Possible Applications of Quantum Computation to High Energy Physics

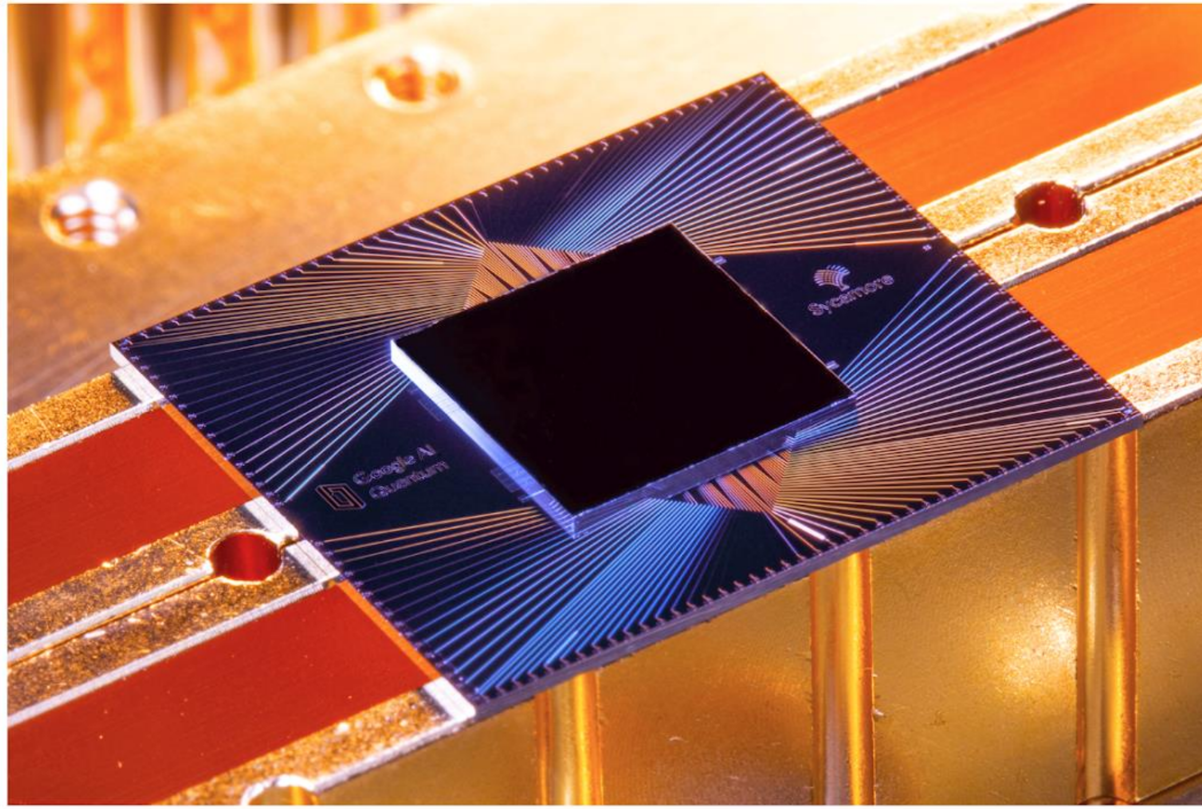
Masazumi Honda

(本多正純)

References:

- arXiv:2011.00485 [hep-lat],
  w/ Bipasha Chakraborty (Cambridge U.), Yuta Kikuchi (BNL),
  Taku Izubuchi (BNL-RIKEN BNL) & Akio Tomiya (RIKEN BNL)
- in preparation,
  w/ Yuta Kikuchi, Etsuko Itou (YITP-Keio U.-Kochi U.-RCNP),
  Lento Nagano (Tokyo U.) & Takuya Okuda (Tokyo U.)
- arXiv:2011.06576 [hep-th],
  w/ Alexander Buser (Caltech), Masanori Hanada (Surrey U.),
  Hrant Gharibyan (Caltech) & Junyu Liu (Caltech)
- arXiv:2011.06573 [hep-th], w/ Masanori Hanada, Hrant Gharibyan & Junyu Liu

Quantum computer sounds growing well…



Article

Quantum supremacy using a programmable superconducting processor

This talk = How can we use it for particle physics?

This talk is on

(practical)

Applications of Quantum Computation
to
Quantum Field Theory (QFT)

(& possibly String/M-theory)

- Generic motivation:

simply would like to use powerful computers?

- Specific motivation:

Quantum computation is suitable for Hamiltonian formalism

$\longrightarrow$ Liberation from infamous sign problem in Monte Carlo?

(next slide)

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi \ \mathcal{O}(\phi) e^{-S[\phi]} \quad \Longrightarrow \quad \int d\phi \ \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi \ \mathcal{O}(\phi) e^{-S[\phi]} \Rightarrow \int d\phi \ \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a probability:

$$\langle \mathcal{O}(\phi) \rangle \simeq \frac{1}{\sharp(\text{samples})} \sum_{i \in \text{samples}} \mathcal{O}(\phi_i)$$

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \ \mathcal{O}(\phi)\underbrace{e^{-S(\phi)}}_{\text{probability}}$$

problematic when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " much worse

Sign problem in Monte Carlo simulation (Cont'd)

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Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " *much worse*

In Hamiltonian formalism,

sign problem is absent from the beginning

($^{\exists}$various approaches within framework of path integral formalism but I'll skip it)

Cost of Hamiltonian formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

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Quantum computers do this job?

In this talk, we mainly focus on

Schwinger model with topological term in Minkowski space

1+1d QED

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{\frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu}}_{\text{topological “theta term”}} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi}$$

supposed to be difficult in the conventional approach:

- real time
- $^\exists$ sign problem even in Euclidean case when θ isn't small

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- $\exists$ sign problem even in Euclidean case when θ isn't small

Results:

[cf. Tensor Network approach:
Banuls-Cichy-Jansen-Saito '16 , Funcke-Jansen-Kuhn '19, etc.]

- Construction of the true vacuum (ground state)
- Computation of $\langle\bar{\psi}\psi\rangle$ & consistency check/prediction [Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]
- Exploration of the screening vs confinement problem [in preparation, MH-Itou-Kikuchi-Nagano-Okuda]
- Estimation of computational resource

(If time is allowed) I'll also discuss

[Gharibyan-Hanada-MH-Liu '20]

possible applications to string/M-theory

In particular,

[Berenstein-Maldacena-Nastase '02]

how to put BMN model on quantum computer

(∼supersymmetric matrix QM coupled to $U(N)$ gauge field)

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possible applications to string/M-theory

In particular, [Berenstein-Maldacena-Nastase '02]

how to put BMN model on quantum computer

(~supersymmetric matrix QM coupled to $U(N)$ gauge field)

$^\exists$Various connections to string/M-theory:

- It is a candidate for a non-perturbative formulation of M-theory on pp-wave spacetime
- It describes worldvolume theories of branes in string/M-theory
- It has holographic duals

Contents

QFT as Quantum Bit (=Qubit) ?

Qubit = Quantum system w/ 2-dim. Hilbert space

(ex. up/down spin system)

Quantum computer = a combination of qubits

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To put QFT on quantum computer,

1. "Regularize" Hilbert space (make it finite-dim.!)
2. Rewrite the regularized theory in terms of qubits

QFT as Quantum Bit (=Qubit) ?

Qubit = Quantum system w/ 2-dim. Hilbert space

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To put QFT on quantum computer,

1. "Regularize" Hilbert space (make it finite-dim.!)
2. Rewrite the regularized theory in terms of qubits

Schwinger model = the simplest nontrivial example w/ gauge interaction in this context

— 1+1d gauge field has only 1-dim. physical Hilbert sp.

— Lattice fermion has finite-dim. Hilbert sp.

Schwinger model w/ topological term

Continuum ①:

(will be used in the confinement vs screening)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

Continuum ②:

(equivalent via "chiral anomaly", used here)

[Fujikawa'79]

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}e^{i\theta\gamma^5}\psi}$$

Schwinger model w/ topological term

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$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

Continuum ②: (equivalent via "chiral anomaly", used here) [Fujikawa'79]

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Taking temporal gauge $A_0 = 0$, $(\Pi = \dot{A}^1)$

$$\hat{H} = \int dx\left[-i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}e^{i\theta\gamma^5}\psi + \frac{1}{2}\Pi^2\right]$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1\Pi - g\bar{\psi}\gamma^0\psi$$

Accessible region by analytic computation

- Massive limit:

The fermion can be integrated out

&

the theory becomes effectively pure Maxwell theory w/ θ

Accessible region by analytic computation

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The fermion can be integrated out

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- Bosonization (duality):

[Coleman '76]

$$\mathcal{L} = \frac{1}{8\pi}\left(\partial_\mu \phi\right)^2 - \frac{g^2}{8\pi^2}\phi^2 + \frac{e^\gamma g}{2\pi^{3/2}} m \cos(\phi + \theta)$$

exactly solvable for $m = 0$

&

small m regime is approximated by perturbation

Sign problem in path integral formalism

In Minkowski space,

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}\big(i\gamma^\mu D_\mu - m\big)\psi \right] + \frac{g\theta}{4\pi} \int F \in \boldsymbol{R}$$

$$\langle \mathcal{O} \rangle = \frac{\int DAD\psi D\bar{\psi}\ \ \mathcal{O}\ e^{iS}}{\int DAD\psi D\bar{\psi}\ e^{iS}}$$

highly oscillating

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$$\langle \mathcal{O} \rangle = \frac{\int DAD\psi D\bar{\psi}\ \mathcal{O}\ e^{iS}}{\int DAD\psi D\bar{\psi}\ e^{iS}}$$

highly oscillating

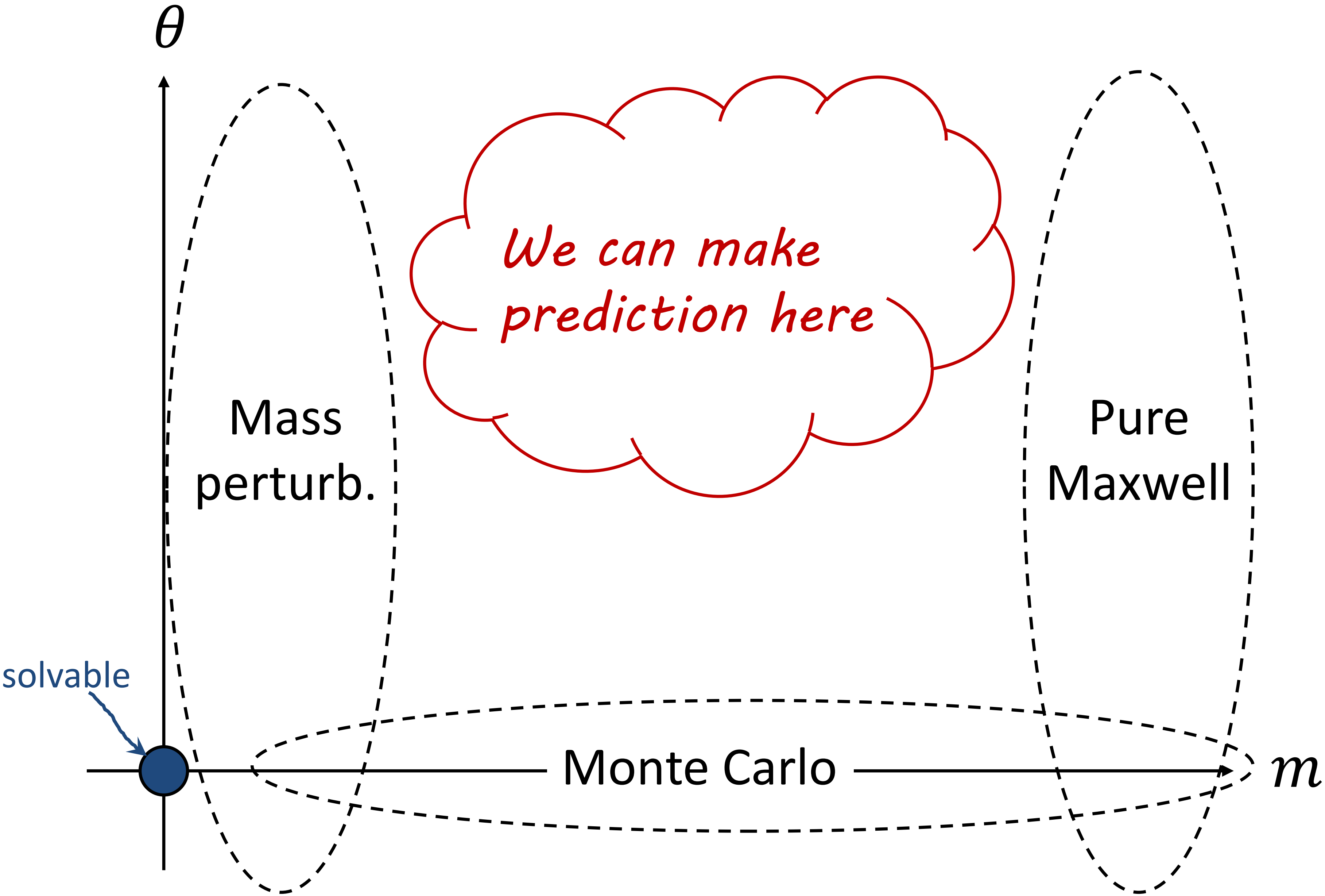
In Euclidean space,

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \right] + i\,\frac{g\theta}{4\pi} \int F \in \boldsymbol{C}$$

