Soft-Hair Symmetries at Horizons

"Recent Progress in Theoretical Physics based on Quantum Information Theory"
YITP workshop 2021 March 1-5

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Based on

M. Hotta, J. Trevison and K. Yamaguchi, Phys. Rev. D94, 083001 (2016).
T. Tomitsuka, K. Yamaguchi and M. Hotta, arXiv:2012.14050.

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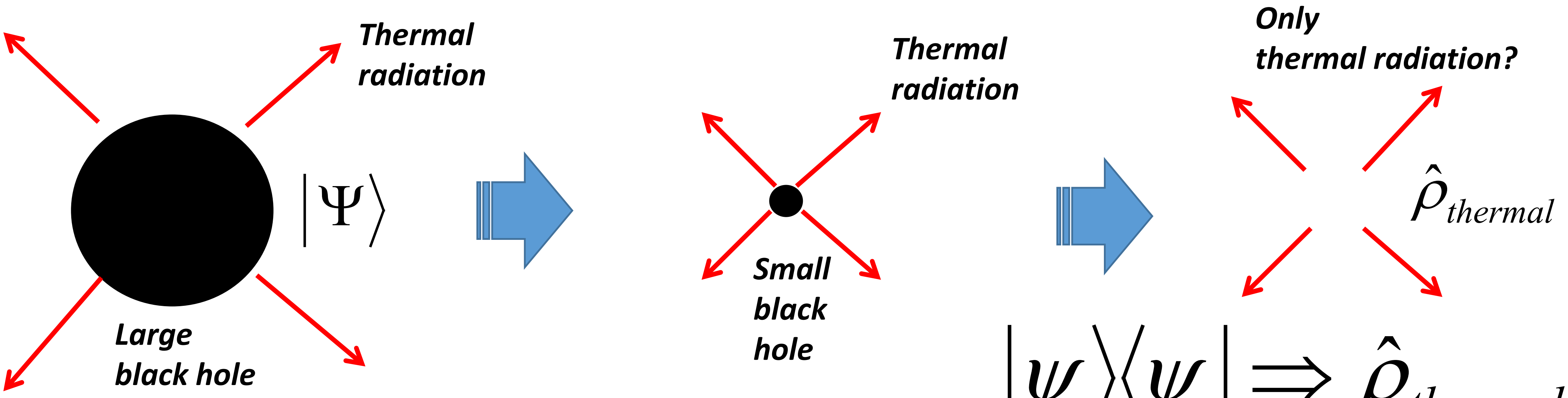
Introduction

Black hole physics has a very close relation to quantum information.

○ Black Hole Information Loss Problem *(S.W. Hawking, 1976)*

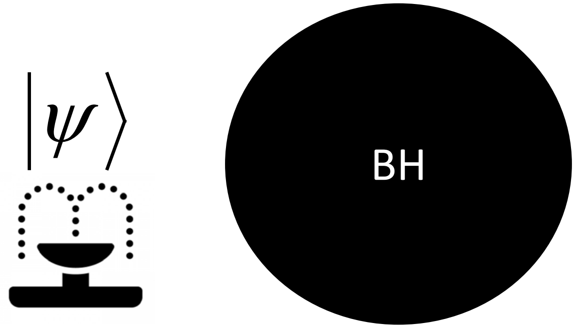


$$|\psi\rangle\langle\psi| \Rightarrow \hat{\rho}_{thermal} \ ?$$

Tough obstacle: Release of an astrophysical amount of quantum information inside of a horizon using small energy of Planck scale order (almost zero energy) at the last burst of a black hole.

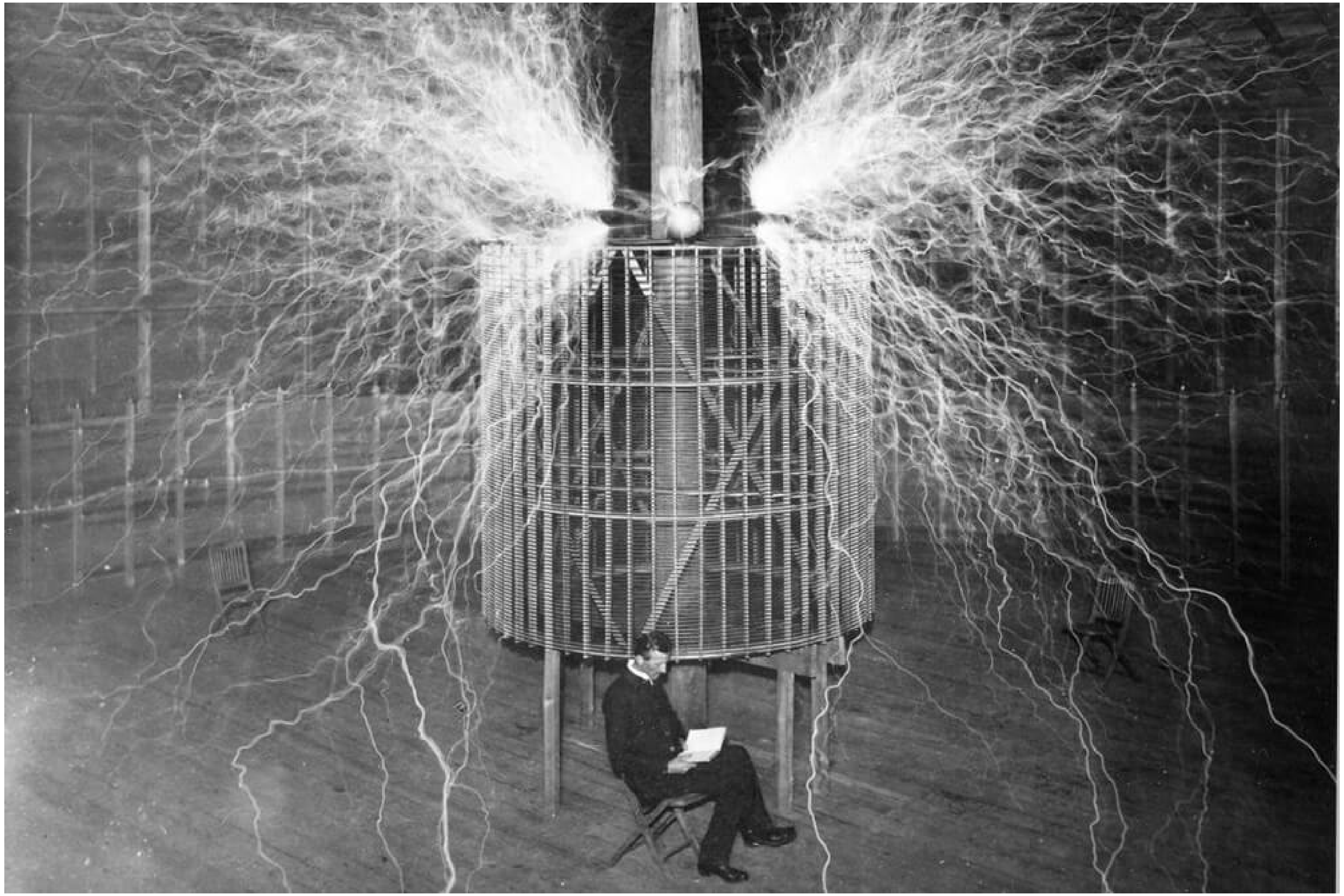
cf. island conjecture.

⇒ information fountain outside horizon?



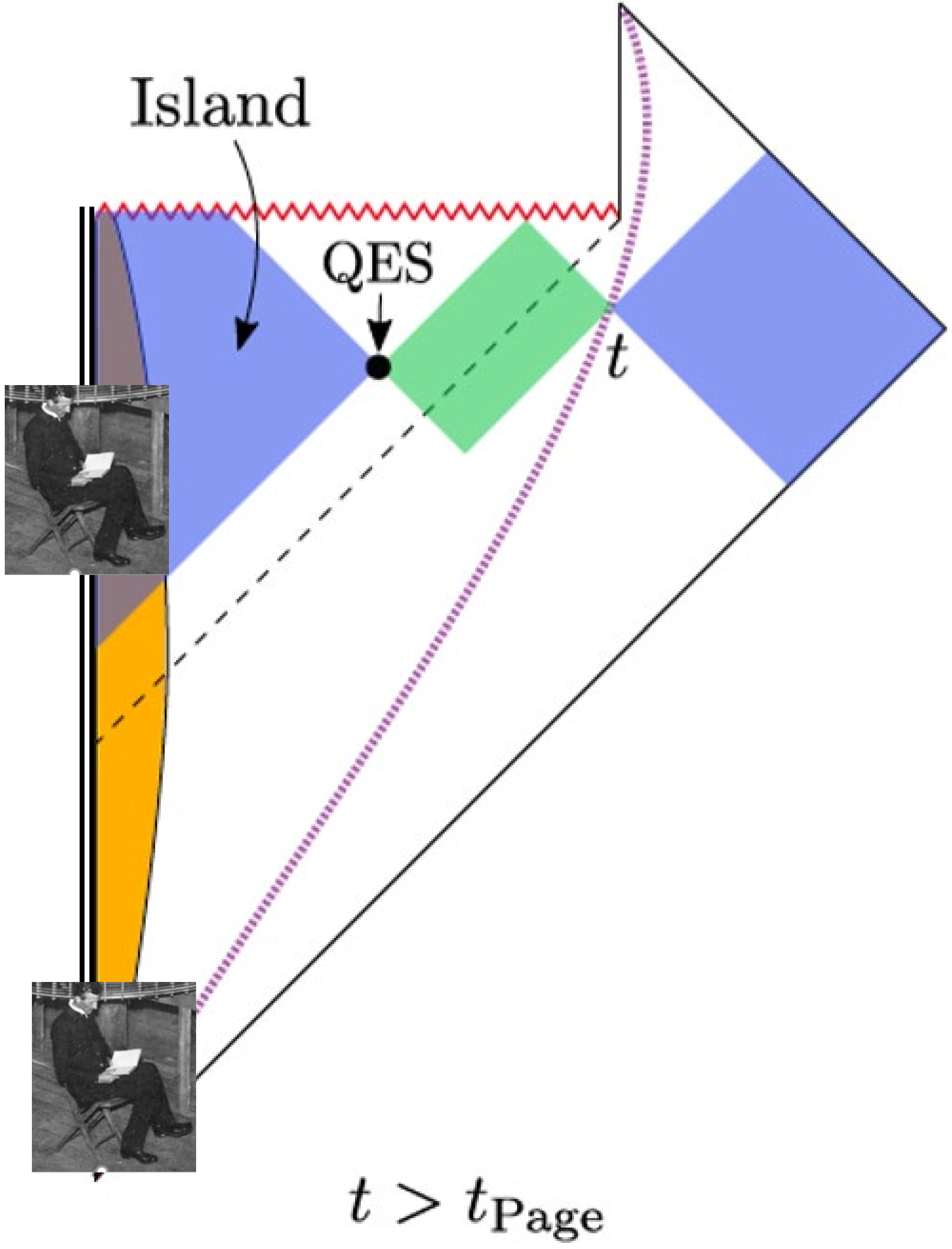
Too early to say that information loss has been completely resolved. It looks like a handmade oracle. We need more exact foundation of quantum gravity based on some physical principles. Especially, quantum measurement theory for BH interior space should be developed.

