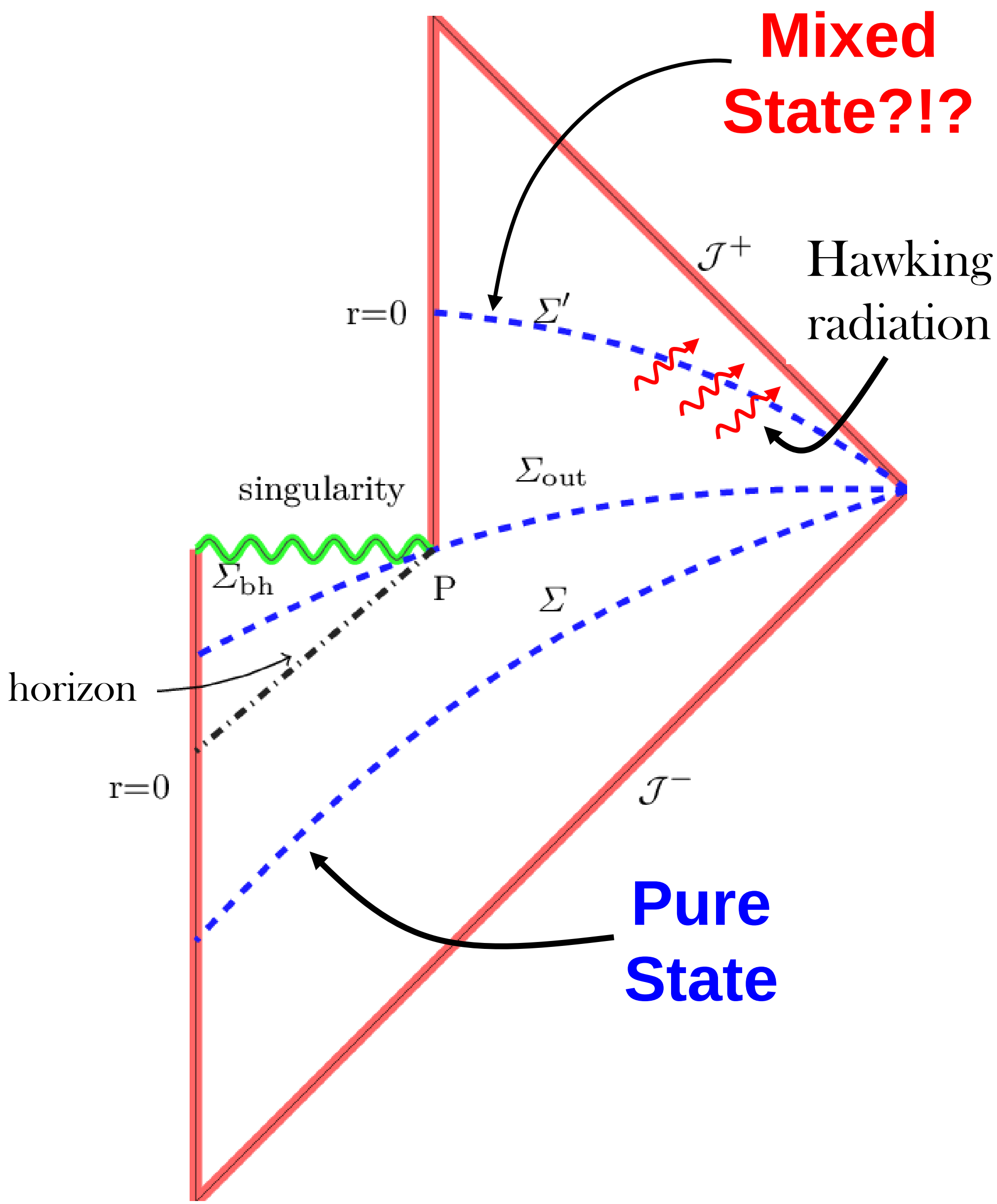
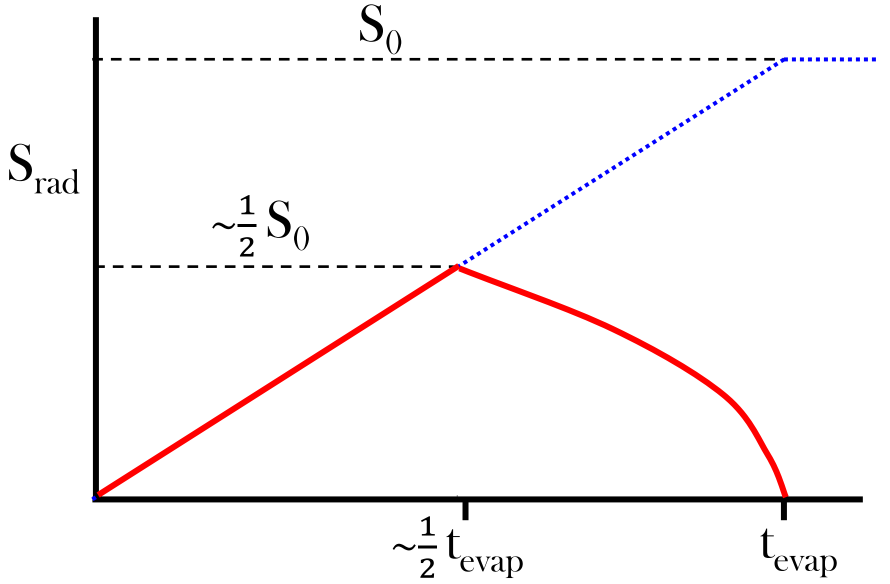
Quantum Extremal Islands Made Easy

with Chen, Neuenfeld, Reyes & Sandor; Hernandez & Ruan
(2006.04851 & 2010.00018; 2010.16398)

Black hole information paradox:



Page Curve



New insights from Holographic EE:

- with recent progress, it is possible to compute the Page curve in a controlled manner!

Penington [arXiv:1905.08255]
Almheiri, Engelhardt, Marolf & Maxfield [arXiv:1905.08762]
* Almheiri, Mahajan, Maldacena & Zhao [arXiv:1908.10996]

➡ Island Rule:

- black hole coupled to an auxiliary non-gravitational reservoir (the "bath"), which captures the Hawking radiation
- (correct) entropy of the Hawking radiation is given by

$$S_{EE}(\mathbf{R}) = \min\left\{ \underset{\text{islands}}{\text{ext}} \left(S_{QFT}(\mathbf{R} \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_N} \right) \right\}$$

- evaluate the (semiclassical) entanglement entropy of quantum fields in the bath region **R** combined with various space-like subregions in the gravitating region, ie, islands, which also contributes the usual Bekenstein-Hawking entropy

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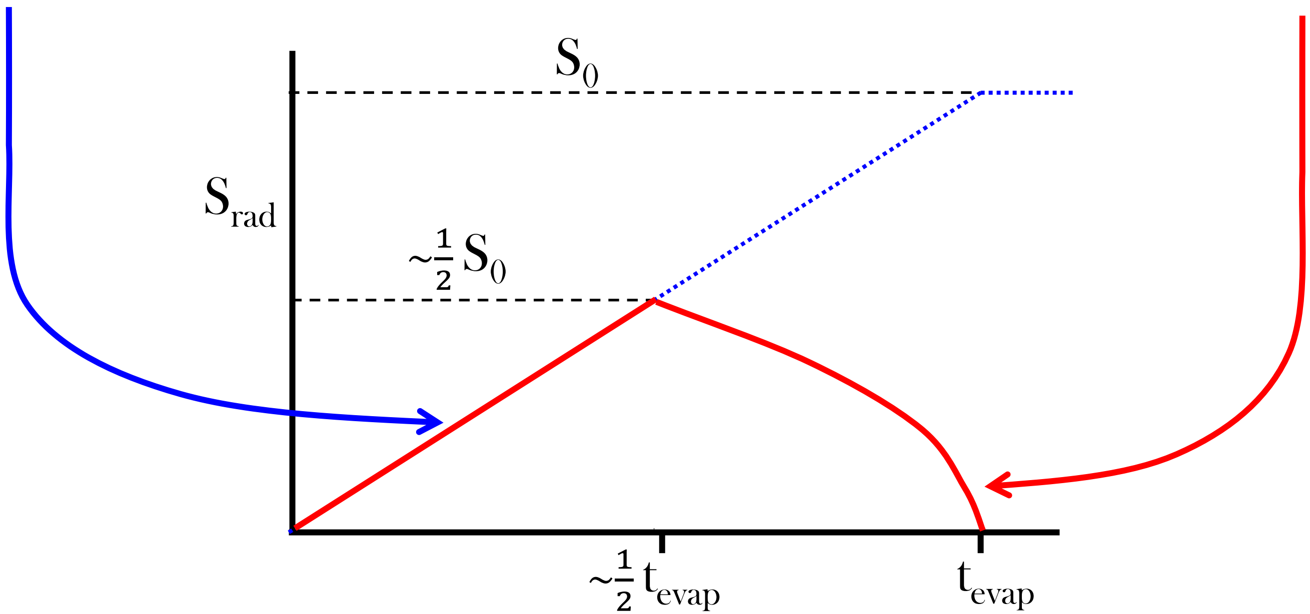
Early: island is the empty set; agrees with Hawking's calculation

Late: large entanglement between radiation and region behind horizon; **new saddle** with nontrivial island



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Key ingredients of early calculations:

$\longrightarrow$ absorbing or transparent b.c.

$\longrightarrow$ two-dimensional JT gravity

$\longrightarrow$ quantum extremal surfaces

Aside:

- Einstein-Hilbert term is topological in two dimensions
- Jackiw-Teitelboim (JT) gravity introduces extra scalar, dilaton

$$I_{\rm bulk}^{\rm JT} = \frac{\Phi_0}{16\pi G_N} \int_{\mathcal{M}} d^2x \sqrt{-g}\, \mathcal{R} + \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^2x \sqrt{-g}\, \Phi \left(\mathcal{R} - 2\Lambda_2\right) \,.$$

$$\underline{+ I_{CFT}(\Psi, g)}$$

→ describes physics of extremal horizons in higher dimensions

→ low-energy sector of Sachdev-Ye-Kitaev (SYK) model:

$$H_S = i^{q_S/2} \sum_{1 \le i_1 < \cdots < i_{q_S} \le N} J_{i_1 \ldots i_{q_S}} \psi_{i_1} \cdots \psi_{i_{q_S}} \quad \text{with} \quad \langle J^2_{i_1 \ldots i_{q_S}} \rangle = \frac{J_S^2 (q_S - 1)!}{N^{q_S - 1}}$$

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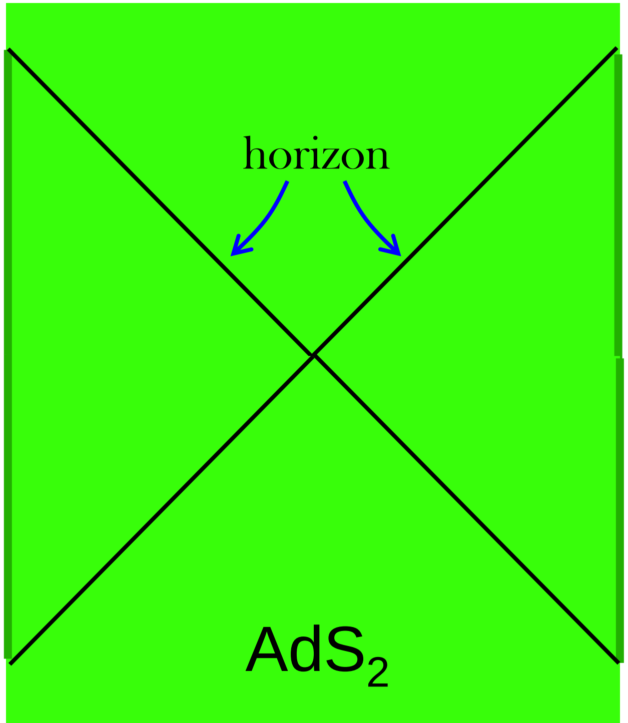
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} \quad \text{with} \quad f(r) \equiv \frac{r^2-\mu^2}{L_2^2}\,.$$

→ AdS$_2$ geometry

and $\Phi = \Phi_b \dfrac{r}{r_c}$



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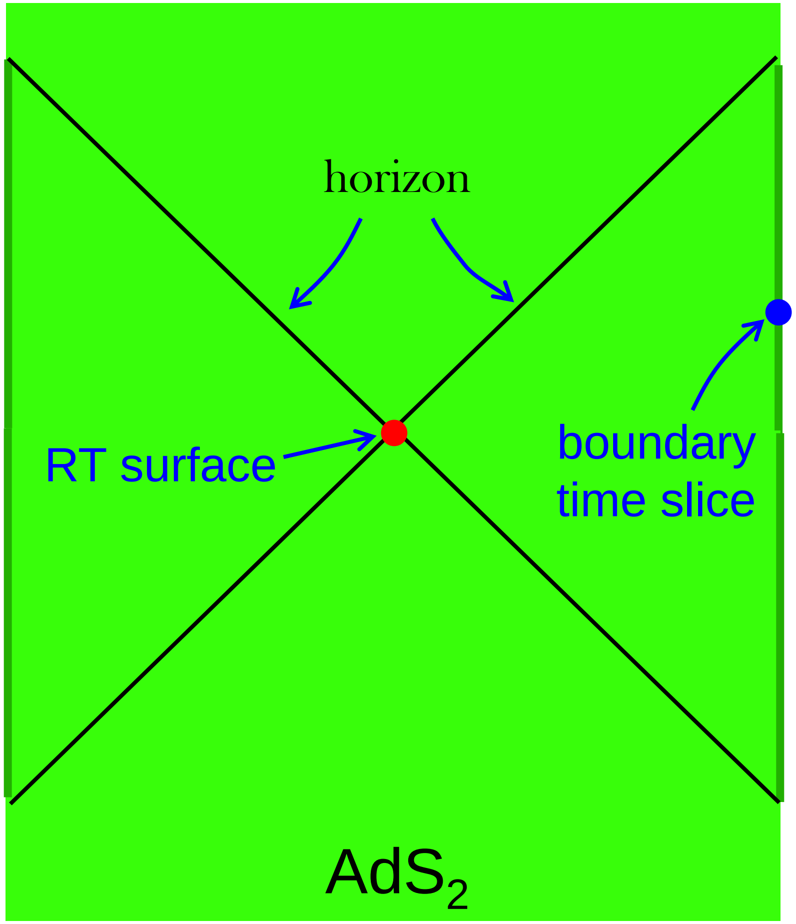

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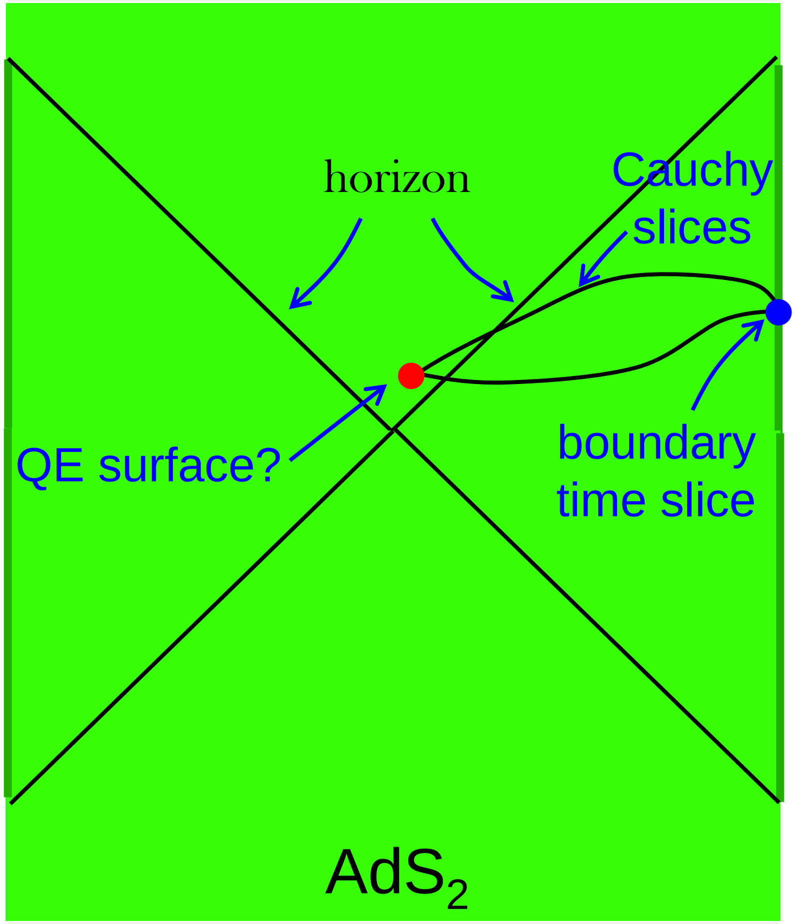

- RT surface: simply a point in bulk extremizing $\Phi(x)/4G_N$

- Quantum Extremal surface: extremizes $\Phi(x)/4G_N$ plus quantum S_{EE} of matter fields (need Cauchy slice connecting to boundary time slice)

(Faulkner, Lewkowycz & Maldacena; Engelhardt & Wall)

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Example: Not Evaporating Black Holes!

