Wormholes and holographic decoherence

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Osaka U.
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Talk based on

- arXiv:2012.03514v2, accepted by *JHEP*
- Collaborations w/ Takanori Anegawa, Kotaro Tamaoka, Tomonori Ugajin

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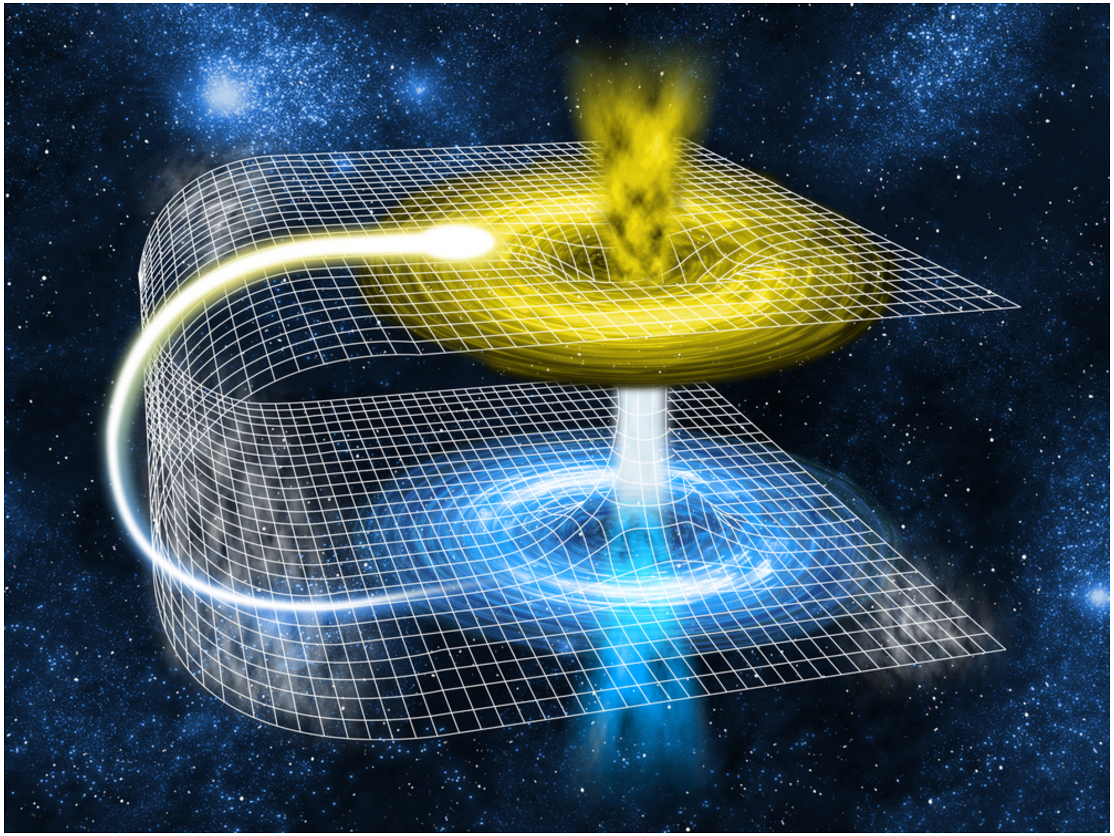
Takanori Anegawa M2 Student @ Osaka U.

Today's contents

- Introduction and key question
- Cooking recipes for wormholes
- Moduli parameter evolutions for decoherence
- Main results

Introduction

- Wormholes are interesting ‘saddle points’ in gravity path integral



Introduction

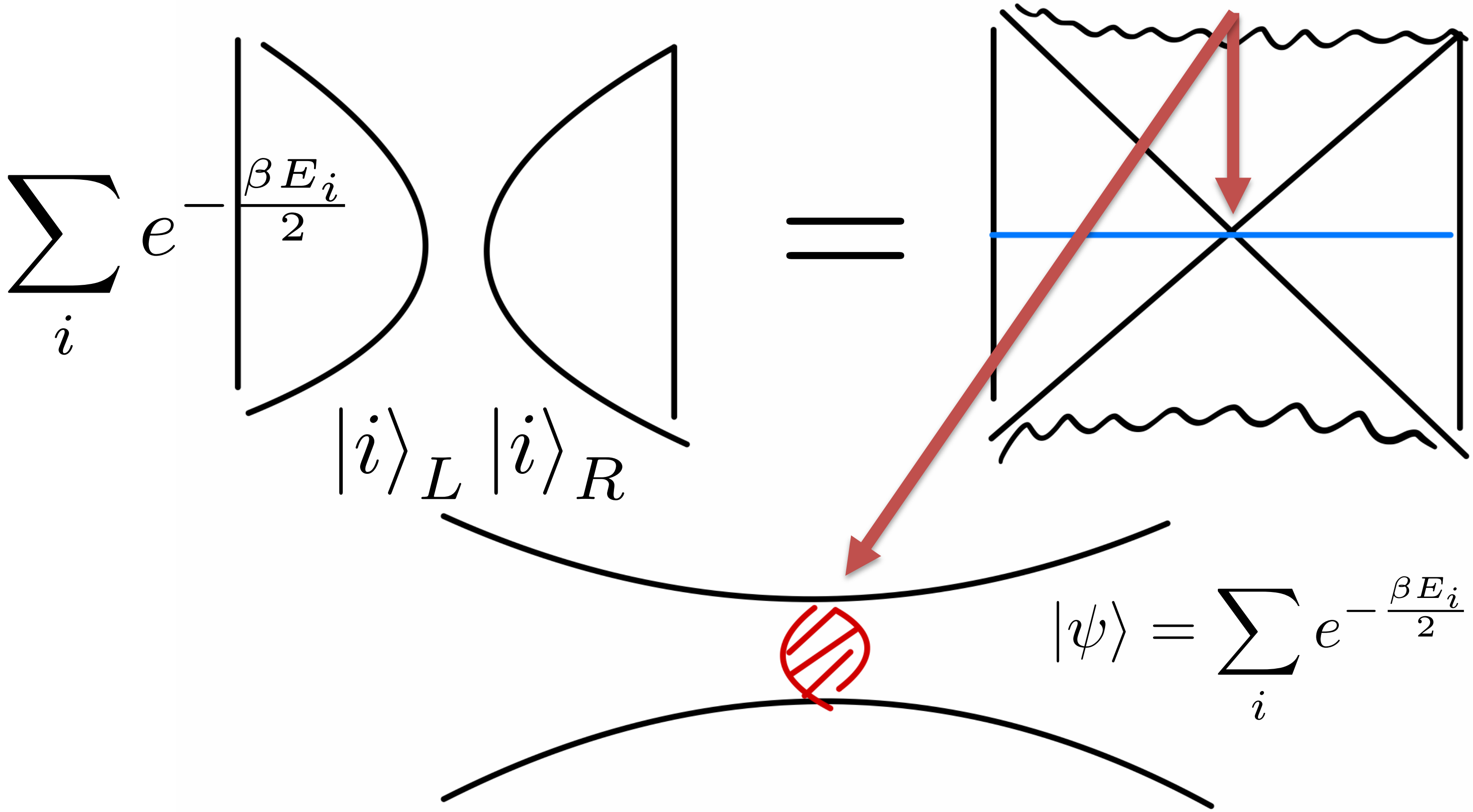
- Even though they play interesting roles in phenomenology (see arXiv:1807.00824), the main focus in today's talk is the implication of spacelike wormhole for holography

Van Raamsdoonk’s view and ER=EPR

Van Raamsdonk
Maldacena-Susskind

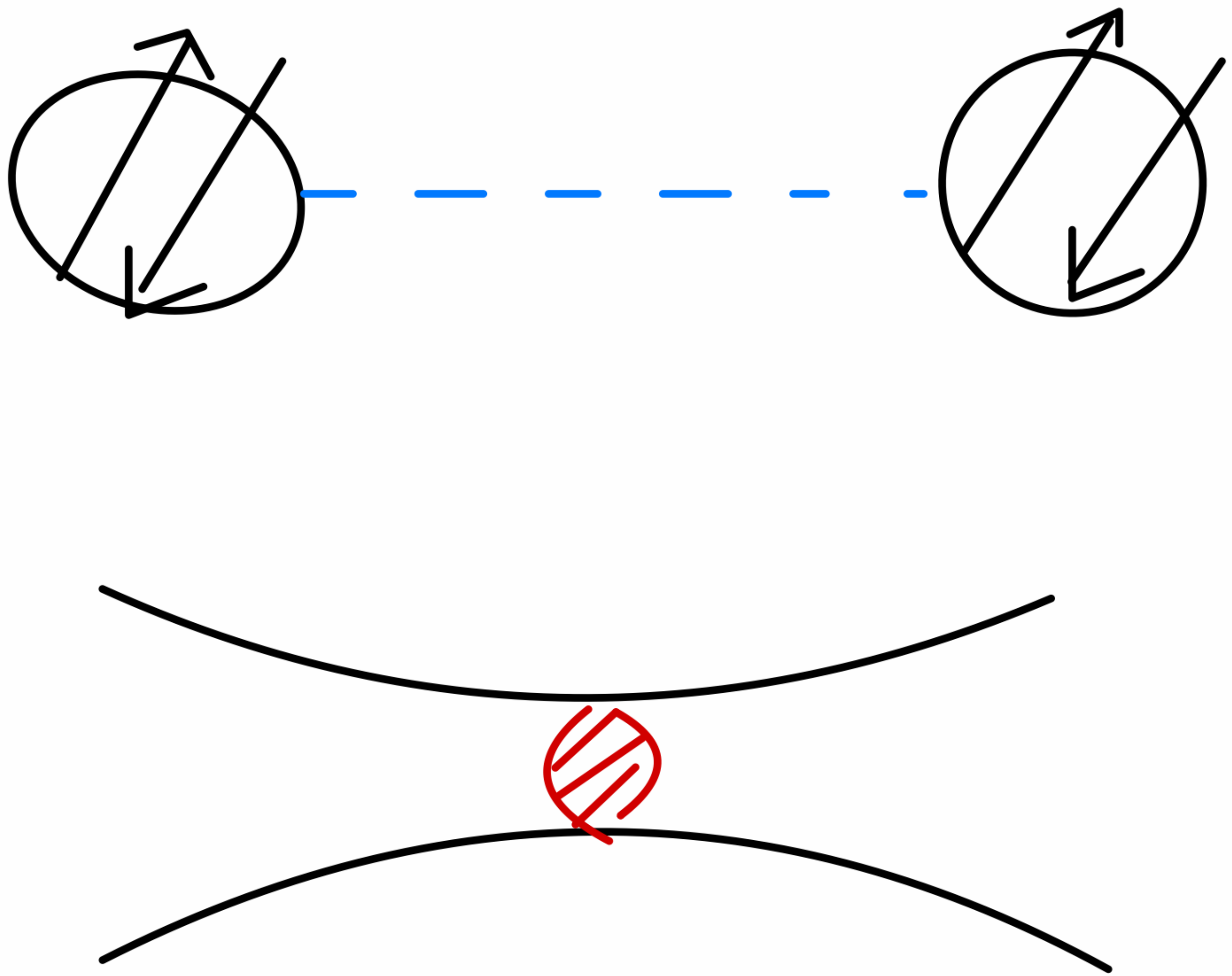
- Quantum entanglement is an indispensable ingredient for the emergence of smooth geometry in the semi-classical limit of gravity
- In other words, without quantum entanglement, we might have only disconnected geometries, instead of smooth connected geometry

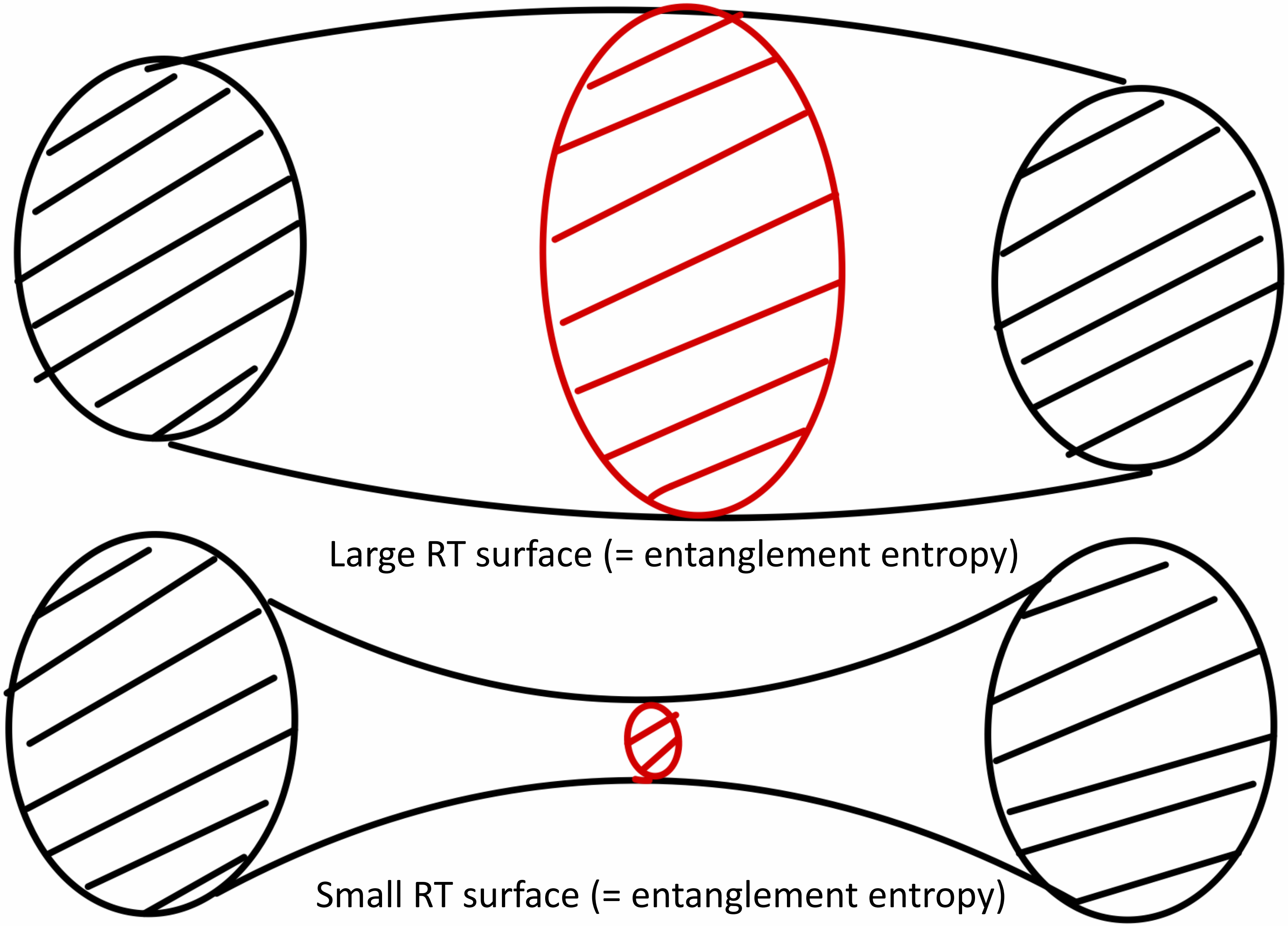
Thermo-field double = ER bridge



$$|\psi\rangle = \sum_i e^{-\frac{\beta E_i}{2}} |i\rangle_L |i\rangle_R$$

Israel, Maldacena, Balasubramanian-Kraus-Lawrence-Trivedi





Van Raamsdonk, Maldacena-Susskind

Our question

- On the other hand, correlation is not always induced quantum mechanically, and it might be possible that classical correlation can induce similar effects (smooth connected geometry).
- The main question in this talk is; *instead of quantum entanglement, can classical correlation have such a smooth geometric description in dual gravity?*