Nikola Tesla



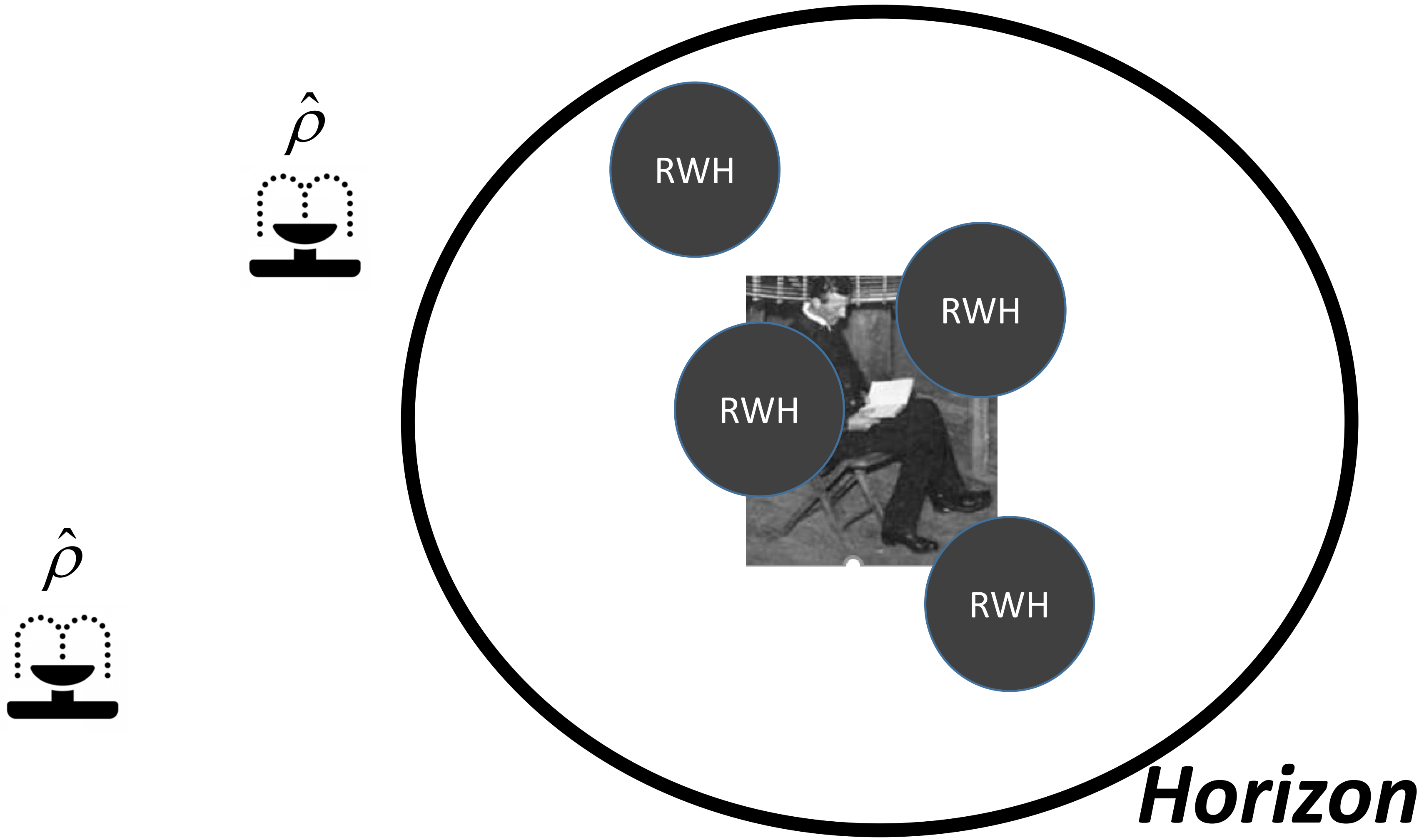
Tesla-in-Island Experiment

Tesla in island ⇒

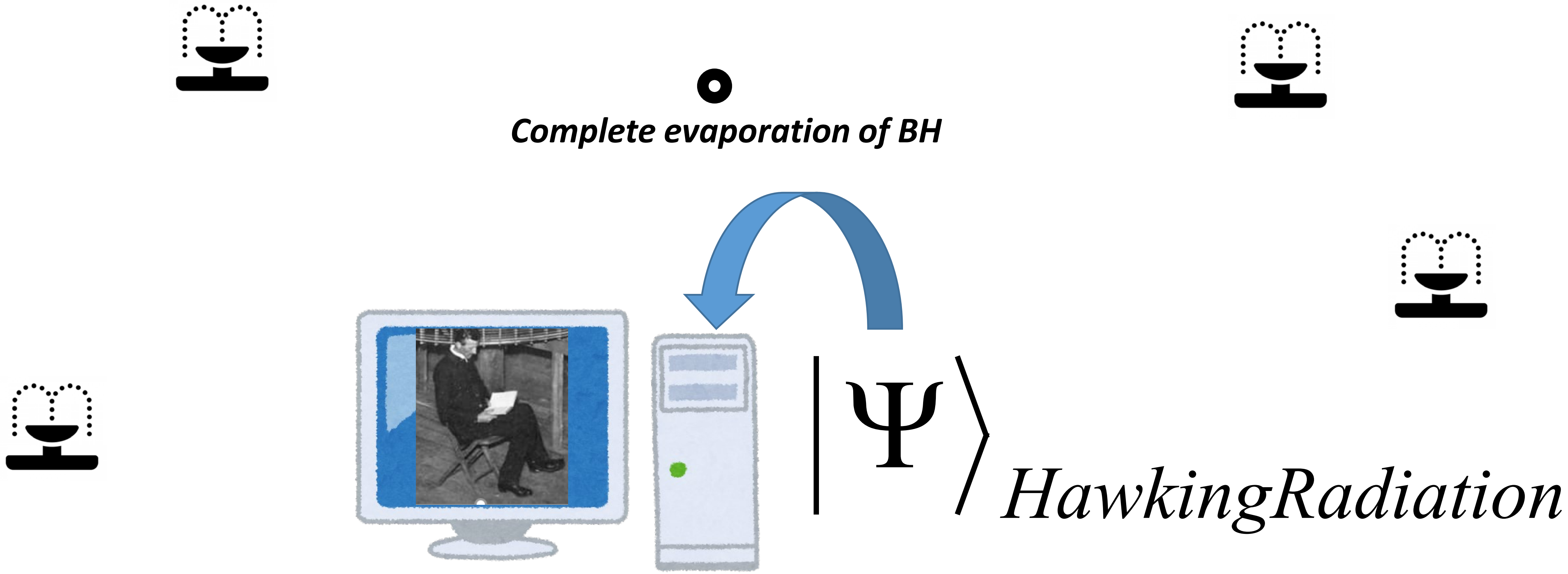


Maldacena's slide modified

Does Tesla notice that he has already landed in Replica-Worm-Hole Island, even though the spacetime curvature is small?



By inputting whole data of emitted Hawking radiation to a quantum computer, is it possible to reproduce what Tesla was thinking in the island of BH interior? Will he say he saw quantum space of low curvature?



In the island conjecture, lack of knowledge about the true partner for early Hawking particle after the last burst of black holes.

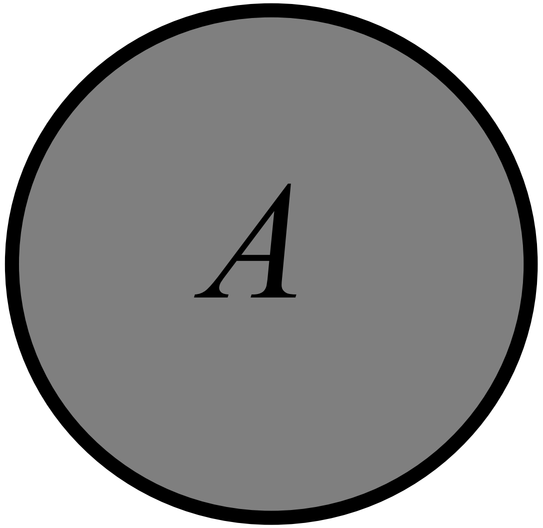
It has not been computed what kind of late Hawking particle is the partner. The partner, instead, may live in gravitational degeneracy like soft hair at null future infinity.

Anyway, nobody has answered many fundamental questions of information loss! *(Please watch the talk video of Rob Myers.)*

○ Black Hole Entropy Problem

Bekenstein-Hawking Entropy:

$$\boxed{S_{bh} = \frac{A}{4G}}$$



What is the statistical mechanical origin of this entropy, especially for non-BPS BH's?

Entanglement entropy?

(Azeyanagi-Nishioka-Takayanagi (2008), …)

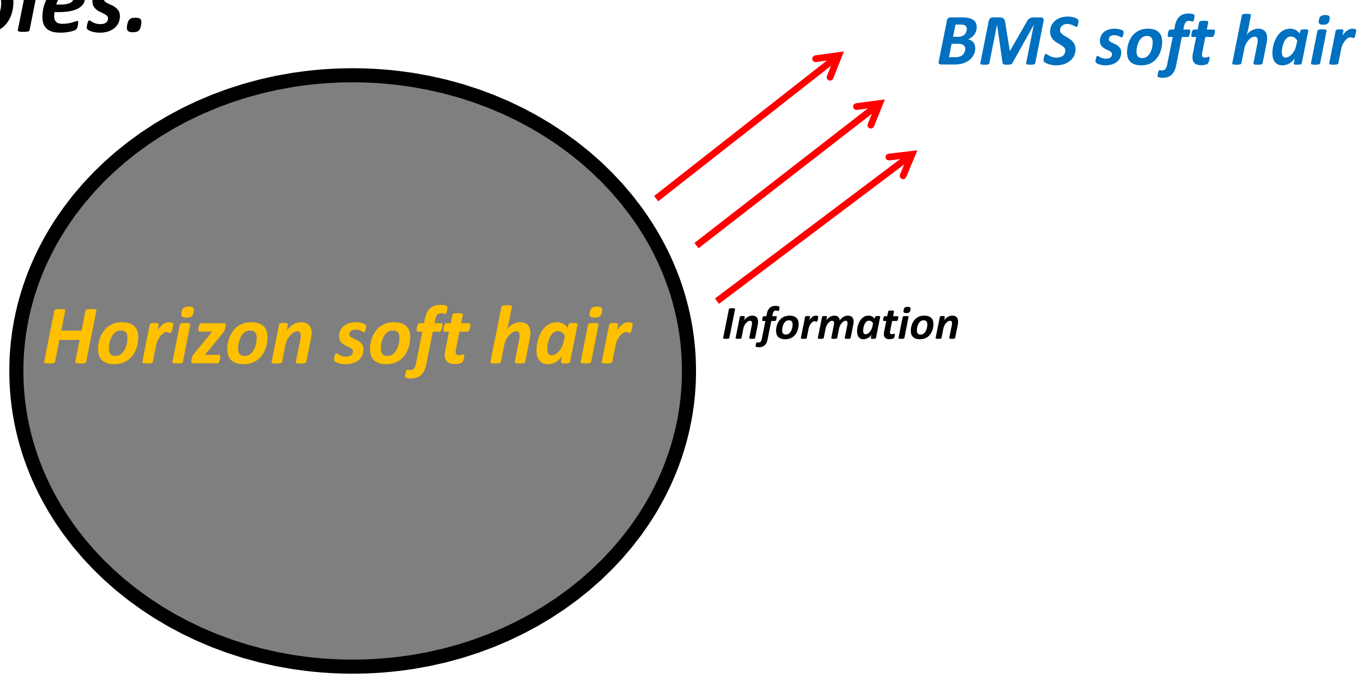
Tough obstacle:

No hair property of classical black hole spacetimes

Hawking, Perry and Strominger (HPS) proposed an interesting symmetry-based conjecture which may simultaneously resolve the information loss problem and the black hole entropy problem, using supertranslation and superrotation at a black hole horizon, and Bondi-Metzner-Sachs (BMS) symmetry at the null future infinity.

S. W. Hawking, "The Information Paradox for Black Holes", arXiv:1509.01147.
S. W. Hawking, M. J. Perry and A. Strominger, PRL 116, 231301 (2016),
S. W. Hawking, M. J. Perry and A. Strominger, JHEP,161 (2017).

Very interestingly, gravitational vacuum degeneracy with zero energy near horizon and at null future infinity plays a crucial role in their scenario. This is recently called soft hair of black holes.

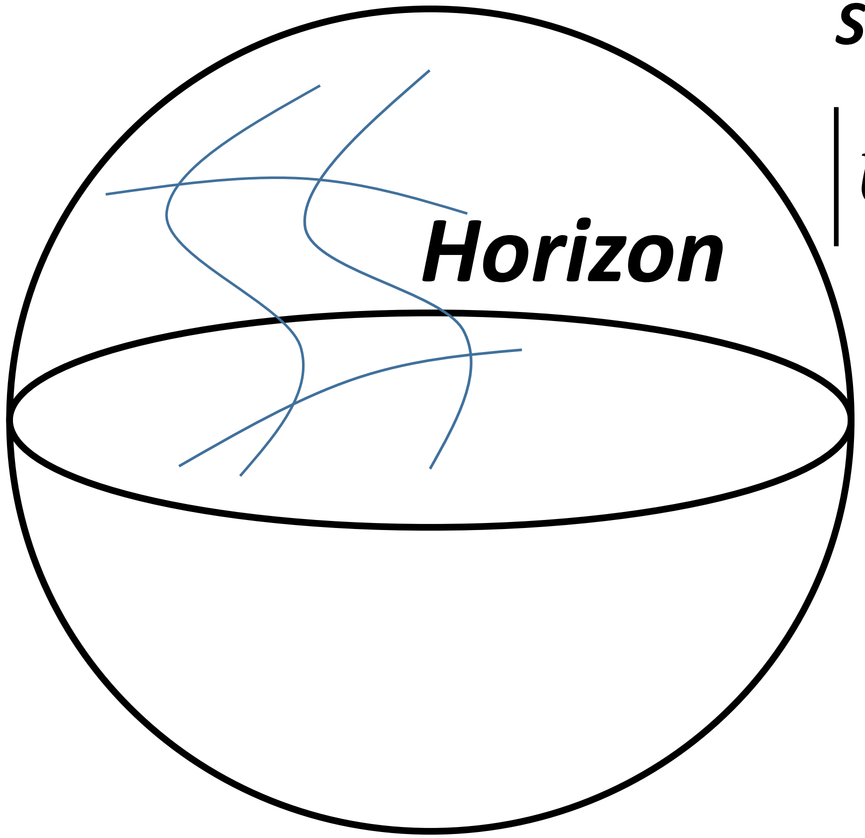


Soft hair may preserve whole the quantum information and, simultaneously, generate horizon microstates for BH entropy. Possible resolution of information loss and the origin of BH entropy.

Remarks

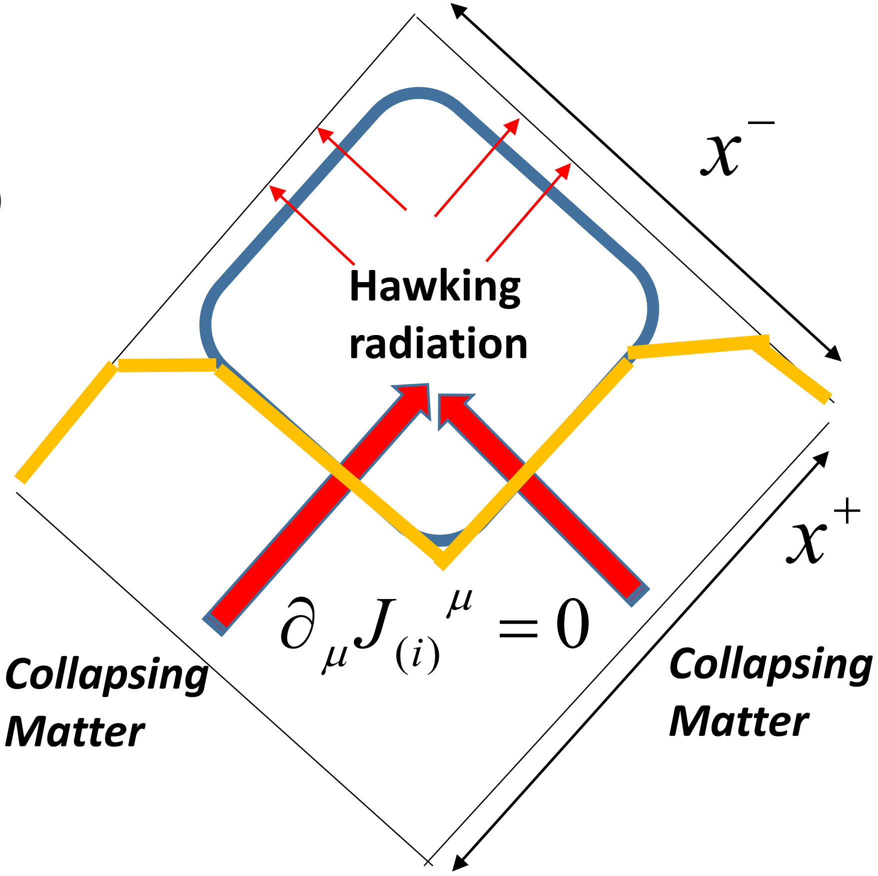
Soft hairs at horizons are expected to play a crucial role for exploration of edge modes at BH horizons. Also soft hairs may have relations to soft modes in in black-hole zone regions of island scenario, and brane-world holography.