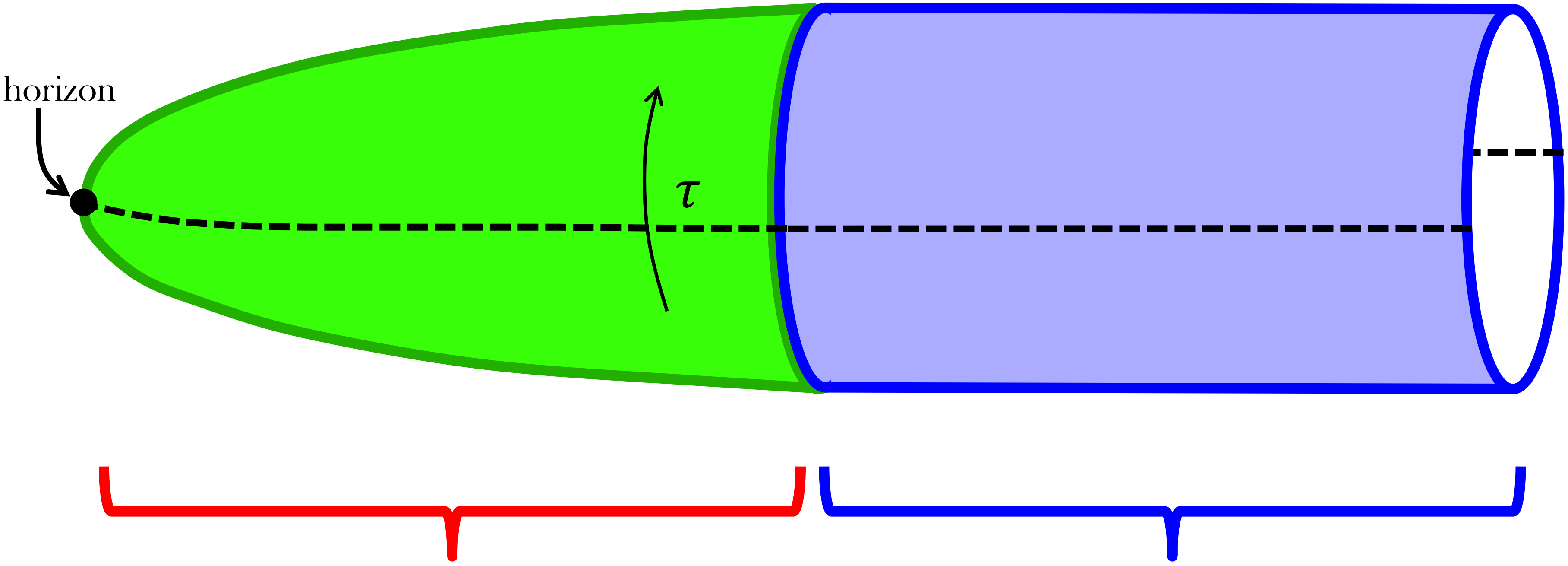
Almheiri, Mahajan & Maldacena [arXiv:1910.11077]

Example: Black Holes in Thermal Equilibrium

Almheiri, Mahajan & Maldacena
(see also: Rozali, Van Raamsdonk, Waddell & Wakeham)

- simple holographic model: 2d JT gravity + CFT = 1d quantum mech's
- prepare state with 2d black hole & bath in thermal equilibrium

→ Hartle-Hawking state



2d Euclidean BH

CFT_2 on thermal cylinder

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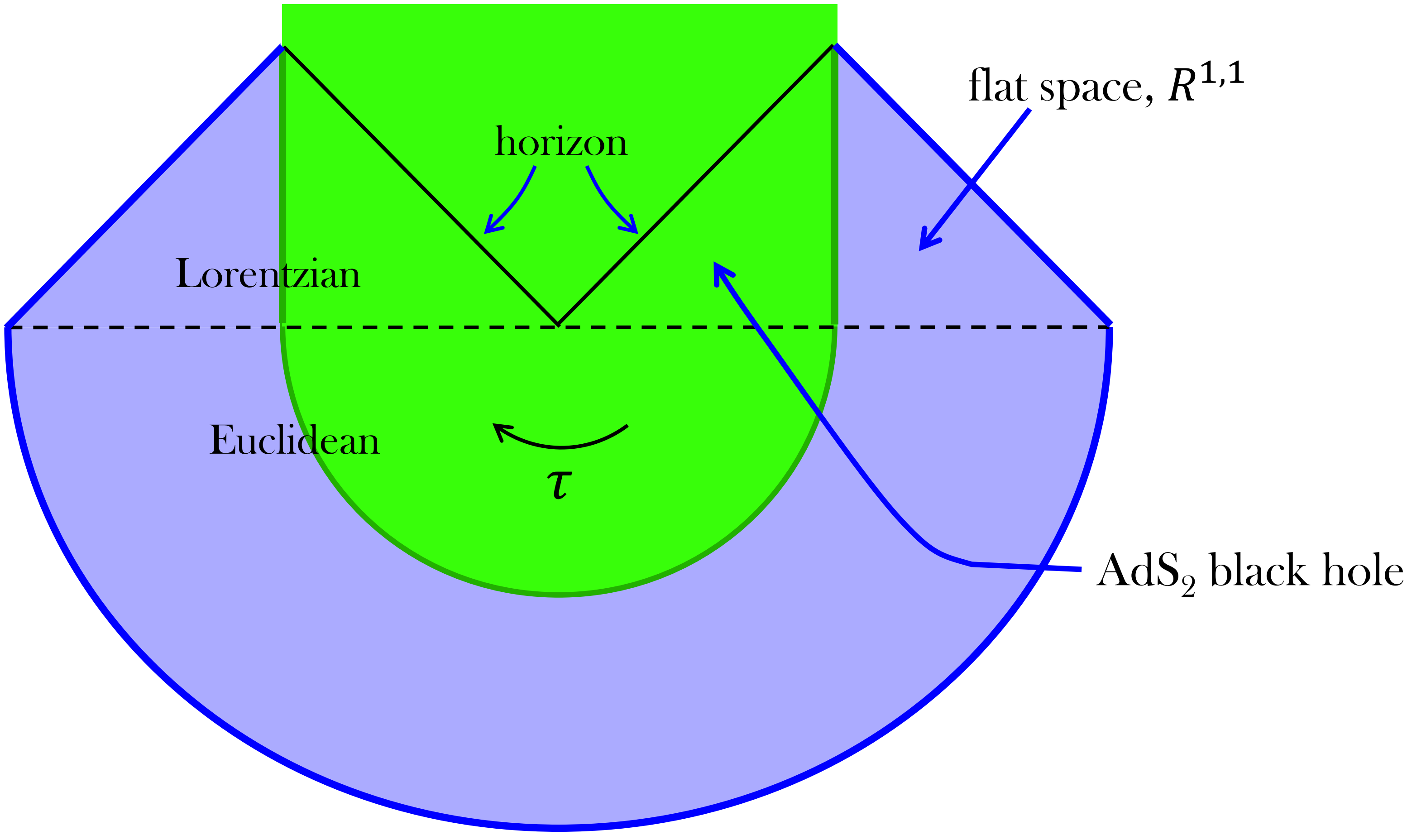

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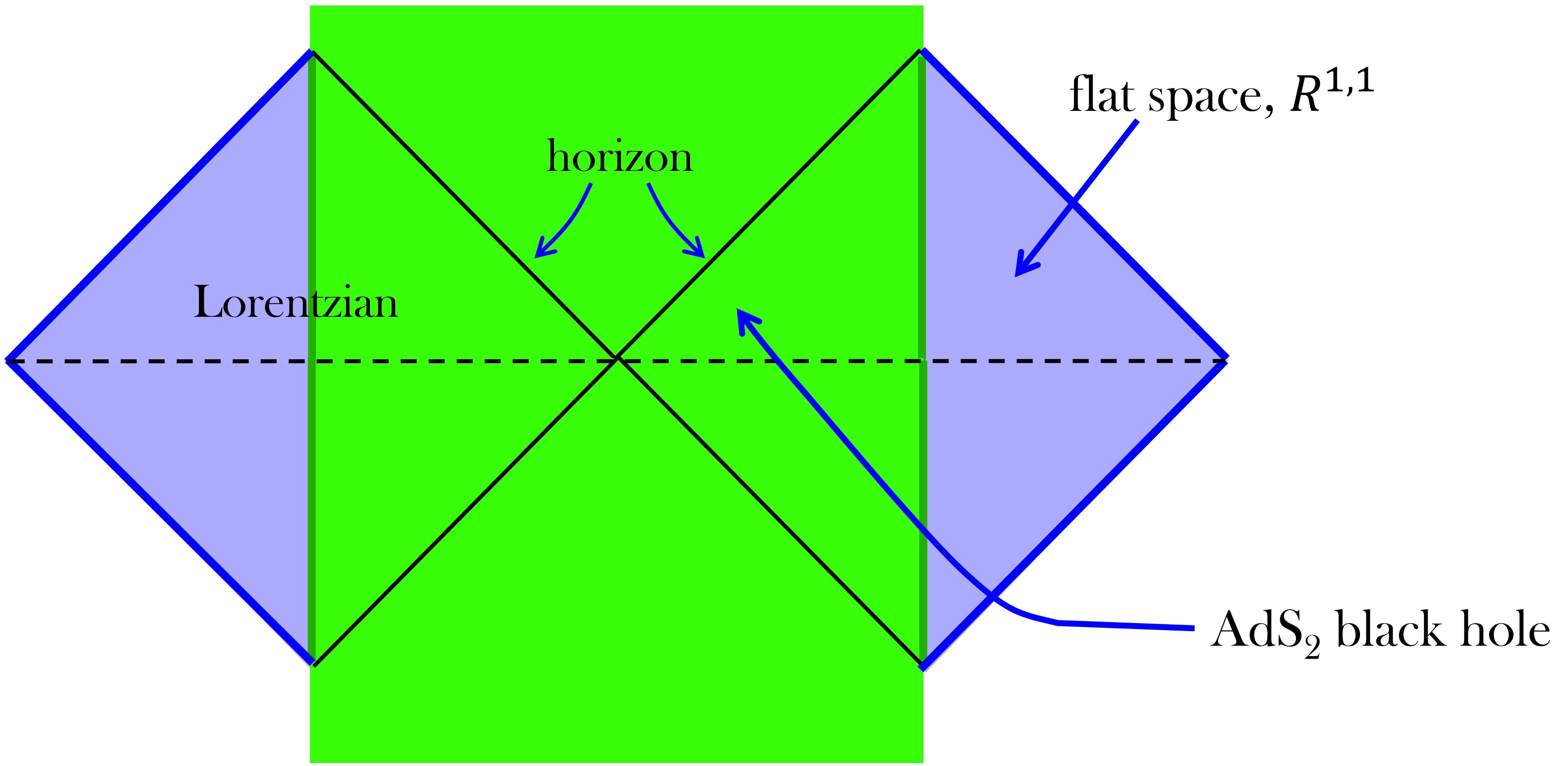

Thermal equilibrium? No information paradox?

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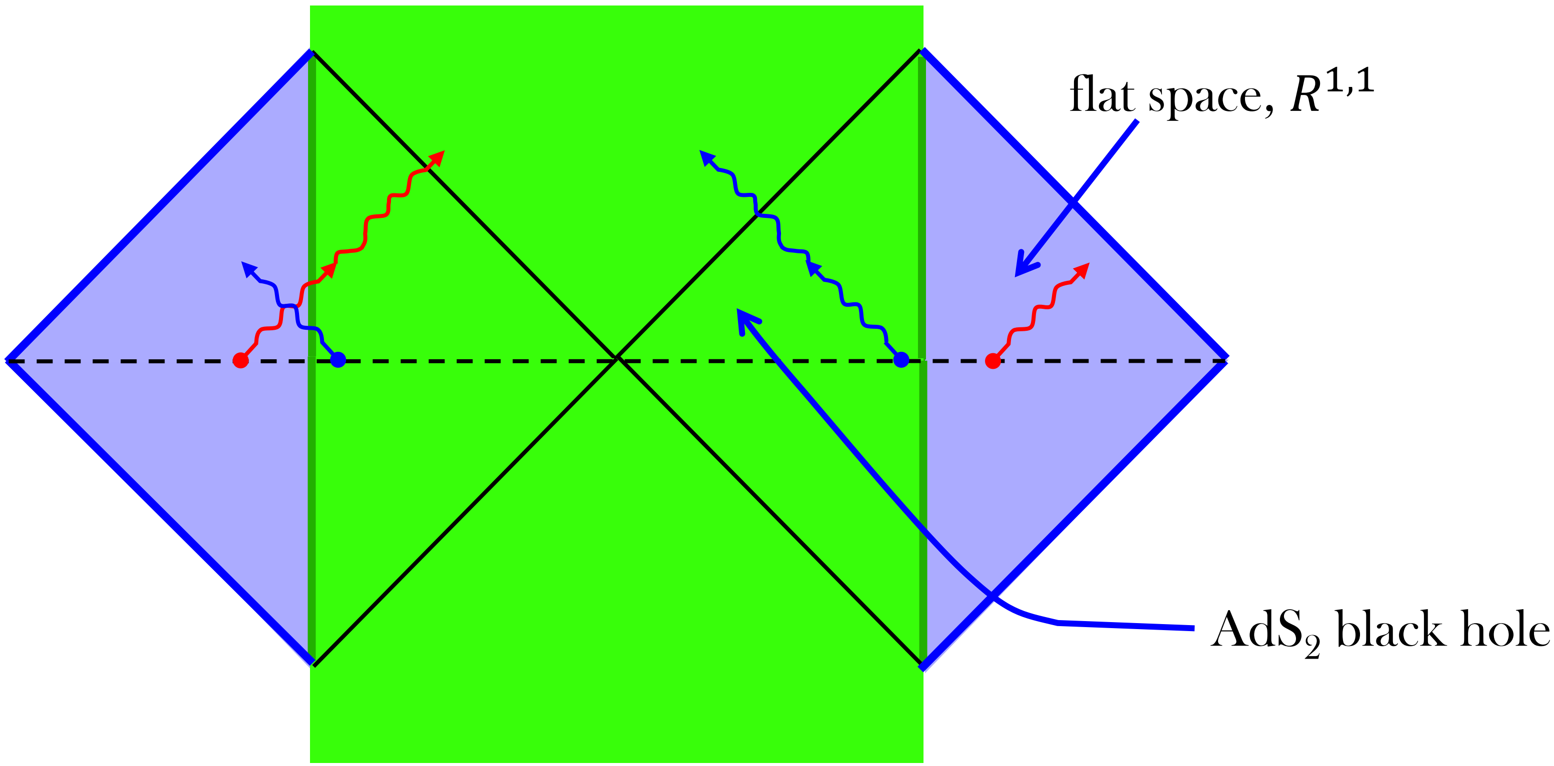

