Exploration of Initial State in Gravity via Entanglement

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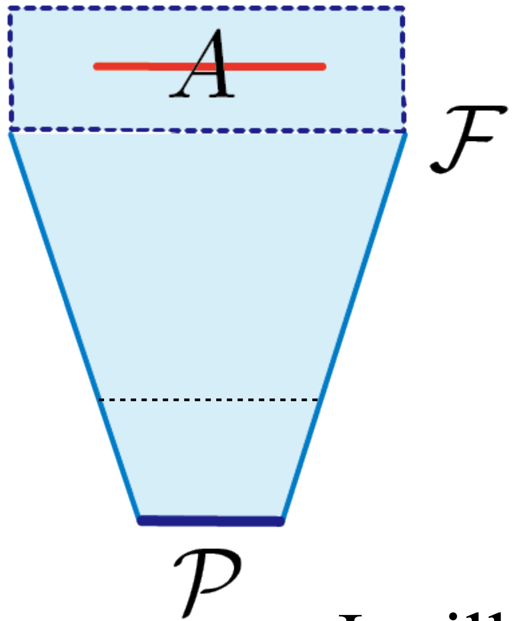
@YITP Kyoto, March 2021

Questions in gravity:



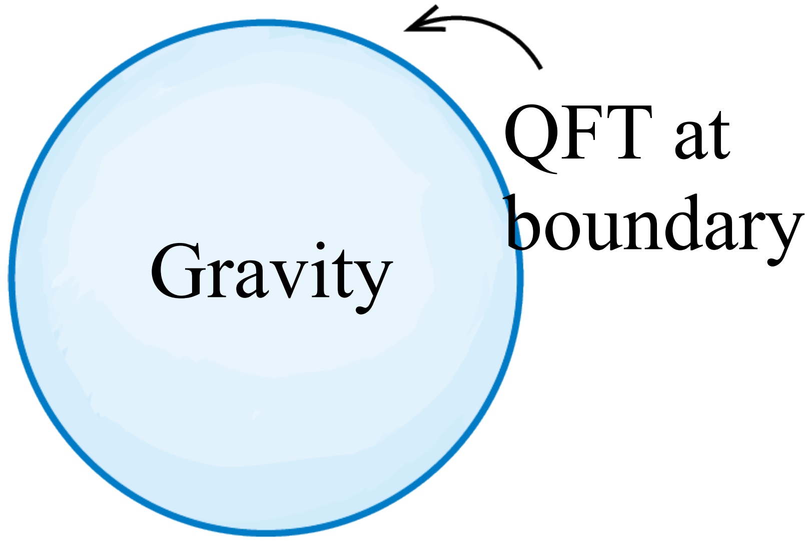
- Where do gravity degrees of freedom live?
- How can we extract them?

I will apply recent developments to the following questions:

- How can we learn about the initial state in gravity from present universe?
- Is entanglement entropy of subsystem gets large corrections from gravity and initial state?

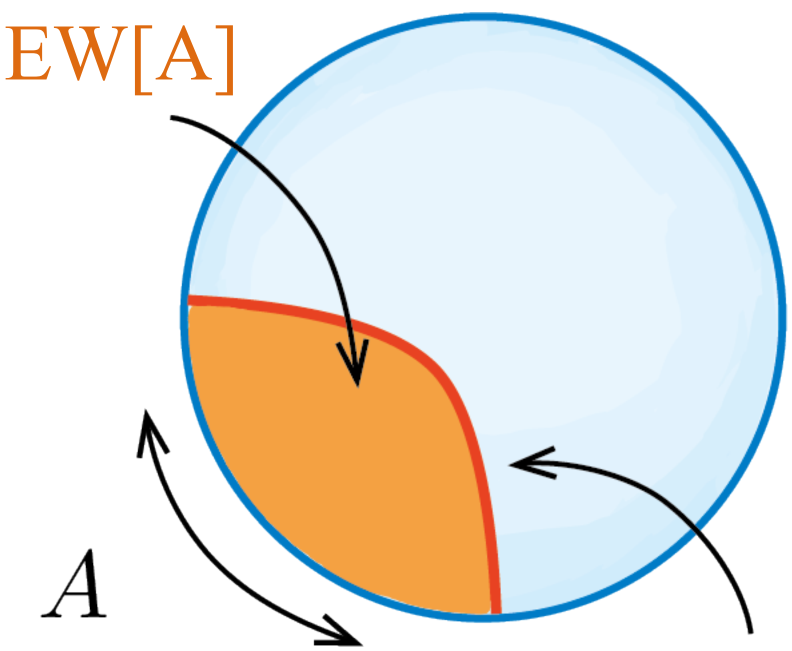
Holographic principle: Gravity degrees of freedom live at the boundary of the spacetime.