HPS Resolution for Information Loss and Entropy Origin



Soft hair of BH

$$\left|\psi\right\rangle^{\otimes Area/(4G)}$$

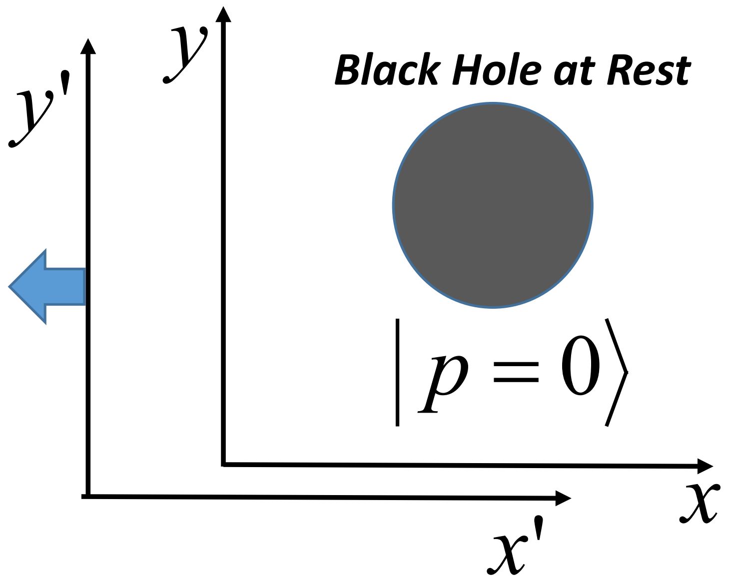


$$\boxed{\ln W_{soft\ hair} \approx Area/(4G)}$$

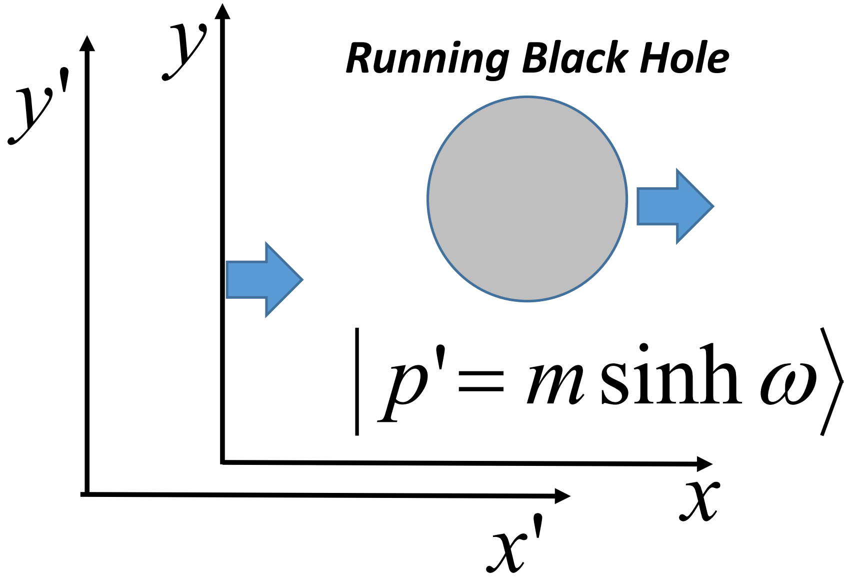
An infinite number of holographic charges on the horizon store whole quantum information of absorbed matter.

⇨ ***Unitarity***

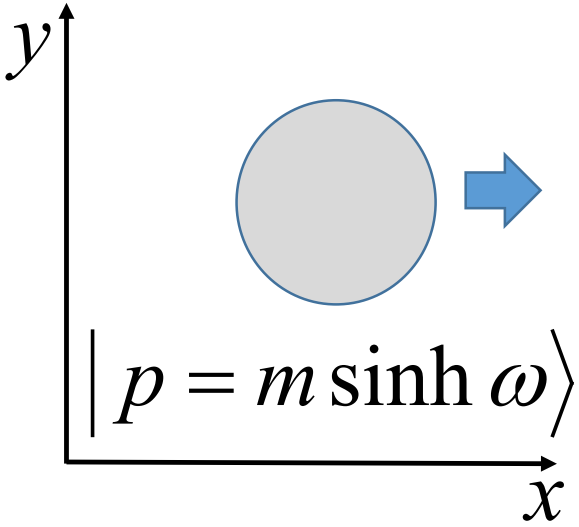
Horizon soft hair of black holes comes from "would-be" gauge freedom of general covariance, diffeomorphisms (Hotta et al, 2001). This is a similar mechanism of emergence of momentum eigenstates orthogonal to each other.



$$x' = x\cosh\omega + t\sinh\omega,$$
$$t' = x\sinh\omega + t\cosh\omega$$



We must have Lorentz covariance in the theory.

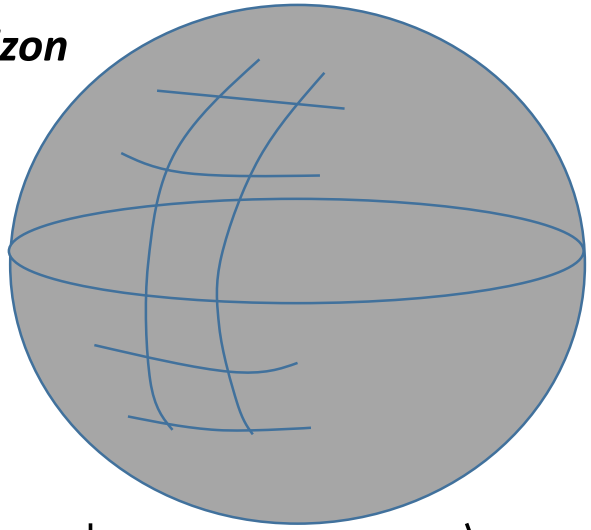


$$\boxed{\left\langle p = 0 \middle| p = m \sinh \omega \right\rangle = 0}$$

Lorentz transformation, one of general coordinate transformations, generates an infinite number of physical states with different values of momentum.

Soft hair of black holes comes from "would-be" gauge freedom of general covariance.

Horizon



$$\left| Q_{horizon} = 0 \right\rangle$$

Supertranslation:

$$\tau' = \tau + T(\theta, \phi)$$

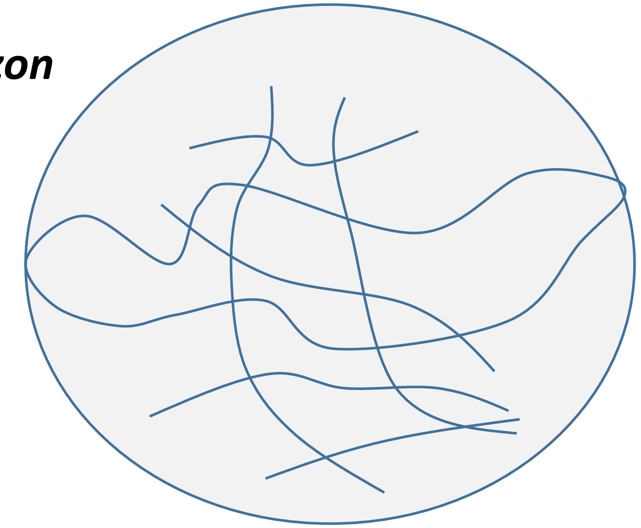
Superrotation:

$$\theta' = \Theta(\theta, \phi),$$

$$\phi' = \Phi(\theta, \phi)$$

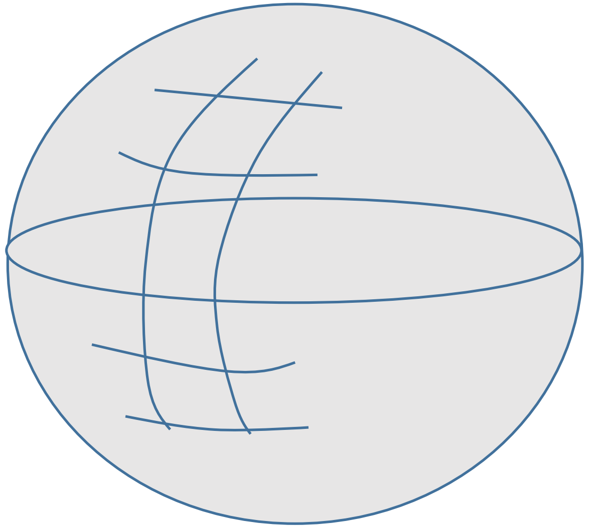
(Hotta-Sasaki-Sasaki, 2001)

Horizon



$$\left| Q'_{horizon} = q \right\rangle$$

We must have asymptotic symmetry as a part of general covariance in the theory.



$$\left|Q_{horizon} = q\right\rangle$$

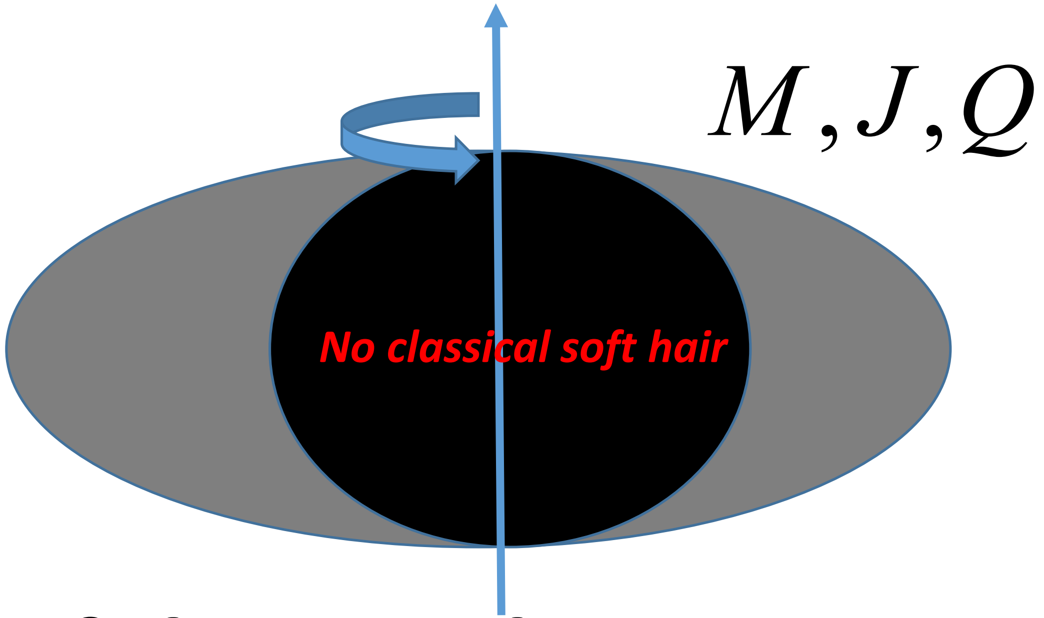
$$\boxed{\left\langle Q_{horizon} = 0 \middle| Q_{horizon} = q \right\rangle = 0}$$

(Hotta-Sasaki-Sasaki, 2001)

Supertranslation and superrotation generate an enormous number of physical states with different values of the charges.

Unfortunately, HPS found in their setting that stationary black holes do not carry classical supertranslation hair at horizon.

$$Q^H{}_f = -\frac{1}{16\pi G}\int_{H^+} dx^2 \sqrt{-g}\, f g^{AB}\partial_v h_{AB} = 0.$$



We may have purely quantum soft hair in this scenario. But, due to the lack of computable quantum gravity theory, the exact quantization has not been achieved so far. ***cf. Haco-Hawking-Perry-Strominger, JHEP12 (2018) 098.***

HPS asymptotic near-horizon metric of stationary BH:

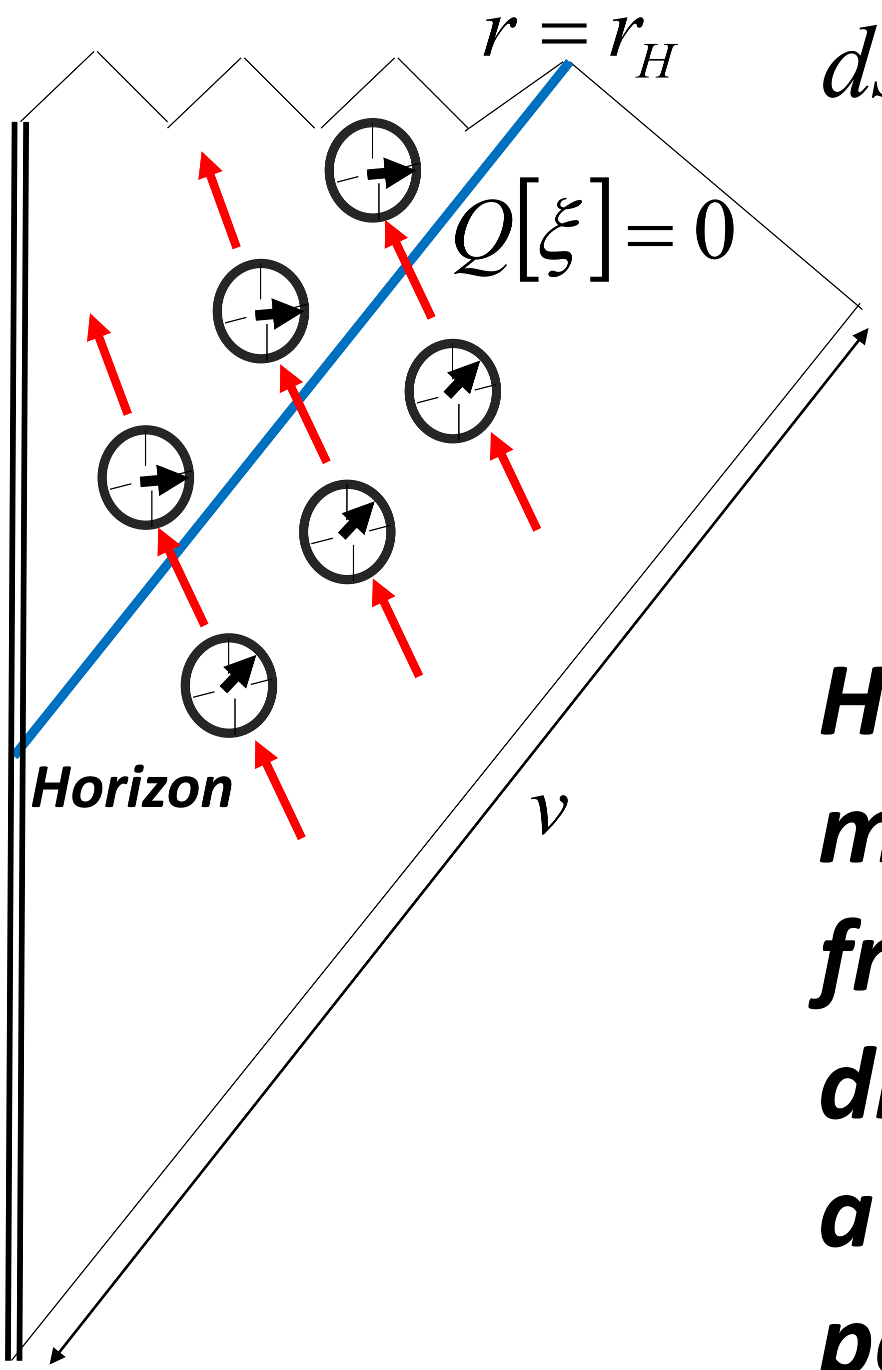
$$ds^2 = 2dvdr + \sum_{A=\theta,\phi} \sum_{B=\theta,\phi} g_{AB} dx^A dx^B + O(r - r_H)$$

