Noise and decoherence induced by gravitons

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Based on
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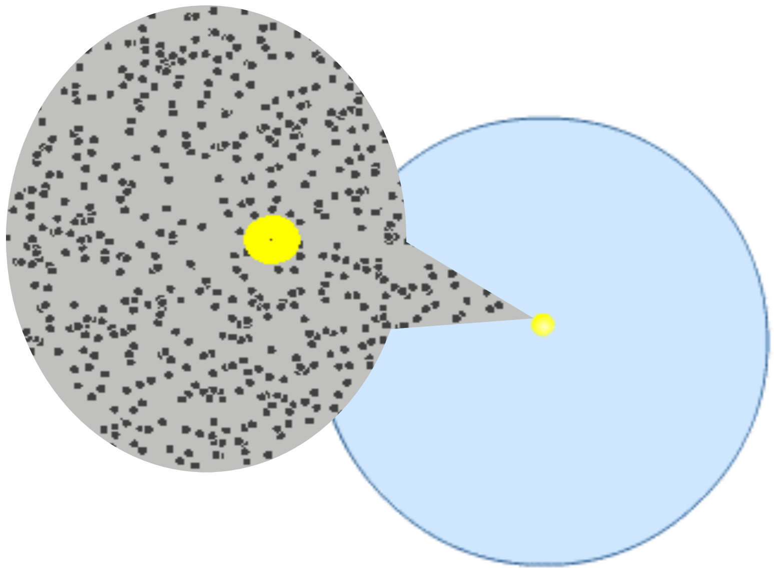
Brownian motion and gravitons



A phonemenon of random motion of minumte particles in water
Brown (1827)

The random motion occurs due to the random collisions of surrounding individual water molecules
Einstein (1905)

The size of the water molecules can be determined by the Brownian motion

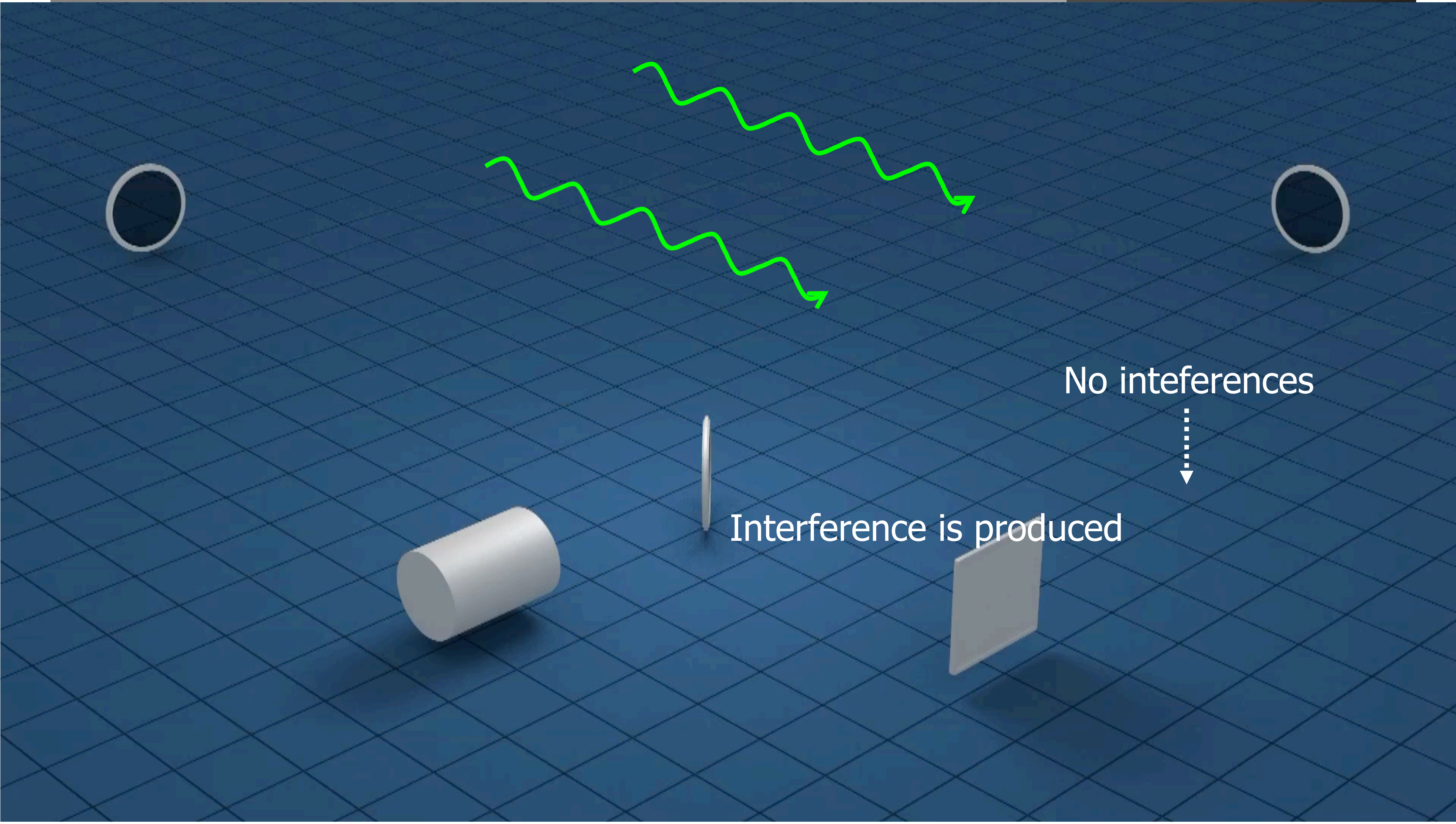
The size was determined by the experiment of the Brownian motion Perrin (1908)