$$\langle \mathcal{O} \rangle = \frac{\int DAD\psi D\bar{\psi}\ \mathcal{O}\ e^{-S}}{\int DAD\psi D\bar{\psi}\ e^{-S}}$$

highly oscillating for non-small θ

Map of accessibility/difficulty



Put the theory on lattice

▪ Fermion (on site):

"Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{\underbrace{a^{1/2}}_{\text{lattice spacing}}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{array}{l} \rightarrow \text{odd site} \\ \rightarrow \text{even site} \end{array}$$

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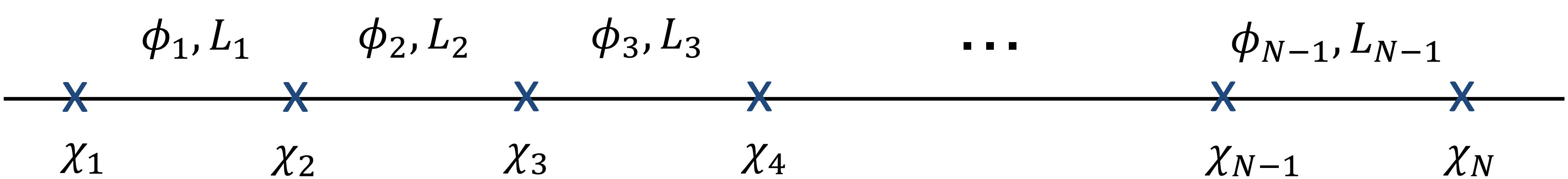
・Fermion (on site): "*Staggered fermion*" [Susskind, Kogut-Susskind '75]

$$\underbrace{\frac{\chi_n}{a^{1/2}}}_{a:\ \text{lattice spacing}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{matrix} \rightarrow \text{odd site} \\ \rightarrow \text{even site} \end{matrix}$$

・Gauge field (on link):

$$\phi_n \leftrightarrow -agA^1(x), \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$



Lattice theory w/ staggered fermion

Hamiltonian:

$$\hat{H} = -i \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[\chi_n^\dagger e^{i\phi_n} \chi_n - \text{h.c.} \right]$$

$$+ m \cos\theta \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n + J \sum_{n=1}^{N-1} L_n^2 \qquad \left(w = \frac{1}{2a}, J = \frac{g^2 a}{2} \right)$$

Commutation relation:

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \ \{\chi_n, \chi_m\} = 0, \ \ [\phi_n, L_m] = i\delta_{mn}$$

Gauss law:

$$L_n - L_{n-1} = \chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2}$$

Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = \sum_{\ell=1}^{n-1}\left[\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2}\right] \qquad \text{(took } L_0 = 0\text{)}$$

2. Redefine fermion to absorb ϕ_n:

$$\chi_n \to \prod_{\ell<n}\left[e^{-i\phi_\ell}\right]\ \chi_n$$

Then,

$$\hat{H} = -i\sum_{n=1}^{N-1}\left(w - (-1)^n \frac{m}{2}\sin\theta\right)\left[\chi_n^\dagger \chi_{n+1} - \text{h.c.}\right] + m\cos\theta \sum_{n=1}^{N}(-1)^n \chi_n^\dagger \chi_n$$
$$+J\sum_{n=1}^{N-1}\left[\sum_{\ell=1}^{n-1}\left(\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2}\right)\right]^2$$

This acts on finite dimensional Hilbert space

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\chi_n = \left(\prod_{\ell<n} iZ_\ell\right) \frac{X_n - iY_n}{2}$$

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Now the system is purely a spin system:

$$\hat{H} = H_{ZZ} + H_\pm + H_Z$$

$$\begin{cases} H_{ZZ} = \dfrac{J}{2} \displaystyle\sum_{n=2}^{N-1} \sum_{1\leq k<\ell\leq n} Z_k Z_\ell, \\ H_\pm = \dfrac{1}{2} \displaystyle\sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta\right) \left[X_n X_{n+1} + Y_n Y_{n+1}\right], \\ H_Z = \dfrac{m\cos\theta}{2} \displaystyle\sum_{n=1}^{N} (-1)^n Z_n - \frac{J}{2} \sum_{n=1}^{N-1} (n \bmod 2) \sum_{\ell=1}^{n} Z_\ell \end{cases}$$

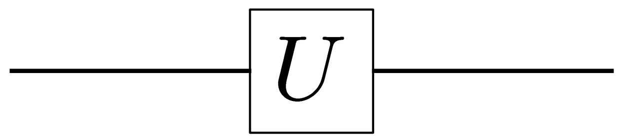
Qubit description of the Schwinger model !!

Contents

“Rule” of Quantum Computation

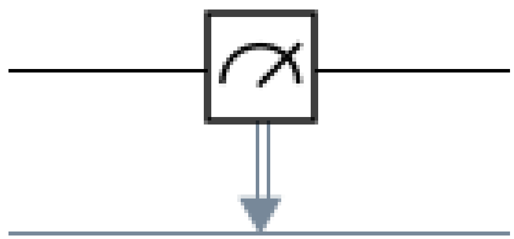
Do something interesting by combining the following 2 operations:

- **Action of unitary operator:** $|\psi\rangle \rightarrow U|\psi\rangle$

$|\psi\rangle$  $U|\psi\rangle$

- **Measurement:**

$|\psi\rangle$
$= \alpha|0\rangle + \beta|1\rangle$



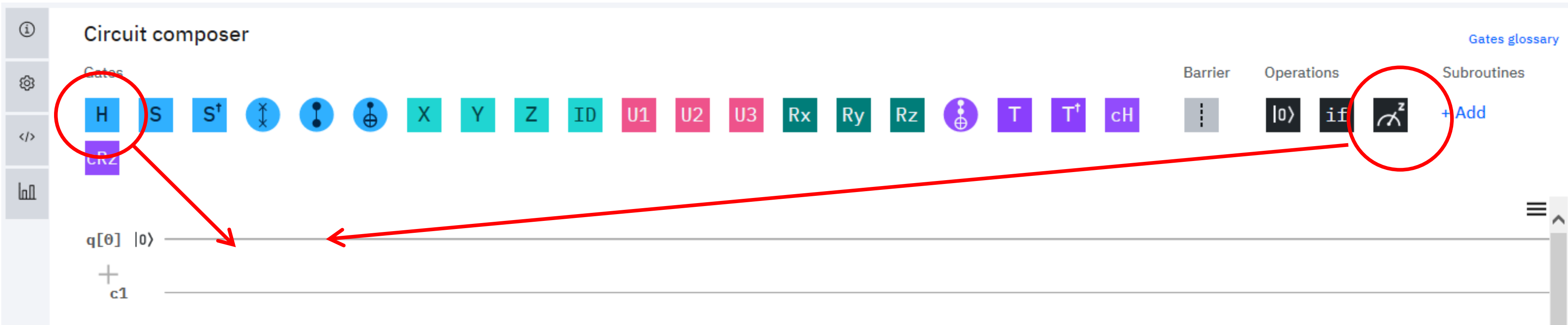
c (classical number)

$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

Atmosphere (?) of using quantum computer…

Suppose we'd like to measure the state: $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Screenshot of IBM Quantum Experience:



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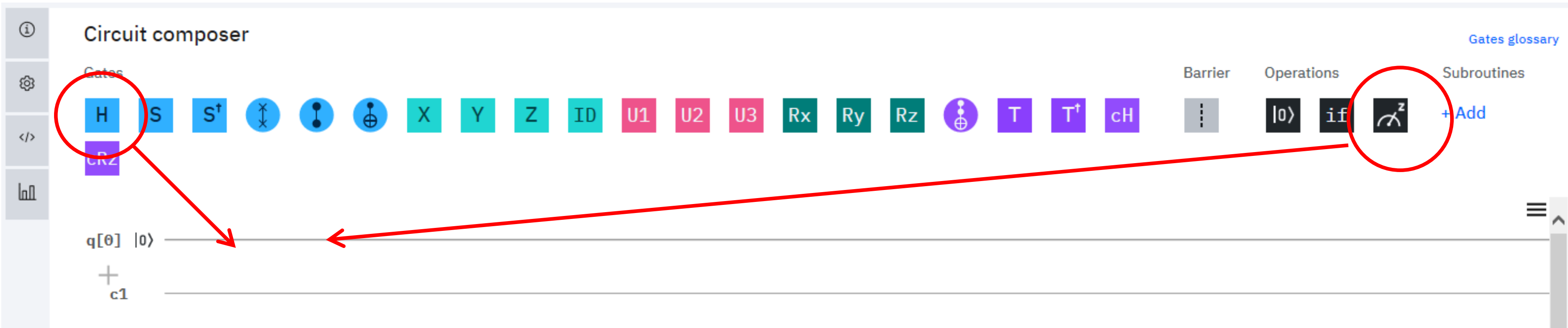
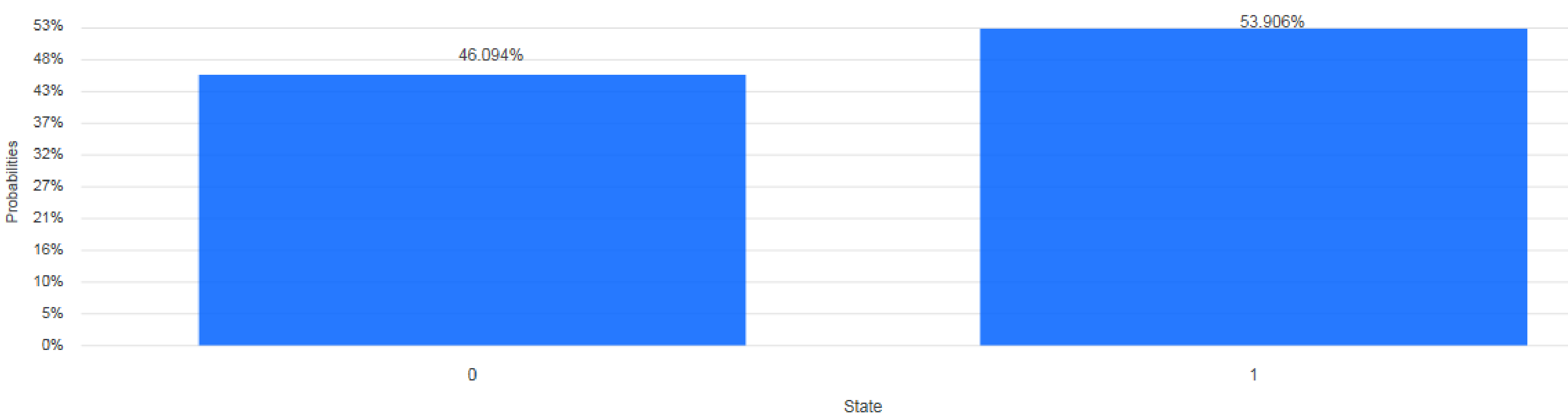


Output of 1024 times measurements ("shots") :



Idea: express physical quantities in terms of "probabilities"
& measure the "probabilities"

VEV of mass operator (chiral condensation)

$$\langle \bar{\psi}(x)\psi(x)\rangle = \langle \mathrm{vac}|\bar{\psi}(x)\psi(x)|\mathrm{vac}\rangle$$

Instead of the local op., we analyze the average over the space:

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle$$

Once we get the vacuum, we can compute the VEV as

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle = \frac{1}{2Na}\sum_{n=1}^{N}(-1)^n \sum_{i_1\cdots i_N=0,1}\langle \mathrm{vac}|Z_n|i_1\cdots i_N\rangle\langle i_1\cdots i_N|\mathrm{vac}\rangle$$

$$= \frac{1}{2Na}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}(-1)^{n+i_n}\,|\langle i_1\cdots i_N|\mathrm{vac}\rangle|^2$$

How can we obtain the vacuum?