This is invariant under HPS horizon supertranslation,

$$\delta v = T(x^A) + \cdots$$

$$\delta r = 0$$

$$\delta x^A = -(r - r_H) g^{AB} \partial_B T + \cdots$$

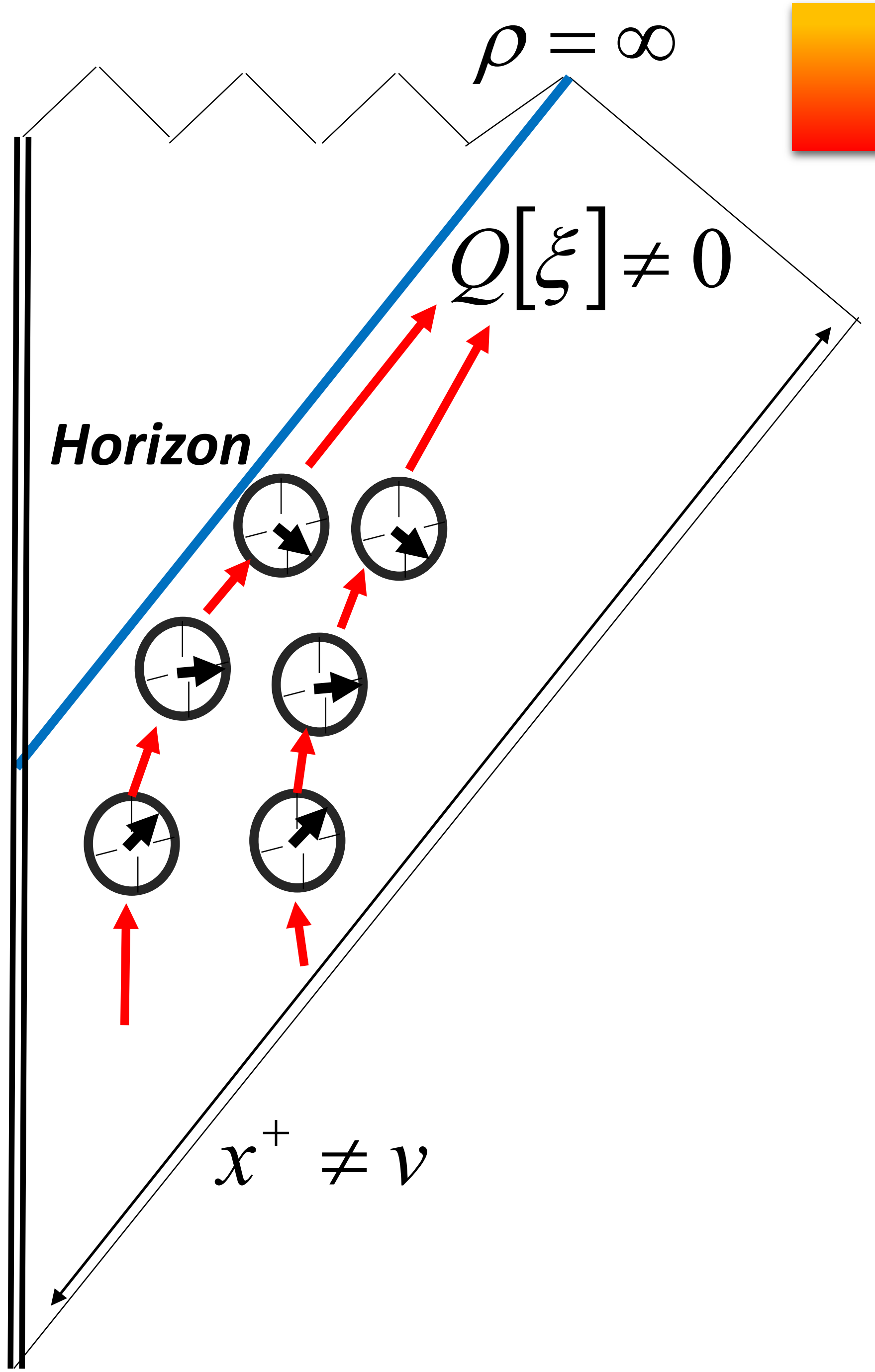


$$ds^2 = 2dvdr + \sum_{\theta,\phi} g_{AB} dx^A dx^B + O(r - r_H)$$

By using the time v , HPS horizon supertranslation is defined as

$$\delta v = T(x^A) + \cdots$$

HPS coordinate system near horizon may be physically implemented by free-falling block-numbered clocks distributed in the space, which play a role of the detector of metric perturbation and soft hair.



MASSAGE (I) of THIS TALK

In a coordinate system implemented by ***accelerating*** ***block-numbered clocks distributed in the space, stationary black holes indeed carry non-vanishing classical charges of supertranslation and superrotation.***

【M. Hotta, K. Sasaki and T. Sasaki, (2001)】

MASSAGE (II) of THIS TALK

We construct a general theory of gravitational holographic charges on Rindler horizons for a 1+3 dimensional linearized gravity field in the Minkowski background. Especially, we give a general formula of holographic charge shift of supertranslation and superrotation induced by infalling matter absorption.

【M. Hotta, J. Trevison and K. Yamaguchi 2016】

MASSAGE (III) of THIS TALK

In the ordinary scheme, it is a very heavy task to find asymptotic symmetries at boundaries in general relativity. Recently, we provide a simple scheme: covariant building-block approach to finding asymptotic symmetries at boundaries. By using our formulation, we have found a new soft-hair symmetry at Rindler horizons: superdilation symmetry.

【*T. Tomitsuka, K. Yamaguchi and M. Hotta 2020*】

Outline:

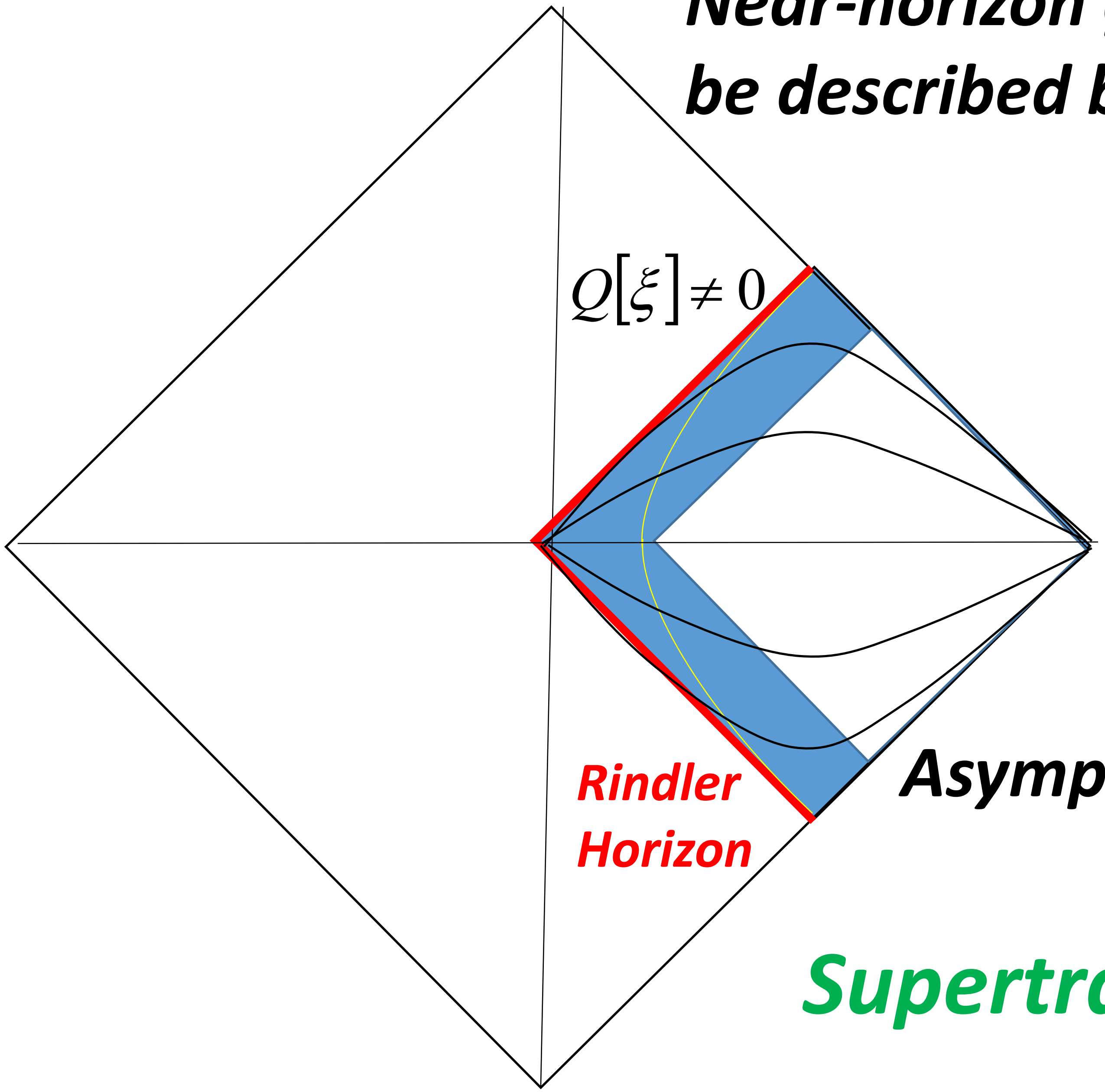
1. Holographic Charge Shift at Rindler Horizons

2. Covariant Building-Block Approach to Asymptotic Symmetries at Boundaries

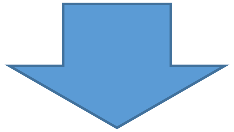
3. Summary

1. Holographic Charge Shift at Rindler Horizons

Near-horizon geometry of infinite-mass BH can be described by Rindler metric of flat spacetime.



Asymptotic symmetry near Rindler horizon



Supertranslation and Superrotation

(Hotta-Trevison-Yamaguchi, 2016)

$$ds^2 = \exp\left(-\frac{\rho}{\kappa}\right)\left[-d\tau^2 + d\rho^2\right] + dy^2 + dz^2 + \kappa\,\varphi_{\mu\nu} d\rho^\mu d\rho^\nu$$

Rindler horizon at $\rho = \infty$.

Asymptotic metric near Rindler horizon

$$\begin{bmatrix} \varphi_{\tau\tau} & \varphi_{\tau\rho} & \varphi_{\tau y} & \varphi_{\tau z} \\ \varphi_{\rho\tau} & \varphi_{\rho\rho} & \varphi_{\rho y} & \varphi_{\rho z} \\ \varphi_{y\tau} & \varphi_{y\rho} & \varphi_{yy} & \varphi_{yz} \\ \varphi_{z\tau} & \varphi_{z\rho} & \varphi_{zy} & \varphi_{zz} \end{bmatrix} = \begin{bmatrix} O(\Delta^2) & O(\Delta^2) & O(\Delta) & O(\Delta) \\ O(\Delta^2) & 0 & O(\Delta^2) & O(\Delta^2) \\ O(\Delta) & O(\Delta^2) & O(\Delta^0) & O(\Delta^0) \\ O(\Delta) & O(\Delta^2) & O(\Delta^0) & O(\Delta^0) \end{bmatrix}$$

$$\Delta = \exp\left(-\frac{\rho}{\kappa}\right) \to 0 \qquad \left(\kappa = \sqrt{16\pi G}\right)$$

$$ds^2 = \exp\left(-\frac{\rho}{\kappa}\right)\left[-d\tau^2 + d\rho^2\right] + dy^2 + dz^2 + \kappa\,\varphi_{\mu\nu} d\rho^\mu d\rho^\nu$$

The metric form is invariant under asymptotic symmetries near horizon.

Supertranslation **(coordinate dependent time translation)**

$$\delta_\xi \tau = \xi^\tau (y,z)$$ **at horizon**

Superrotation **(2 dim general coordinate transformation at a Rindler horizon)**

$$\delta_\xi y = \xi^y (y,z), \quad \delta_\xi z = \xi^z (y,z)$$ **at horizon**

Due to the asymptotic metric form, Regge-Teitelboim charges are indeed integrable for supertranslation and superrotation.

$$Q[\xi] = \int \delta Q[\xi]$$

$$Q[\xi] = \int dydz \left[-\frac{\xi^{\tau}(y,z)}{\kappa^{3}} \left(\sqrt{\det[h_{AB}]} - 1 \right) + 2\xi^{A}(y,z) \Pi^{\rho}{}_{A} \right]$$

(Hotta Trevison Yamaguchi 2016)

Supertranslation and superrotation generate excited metrics from ground-state Rindler metric:

$$ds^2 = \exp\left(-\frac{\rho'}{\kappa}\right)\left[-d\tau'^2 + d\rho'^2\right] + dy'^2 + dz'^2.$$