Hard to observe water molecules directly → Proved the existence of water molecules indirectly through the minute particles

Hard to observe gravitons directly → Prove the existence of gravitons indirectly through something

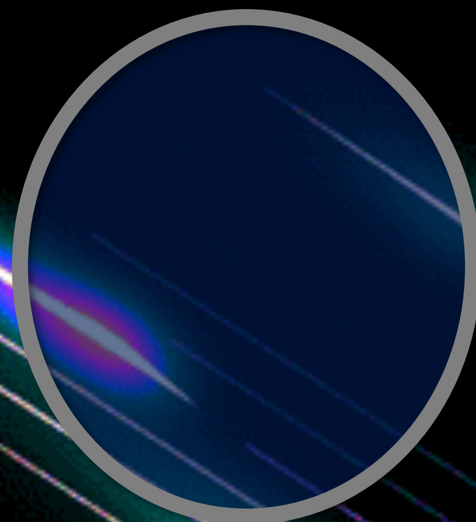
What should we target as the “something”

Laser interferometers



A suspended mirror

Gravitons :←Environment
Water molecules



:←System
Mirror
Minute particle

The mirror

The same setup as the Brownian motion

The system of the mirror couples to an huge environment of gravitons

⬇

Noise due to gravitons should be induced on the mirror

Can we prove the existence of gravitons indirectly through this noise ?

What should we calculate?

Derive the Langevin equation
(stocastic differential equation)
of motion of this system

We first derive the total action of the mirror and the gravitational waves

How the mirror feels the effect of gravitational waves?

