Entanglement Partner of Super Horizon Modes

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Recent progress in theoretical physics
based on quantum information theory@YITP zoom

March 5, 2021

In this talk:

I discuss on entanglement in cosmological situation

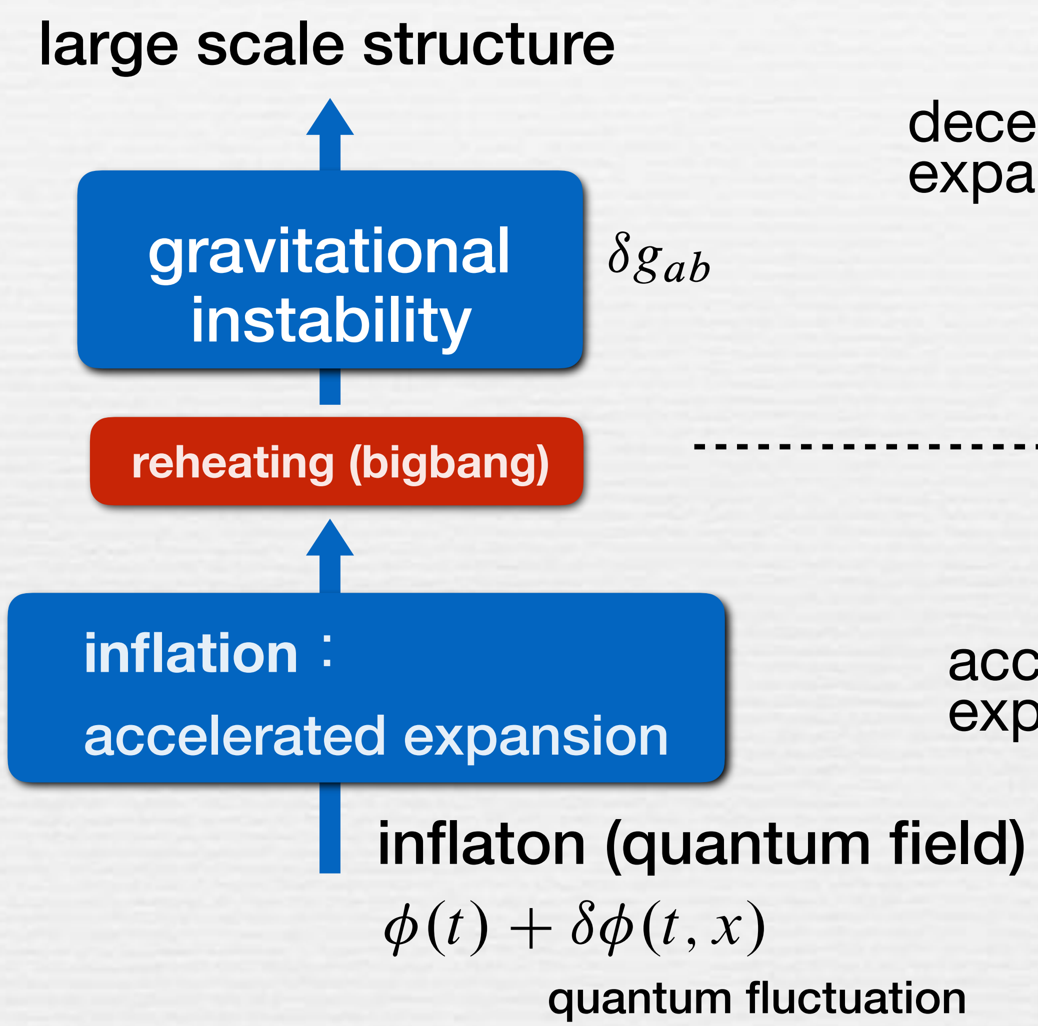
entanglement structure of de Sitter inflation

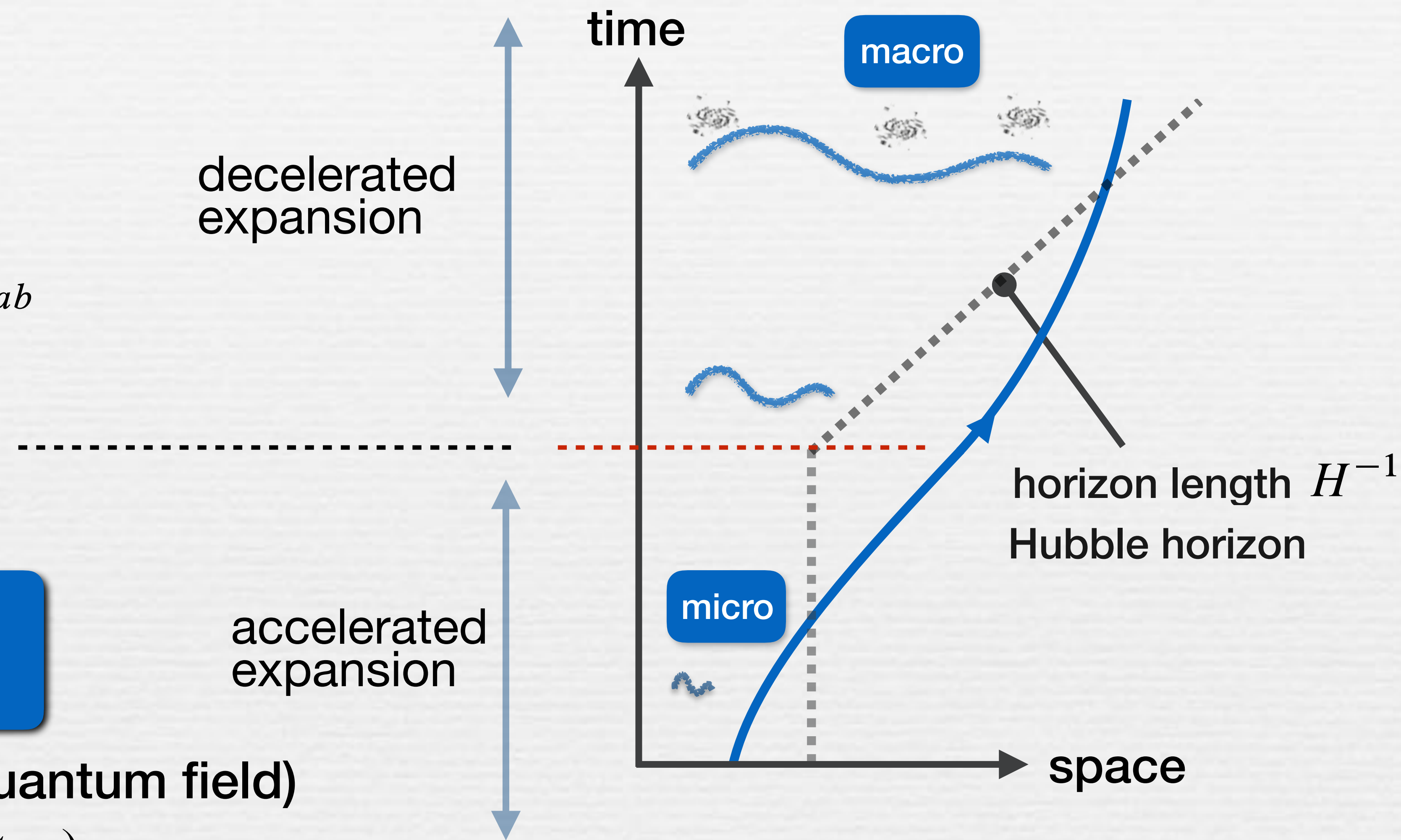
- Introduction
- Entanglement harvesting in de Sitter space
- Entanglement partner in de Sitter space
- Summary

Scenario of structure formation

One important purpose in cosmology is to explain origin of structures of the Universe.

In modern cosmology, the origin of the large scale structure in our universe is explained by inflation

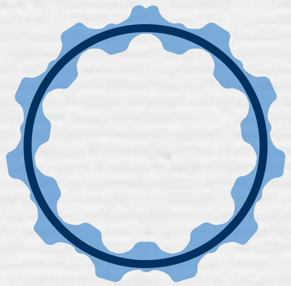




duration of inflation

within 10^{-36}-10^{-34}s, scale increases e^{60}=10^{24}

0.1mm ⇒ size of galaxy



energy density

curvature fluctuation

inflation prepares
seed of structure formation

Quantum Effect in Expanding Universe

The quantum fluctuation of the inflaton field is calculated by using quantum field theory in curved spacetime.

- Quantum field (scalar field, EM field, GW)

$$\hat{\varphi}(\eta, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \hat{\varphi}_{\boldsymbol{k}}(\eta) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\hat{\varphi}_{\boldsymbol{k}} = \frac{1}{\sqrt{2k}}\left(\hat{c}_{\boldsymbol{k}} + \hat{c}^{\dagger}_{-\boldsymbol{k}}\right)$$

$$\hat{p}_{\boldsymbol{k}} = i\sqrt{\frac{k}{2}}\left(\hat{c}^{\dagger}_{\boldsymbol{k}} - \hat{c}_{-\boldsymbol{k}}\right)$$

Hamiltonian in expanding universe

$$\hat{H} = \int d^3k \left[\frac{k}{2}\left(\hat{c}^{\dagger}_{\boldsymbol{k}}\hat{c}_{\boldsymbol{k}} + \hat{c}_{\boldsymbol{k}}\hat{c}^{\dagger}_{\boldsymbol{k}}\right) + i\frac{a'}{a}\left(\hat{c}^{\dagger}_{\boldsymbol{k}}\hat{c}^{\dagger}_{-\boldsymbol{k}} - \hat{c}_{\boldsymbol{k}}\hat{c}_{-\boldsymbol{k}}\right)\right]$$

squeezing op: generate entanglement between $\boldsymbol{k}, -\boldsymbol{k}$

scale factor: a

$$\begin{pmatrix}\hat{c}_{\boldsymbol{k}}(\eta) \\ \hat{c}^{\dagger}_{-\boldsymbol{k}}(\eta)\end{pmatrix} = \begin{pmatrix}\alpha_k & \beta_k \\ \beta_k^* & \alpha_k^*\end{pmatrix}\begin{pmatrix}\hat{c}_{\boldsymbol{k}}(\eta_0) \\ \hat{c}^{\dagger}_{-\boldsymbol{k}}(\eta_0)\end{pmatrix}$$

Bogoluibov tranformation

$$\alpha_k = \cosh r_k, \quad \beta_k = \sinh r_k$$

particle number

$$\langle \hat{c}^{\dagger}_{\boldsymbol{k}}(\eta)\hat{c}_{\boldsymbol{k}}(\eta)\rangle = |\beta_k|^2$$

$$|\psi\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty}(\tanh r_k)^n |n_{\boldsymbol{k}}\rangle \otimes |n_{-\boldsymbol{k}}\rangle$$

squeezing parameter

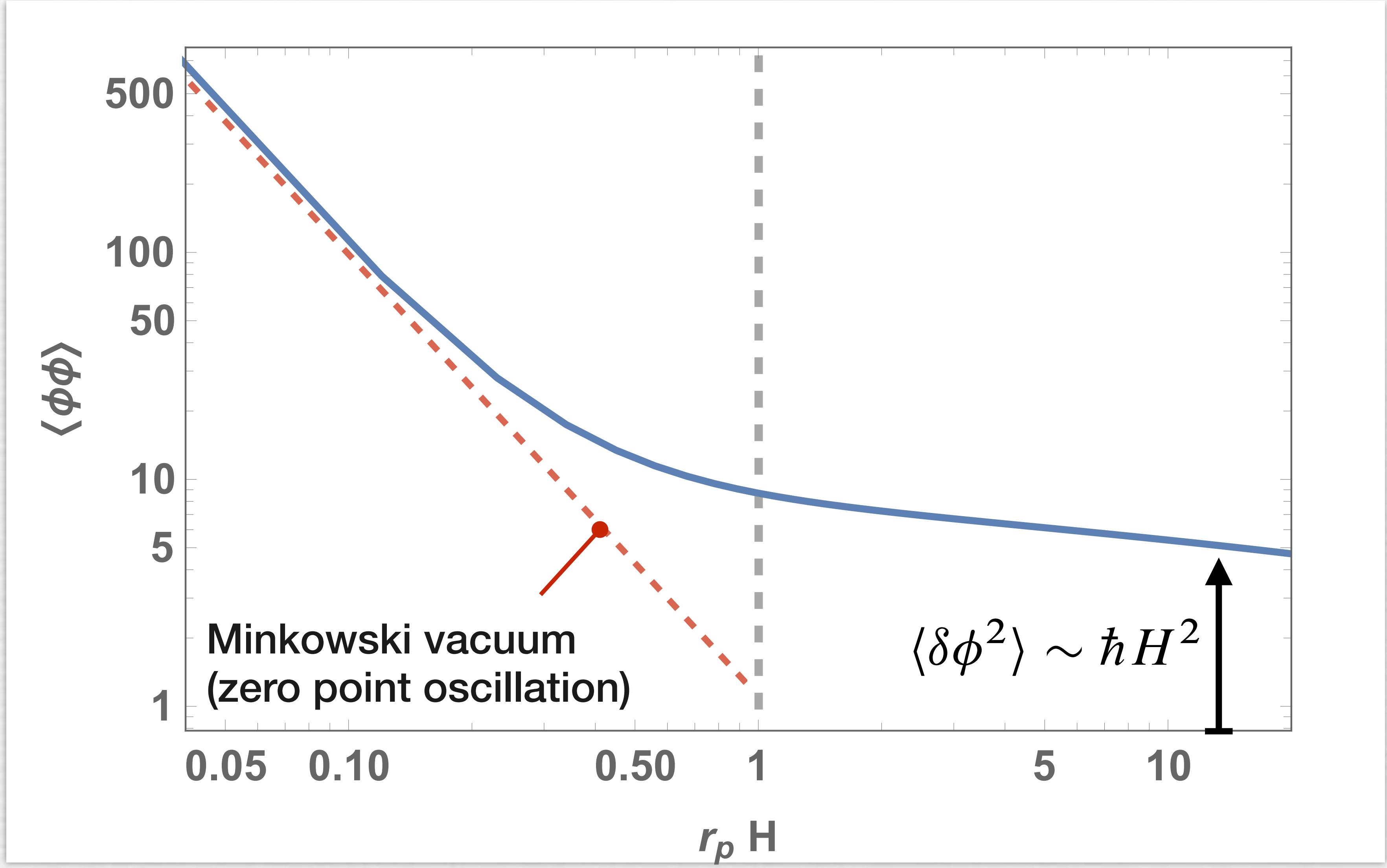
$r_k \sim 100$ for cosmic inflation

vacuum state evolves to 2 mode squeezed state by accelerated expansion of the universe

r→∞ EPR state

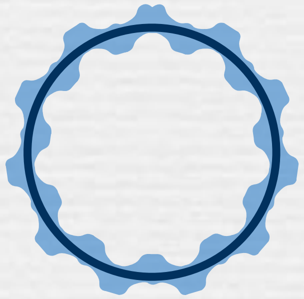
Correlation Function in de Sitter Space massless minimal scalar

$$\langle \delta\phi(\boldsymbol{x}_1)\delta\phi(\boldsymbol{x}_2)\rangle = \frac{\hbar}{4\pi^2 r_p^2} - \frac{\hbar H^2}{4\pi^2}\log\left(H r_p\right) + \frac{\hbar H^3 t}{4\pi^2}$$



curvature fluctuation

$$\langle |\hat{\mathcal{R}}_k|^2\rangle \sim \left(\frac{H}{\dot{\phi}}\right)^2 \times \langle |\delta\hat{\phi}_{\boldsymbol{k}}|^2\rangle$$



Cosmic inflation generate large scale quantum fluctuations beyond horizon scale of deSitter space $r_p > H^{-1}$ (origin of primordial fluctuation)

Classical property of primordial fluctuations

- If we adopt quantum fluctuation as initial seed for structure formation, some conceptual difficulty appears.
- Usually, quantum expectation value is adopted as initial condition for classical evolution

assumption of classicality

$\langle \hat{\mathcal{R}}\hat{\mathcal{R}} \rangle$ → $\langle \cdots \rangle$ as statistical average

$\hat{\mathcal{R}}$ → c-number random Gaussian variable

quantum mechanical variable

existence of classical probability distribution reproducing quantum expectation values

- condition of classicality

Assumption: existence of local probability distribution reproducing quantum correlations

correlations can be represented as local hidden variable (LHV) theory

→ Bell's inequality is satisfied

For bipartite system, the condition is equivalent to the state is separable (no bipartite entanglement)

Although we want to prepare classical initial condition for structure formation, seed fluctuation is quantum! We must check validity of assumption of classicality.