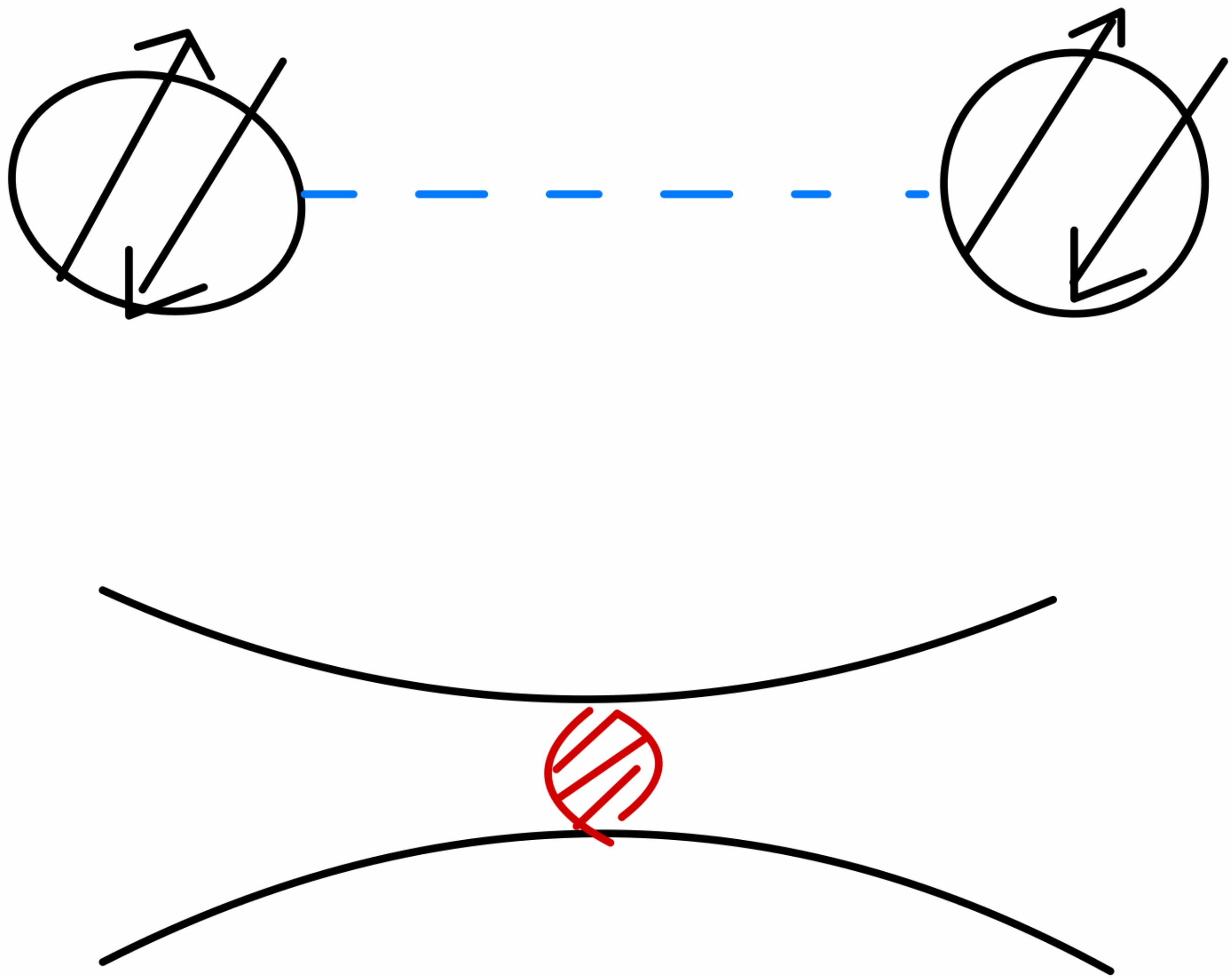
Verlinde

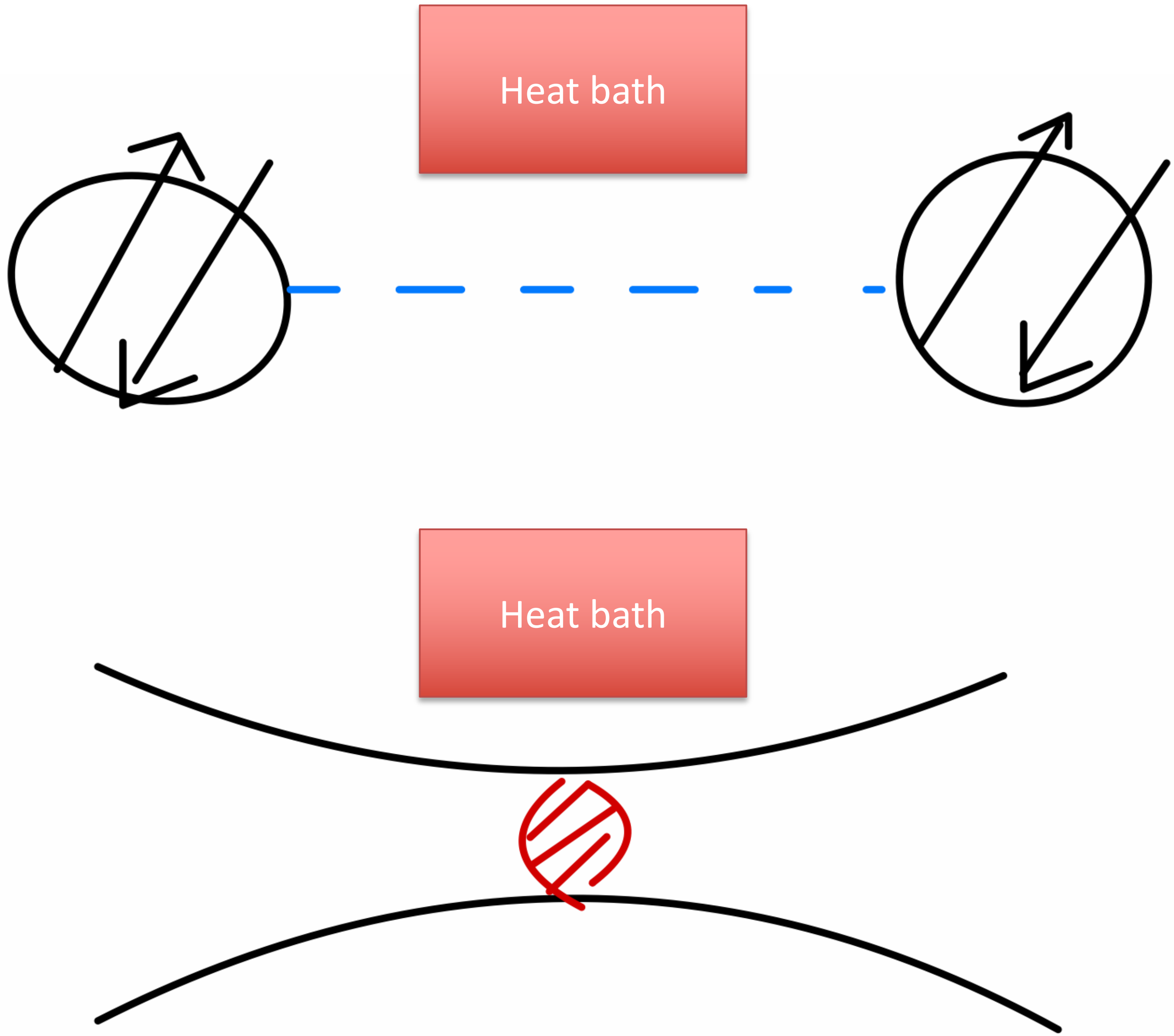
Our idea

- To understanad this, we consider following decoherence process;
- Start with an entangled state, which is dual to smooth connected wormhole

Our idea

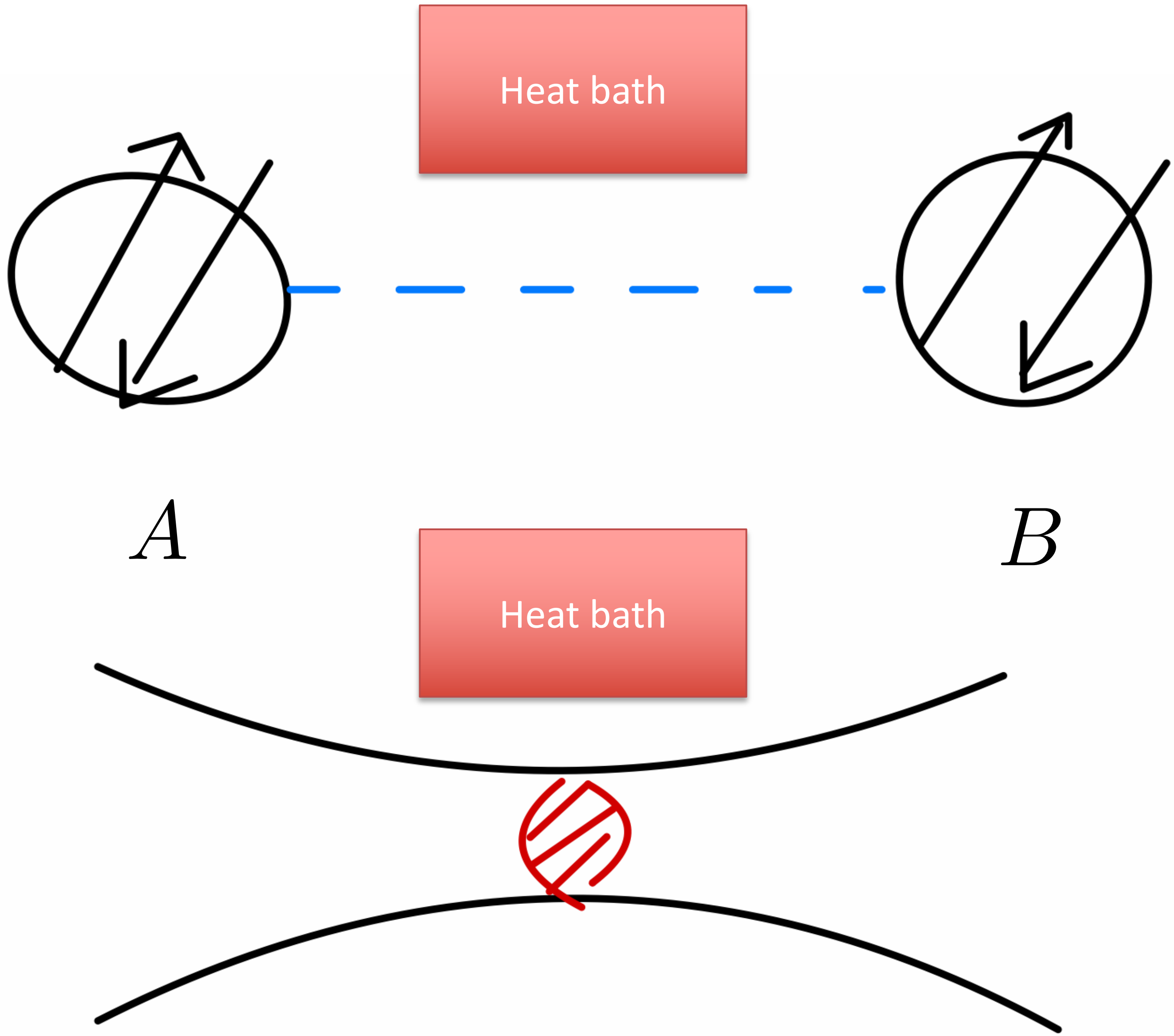
- Now we disturb the system to destroy the entanglement (i.e., decoherence process)
- We do this decoherence by attaching the sysmtem to external d.o.f. (which is heat bath/ environment)




Heat bath
Heat bath

Our setup

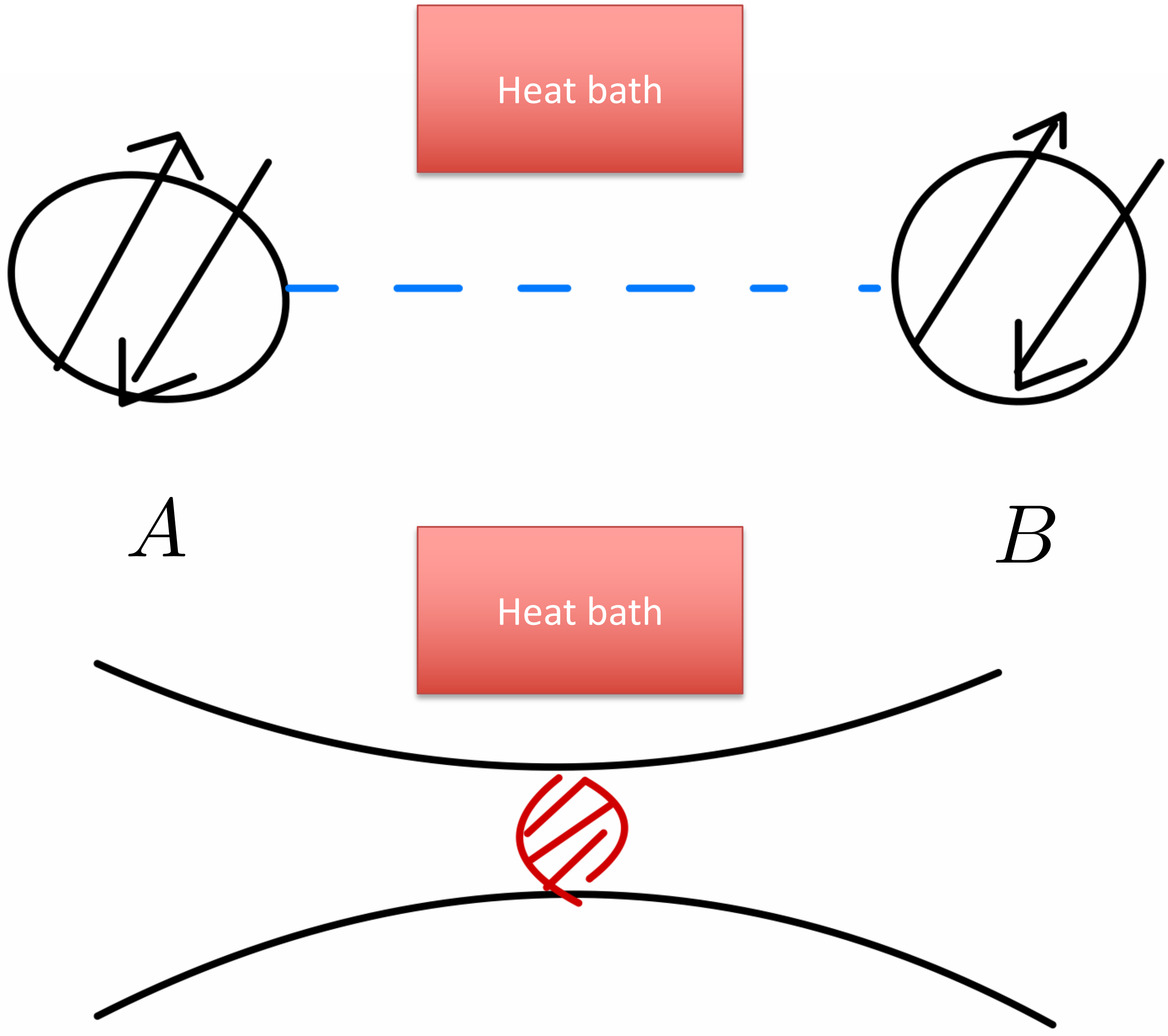
- Let us start from an AdS eternal black hole. The ER bridge of the eternal black hole is induced purely by quantum entanglement, since this two-sided eternal black hole is dual to a thermo-field double state on a bipartite system.
- Let us call this bi-partite system as A and B.