For simplicity, regard the mirror as a point particle

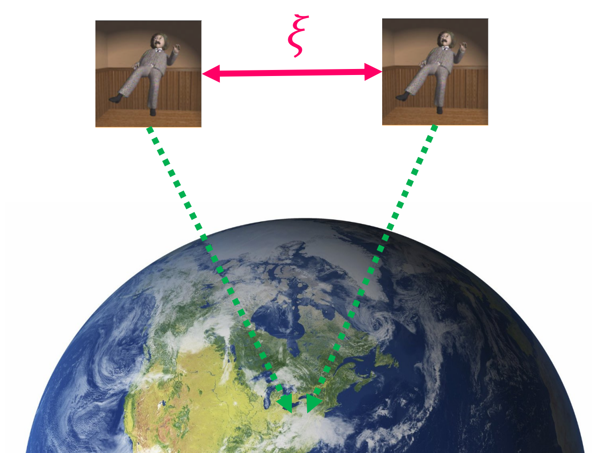
A single particle (mirror)? *Einstein's equivalence principle*

A free falling object feels no gravity

local inertial frame

How to measure the gravitational force? *Geodesic deviation* ξ

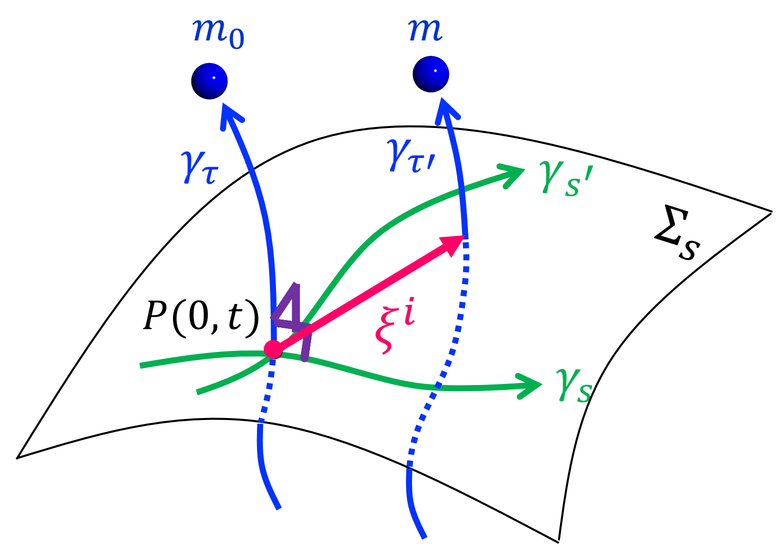
Observe the geodesics of two free falling particles deviating relative to each other



The total action for the GWs and the mirrors

Total Action

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\,R\; - m_0 \underbrace{\int d\tau}_{\text{No dynamical variables}} - m\int d\tau$$



Fermi normal coordinates

$$\begin{cases} h^A_{\boldsymbol{k}} \text{ : Fourier modes of metric field (GWs)} \\ \xi^i \text{ : Geodesic deviation between the mirros} \end{cases}$$

$$S = \int dt \sum_{\boldsymbol{k}} \sum_{A=+,\times} \left[\frac{1}{2}\dot{h}^A_{\boldsymbol{k}}\,\dot{h}^A_{\boldsymbol{k}} - \frac{1}{2}k^2 h^A_{\boldsymbol{k}}\,h^A_{\boldsymbol{k}} \right]$$

$$+\int dt \left[\frac{m}{2}(\dot{\xi}^i)^2 + \frac{1}{2}\frac{m}{M_{\rm pl}}\frac{1}{\sqrt{V}}\sum_{\boldsymbol{k},A}\left[e^A_{ij}(\boldsymbol{k})\,\boxed{\ddot{h}^A_{\boldsymbol{k}}\,\xi^i\xi^j}\,\right]\right]$$

qubic derivative interaction

Quantize the GWs and the deviation between mirrors

For simplicity, work in the interaction picture below

We promote the free metric field $h_{\boldsymbol{k}}^{A}(t)$ to the operator $\hat{h}_{\boldsymbol{k}}^{A}(t)$