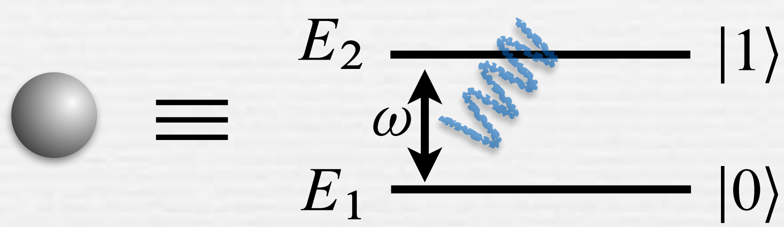
Entanglement Harvesting in de Sitter space

Entanglement harvesting in de Sitter space:

Detection of entanglement of quantum field

One theoretical setup to check entanglement of quantum field is the protocol of entanglement harvesting.
This method measures entanglement of quantum field using entanglement between UD detectors

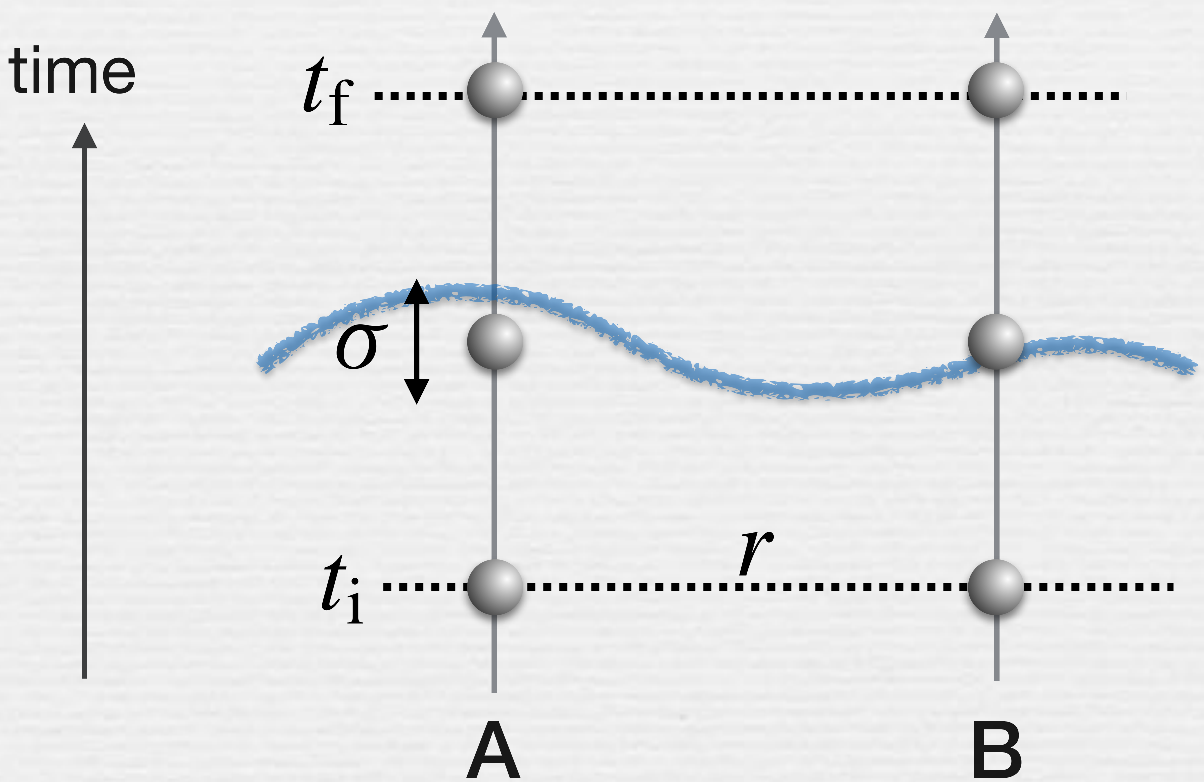
two level atom (qubit)

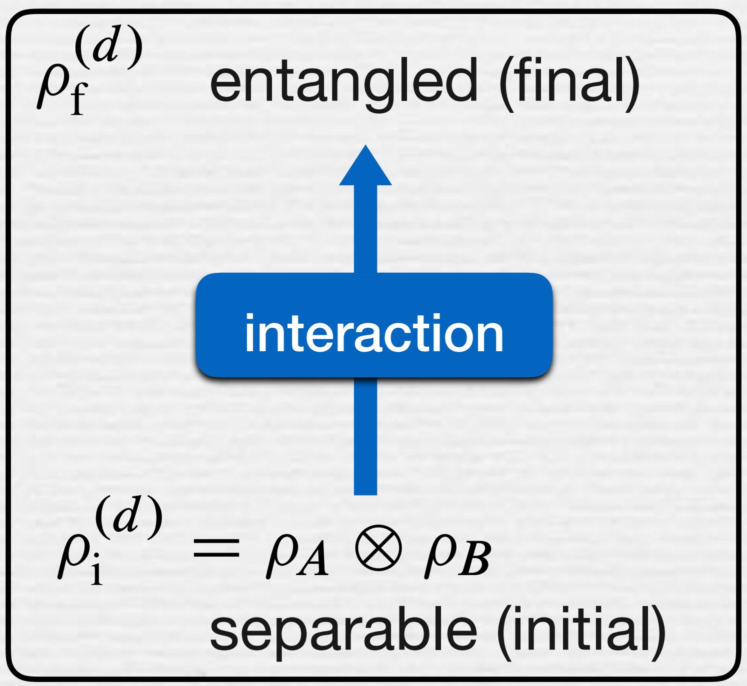


$$\hat{H}_{\mathrm{int}} = g(\hat{\sigma}_+ + \hat{\sigma}_-)\phi(x(\tau))$$

$$\hat{\sigma}_+ = |1\rangle\langle 0| \qquad \hat{\sigma}_- = |0\rangle\langle 1|$$

g: coupling with finite interval of time





harvesting protocol

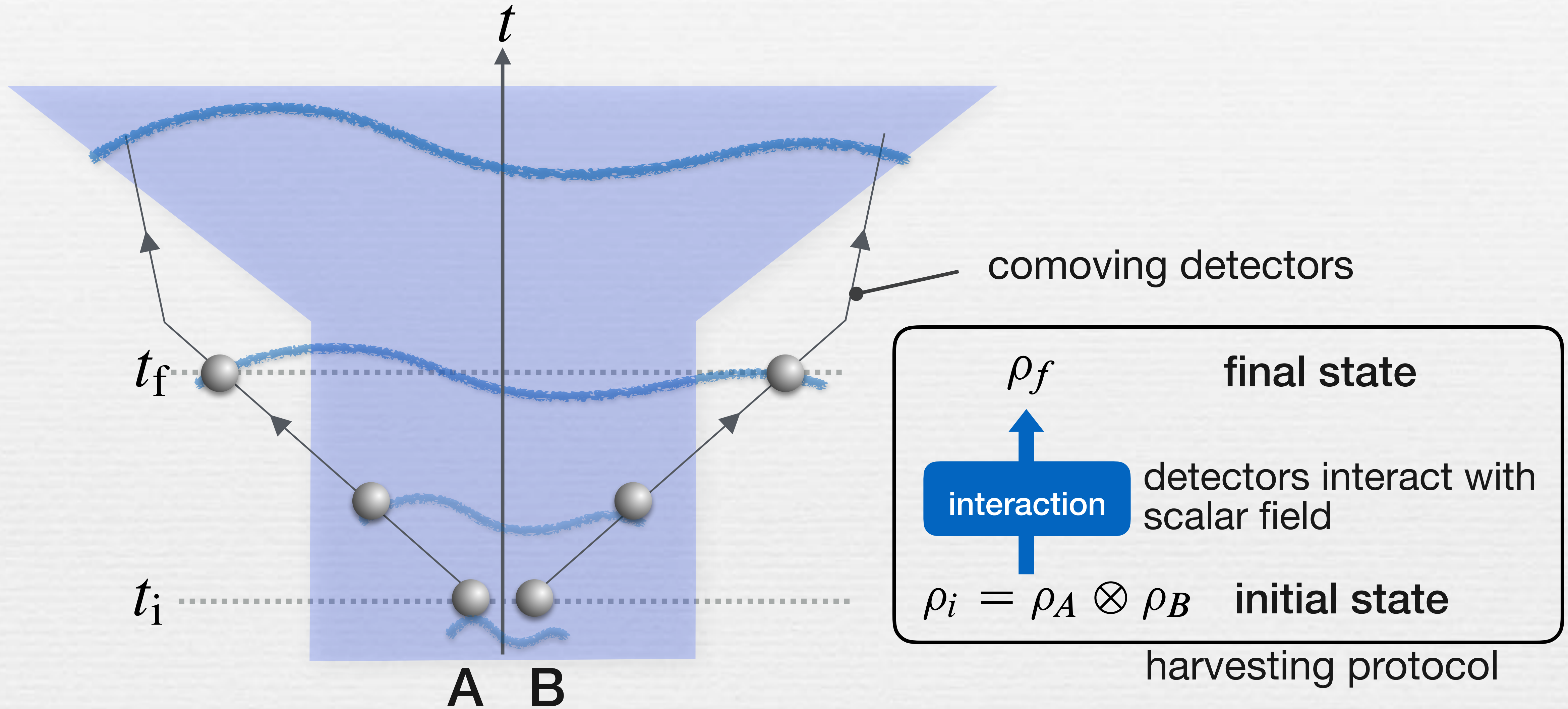
Local operation cannot generate entanglement

➡ a pair of detectors measures entanglement of quantum field (entanglement harvesting)

Large Scale Entanglement in Cosmic Inflation

and entanglement harvesting: thought experiment

We apply this protocol in de Sitter universe

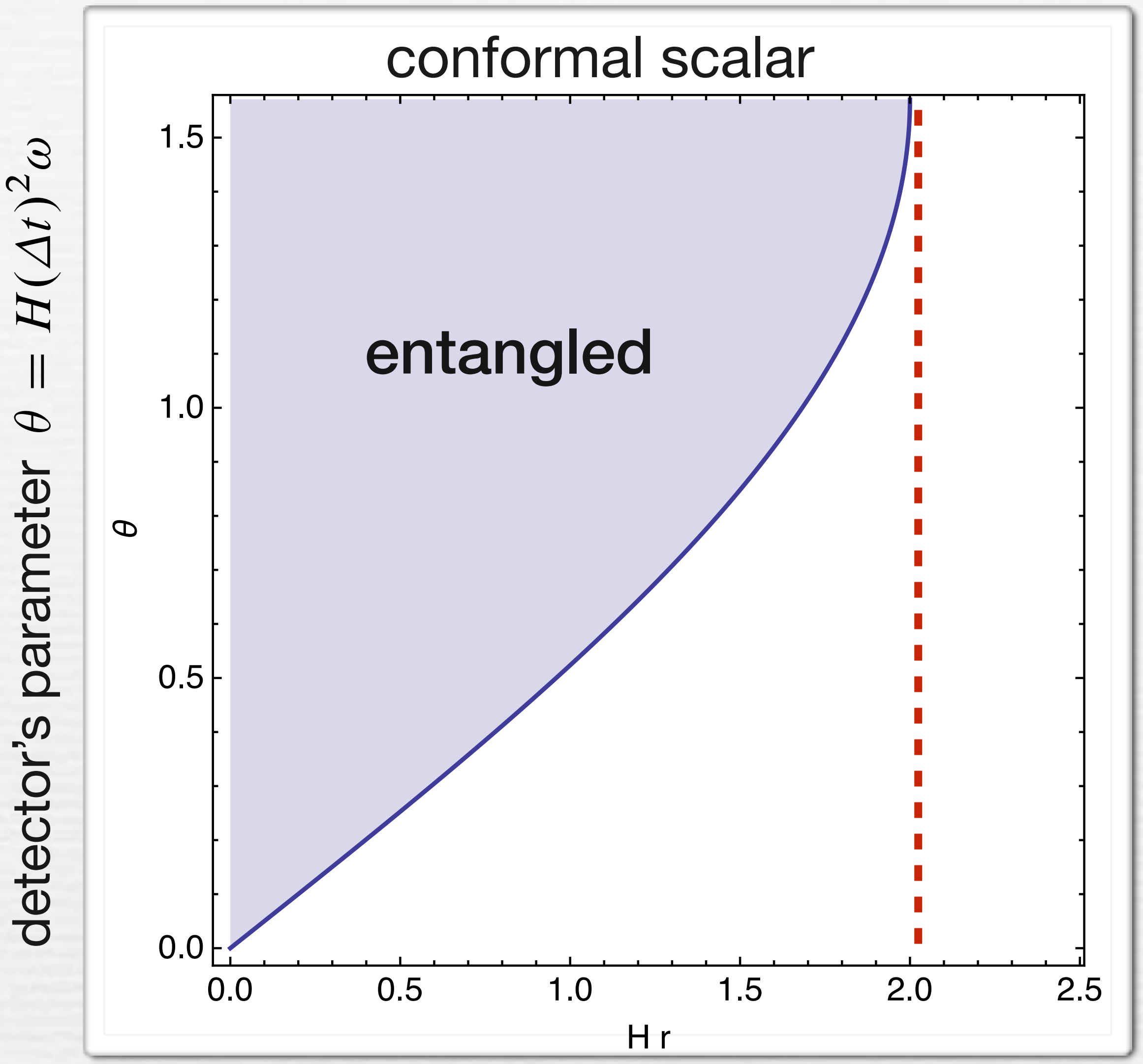


Based on this thought experiment, we can investigate quantum correlation in deSitter universe.

- theoretical justification of origin of "classical" primordial fluctuations
- experimental verification of "quantumness" of primordial fluctuations

Entanglement between 2 quit detectors

G.V.Steeg and N.C.Menicucci 2009
Y. Nambu and Y.Ohsumi 2011
E. Martin-Martinez and N.C.Menicucci 2016



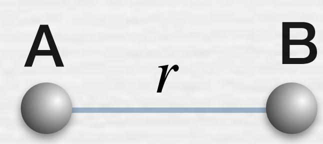
scale (separation)

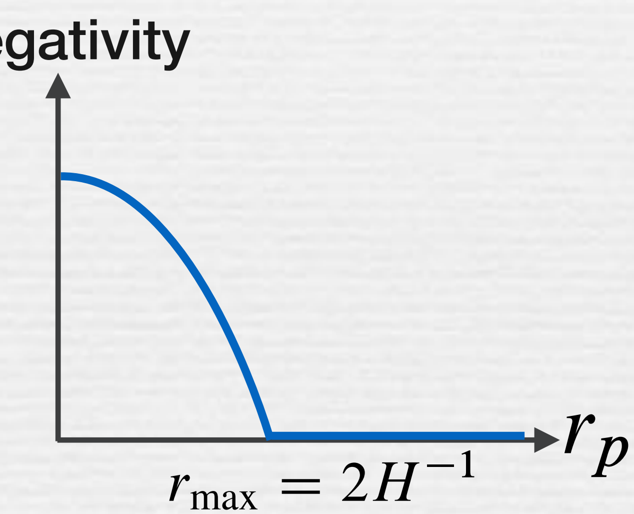
negativity

$$\mathcal{N} = \left|C^{(r)}_{++}\right| - C^{(0)}_{-+}$$

non-local correlation   local noise

$|00\rangle \to |11\rangle$ $|00\rangle \to |10\rangle, |01\rangle$





- For large r, local noise destroys quantum correlation and system becomes separable
- A pair of qubit detectors cannot access entanglement of super horizon scale

typical scale of entanglement $\sim H^{-1}$

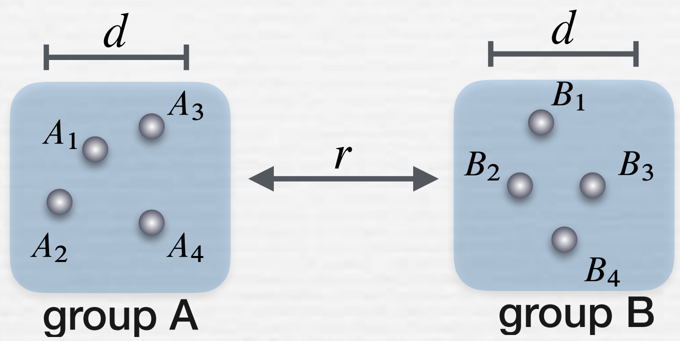
At first glance, this imply there is no quantum correlation in super horizon scale (quantum to classical transition occurs) No. This is only one aspect of entanglement