- AdS/CFT correspondence.



Entanglement wedge reconstruction tells us where and how to extract bulk excitations from CFT in AdS/CFT.

Entanglement wedge: EW[A]



Ryu-Takayanagi surface: RT

Ryu-Takayanagi formula (or FLM)

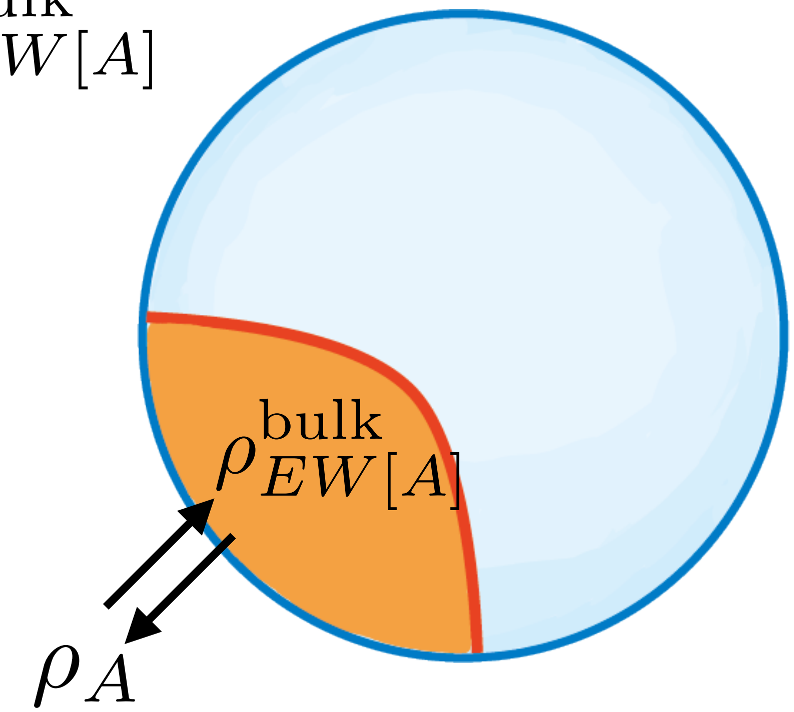
$$S_A = \frac{\text{Area}[RT]}{4G_N} + S^{\text{matter}}_{EW[A]}$$

[Ryu, Takayanagi][Hubeny, Rangamani, Takayanagi]
[Faulkner, Lewkowicz, Maldacena][Jafferis, Lewkowycz, Maldacena, Suh] See also [Akers, Penington]

Entanglement Wedge Reconstruction:

Map between bulk matter state on entanglement wedge and state at the boundary.

$$\mathcal{N} : \rho^{\text{bulk}}_{EW[A]} \to \rho_A \qquad \mathcal{R} : \rho_A \to \rho^{\text{bulk}}_{EW[A]}$$



Key formula:
Bulk relative entropy = Boundary relative entropy
[Jafferis, Lewkowycz, Maldacena, Suh]

$$S(\rho^{\text{bulk}}_{EW[A]} | \sigma^{\text{bulk}}_{EW[A]}) \approx S(\rho_A | \sigma_A)$$

Then Petz recovery map is an exact recovery*.

$$\mathcal{R} : \sigma_A \to \rho^{1/2}_{EW[A]} \mathcal{N}^* \left[\rho_A^{-1/2} \sigma_A \rho_A^{-1/2} \right] \rho^{1/2}_{EW[A]}$$