$$Q[\xi] = 0.$$

$$\tau' = \tau + T(y, z),$$

$$\rho' = \rho,$$

$$x'_A = X_A(y, z),$$

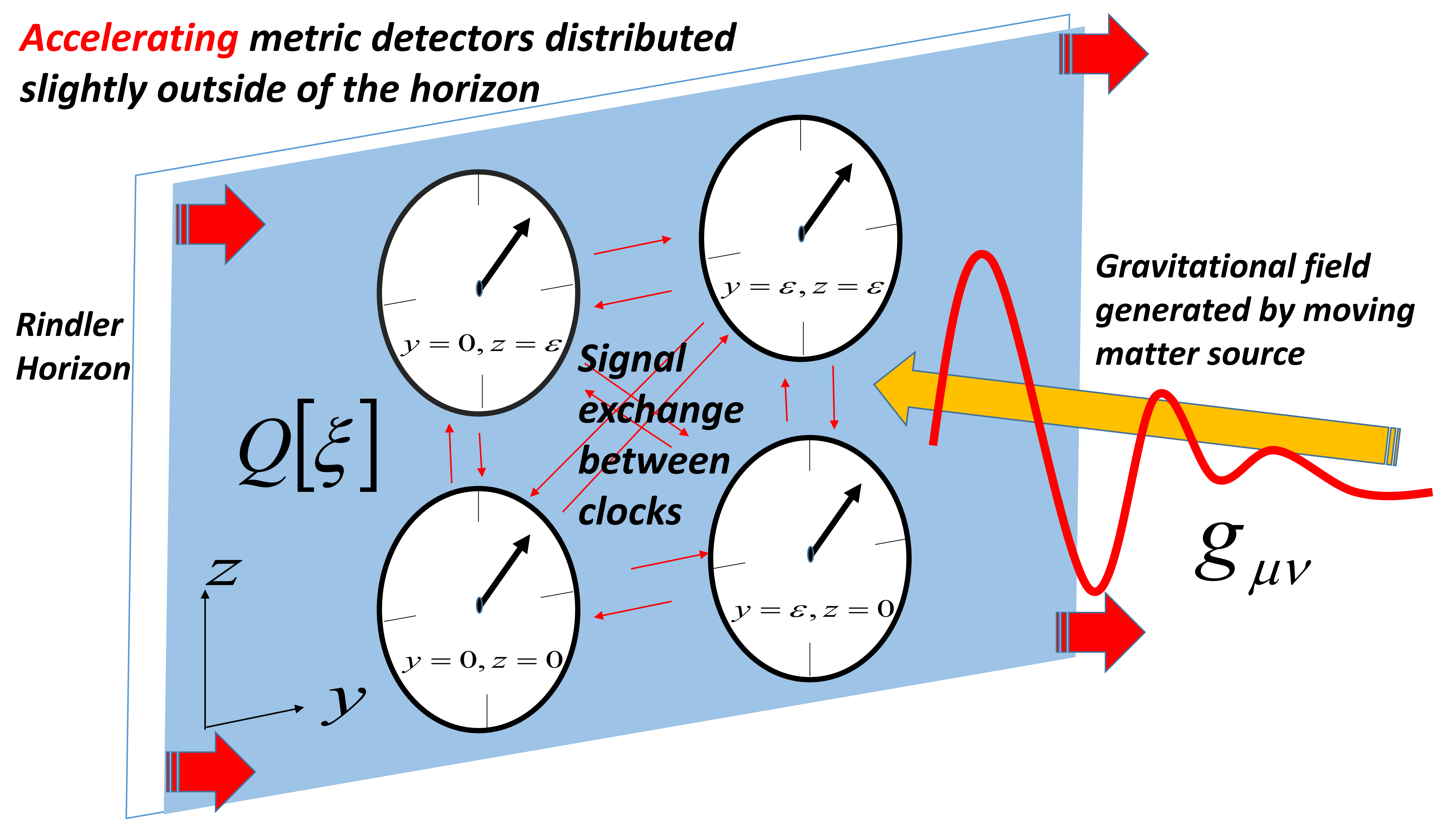
$$ds^2 = \Delta\left(-d\tau^2 - 2\partial_A T(y,z)\, d\tau dx^A + d\rho^2\right) + \left(\partial_A X^C(y,z)\partial_B X^C(y,z) - \Delta\partial_A T\partial_B T\right)dx^A dx^B.$$

$$Q[\xi] \neq 0.$$

$$Q[\xi] = -\frac{1}{\kappa^3}\int dydz\left[\begin{array}{l}\xi^\tau(y,z)\left(\sqrt{\det\left[\partial_A X^C \partial_B X^C\right]}-1\right)\\ +\xi^C(y,z)\sqrt{\det\left[\partial_A X^{C'} \partial_B X^{C'}\right]}\partial_C T\end{array}\right] \neq 0$$

Non-singlet representation as soft hair.

The asymptotic metrics and charges are observed by metric detectors which consist of accelerating clocks exchanging signals and accumlating the metric information.



Accelerating metric detectors distributed slightly outside of the horizon
Rindler Horizon
$Q[\xi]$
$y=0, z=\varepsilon$
$y=\varepsilon, z=\varepsilon$
$y=0, z=0$
$y=\varepsilon, z=0$
Signal exchange between clocks
Gravitational field generated by moving matter source
$g_{\mu\nu}$
z
y

Rindler Gauge Fixing of Weak Gravity Field

$$ds^2 = dx^+ dx^- + dy^2 + dz^2 + \underline{\kappa\, h_{\mu\nu} dx^\mu dx^\nu} \qquad \left(\kappa = \sqrt{16\pi G}\,\right)$$

Conformal Rindler Coordinates

$$x^\pm = 2\kappa \exp\left(-\frac{\rho \mp \tau}{2\kappa}\right)$$

$$\left(x^\mu\right) = \left(x^+, x^-, y, z\right) \Leftrightarrow \left(\rho^\mu\right) = \left(\tau, \rho, y, z\right)$$

$$ds^2 = \exp\left(-\frac{\rho}{\kappa}\right)\left[-d\tau^2 + d\rho^2\right] + dy^2 + dz^2 + \underline{\kappa\, \varphi_{\mu\nu} d\rho^\mu d\rho^\nu}$$

$$\boxed{\varphi^{(R)}{}_{\rho\mu} = 0}$$

$$\varphi^{(R)}{}_{\rho\rho} = \frac{1}{4\kappa^2}\left[\left(x^+\right)^2 h^{(R)}{}_{++} + 2x^+x^-h^{(R)}{}_{+-} + \left(x^-\right)^2 h^{(R)}{}_{--}\right] = 0$$

$$\varphi^{(R)}{}_{\rho\tau} = \frac{1}{4\kappa^2}\left[\left(x^+\right)^2 h^{(R)}{}_{++} - \left(x^-\right)^2 h^{(R)}{}_{--}\right] = 0$$

$$\varphi^{(R)}{}_{\rho A} = -\frac{1}{2\kappa}\left[x^+h^{(R)}{}_{+A} + x^-h^{(R)}{}_{-A}\right] = 0$$

From the gauge fixing, it turns out that

$$\varphi^{(R)}{}_{\tau\tau} = \frac{1}{\kappa^2}\left(x^-\right)^2 h^{(R)}{}_{--} = O\left(\left(x^-\right)^2\right),$$

$$\varphi^{(R)}{}_{\tau A} = -\frac{1}{\kappa} x^- h^{(R)}{}_{--} = O(x^-).$$

In summary,

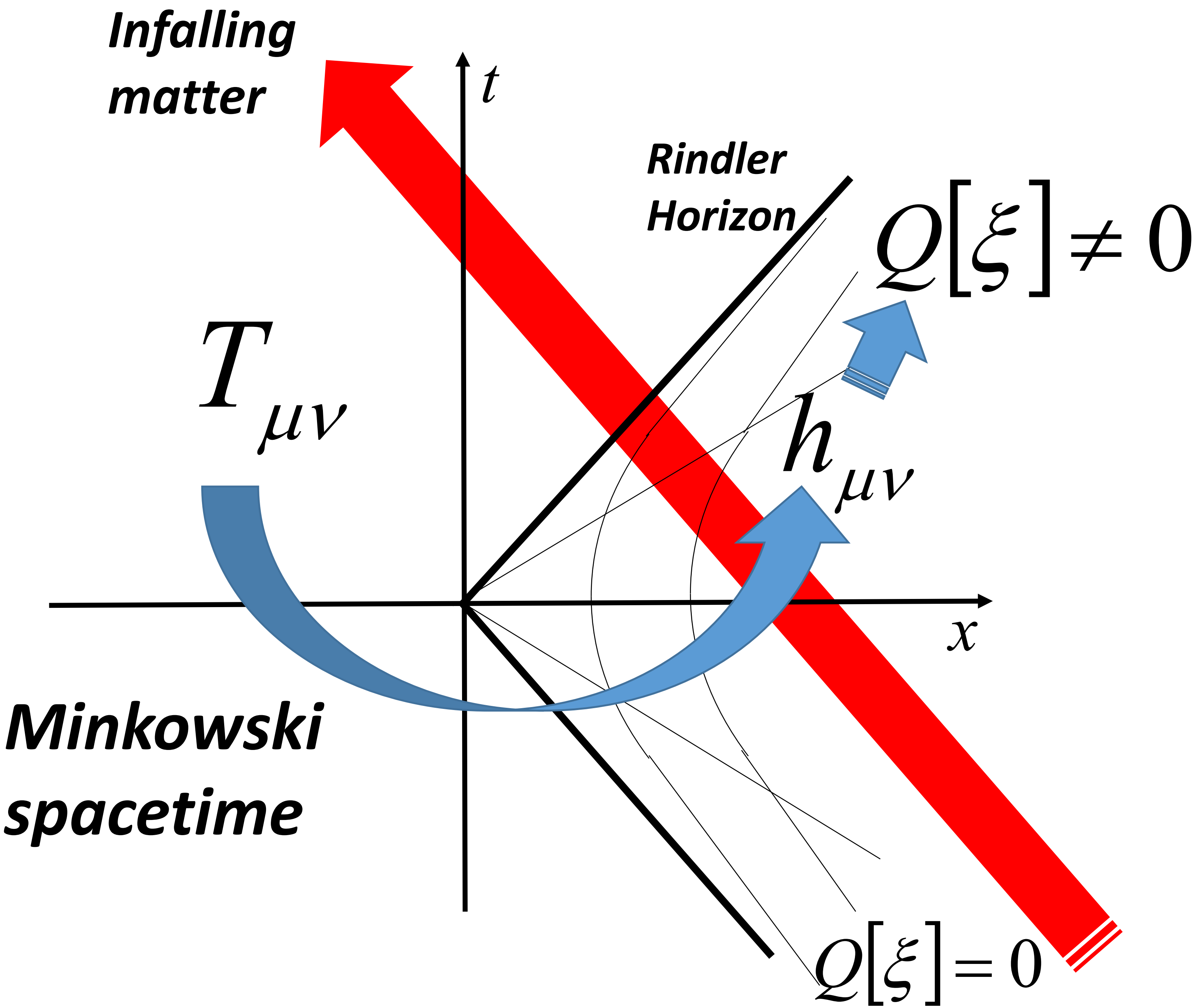
$$\begin{bmatrix} \varphi^{(R)}{}_{\tau\tau} & \varphi^{(R)}{}_{\tau\rho} & \varphi^{(R)}{}_{\tau y} & \varphi^{(R)}{}_{\tau z} \\ \varphi^{(R)}{}_{\rho\tau} & \varphi^{(R)}{}_{\rho\rho} & \varphi^{(R)}{}_{\rho y} & \varphi^{(R)}{}_{\rho z} \\ \varphi^{(R)}{}_{y\tau} & \varphi^{(R)}{}_{y\rho} & \varphi^{(R)}{}_{yy} & \varphi^{(R)}{}_{yz} \\ \varphi^{(R)}{}_{z\tau} & \varphi^{(R)}{}_{z\rho} & \varphi^{(R)}{}_{zy} & \varphi^{(R)}{}_{zz} \end{bmatrix} = \begin{bmatrix} O\left(\left(x^-\right)^2\right) & 0 & O\left(x^-\right) & O\left(x^-\right) \\ 0 & 0 & 0 & 0 \\ O\left(x^-\right) & 0 & O\left(\left(x^-\right)^0\right) & O\left(\left(x^-\right)^0\right) \\ O\left(x^-\right) & 0 & O\left(\left(x^-\right)^0\right) & O\left(\left(x^-\right)^0\right) \end{bmatrix}$$

⇒ Asymptotic metric near Rindler horizon at $x^- = 0$.

Note that this asymptotic metric form is also invariant under supertranslation and superroration. Simultaneously, the corresponding charges evolve from zero to nonzero values during matter passing.

(Hotta (2002), Hotta-Trevison-Yamaguchi (2016))

Let us consider the charges of supertranslation and superrotation for weak gravity perturbation.



Infalling matter
t
Rindler Horizon
$Q[\xi] \neq 0$
$T_{\mu\nu}$
$h_{\mu\nu}$
x
Minkowski spacetime
$Q[\xi] = 0$

Main Result

Supertranslation Charge at Horizon

$$Q_{st}\left[\xi^{\tau}\right]=-\frac{1}{2\kappa}\int dydz\,\xi^{\tau}\left(y,z\right)\int_{0}^{\infty}x^{+}T_{++}\left(x^{+},x^{-}=0,y,z\right)dx^{+}$$

Superrotation Charge at Horizon

$$\left(\kappa=\sqrt{16\pi G}\right)$$

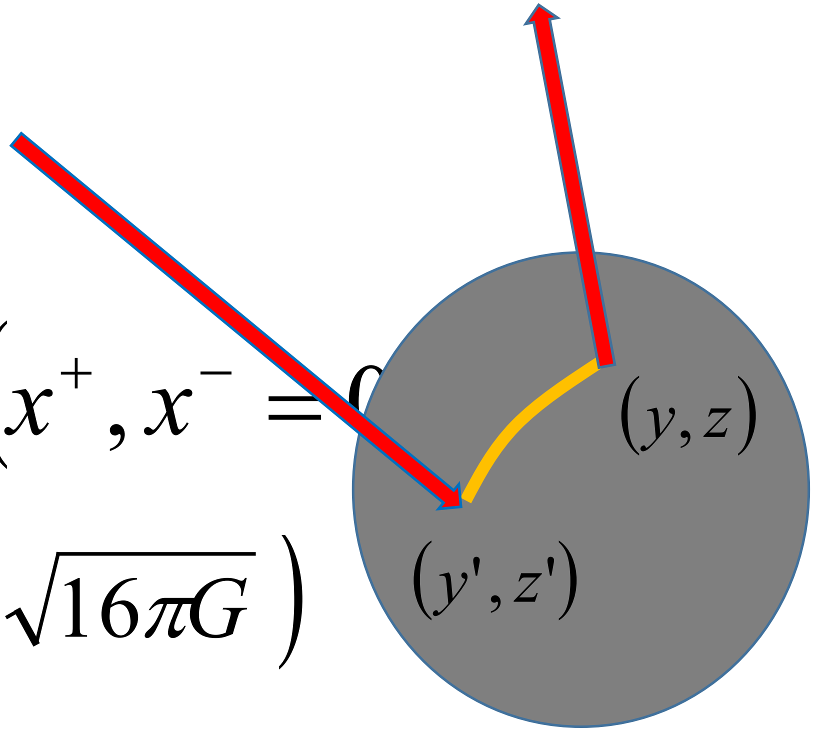
$$Q_{sr}\left[\xi^{y},\xi^{z}\right]$$

$$=\frac{1}{4\pi}\int dydz\int dy'dz'\xi^{A}\left(y,z\right)\partial_{A}\ln\left[\frac{\left(y-y'\right)^{2}+\left(z-z'\right)^{2}}{\kappa^{2}}\right]$$

$$\times\int_{0}^{\infty}\partial_{-}T_{++}\left(x^{+},x^{-}=0,y',z'\right)dx^{+}$$

Main Result

Supertranslation Charge at Horizon



$$Q_{st}\left[\xi^{\tau}\right] = -\frac{1}{2\kappa}\int dydz\,\xi^{\tau}(y,z)\int_0^{\infty} x^{+}T_{++}\left(x^{+},x^{-}=0\right.$$

Superrotation Charge at Horizon

$$\left(\kappa = \sqrt{16\pi G}\right)$$

$$Q_{sr}\left[\xi^{y},\xi^{z}\right]$$

$$= \frac{1}{4\pi}\int dydz\int dy'\,dz'\,\xi^{A}(y,z)\partial_{A}\boxed{\ln\left[\frac{(y-y')^2+(z-z')^2}{\kappa^2}\right]}$$

$$\times\int_0^{\infty}\partial_{-}T_{++}\left(x^{+},x^{-}=0,y',z'\right)dx^{+}$$