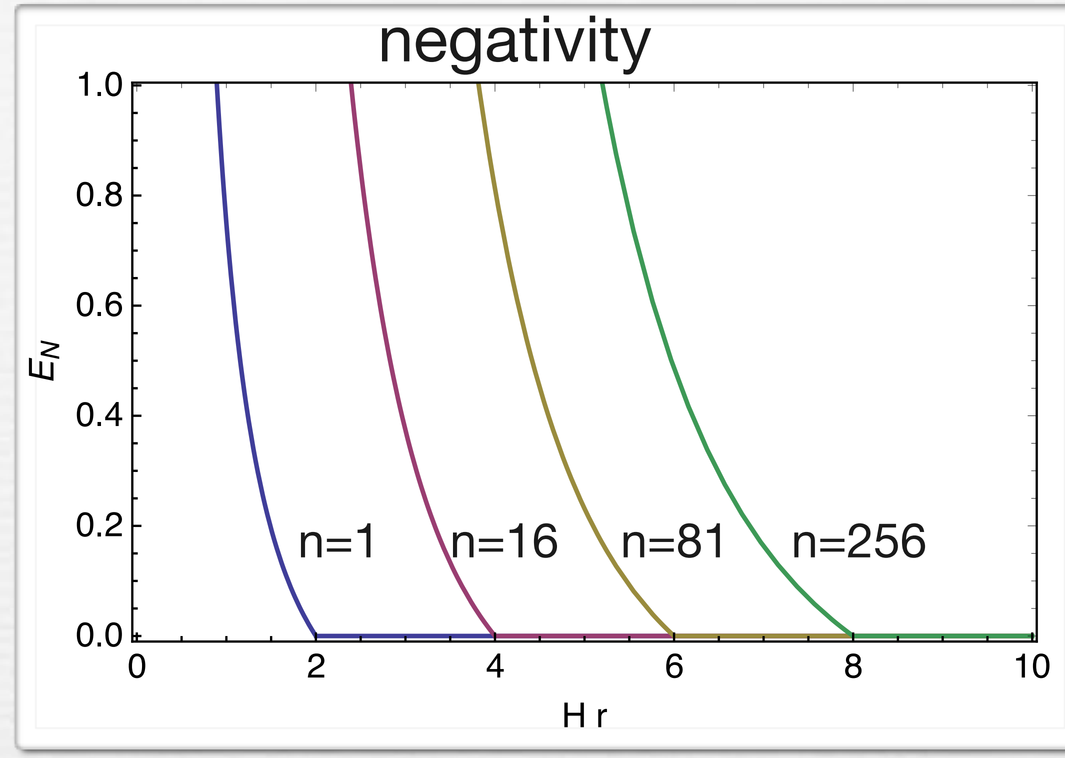
➡ We must check multi-partite entanglement effect

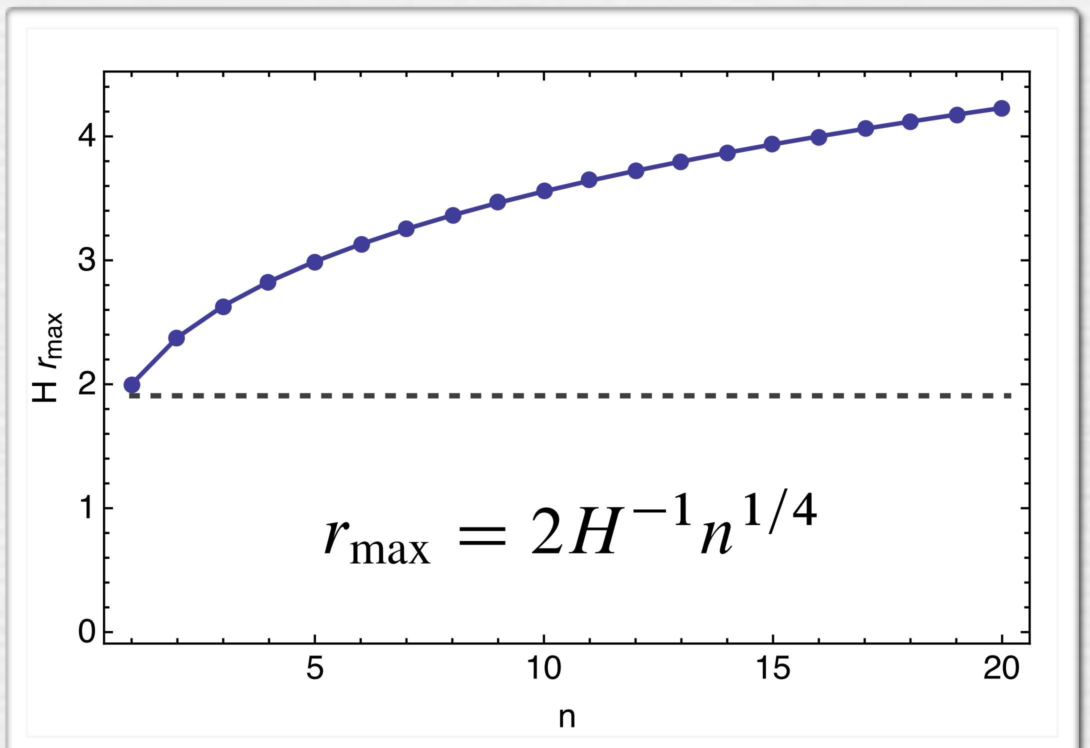
Entanglement of (n+n)-quit detectors

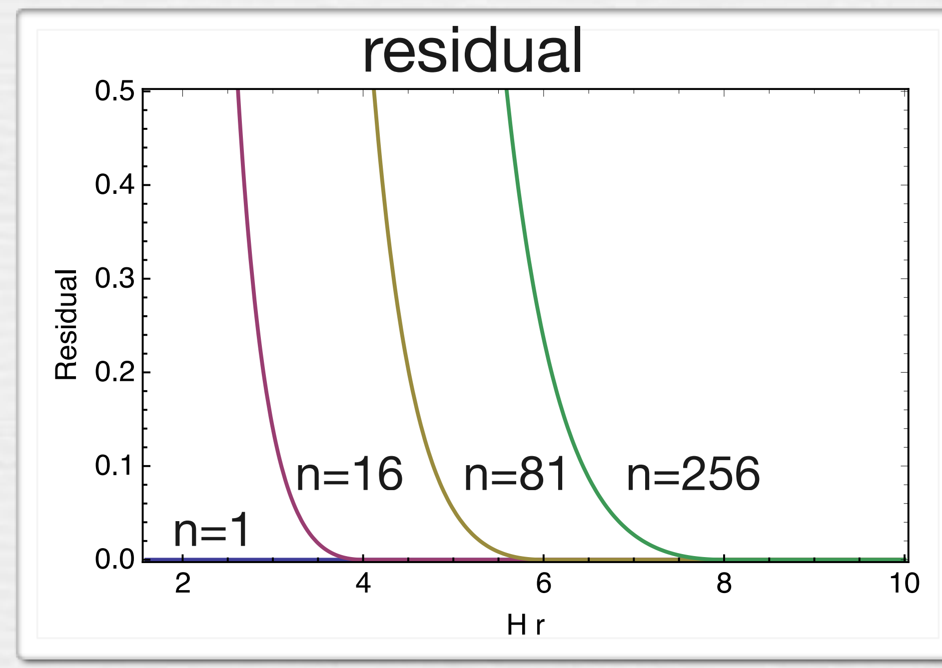
S. Kukita & YN, Entropy 19 (2017), 449

To check multi-partite effect of entanglement,
we consider harvesting with multiple detectors









- Effect of multipartite entanglement:
  enhances non-local correlation and reduces local noise
- Detection of super horizon scale entanglement is possible
  with multiple detectors

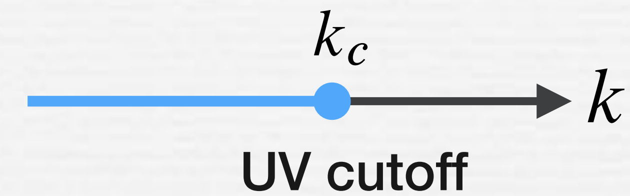
Entanglement of Long Wavelength Modes

Y.Nambu and Y.Ohsumi
PRD 80(2009)124031

For later usage, we consider different setup of entanglement harvesting based on coarse-graining of field variable.

- **coarse-grained field** (with momentum cutoff)

$$\delta_c = \frac{2\pi}{k_c}$$



$$\hat{Q}_{A,B} = \int \frac{d^3k}{(2\pi)^{3/2}} W_0\theta(k_c - k)\hat{\varphi}_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}_{A,B}}$$

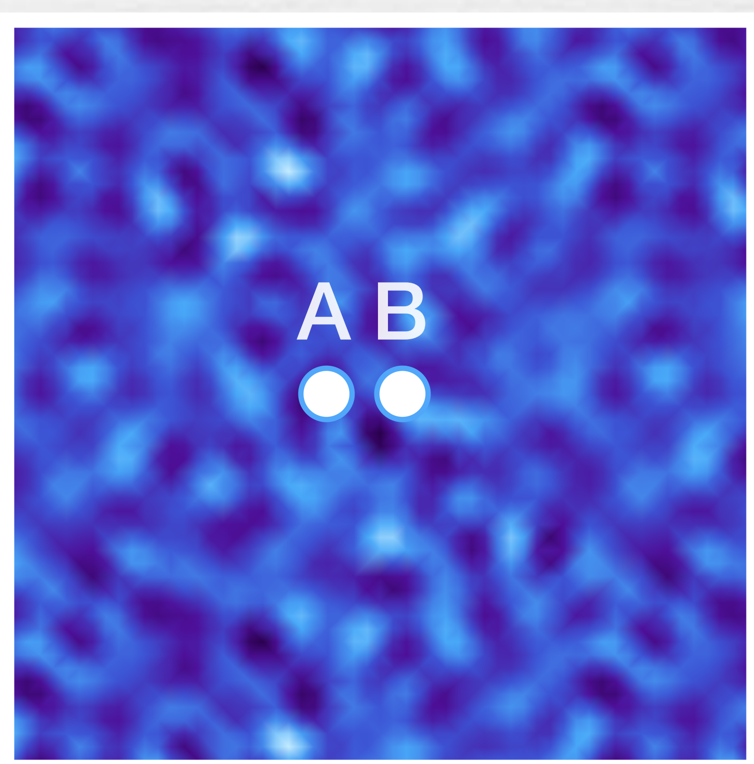
window function

$$\hat{P}_{A,B} = \int \frac{d^3k}{(2\pi)^{3/2}} W_0\theta(k_c - k)\,\hat{p}_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}_{A,B}}$$

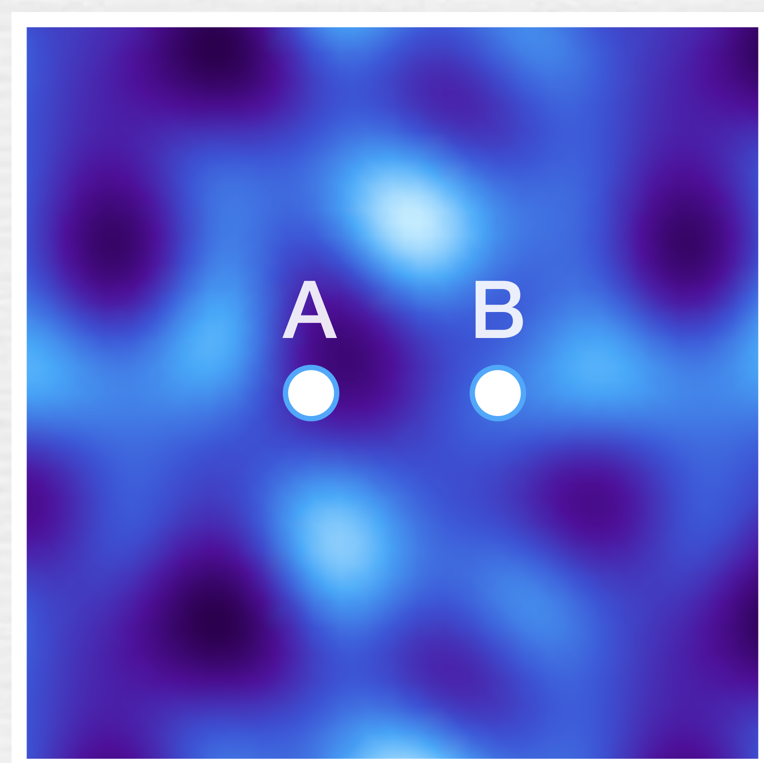
$$f_k'' + \left(k^2 + m^2a^2 - \frac{a''}{a}\right) f_k = 0, \quad g_k = i\left(f_k' - \frac{a'}{a} f_k\right)$$

$$\hat{\varphi}_k = f_k \hat{a}_{\boldsymbol{k}} + f_k^* \hat{a}^\dagger_{-\boldsymbol{k}}$$
$$\hat{p}_k = g_k \hat{a}_{\boldsymbol{k}} + g_k^* \hat{a}^\dagger_{-\boldsymbol{k}}$$
$$W_0^2 = \frac{6\pi^2}{k_c^3}$$