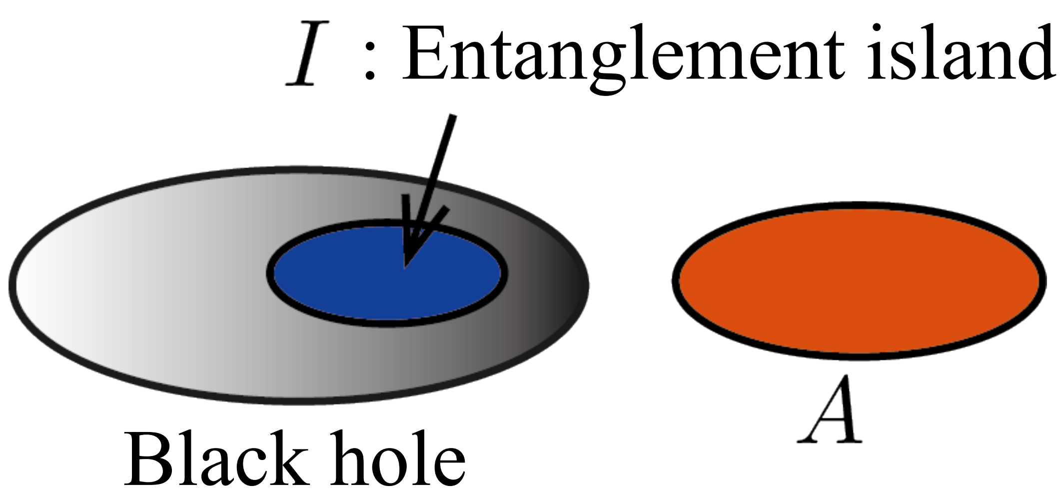
[Petz][Ohya, Petz][Junge, Renner, Sutter, Wilde, Winter]
[Cotler, Hayden, Penington, Salton, Swingle, Walter]

*) Twirled Petz map can be used as approximate recovery when the equality of relative entropy is only approximate.

Hawking's black hole information paradox was resolved!*

[Penington][Almheiri, Engelhardt, Marolf, Maxfield][Almheiri, Mahajan, Maldacena, Zhao]
[Penington, Shenker, Stanford, Yang][Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini]

Entanglement entropy of a reservoir A, sharing many EPR pairs with a black hole, is captured by the Island formula. It is the generalized entropy of A+I, I is the entanglement island.