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|vac_0\rangle$ is known and unique

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Step 2: Consider the time evolution

$$\mathcal{T}\exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0> \quad \text{w/} \quad H_A(0)=H_0,\ H_A(T)=\hat{H}$$

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Step 3: Use the adiabatic theorem

If the system w/ the Hamiltonian $H_A(t)$ has a unique gapped vacuum, then the desired ground state is obtained by

$$|\mathrm{vac}>=\lim_{T\to\infty}\mathcal{T}\exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0>$$

Adiabatic state preparation of vacuum (Cont'd)

$$|\mathrm{vac}> = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0>$$

$$\simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\mathrm{vac}_0>$$

$$\left(U(t) = e^{-iH_A(t)\delta t}\right)$$

Here we choose

$$\begin{cases} H_0 = H_{ZZ} + H_Z|_{m\to m_0, \theta\to 0} \quad \Rightarrow \quad |\mathrm{vac}_0\rangle = |0101\cdots 01\rangle \\ H_A(t) = \hat{H}\Big|_{w\to w(t), \theta\to\theta(t), m\to m(t)} \\ w(t) = \frac{t}{T}w,\ \theta(t) = \frac{t}{T}\theta,\ m(t) = \left(1-\frac{t}{T}\right)m_0 + \frac{t}{T}m \end{cases}$$

m_0 can be any positive number in principle

but it is practically chosen to have small systematic error

Time evolution operator

Suzuki-Trotter decomposition: (more precisely, we actually use its improvement but I skip it)

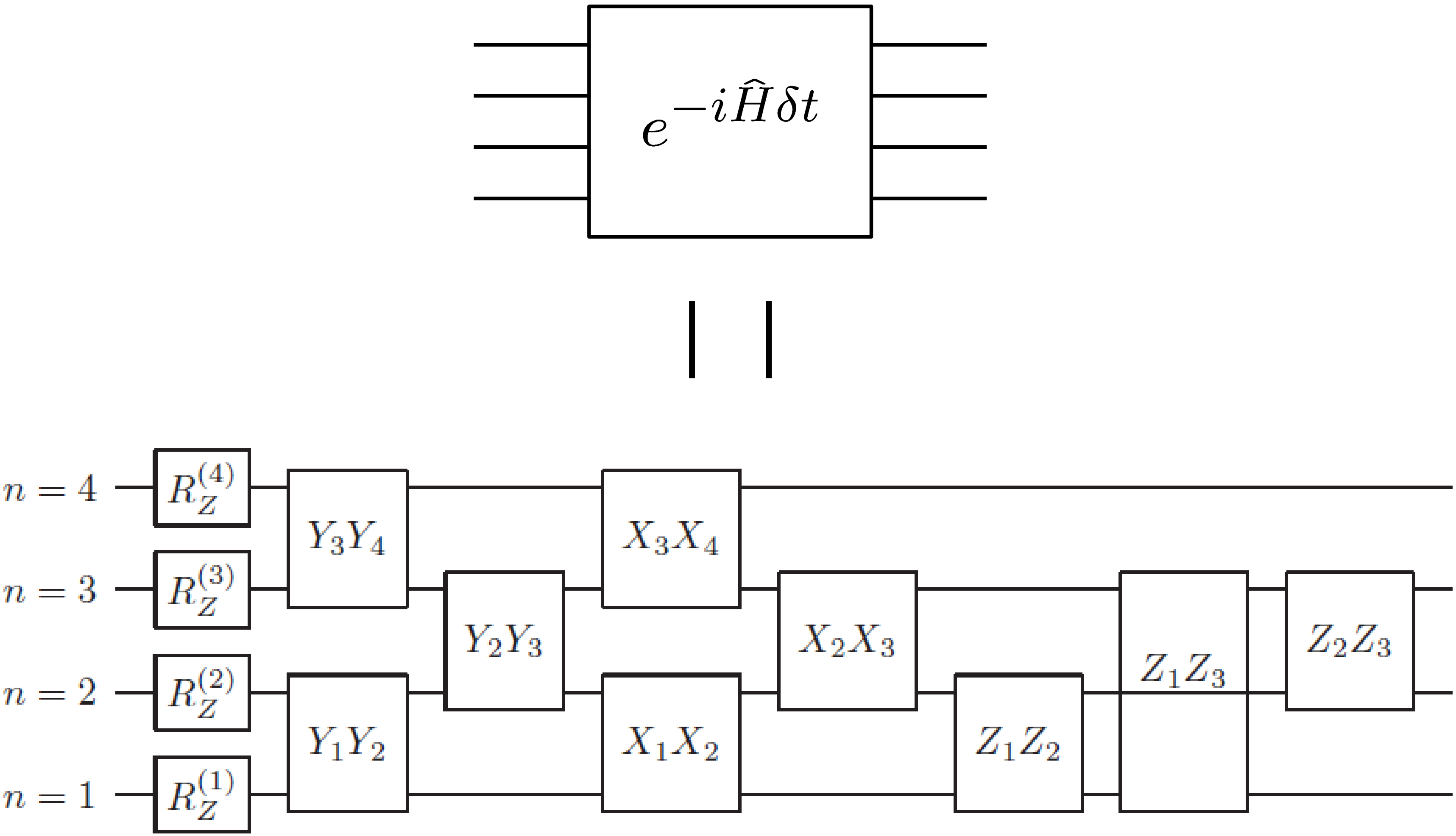
$$e^{-i\hat{H}t} = \left(e^{-i\hat{H}\frac{t}{M}}\right)^M \qquad (M \in \boldsymbol{Z}, M \gg 1)$$

$$\simeq \left(e^{-iH_Z\frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}} e^{-iH_{XX}\frac{t}{M}} e^{-iH_{YY}\frac{t}{M}}\right)^M + \mathcal{O}(1/M)$$

$$\begin{cases} H_Z = \dfrac{m\cos\theta}{2}\displaystyle\sum_{n=1}^{N}(-1)^n Z_n - \dfrac{J}{2}\sum_{n=1}^{N-1}(n \bmod 2)\sum_{\ell=1}^{n} Z_\ell \\ H_{ZZ} = \dfrac{J}{2}\displaystyle\sum_{n=2}^{N-1}\sum_{1\le k<\ell\le n} Z_k Z_\ell, \\ H_{XX} = \dfrac{1}{2}\displaystyle\sum_{n=1}^{N-1}\left(w - (-1)^n\frac{m}{2}\sin\theta\right) X_n X_{n+1} \\ H_{YY} = \dfrac{1}{2}\displaystyle\sum_{n=1}^{N-1}\left(w - (-1)^n\frac{m}{2}\sin\theta\right) Y_n Y_{n+1} \end{cases}$$

These operations can be easily implemented (details skipped)

Quantum circuit for time evolution op. (N=4)



Results on chiral condensate

Skipped contents:

- processes of taking ∞ volume & continuum limits
- how to estimate systematic errors, etc...

(Classical) simulator for Quantum computer

In real quantum computer,

Qubits in quantum circuit ≠ isolated system

→ Interactions w/ environment cause errors

(Classical) simulator for Quantum computer

In real quantum computer,

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Here we use

Simulator = tool to simulate quantum computer by classical computer

- Doesn't have errors → ideal answers

  (More precisely, classical computer also has errors but its error correction is established)

- The same code can be run in quantum computer w/ speed-up

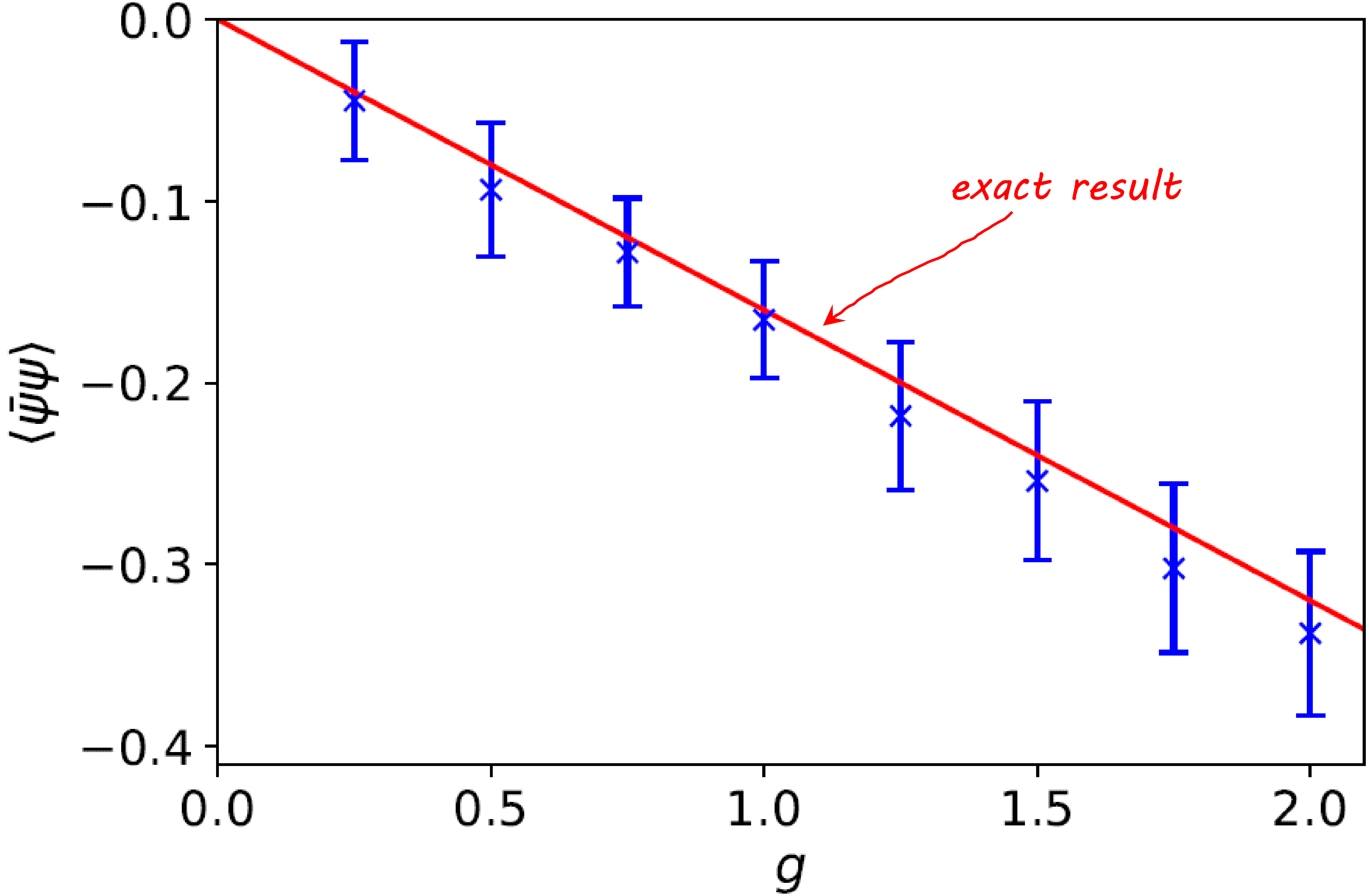
Useful to test algorithm & estimate computational resources