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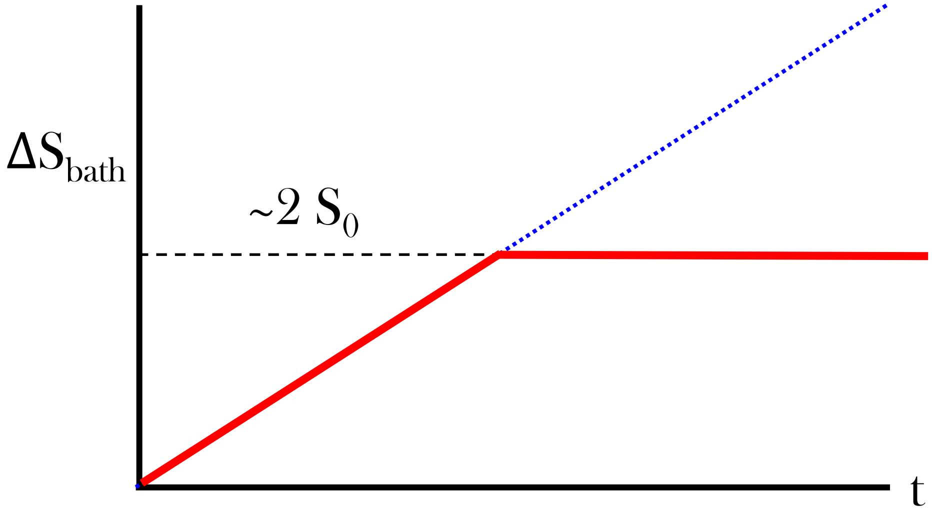
What does Page curve look like for eternal black hole?

- eternal BH and bath are continuously exchanging radiation



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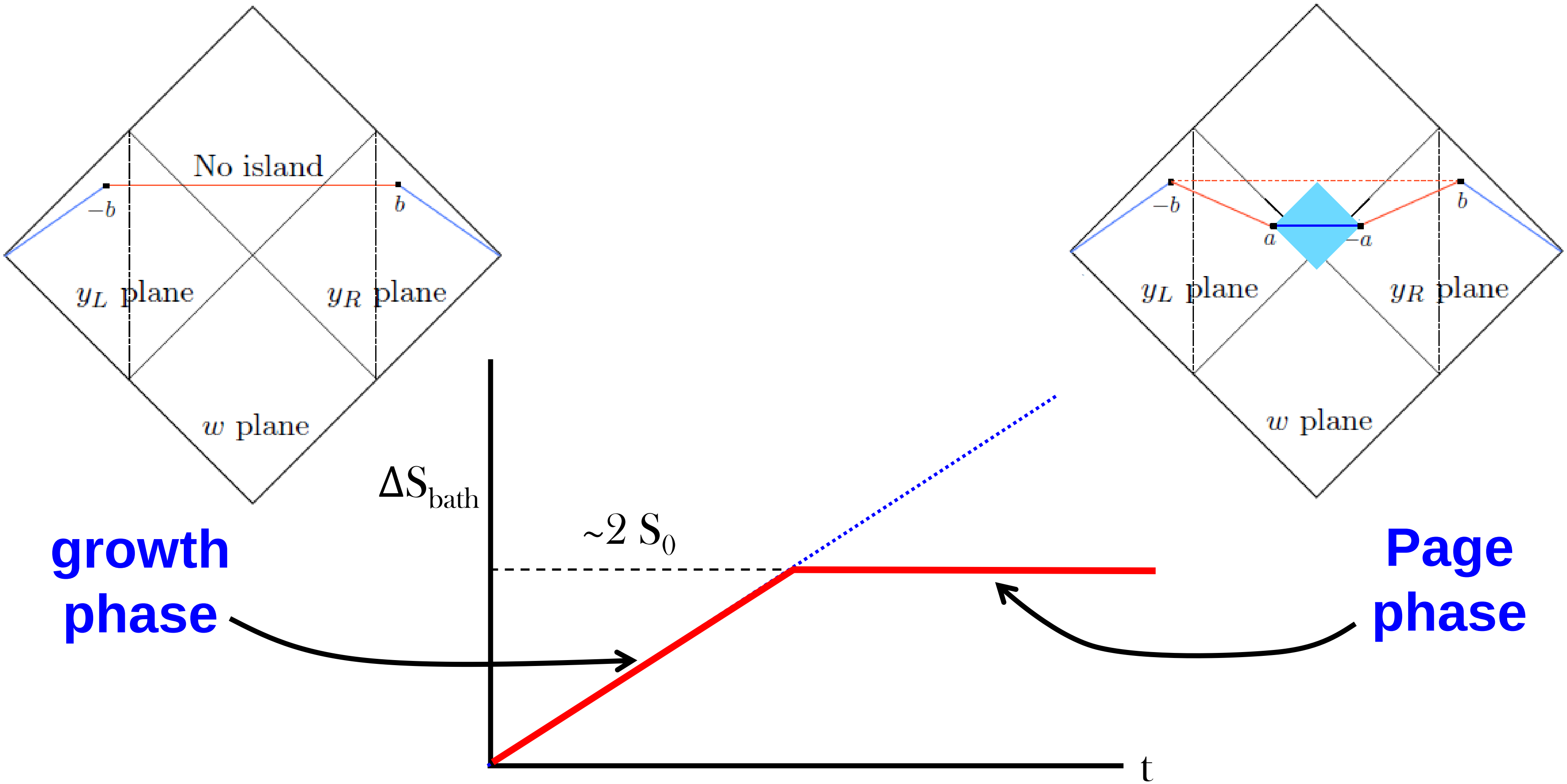

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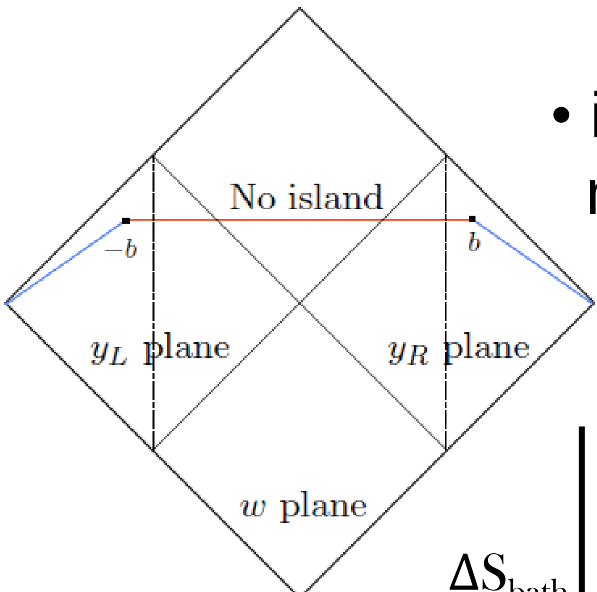
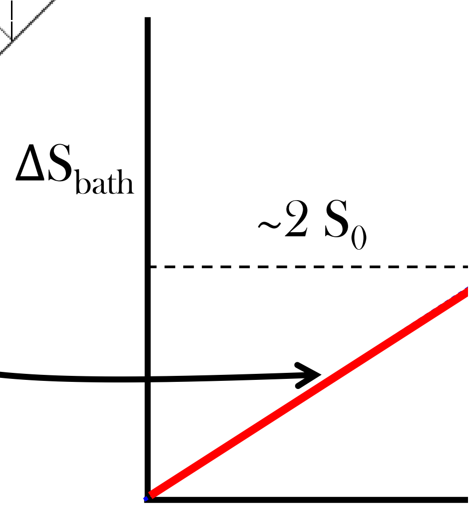
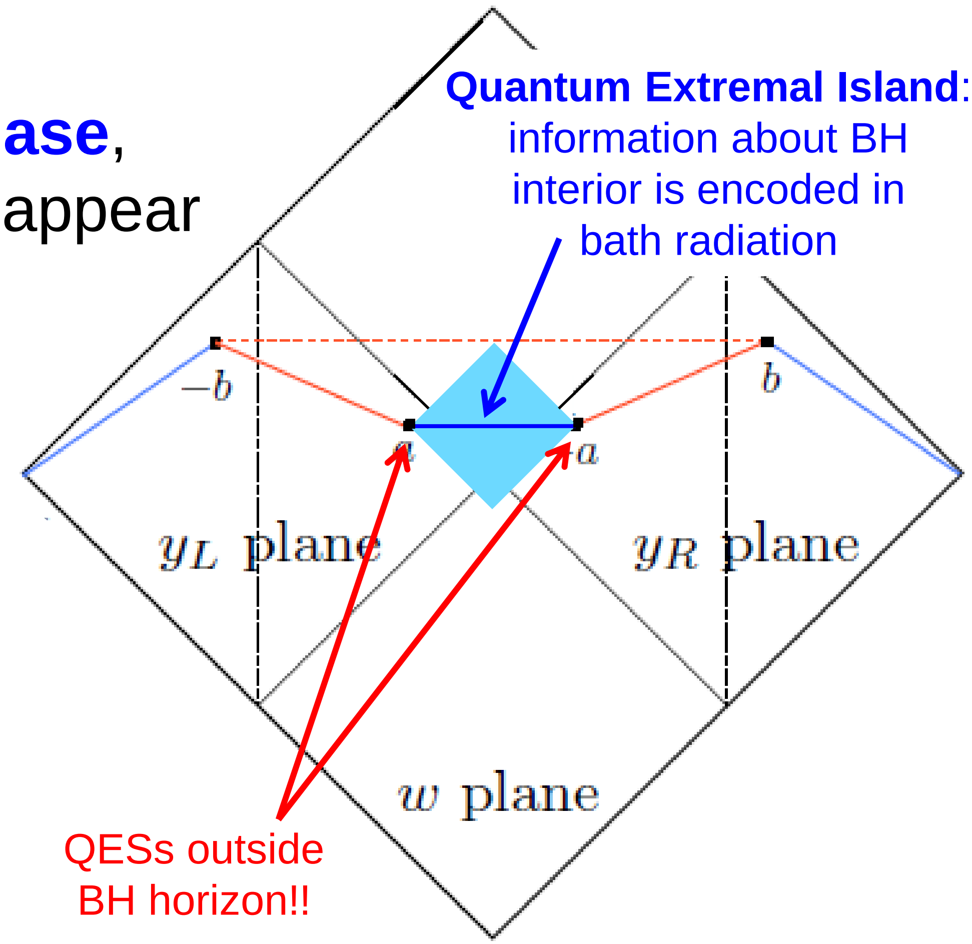


- in **Page phase**, new QESs appear



growth phase



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Questions, Questions, Questions:

- how important is two dimensions?
- are dof on Planck brane part of boundary or bulk?
- was JT gravity important?
- was ensemble average of SYK model important?
- how was information encoded in Hawking radiation?

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Many of new insights can be understood as familiar properties of holographic entanglement entropy

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Randall-Sundrum gravity (quick review):

- introduce d-dim. brane in (d+1)-dim. AdS geometry, backreaction creates extra d-dim. graviton mode localized on brane:

$$I_{\text{bulk}} = \frac{1}{16\pi G_{\text{bulk}}} \int d^{d+1}x \, \sqrt{-g} \left[\frac{d(d-1)}{L^2} + R(g) \right]$$

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- “position” of brane can be determined by:
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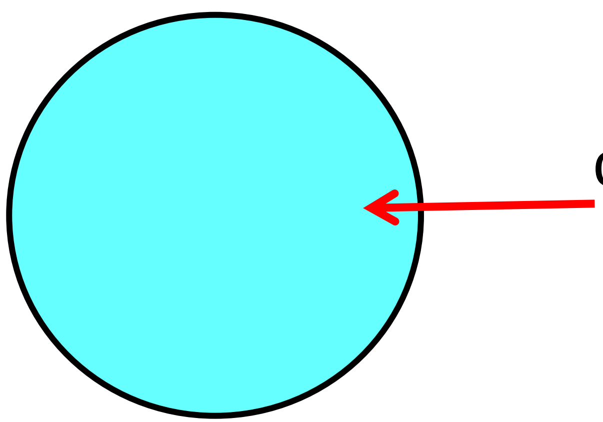

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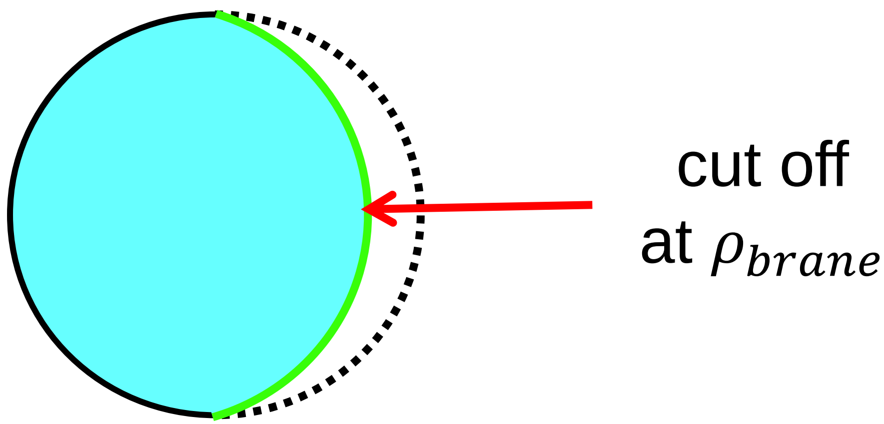

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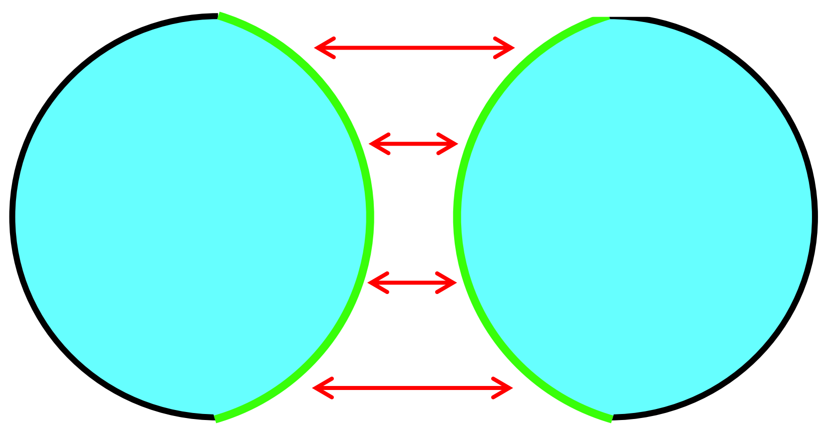