Two-dim Green function at the horizon

Cf: 't Hooft's blackhole S matrix

Main Result

Supertranslation Charge on Horizon

$$Q_{st}\left[\xi^{\tau}\right] = -\frac{1}{2\kappa}\int dydz\,\xi^{\tau}(y,z)\boxed{\int_0^{\infty} x^{+}T_{++}\left(x^{+}, x^{-}=0, y, z\right)dx^{+}}$$

Superrotation Charge on Horizon

$$\left(\kappa = \sqrt{16\pi G}\right)$$

Time integration
⇒ Long-term memory effect

$$Q_{sr}\left[\xi^{y}, \xi^{z}\right]$$

$$= \frac{1}{4\pi}\int dydz\int dy'dz'\,\xi^{A}(y,z)\partial_{A}\ln\left[\frac{(y-y')^{2}+(z-z')^{2}}{\kappa^{2}}\right]$$

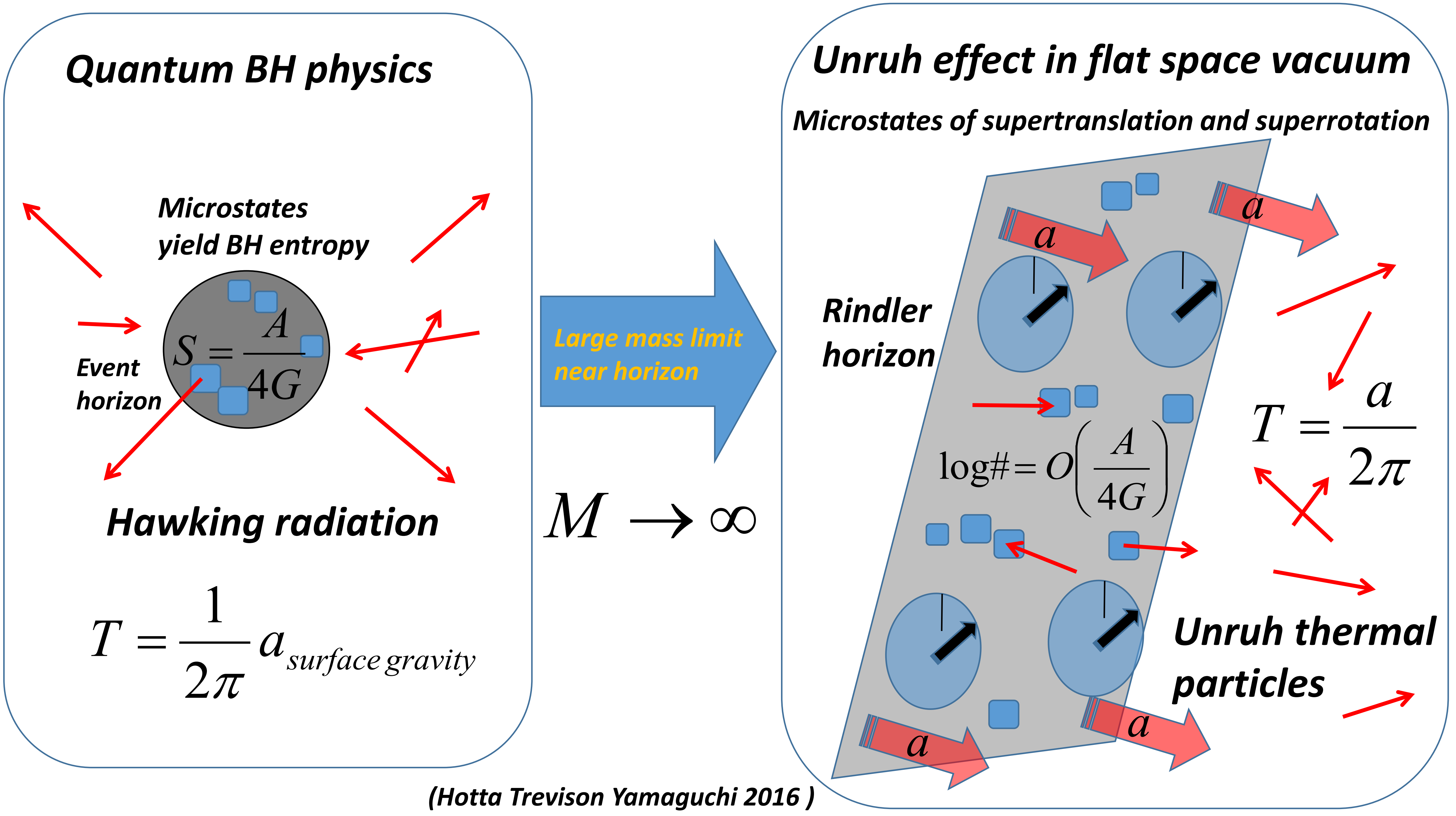
$$\times \boxed{\int_0^{\infty}\partial_{-}T_{++}\left(x^{+}, x^{-}=0, y', z'\right)dx^{+}}$$

Our results resolve two paradoxs:

(1) No-cloning paradox of gravitational memories at two Rindler horizons

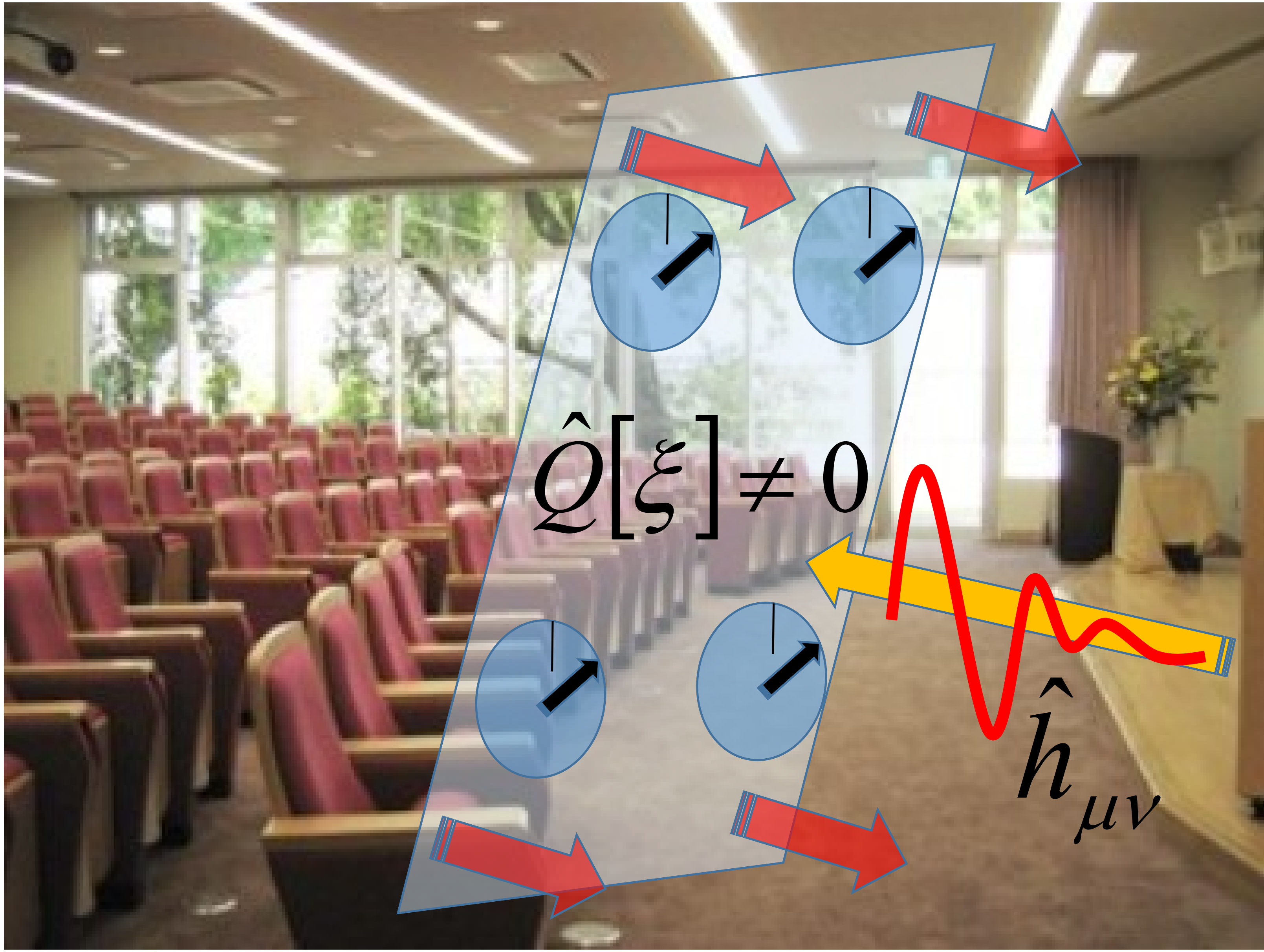
(2) Decoherence paradox of absorbed matter and gravitational memory

Please see our paper, M. Hotta, J. Trevison and K. Yamaguchi, Phys. Rev. D94, 083001 (2016).

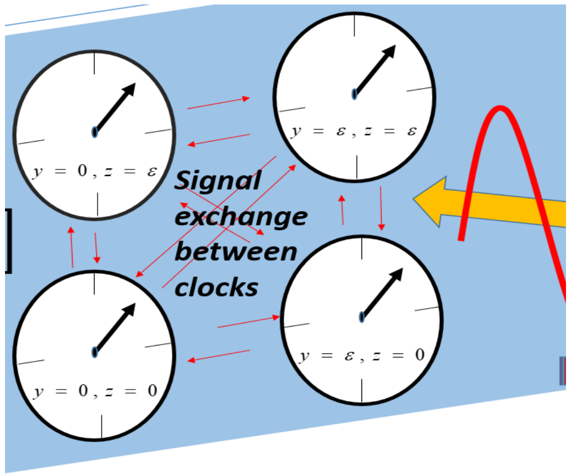

Quantum BH physics
Microstates
yield BH entropy
$S = \frac{A}{4G}$
Event
horizon
Hawking radiation
$T = \frac{1}{2\pi} a_{surface\,gravity}$
Large mass limit
near horizon
$M \to \infty$
Unruh effect in flat space vacuum
Microstates of supertranslation and superrotation
Rindler
horizon
a
a
a
a
$\log\# = O\left(\frac{A}{4G}\right)$
$T = \frac{a}{2\pi}$
Unruh thermal
particles
(Hotta Trevison Yamaguchi 2016)

Accelerating block-numbered clocks as metric detector observe BH soft hair in a room, as well as Hawking-Unruh thermal bath.

Each metric detector determines different physical reality of spacetime for it.

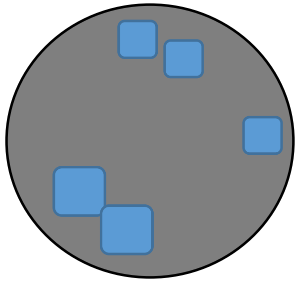


Metric Detector



Observed Asymptotic Metrics

$$\{g_{\mu\nu}\}$$



Microstates of Spacetime as Reality for Detector

Different metric detectors experience different reality of spacetime.

2. Covariant Building-Block Approach to Asymptotic Symmetries at boundaries

T. Tomitsuka, K. Yamaguchi and M. Hotta, arXiv:2012.14050.

In the ordinary scheme to find asymptotic symmetries, firstly we have to choose asymptotic metrics as a trial. If the choice is bad without luck, we encounter the cases:

(I) Integrability of charges is lost.

(To pass the test, many trials are usually requested.)

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(To pass the test, many trials are usually requested.)

(II) After passing this hard integrability test, it is noticed that the charges take zero values in the final result. (pity!)

The choice of asymptotic metrics determines the size of symmetry.

Example: 3-dim AdS background

$$\begin{pmatrix} \bar{g}_{tt} & \bar{g}_{tr} & \bar{g}_{t\phi} \\ \bar{g}_{rt} & \bar{g}_{rr} & \bar{g}_{r\phi} \\ \bar{g}_{\phi t} & \bar{g}_{\phi r} & \bar{g}_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -\left(\frac{r^2}{l^2}+1\right) & 0 & 0 \\ 0 & \left(\frac{r^2}{l^2}+1\right)^{-1} & 0 \\ 0 & 0 & r^2 \end{pmatrix}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

Brown-Henneaux (1986)

First Trial

→Time Translation + Rotation sym.

$$N = \alpha\left[\frac{r^2+R^2}{\alpha^2R^2-A^2}\right]^{1/2}\left[1-\frac{A^2R^2}{(\alpha^2R^2-A^2)r^2}\right]^{-1/2},$$