Positive freq. mode in the Minkowski space

The Minkowski vacuum

$$\hat{h}_{\boldsymbol{k}}^{A}(t) = \hat{a}_{\boldsymbol{k}}^{A} v_k(t) + \hat{a}_{-\boldsymbol{k}}^{A\dagger} v_k^{*}(t) \qquad \hat{a}_{\boldsymbol{k}}^{A}|0\rangle = 0 \qquad \left[\hat{a}_{\boldsymbol{k}}^{A}, \hat{a}_{\boldsymbol{p}}^{B\dagger}\right] = \delta^{AB}\delta_{\boldsymbol{kp}}$$

Since $\xi^{i}(t)$ is just a position, we promote $\hat{\xi}^{i}(t)$ to the operator as well

Solve the EOMs of the system of $\hat{h}_{\boldsymbol{k}}^{A}$ and $\hat{\xi}^{i}$ to derive the Langevin equation of geodesic deviation $\hat{\xi}^{i}$

Langevin equation of geodesic deviation

Kanno, Soda & Tokuda (2020)
Parikh, Wilczek & Zahariade (2020)

We obtained

$$\ddot{\hat{\xi}}^i + \frac{m}{40\pi M_{\rm pl}^2}\left(\delta_{ik}\delta_{j\ell}+\delta_{i\ell}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{k\ell}\right)\hat{\xi}^i\,\frac{d^5}{dt^5}\left(\hat{\xi}^k\hat{\xi}^\ell\right)$$

frictional force (radiation reaction force)

$$= -\hat{N}_{ij}(0,t)\,\hat{\xi}^j(t)$$

random force (noise)

$$\left\langle \hat{h}^A_{\boldsymbol{k}}(0)\right\rangle = 0$$

classical free GWs

where

$$\hat{N}_{ij}(t) = \frac{1}{M_{\rm pl}\sqrt{V}}\sum_{\boldsymbol{k},A} k^2\, e^A_{ij}(\boldsymbol{k})\left\{\hat{h}^A_{\boldsymbol{k}}(0)\cos kt + \dot{\hat{h}}^A_{\boldsymbol{k}}(0)\,\frac{\sin kt}{k}\right\}$$

The noise consists of quantum fluctuations of gravitational fields (gravitons)

Is this quantum noise detectable by gravitational detectors?

Noise due to gravitons

Detectability is discussed by effective strain in the frequency domain

$$h_{\rm eff} = \frac{N(f)}{(2\pi f)^2} \equiv \frac{1}{(2\pi f)^2}\left(\int_{-\infty}^{\infty} dt \left\langle \{\hat{N}^{ij}(t), \hat{N}_{ij}(0)\} \right\rangle e^{2\pi i f t}\right)^{\frac{1}{2}}$$