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$$\Delta K_{ij}-\tilde g_{ij}\,\Delta K^{\ell}{}_{\ell}=-8\pi G_{\rm bulk}T_0\,\tilde g_{ij}$$

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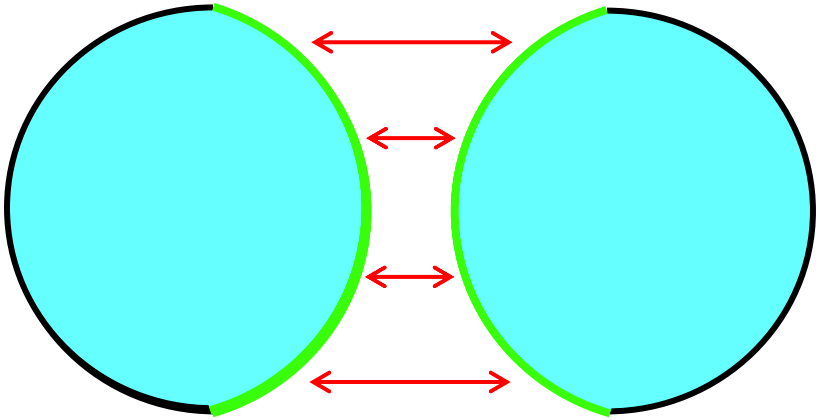
- “position” of brane can be determined by:
  using Israel junction conditions or solving brane gravity eom

$$\frac{1}{\ell_{\rm eff}^2} = \frac{1}{\ell_{\rm B}^2}\left[1 + \frac{1}{4}\,\frac{L^2}{\ell_{\rm B}^2} + \cdots\right]$$

with $\ell_{\rm B} = L\,\cosh\rho_{\rm brane}$



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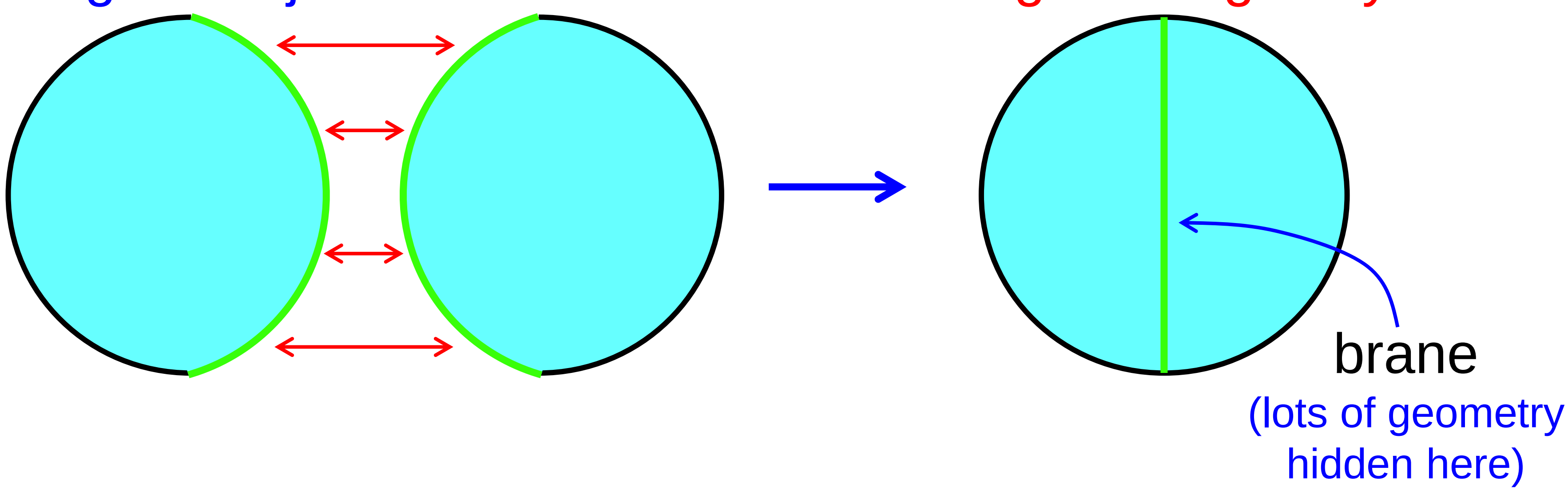
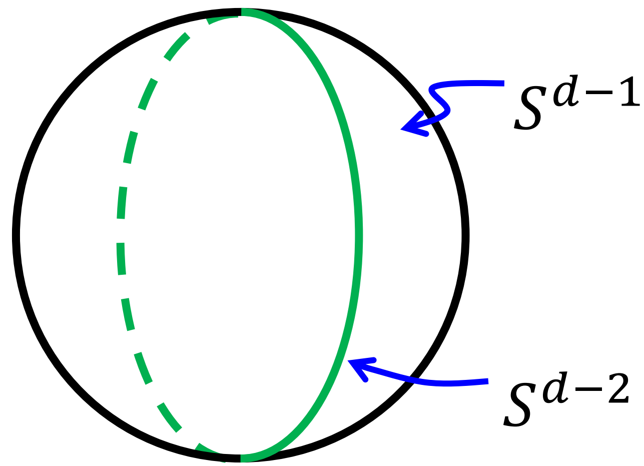
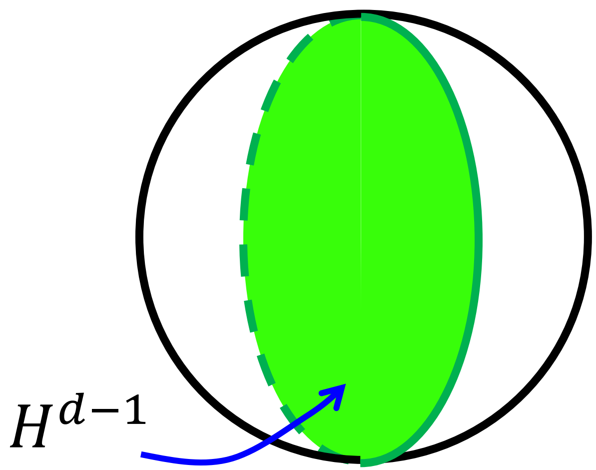


Randall-Sundrum gravity:

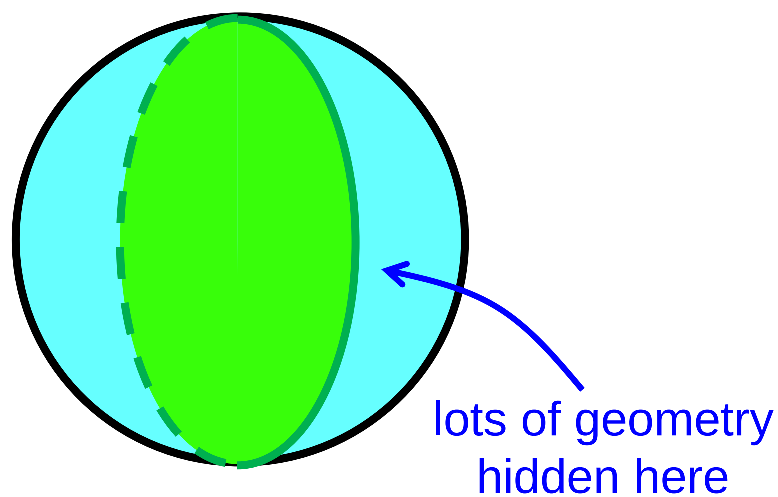
(a) holographic CFT_d coupled to conformal defect (ie, boundary CFT_{d-1})





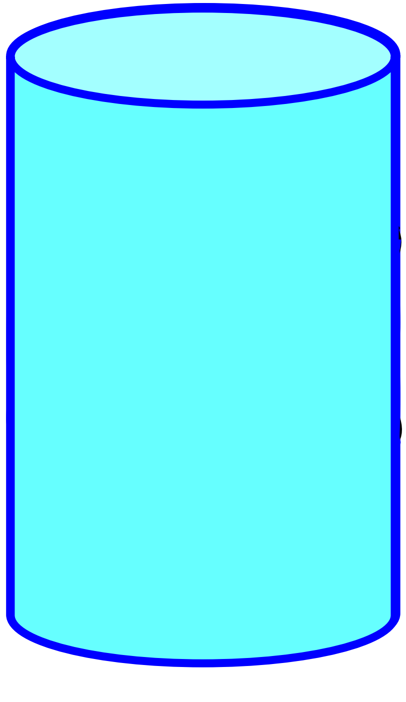
(b) holographic CFT_d coupled to CFT_d with gravity on AdS_d

(c) AdS_{d+1} gravity coupled to brane with AdS_d geometry



Black Holes in Thermal Equilibrium:

- AdS_{d+1} gravity coupled to brane with AdS_d geometry

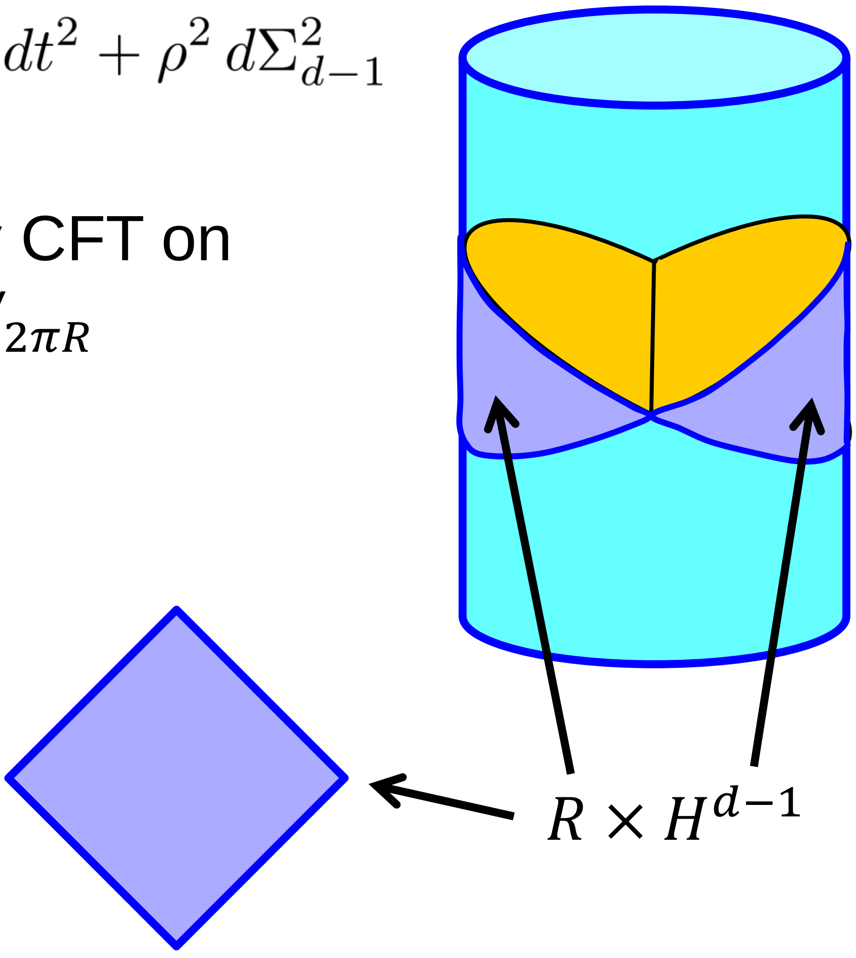


Black Holes in Thermal Equilibrium:

- AdS_{d+1} gravity coupled to brane with AdS_d geometry
- empty AdS_{d+1} space can be described as “hyperbolic” black hole

$$ds^2 = \frac{L^2\, d\rho^2}{(\rho^2 - L^2)} - \frac{\rho^2 - L^2}{R^2}\, dt^2 + \rho^2\, d\Sigma^2_{d-1}$$

- describes TFD state of boundary CFT on $R \times H^{d-1}$ at temperature $T = {}^{1}/_{2\pi R}$



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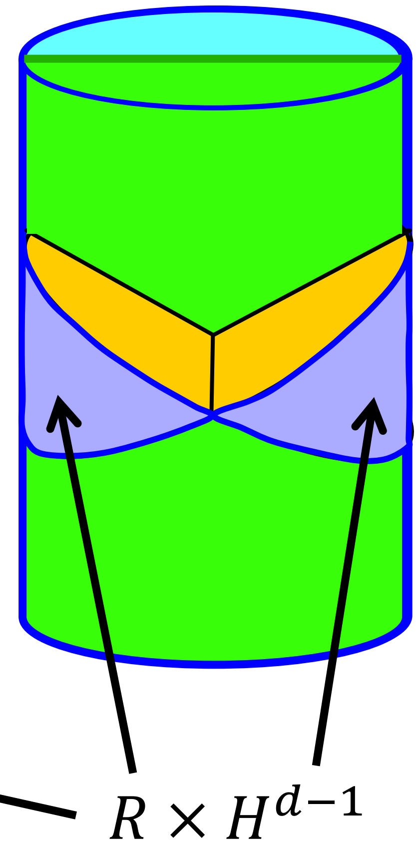
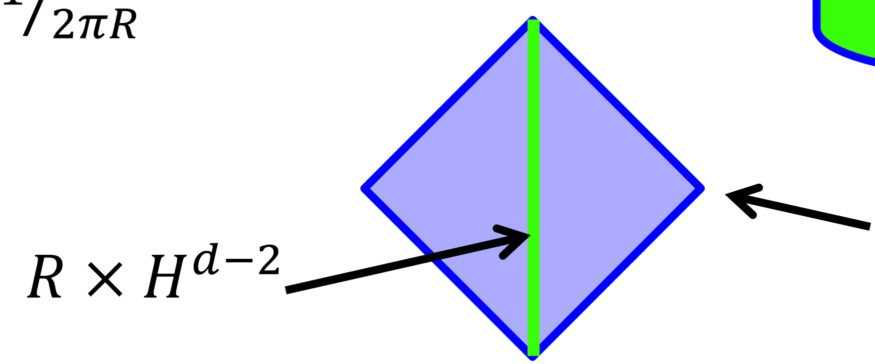


- describes TFD state of boundary CFT on $R \times\ H^{d-1}$ at temperature $T =\ ^1/_{2\pi R}$
- insert brane, describes TFD state of boundary CFT coupled to conformal defect on $R \times\ H^{d-1}$ at temperature $T =\ ^1/_{2\pi R}$



Black Holes in Thermal Equilibrium:

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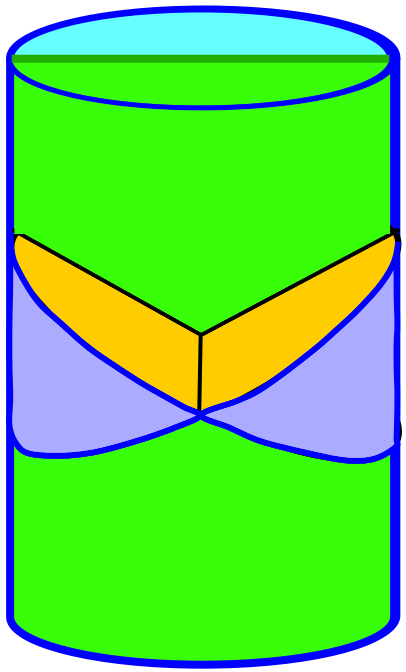