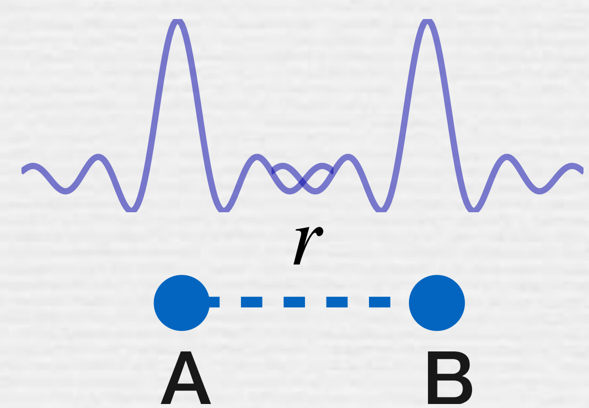
small coarse graining scale δ_c



large coarse graining scale δ_c



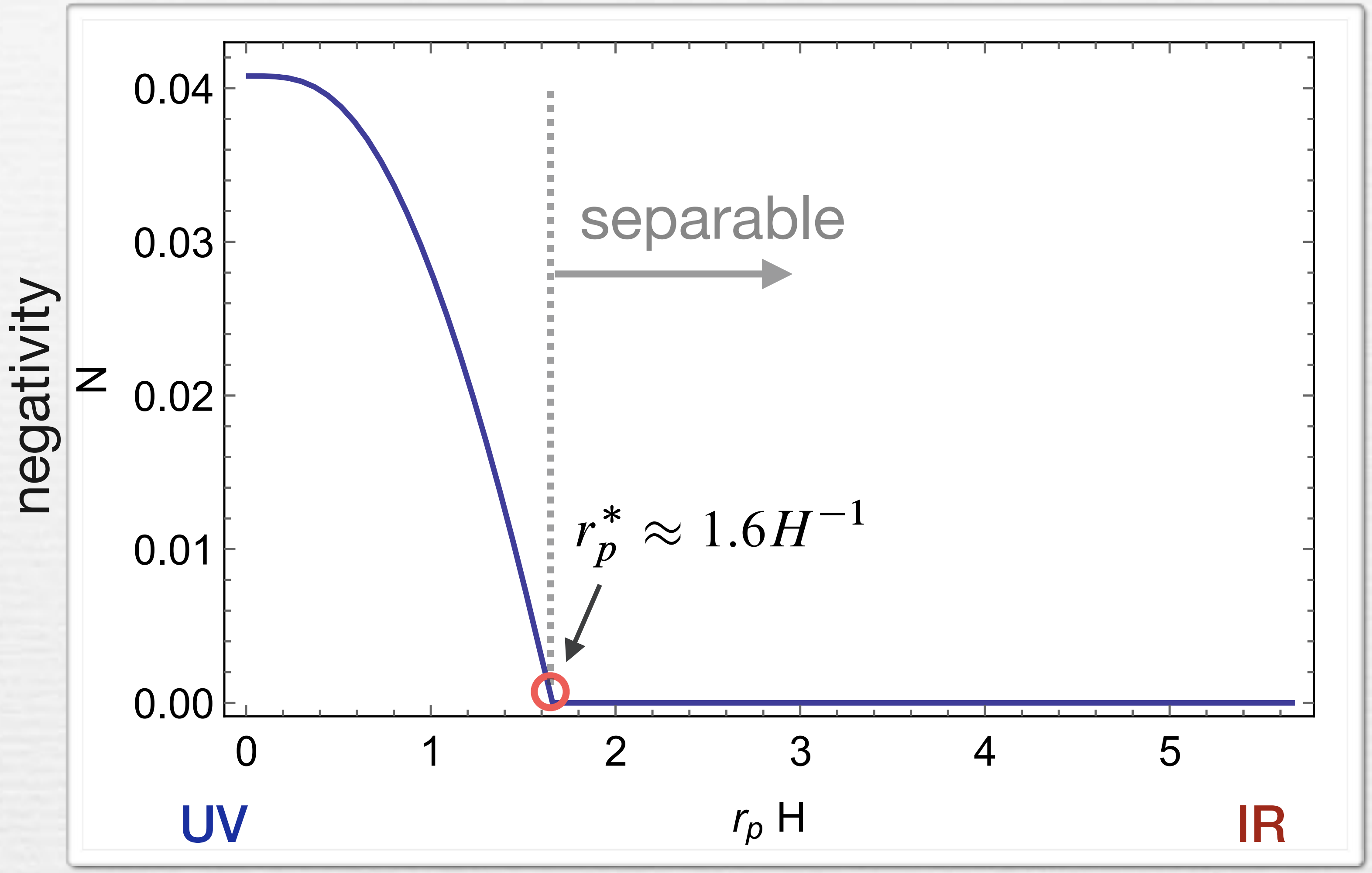
profile of mode: localize at A,B



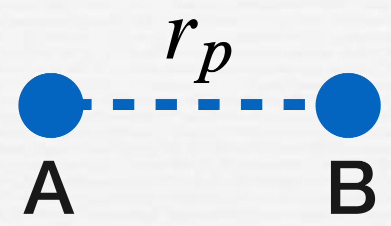
$$r = r(\delta_c)$$

Scale dependence of Entanglement

With this setup, we can evaluate entanglement negativity between A and B

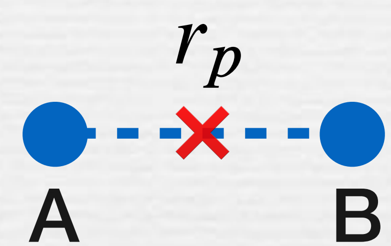


coarse graining scale



$0 < r_p < 1.6H^{-1}$

AB is entangled



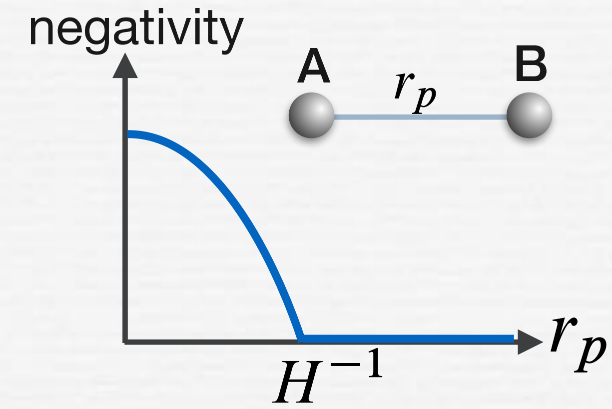
$1.6H^{-1} < r_p$

AB is separable

critical coarse graining scale $r_p^* \approx 1.6H^{-1}$

The mode AB becomes separable for distance larger than the Hubble horizon and this behavior is the same as qubit-detector case.

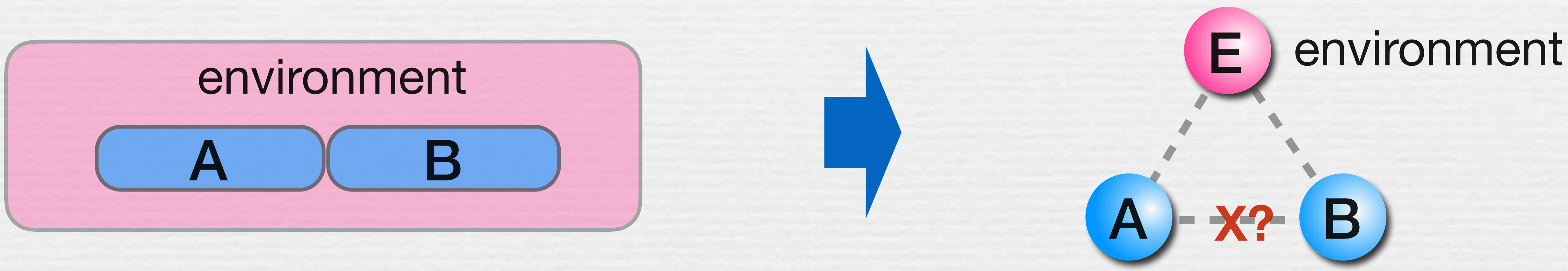
Why the state is separable on large scale?



From these observation, we pose the question why the mode AB becomes separable for super-horizon scales.

- Thermal noise with Gibbons-Hawking temperature $T_{\mathrm{H}} = H/2\pi$ destroys quantum correlations
- Effect of multi-partite entanglement
- Both are related

We consider this behavior from the viewpoint of multi-partite entanglement



- This separability can be understood based on entanglement monogamy (sharing structure of entanglement)

  we expect as correlation between AE (BE) increases, correlation between AB is reduced
- What is the role of “partner” mode E?
- Is it possible to understand emergence of classicality deSitter space?

We will try to find out explicit form of partner for two modes AB.

Entanglement Partner in de Sitter Space

in collaboration with Koji Yamaguchi
(Tohoku Univ, Japan)

Partner modes and entanglement structure

First, we shortly review concept of purification and entanglement partner
(basic concept in QM)

- **purification**

For arbitrary mixed state

$$\hat{\rho}_A = \sum_{n_A} p_{n_A} |n_A\rangle \langle n_A|$$

It is possible to embed $\hat{\rho}_A$ in a pure state by adding an ancilla mode $\bar{A}$:

$$|A\bar{A}\rangle = \sum_n \sqrt{p_n} |n_A\rangle |n_{\bar{A}}\rangle$$

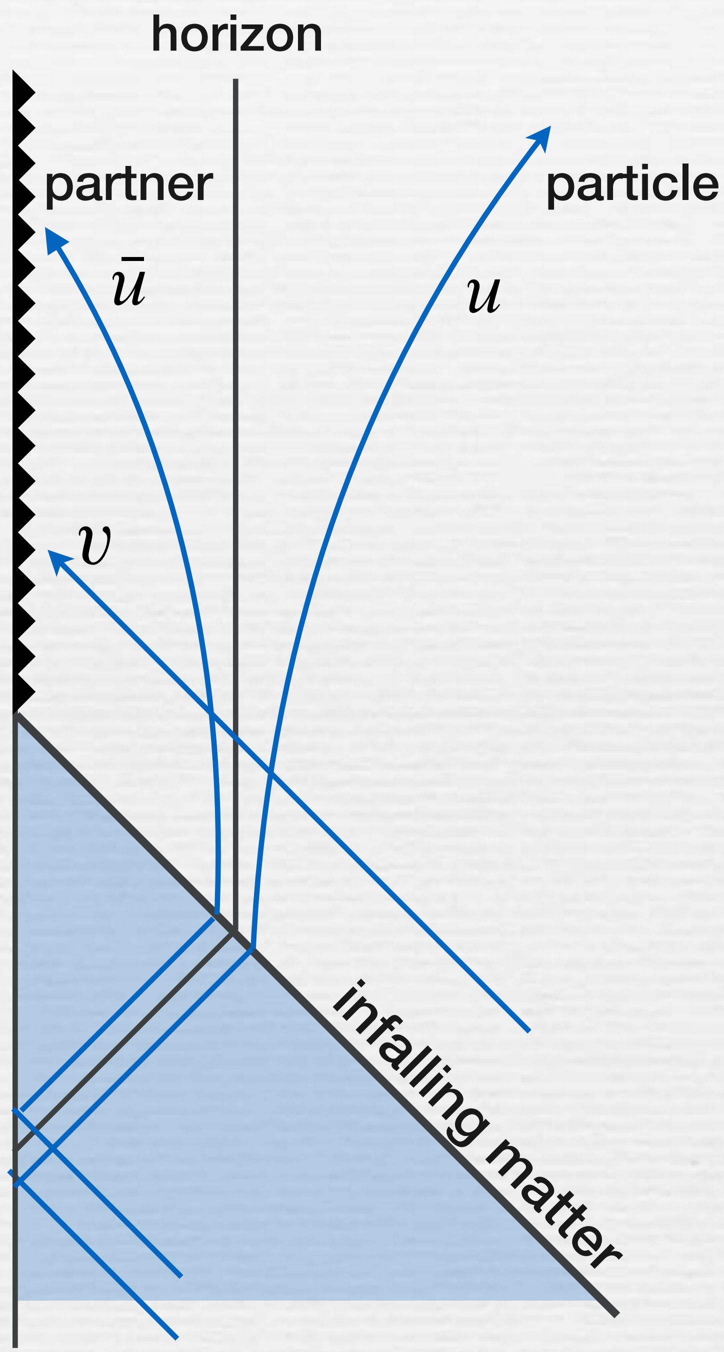
$$\begin{aligned} \mathrm{tr}_{\bar{A}}\left(|A\bar{A}\rangle\langle A\bar{A}|\right) &= \sum_{m,n} \sqrt{p_m p_n} |m_A\rangle \langle n_A| \,\mathrm{tr}\left(|m_{\bar{A}}\rangle\langle n_{\bar{A}}|\right) \\ &= \sum_n p_n |n_A\rangle \langle n_A| \\ &= \hat{\rho}_A \end{aligned}$$

- $\bar{A}$ is purification (entanglement) partner of A
- If A is pure, $A\bar{A}$ is a product state （no correlation between A and $\bar{A}$）

Entanglement structure of Hawking radiation

Well known example of concept of partner appears in Hawking radiation.

- **Standard scenario (gravitational collapse)** *S.W.Hawking 1975*



2 mode Bogoliubov transformation

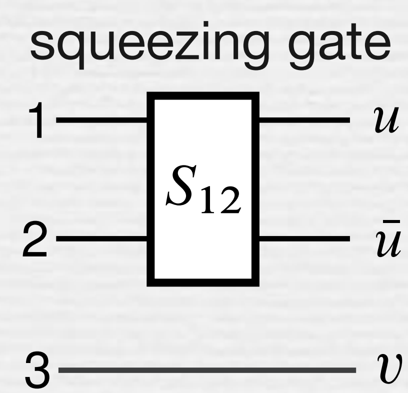
in-vacuum state 2 mode squeezed sate (entangled state)

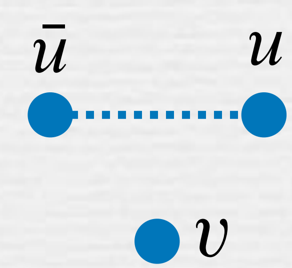
$$|\psi_0\rangle \propto \sum_n e^{-n\omega/(2T_H)} |n\rangle_u |n\rangle_{\bar{u}}$$

reduced state

$$\hat{\rho}_0 \propto \sum_n e^{-n\omega/T_H} |n_u\rangle \langle n_u|$$

thermal state with Hawking temperature $T_H = \dfrac{\kappa}{2\pi}$



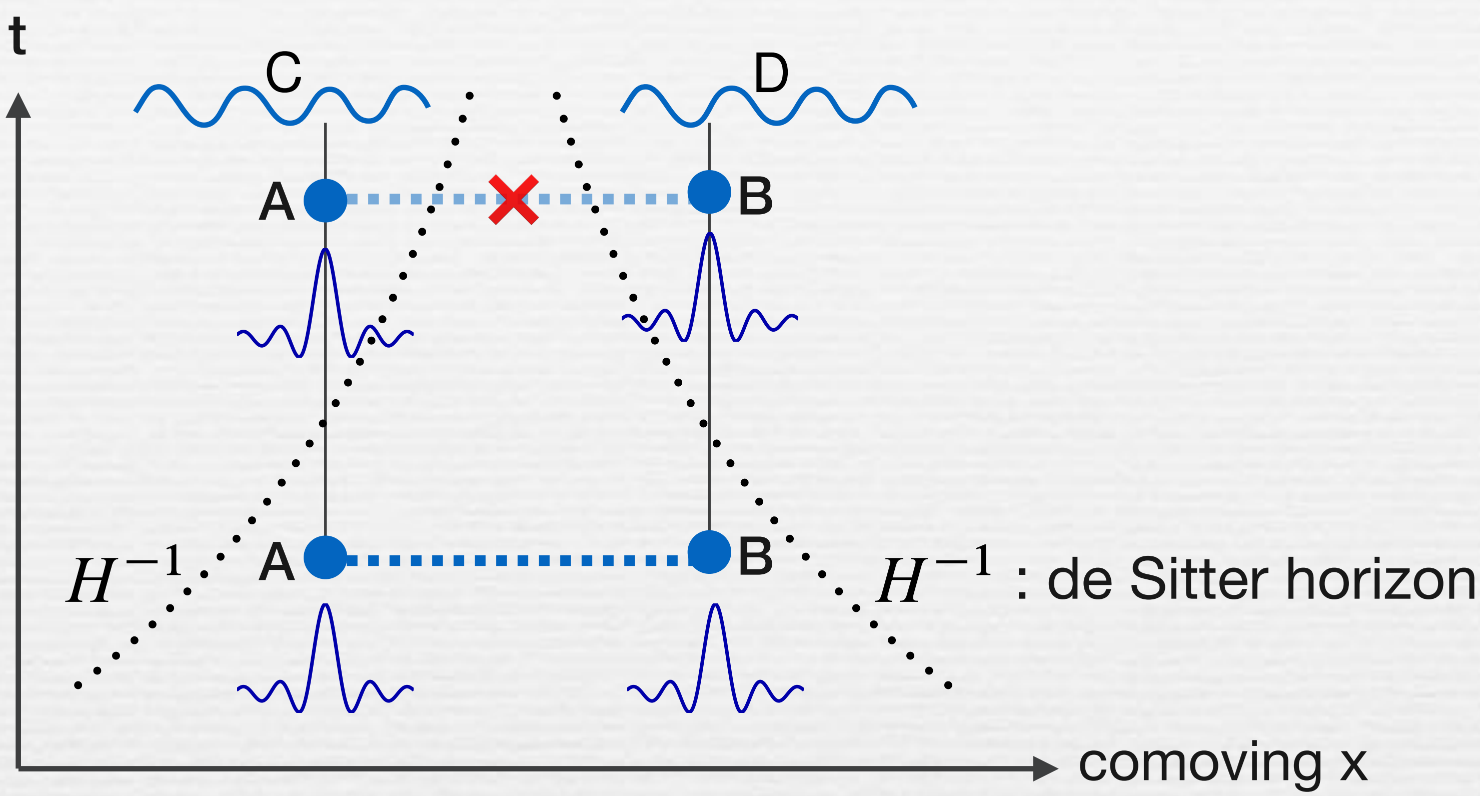


infalling $\bar{u}$ is a purification partner of u

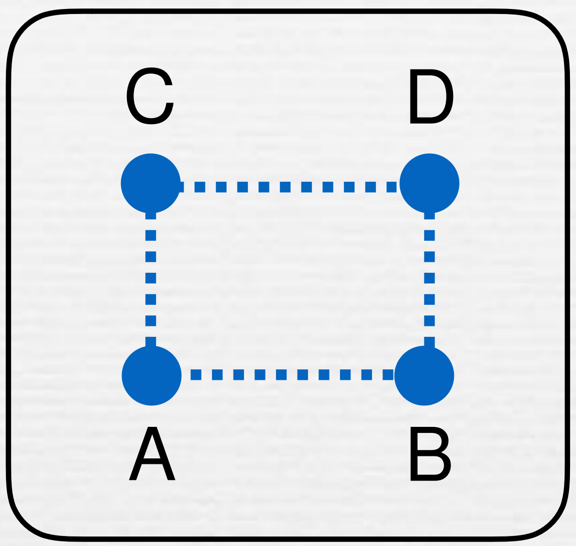
origin of $u\,\bar{u}$ is vacuum fluctuation before horizon formation

What is partner mode of inflaton fluctuation?