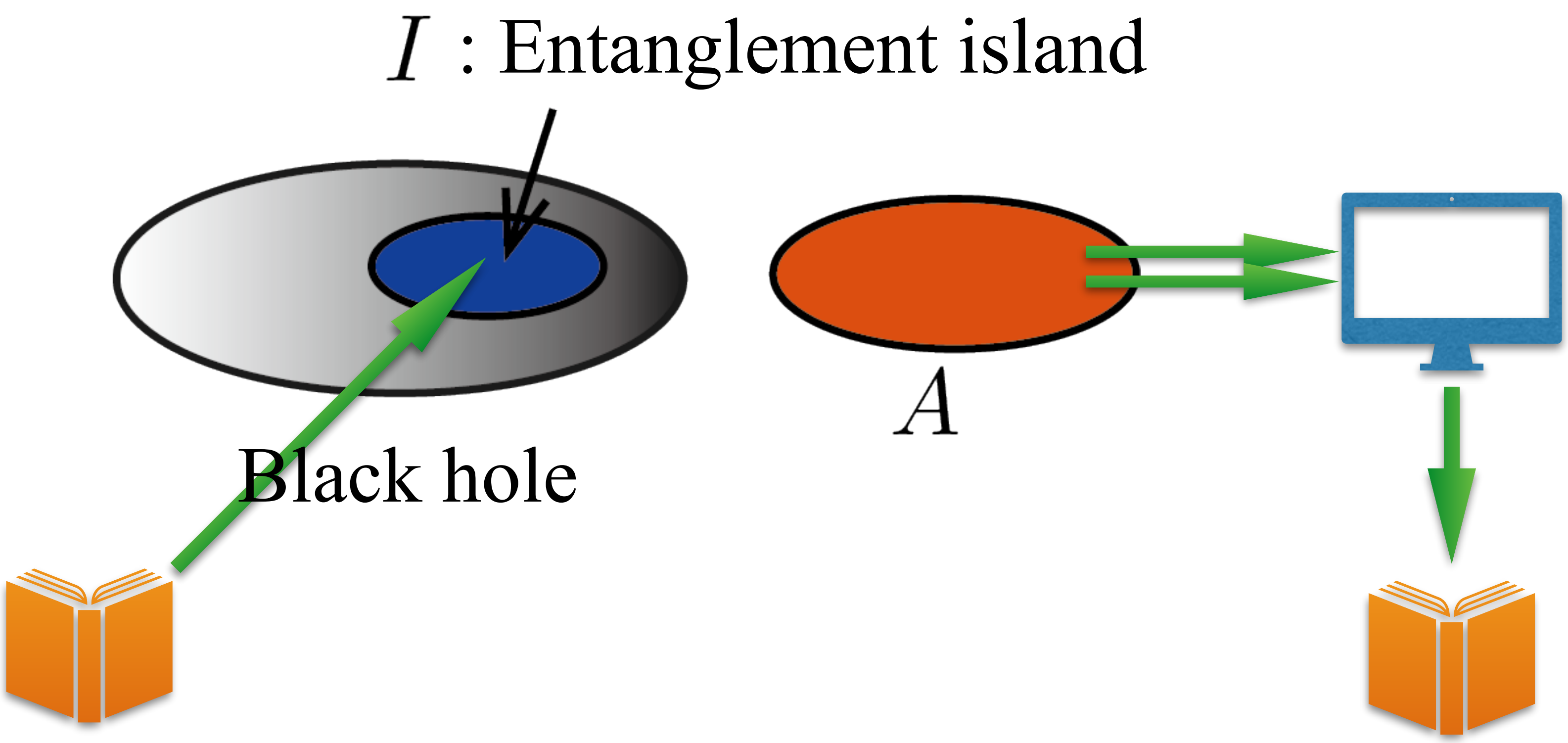
Island formula

$$S_A = \underset{I}{\mathrm{Min}}\,\mathrm{Ext}\left[S^{\mathrm{matter}}_{A\cup I} + \frac{\mathrm{Area}[\partial I]}{4G_N}\right]$$

Gravitationally suppressed small corrections are now captured by the island formula, when large number qubits are collected.

*While factorization paradox and how to interpret ensemble average raised new questions.
[Coleman][Maldacena, Maoz][Saad, Shenker, Stanford][Saad][Nomura][Marolf, Maxfield][Bousso, Tomasevic]
[Pollack, Rozali, Sully, Wakeham][Belin, de Boer][Stanford][Bousso, Wildenhain]

Entanglement island I is contained in the entanglement wedge of reservoir A.



Small book thrown into a black hole after Page time, can be recovered from the reservoir A, which has collected enough Hawking radiation. (Hayden-Preskill protocol)

Objective of this work:

- Evaluation of entanglement entropy at late time slice in universes with given initial state/conditions.

    - JT gravity on AdS and dS, gravity is turned off at late time.
    - Explicit initial conditions and state (with no bra-ket wormhole [Dong, Qi, Shangnan, Yang][Chen, Gorbenko, Maldacena] connecting bra and ket).
    - Entanglement entropy of large subsystem is given either by boundary entropy or the island formula.

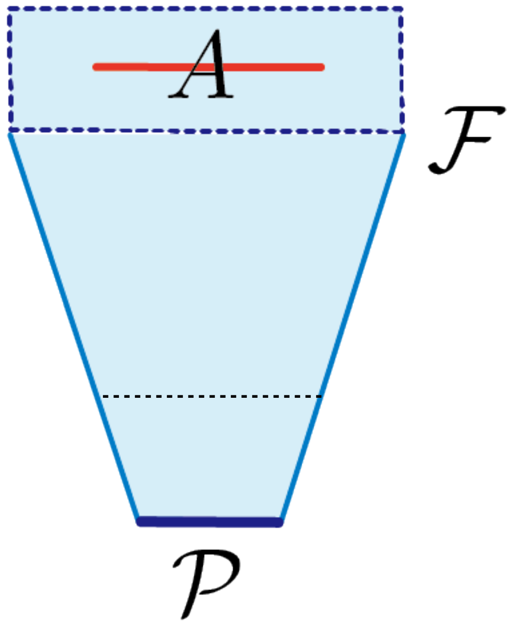
- Study reconstruction scheme for initial state in gravity. (Work in progress)