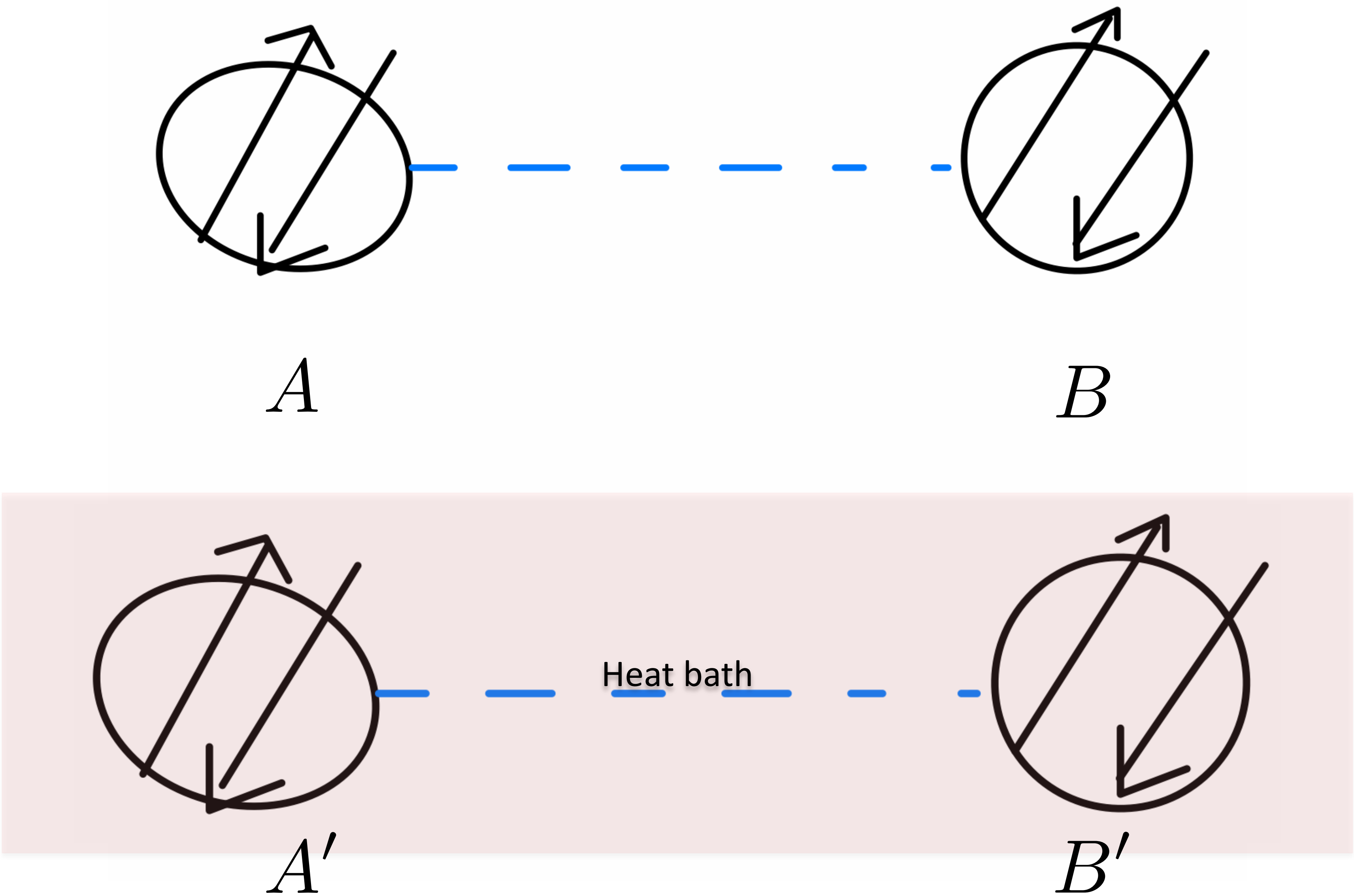


Heat bath
A
B
Heat bath

Our setup

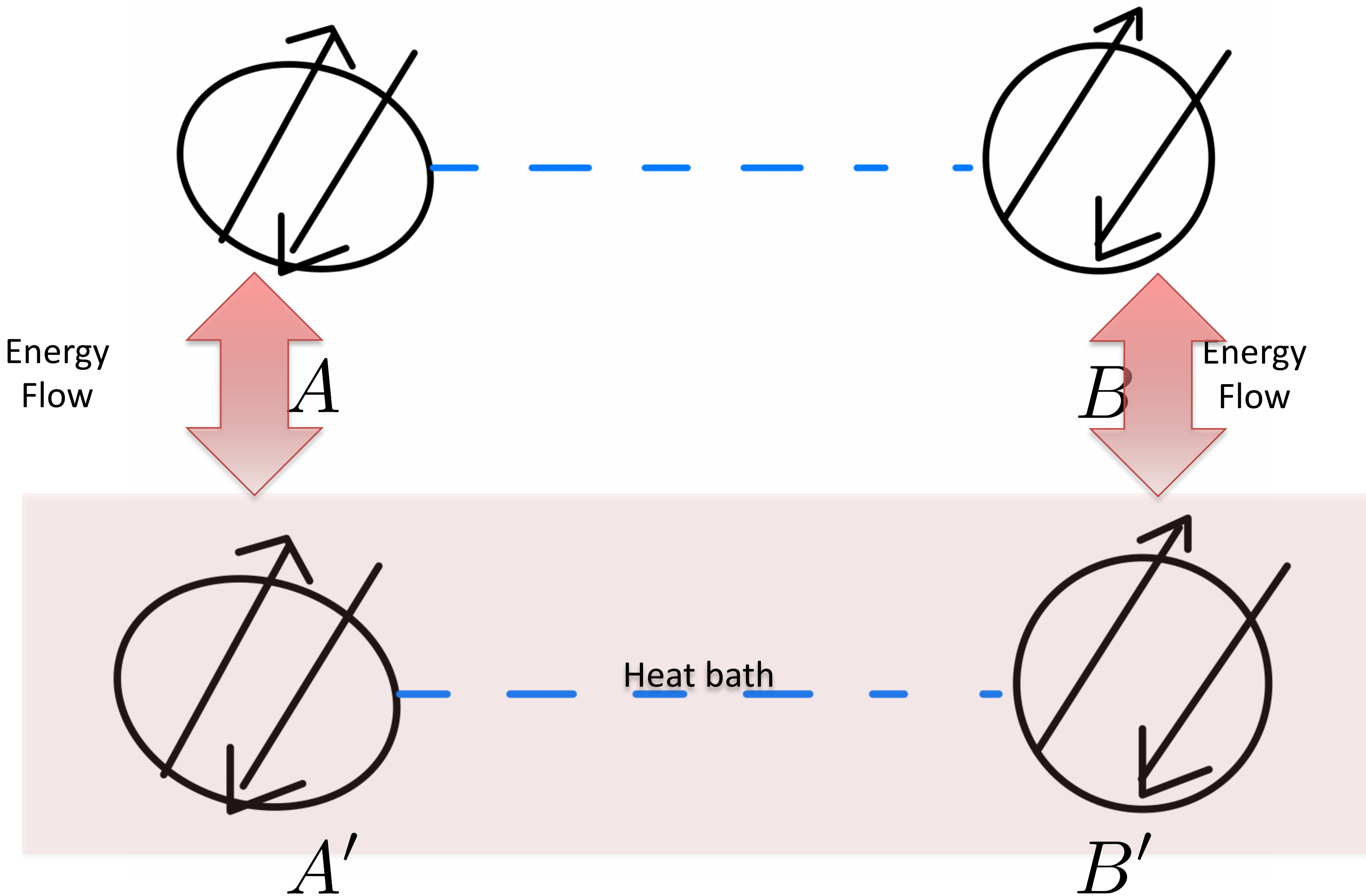
- We then prepare an auxiliary bipartite system A′ and B′ which is again modeled by another eternal black hole.
- This auxiliary bipartite system A′ and B′ plays the role of heat bathes/environment.


Heat bath
A
B
Heat bath

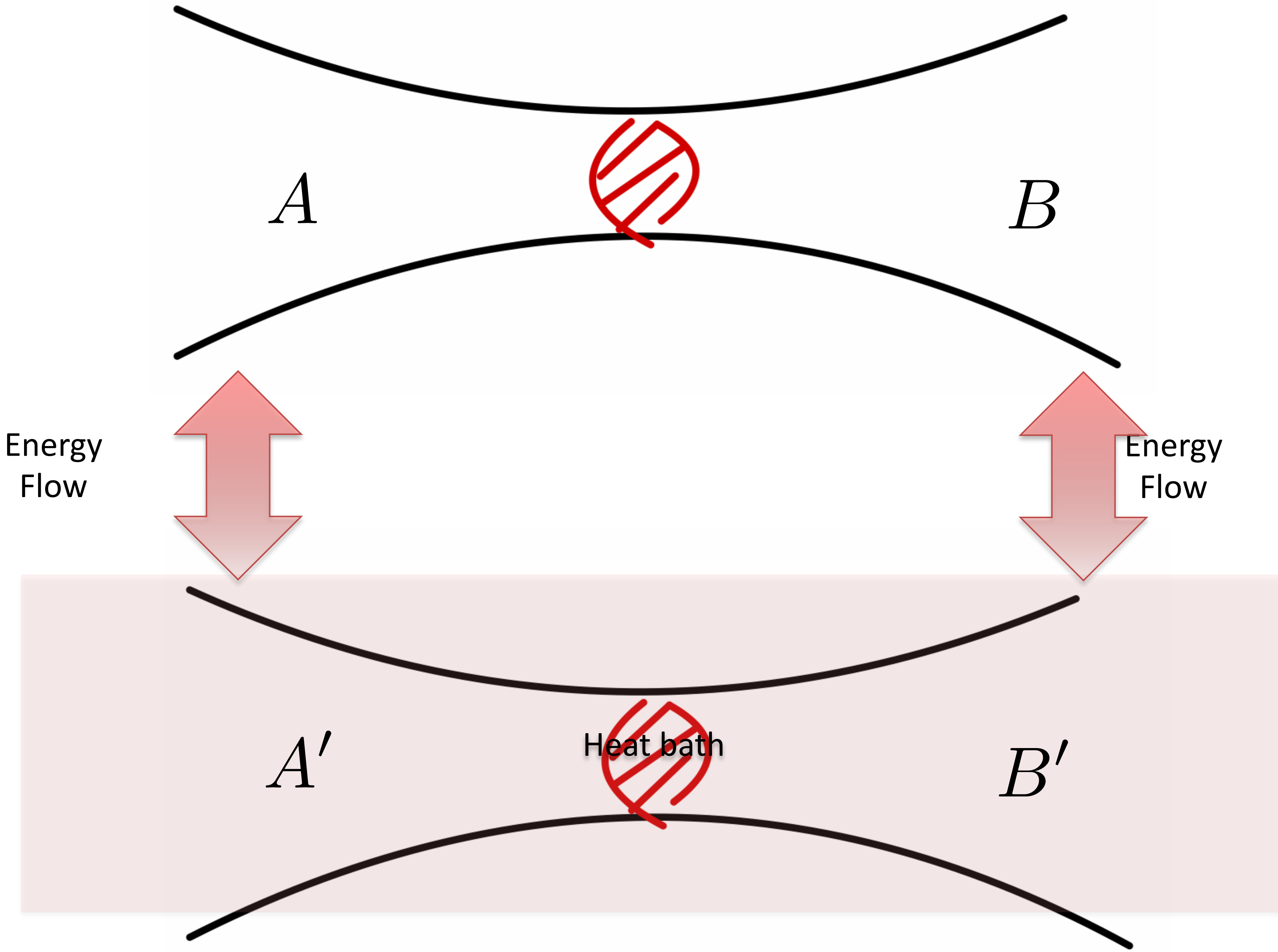

A
B
Heat bath
A'
B'

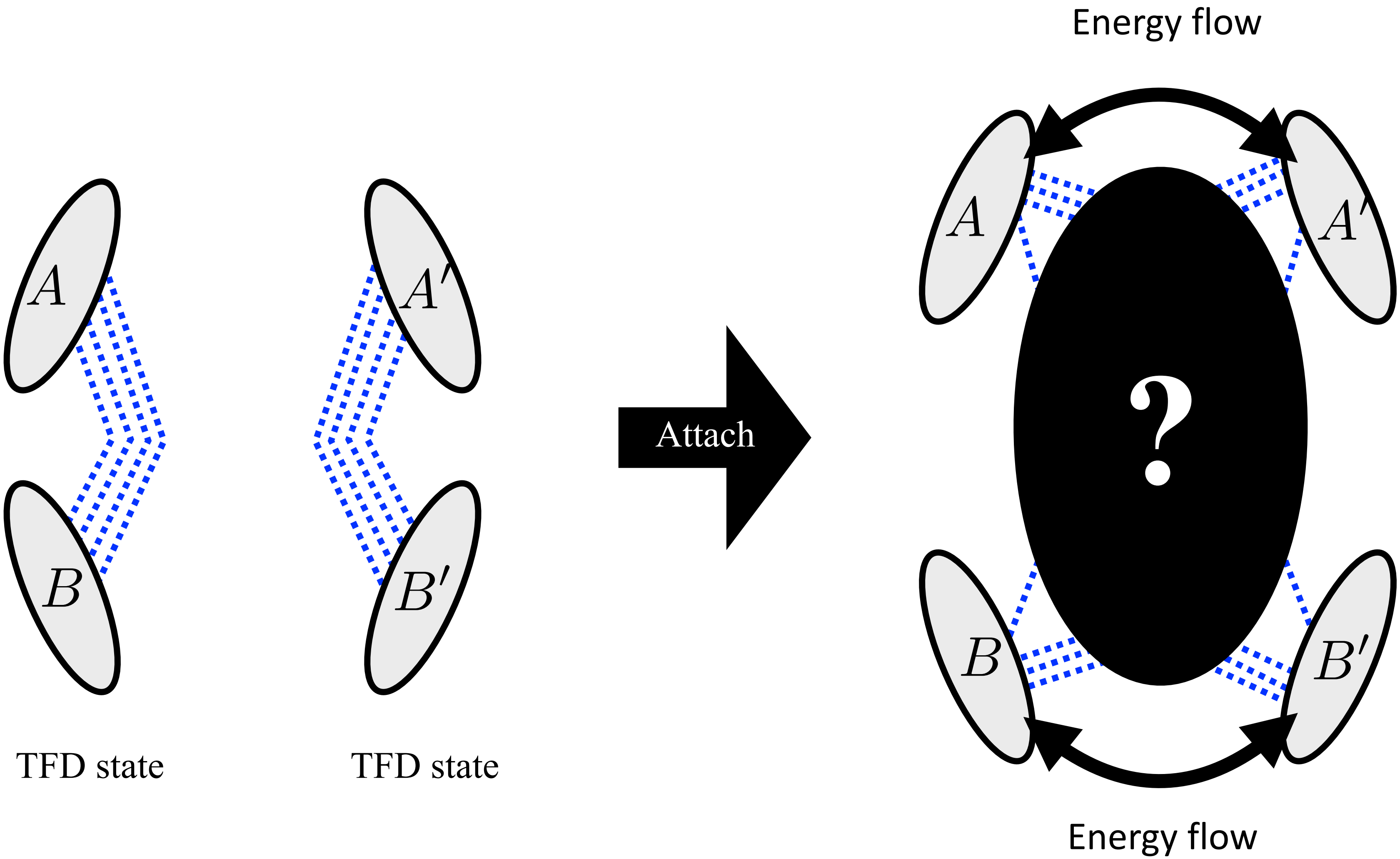
Our setup

- We then attach this auxiliary black hole (A′ and B′) to the original two sided black hole (A and B) and allow the energy flow from A to A′, and similarly, from B to B′.
- In the dual conformal field theory point of view, this process induces equilibration between A and A′ and similarly B and B′ and simultaneously, induces decoherence between A and B.

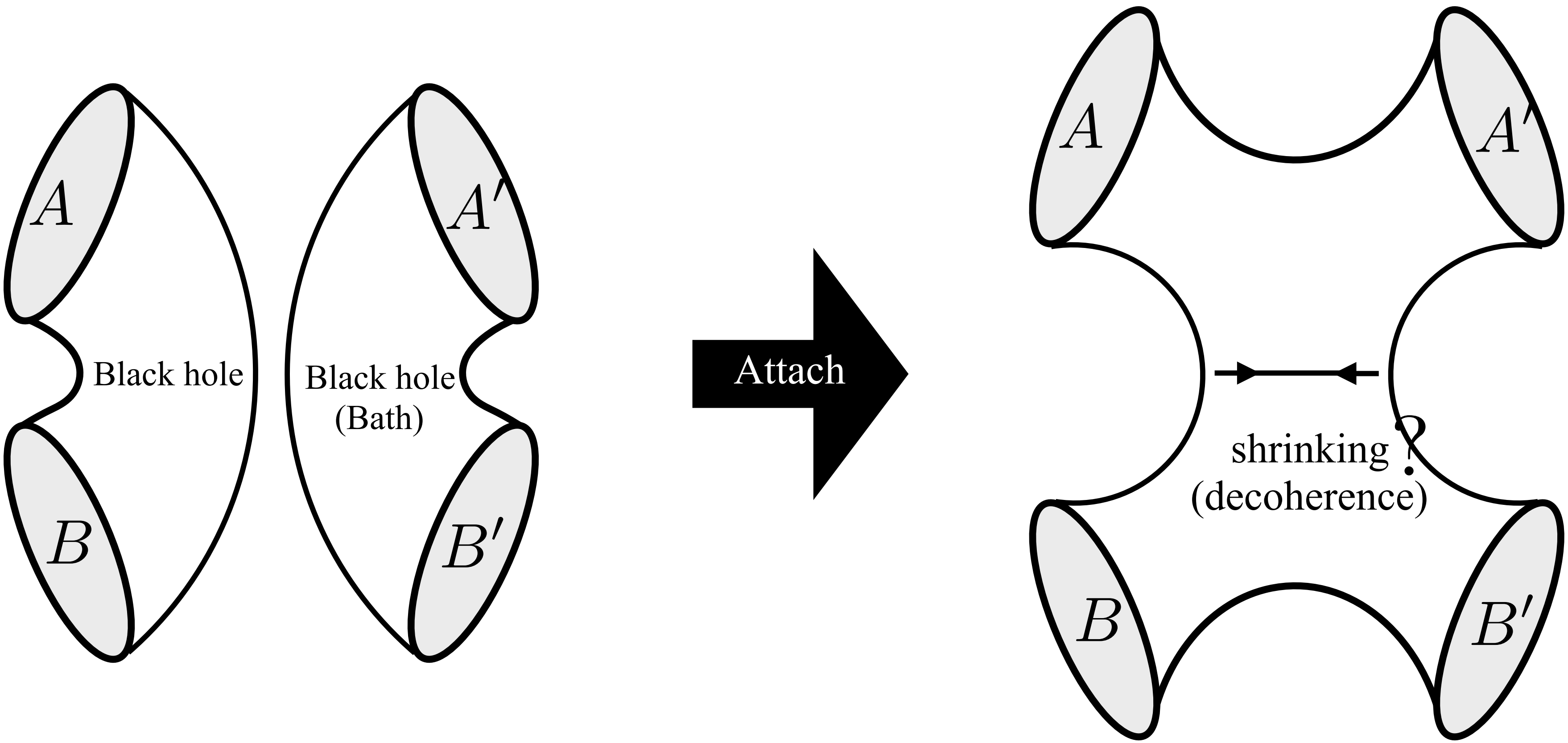


Energy
Flow
A
B
Energy
Flow
Heat bath
A'
B'


A
B
Energy Flow
Energy Flow
A'
Heat bath
B'



Energy flow
A
A'
B
B'
TFD state
TFD state
Attach
?
A
A'
B
B'
Energy flow



A
B
Black hole
A'
B'
Black hole
(Bath)
Attach
A
A'
shrinking ?
(decoherence)
B
B'

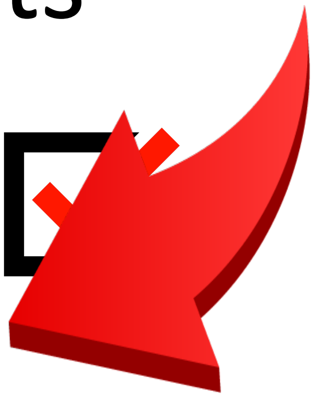
Our setup

- What we would like to see is, as the initial thermo-field double state (A and B) interacts with heat bathes (A’ and B’), how the original quantum entanglement between A and B can be washed out, and leave, even if exist, only classical correlation.
- In this talk, we concretely study this decoherence process in the AdS3/CFT2 setup.