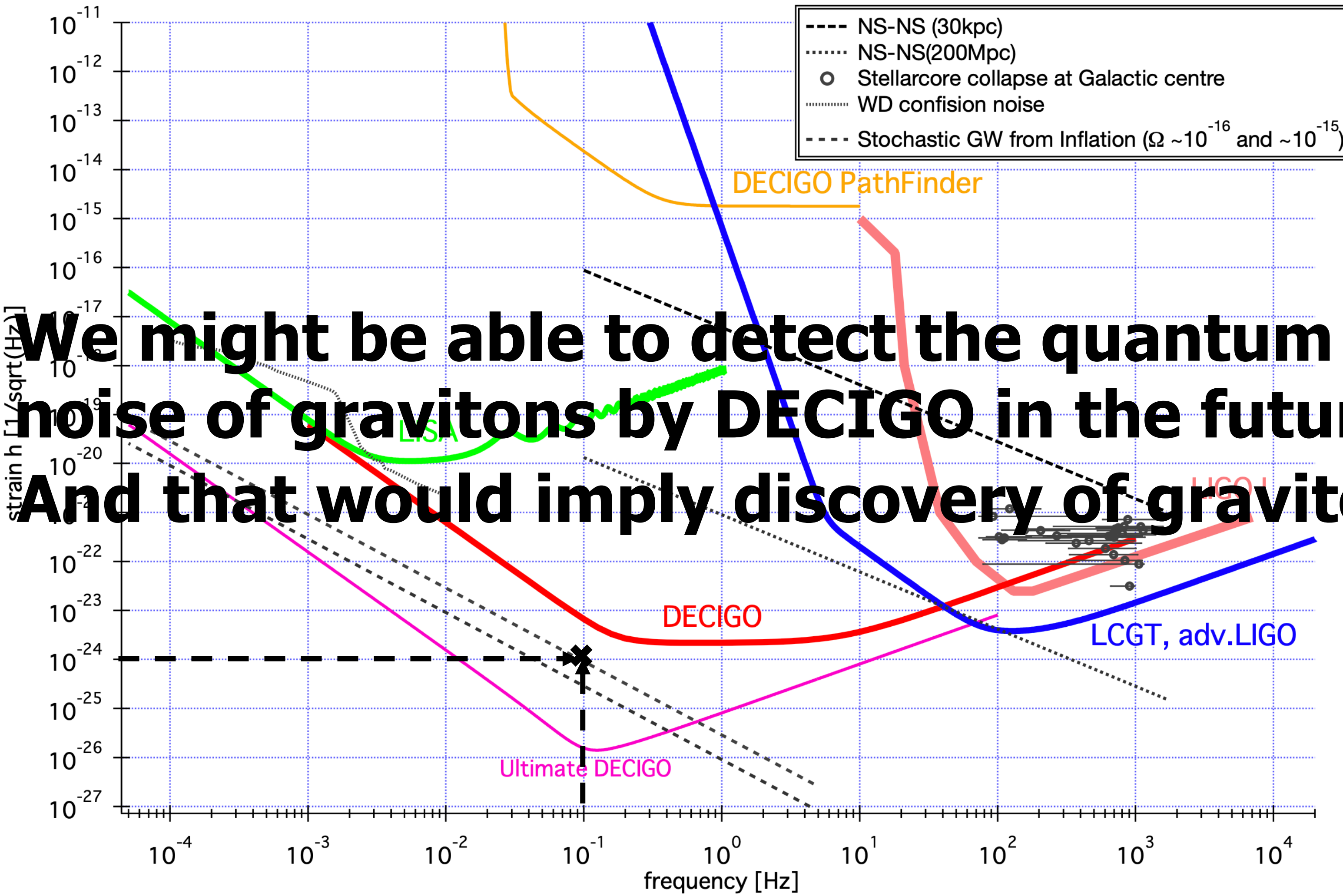
For primordial GWs

$$h_{\rm eff} \approx 2\times10^{-42}\left(\frac{f}{1\ {\rm Hz}}\right)\left(\frac{f_c}{f}\right)^2 \qquad {\rm Hz}^{-\frac{1}{2}}$$

, f_c: cutoff frequency $\sim 2\times10^8$ Hz

The latest sensitivity curves

Kanno, Soda & Tokuda (2020)



We might be able to detect the quantum noise of gravitons by DECIGO in the future