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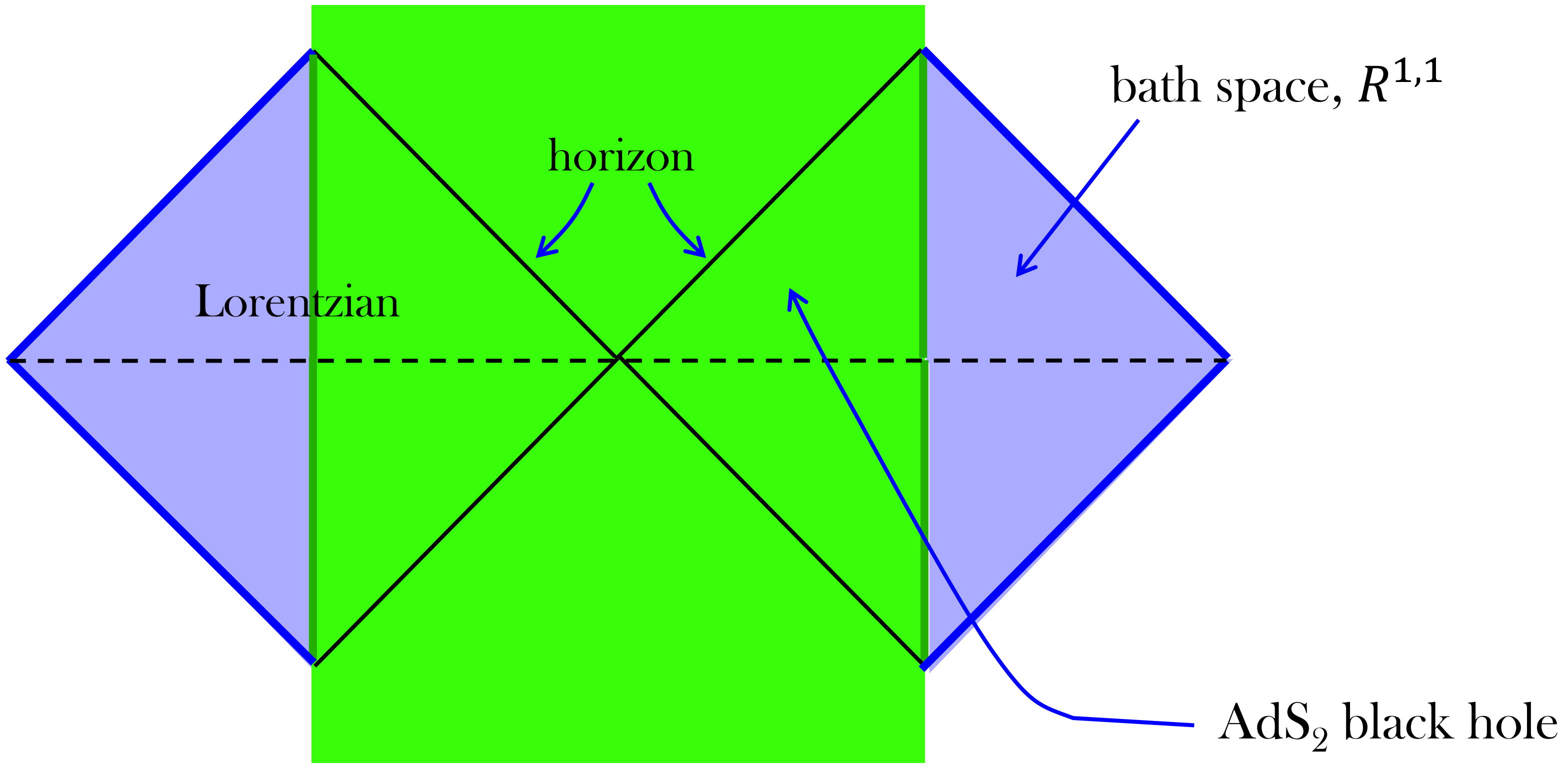
- insert brane, describes TFD state of boundary CFT coupled to conformal defect on $R \times H^{d-1}$ at temperature $T = {}^1/_{2\pi R}$
- induced brane metric "inherits" hyperbolic black hole geometry

$$ds^2 = \frac{\ell_B^2\, d\tilde{\rho}^2}{\tilde{\rho}^2 - \ell_B^2} - \frac{\tilde{\rho}^2 - \ell_B^2}{R^2}\, dt^2 + \tilde{\rho}^2\, d\Sigma^2_{d-2}$$

Black Holes in Thermal Equilibrium:

- previous discussion lifts to higher dim'l holographic model with d=2 JT gravity replaced by induced d-dim. Einstein gravity



Black Holes in Thermal Equilibrium:

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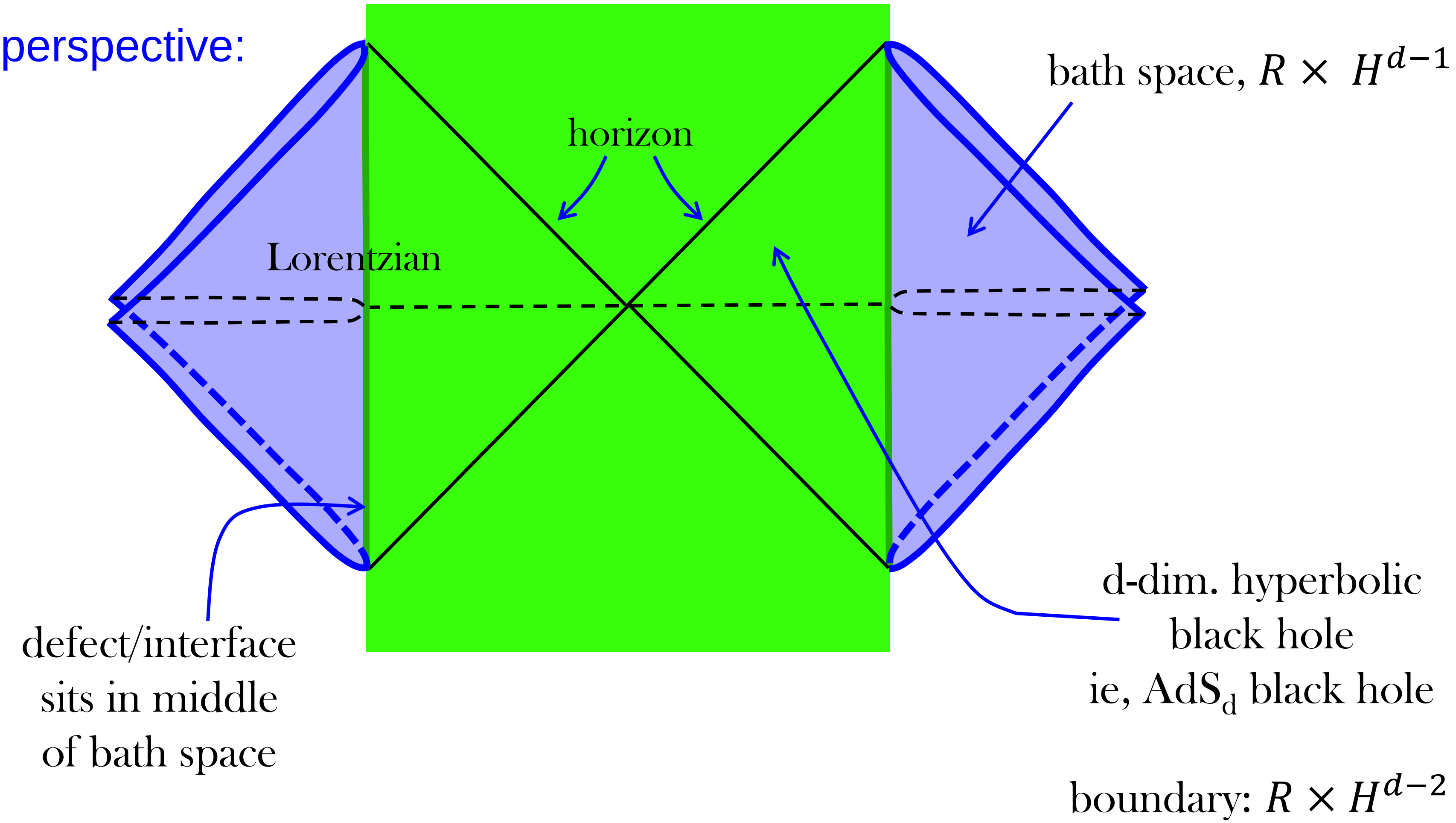
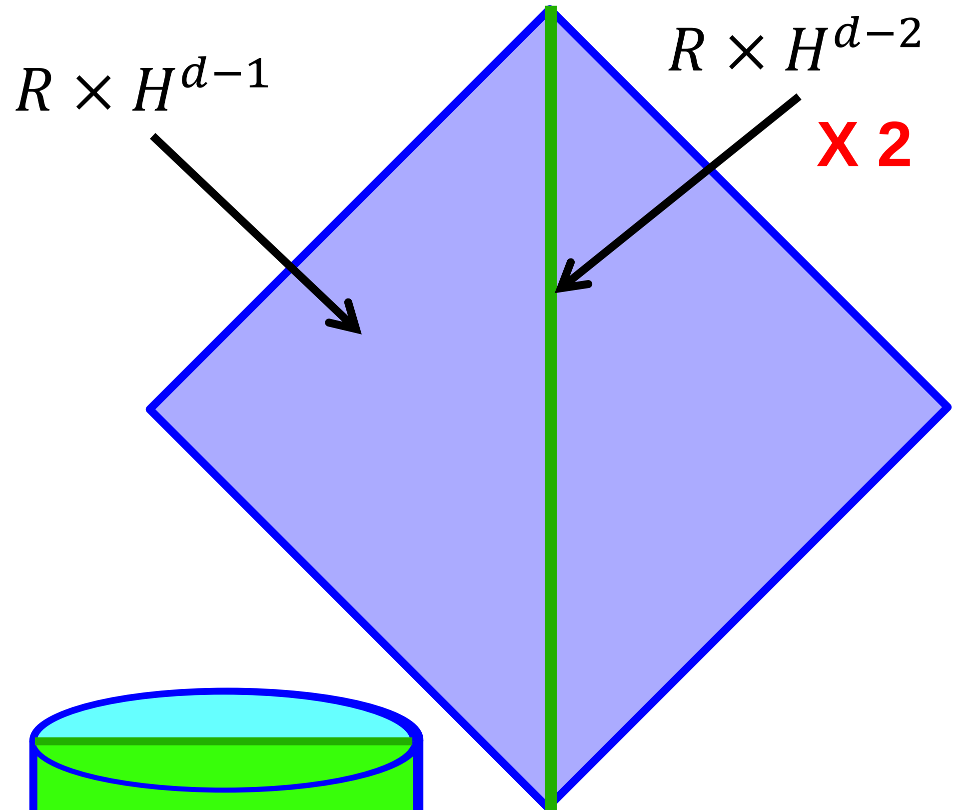
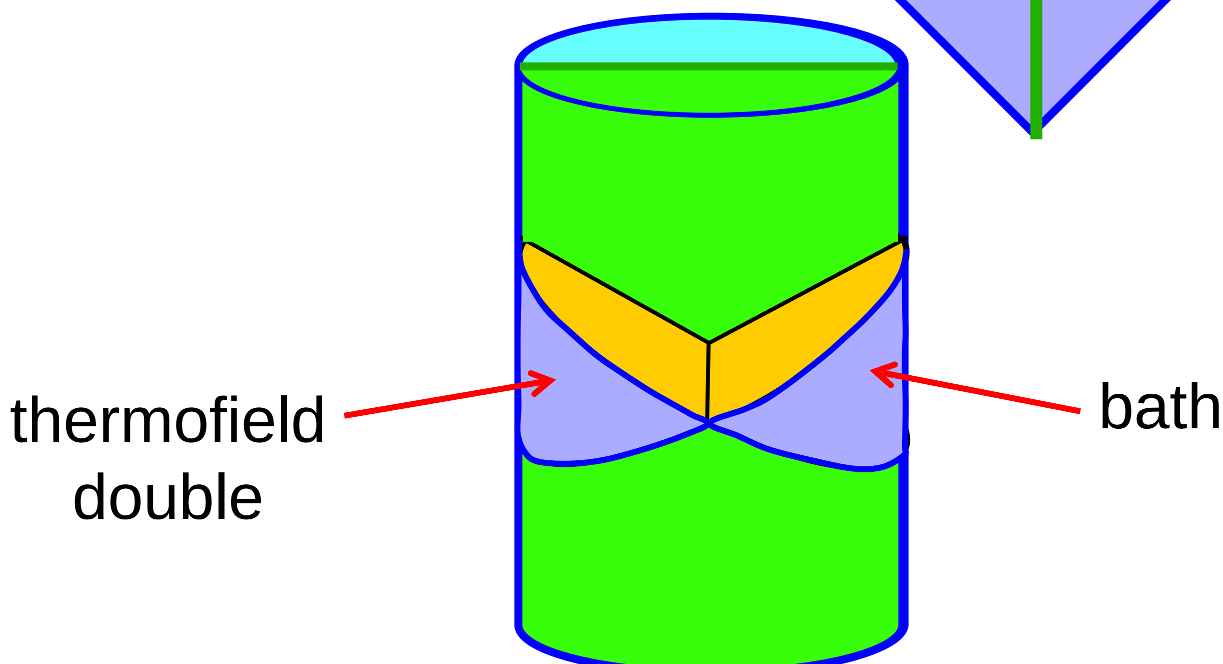
brane perspective:



• boundary CFT coupled to conformal defect:

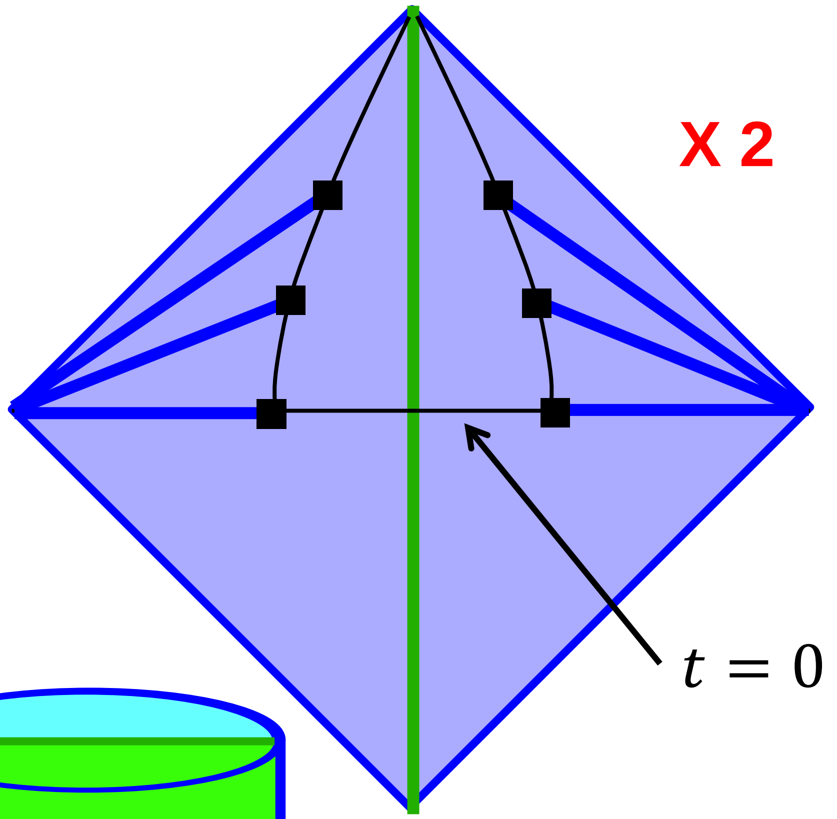


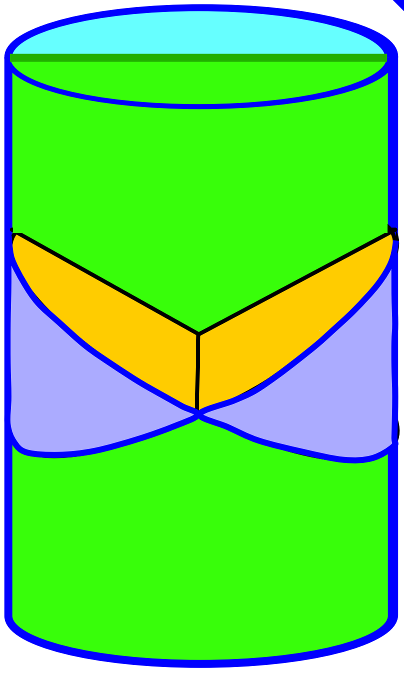


• boundary CFT coupled to conformal defect:

• EE of a bath (and thermal copy), some distance from defect:

• evolve in time; end points drawn closer to the defect





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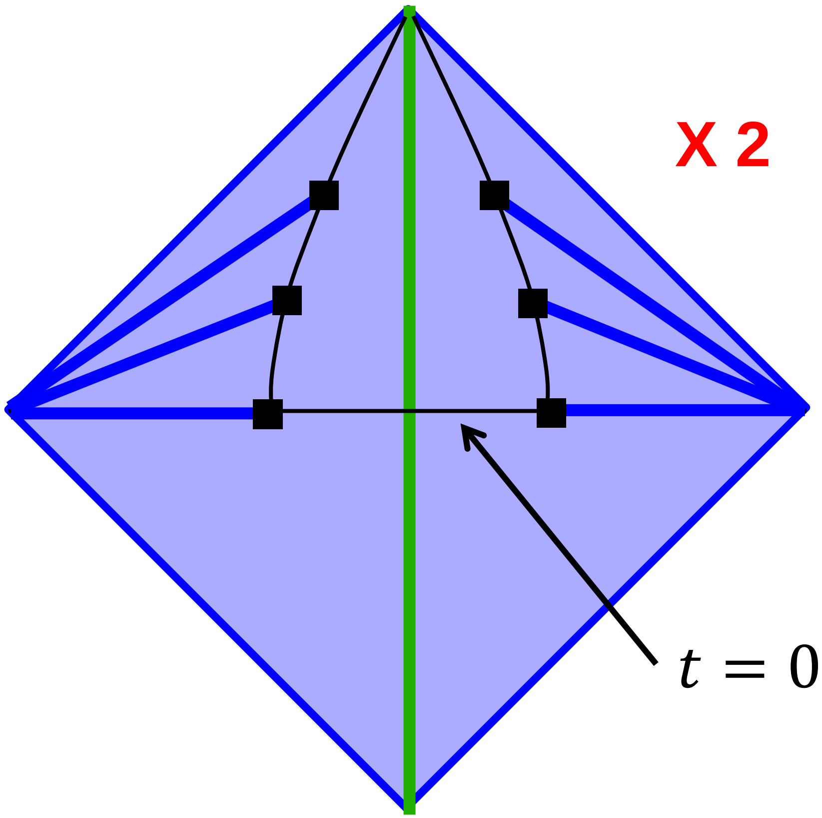

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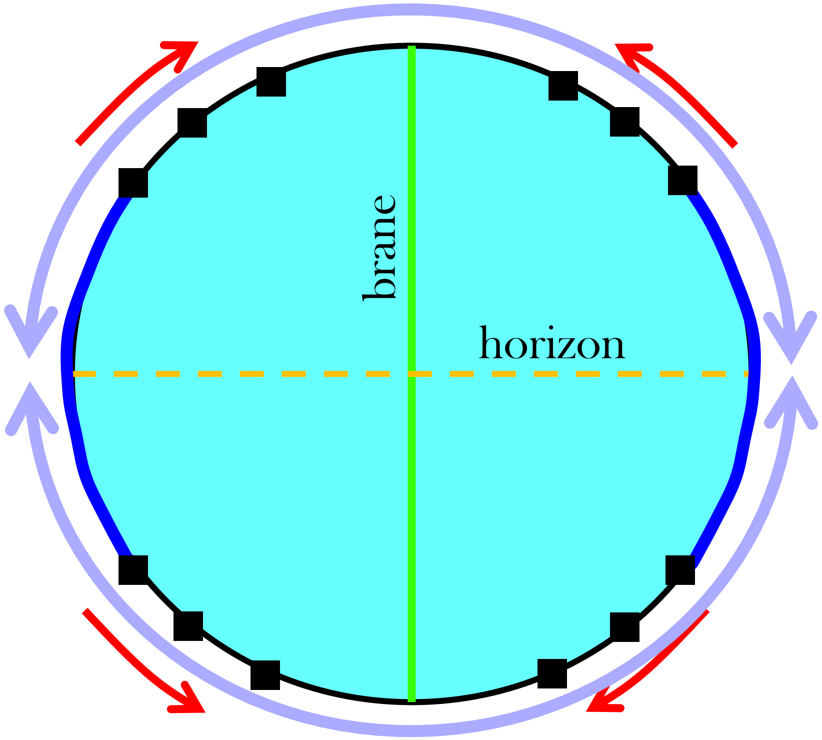
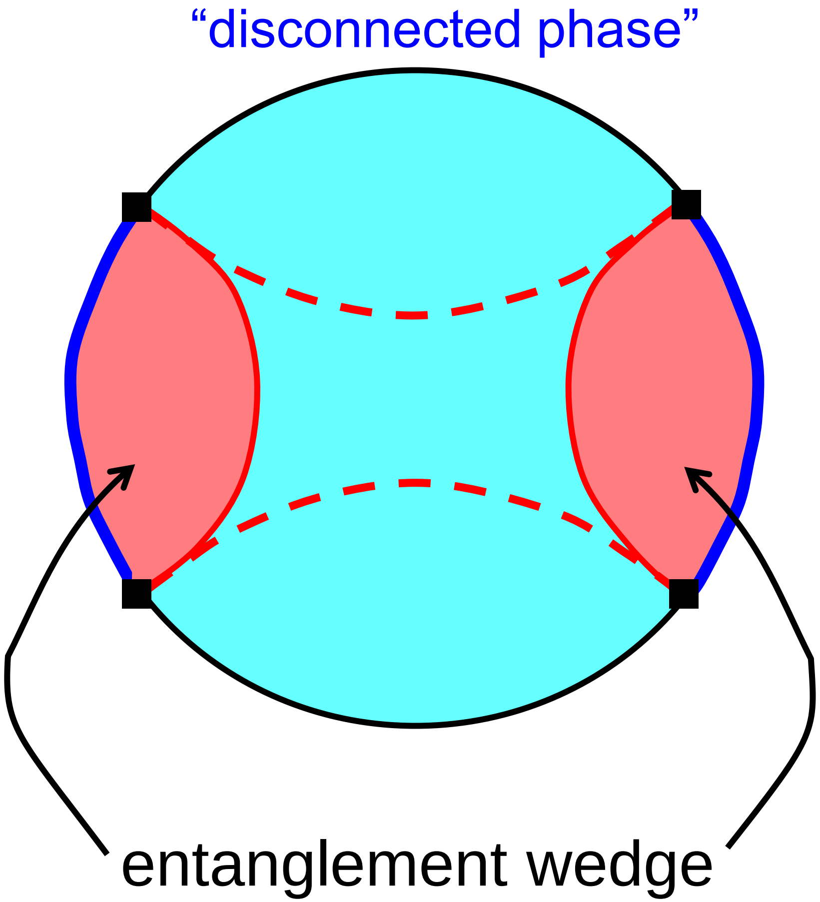
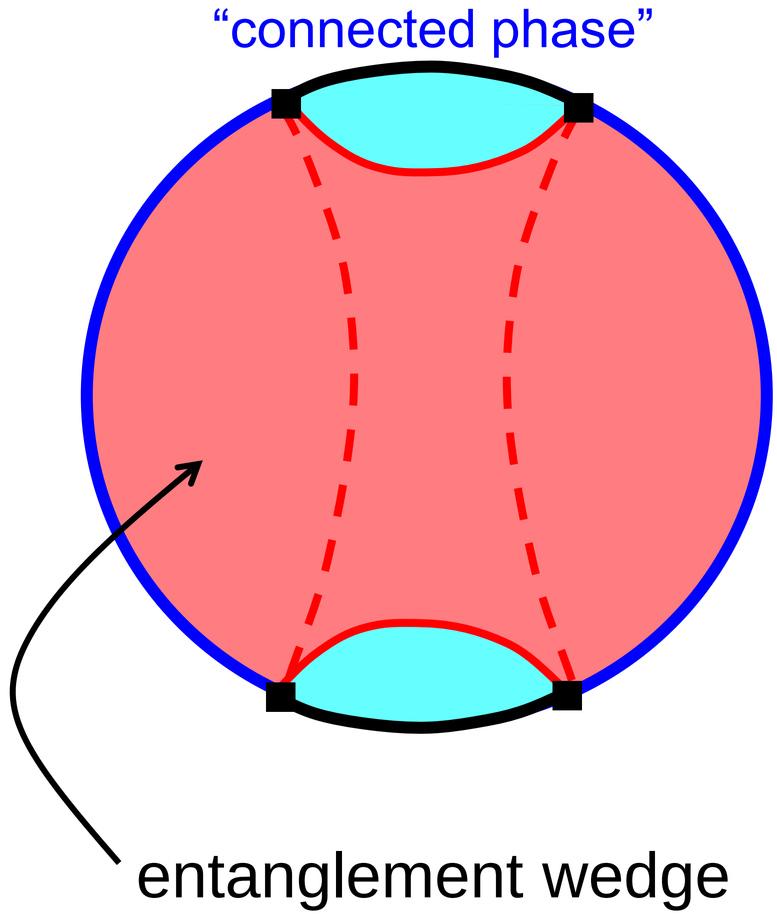


- evolution of endpoints on AdS boundary

→ reminiscent of familiar holographic EE scenario

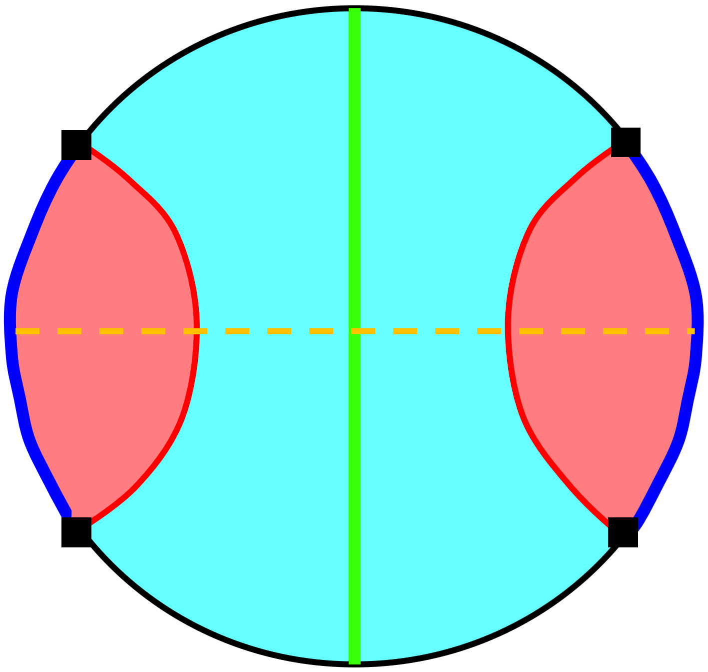
→ reminiscent of familiar holographic EE scenario:
two saddles compete to give minimal RT surface

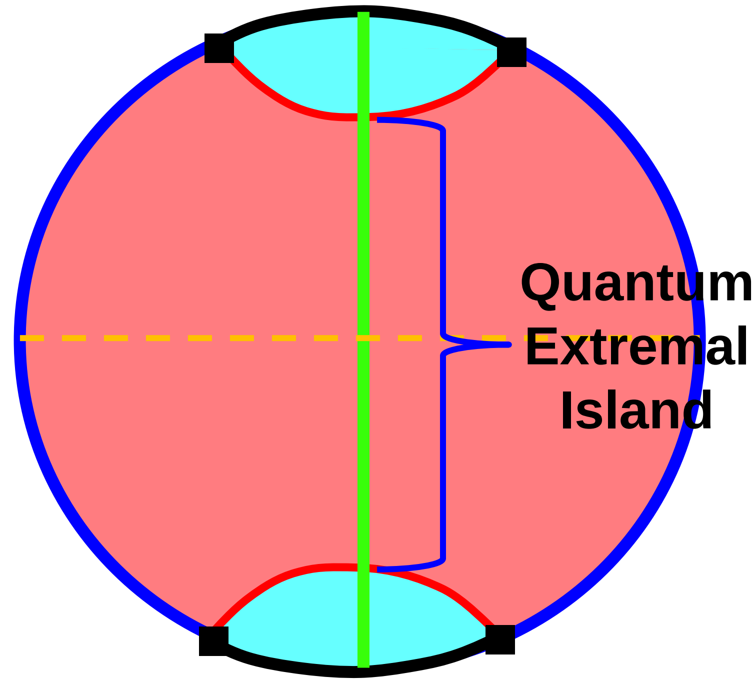




- entanglement wedge reconstruction: can recover bulk operators (within code subspace) inside entanglement wedge with boundary CFT operators in corresponding boundary subregion

→ reminiscent of familiar holographic EE scenario





Early times:

- RT surfaces join opposite sides of BH → EE grows with time
- entanglement wedge close to boundary

→ **growth phase**

Late times:

- RT surfaces on single side of BH → EE fixed in time
- entanglement wedge extends through brane → QE island

→ **Page phase**

Question: How did we get from RT entropy

$$S_{EE}(\mathbf{R}) = \min\left\{ \underset{\Sigma_{\mathbf{R}}}{\mathrm{ext}} \left(\frac{A(\Sigma_{\mathbf{R}})}{4G_{\mathrm{bulk}}} + \frac{A(\sigma_{\mathbf{R}})}{4G_{\mathrm{brane}}} \right) \right\}$$

(Bulk perspective)