(~# of qubits, gates)

Result for massless case (after continuum limit)

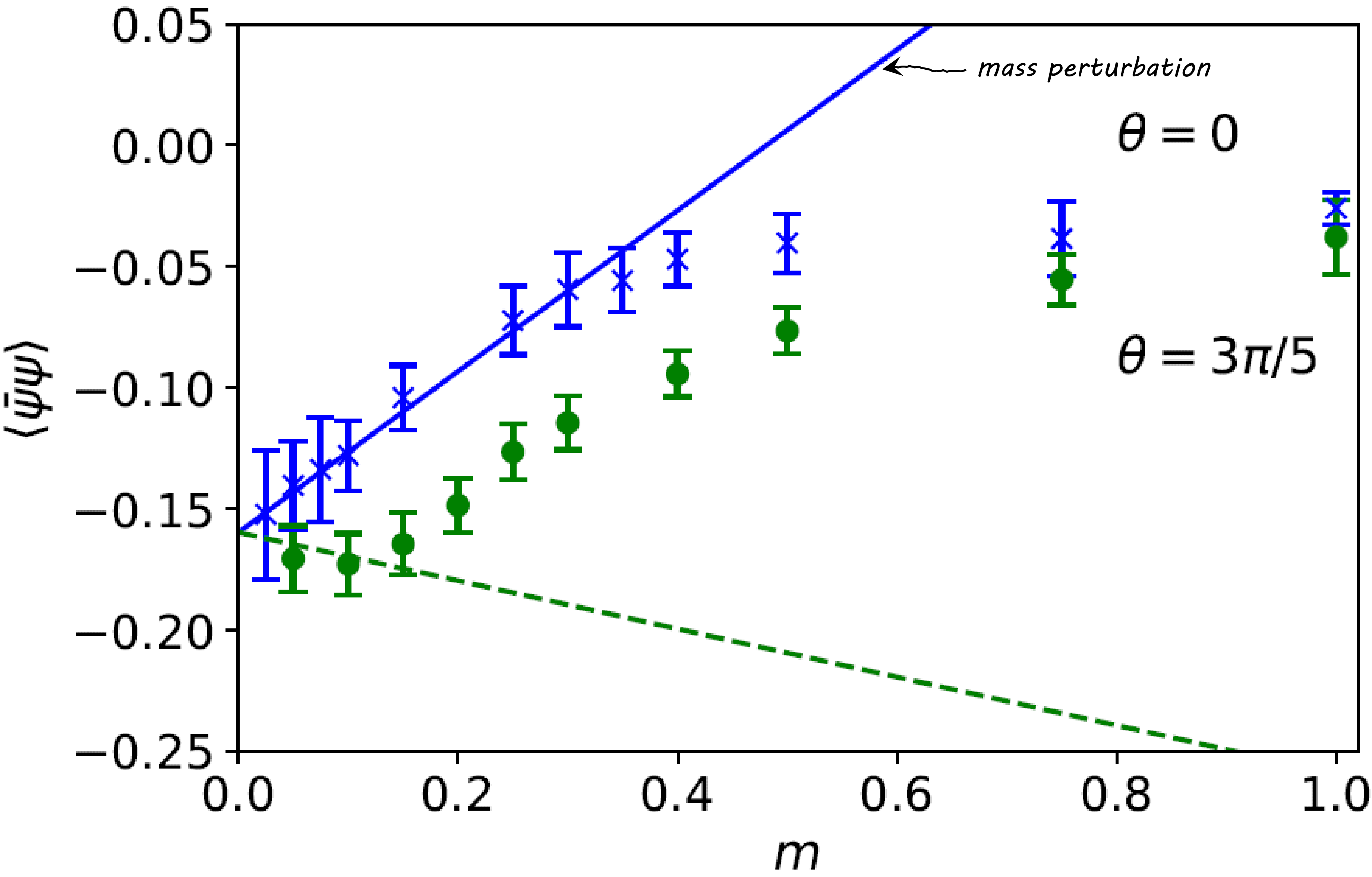
$T = 100, \delta t = 0.1, N_{\text{max}} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

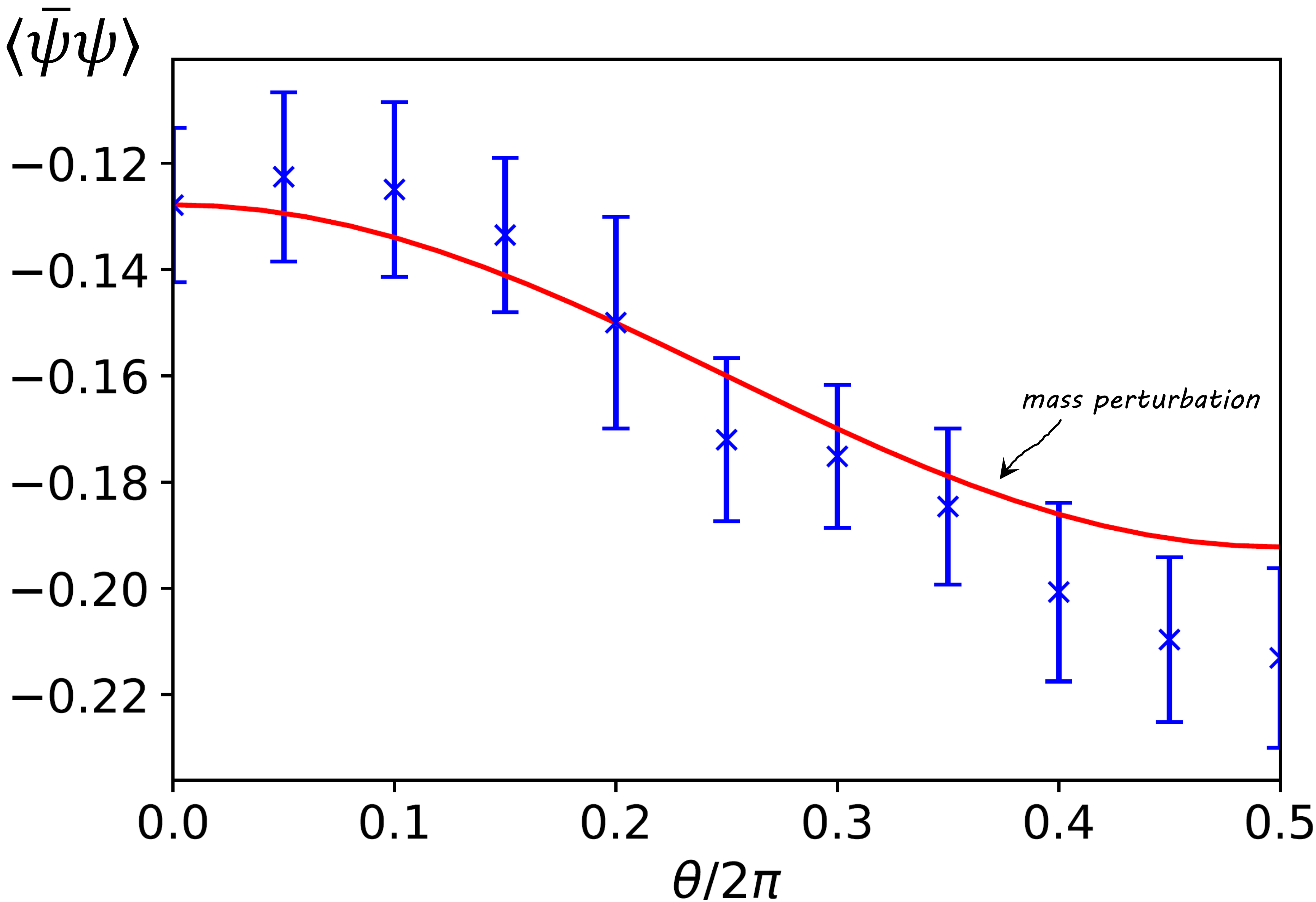


Result for massive case at g=1

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]



θ dependence at $m = 0.1$ & $g = 1$



Contents

Expectations from previous analyzes

Potential between probe charges $\pm q$ has been analytically computed for ∞ volume.

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q^2 g^2}{2\mu}(1 - e^{-\mu x})$$

screening

- massive case:

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- massless case: $\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q^2 g^2}{2\mu}(1 - e^{-\mu x})$$ *screening*

- massive case: $\Sigma \equiv e^{1+\gamma}/2\pi^{3/2}$

$$V(x) \sim m\Sigma(1 - \cos(2\pi q))\, x$$ $(\mathrm{m} \ll g,\ |x| \gg 1/g\ )$

$$\begin{cases} = \text{Const.} & \text{for } q \in \boldsymbol{Z} \quad \textit{screening} \\ \propto x & \text{for } q \notin \boldsymbol{Z} \quad \textit{confinement} \end{cases}$$

Let's explore this aspect by quantum simulation!

Our strategy

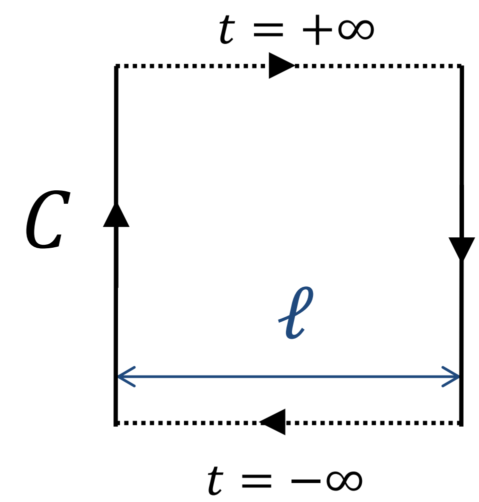
① Introduce the probe charges $\pm$q:

$$e^{iqg\int_C A}$$

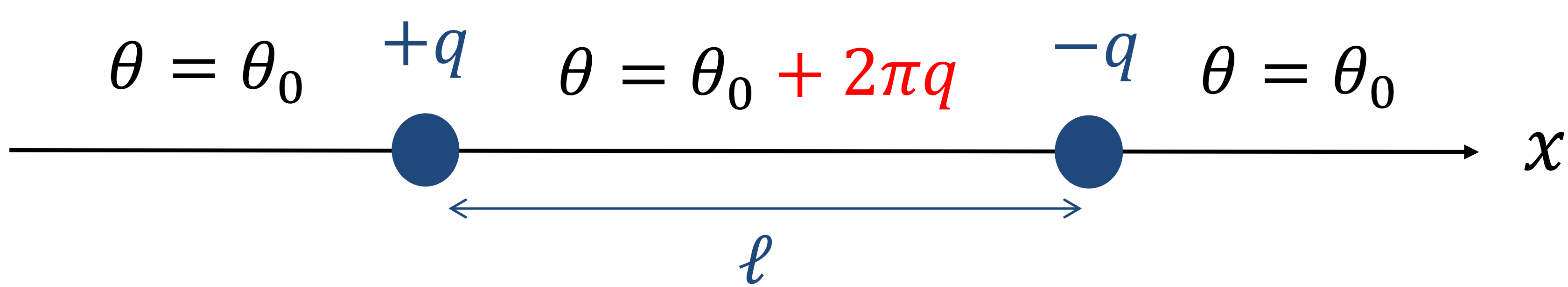
$$||$$

$$e^{iqg\int_{S,\partial S=C} F}$$

local θ-term w/ $\theta = 2\pi q$!!