In deSitter case, we want to identify partner modes for AB,
which are defined as two spatially local modes at two points.



purification of AB



4-mode pure Gaussian state

- Entanglement between region A and region B is lost on super horizon scale
- By embedding the target modes AB in 4-mode pure Gaussian state, it is possible to understand structure of entanglement sharing between these 4 modes (looking for partner modes of AB)

Construction of partner mode and QIC

- As already introduced, we define local mode of the quantum field using window function, which determines spatial profile of the modes.
- "Information" of the local mode at x_j can be measured by Unruh-DeWitt type detectors:

$$\hat{O}_j = \int d^3x \left(\underbrace{w_1(x, x_j)}\hat{\varphi}(x) + \underbrace{w_2(x, x_j)}\hat{\pi}(x) \right) \qquad H_{\mathrm{int}} = \lambda \hat{\mu}(x) \otimes \sum_j \hat{O}_j$$

window functions

- For given set of operators $\{\hat{O}_j\}$ $(j = 1, \cdots, 2k)$, it is possible to identify 2k independent modes

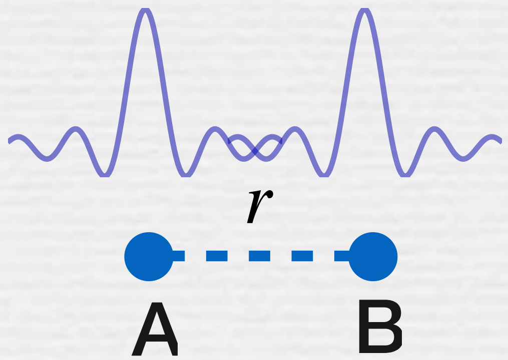
(1) $\hat{\xi}_j = (\hat{q}_j, \hat{p}_j), \ j = 1, \cdots k \qquad [\hat{q}_j, \hat{p}_j] = i\delta_{kj}$

(2) Modes $\{\hat{\xi}_j\}$ constitute a pure state (provides a method of purification)

QIC : quantum information capsule (by M.Hotta and K.Yamaguchi)

Information associated with original mode $\{\hat{O}_j\}$ is completely contained in QIC (purification of target modes)

target modes: localize at A,B



$r = r(\delta_c)$

We adopt the method formulated in these papers

J.Trevision et al. PTEP 2018, 103A03
K.Yamaguchi et al. Phys.Lett.A 383 (2019)1255
K.Yamaguchi et al. PRD 101(2020)105009

Construction of partner mode

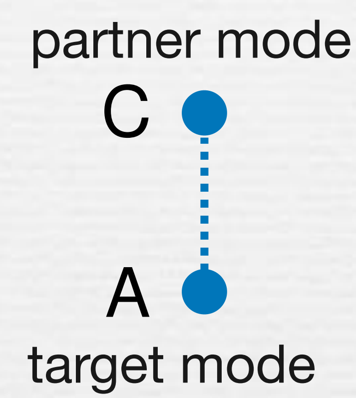
As a demonstration, we first consider purification of local mode A in de Sitter space

- **Single mode case**

Using coarse-grained field, we assign canonical operator at a spatial point x_1

$$\hat{\xi}_A = (\hat{q}_1, \hat{p}_1)^T \qquad \hat{q}_1 = \int_{-\infty}^{\infty} dk W_k (f_k \hat{a}_k + f_k^* \hat{a}_{-k}{}^\dagger) e^{ikx_1},$$

$$[\hat{q}_1, \hat{p}_1] = i \qquad \hat{p}_1 = \int_{-\infty}^{\infty} dk W_k (-i)(g_k \hat{a}_k - g_k^* \hat{a}_{-k}{}^\dagger) e^{ikx_1},$$



We define the following linear map for operators:

$$\boxed{f_\psi(\hat{a}_k) = -i\hat{a}_k, \quad f_\psi(\hat{a}_k^\dagger) = i\hat{a}_k^\dagger.}$$

$$f_\psi(\hat{q}_1) := \int dk W_k (-i)(f_k \hat{a}_k - f_k^* \hat{a}_{-k}{}^\dagger) e^{ikx_1},$$

$$f_\psi(\hat{p}_1) := -\int dk W_k (g_k \hat{a}_k + g_k^* \hat{a}_{-k}{}^\dagger) e^{ikx_1}.$$

Thus we introduced 2-mode system characterized by the following 4 operators:

$$\hat{q}_1, \hat{p}_1, f_\psi(\hat{q}_1), f_\psi(\hat{p}_1).$$

The map f preserves commutation relation and covariance:

target mode: $\hat{\xi}_i = (\hat{q}_1, \hat{p}_1)^T \qquad \hat{\xi}_i, \ f_\psi(\hat{\xi}_i)$

commutators $[\hat{\xi}_i, \hat{\xi}_j] = [f_\psi(\hat{\xi}_i), f_\psi(\hat{\xi}_j)] = i(J)_{ij}, \quad [\hat{\xi}_i, f_\psi(\hat{\xi}_j)] = i(m)_{ij} = ia\delta_{ij}.$

covariance $\left\langle \{\hat{\xi}_i, \hat{\xi}_j\} \right\rangle = \left\langle \{f_\psi(\hat{\xi}_i), f_\psi(\hat{\xi}_j)\} \right\rangle = (m)_{ij}, \quad \left\langle \{\hat{\xi}_i, f_\psi(\hat{\xi}_j)\} \right\rangle = -(J)_{ij}$

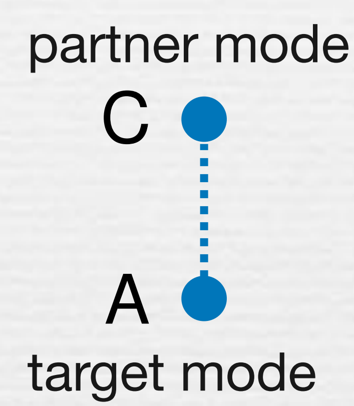
covariance matrix of target mode

$\langle \hat{q}_1^2 \rangle = \langle \hat{p}_1^2 \rangle = a/2 \qquad m = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} = aI, \quad J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$

By re-defining combination of 4 ops., it is possible to idenfity independent two mode

$$\boxed{\begin{array}{c} \underset{\text{target mode}}{\hat{\xi}_A = \hat{\xi}}, \quad \underset{\text{partner mode}}{\hat{\xi}_C = \dfrac{1}{\sqrt{a^2-1}} X(f_\psi(\hat{\xi}) - mJ\hat{\xi})} \\[2ex] [\hat{\xi}_A, \hat{\xi}_A] = iJ, \quad [\hat{\xi}_C, \hat{\xi}_C] = iJ, \quad [\hat{\xi}_A, \hat{\xi}_C] = 0 \end{array}}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



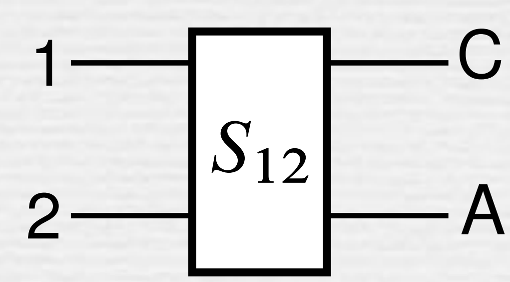
covariance of total 2 mode system (pure)

$$M_{AC} = \begin{bmatrix} aI & \sqrt{a^2-1}Z \\ \sqrt{a^2-1}Z & aI \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

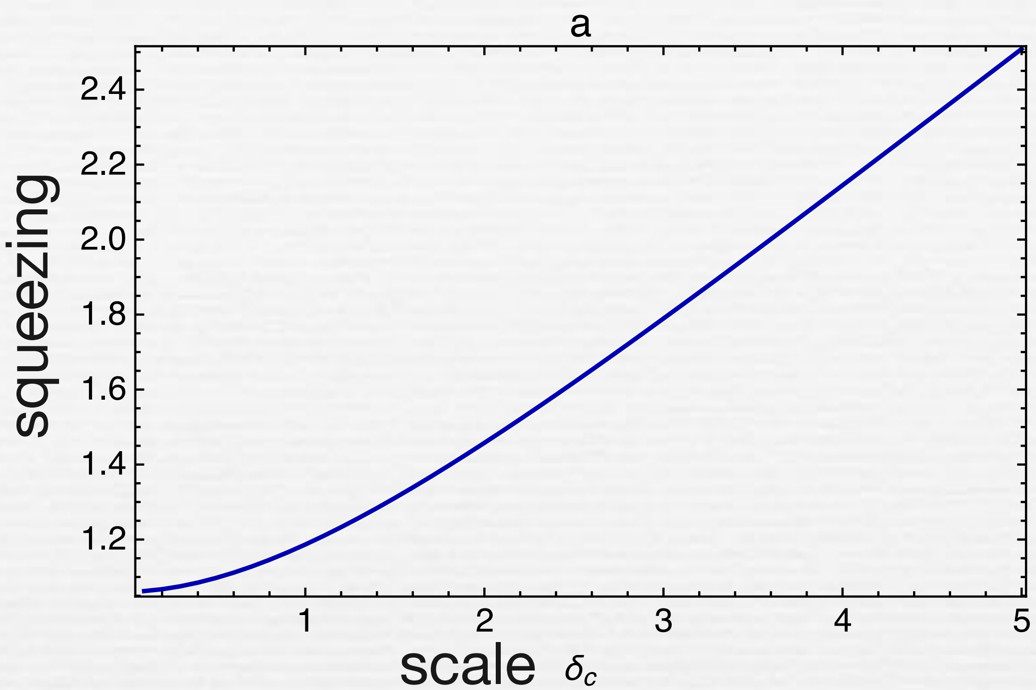
This is the standard form of two mode squeezed state (pure)

Partner of A is represented as $\hat{\xi}_C = (\hat{q}_C, \hat{p}_C)$

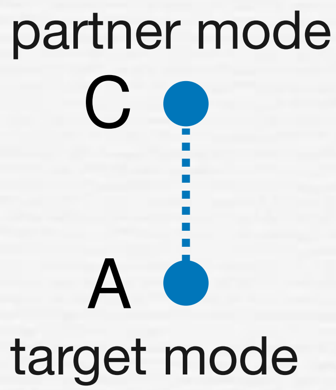
As we will see, this method provides spatial profile of partner mode



- **degrees of squeezing**



$$M_{AC} = \begin{bmatrix} aI & \sqrt{a^2-1}Z \\ \sqrt{a^2-1}Z & aI \end{bmatrix}$$



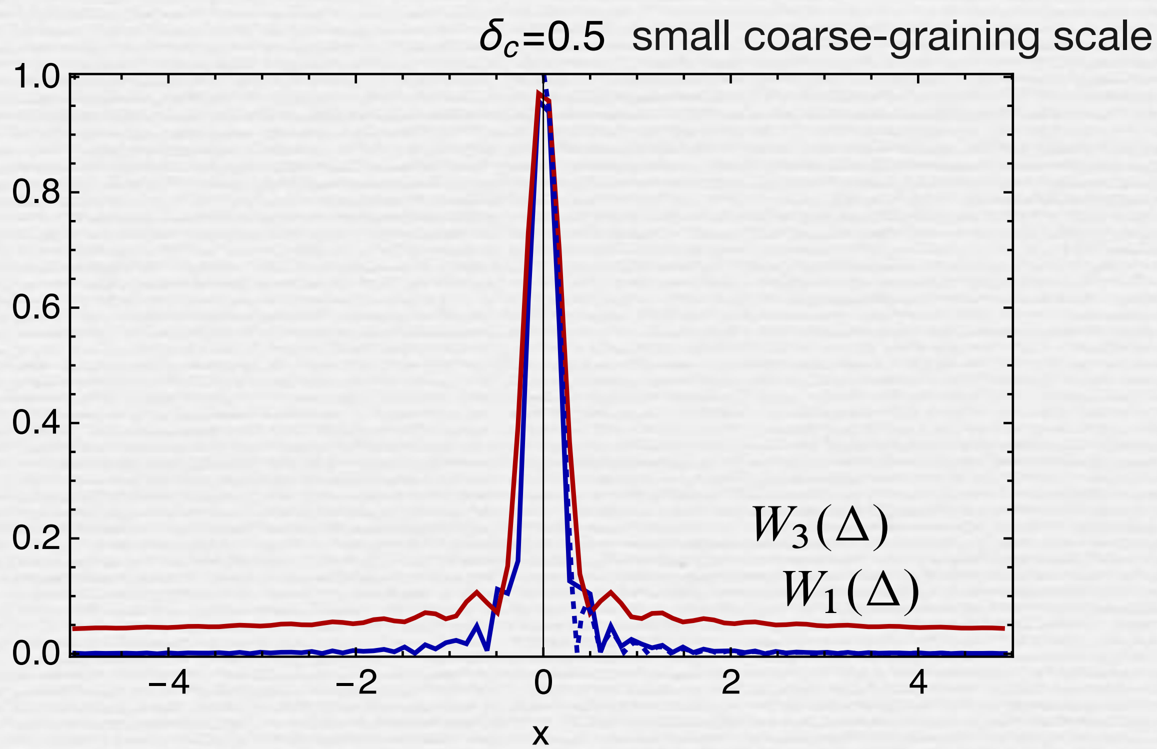
entanglement entropy

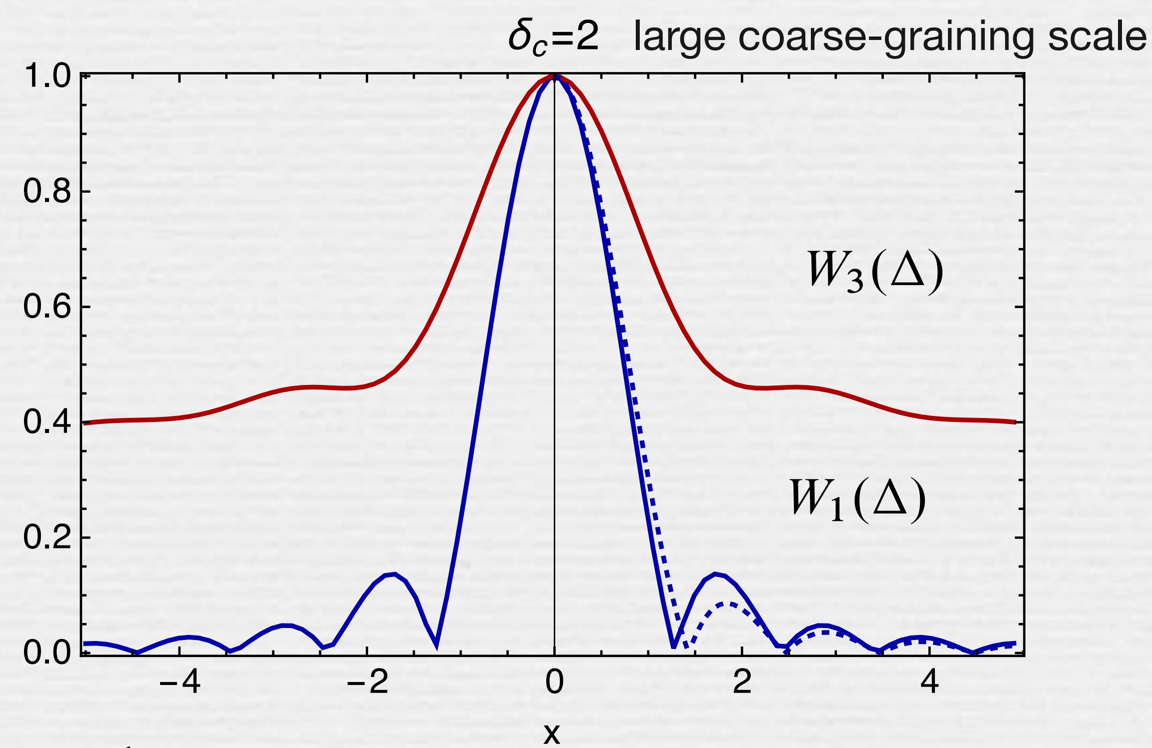
$$S_A = \left(\frac{a+1}{2}\right)\log\left(\frac{a+1}{2}\right) - \left(\frac{a-1}{2}\right)\log\left(\frac{a-1}{2}\right)$$

Entanglement entropy of mode A monotonically increases with scale

As we identify operator form of partner, it is possible to reconstruct spatial shape of it

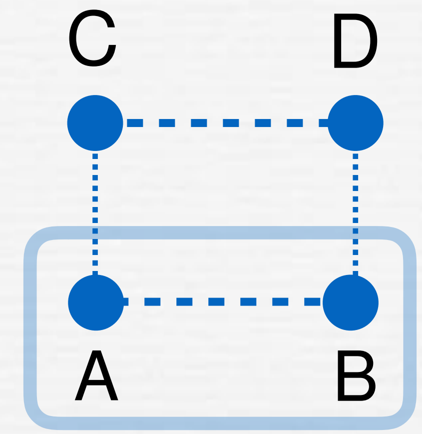
- **spatial profile of partner mode C** $\hat{Q}_C = \int d^3x W_1(x_1 - x)\hat{\varphi}(x)$ $\hat{P}_C = \int d^3x W_3(x_1 - x)\hat{\pi}(x)$





For large coarse-graining scale $\delta_c > H^{-1}$, partner mode spreads over super horizon scale (de-localize)