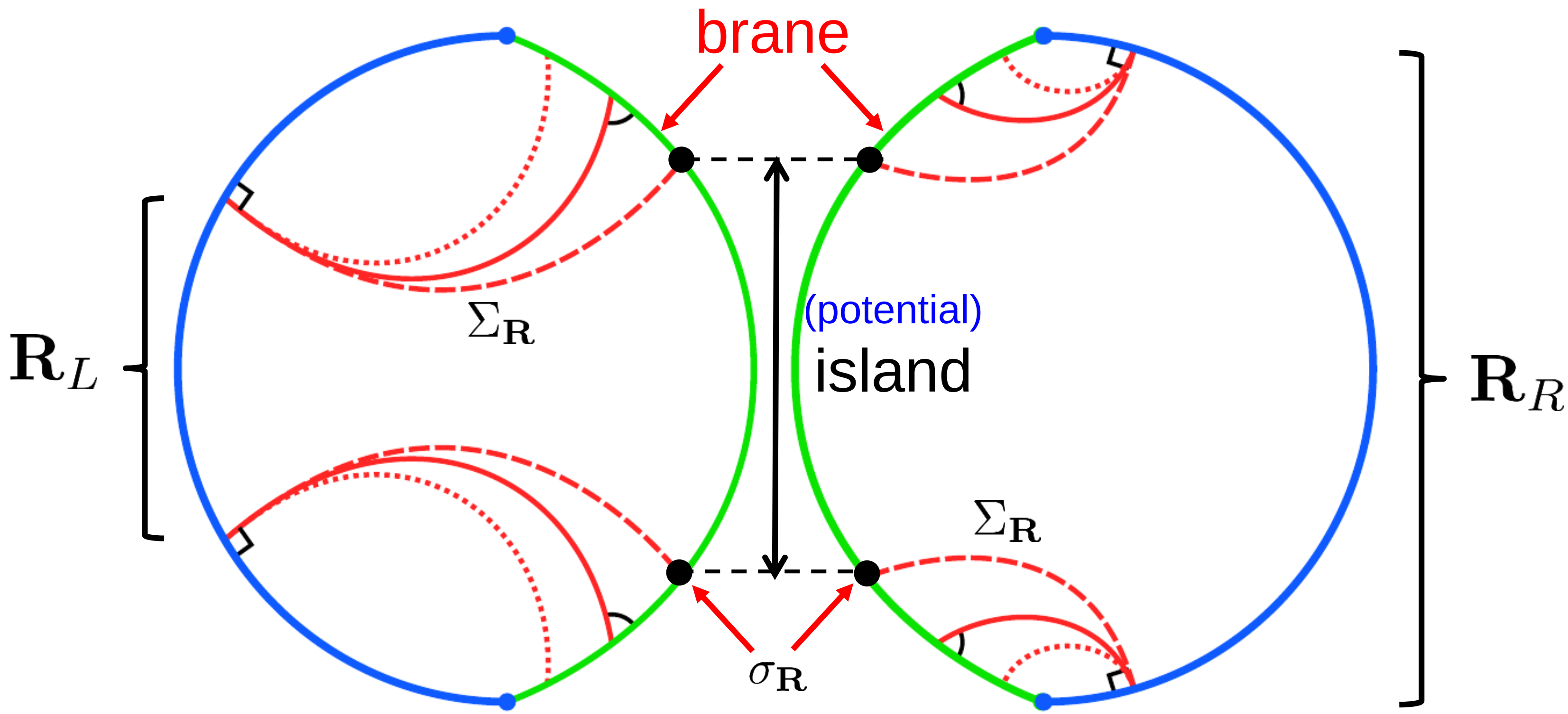
to the island rule?

$$S_{EE}(\mathbf{R}) = \min\left\{ \underset{\mathrm{islands}}{\mathrm{ext}} \left(S_{QFT}(\mathbf{R} \cup \mathrm{islands}) + \frac{A(\partial(\mathrm{islands}))}{4G_N} \right) \right\}$$

(Brane perspective)

Evaluating RT entropy:

1) extremize RT surfaces away from the brane: $h^{\alpha\beta}\,\mathcal{K}^{\mu}_{\alpha\beta}=0$



$\mathbf{R}=\mathbf{R}_L\cup\mathbf{R}_R$

Evaluating RT entropy:

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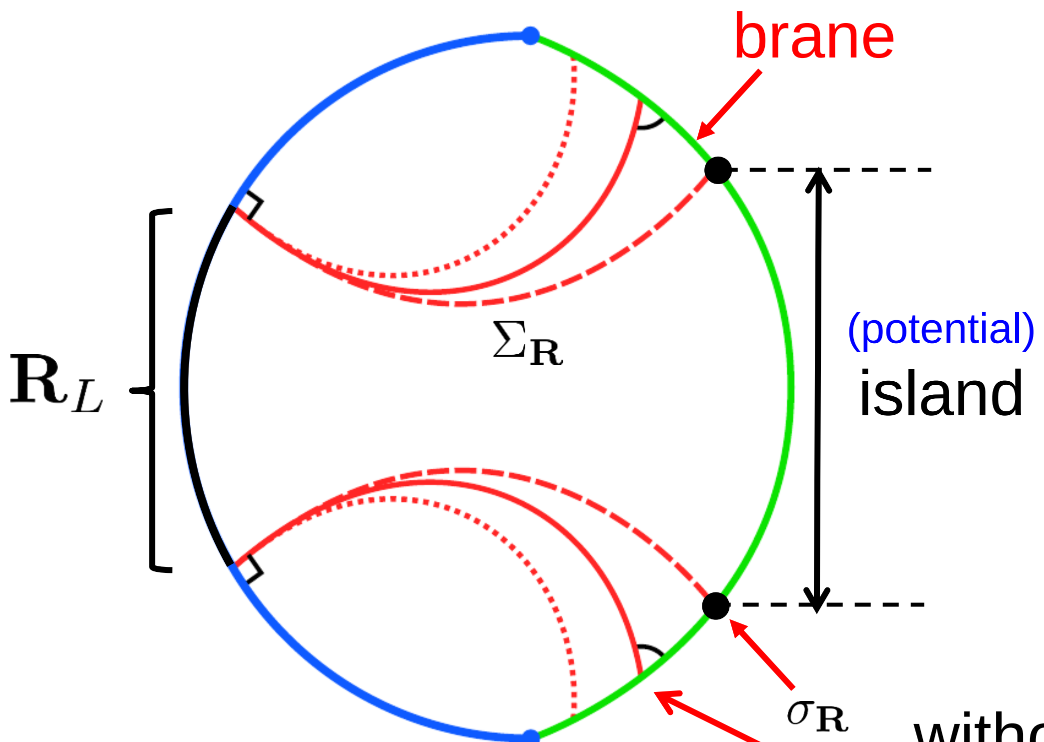

without brane, $\sigma_{\mathbf{R}}$ would be fixed by data on other ∂AdS

- many candidate extremal RT surfaces crossing brane with different profiles $\sigma_{\mathbf{R}}$

Evaluating RT entropy:

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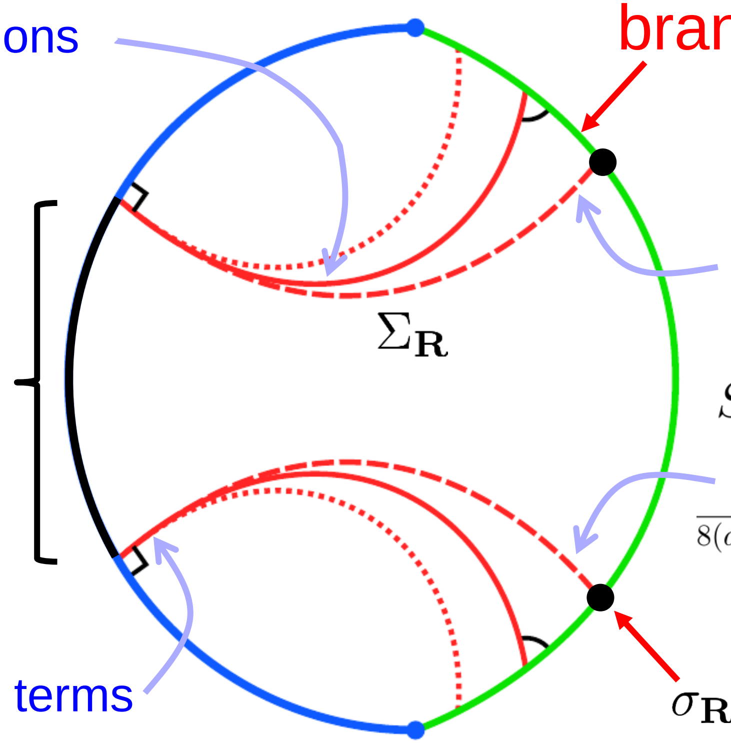
finite IR contributions

QFT entropy
$S_{QFT}(\mathbf{R}\cup\text{island})$

(identical)

UV divergent boundary terms

$\#\,\frac{A(\partial\mathbf{R})}{\delta^{d-2}}+\frac{\#'}{\delta^{d-4}}\int_{\partial\mathbf{R}} d^{d-2}y\sqrt{h}\,(\text{curvature})+\cdots$



Wald-Dong entropy
for brane gravity

$$S_{WD}=\frac{A(\sigma_{\mathbf{R}})}{4G_{eff}}+\frac{L^2}{8(d-2)(d-4)G_{RS}}\int_{\sigma_{\mathbf{R}}} d^{d-2}y\sqrt{\tilde{h}}\left(2\tilde{R}_{ij}n^{im}n^{j}{}_{m}-\frac{d}{d-1}\tilde{R}-\tilde{\mathcal{K}}_i\tilde{\mathcal{K}}^i\right)+\cdots$$

- at this step have evaluated $S_{QFT}(\mathbf{R}\cup\text{islands})+\frac{A(\partial(\text{islands}))}{4G_N}+\cdots$

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finite IR contributions

QFT entropy

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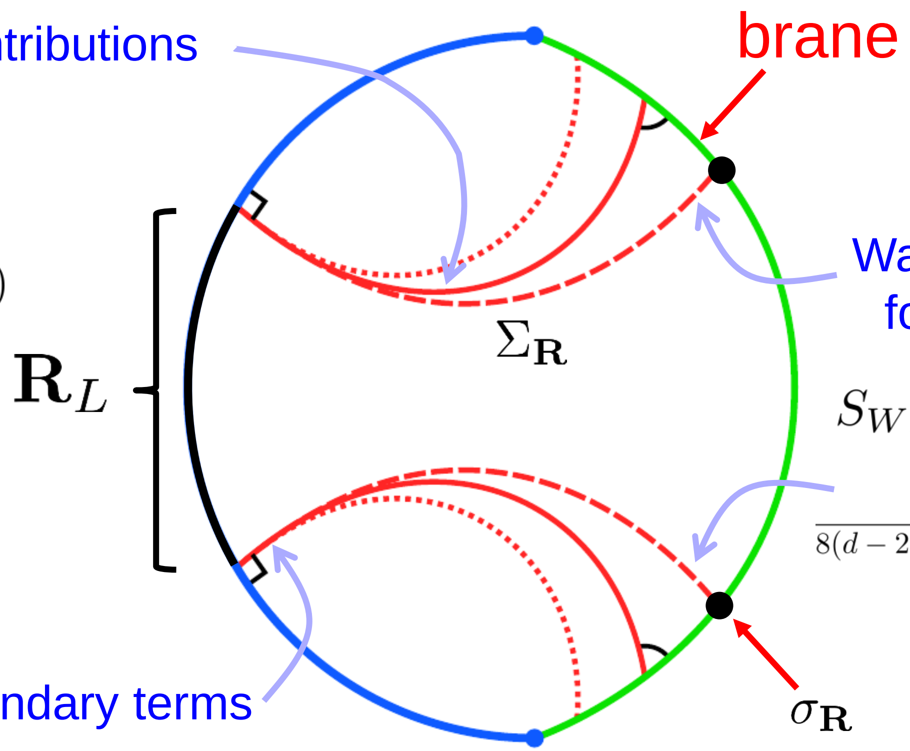

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- at this step have evaluated $S_{QFT}(\mathbf{R} \cup \text{islands}) + \dfrac{A(\partial(\text{islands}))}{4G_N} + \cdots$

2) extremize RT entropy by varying the island boundary $\sigma_{\mathbf{R}}$!

⟶ extremizing over possible islands!

How did we get from RT entropy

$$S_{EE}(\mathbf{R}) = \min \left\{ \operatorname*{ext}_{\Sigma_{\mathbf{R}}} \left(\frac{A(\Sigma_{\mathbf{R}})}{4G_{\text{bulk}}} + \frac{A(\sigma_{\mathbf{R}})}{4G_{\text{brane}}} \right) \right\}$$

(Bulk perspective)

to the island rule?

$$S_{WD}\left(\partial(\text{islands})\right)$$

$$S_{EE}(\mathbf{R}) = \min \left\{ \operatorname*{ext}_{\text{islands}} \left(S_{QFT}(\mathbf{R} \cup \text{islands}) + \cancel{\frac{A(\partial(\text{islands}))}{4G_N}} \right) \right\}$$

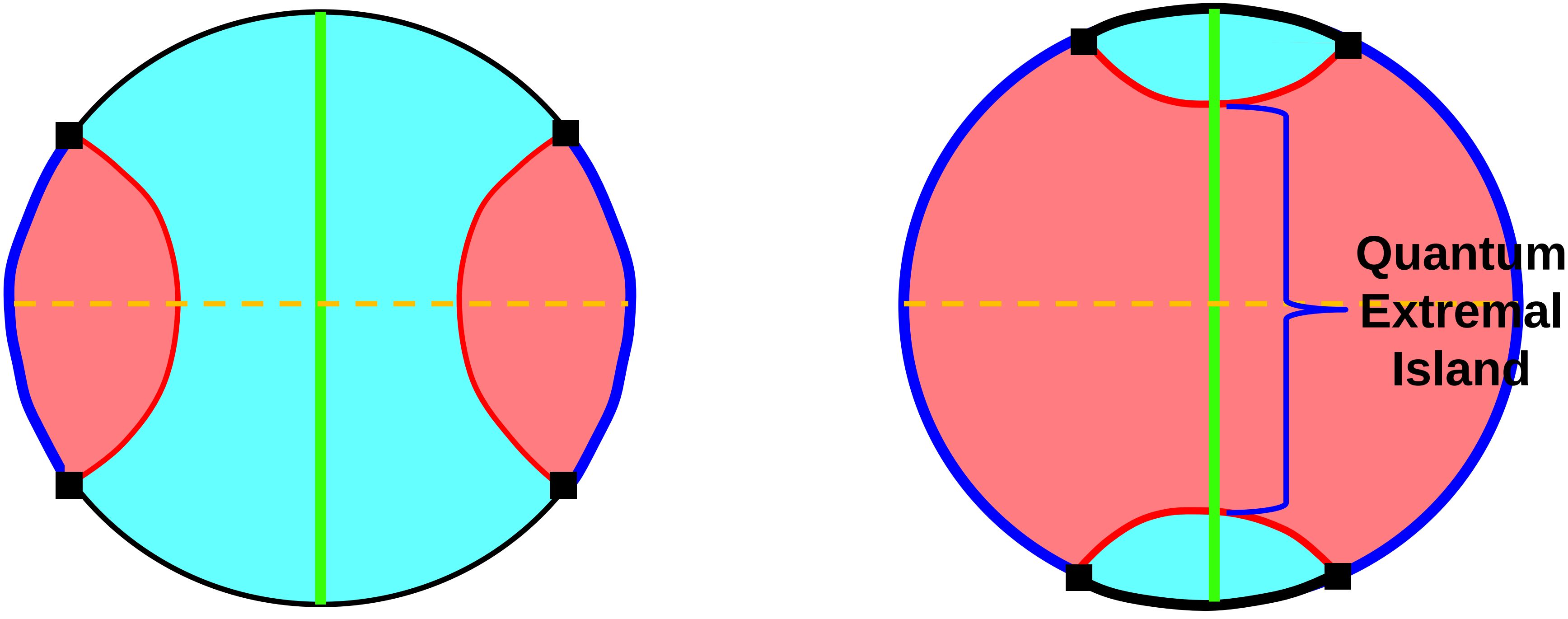
(Brane perspective)

3) connected vs disconnected phase

1) extremize RT surfaces away from the brane

2) extremize RT entropy by varying the island boundary $\sigma_{\mathbf{R}}$!!!!