$$N^r = 0,$$

$$N^\phi = \frac{A(r^2+R^2)}{r^2(\alpha^2R^2-A^2)-A^2R^2},$$

$$g_{rr} = \left(\frac{r^2}{R^2}+1\right)^{-1},$$

$$g_{\phi\phi} = \alpha^2r^2 - A^2\left(\frac{r^2}{R^2}+1\right),$$

$$\pi^r_\phi = \alpha A,$$

$$J[d/dt] = 4\pi(1-\alpha),$$

$$J[d/d\phi] = 4\pi\alpha A.$$

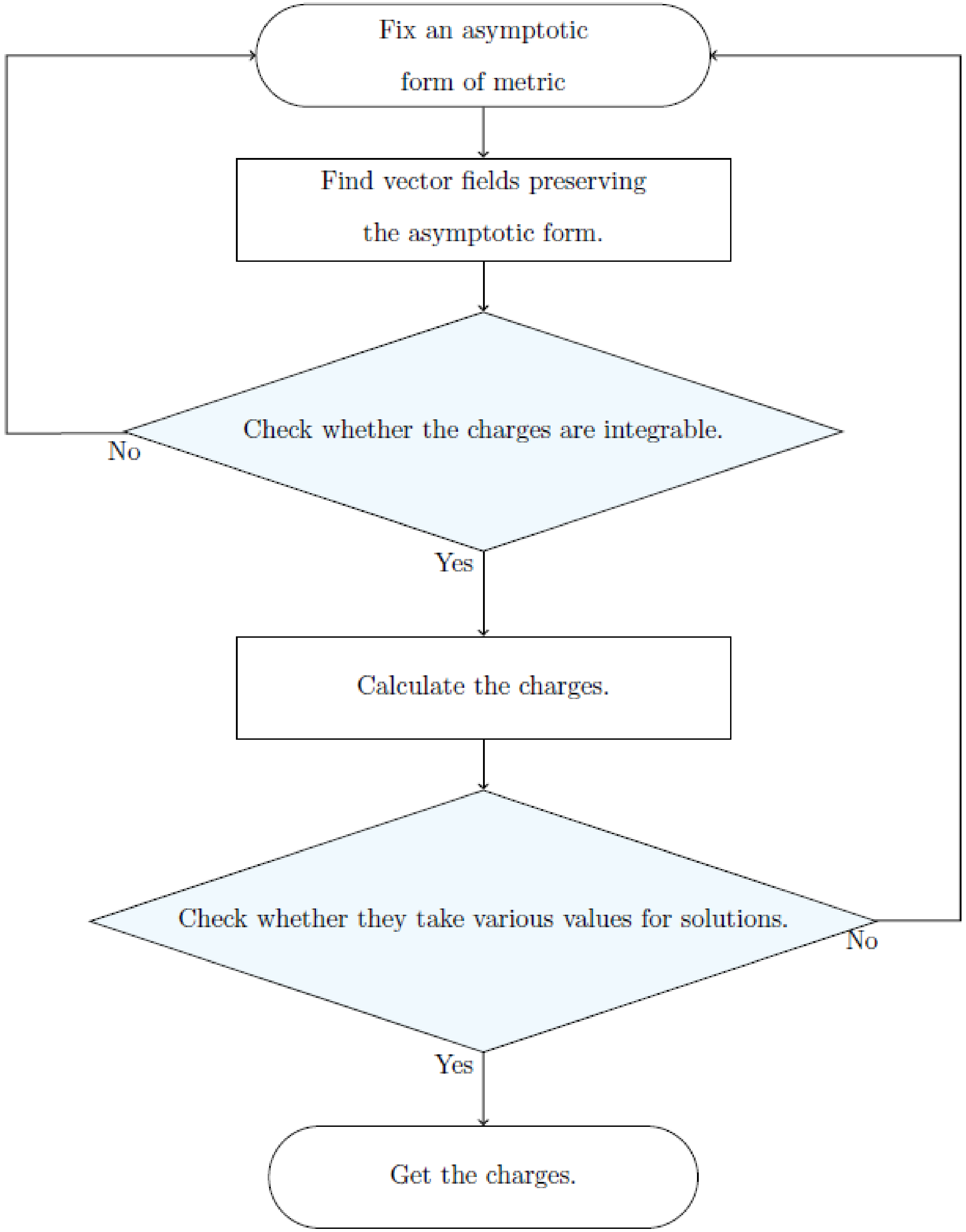
Second Trial

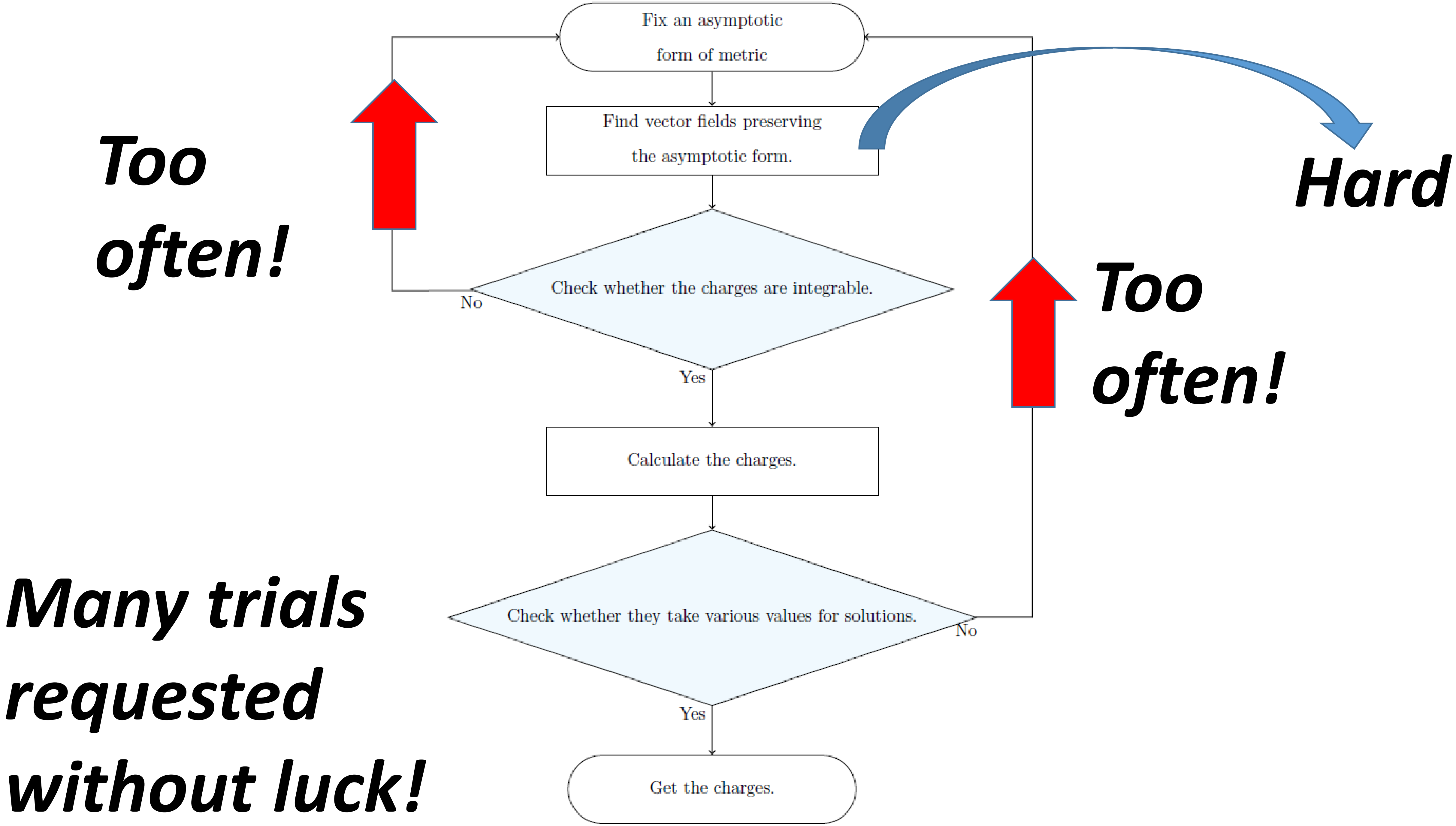
→Virasoro Sym.

$$(\delta g_{\mu\nu}) = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(r^{-3}) & \mathcal{O}(1) \\ \mathcal{O}(r^{-3}) & \mathcal{O}(r^{-4}) & \mathcal{O}(r^{-3}) \\ \mathcal{O}(1) & \mathcal{O}(r^{-3}) & \mathcal{O}(1) \end{pmatrix}$$

$$\xi = \begin{pmatrix} \xi^t \\ \xi^r \\ \xi^\phi \end{pmatrix} = \begin{pmatrix} lT(t,\phi) + \frac{l^3}{r^2}\overline{T}(t,\phi) + \mathcal{O}(r^{-4}) \\ rR(t,\phi) + \mathcal{O}(r^{-1}) \\ \Phi(t,\phi) + \frac{l^2}{r^2}\overline{\Phi}(t,\phi) + \mathcal{O}(r^{-4}) \end{pmatrix}$$

Ordinary scheme to find asymptotic symmetries (hard task)




Fix an asymptotic form of metric
Find vector fields preserving the asymptotic form.
Check whether the charges are integrable.
No
Yes
Calculate the charges.
Check whether they take various values for solutions.
No
Yes
Get the charges.
Too often!
Too often!
Hard
Many trials requested without luck!

Covariant Phase Space Formalism (*Wald, …*)

$$\mathcal{L}_{EH} := \frac{1}{16\pi G}\sqrt{-g}R$$

$$\delta\mathcal{L}_{EH} = -\frac{\sqrt{-g}}{16\pi G}G^{\mu\nu}\delta g_{\mu\nu} + \partial_\mu\Theta^\mu(g,\delta g)$$

Pre-Symplectic Potential: $$\Theta^\mu(g,\delta g) = \frac{\sqrt{-g}}{16\pi G}\left(g^{\mu\alpha}\nabla^\beta\delta g_{\alpha\beta} - g^{\alpha\beta}\nabla^\mu\delta g_{\alpha\beta}\right)$$

Pre-Symplectic Current: $$\omega^\mu(g,\delta_1 g,\delta_2 g) := \delta_1\Theta^\mu(g,\delta_2 g) - \delta_2\Theta^\mu(g,\delta_1 g)$$

Pre-Symplectic Form: $$\Omega(g,\delta_1 g,\delta_2 g) := \int_\Sigma (\mathrm{d}^{d-1}x)_\mu\omega^\mu(g,\delta_1 g,\delta_2 g)$$

Charge Variation: $$\delta H[\xi] = \Omega(g,\delta g,\pounds_\xi g) = \int_\Sigma (\mathrm{d}^{d-1}x)_\mu\omega^\mu(g,\delta g,\underline{\pounds_\xi g})$$

Lie derivative w.r.t. the vector field

Let us introduce

$$
\begin{aligned}
S^{[\mu\nu]}(g, \delta g, \pounds_\xi g) &:= \delta Q^{\mu\nu}[\xi] + 2\xi^{[\mu}\Theta^{\nu]}(g, \delta g) \\
&= \frac{\sqrt{-g}}{8\pi G}\Bigg[-\left(\frac{1}{2}g^{\mu\gamma}g^{\alpha\beta} - g^{\mu\alpha}g^{\beta\gamma}\right)\delta g_{\alpha\beta}\nabla_\gamma \xi^\nu - g^{\mu\alpha}g^{\nu\beta}\nabla_\alpha \delta g_{\beta\gamma}\xi^\gamma \\
&\qquad + \xi^\mu\left(g^{\nu\alpha}g^{\beta\gamma} - g^{\nu\gamma}g^{\alpha\beta}\right)\nabla_\gamma \delta g_{\alpha\beta}\Bigg].
\end{aligned}
$$

Then, pre-symplectic current is computed as

$$
\omega^\mu(g, \delta g, \pounds_\xi g) = \partial_\nu S^{[\mu\nu]}(g, \delta g, \pounds_\xi g)
$$

In nontrivial representations,

$$\delta_\eta H[\xi] \neq 0,$$

$$\Updownarrow$$

$$\int_{\partial\Sigma} (\mathrm{d}^{d-2}x)_{\mu\nu} S^{[\mu\nu]}(g, \pounds_\eta g, \pounds_\xi g) \neq 0.$$

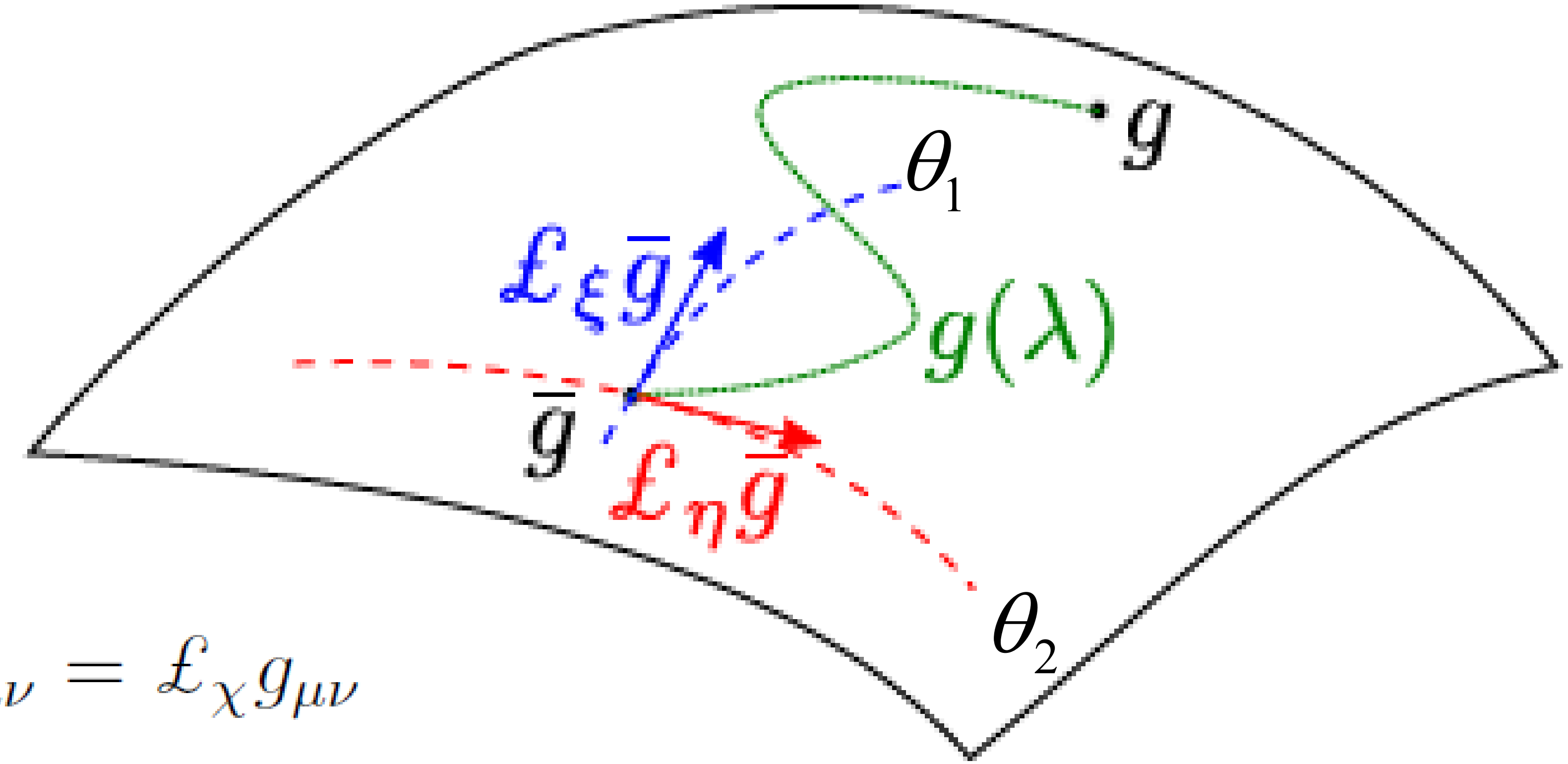
Tomitsuka Condition:

$$\int_{\partial\Sigma} (\mathrm{d}^{d-2}x)_{\mu\nu} S^{[\mu\nu]}(\bar{g}, \pounds_\eta \bar{g}, \pounds_\xi \bar{g}) \neq 0$$

Background metric

This guarantees nontrivial values of the charges. (Non-singlet.)
The task of finding asymptotic symmetry becomes much easier.