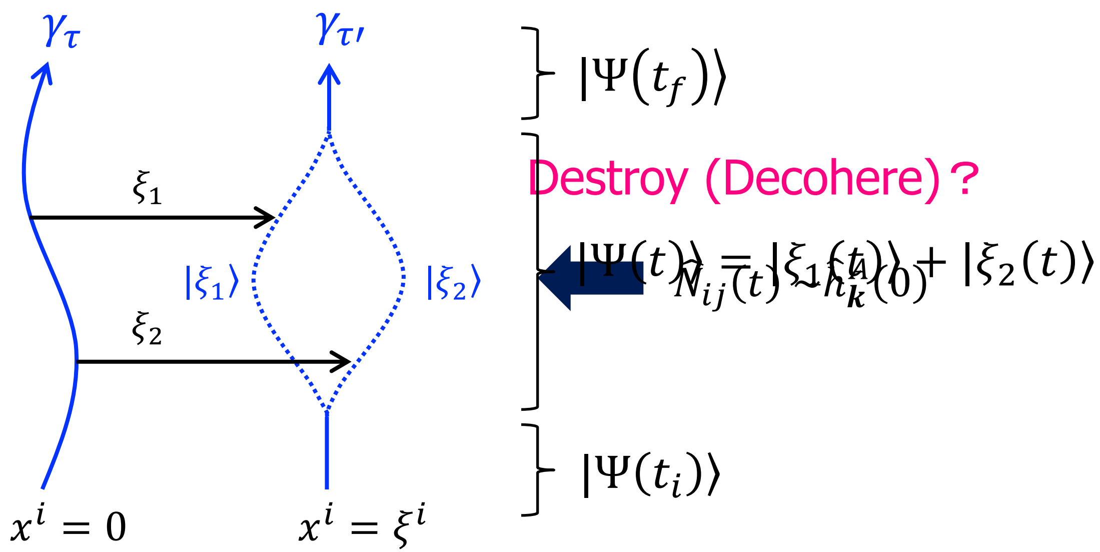
And that would imply discovery of gravitons

The strain: $h_{\text{eff}} \sim 10^{-24} \quad \text{Hz}^{-\frac{1}{2}}$ at $f \sim 0.1\ \text{Hz}$

Can we detect the noise of gravitons in a laboratory?

Effect of noise on a superposition state in a lab

It’s possible to create a superposition state of two spatially separated locations



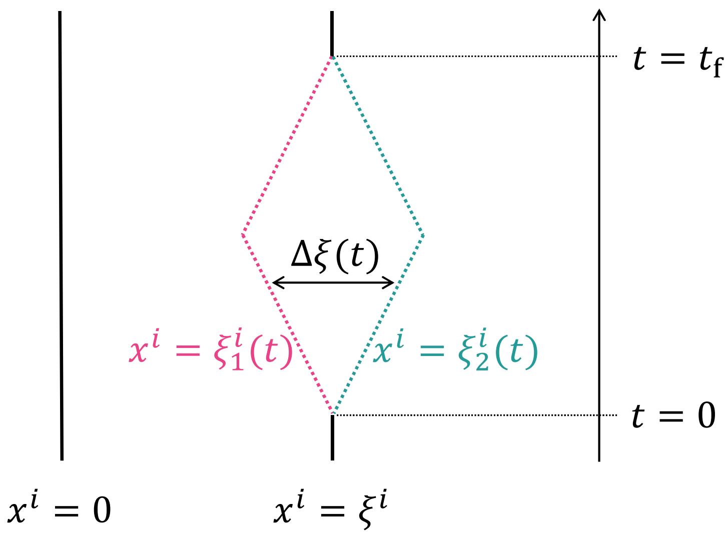
The decoherence factor

$$\Gamma \approx \frac{m^2}{8}\int_0^{\tau_{\rm dec}} dt\, \Delta\left(\xi^i\xi^j\right)(t) \int_0^{\tau_{\rm dec}} dt' \Delta\left(\xi^k\xi^\ell\right)(t')\left\langle\left\{\hat{N}_{ij}(t), \hat{N}_{k\ell}(t')\right\}\right\rangle \geq 1$$

The loss of quantum coherence between $|\xi_1(t)\rangle$ and $|\xi_2(t)\rangle$ occurs

By using this decoherence factor, we can predict the decoherence time due to gravitons

A configuration of the superposition state



$$\Delta\xi(t) = \begin{cases} 2vt & \text{for } 0 < t \le {}^{t_{\mathrm{f}}}/_{2} \\ 2v(t_{\mathrm{f}} - t) & \text{for } {}^{t_{\mathrm{f}}}/_{2} < t \le t_{\mathrm{f}} \end{cases}$$