• Two mode case



target mode

target modes $\hat{\xi}_{AB} = (\hat{Q}_1, \hat{P}_1, \hat{Q}_2, \hat{P}_2)^T$

partner modes $\hat{\xi}_{CD} = A^{-1}(f_\psi(\hat{\xi}_{AB}) - M\Omega_2\hat{\xi}_{AB})$

$$A\Omega_2 A^T = \Omega_2 - M\Omega_2 M. \qquad \Omega_2 = \begin{bmatrix} J & \\ & J \end{bmatrix}$$

covariance of target mode (we assume standard form of Gaussian two mode state)

$$M_{AB} = \begin{bmatrix} a & 0 & d_1 & 0 \\ 0 & a & 0 & d_2 \\ d_1 & 0 & a & 0 \\ 0 & d_2 & 0 & a \end{bmatrix}$$

commutators of 4-mode $[\hat{\xi}_{AB}, \hat{\xi}_{AB}] = i\Omega_2, \quad [\hat{\xi}_{CD}, \hat{\xi}_{CD}] = i\Omega_2, \quad [\hat{\xi}_{AB}, \hat{\xi}_{CD}] = 0$

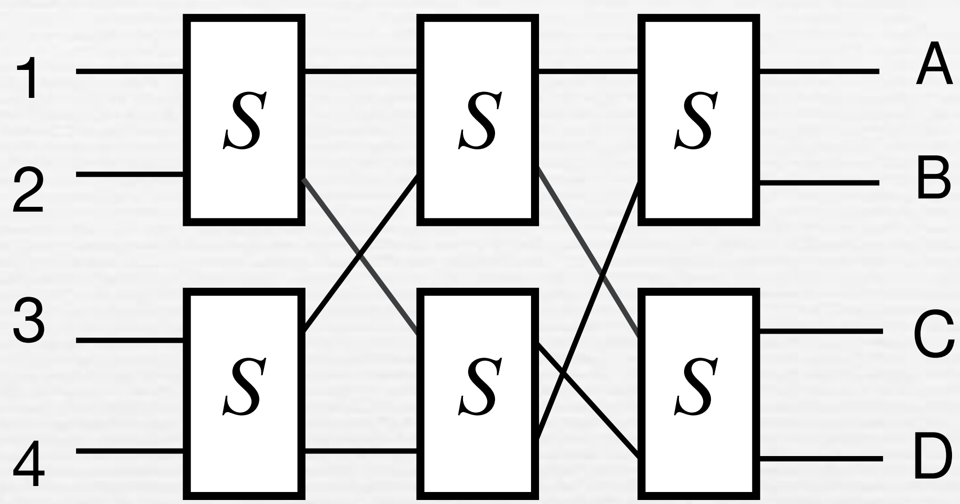
covariance of total 4 mode system (pure)

$$M_{ABCD} = \begin{bmatrix} a & 0 & d_1 & 0 & g & 0 & h & 0 \\ 0 & a & 0 & d_2 & 0 & -g & 0 & -h \\ & & a & 0 & h & 0 & g & 0 \\ & & 0 & a & 0 & -h & 0 & -g \\ & & & & a & 0 & d_2 & 0 \\ & & & & 0 & a & 0 & d_1 \\ & & & & & & a & 0 \\ & & & & & & 0 & a \end{bmatrix}$$

$$g^2 = \frac{x}{2} + \frac{1}{2}\sqrt{x^2 - y^2}, \quad h^2 = \frac{x}{2} - \frac{1}{2}\sqrt{x^2 - y^2},$$

$$x = a^2 + d_1 d_2 - 1, \quad y = a(d_1 + d_2).$$

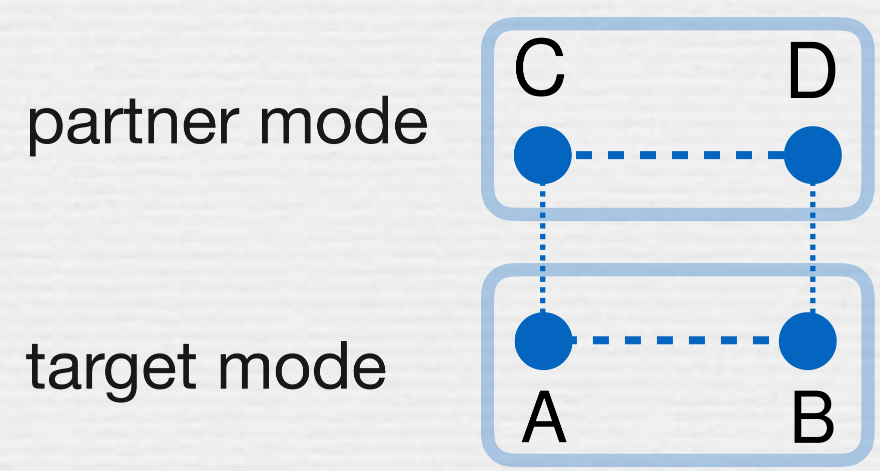
Structure of 4 mode state ABCD



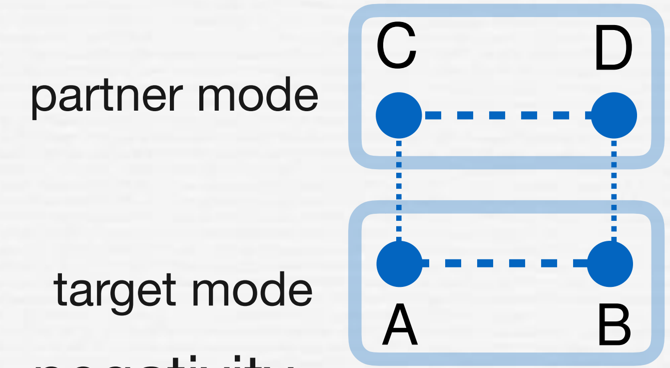
covariance matrix

$$M_{ABCD} = \begin{array}{cc} \begin{array}{cccc} \quad A \quad & \quad B \quad & \quad C \quad & \quad D \quad \end{array} & \\ \begin{bmatrix} a & 0 & d_1 & 0 & g & 0 & h & 0 \\ 0 & a & 0 & d_2 & 0 & -g & 0 & -h \\ & & a & 0 & h & 0 & g & 0 \\ & & 0 & a & 0 & -h & 0 & -g \\ & & & & a & 0 & d_2 & 0 \\ & & & & 0 & a & 0 & d_1 \\ & & & & & & a & 0 \\ & & & & & & 0 & a \end{bmatrix} & \begin{array}{c} A \\ \\ B \\ \\ C \\ \\ D \end{array} \end{array}$$

- ABCD is obtained by applying 6 squeezing gates to vacuum states 1,2,3,4
- CD is partner of AB
- total system ABCD is pure

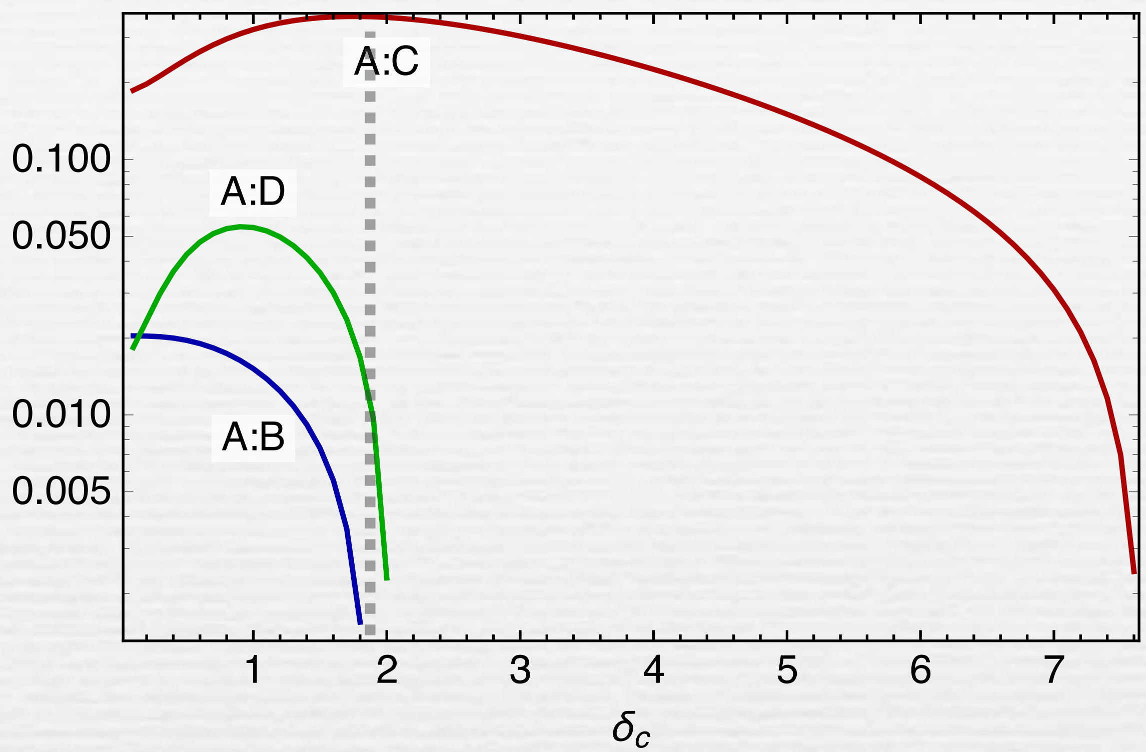


Structure of entanglement

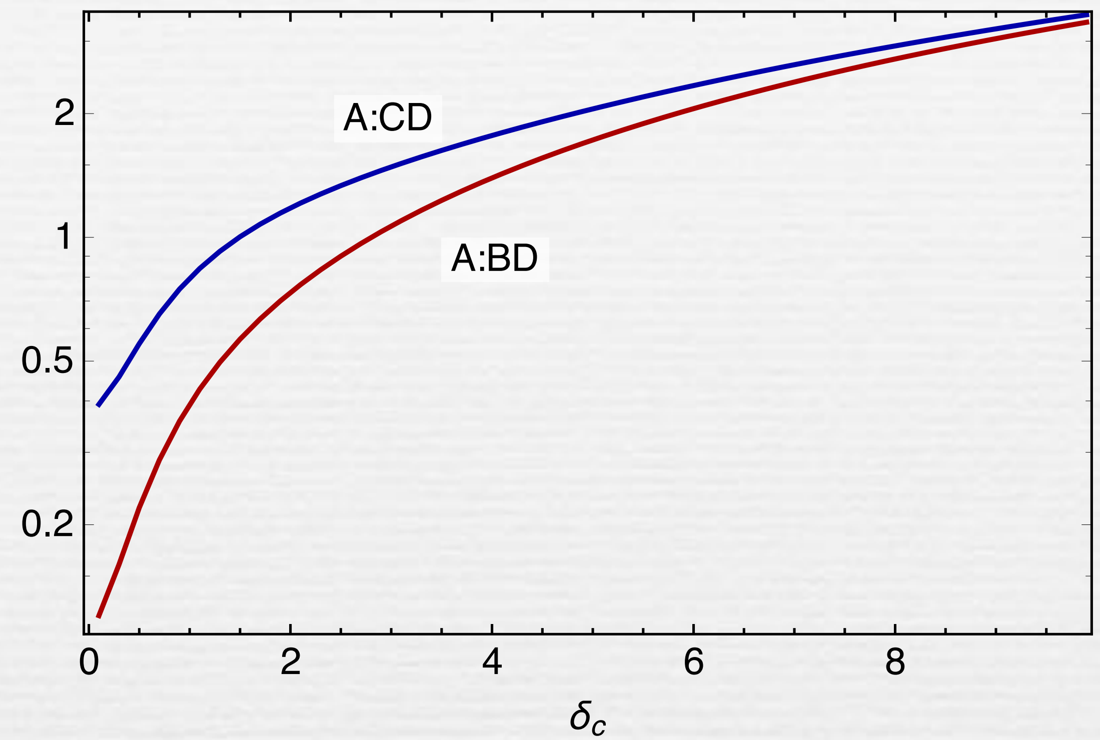


We show scale dependence of negativity of state ABCD

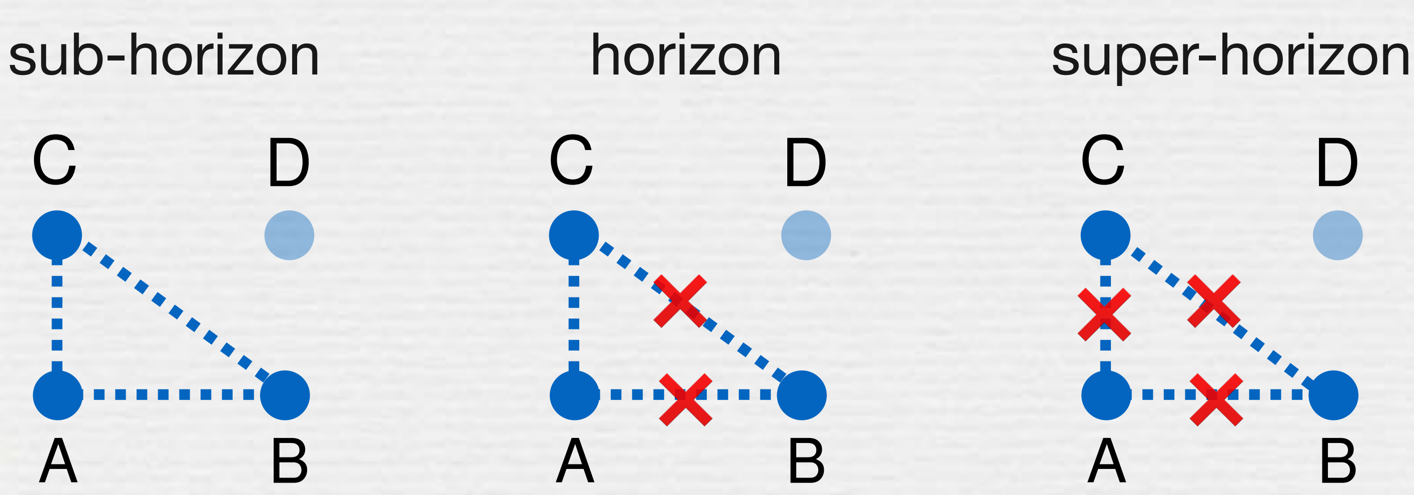
reduced two modes negativity



reduced three modes negativity



reduced 3 modes system



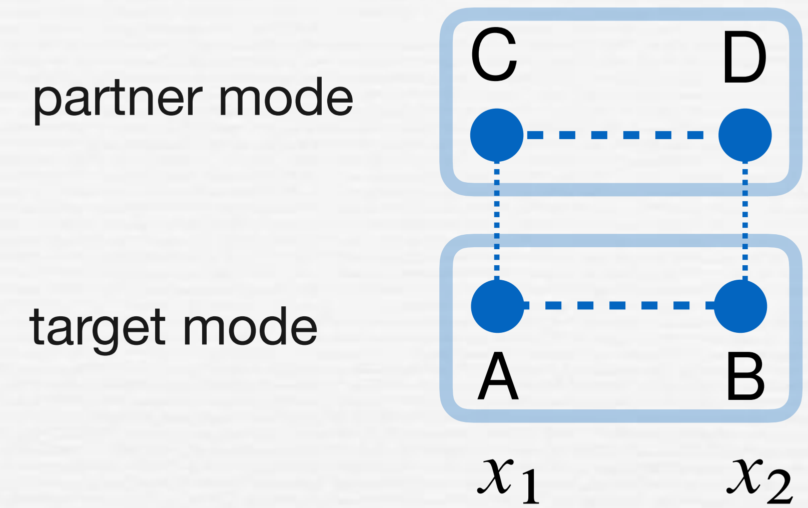
- Target modes AB become separable on super horizon scale
- Entanglement between AC increases until AB become separable
- Trade off relation between N(A:B) and N(A:C) (monogamy property of entanglement)
- on super-horizon scale, reduced 3 mode has genuine tripartite entanglement (GHZ type)

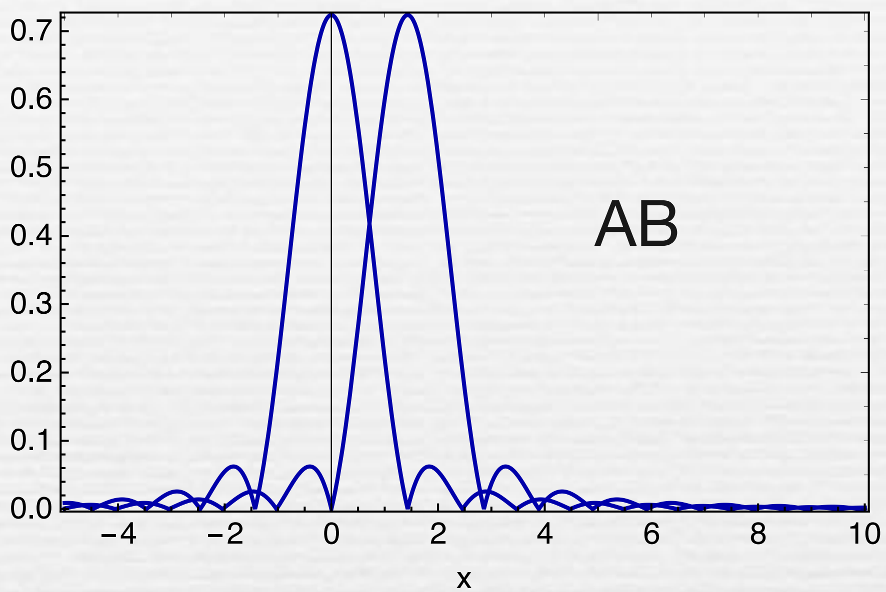
$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$

