partial_\tau R(A;x_n) = i(T_{yy}-T_{\bar{y}\bar{y}})$

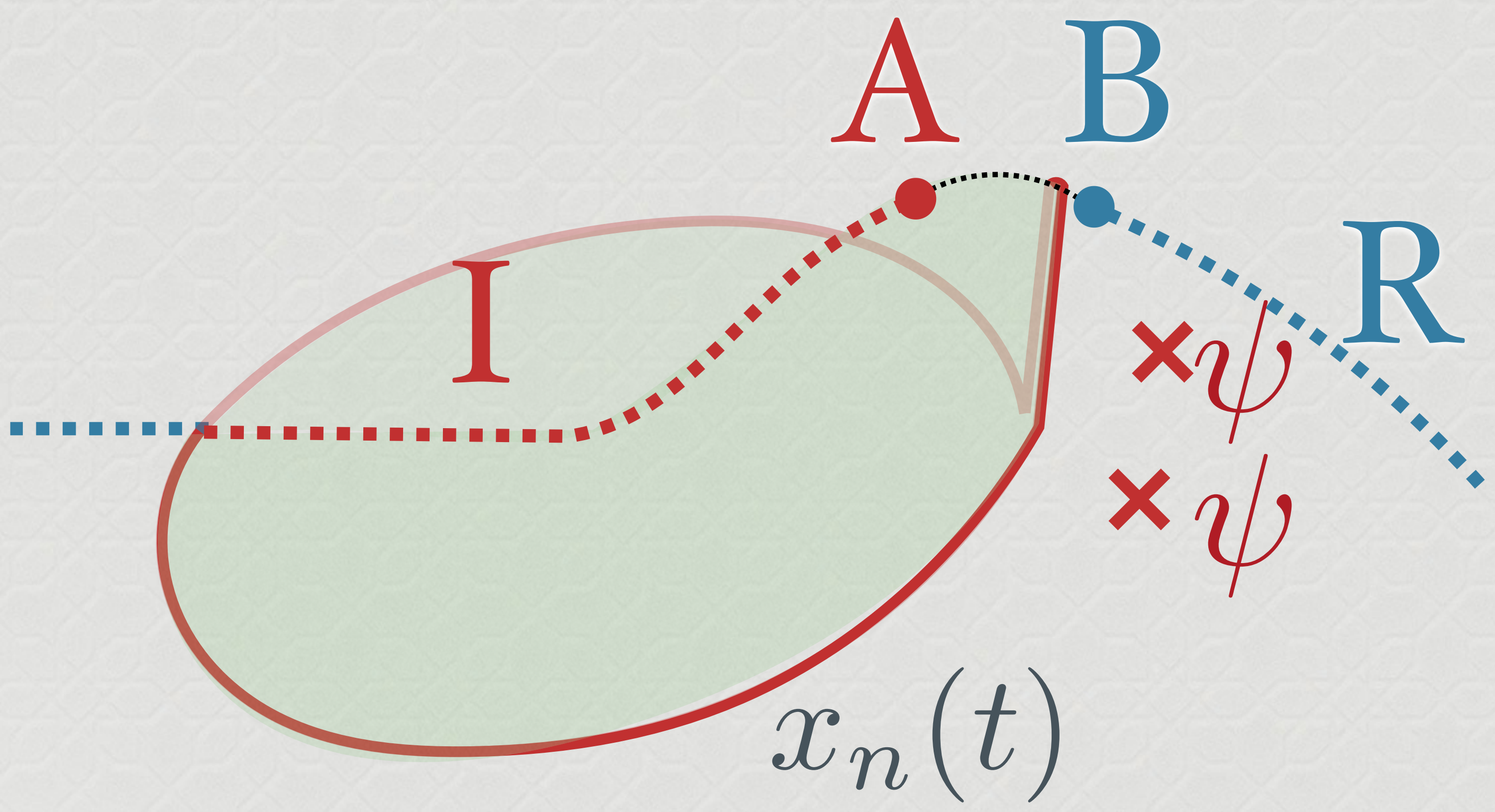
Additional contribution due to back-reaction of twist defect A

Stress tensor: $T_{yy} \sim T_{yy}^{n=1} + (n-1)\delta T_{yy}^{\text{twist}}(A,B;x_n) + (n-1)\delta T_{yy}^{\text{welding}}(x_n)$

From twist operators A & B

Twist operator insertion changes the welding solution

QES condition from replica wormhole equation



Look at the singularity at A

$$\text{E.O.M: } \mathrm{Res}_A\left[\partial_\tau R(A;x_n) + i\delta T^{\text{twist}}(A,B;x_n)\right] = 0$$

Conformal Ward identity

$$\text{QES condition: } \partial_A \frac{\mathrm{Area}(A)}{4G_N} + \partial_A S_{\mathrm{CFT}}(A,B) = 0$$

Island formula

$$S_{\mathrm{Rad}} = \min \mathrm{ext}_I \left[\frac{\mathrm{Area}(\partial I)}{4G_N} + S_{\mathrm{matter}}(I\cup R)\right]$$

Einstein equation of replica wormhole

Back-reaction of the dynamical twist A

Matter entropy from replica wormhole

Summary

- We derived the “island formula” for an evaporating black hole from the gravitational path-integral

Background geometry (n=1)

- Constructed an evaporating BH from the Euclidean path-integral

  Solved the conformal welding problem on a Schwinger-Keldysh contour and obtained a local E.O.M.

Replica geometry (n=1+ε)

- Replica wormhole equation gives the QES condition