② Include it to the action & switch to Hamilton formalism

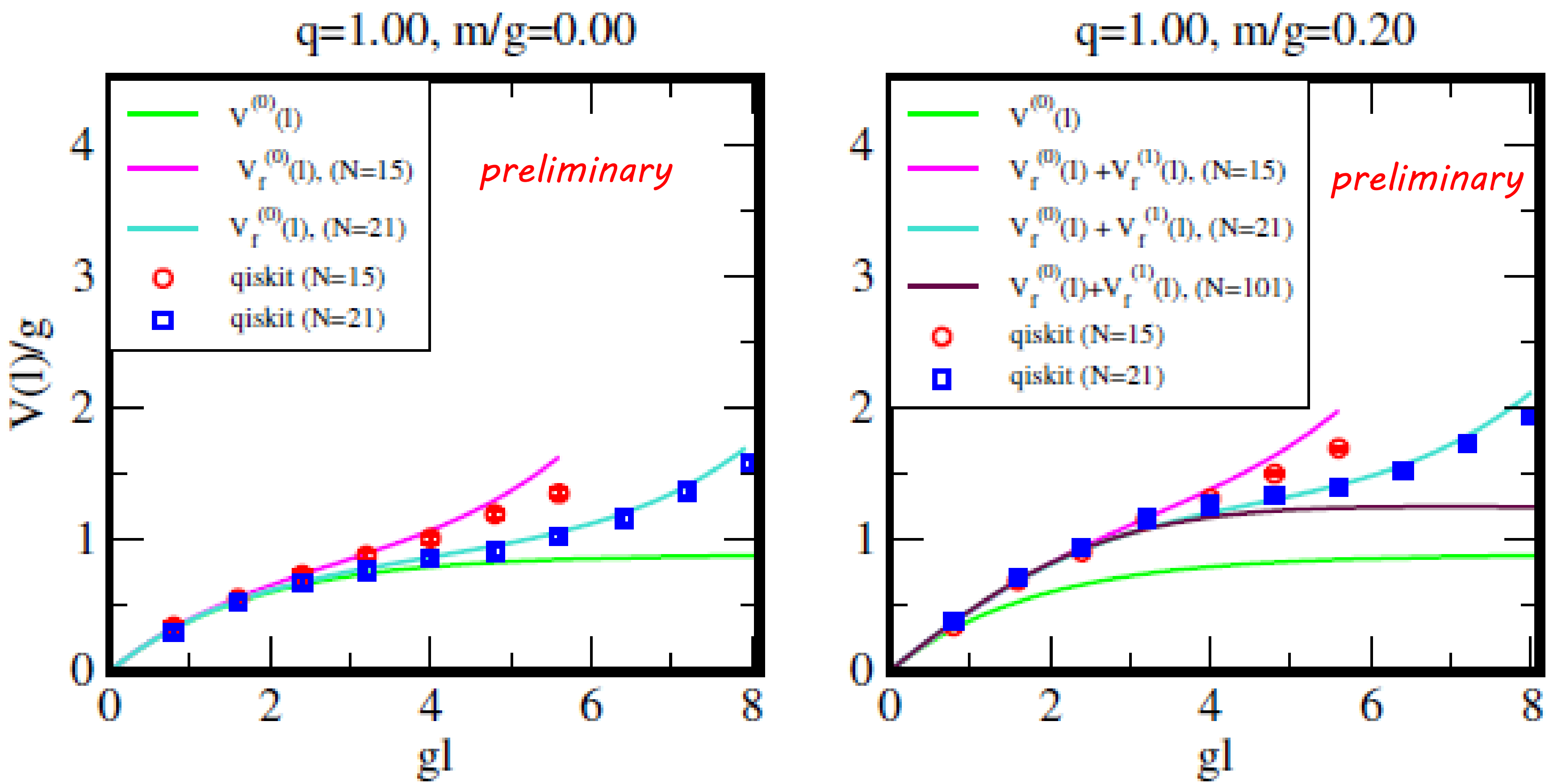


③ Compute the ground state energy (in the presence of the probes)

Results for $\theta_0 = 0$ & $q \in \boldsymbol{Z}$

Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q = 1$

Lines: analytical results in the continuum limit (finite & ∞ vols.)



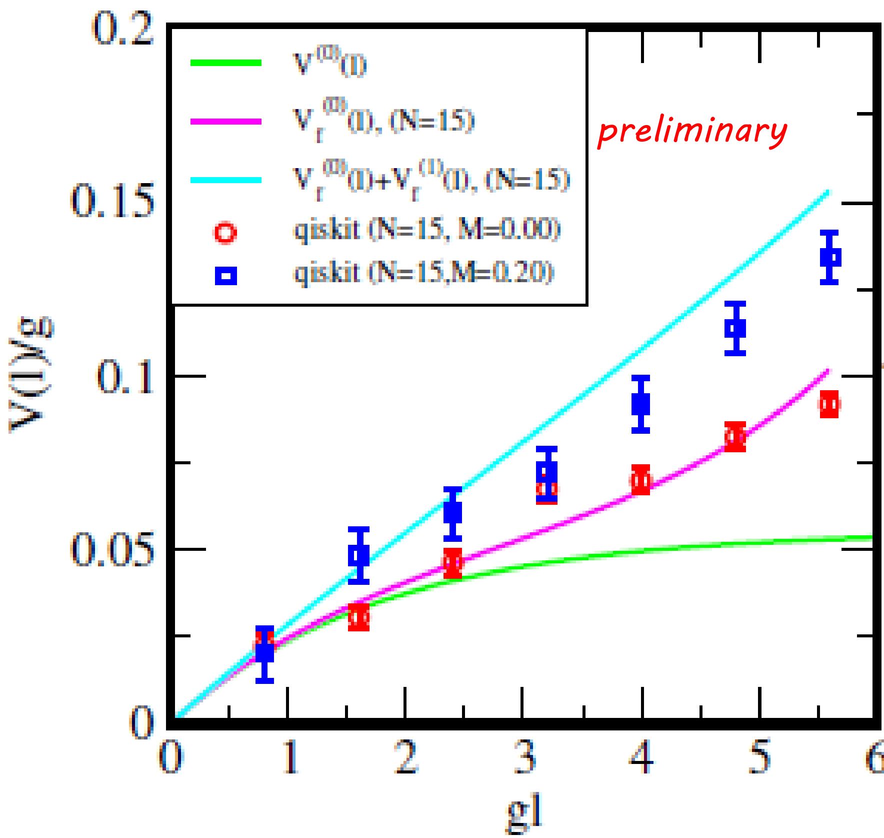
Consistent w/ expected screening behavior

Results for $\theta_0 = 0$ & $q \notin \boldsymbol{Z}$

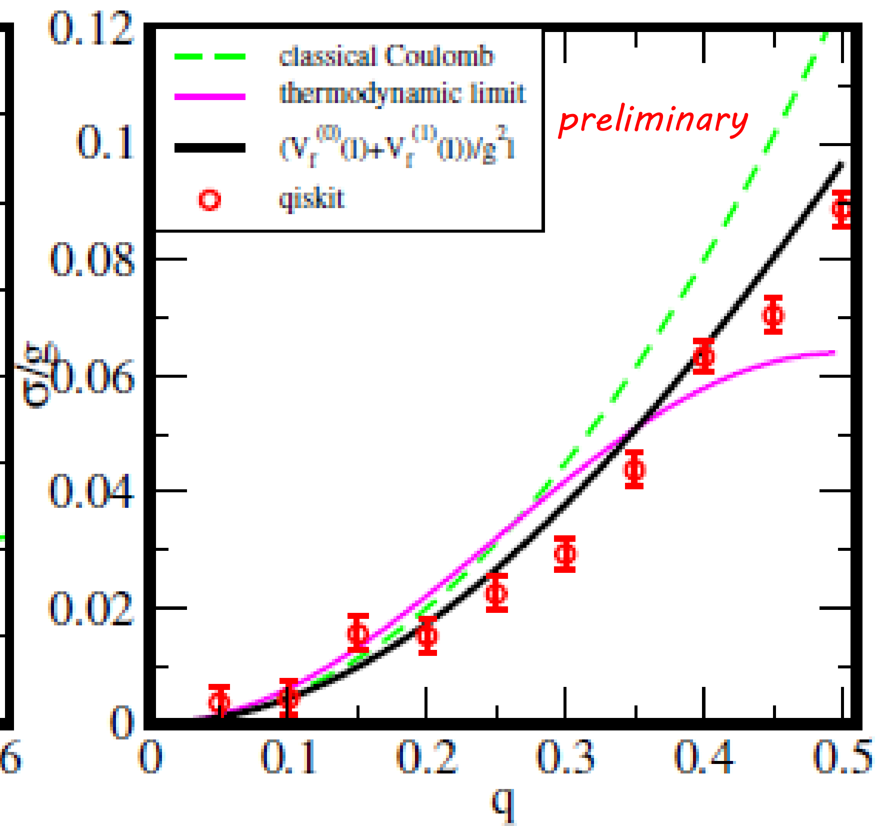
Parameters: $g = 1, a = 0.4, N = 15, T = 99, q = 0.25, m/g = 0$ & 0.2

Lines: analytical results in the continuum limit (finite & ∞ vol.)

"Potential":



Slope for large gl ("string tension"):