The decoherence factor $\Gamma(t_{\mathrm{f}}) \approx \dfrac{2m^2v^2}{5\pi^2 M_{\mathrm{pl}}^2} G(\Omega_{\mathrm{m}} t_{\mathrm{f}})$ $\quad G(\Omega_{\mathrm{m}} t_{\mathrm{f}}) \sim \mathcal{O}(1)$

$mv \ll M_{\mathrm{pl}} \approx 4\times10^{-6}$ g ➡ $\Gamma(t_{\mathrm{f}}) \ll 1$: Decoherence does not occur

10^{-20}

$mv \gg M_{\mathrm{pl}} \approx 4\times10^{-6}$ g ➡ $\Omega_{\mathrm{m}}\,\tau_{\mathrm{dec}} \cong 10\times\left(\dfrac{M_{\mathrm{pl}}}{mv}\right)^{\frac{1}{2}}$ s

Need to exceed the speed of light…

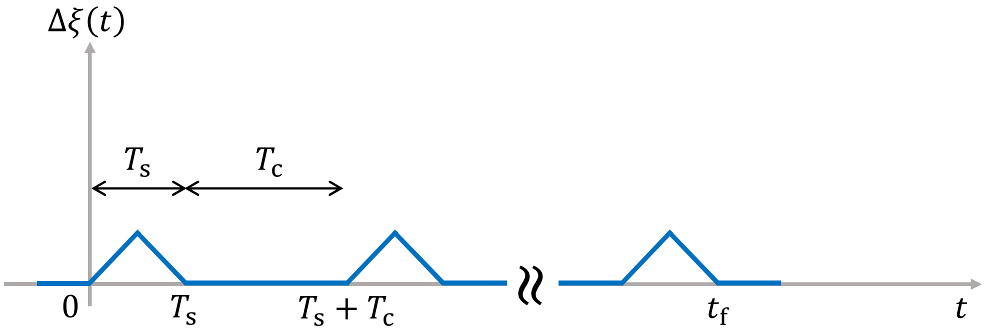
$(c = 1)$

Currently, the spatial superposition state is only created up to $m = 10^{-20}$ g in a lab

Repeated configuration of the superposition state

Kanno, Soda & Tokuda (2020)

Previous configuraton repeats n times



$(t_\mathrm{f} = NT_\mathrm{s} + (N-1)T_\mathrm{c})$

The decoherence factor $\Gamma(t_\mathrm{f}) \approx n \times \dfrac{2m^2v^2}{5\pi^2 M_\mathrm{pl}^2} G(\Omega_\mathrm{m} t_\mathrm{f})$ $\quad G(\Omega_\mathrm{m} t_\mathrm{f}) \sim \mathcal{O}(1)$

If we take sufficiently large n, decoherence occurs even for $mv \ll M_\mathrm{pl} \approx 4\times10^{-6}$ g