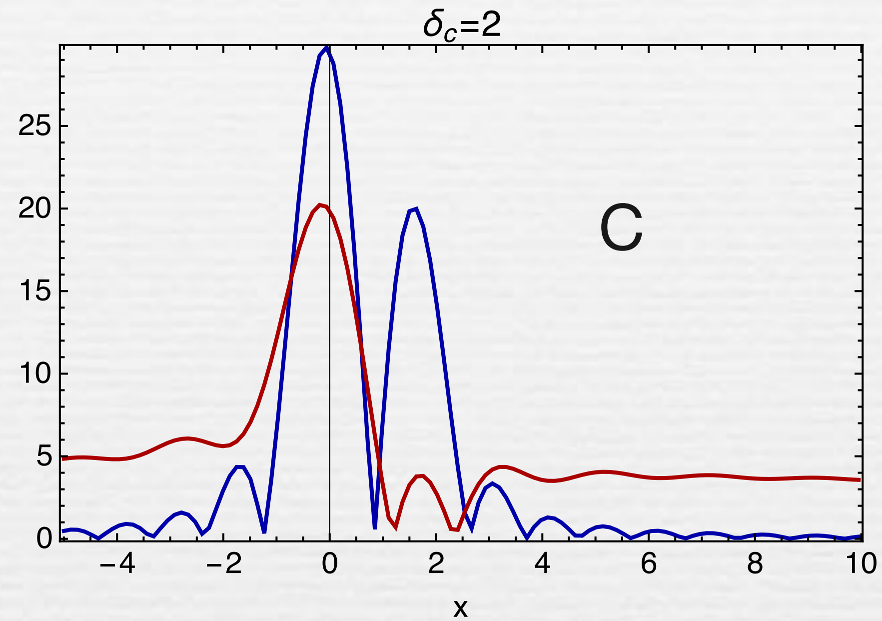
Spatial profile of partner mode

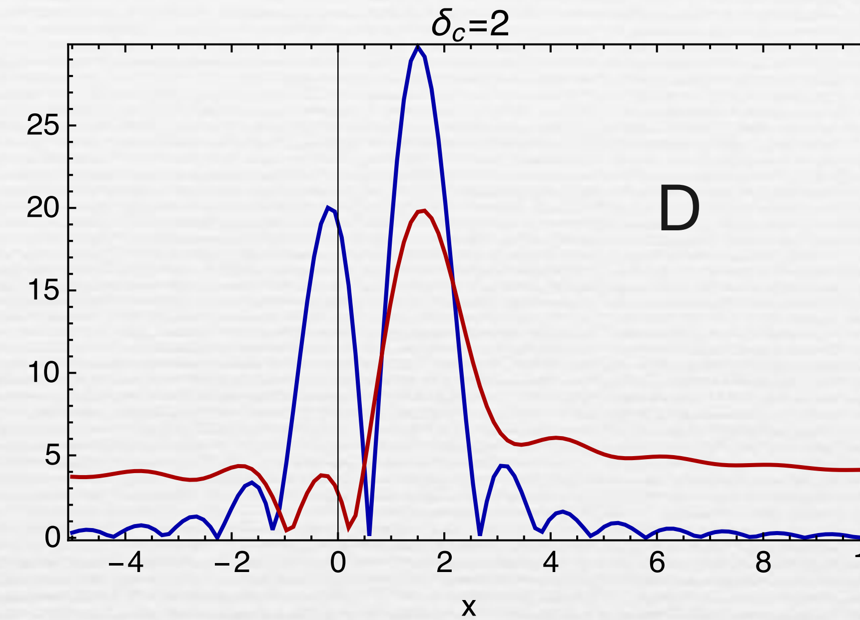
$$\hat{Q}_{C,D} = \int d^3y \left[W_1^{(C,D)}(x_1 - y) + W_2^{(C,D)}(x_2 - y) \right] \hat{\varphi}(y)$$

$$\hat{P}_{C,D} = \int d^3y \left[W_3^{(C,D)}(x_1 - y) + W_4^{(C,D)}(x_2 - y) \right] \hat{\pi}(y)$$



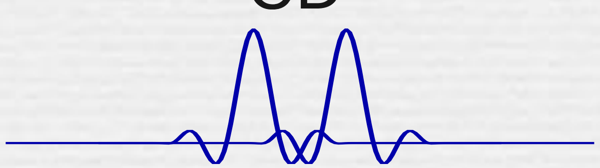






small coarse-graining scale

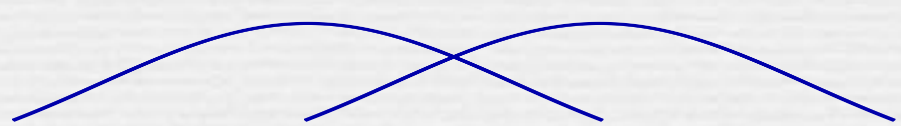
CD



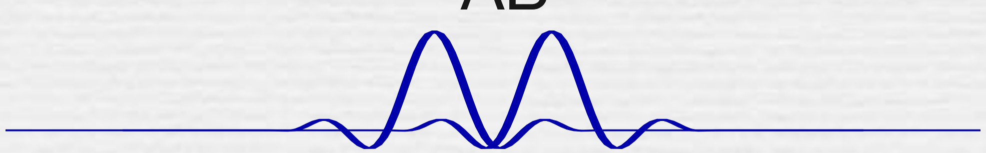
AB

large coarse-graining scale

CD



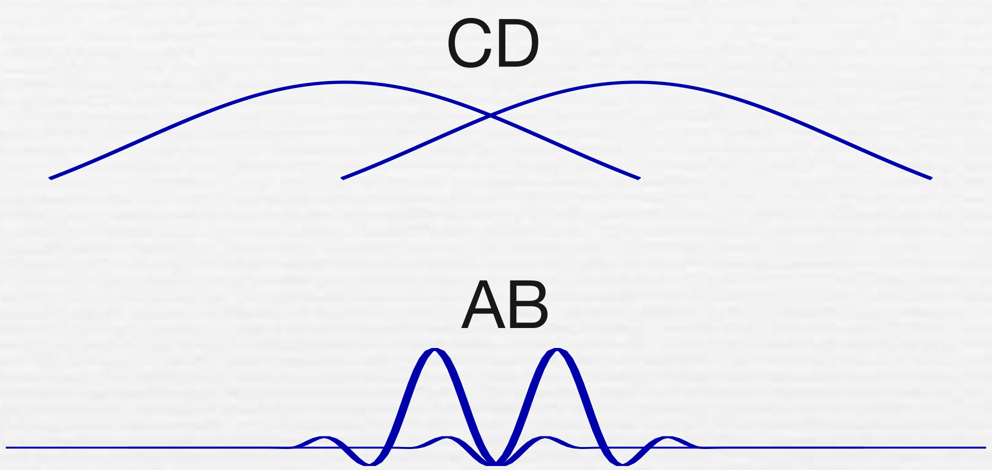
AB



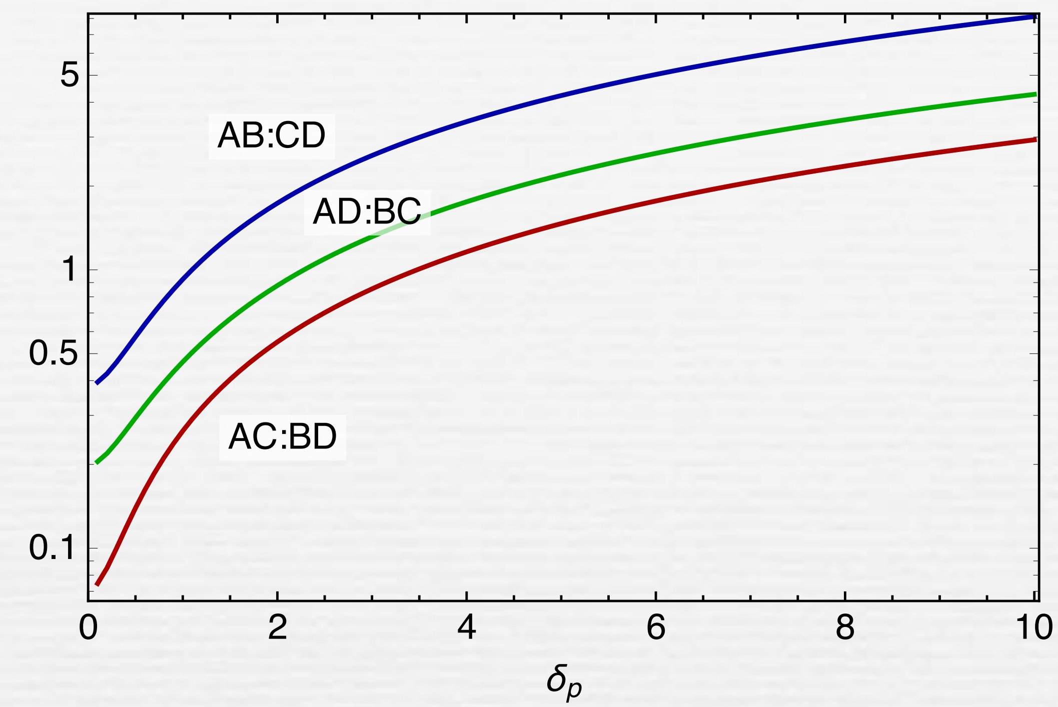
Profiles of parter CD spread over super horizon scale

Large scale entanglement

large coarse-graining scale



negativity

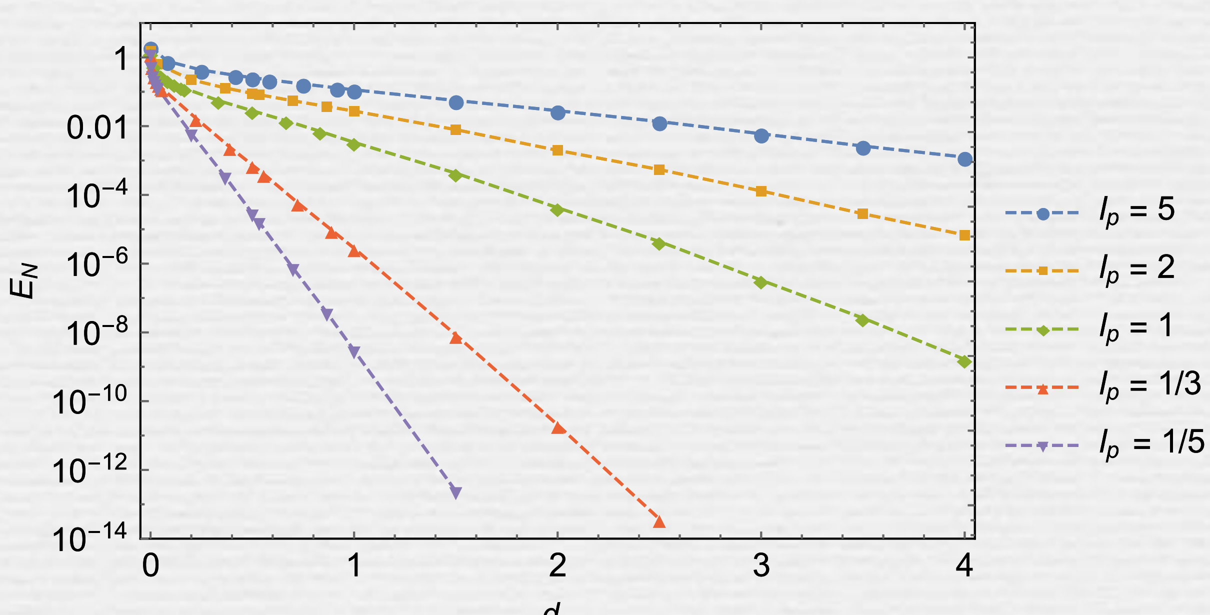


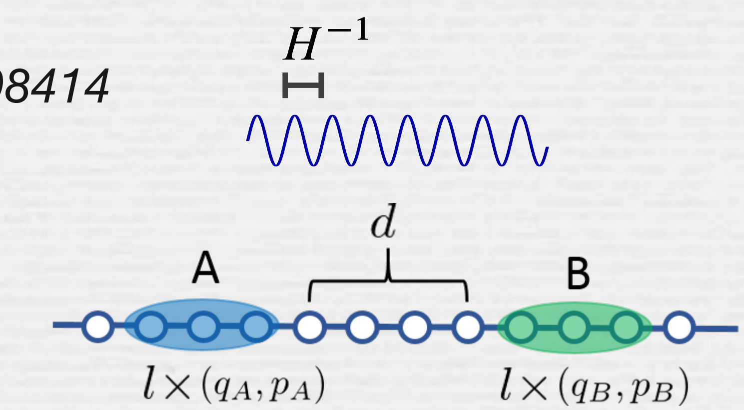
- Entanglement between AB and CD increases with scale and captures large scale entanglement (multi-partite effect)

Reduced ABC system is GHZ type state

- Existence of large scale entanglement in de Sitter space

Numerical calculation of 1+1 harmonic chain in de Sitter space *A.Matsumura and Y.Nambu 1707.08414*





Two spatial regions are entangled even for super-horizon scale separation

(effect of multi-partite entanglement)

Summary

Structure of entanglement in de Sitter inflation

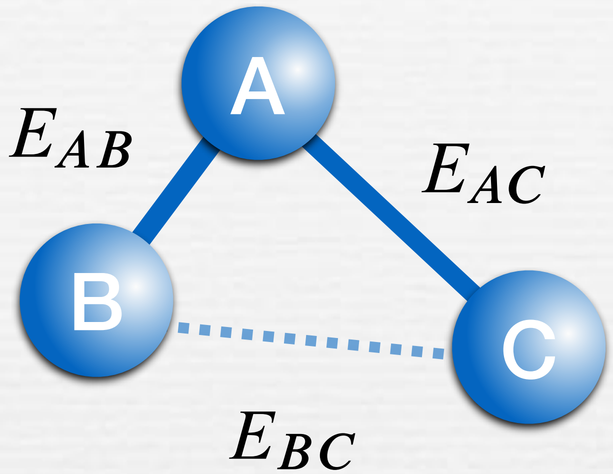
- entanglement harvesting with two detectors (two modes)
  - Harvesting with two modes AB is impossible for super horizon scale (two spatial region becomes separable)
  - Local noise of de Sitter space kills quantum correlation (but this is related to monogamy of entanglement)
  - This behavior can be understood from monogamous property by introducing partner modes

- 4 mode Gaussian state as total system
  - Spatial profile of partner modes unlocalized over super horizon scale
  - Negativity with 2x2 bipartition and 1x2 bipartition (GHZ type) increases with scale. This behavior reflects structure of multi-partite entanglement in de Sitter space

Back Up

Entanglement Monogamy

basic properties



E :entanglement measure

qubit system

square of concurrence, negativity

Gaussian system

square of negativity

monogamy relation of entanglement

$$E_{\mathrm{A|BC}} \geq E_{\mathrm{AB}} + E_{\mathrm{AC}}$$

V.Coffman, J.Kundu, W.K.Wootters 1998

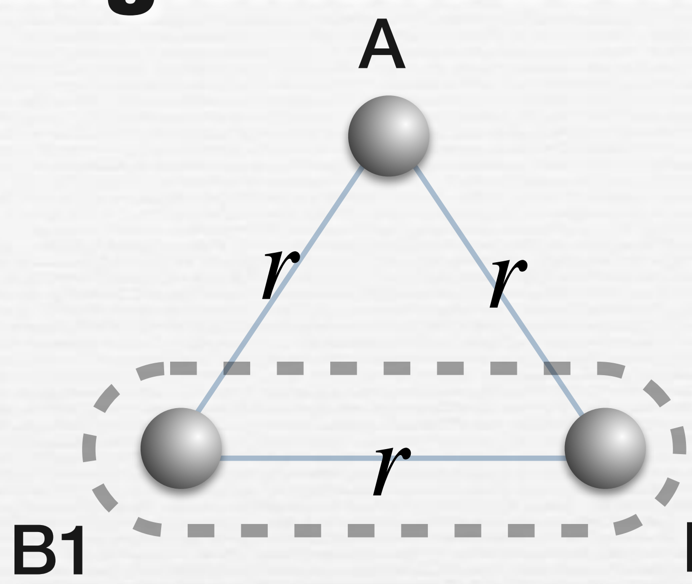
T.J.Osbone, F.Verstraete 2006

G.Adesso, F.Illuminati 2006

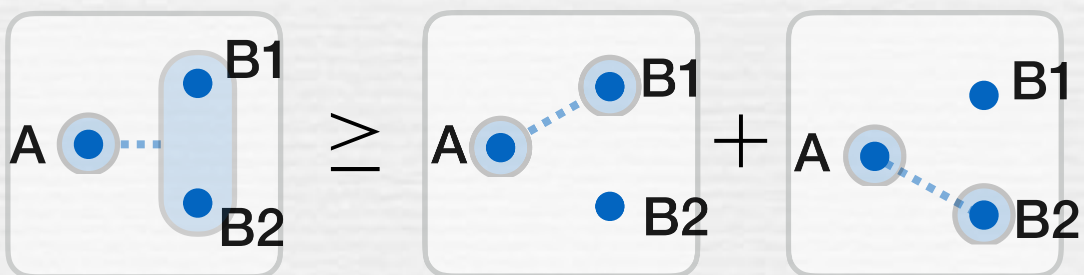
trade off relation between E_{AB} and E_{AC}