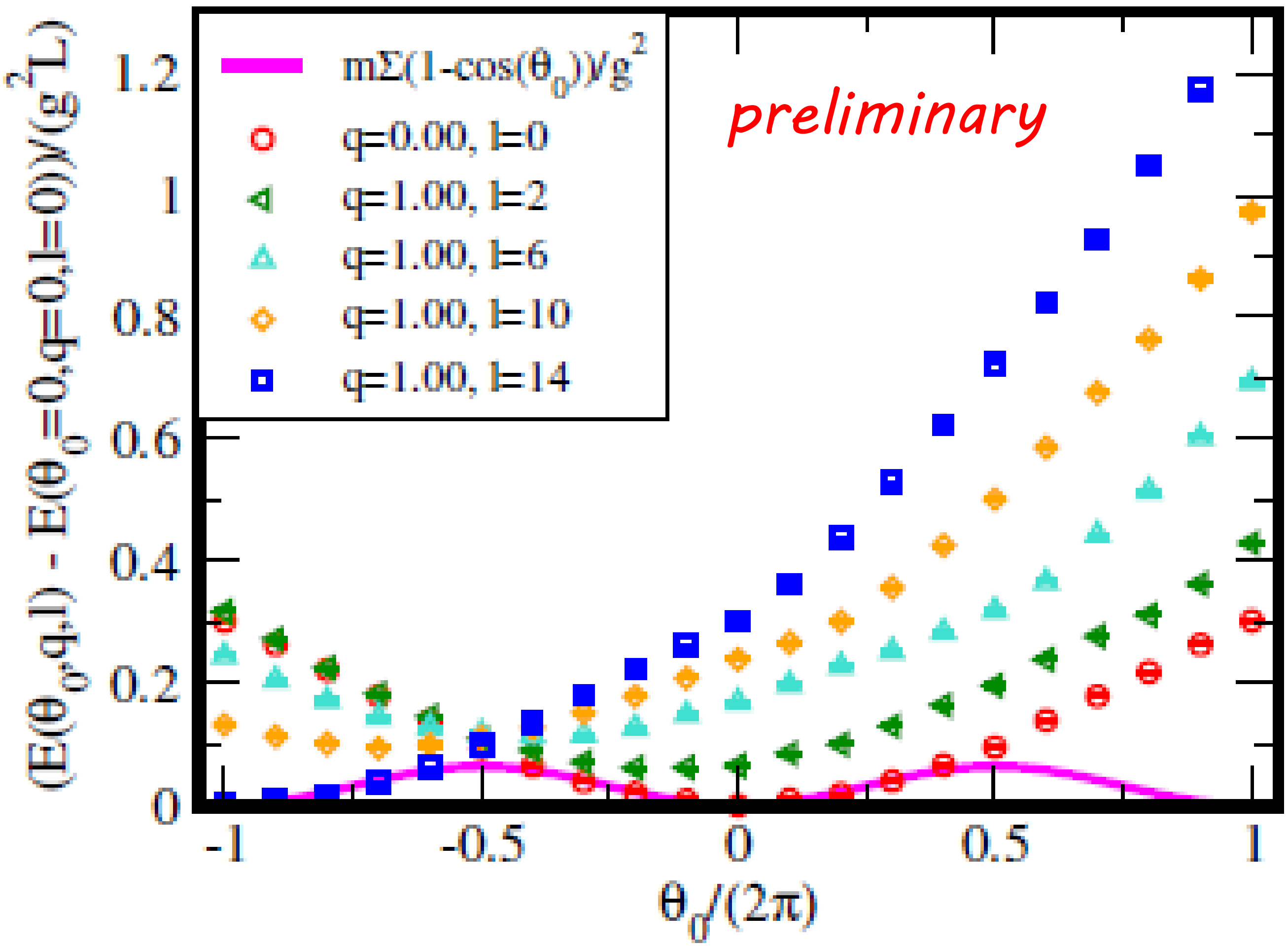


Consistent w/ expected confinement behavior

Results for $\theta_0 \neq 0$

(difficult to explore by the conventional Monte Carlo approach)

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q = 1, m/g = 0.2$



Contents

BMN matrix model ($U(N)$ gauged matrix QM)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k \right.$$
$$\left. + \frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi \right\},$$

- (0+1) dim. $U(N)$ gauge theory
- all the fields are $N \times N$ Hermitian matrices
- X_I: bosonic matrices ($I = 1, \cdots, 9$)
- Ψ: 16 component Majorana-Weyl fermion
- $i = 1,2,3, a = 4, \cdots, 9$

BMN matrix model (cont'd)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k \right.$$
$$\left. + \frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi \right\},$$

related to various interesting "stringy" theories:

- M-theory on pp-wave spacetime
- 3d $\mathcal{N} = 8$ SYM on $\boldsymbol{R} \times S^2$ ~ D2-branes in IIA string theory
- 4d $\mathcal{N} = 4$ SYM on $\boldsymbol{R} \times S^3$ ~ D3-branes in IIB string theory

[Ishii-Ishiki-Shimasaki-Tsuchiya '08, etc…]

- 6d $\mathcal{N} = (2,0)$ theory on $\boldsymbol{R} \times S^5$ ~ M5-branes in M-theory

[Maldacena-Sheikh-Jabbari-Van Raamsdonk '02]

- holographic duals

SUSY QFTs from BMN matrix model

X_i part:

$$L|_{A_t=X_a=\Psi=0} = \mathrm{Tr}\left\{\frac{1}{2}(\partial_t X_i)^2 + \frac{g^2}{4}[X_i, X_j]^2 - \frac{\mu^2}{18}X_i^2 - \frac{i\mu g}{3}\epsilon^{ijk}X_i X_j X_k\right\}$$
$$= \mathrm{Tr}\left\{\frac{1}{2}(\partial_t X_i)^2 + \frac{g^2}{4}\left([X_i, X_j] - \frac{i\mu}{3g}\epsilon_{ijk}X_k\right)^2\right\}.$$

SUSY vacua:

"Fuzzy sphere"

$$X_i = \frac{\mu}{3g}J_i, \qquad [J_i, J_j] = i\epsilon^{ijk}J_k$$

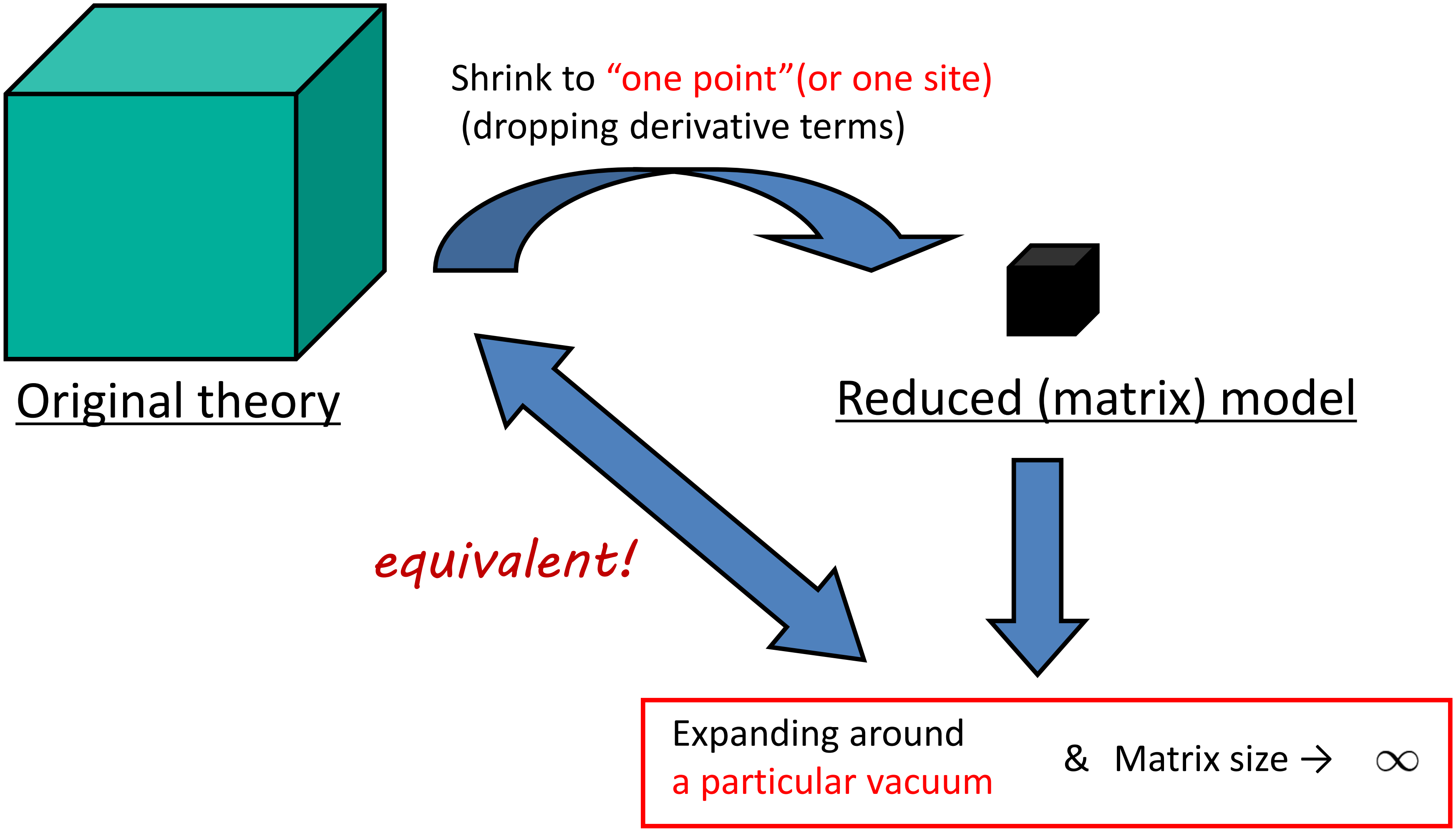
J_i: $SU(2)$ generator in N-dim. (ir)reducible rep.

Expanding the theory around fuzzy sphere sols. w/ appropriate reps., we can obtain SUSY QFTs in the large-N limit via "large-N reduction"

[Maldacena-Sheikh-Jabbari-Van Raamsdonk '02, Ishii-Ishiki-Shimasaki-Tsuchiya '08, etc...]

Concept of Large N reduction

[Eguchi-Kawai, Bhanot-Heller-Neuberger, Gonzalez-Arroyo-Okawa, Gross-Kitazawa, etc.]



Here we apply it only for space and leave time continuous

SUSY QFTs from BMN matrix model

$$X_i = \frac{\mu}{3g} J_i$$ fuzzy sphere

- 3d $\mathcal{N} = 8$ SYM on $\boldsymbol{R} \times S^2$:

($J_i^{(s)}$: $SU(2)$ generator of $2s+1$ dim. representation)

$$J_i = J_i^{(s)} \otimes \mathbf{1}_{N_2} \qquad s = \frac{N_5 - 1}{2},\ N_5 \to \infty$$

- 6d $\mathcal{N} = (2,0)$ theory on $\boldsymbol{R} \times S^5$:

$$J_i = J_i^{(s)} \otimes \mathbf{1}_{N_2} \qquad s = \frac{N_5 - 1}{2},\ N_2 \to \infty$$

- 4d $\mathcal{N} = 4$ SYM on $\boldsymbol{R} \times S^3$:

$$J_i = \bigoplus_{s=n-T}^{n+T} \left(J_i^{(s)} \otimes \mathbf{1}_{\boldsymbol{k}}\right) \qquad k, n, T, n-T \to \infty$$

Hamiltonian formalism

$$\Psi = \begin{pmatrix} \psi_{Ip} \\ \epsilon_{pq}\psi^{\dagger Iq} \end{pmatrix}$$

$$\hat{H} = \mathrm{Tr}\Big\{ \frac{1}{2}(\hat{P}_I)^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2 + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k$$
$$+ g\hat{\psi}^{\dagger Ip}\sigma_p^{iq}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip}g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip}\Big\}$$

Commutation relations:

(α, β: gauge indices)

$$\left[\hat{X}_{I\alpha}, \hat{P}_{J\beta}\right] = i\delta_{IJ}\delta_{\alpha\beta}, \qquad \left\{\hat{\psi}^{\dagger Ip\alpha}, \hat{\psi}^{\beta}_{Jq}\right\} = \delta_{IJ}\delta^{pq}\delta^{\alpha\beta}$$

Gauss law:

$$\hat{G}_\alpha|\mathrm{phys}\rangle = 0 \quad \text{w/} \quad \hat{G}_\alpha = \sum_{\beta,\gamma=1}^{N^2}\left(\sum_{I=1}^{9}\hat{X}_I^\beta\hat{P}_I^\gamma - i\sum_{I,p}\hat{\psi}^{\dagger Ip\alpha}\hat{\psi}^{\gamma}_{Ip}\right)$$

Hilbert space is ∞-dimensional → regularize it!