Main results

- We will see that the final state of the holographic decoherence process can not have any correlation between A and B, both classically and quantum mechanically.
- This in particular means that we cannot construct an ER bridge which only contains classical correlation, at least in the moduli space we studied.

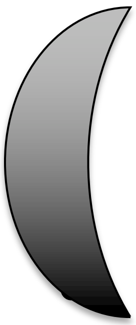
Today's contents

- Introduction and key question 
- Cooking recipes for wormholes
- Moduli parameter evolutions for decoherence
- Main results

Cooking recipes for 3D wormholes

Cooking recipes for 3D wormholes

- First, you need to prepare followings;



  ∗ 1 piece of AdS3

  ∗ One pair of scissors (to cut space)

  ∗ Glue (to attach space)

Cooking recipes for 3D wormholes

- First, you need to prepare followings;
  - ∗ 1 piece of AdS3
  - ∗ One pair of scissors (to cut space)
  - ∗ Glue (to attach space)

This recipe is a bit technical!

Cooking recipes for 3D wormholes

- First, you need to prepare followings;
  * 1 piece of AdS3
  * One pair of scissors (to cut space)
  * Glue (to attach space)

This recipe is a bit technical!

1 piece of AdS3

- AdS3 can be embedded by following 4-dim space-time;

$$ds^2 = -dU^2 - dV^2 + dX^2 + dY^2\,,$$

$$-U^2 - V^2 + X^2 + Y^2 = -1\,,$$

- Two (famous) expressions for AdS are known, global coord., and Poincare coord.

1 piece of AdS3

- In this talk, we use only Poincare coord.,

$$U = \frac{1}{2z}\left(x^2 - t^2 + z^2 + 1\right), \quad V = \frac{t}{z},$$

$$X = \frac{1}{2z}\left(x^2 - t^2 + z^2 - 1\right), \quad Y = \frac{x}{z},$$

- Then the metric for AdS3 becomes

$$ds^2 = \frac{-dt^2 + dx^2 + dz^2}{z^2}$$

1 piece of AdS3

- Especially, the $t = 0$ slice is

$$U = \frac{1}{2z}\left(x^2 + z^2 + 1\right), \quad V = 0,$$
$$X = \frac{1}{2z}\left(x^2 + z^2 - 1\right), \quad Y = \frac{x}{z},$$

- Then the metric becomes $Z \equiv x + iz$

$$ds^2 = \frac{dx^2 + dz^2}{z^2} = \frac{dZ\, d\bar{Z}}{|\mathrm{Im} Z|^2}.$$

1 piece of AdS3

- Since AdS3 has a boundary at $z = 0$, the geometry is bounded as

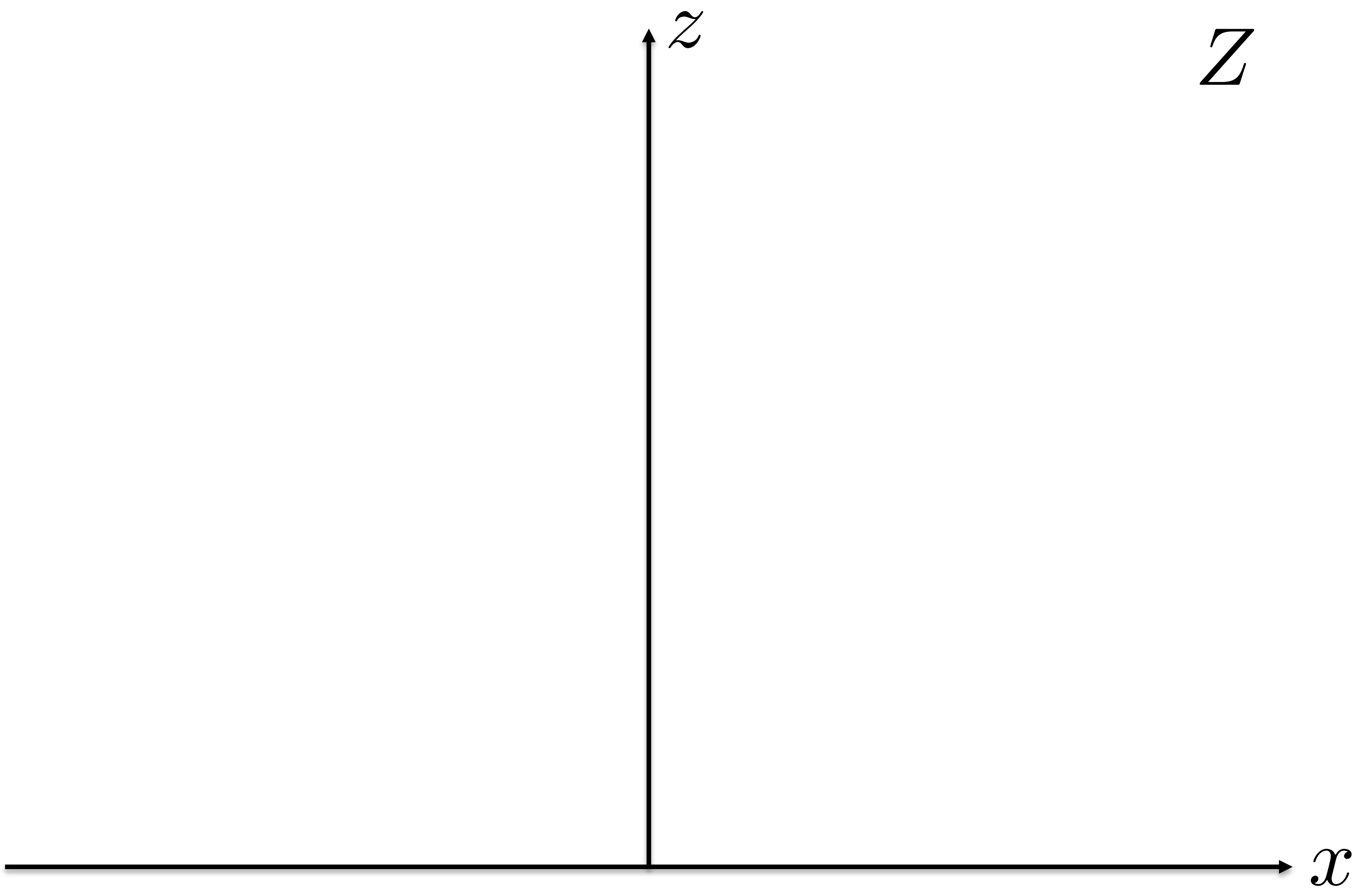
$$z > 0$$

- Then it is clear that AdS3 is *conformally* equivalent to complex plane with

$$\mathrm{Im} Z \equiv \mathrm{Im}\left(x + iz\right) = z > 0$$

- So AdS3 is just an upper half plane in Z

AdS3 in Z



1 piece of AdS3

- We can show that 3D gravity with negative cosmological constant is so simple such that any locally AdS3 geometry is a solution of the Einstein equations for pure gravity w/ $\Lambda < 0$
- The easiest way to understand this is that in 3D, there is no local degrees of freedom, therefore it has at most global structure to have non-trivial geometries
- Any nontrivial sol'ns are nontrivial oly globally

1 piece of AdS3

- Construction method of any sol'ns of pure gravity in 3D
- Given AdS3, find isometry of AdS3, and devide the space by its isometry then we obtain new solution, to see this, I illustrate the simplest example;
- Consider 1D flat space w/ translational isometry;
- Now identify by this isometry $x \sim x + L$
- This is exactly the circle compactification, global structure is modified but locally it is the same.

1 piece of AdS3

- Similarly one can identify the isometry of AdS3 and devide the space by its isometry; in this way we obtain different spacetime soln.
- This is quotient of two-dimensional hyperbolic space H^2/Γ
- We focus on $t=0$ slice which is H^2

$$Z \equiv x + iz$$

$$ds^2 = \frac{dx^2 + dz^2}{z^2} = \frac{dZ d\bar{Z}}{|\mathrm{Im} Z|^2} .$$

Cooking recipes for 3D wormholes

- First, you need to prepare followings;
  - ∗ 1 piece of AdS3
  - ∗ One pair of scissors (to cut space-time)
  - ∗ Glue (to attach space-time)

Isometry of AdS3 w/ $t = 0$

- To see the isometry of AdS3 w/ $t = 0$, it is useful to re-express AdS3 by matrix rep'n;

$$\hat{M} := \begin{pmatrix} U + X & Y + V \\ Y - V & U - X \end{pmatrix} , \quad d\hat{M} = \begin{pmatrix} dU + dX & dY + dV \\ dY - dV & dU - dX \end{pmatrix} ,$$

$$ds^2 = -\det d\hat{M} \,, \quad \text{where} \quad \det \hat{M} = 1 \,.$$

$$\Leftrightarrow \quad \boxed{\begin{aligned} ds^2 &= -dU^2 - dV^2 + dX^2 + dY^2 \,, \\ & - U^2 - V^2 + X^2 + Y^2 = -1 \,, \end{aligned}}$$

Isometry of AdS3 w/ $t = 0$

- We are interested in $t = 0 \Leftrightarrow V = 0$

Isometry of AdS3 w/ $t = 0$

- Especially, the $t = 0$ slice is

$$U = \frac{1}{2z}\left(x^2 + z^2 + 1\right)\,, \qquad V = 0\,,$$
$$X = \frac{1}{2z}\left(x^2 + z^2 - 1\right)\,, \qquad Y = \frac{x}{z}\,,$$

- Then the metric becomes $Z \equiv x + iz$

$$ds^2 = \frac{dx^2 + dz^2}{z^2} = \frac{dZ\,d\bar{Z}}{|\mathrm{Im}Z|^2}\,.$$

Isometry of AdS3 w/ $t = 0$

- Clearly followings are isometry of AdS3

$$\boxed{\begin{pmatrix} U+X & Y+V \\ Y-V & U-X \end{pmatrix} \mapsto \gamma_1 \begin{pmatrix} U+X & Y+V \\ Y-V & U-X \end{pmatrix} \gamma_2^T}$$

$$\gamma_i \in SL(2, \mathrm{Real})$$

$$\hat{M} := \begin{pmatrix} U+X & Y+V \\ Y-V & U-X \end{pmatrix}, \quad d\hat{M} = \begin{pmatrix} dU+dX & dY+dV \\ dY-dV & dU-dX \end{pmatrix},$$

$$ds^2 = -\det d\hat{M}\,, \quad \text{where} \quad \det \hat{M} = 1\,.$$

- We are interested in $t = 0 \Leftrightarrow V = 0$

Isometry of AdS3 w/ $t = 0$

- One can show that for $t = 0 \Leftrightarrow V = 0$, only $\gamma_1 = \gamma_2 \equiv \gamma$ type is isometry

$$\boxed{\begin{aligned}\begin{pmatrix} U+X & Y \\ Y & U-X \end{pmatrix} &\mapsto \gamma \begin{pmatrix} U+X & Y \\ Y & U-X \end{pmatrix} \gamma^T \\ &= \begin{pmatrix} U'+X' & Y' \\ Y' & U'-X' \end{pmatrix}\end{aligned}}$$

- In other words,

$$\text{if } \gamma_1 \neq \gamma_2\,,\text{ then } V = 0 \to V' \neq 0$$

Isometry of AdS3 w/ $t = 0$

- Just as the simplest example, where we obtain S^1 by quotient of R^1, we devide AdS3 by its isometry

$$\gamma \in SL(2, \mathrm{Real})$$

- Again this does not change locally, so it gives solutions of the pure gravity Einstein equations (it changes only global structure)
- Before we proceed, we can classify

$$\gamma \in SL(2, \mathrm{Real})$$

$$\gamma \in SL(2, \mathrm{Real})$$

- We set $$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \; ad - bc = 1$$
- Then after some calculations, one can show

$$\boxed{\begin{aligned} \begin{pmatrix} U+X & Y \\ Y & U-X \end{pmatrix} &\mapsto \gamma \begin{pmatrix} U+X & Y \\ Y & U-X \end{pmatrix} \gamma^T \\ &= \begin{pmatrix} U'+X' & Y' \\ Y' & U'-X' \end{pmatrix} \end{aligned}}$$

$$\Updownarrow$$

$$Z \to Z' = x' + iz' = \frac{aZ+b}{cZ+d} = \frac{\{(ax+b)(cx+d)+acz^2\}+iz}{(cx+d)^2+(cz)^2} .$$

$$\gamma \in SL(2, \mathrm{Real})$$

- In summary, the isometry of AdS3 t=0 slice is

$$\boxed{\begin{aligned}\begin{pmatrix} U+X & Y \\ Y & U-X \end{pmatrix} &\mapsto \gamma \begin{pmatrix} U+X & Y \\ Y & U-X \end{pmatrix} \gamma^T \\ &= \begin{pmatrix} U'+X' & Y' \\ Y' & U'-X' \end{pmatrix}\end{aligned}}$$

- This is equivalent to

$$\Big(t = 0\,, Z = x + iz\Big) \to \left(t' = 0\,, Z' = \frac{aZ+b}{cZ+d}\right) .$$

Classification of $\gamma \in SL(2, \mathrm{Real})$

$\gamma \in SL(2, \mathrm{Real})$ has manifestly 3 independent parameters (therefore actions);

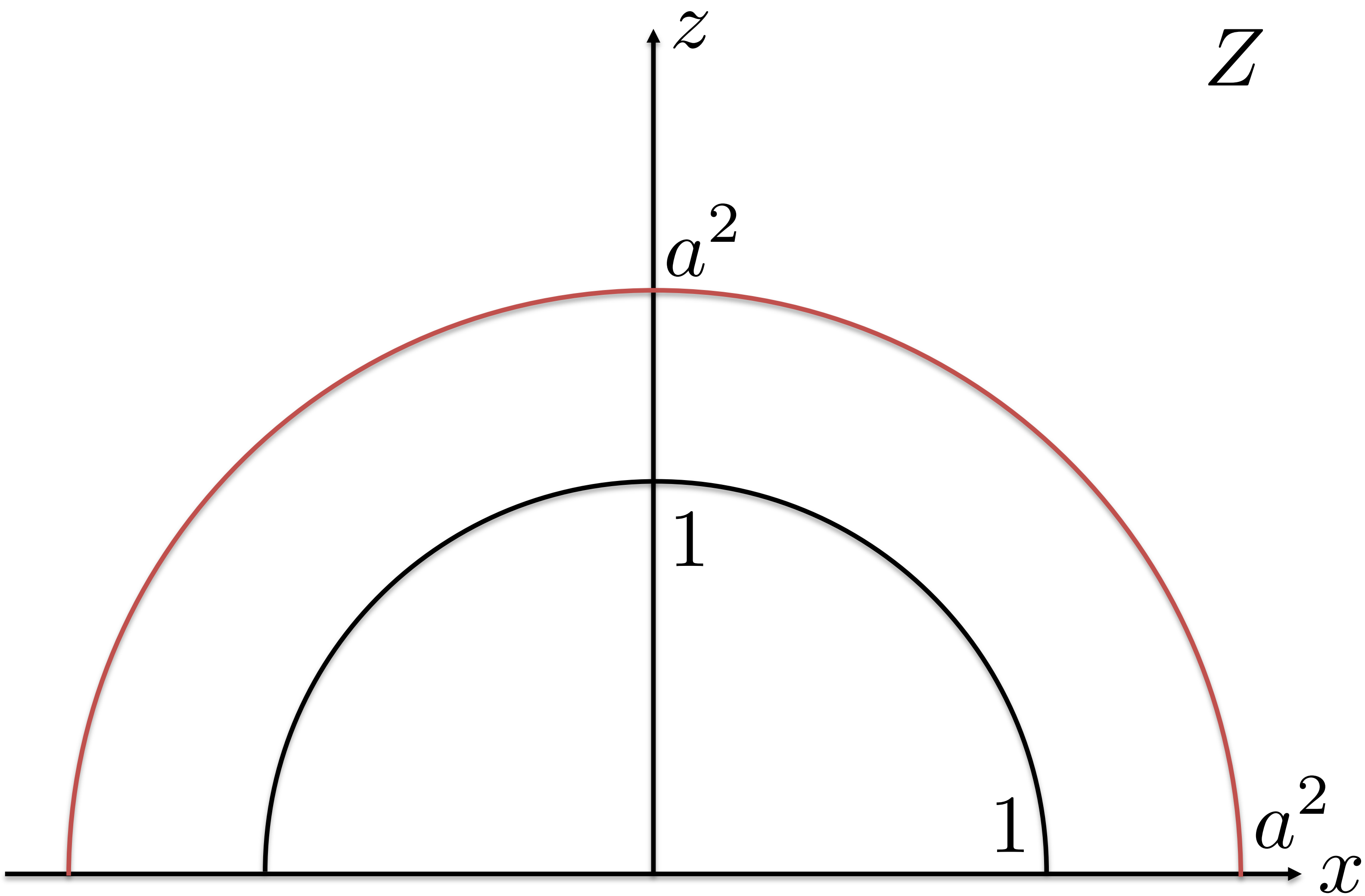
- Dilatation: this corresponds to the Mobius transformation w/

$$\gamma_D(a) = \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}$$

- Under this, the hyperbolic space coordinate transforms as

$$Z \to a^2 Z$$

AdS3 in Z



Dilatation

- In fact one can obtain non-rotating BTZ black hole from AdS3 as quotient by setting