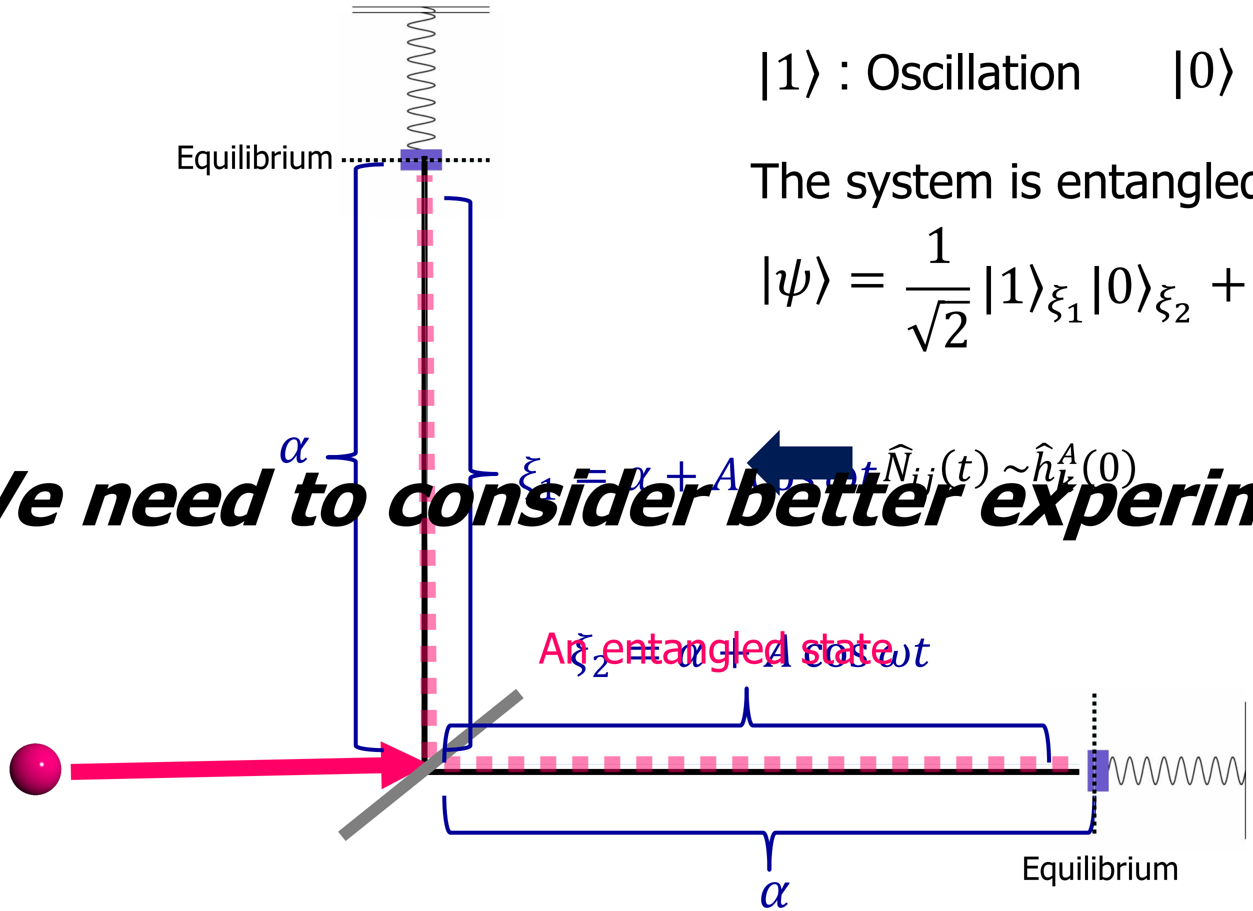
If the decoherenece is observed and the decoherence time in a lab agrees with the theoretial prediction, it turns out to be a discovery of gravitons

Effect of noise on an entangled state in a lab

$|1\rangle$: Oscillation $|0\rangle$: Equilibrium

The system is entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1\rangle_{\xi_1}|0\rangle_{\xi_2} + \frac{1}{\sqrt{2}}|0\rangle_{\xi_1}|1\rangle_{\xi_2}$$



We need to consider better experimental designs

The decoherence factor $\Gamma(t_{\mathrm{f}}) \approx \left(\frac{m}{M_{\mathrm{pl}}}\right)^2 \left(\frac{A}{\alpha}\right)^2 N \left(\frac{f_c}{f}\right)^4 \sim 1$

Decoherence time $\approx$ 1 y ***Preliminary***

$m \sim 40$ kg
$A \sim 10^{-17}$ cm
$\alpha \sim 40$ km
$f \sim 10^{4}$ Hz
$N \sim 10^{10}$

Summary and future work

We estimated the quantum noise of gravitons on the mirrors of gravitational wave interferometers

We found that the quantum noise of gravitons might be detectable by the future DECIGO

By using the decoherence time due to gravitons in a lab, we might be able to discover gravitons indirectly in the future