The essence is common w/ single particle QM

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{\omega^2}{2}\hat{x}^2 + V_{\text{int}}(\hat{x})$$

Most naïve approach = truncation in harmonic osc. basis:

$$\hat{a} = \sqrt{\frac{\omega}{2}}\,\hat{x} + \frac{i}{\sqrt{2\omega}}\,\hat{p} \;=\; \sum_{n=0}^{\infty} \sqrt{n+1}\,|n\rangle\langle n+1|$$

$$\xRightarrow[\textit{regularize!}]{} \quad \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

Then replace $\hat{p}$ & $\hat{x}$ by

$$\hat{x}\Big|_{\text{regularized}} \equiv \frac{1}{\sqrt{2\omega}}(\hat{a}+\hat{a}^\dagger)\Big|_{\text{regularized}}$$

$$\hat{p}\Big|_{\text{regularized}} \equiv \frac{1}{i}\sqrt{\frac{\omega}{2}}\,(\hat{a}-\hat{a}^\dagger)\Big|_{\text{regularized}}$$

The essence is common w/ single particle QM (Cont'd)

$$\hat{a}\Big|_{\text{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

We can rewrite the Fock basis in terms of qubits:

$$|n\rangle = |b_{K-1}\rangle|b_{K-2}\rangle\cdots|b_0\rangle \qquad K \equiv \log_2 \Lambda$$

$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Then,

$$|n\rangle\langle n+1| = \otimes_{\ell=0}^{K-1} \underbrace{(|b'_\ell\rangle\langle b_\ell|)}_{\text{either one of}}$$

$$\left(\begin{array}{ll} |0\rangle\langle 0| = \dfrac{1_2 - \sigma_z}{2}, & |1\rangle\langle 1| = \dfrac{1_2 + \sigma_z}{2}, \\ |0\rangle\langle 1| = \dfrac{\sigma_x + i\sigma_y}{2}, & |1\rangle\langle 0| = \dfrac{\sigma_x - i\sigma_y}{2} \end{array}\right)$$

The essence is common w/ single particle QM (Cont'd)

The ground state of the truncated system can be constructed by e.g. adiabatic state preparation:

$$\hat{H}_A(t) = \frac{1}{2}\hat{p}^2 + \frac{\omega^2}{2}\hat{x}^2 + \frac{t}{T}V_{\text{int}}(\hat{x})$$

$$|\text{vac}> = \lim_{T\to\infty} \mathcal{T}\exp\left(-i\int_0^T dt\ H_A(t)\right)\underbrace{|\text{vac}_0>}_{=|0\rangle}$$

The BMN model has much more variables

but we can regularize it in essentially the same way

Preparation of fuzzy sphere state

To get the SUSY QFTs., we need to construct states corresponding to the fuzzy spheres at finite coupling

Steps:

① Expand the theory around the fuzzy sphere

② Take its Fock vacuum at weak coupling limit

$$|J_i\rangle_{g\to 0}$$

③ Starting w/ the Fock vacuum, adiabatically turn on the coupling & apply the adiabatic time evolution

$$|J_i\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\ H_A(t)\right) |J_i\rangle_{g\to 0}$$

Computational costs

of qubits:

- Single particle QM w/ truncation Λ requires $\log_2 \Lambda$ qubits
- The BMN model has 9 scalars & 16 component real fermion which are $N \times N$ matrices

$$\Rightarrow \quad 9N^2 \log_2 \Lambda + 8N^2 \text{ qubits}$$

Computational costs

of qubits:

- Single particle QM w/ truncation Λ requires $\log_2 \Lambda$ qubits
- The BMN model has 9 scalars & 16 component real fermion which are $N \times N$ matrices

$$\Rightarrow \quad 9N^2 \log_2 \Lambda + 8N^2 \text{ qubits}$$

of spin ops. in Hamiltonian:

- each annihilation/creation op. has less than $\mathcal{O}(\Lambda^2)$ spin ops.
- we have 4-pt. interaction at most
- $^\exists \mathcal{O}(N^4)$ combinations regarding the color indices

$$\Rightarrow \quad < \mathcal{O}(\Lambda^8 N^4) \text{ spin ops.}$$

Possible applications

various real time physics such as

- Testing holography for real time
- Out of time order correlator
- Black hole thermalization
- decay of fuzzy sphere for non-SUSY cases

etc...

Summary & Outlook

Summary

- Quantum computation is suitable for Hamiltonian formalism which is free from sign problem

- Instead we have to deal with huge vector space. Quantum computers in future may do this job.

- We've constructed the vacuum of Schwinger model w/ the topological term by adiabatic state preparation

- found agreement in the chiral condensate with the exact result for $m = 0$ & mass perturbation theory for small m

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

- explored the screening vs confinement problem

[in preparation, MH-Itou-Kikuchi-Nagano-Okuda]

- string/M-theory on quantum computer via BMN model

[Gharibyan-Hanada-MH-Liu '20]

Other progress & Outlook

- Searching critical point [work in progress, Chakraborty-MH-Kikuchi-Izubuchi-Tomiya]
- Other ways to prepare vacuum (e.g. variational method, imaginary time evolution) [work in progress, MH-Kikuchi-Rendon]
- Finite temperature & Real time?
- Scattering amplitude?
- Alternative way to put gauge theory on quantum computer using matrix QM via "orbifold lattice" [Buser-Gharibyan-Hanada-MH-Liu '20]
- Simulation of matrix QM
- Including quantum error correction/mitigation?

Thanks!

Appendix

Basics of quantum computation

Qubit = Quantum Bit

Qubit = Quantum system w/ 2 dim. Hilbert space

Basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Generic state:

$$\alpha|0\rangle + \beta|1\rangle \qquad \text{w/} \quad |\alpha|^2 + |\beta|^2 = 1$$

Ex.) Spin 1/2 system:

$$|0\rangle = |\uparrow\rangle, \qquad |1\rangle = |\downarrow\rangle$$

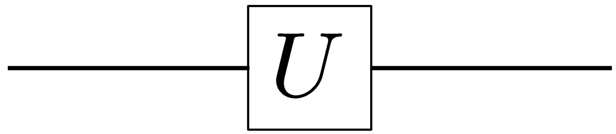
(We don't need to mind how it is realized as "users")

Single qubit operations

- **Acting unitary operator:** $|\psi\rangle \;\rightarrow\; U|\psi\rangle$ (multiplying 2x2 unitary matrix)

  In quantum circuit notation,

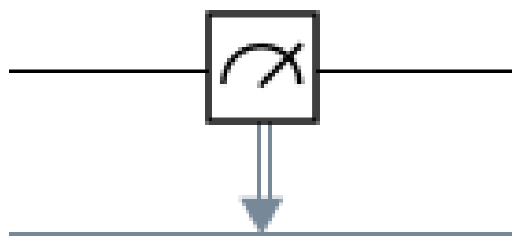
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$U|\psi\rangle = \alpha U|0\rangle + \beta U|1\rangle$$

- **Measurement:**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



c (classical number)

$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

Single qubit gates used here

X, Y, Z gates : (just Pauli matrices)

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

X is "NOT": $X|0\rangle = |1\rangle, \; X|1\rangle = |0\rangle$

R_X, R_Y, R_Z gates :

$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \quad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \quad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

Multiple qubits

2 qubits – 4 dim. Hilbert space:

$$|\psi\rangle = \sum_{i,j=0,1} c_{ij}|ij\rangle, \qquad |ij\rangle \equiv |i\rangle \otimes |j\rangle$$

$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \quad |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

N qubits – 2^N dim. Hilbert space:

$$|\psi\rangle = \sum_{i_1,\cdots i_N=0,1} c_{i_1\cdots i_N}|i_1\cdots i_N\rangle,$$

$$|i_1 i_2\cdots i_N\rangle \equiv |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

Only one 2-qubit gate is used here

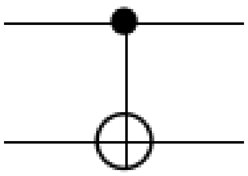
Controlled X (NOT) gate:

$$\begin{cases} CX|00\rangle = |00\rangle, & CX|01\rangle = |01\rangle, \\ CX|10\rangle = |11\rangle, & CX|11\rangle = |10\rangle \end{cases}$$

or equivalently

$$CX|0\rangle \otimes |\psi\rangle = |0\rangle \otimes |\psi\rangle, \qquad CX|1\rangle \otimes |\psi\rangle = |1\rangle \otimes X|\psi\rangle$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



Schwinger model as qubits

Time evolution operator

Suzuki-Trotter decomposition:

(more precisely, we actually use its improvement but I skip it)

$$e^{-i\widehat{H}t} = \left(e^{-i\widehat{H}\frac{t}{M}}\right)^M \qquad (M \in \boldsymbol{Z}, M \gg 1)$$

$$\simeq \left(e^{-iH_Z\frac{t}{M}}e^{-iH_{ZZ}\frac{t}{M}}e^{-iH_{XX}\frac{t}{M}}e^{-iH_{YY}\frac{t}{M}}\right)^M + \mathcal{O}(1/M)$$

$$\left\{\begin{aligned}
H_Z &= \frac{m\cos\theta}{2}\sum_{n=1}^{N}(-1)^n Z_n - \frac{J}{2}\sum_{n=1}^{N-1}(n \bmod 2)\sum_{\ell=1}^{n} Z_\ell \\
H_{ZZ} &= \frac{J}{2}\sum_{n=2}^{N-1}\sum_{1\le k<\ell\le n} Z_k Z_\ell, \\
H_{XX} &= \frac{1}{2}\sum_{n=1}^{N-1}\left(w - (-1)^n\frac{m}{2}\sin\theta\right) X_n X_{n+1} \\
H_{YY} &= \frac{1}{2}\sum_{n=1}^{N-1}\left(w - (-1)^n\frac{m}{2}\sin\theta\right) Y_n Y_{n+1}
\end{aligned}\right.$$

Can we express it in terms of elementary gates?

Time evolution operator (cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_Z\frac{t}{M}}e^{-iH_{ZZ}\frac{t}{M}}e^{-iH_{XX}\frac{t}{M}}e^{-iH_{YY}\frac{t}{M}}\right)^M$$

- The 1st one is trivial:

$$e^{-icZ} = R_Z(2c)$$

- For the others, use the identities: (proof skipped)

$$\left\{\begin{array}{l} e^{-icZ_1Z_2} = CX R_Z^{(2)}(2c) CX \\ e^{-icX_1X_2} = CX R_X^{(1)}(2c) CX \\ e^{-icY_1Y_2} = R_Z^{(1)}\left(-\frac{\pi}{2}\right) R_Z^{(2)}\left(-\frac{\pi}{2}\right) e^{-icX_1X_2} R_Z^{(2)}\left(\frac{\pi}{2}\right) R_Z^{(1)}\left(\frac{\pi}{2}\right) \end{array}\right.$$

Only elementary gates !!