→ reminiscent of familiar holographic EE scenario



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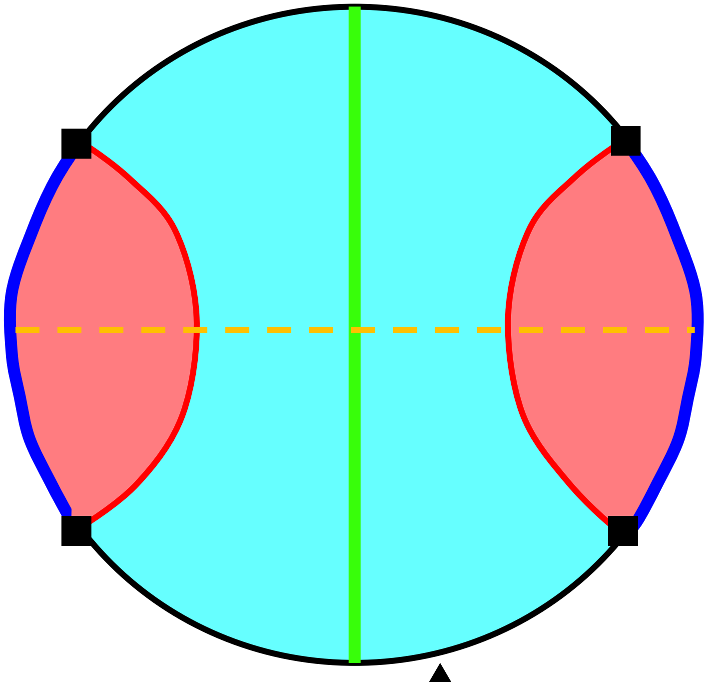
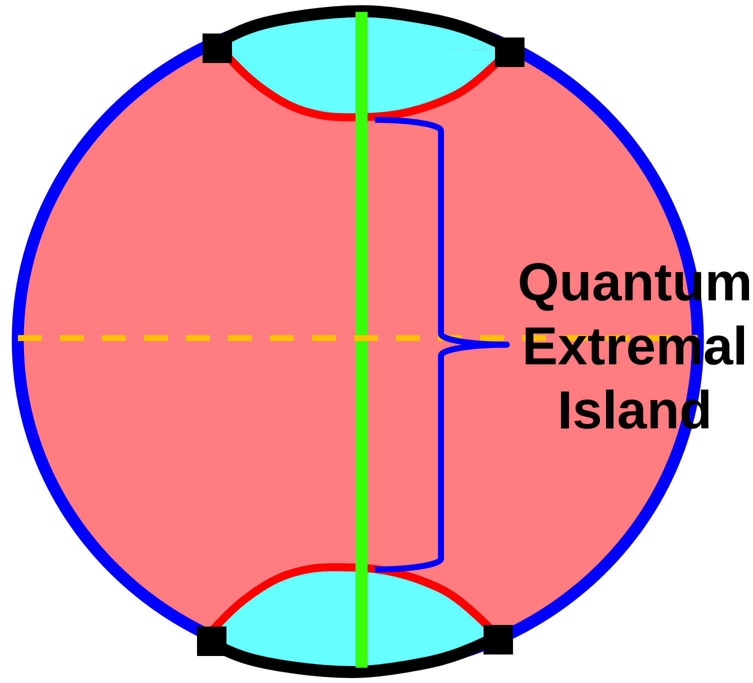




Early times:

- RT surfaces
  of BH → EE
- entanglemen
  boundary

Late times:

n single side
xed in time

wedge extends
→ QE island

→ **g ... ge phase**

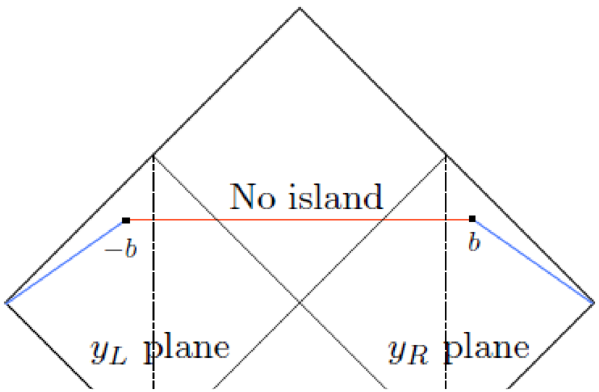
- consider **complementary** regions:

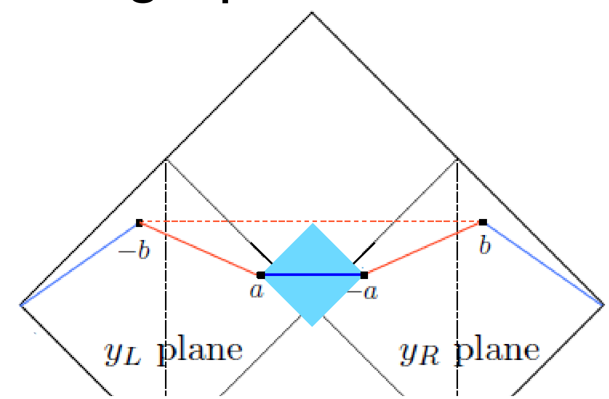
a) compare Hartman & Maldacena
→ system rapidly thermalizes

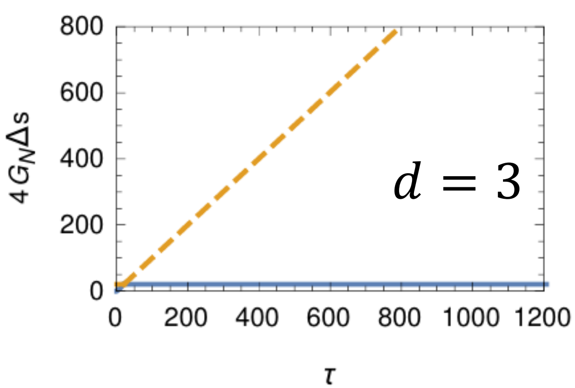
b) outside the horizon?
→ entanglement wedge nesting

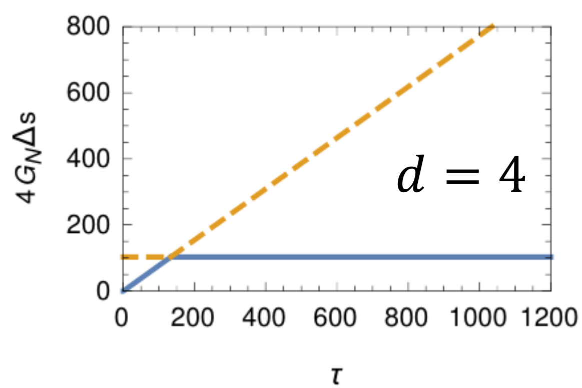
New insights from Holographic EE:

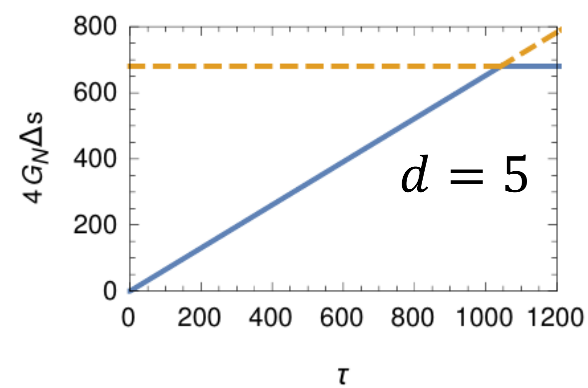
- previous discussion lifts to higher dim'l holographic model with d=2 JT gravity replaced by induced d-dim. Einstein gravity
- new model reproduces precisely the behaviour originally seen with d=2 model from familiar properties of holographic EE











Questions, Questions, Questions:

- how important is two dimensions?

→ **not at all**, our construction extends discussion to gravity and black holes in d dimensions

(see also: Almheiri, Mahajan & Santos)

- was JT gravity important?

→ **no**, our construction extends discussion to Einstein gravity and black holes in d dimensions

- was ensemble average of SYK model important?

→ **no**, our construction relies on standard rules of AdS/CFT correspondence, ie, do not average over couplings in boundary CFT

(Note top-down construction with D3 $\perp$ D5 by Karch & Randall)

Questions, Questions, Questions:

- Almheiri, Mahajan & Maldacena distinguish "*full quantum description*" of radiation and "*semiclassical description*" which includes outgoing radiation and purifying partners on QE island (ie, boldface notation)

Island Rule:

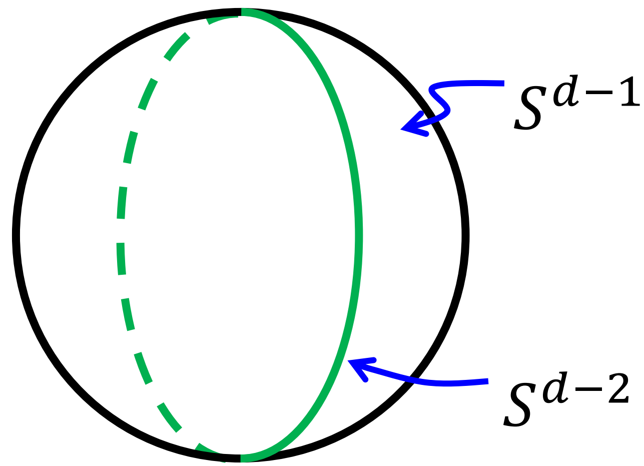
$$S_{EE}(\mathbf{R}) = \min \left\{ \underset{\text{islands}}{\text{ext}} \left(S_{QFT}(\mathbf{R} \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_N} \right) \right\}$$

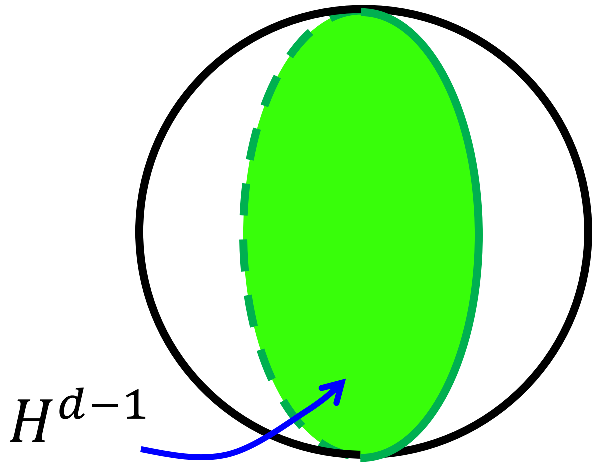
"*full quantum description*" "*semiclassical description*"

- what's up with that?

Randall-Sundrum gravity:

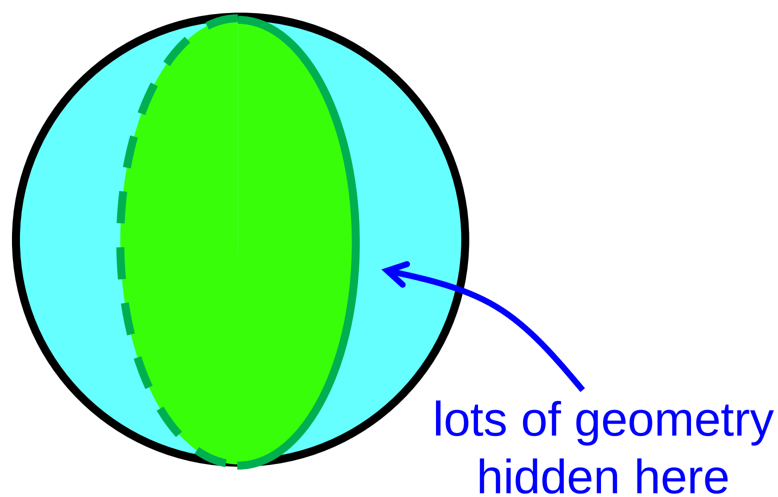
(a) holographic CFT_d coupled to conformal defect (ie, boundary CFT_{d-1})





(b) holographic CFT_d coupled to CFT_d with gravity on AdS_d

(c) AdS_{d+1} gravity coupled to brane with AdS_d geometry



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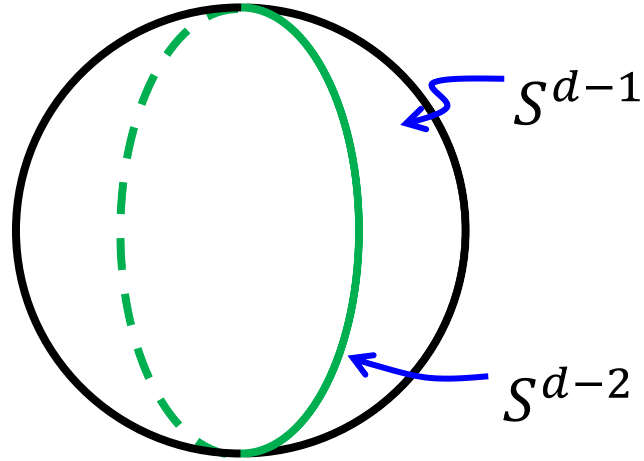
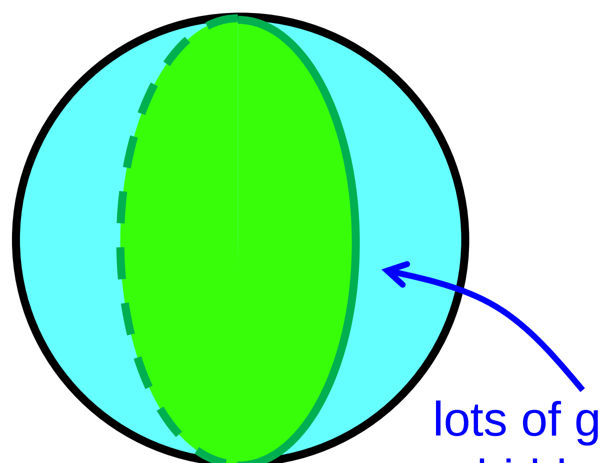


AdS/CFT correspondence

- **these descriptions provide UV complete framework; provide “full quantum description” of radiation**

(c) AdS_{d+1} gravity coupled to brane with AdS_d geometry



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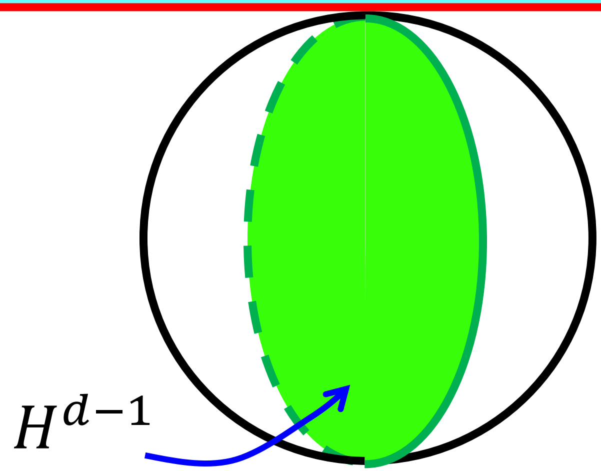
Island rule: $S_{EE}(\mathbf{R}) = \min \left\{ \underset{\text{islands}}{\text{ext}} \left(S_{QFT}(\mathbf{R} \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_N} \right) \right\}$

- becomes mnemonic for this “effective” gravitational theory
- within this framework, can not reveal “hidden”correlations

compare: Akers, Engelhardt & Harlow



(b) holographic CFT_d coupled to CFT_d with gravity on AdS_d

- **this description provides effective low energy framework, eg, cut-off in CFT_d with gravity theory**
- **also only keep “local” interactions between brane and boundary**
- **provides “semiclassical description” of radiation and Hawking partners**
- **framework for calculations in** Almheiri, Mahajan & Maldacena

Conclusions:

- simple holographic model illustrates the appearance of quantum extremal islands
- new insights viewed as familiar properties of holographic EE → are insights universal??
- Page phase can be described by saddle point without revealing microscopic details with large-N!!
  → what/how learn about microstates and information?
- what about: evaporating BHs? massive BHs?
  further insights to holographic EE and complexity?
  how is information encoded in radiation? . . . . .

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Still lots to explore!