Asymptotic Metrics $\left\{ g_{\mu\nu}\left(x, \theta_1\xi + \theta_2\eta\right) = \frac{\partial \bar{x}^{\alpha}}{\partial x^{\mu}}\left(x, \theta_1, \theta_2\right)\frac{\partial \bar{x}^{\beta}}{\partial x^{\nu}}\left(x, \theta_1, \theta_2\right)\bar{g}_{\alpha\beta}\left(\bar{x}\right) \right\}$



$$\delta g_{\mu\nu} = \pounds_{\chi} g_{\mu\nu}$$

Full Integrability Condition:

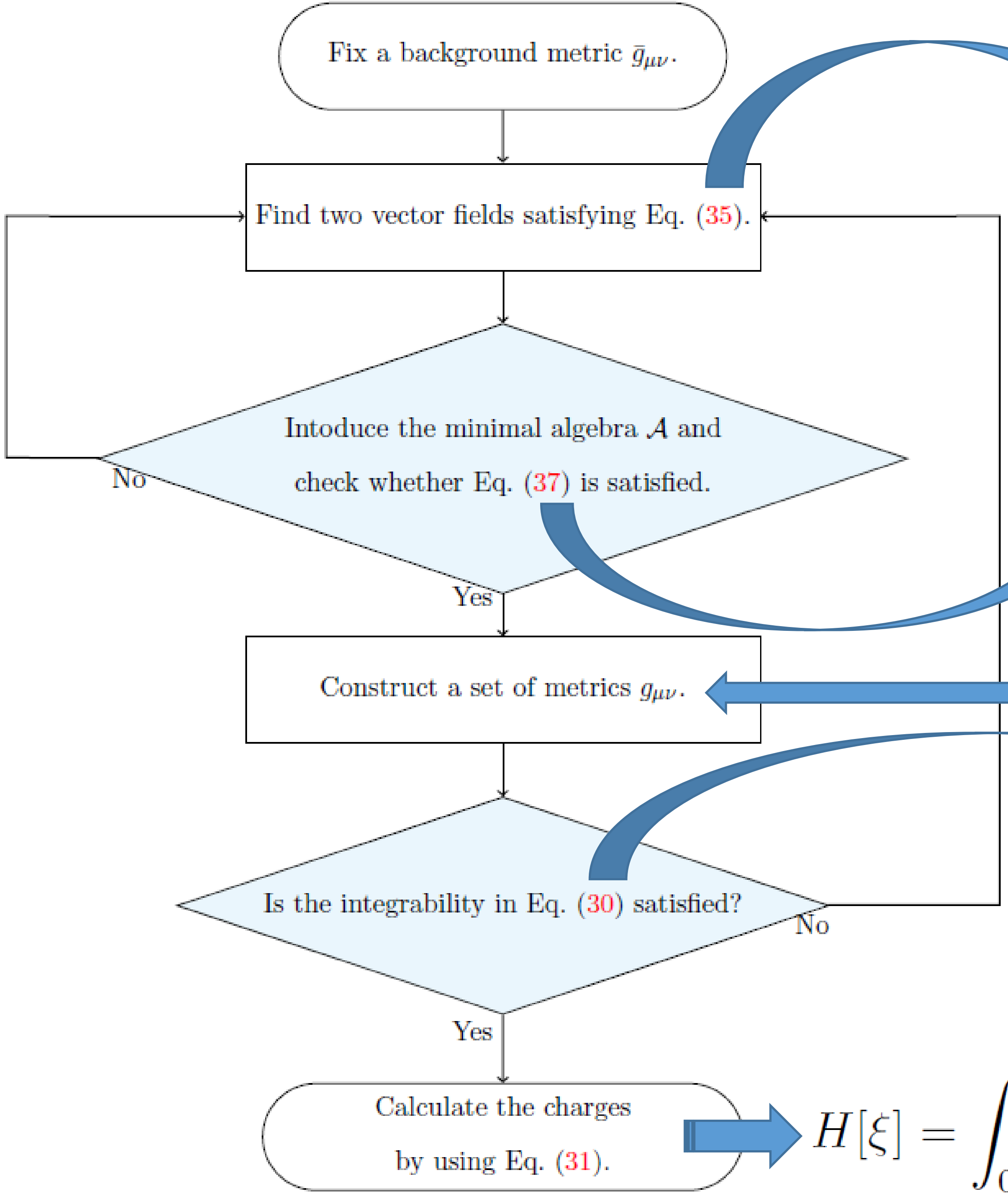
$$0 = \int_{\partial\Sigma} \left(\mathrm{d}^{d-2}x\right)_{\mu\nu} \xi^{[\mu} \partial_\alpha S^{\nu]\alpha} \left(g, \pounds_\eta g, \pounds_\chi g\right), \quad \forall \xi, \eta, \chi \in \mathcal{A}$$

We often fail to pass the test of this condition for our selected vector fields. In order to pass this integrability test easily, we should first check a simpler test for our vector fields. Just check the integrability at the back ground metric!

$$\int_{\partial\Sigma} \left(\mathrm{d}^{d-2}x\right)_{\mu\nu} \xi^{[\mu} \partial_\alpha S^{\nu]\alpha} \left(\bar{g}, \pounds_\eta \bar{g}, \pounds_\chi \bar{g}\right) = 0, \quad \forall \xi, \eta, \chi \in \mathcal{A}$$

If our choice of the vector fields is bad, we can notice the fact and go back soon to Tomitsuka condition equation!!

Our Approach



$$\int_{\partial\Sigma}(\mathrm{d}^{d-2}x)_{\mu\nu}S^{[\mu\nu]}(\bar{g},\pounds_{\eta}\bar{g},\pounds_{\xi}\bar{g})\neq 0$$

$$\int_{\partial\Sigma}(\mathrm{d}^{d-2}x)_{\mu\nu}\,\xi^{[\mu}\partial_{\alpha}S^{\nu]\alpha}(\bar{g},\pounds_{\eta}\bar{g},\pounds_{\chi}\bar{g})=0,\quad \forall\xi,\eta,\chi\in\mathcal{A}$$

Integrability check at background as easy work.

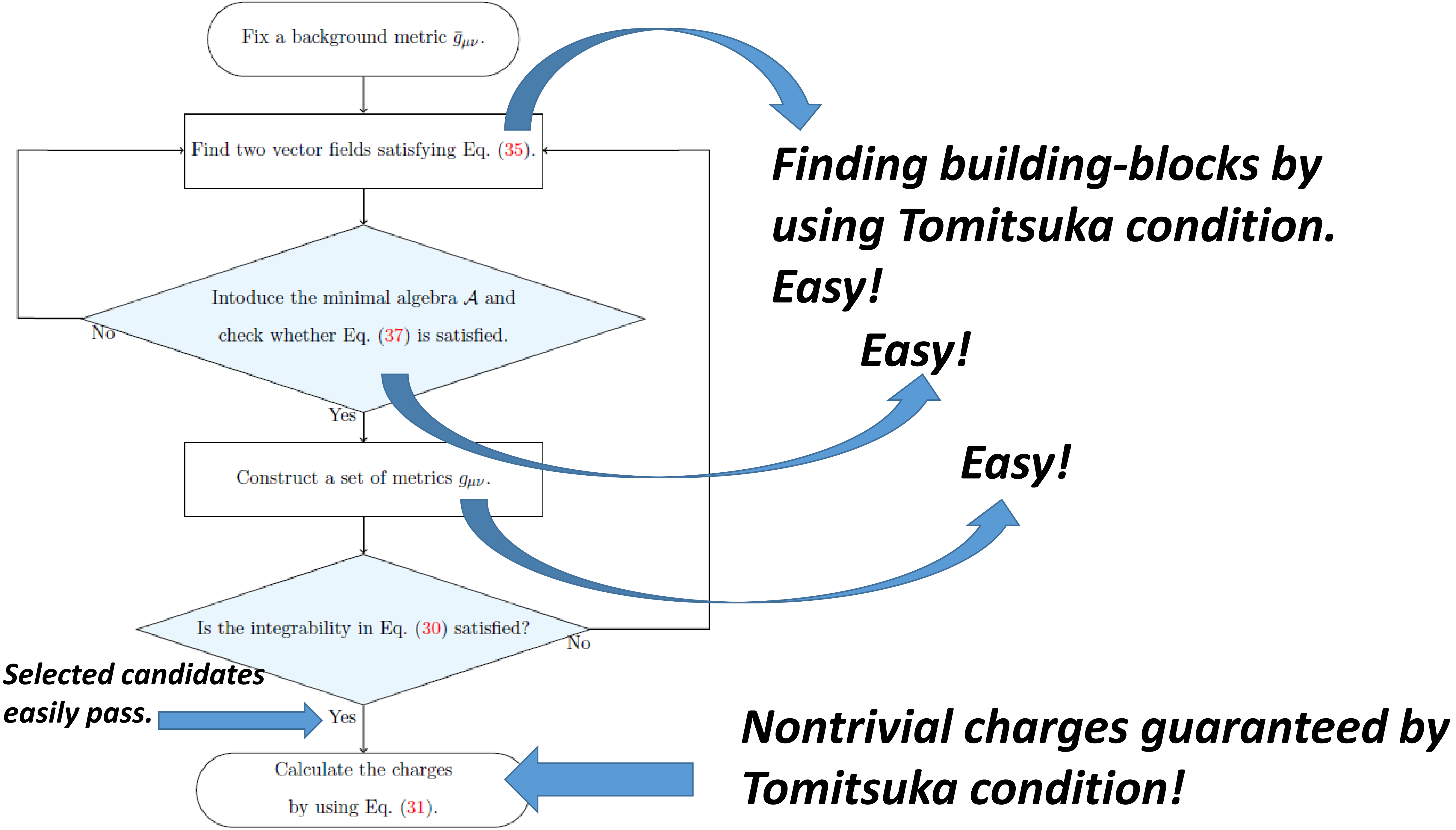
Candidates of the vector fields are selected.

by coordinates transformation
generated by the two vector fields.

$$0=(\delta_1\delta_2-\delta_2\delta_1)H[\xi]$$

Integrability check for all asymptotic metrics.

$$H[\xi]=\int_0^1\mathrm{d}\lambda\int_{\partial\Sigma}(\mathrm{d}^{d-2}x)_{\mu\nu}(\partial_{\lambda}Q^{\mu\nu}[\xi](g,\partial_{\lambda}g)+2\xi^{[\mu}\Theta^{\nu]}(g,\partial_{\lambda}g)).$$



Fix a background metric $\bar{g}_{\mu\nu}$.
Find two vector fields satisfying Eq. (35).
Intoduce the minimal algebra $\mathcal{A}$ and check whether Eq. (37) is satisfied.
No
Yes
Construct a set of metrics $g_{\mu\nu}$.
Is the integrability in Eq. (30) satisfied?
No
Yes
Calculate the charges by using Eq. (31).
Finding building-blocks by using Tomitsuka condition. Easy!
Easy!
Easy!
Selected candidates easily pass.
Nontrivial charges guaranteed by Tomitsuka condition!

After finding the building-block charges,
we can explore whether more nontrivial charges exist or not.

Rindler Soft Hair

$$\mathrm{d}\bar{s}^2 = -\kappa^2\rho^2\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \mathrm{d}y^2 + \mathrm{d}z^2.$$ **Horizon at** $\rho = 0.$

Horizon Condition: $\xi^\tau = \mathcal{O}(1), \quad \xi^\rho = \mathcal{O}(\rho), \quad \xi^y = \mathcal{O}(1), \quad \xi^z = \mathcal{O}(1).$

$$\xi \Rightarrow V_1 = (X^\tau(\tau, y, z) + \mathcal{O}(\rho), X^\rho(\tau, y, x)\rho + \mathcal{O}(\rho^2), X^A(\tau, y, z) + \mathcal{O}(\rho)),$$

$$\eta \Rightarrow V_2 = (Y^\tau(\tau, y, z) + \mathcal{O}(\rho), Y^\rho(\tau, y, z)\rho + \mathcal{O}(\rho^2), Y^A(\tau, y, z) + \mathcal{O}(\rho)),$$

Tomitsuka Condition:

$$\begin{aligned} 0 \neq \; & \int_{\partial\Sigma} (\mathrm{d}^{d-2}x)_{\mu\nu} S^{\mu\nu}(\bar{g}, \pounds_{V_2}\bar{g}, \pounds_{V_1}\bar{g}) \\ &= \frac{1}{16\pi G\kappa}\int_{\mathbb{R}^2} \Big(2\kappa^2(Y^A\partial_A X^\tau + Y^\tau\partial_A X^A) + \partial_A X^A \partial_\tau Y^\rho - \partial_\tau X^\tau \partial_\tau Y^\rho \\ &\qquad + \partial_\tau X^\rho \partial_\tau Y^\tau + Y^A \partial_\tau \partial_A X^\rho - \partial_A(Y^\rho \partial_\tau X^A)\Big)\mathrm{d}y\mathrm{d}z. \end{aligned}$$

New example satisfying Tomitsuka condition: superdilatation algebra (not SUSY!)

$$V_1 = \tau\partial_\tau + \mathcal{O}(\rho), \quad V_2 = \tau\rho F(y,z)\partial_\rho + \mathcal{O}(\rho^2)$$

$$[V_1, V_2] = V_2,$$

$$\xi^\mu_{(\theta_1,\theta_2)} := \theta_1 V_1 + \theta_2 V_2,$$

Generated asymptotic metrics by the building blocks

$$(g_{\mu\nu}(x)) = \left(\frac{\partial \phi^{\alpha}_{(\theta_1,\theta_2)}}{\partial x^{\mu}} \frac{\partial \phi^{\beta}_{(\theta_1,\theta_2)}}{\partial x^{\nu}} \bar{g}_{\alpha\beta}(\phi_{(\theta_1,\theta_2)}(x)) \right)$$

$$= \begin{pmatrix} \rho^2 e^{2ft}\left(-\kappa^2 e^{2\theta_1} + f^2\right) & \rho f e^{2ft} & 0 & 0 \\ \rho f e^{2ft} & e^{2ft} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$f = \frac{\theta_2}{\theta_1}(e^{\theta_1} - 1)$$

It turns out that the charges of superdilatation are integrable for the asymptotic metrics.

Charges of Superdilatation:

$$H\left[V_1\right] = \frac{1}{8\pi G}\int \frac{\theta_2}{2\kappa\theta_1}\left(1 - e^{-\theta_1}\right)dydz,$$

$$H\left[V_2\right] = \frac{1}{16\pi\kappa G}\int \left(e^{-\theta_1} - 1\right)dydz.$$

Open Question: Application to Brane-World Holography?

Our building-block approach is easy and powerful, and provides successful exploration for new asymptotic symmetries at horizons!

Our building-block approach is applicable to other boundaries in GR.

Actually, new results for finite-mass BH horizon and de Sitter horizon will be reported in our forthcoming papers.

Summary

○ *Stationary black holes actually carry non-vanishing classical charges of supertranslation and superrotation as asymptotic symmetries.*

○ *General formula of charge shift of supertranslation and superroration at Rindler horizons (infinite-mass-BH horizons) is available. ⇒ possible to resolve two pardoxs using the results.*

○ *Covariant building-block approach to asymptotic symmetry at boundary (Tomitsuka condition) is available. This provides superdilatation as a new soft-hair symmetry at Rindler horizons.*