Time evolution operator (Cont'd)

$$e^{-icZ_1Z_2} = CXR_Z^{(2)}(2c)CX$$

Proof:

$$CXR_Z^{(2)}(2c)CX|0\rangle \otimes |\psi\rangle$$

$$= CXR_Z^{(2)}(2c)|0\rangle \otimes |\psi\rangle = CX|0\rangle \otimes R_Z(2c)|\psi\rangle$$

$$= |0\rangle \otimes R_Z(2c)|\psi\rangle = \cos c|0\rangle \otimes |\psi\rangle - i\sin c\ Z|0\rangle \otimes Z|\psi\rangle$$

$$CXR_Z^{(2)}(2c)CX|1\rangle \otimes |\psi\rangle$$

$$= CXR_Z^{(2)}(2c)|1\rangle \otimes X|\psi\rangle = CX|1\rangle \otimes R_Z(2c)X|\psi\rangle = |1\rangle \otimes XR_Z(2c)X|\psi\rangle$$

$$= \cos c|1\rangle \otimes XX|\psi\rangle - i\sin c\ |1\rangle \otimes XZX|\psi\rangle$$

$$= \cos c|1\rangle \otimes |\psi\rangle - i\sin c\ Z|1\rangle \otimes Z|\psi\rangle$$

Thus,

$$CXR_Z^{(2)}(2c)CX|\varphi\rangle \otimes |\psi\rangle = \cos c|\varphi\rangle \otimes |\psi\rangle - i\sin c\ Z|\varphi\rangle \otimes Z|\psi\rangle$$

$$= e^{-icZ_1Z_2}|\varphi\rangle \otimes |\psi\rangle$$

Time evolution operator (Cont'd)

$$e^{-icX_1X_2} = CXR_X^{(1)}(2c)CX$$

Proof:

$$CXR_X^{(1)}(2c)CX|0\rangle \otimes |\psi\rangle$$

$$= CXR_X^{(1)}(2c)|0\rangle\otimes|\psi\rangle = CX\left[\cos c|0\rangle\otimes|\psi\rangle - i\sin c|1\rangle\otimes|\psi\rangle\right]$$

$$= \cos c|0\rangle\otimes|\psi\rangle - i\sin c|1\rangle\otimes X|\psi\rangle = \cos c|0\rangle\otimes|\psi\rangle - i\sin c\, X|0\rangle\otimes X|\psi\rangle$$

$$CXR_X^{(1)}(2c)CX|1\rangle \otimes |\psi\rangle$$

$$= CXR_X^{(1)}(2c)|1\rangle\otimes X|\psi\rangle = CX\left[\cos c|1\rangle\otimes X|\psi\rangle - i\sin c|0\rangle\otimes X|\psi\rangle\right]$$

$$= \cos c|1\rangle\otimes|\psi\rangle - i\sin c|0\rangle\otimes X|\psi\rangle = \cos c|1\rangle\otimes|\psi\rangle - i\sin c\, X|1\rangle\otimes X|\psi\rangle$$

Thus,

$$CXR_X^{(1)}(2c)CX|\varphi\rangle\otimes|\psi\rangle = \cos c|\varphi\rangle\otimes|\psi\rangle - i\sin c\, X|\varphi\rangle\otimes X|\psi\rangle$$

$$= e^{-icX_1X_2}|\varphi\rangle \otimes |\psi\rangle$$

Improvement of Suzuki-Trotter decomposition

The leading order decomposition:

$$e^{-i(H_1+H_2)\delta t} = e^{-iH_1\delta t}e^{-iH_2\delta t} + \mathcal{O}(\delta t^2)$$

The 2nd order improvement:

$$e^{-i(H_1+H_2)\delta t} = e^{-iH_1\frac{\delta t}{2}}e^{-iH_2\delta t}e^{-iH_1\frac{\delta t}{2}} + \mathcal{O}(\delta t^3)$$

(cf. Baker-Campbell-Hausdorff formula:

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{12}[A,[A,B]]+\cdots}$$
)

This increases the number of gates at each time step
but we can take larger δt (smaller M) to achieve similar accuracy.
Totally we save the number of gates.

Details on chiral condensate

Estimation of systematic errors

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

Approximation of vacuum:

$$|\text{vac}> \simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\text{vac}_0> \equiv |\text{vac}_A\rangle$$

Approximation of VEV:

$$\langle\mathcal{O}\rangle \equiv \langle\text{vac}|\mathcal{O}|\text{vac}\rangle \simeq \langle\text{vac}_A|\mathcal{O}|\text{vac}_A\rangle$$

Introduce the quantity

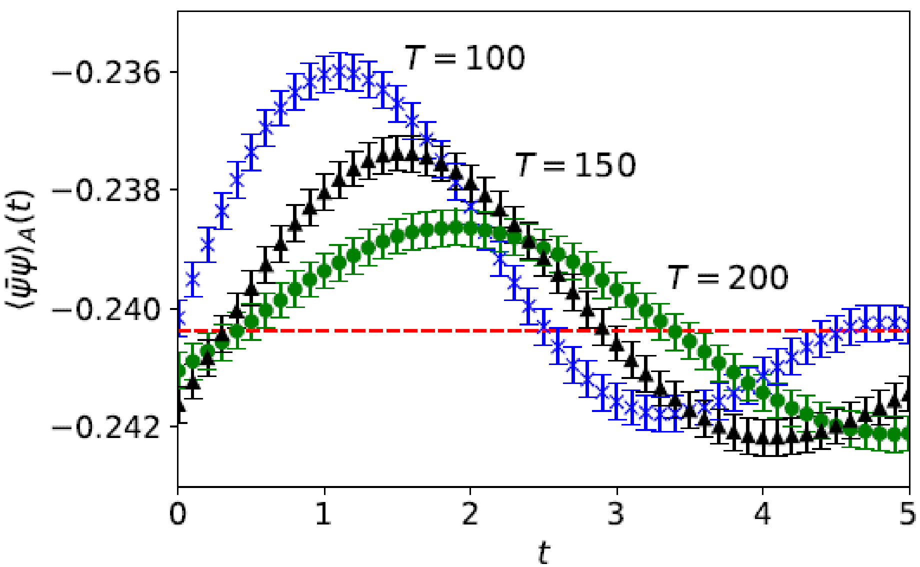
$$\langle\mathcal{O}\rangle_A(t) \equiv \langle\text{vac}_A|e^{i\hat{H}t}\mathcal{O}e^{-i\hat{H}t}|\text{vac}_A\rangle$$

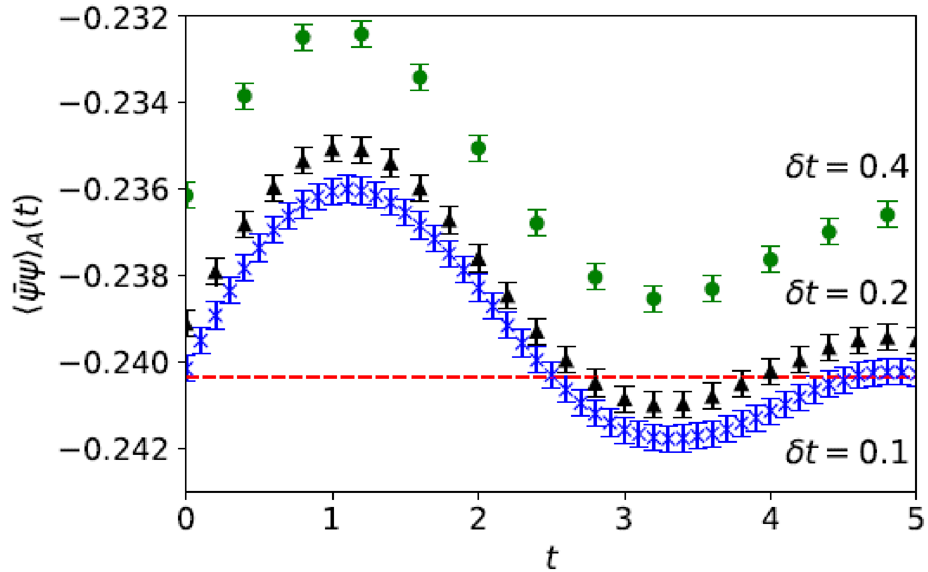
$$\begin{cases} \text{independent of t if } |\text{vac}_A\rangle = |\text{vac}\rangle \\ \text{dependent on t if } |\text{vac}_A\rangle \neq |\text{vac}\rangle \end{cases}$$

This quantity describes intrinsic ambiguities in prediction

→ Useful to estimate systematic errors

Estimation of systematic errors (Cont'd)





Oscillating around the correct value

➡ Define central value & error as

$$\frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)+\min\langle\mathcal{O}\rangle_A(t)\right) \quad \& \quad \frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)-\min\langle\mathcal{O}\rangle_A(t)\right)$$

Massless case

For massless case,

θ is absorbed by chiral rotation $\Rightarrow$ $\theta = 0$ w/o loss of generality

No sign problem

Nevertheless,

it's difficult in conventional approach because computation of fermion determinant becomes very heavy

$^\exists$ Exact result:

[Hetrick-Hosotani '88]

$$\langle \bar{\psi}(x)\psi(x)\rangle = -\frac{e^{\gamma}}{2\pi^{3/2}} g \simeq -0.160 g$$

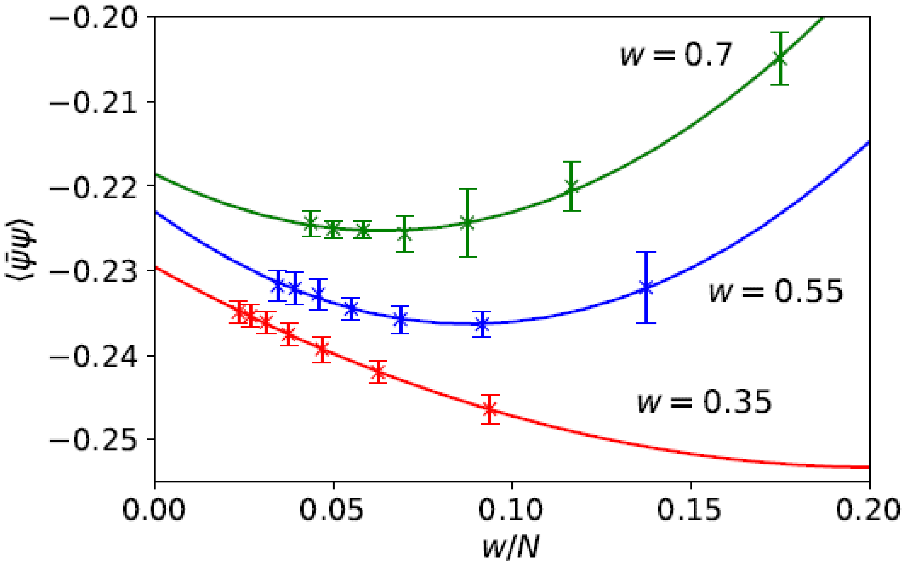
Can we reproduce it?

Thermodynamic & Continuum limit

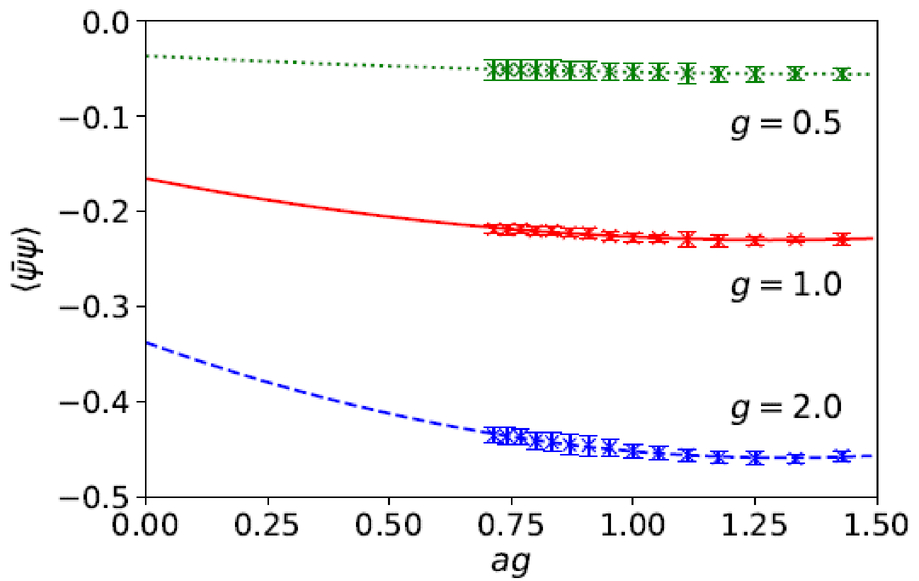
$g = 1, m = 0, N_{\text{max}} = 16, T = 100, \delta t = 0.1, 1M$ shots

#(measurements)

Thermodynamic limit (w/ fixed a)



Continuum limit (after $V \to \infty$)



Massive case

Result of mass perturbation theory: [Adam '98]

$$\langle \bar{\psi}(x)\psi(x) \rangle \simeq -0.160g + 0.322m \cos\theta + \mathcal{O}(m^2)$$

However,

∃ subtlety in comparison: this quantity is UV divergent

$(\sim m \log \Lambda)$

⇒ Use a regularization scheme to have the same finite part

Here we subtract free theory result before taking continuum limit:

$$\lim_{a \to 0} \left[\langle \bar{\psi}\psi \rangle - \langle \bar{\psi}\psi \rangle_{\text{free}} \right]$$