Our Set-up:

Theory: JT gravity on dS (or Euclidean AdS) + conformal matter.

$$I_{\text{Bulk}} = \frac{1}{16\pi G_N}\int_M d^2x\sqrt{-g_M}\phi(R-2) - \frac{1}{8\pi G_N}\int_{\mathcal{F}} dx\sqrt{h_{\mathcal{F}}}\phi(K_{\mathcal{F}}-1)$$

$$I_{\text{Top}} = \frac{\phi_0}{16\pi G_N}\int_M d^2x\sqrt{-g_M}R - \frac{\phi_0}{8\pi G_N}\int_{\mathcal{F}\cup\mathcal{P}} dx\sqrt{h}K$$



We fix (two) independent canonical variables of JT gravity and matter state at initial time slice $\mathcal{P}$.

- We fix spacial metric and extrinsic curvature (with appropriate weight, so that we can use saddle point approximation).
- $\mathcal{P}$ is set at positive imaginary time (as in Hartle-Hawking).
- Matter initial state: $|B(\beta)\rangle := \mathrm{e}^{-\beta H/4}|B\rangle$

Our Set-up:

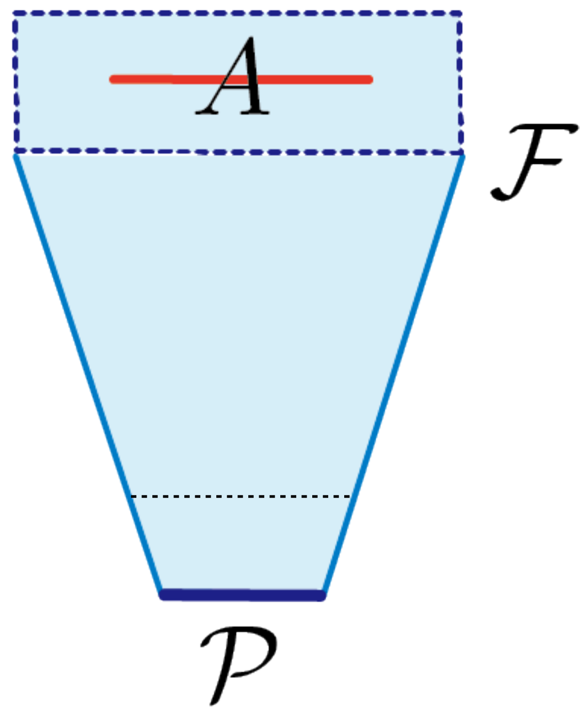
We use regularized boundary state as the initial matter state.

$$|B(\beta)\rangle := \mathrm{e}^{-\beta H/4}|B\rangle$$

- Pure state, while locally behaves as a thermal state with inverse temperature β .
- IR modification of the vacuum.
- Entanglement entropy of large subsystem is given only by the boundary entropy. [M.M, Ryu, Takayanagi, Wen]
- Modeling global quantum quench. [Calabrese, Cardy]

Our Set-up:

Gravitational interaction is turned off at final time slice, transitions to flat Minkowski space. (Modeling of phase transition from suitable potential)



Minkowski: $ds^2 = \frac{-dt_M^2 + dx_M^2}{\epsilon^2}$

dS (or AdS): $ds^2 = \frac{-d\eta^2 + dx^2}{\sinh^2\eta}$

Glueing condition: $\phi = \frac{\phi_b}{\epsilon} = \frac{\phi_r}{-\eta_c}$ $\qquad \frac{dx_M}{\epsilon} = \frac{dx}{-\eta_c}$

We will consider entanglement entropy of large subregion A, in the Minkowski region (where we have infinitely many d.o.f available).