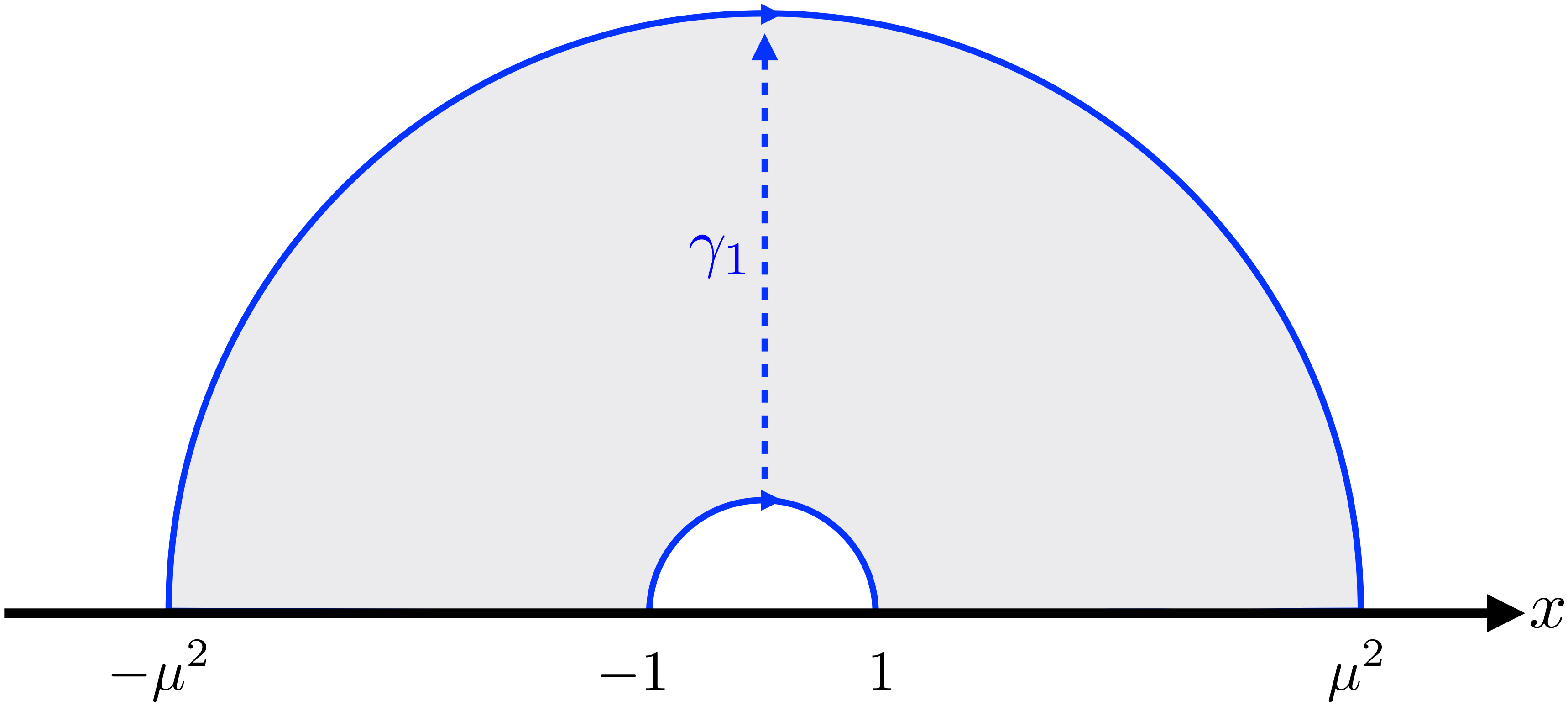
$$a^2 = \mu^2 = e^{2\pi r_h}$$

- Since then, the horizon "area" is given by

$$L_h = \int_1^{a^2} \frac{dz}{z} = \log a^2 = 2\pi r_h$$

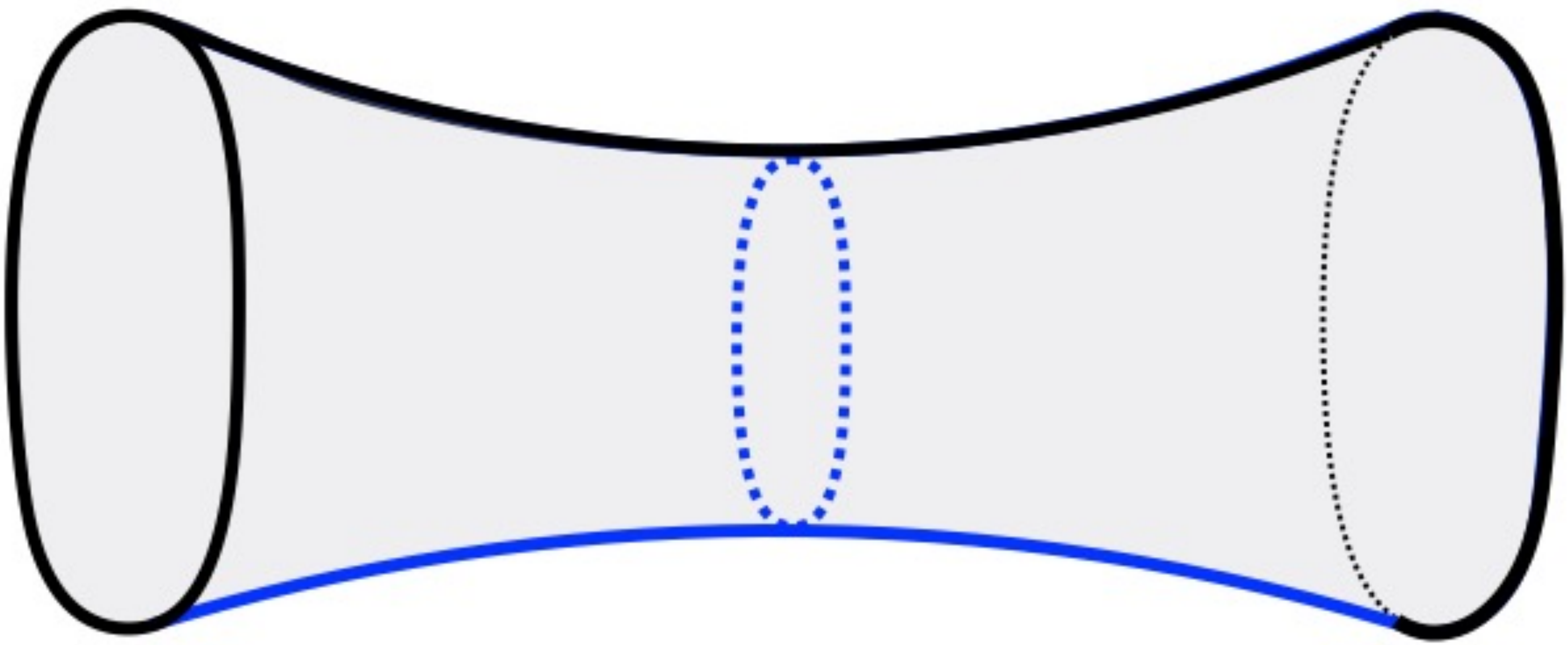
- This is exactly Einstein-Rosen bridge wormhole

(static) Eternal BTZ geometry



$$\boxed{a^2 = \mu^2 = e^{2\pi r_h}}$$

(static) Eternal BTZ geometry



Classification of $\gamma \in SL(2, \mathrm{Real})$

$\gamma \in SL(2, \mathrm{Real})$ has manifestly 3 independent parameters (therefore actions);

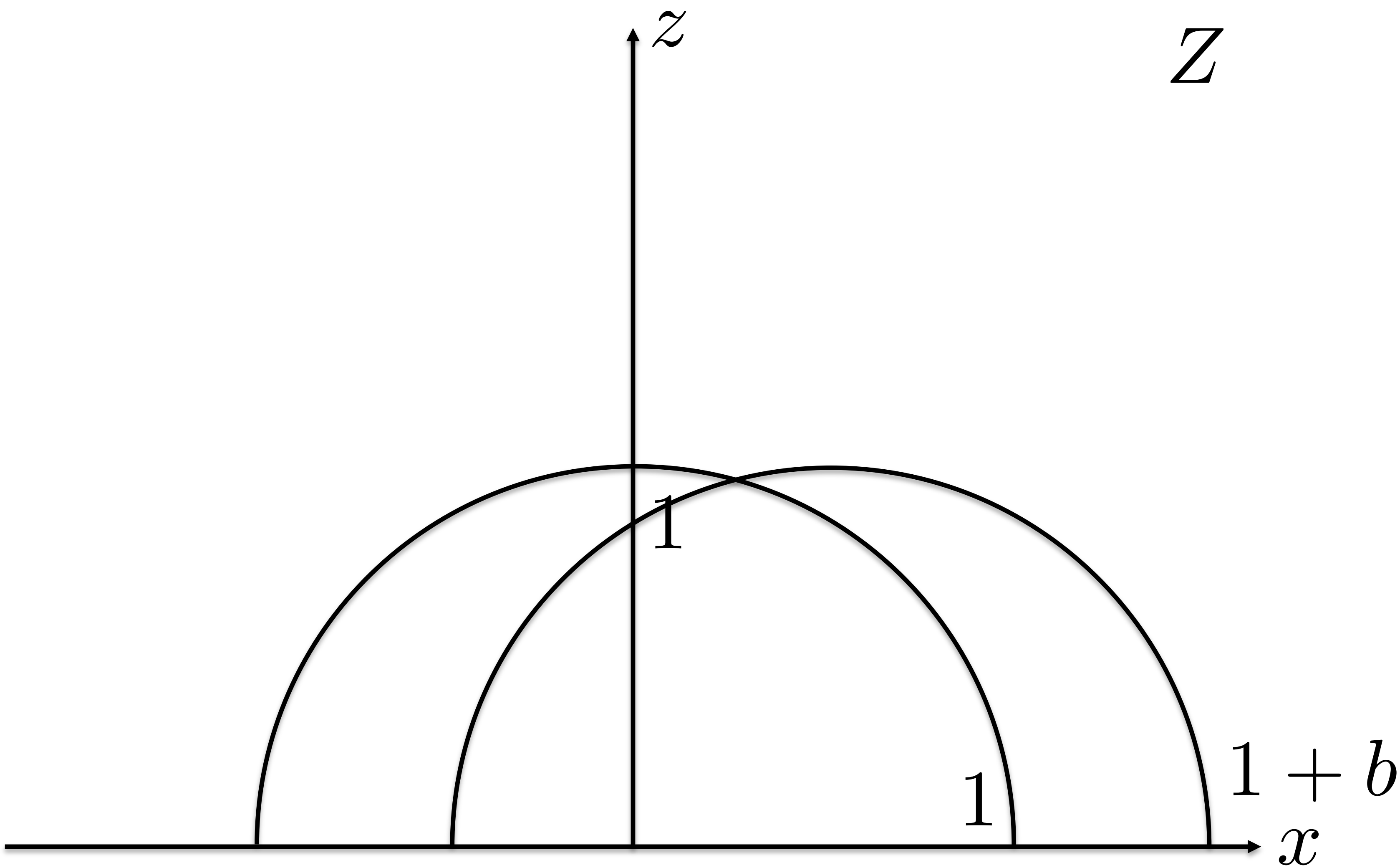
- Translation: this corresponds to the Mobius transformation w/

$$\gamma_T(b) = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$$

- Under this, the hyperbolic space coordinate transforms as

$$Z \to Z + b$$

AdS3 in Z



Classification of $\gamma \in SL(2, \mathrm{Real})$

$\gamma \in SL(2, \mathrm{Real})$ has manifestly 3 independent parameters (therefore actions);

- Special conformal transformation: this corresponds to the Mobius transformation w/

$$\gamma_{SC}(c) = \begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}$$

- Under this, the hyperbolic space coordinate transforms as

$$Z \to \frac{Z}{cZ+1}$$

Classification of $\gamma \in SL(2, \mathrm{Real})$

- Instead of special conformal transformation; it is more easy & convenient to define the third action as 'Inversion':

$$I(R) \equiv \gamma_T(R)\,\gamma_{SC}\left(-\frac{1}{R}\right)\,\gamma_T(R) = \begin{pmatrix} 0 & R \\ -\frac{1}{R} & 0 \end{pmatrix}$$

- The action of the inversion is

$$Z = x + iz \to -\frac{R^2}{Z} = R^2\left[-\frac{x}{x^2+z^2} + i\,\frac{z}{x^2+z^2}\right]$$

Classification of $\gamma \in SL(2, \mathrm{Real})$

- Therefore the inversion

$$Z = x + iz \to -\frac{R^2}{Z} = R^2 \left[-\frac{x}{x^2 + z^2} + i \frac{z}{x^2 + z^2} \right]$$

- Maps the circle

$$x^2 + z^2 = R^2$$

- To itself, but *flip* the orientation

$$x \leftrightarrow -x$$

- Similarly the inversion switches exterior and interior

Classification of $\gamma \in SL(2, \mathrm{Real})$

- Using these, one can construct isometry switching 2 circles;

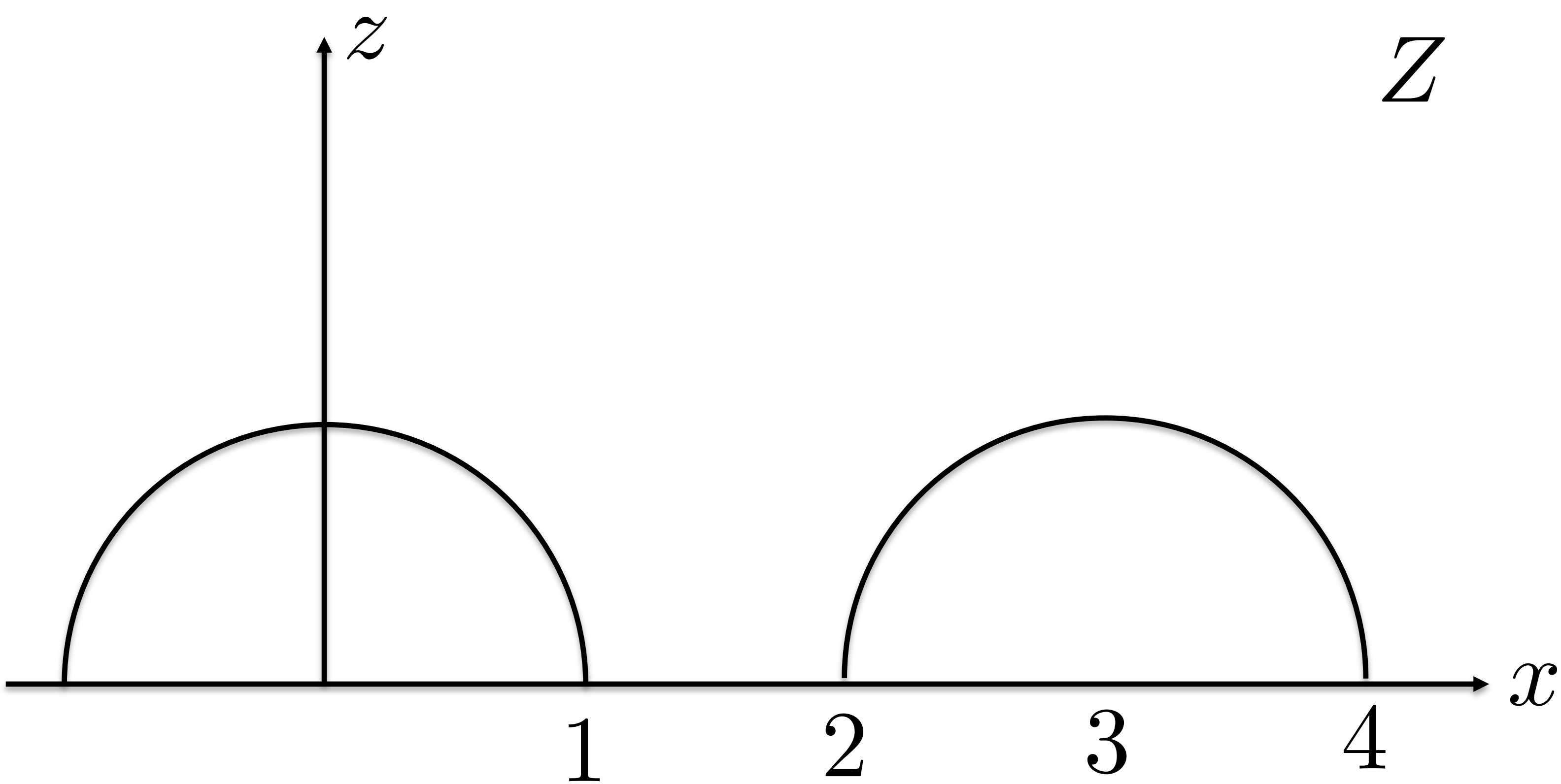
$$C_1 : (x - c_1)^2 + z^2 = R_1^2 \,, \quad C_2 := (x - c_2)^2 + z^2 = R_2^2 \,.$$

- A simple example;

$$R_1 = R_2 = 1 \,, c_1 = 0 \,, c_2 = 3$$

$$\gamma = \begin{pmatrix} -\frac{c_2}{\sqrt{R_1 R_2}} & \frac{c_1 c_2 + R_1 R_2}{\sqrt{R_1 R_2}} \\ -\frac{1}{\sqrt{R_1 R_2}} & \frac{c_1}{\sqrt{R_1 R_2}} \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ -1 & 0 \end{pmatrix}$$

AdS3 in Z



Fundamental domain: $1 \leq x < 2$

By just focusing on z = 0 line, Or $\dfrac{1}{2} \leq x < 1$ or $2 \leq x < \dfrac{5}{2}$ or etc…

Classification of $\gamma \in SL(2, \mathrm{Real})$

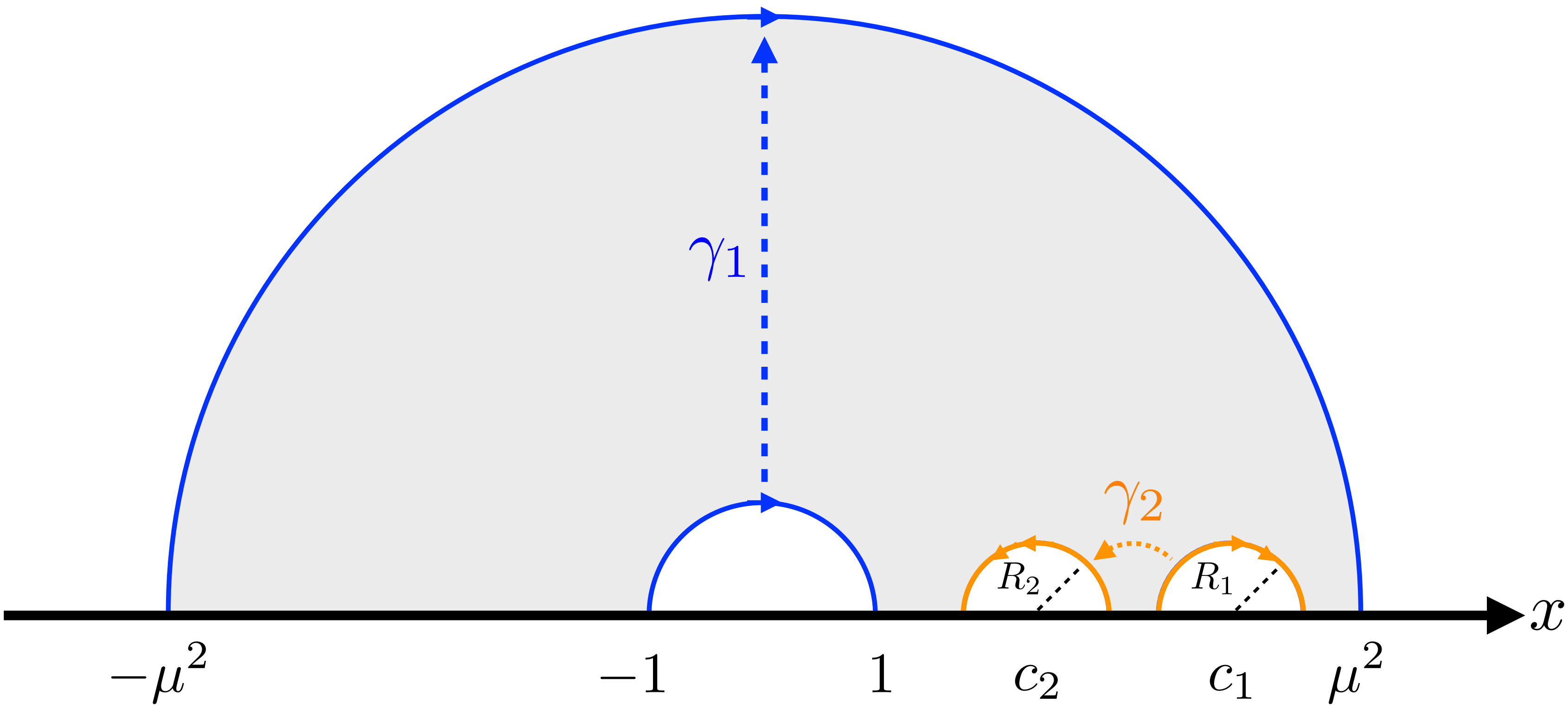
- Using these, one can construct isometry switching 2 circles;

$$C_1 : (x - c_1)^2 + z^2 = R_1^2\,, \quad C_2 := (x - c_2)^2 + z^2 = R_2^2\,.$$

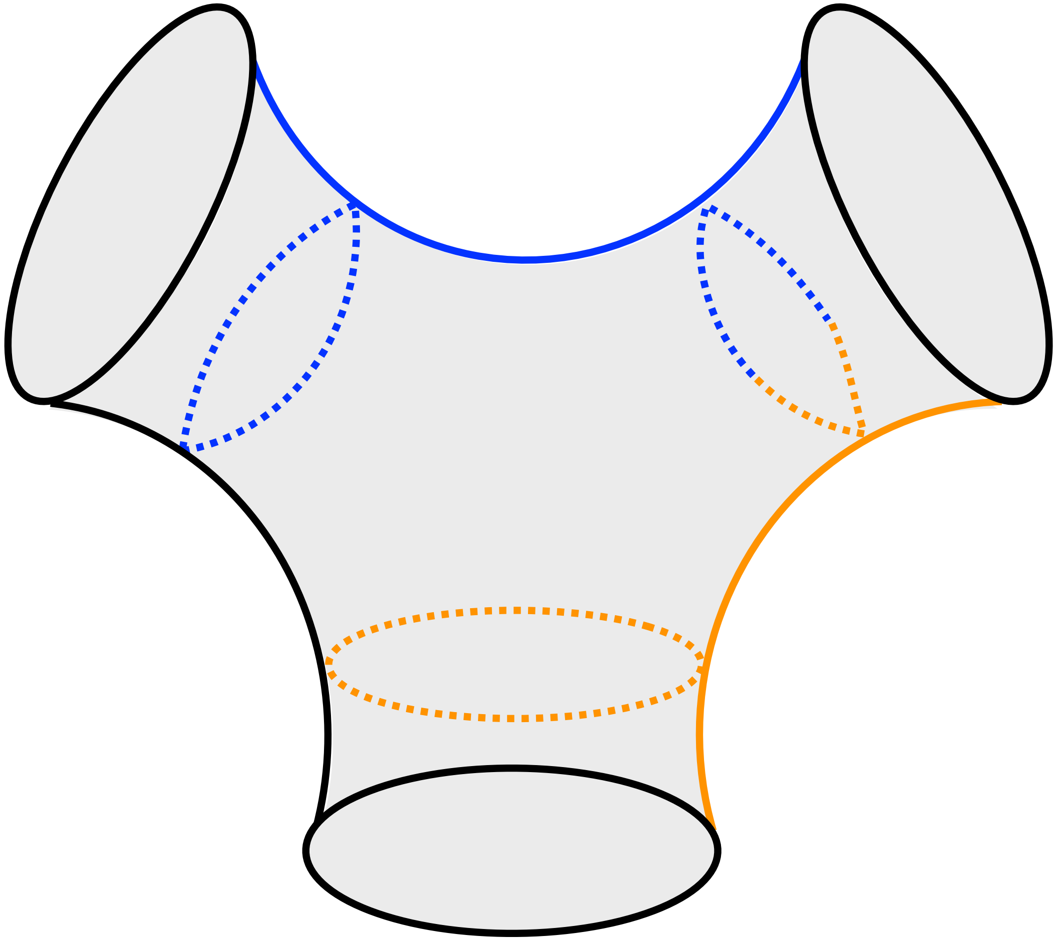
- We devide the AdS3 by γ_2

$$\gamma_2 = \begin{pmatrix} -\frac{c_2}{\sqrt{R_1 R_2}} & \frac{c_1 c_2 + R_1 R_2}{\sqrt{R_1 R_2}} \\ -\frac{1}{\sqrt{R_1 R_2}} & \frac{c_1}{\sqrt{R_1 R_2}} \end{pmatrix}$$

Three boundary-wormhole



Three boundary-wormhole



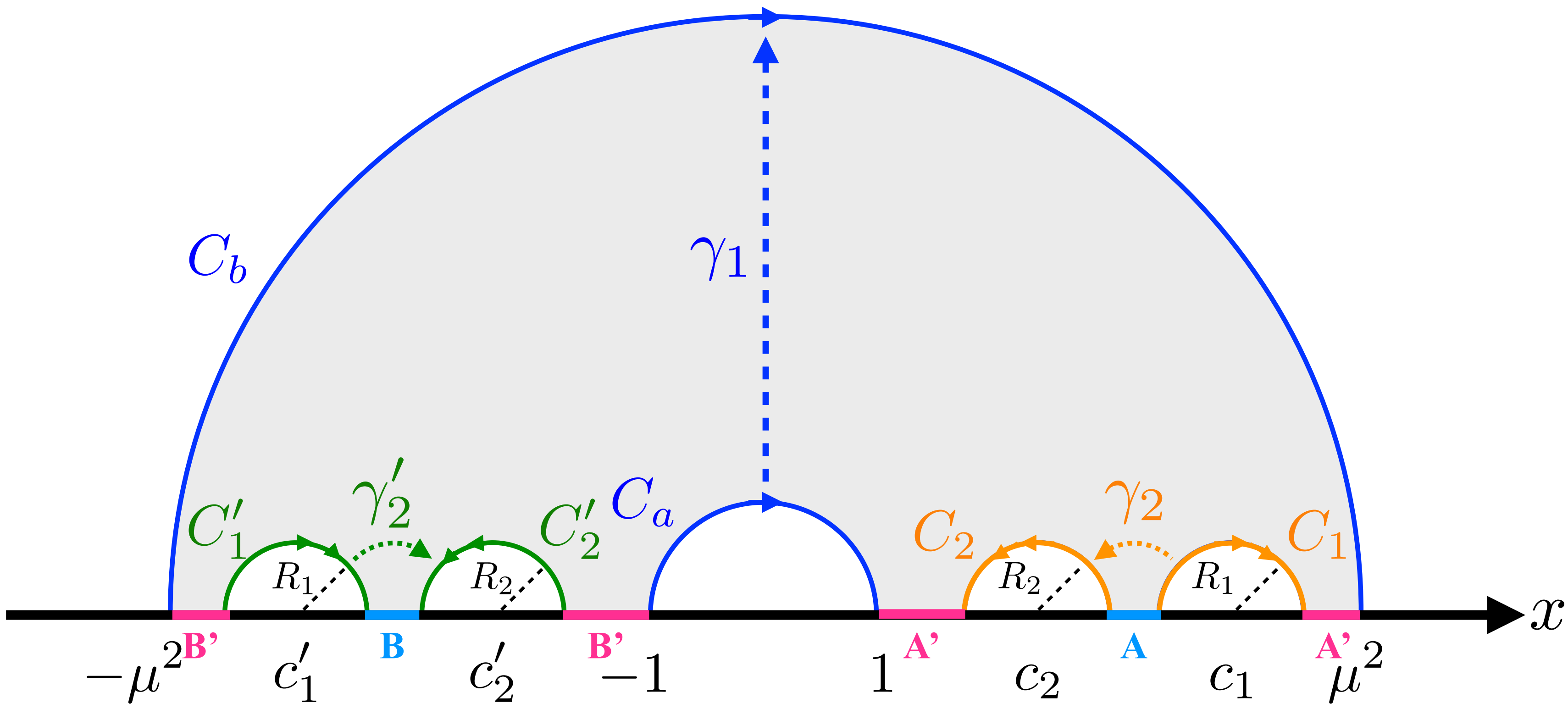
Four boundary-wormhole

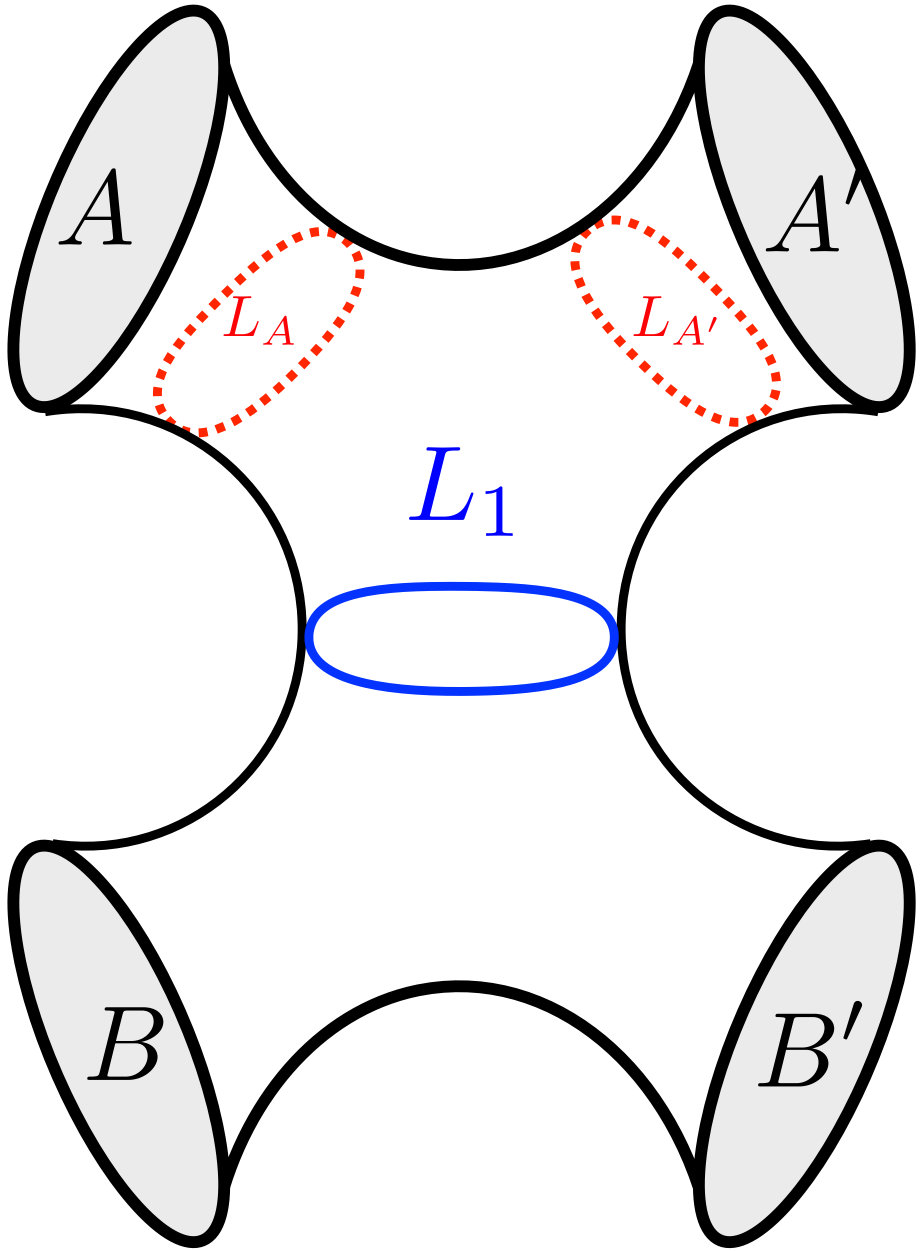
- Furthermore by deviding the space by

$$\gamma_2' = \begin{pmatrix} -\frac{c_2'}{\sqrt{R_1' R_2'}} & \frac{c_1' c_2' + R_1' R_2'}{\sqrt{R_1' R_2'}} \\ -\frac{1}{\sqrt{R_1' R_2'}} & \frac{c_1'}{\sqrt{R_1' R_2'}} \end{pmatrix}$$

- We obtain 4 boundary wormholes

Four boundary-wormhole




A
A'
L_A
$L_{A'}$
L_1
B
B'