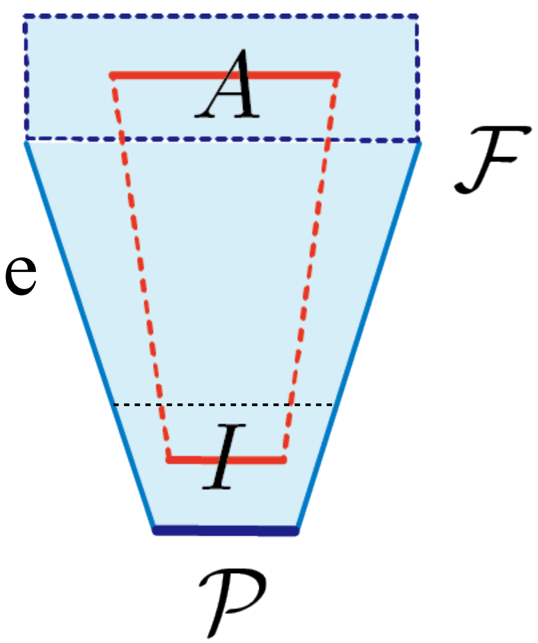
Results: Entanglement entropy

- Small subsystem: Thermal phase, volume law.

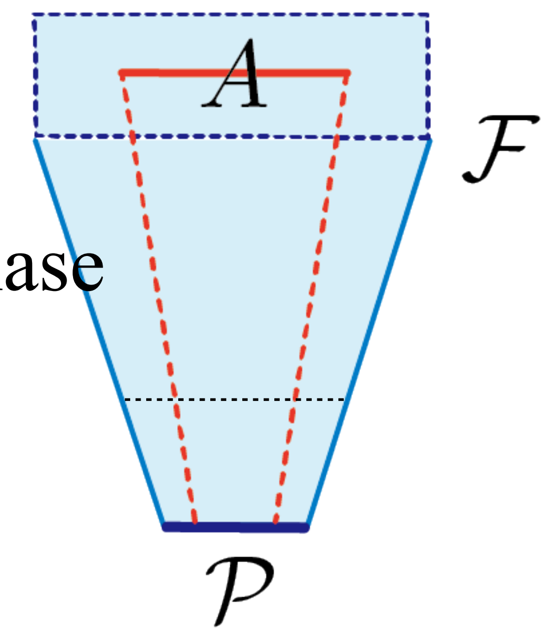
$$S_A \approx \frac{c}{3}\frac{\pi \Delta l}{\beta_M}$$

- Large subsystem: either Island phase or Boundary phase.

Island phase



Boundary phase

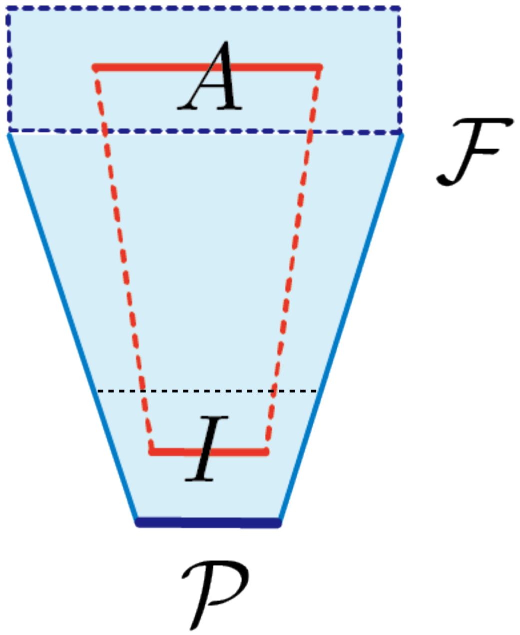


- In both scenarios, entanglement entropy is bounded from above by the area bound[Gibbons, Hawking][Bousso] of the past universe.

Island phase

- When boundary entropy must be larger than the area term.

$$\frac{\phi_0}{4G_N} < S_B = \log\langle 0|B\rangle$$



- When the subsystem thermal entropy is as large as the area term, then

$$S_A \approx 2\frac{\phi(\eta_I)}{4G_N} + S^{\text{matter}}_{A\cup I} \approx 2\frac{\phi_0}{4G_N} + \cdots$$

- Entanglement wedge reconstruction suggests that we can reconstruct excitations on I (Work in progress).