Comments on our 4-bdr wormhole

- From our cooking recipe, I hope it is clear that our 4 boundary wormhole are totally specified once following parameters are given;

$$\mu\,,\, c_1\,,\, c_2\,,\, R_1\,,\, R_2$$

and

$$c_1'\,,\, c_2'\,,\, R_1'\,,\, R_2'$$

Comments on our 4-bdr wormhole

- It is straightforward to compute the minimal area of the horizons; (moduli-area relation)

$$L_A \equiv L(\gamma_2) = 2\cosh^{-1}\left[\frac{c_1 - c_2}{2\sqrt{R_1 R_2}}\right],$$

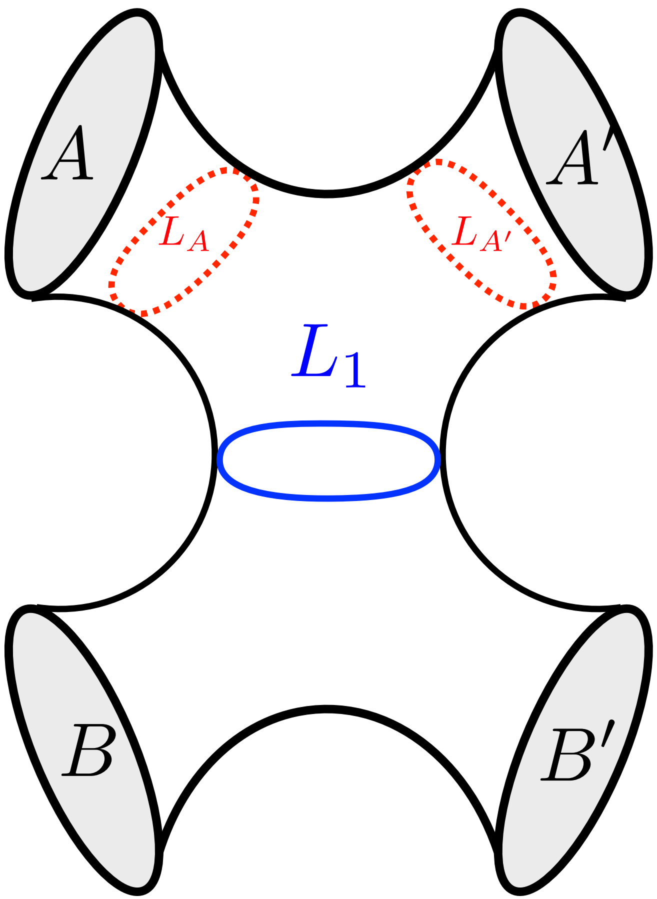
$$L_{A'} \equiv L(\gamma_1\gamma_2) = 2\cosh^{-1}\left[\left|\frac{c_1\mu^{-1} - c_2\mu}{2\sqrt{R_1 R_2}}\right|\right],$$

$$L_B \equiv L(\gamma_2') = 2\cosh^{-1}\left[\frac{-c_1' + c_2'}{2\sqrt{R_1' R_2'}}\right],$$

$$L_{B'} \equiv L(\gamma_1\gamma_2') = 2\cosh^{-1}\left[\left|\frac{(-c_1'\mu^{-1} + c_2'\mu)}{2\sqrt{R_1' R_2'}}\right|\right].$$

Comments on our 4-bdr wormhole

- It is straightforward to compute the minimal area of the horizons; (moduli-area relation)



$$L_1 = 2\log\mu$$

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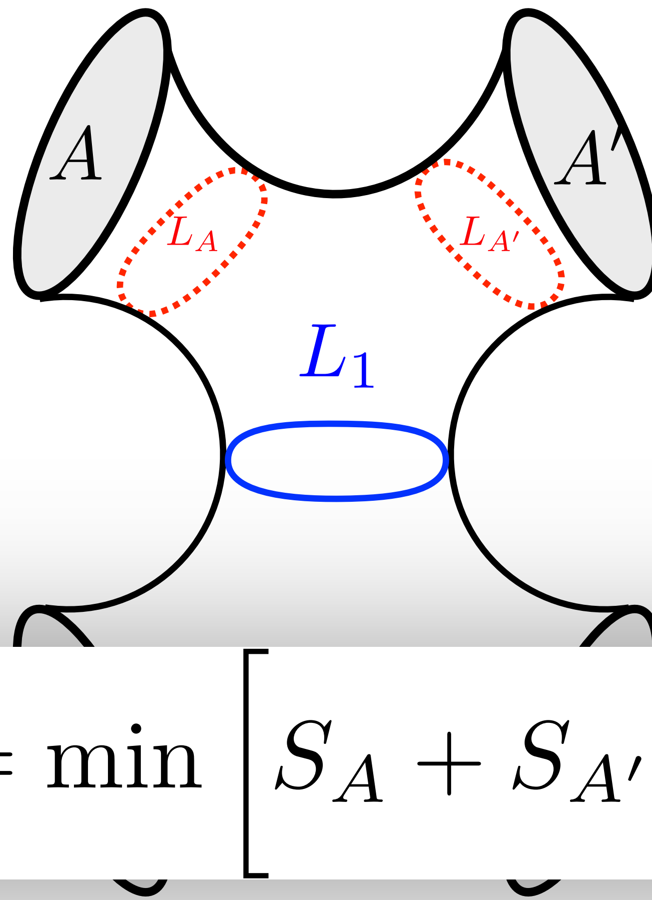


$$L_1 = 2\log\mu$$

$$S_{AA'} = \min\left[S_A + S_{A'},\ \frac{L_1}{4G_N}\right]$$

Comments on our 4-bdr wormhole

- All areas are completely determined given moduli, so one can calculate any mutual informations, for ex., between AA' and BB' and also A and B, etc.

$$S_{AA'} = \min\left[S_A + S_{A'},\, \frac{L_1}{4G_N}\right],$$

Ryu-Takayanagi

$$I(AA' : BB') = S_{AA'} + S_{BB'} - S_{AA'BB'}$$

Comments on our 4-bdr wormhole

- From our cooking recipe, I hope it is clear that our 4 boundary wormhole are totally specified once following parameters are given;

$$\mu\, , c_1\, , c_2\, , R_1\, , R_2$$

and

We consider only Z_2 invariant model

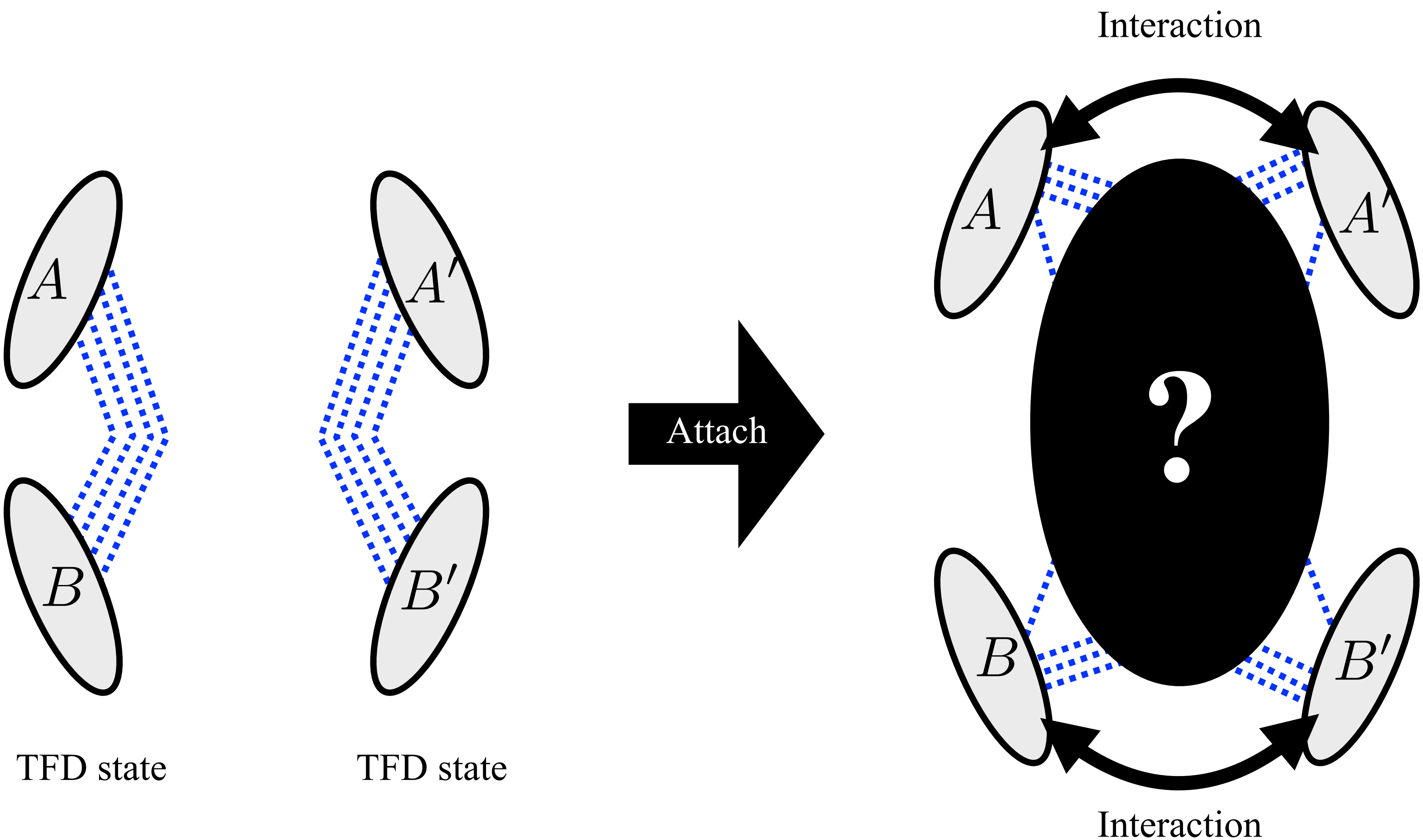
$$\cancel{c_1'\, , c_2'\, , R_1'\, , R_2'}$$

Today's contents

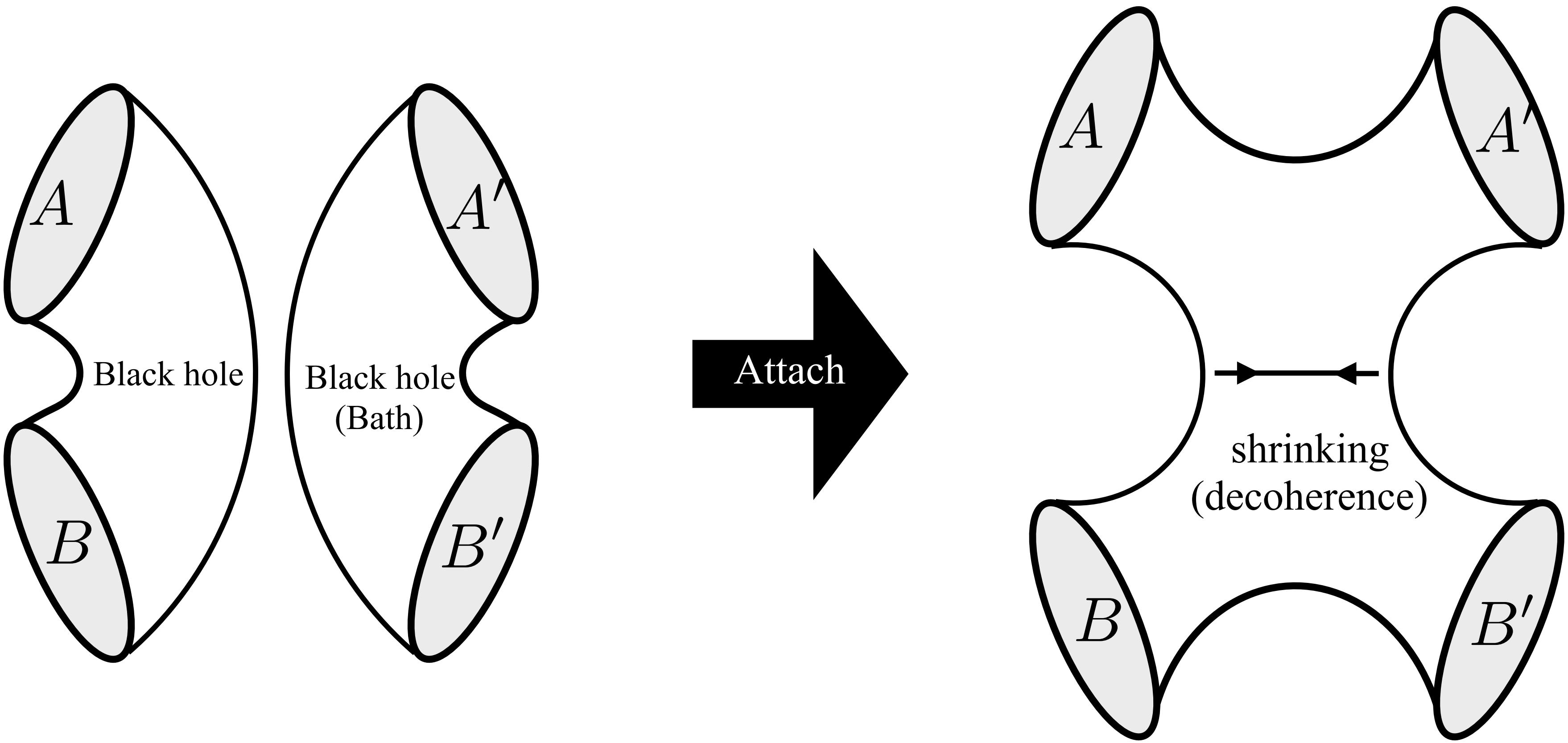
- Introduction and key question
- Cooking recipes for wormholes
- Moduli parameter evolutions for decoherence
- Main results

Moduli parameter evolutions

- As we mentioned, we allow energy from between bath (A’ and B’) and original thermo-field double (A and B) to decohere the entanglement of original thermo-field double between A and B



A
B
A'
B'
TFD state
TFD state
Attach
Interaction
?
Interaction



A
B
Black hole
Black hole
(Bath)
A'
B'
Attach
A
A'
shrinking
(decoherence)
B
B'

Moduli parameter evolutions

- As we mentioned, we allow energy from between bath (A’ and B’) and original thermo-field double (A and B) to decohere the entanglement of original thermo-field double between A and B
- As mentioned, we restrict to Z_2 invariant model for simplicity

Moduli parameter evolutions

- For that, we allow following moduli-evolution as "time"- evolution

$$S_A = \frac{L_A}{4G_N} = \frac{2\pi\sqrt{8G_N(M_{0A} - \alpha t)}}{4G_N},$$

$$S_{A'} = \frac{L_{A'}}{4G_N} = \frac{2\pi\sqrt{8G_N(M_{0A'} + \alpha t)}}{4G_N},$$

- We change moduli by hand as above to decohere

Moduli parameter evolutions

- For that, we allow following moduli-evolution as "time"- evolution

$$S_B = \frac{L_B}{4G_N} = \frac{\sqrt{(M_{0B} - \alpha t)}}{4G_N},$$

$$S_{B'} = \frac{L_{B'}}{4G_N} = \frac{\sqrt{(M_{0B'} + \alpha t)}}{4G_N},$$

- We change moduli by hand as above to decohere

- From previous formula of the moduli-area relationship, this implies 2 constraints for moduli;

$$\cosh\frac{L_A}{2} = \frac{c_1 - c_2}{2\sqrt{R_1 R_2}} = \cosh\left(\frac{\sqrt{M_{0A} - \alpha t}}{2}\right),$$

$$\cosh\frac{L_{A'}}{2} = \frac{c_1 \mu^{-1} - c_2 \mu}{2\sqrt{R_1 R_2}} = \cosh\left(\frac{\sqrt{M_{0A'} + \alpha t}}{2}\right).$$

- We consider decoherence process until it reaches equilibrium, so we consider only following range of "time";

$$M_{0A} - \alpha t \geq M_{0A'} + \alpha t \quad \Leftrightarrow \quad 0 \leq t \leq \frac{M_{0A} - M_{0A'}}{2\alpha}$$

Moduli parameter evolutions

- Now in this way, we specify 2 parameters
- However, we still have plenty of moduli to be fixed
- More precisely we still have to specify 3 more moduli

Comments on our 4-bdr wormhole

- From our cooking recipe, I hope it is clear that our 4 boundary wormhole are totally specified once following parameters are given;

$$\mu \,, c_1 \,, c_2 \,, R_1 \,, R_2$$

and

We consider only Z_2 invariant model

$$\cancel{c_1' \,, c_2' \,, R_1' \,, R_2'}$$

Moduli parameter evolutions

- To understand consistent conditions for these 3 moduli, remember that from the figure, we need following inequality for consistency;

$$1 < c_2 - R_2, \quad c_2 + R_2 < c_1 - R_1, \quad c_1 + R_1 < \mu^2,$$

Moduli parameter evolutions

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$$1 < c_2 - R_2, \quad c_2 + R_2 < c_1 - R_1, \quad c_1 + R_1 < \mu^2,$$