- Strong subadditivity is satisfied, if all decoupled small degrees of freedom are always thermal:

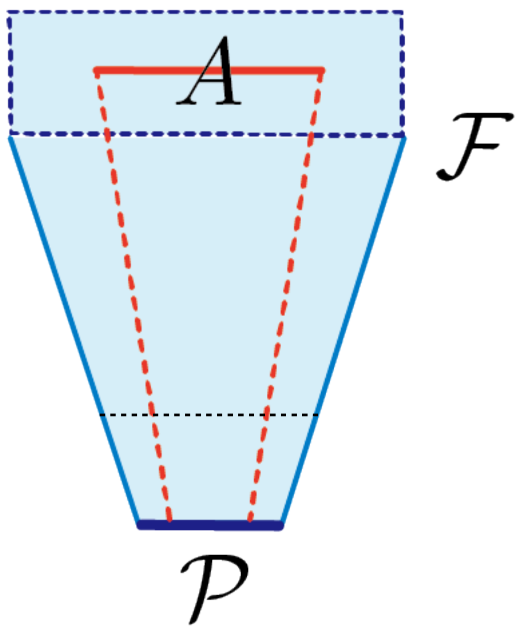
For $\quad \text{CFT}_{c+\tilde{c}} = \text{CFT}_c \otimes \text{CFT}_{\tilde{c}} \quad (\tilde{c} \ll c)$

$$S_A\left(\text{Tr}_{\text{CFT}_c}\left(|B(\beta)\rangle\langle B(\beta)|\right)_{\text{CFT}_c\otimes\text{CFT}_{\tilde{c}}}\right) \approx S_A\left(\rho^{\text{thermal}}_{\text{CFT}_{\tilde{c}}}(\beta)\right)$$

Boundary phase

- Boundary entropy must be smaller than the area term.

$$\frac{\phi_0}{4G_N} > S_B = \log\langle 0|B\rangle$$



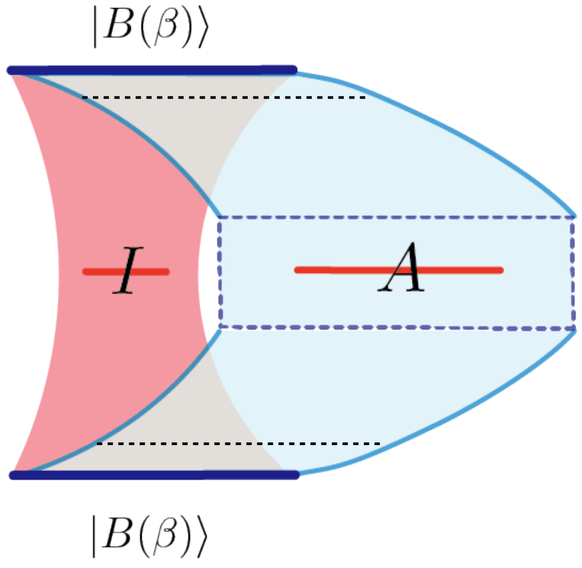
- When the subsystem thermal entropy is as large as the boundary entropy, then

$$S_A \approx 2S_B + \cdots$$

- Subset of degrees of freedom on the boundary can be reconstructed from A (Work in progress).

Special example:

- When the boundary state has holographic dual, and is prepared by a wormhole connecting bra and ket are connected → Bra-ket wormhole[Dong, Qi, Shangnan, Yang][Chen, Gorbenko, Maldacena].



Comments and Outlooks

- Entanglement entropy of large subregion, can probe the bound of entanglement entropy of the past universe. We found island phase and boundary phase.
- Generalizing Hartle-Hawking no boundary proposal by adding boundary.
- Our analysis can be applied to generic spacetimes without wormholes as background (but with replica wormholes!).
  - Known Euclidean wormhole solutions in higher dimensions are unstable, for instance to brane-anti brane nucleation[Maoz, Maldacena].
  - Higher dimensional spacetimes. (Work in progress)
- Choice of contour = choice of initial condition and weight.
  - Semiclassical geometry may be translated into path integral via Lefschetz thimbles[Feldbrugge, Lehners, Turok][Di Tucci, Lehners, Sberna].