- This can be written using 3 unknown positive functions g_i as follows;

$$1 + g_1 = c_2 - R_2\,, c_2 + R_2 + g_2 = c_1 - R_1, c_1 + R_1 + g_3 = \mu^2.$$

Moduli parameter evolutions

- For simplicity, we set all g_i the same and set

$$g_i = g = g(t) = \text{decreasing function}$$
$$= \epsilon + \mathrm{e}^{-\alpha t}$$

- Now all moduli are set as a function of "time"
- Parameters we choose are;

$$M_{0A} = 1.0 \times 10^7, M_{0A'} = 2.0 \times 10^6, \alpha = 1.0 \times 10^6,$$
$$\epsilon = 1.0 \times 10^{-4}, 4G_N = 1.$$

Moduli parameter evolutions

- Remember that we constraint the time evolution till it reaches equibrilium,

$$M_{0A} - \alpha t \geq M_{0A'} + \alpha t \quad \Leftrightarrow \quad 0 \leq t \leq \frac{M_{0A} - M_{0A'}}{2\alpha}$$

- In our parameter choises, this means

$$\Leftrightarrow \quad 0 \leq t \leq 4$$

- From previous formula of the moduli-area relationship, this implies 2 constraints for moduli;

$$\cosh\frac{L_A}{2} = \frac{c_1 - c_2}{2\sqrt{R_1 R_2}} = \cosh\left(\frac{\sqrt{M_{0A} - \alpha t}}{2}\right),$$

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Moduli parameter evolutions

- Now we can solve all moduli as a function of time, I will show the results

Moduli parameter evolutions

- Now we can solve all moduli as a function of time, I will show the results
- Caution: due to technical problems we found, the results we show now are temporal results

Today's contents

- Introduction and key question
- Cooking recipes for wormholes
- Moduli parame[illegible]volutions for decoherence
- Main results

Eqs to solve

- The eqs we solved are

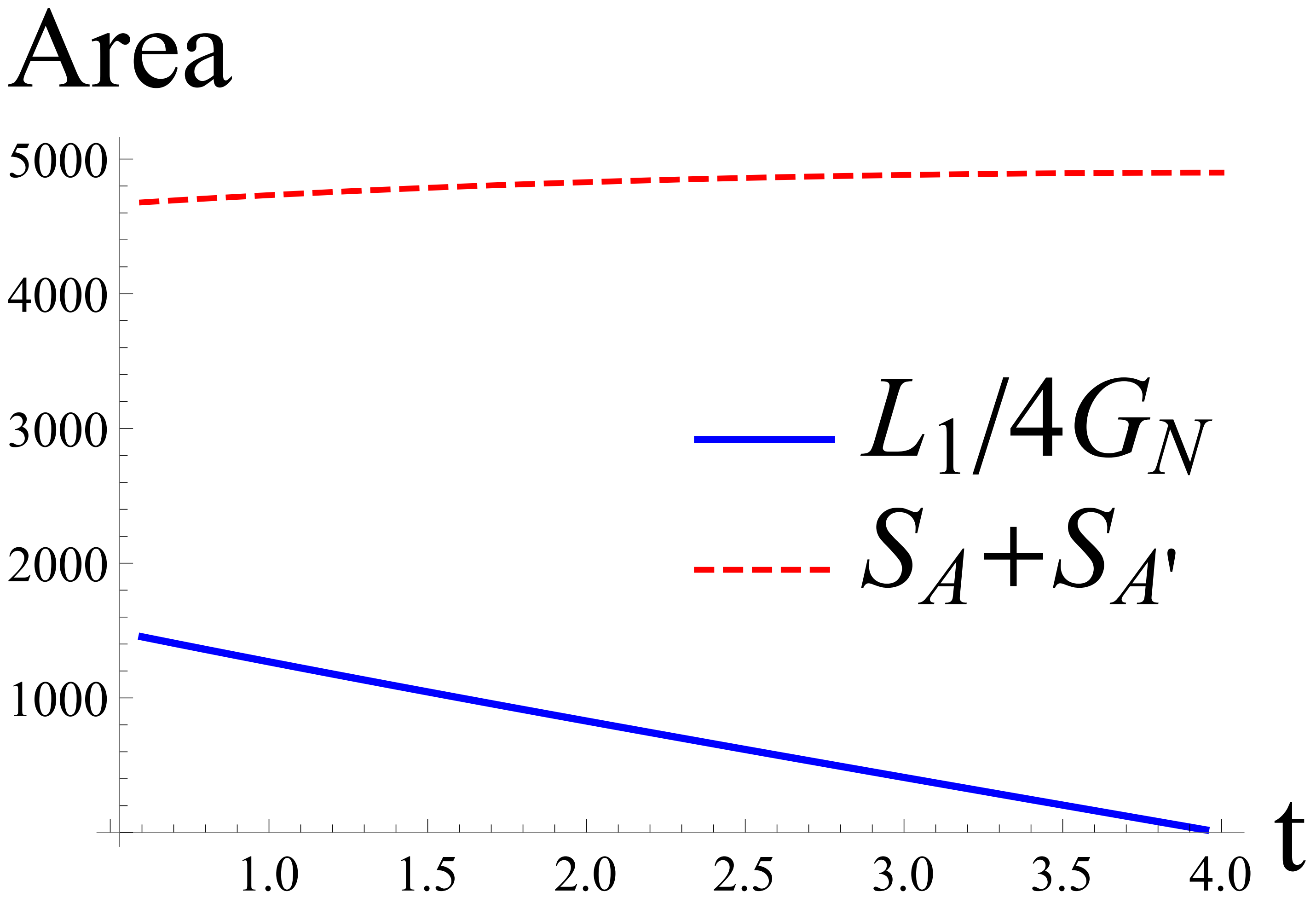
$$1 + g_1 = c_2 - R_2\,, c_2 + R_2 + g_2 = c_1 - R_1, c_1 + R_1 + g_3 = \mu^2.$$

$$\frac{c_1 - c_2}{2\sqrt{R_1 R_2}} = \cosh\left(\frac{\sqrt{M_{0A} - \alpha t}}{2}\right),$$

$$\frac{c_1 \mu^{-1} - c_2 \mu}{2\sqrt{R_1 R_2}} = \cosh\left(\frac{\sqrt{M_{0A'} + \alpha t}}{2}\right).$$

- Our main concerns are how μ behaves by *t*

(temporal) result 1



Comments 1

- As we have mentioned,

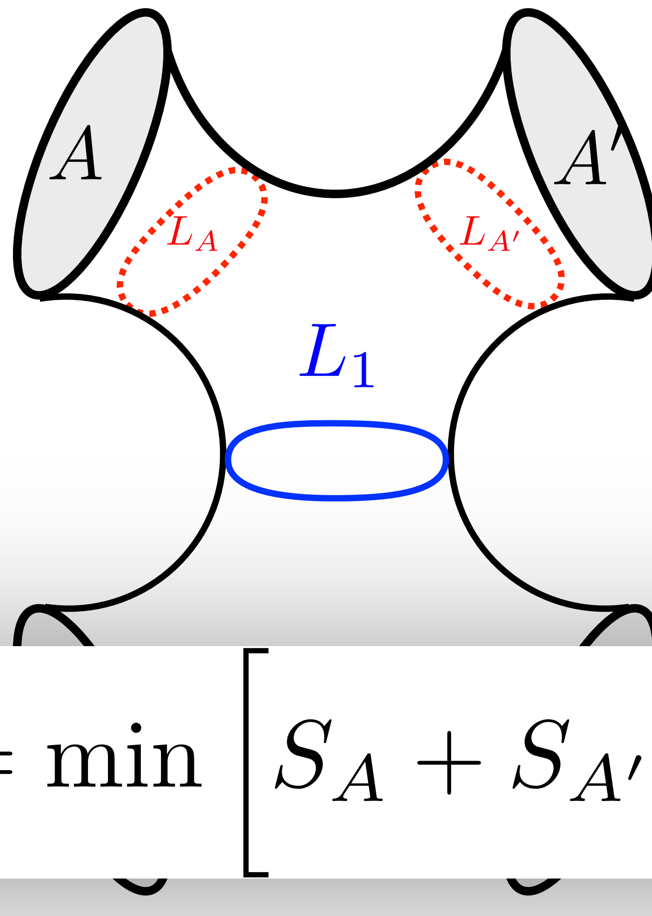
$$S_{AA'} = \min\left[S_A + S_{A'},\ \frac{L_1}{4G_N}\right]$$

- And the result 1 shows that during the decoherence process,

$$S_{AA'} = \frac{L_1}{4G_N} \to 0 \quad \text{as} \quad t \to 4$$

Comments on our 4-bdr wormhole

- It is straightforward to compute the minimal area of the horizons; (moduli-area relation)



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- And the result 1 shows that during the decoherence process,

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Comments 2

- This means at the end of the decoherence,

$$L_1 = 2\log\mu \to 0 \Leftrightarrow \mu \to 1$$

- In other words, the neck L_1 shrinks to zero and *the wormhole pinches off!*

Comments on our 4-bdr wormhole

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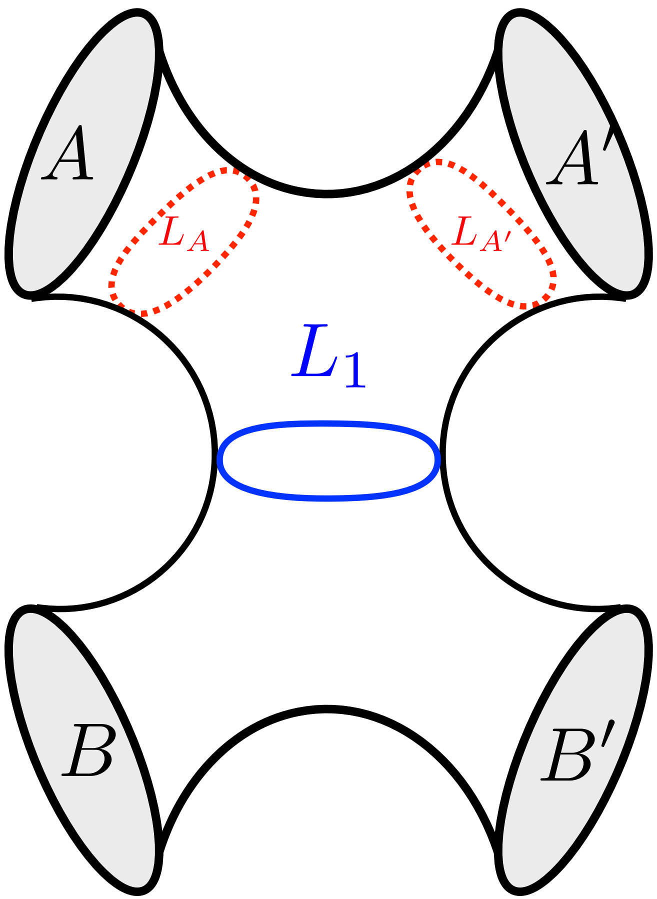


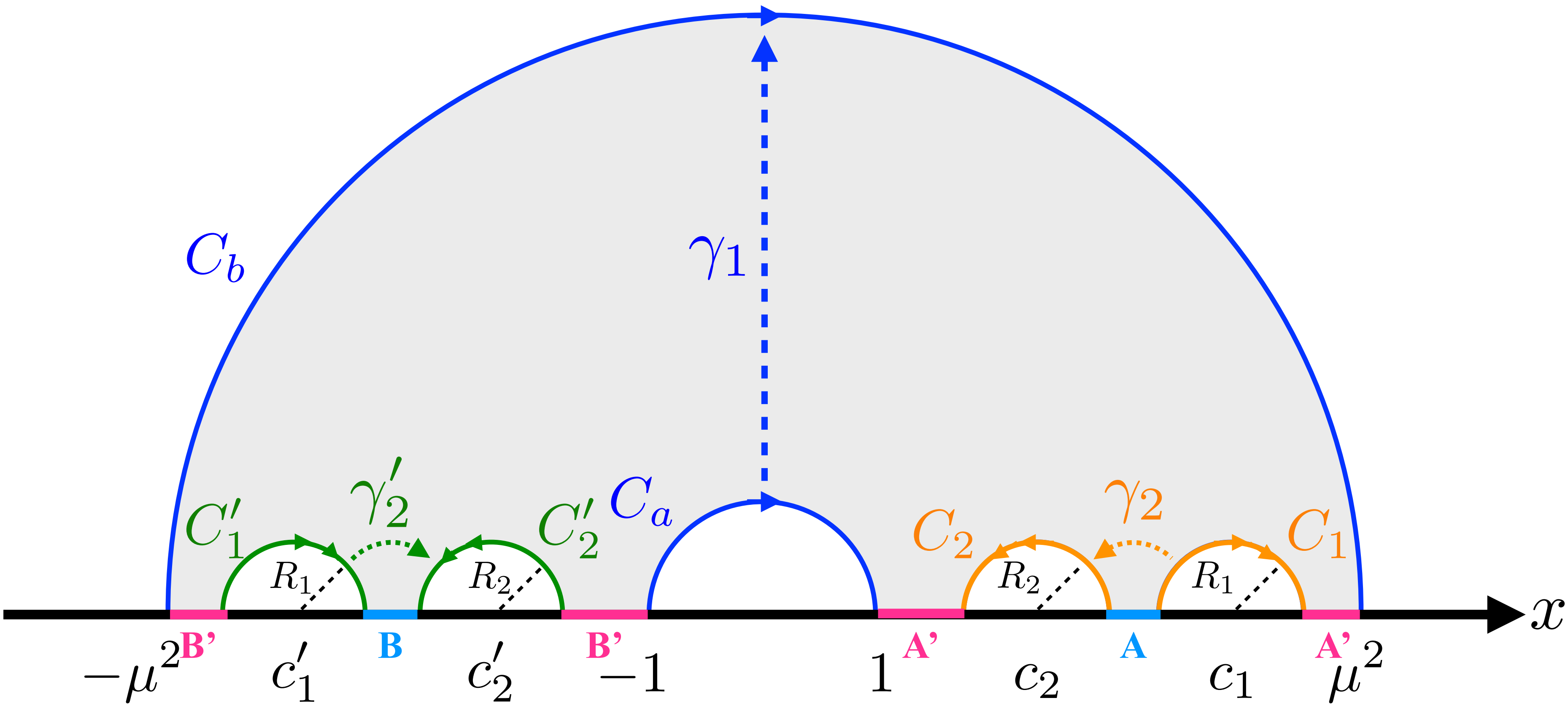
$$L_1 = 2 \log \mu$$

Comments 3

- Now one might wonder at the limit

$$L_1 = 2\log\mu \to 0 \Leftrightarrow \mu \to 1$$

- How in such a limit, the area of A for S_A and A' for $S_{A'}$ does not vanish, since it is apparently a degenerate limit (outer and inner circle coincides)!



C_b
γ_1
C_a
C'_1
γ'_2
C'_2
C_2
γ_2
C_1
R_1
R_2
R_2
R_1
x
B'
B
B'
A'
A
A'
$-\mu^2$
c'_1
c'_2
-1
1
c_2
c_1
μ^2

Comments 3

- The answer is that previous upper half plane figure of Z neglects the warping factor.
- The correct metric is for $Z \equiv x + iz$

$$ds^2 = \frac{dx^2 + dz^2}{z^2} = \frac{dZ d\bar{Z}}{|\mathrm{Im}Z|^2} \,.$$

- It turns out this warping makes all area well-behaved even at this degenerate limit.

No correlation between A and B!

- Given these, one can see that *there is no correlation between A and B, both classically and also quantum mechanically.*
- To see this, from Z_2 symmetry at the end of decoherence we have seen

$$S_{AA'} \to 0$$

$$S_{BB'} \to 0$$

$$I(AA' : BB') = S_{AA'} + S_{BB'} - S_{AA'BB'} \to 0$$

No correlation between A and B!

- One can show that due to strong sub-additivity inequality, the mutual information never increases by tracing out subsystems
- Subsystems to trace out in this case are A′ and B′,

$$I(AA' : BB') \geq I(A : B) = 0$$

- We showed that mutual infomation between A and B becomes zero at the end of process

No correlation between A and B!

- Since mutual information captures both classical and quantum correlations (entanglement),

$$I(AA' : BB') \geq I(A : B) = 0$$

- This implis that there is *no correlation between A and B both classically and quantum mechanically*

Main results

- We will see that the final state of the holographic decoherence process can not have any correlation between A and B, both classically and quantum mechanically.
- This in particular means that we cannot construct an ER bridge which only contains classical correlation, at least in the moduli space we studied.

Comments 1

- As is clearly seen in our set-up, we have NOT studied all of the moduli space
- We assumed Z_2 symmetry. In addition, we have restricted to only a specific set of moduli-evolution
- It is fair to say that once we relax these conditions, we do not precisely know how the decoherence process goes (open questions)

Comments 2

- We have studied the modulie evolutions of the multi-boundary wormholes
- However it is a bit unfortunate that we do not fully understand what is the dual microscopic structure of these mulit-boundary wormholes
- Once we know that, in principle we should be able to conduct that without relying on holographic set-up, that analysis is interesting

Thanks!