- universal relation characterizing multi-partite entanglement
- may provide upper bound of E_{AB} and E_{AC}
- sharing of quantum information, no-cloning theorem

Entanglement between 3-quit detectors



For $2 \leq rH \leq 2.2$, GHZ type state appears



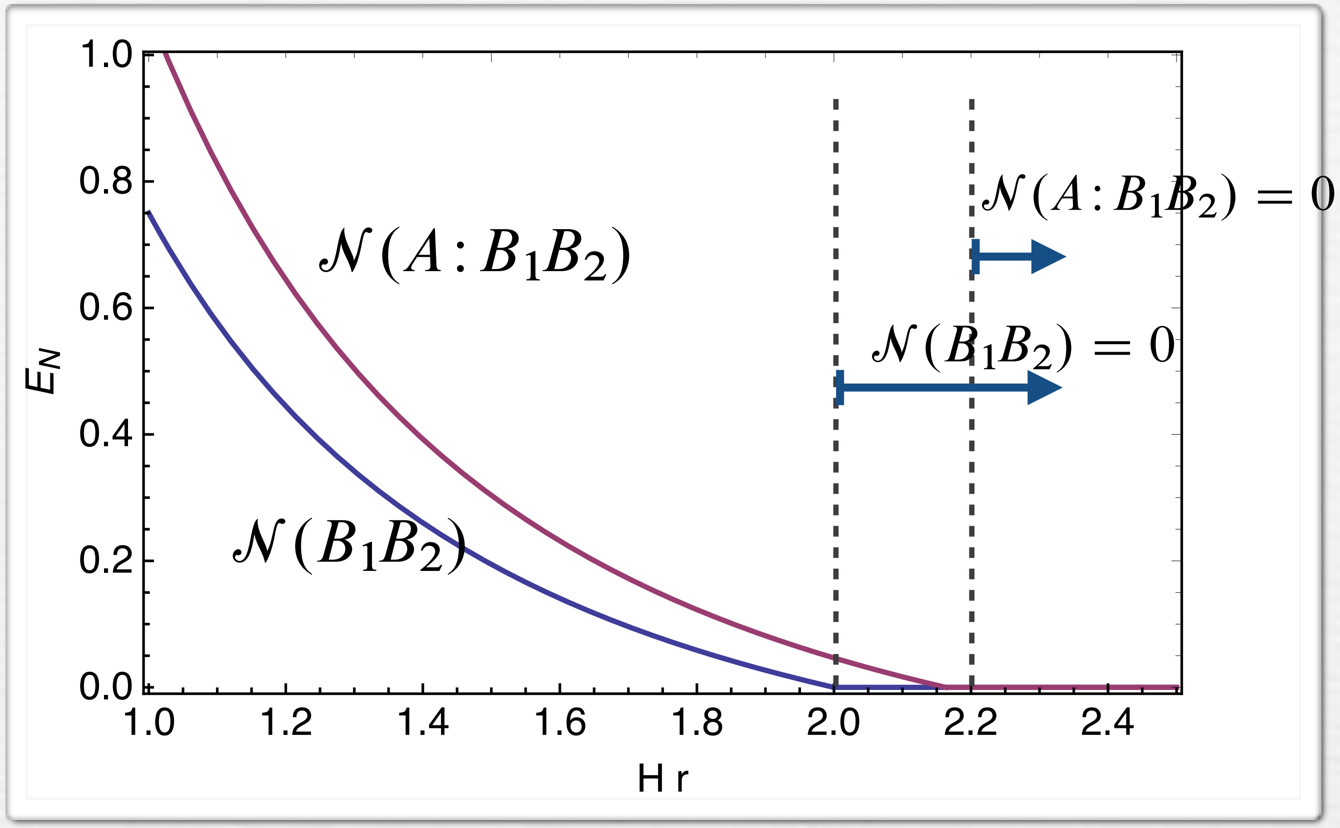
residual:=

$$\mathcal{N}(A:B_1B_2)^2 - \mathcal{N}(A:B_1)^2 - \mathcal{N}(A:B_2)^2$$

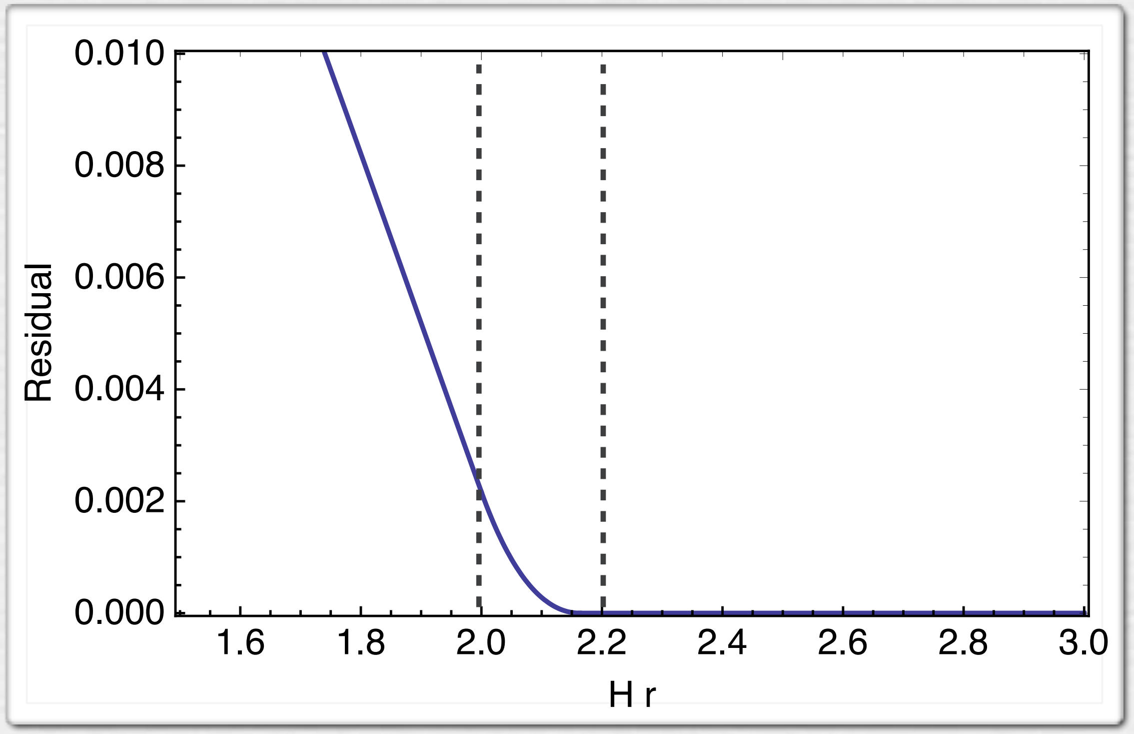
This quantity is positive by monogamy relation for qubits system

contribution of purely tripartite entanglement increases $r_{\max}$

GHZ type structure



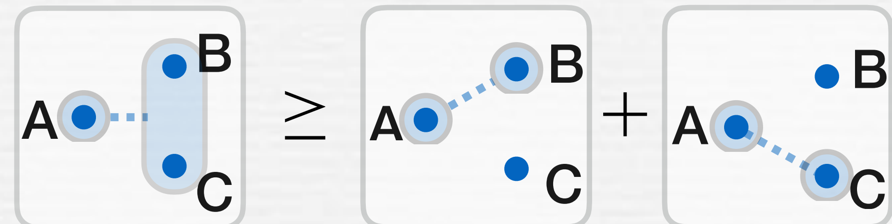
pure tripartite entanglement



Structure of Multipartite Entanglement: Monogamy

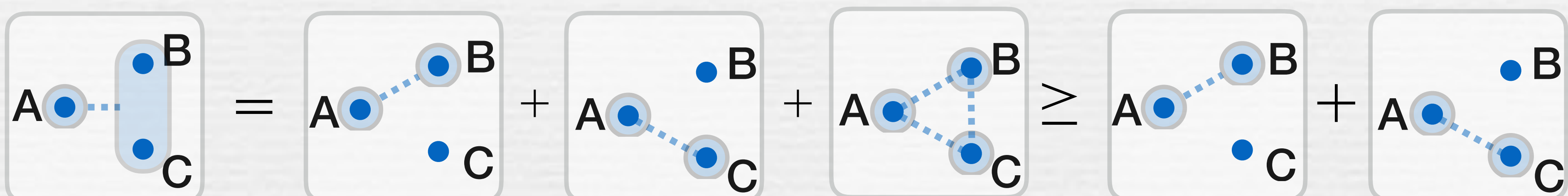
Monogamy of entanglement: property of entanglement sharing
prohibits cloning of an unknown quantum state

$$\mathcal{N}^2_{A|BC} \geq \mathcal{N}^2_{A|B} + \mathcal{N}^2_{A|C}$$

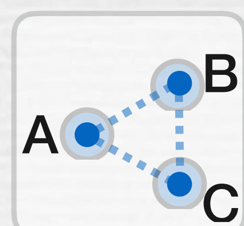


group1 group2

Residual: $\mathcal{N}^2_{A|BC} - \mathcal{N}^2_{A|B} - \mathcal{N}^2_{A|C}$ measure of multipartite entanglement

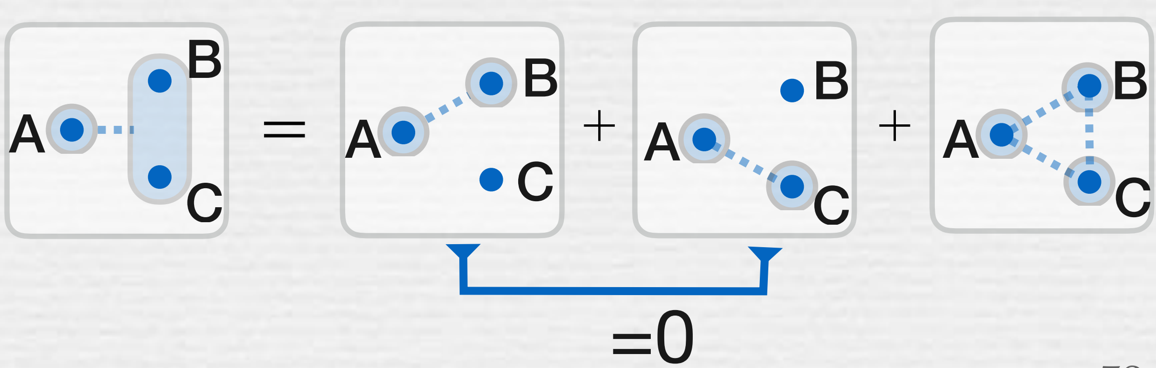


group1 group2

∴ **Residual =** 

pure tripartite entanglement

For super horizon mode in de Sitter space,

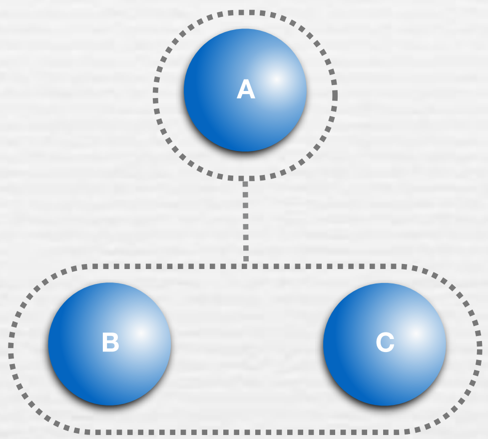


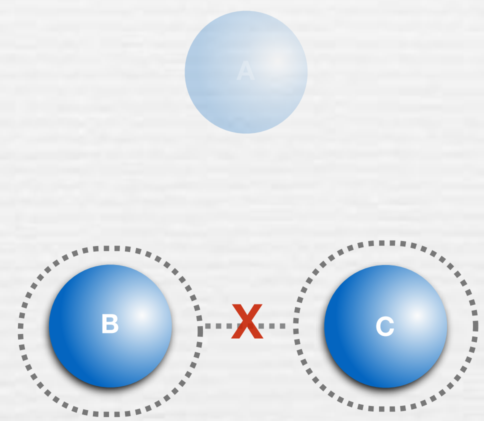
Multipartite effect is responsible
for large scale entanglement

Monogamy and Separability

- Is it possible to say something about emergence of separable state just applying monogamy inequality?

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|111\rangle + |000\rangle\right)$$





$\mathcal{N}(\mathrm{A} : \mathrm{BC}) = 1/2$

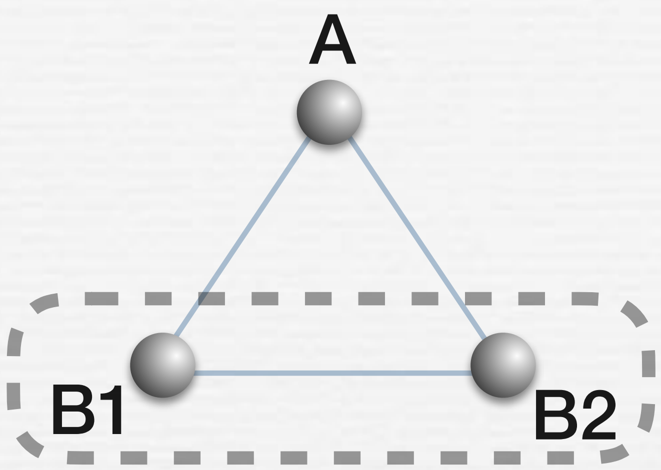
$\mathcal{N}(\mathrm{B} : \mathrm{C}) = 0$

$$\mathcal{N}^2_{B|CA} \geq \mathcal{N}^2_{BC} + \mathcal{N}^2_{AB} \qquad \mathcal{N}^2_{A|BC} \geq \mathcal{N}^2_{AB} + \mathcal{N}^2_{AC}$$

These inequalities are trivially satisfied and do not proved any useful information on relation between separability and strength of entanglement

Standard monogamy relation:

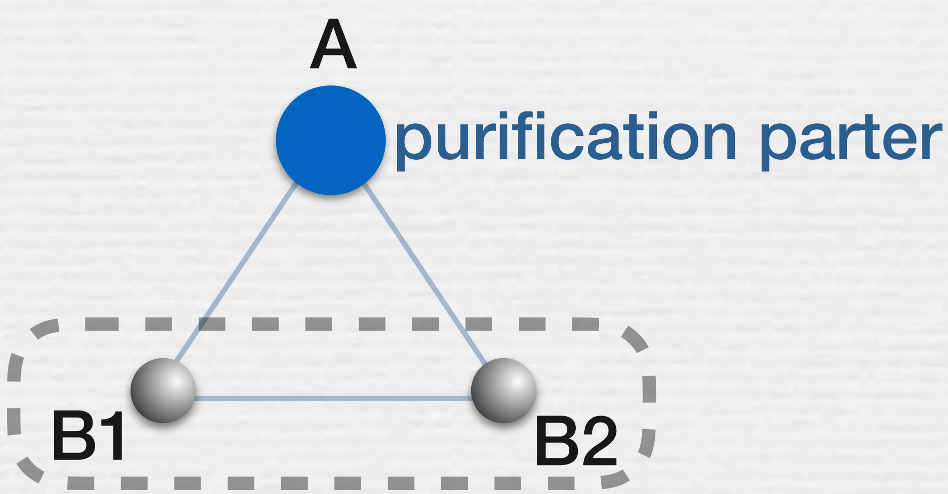
$$\mathcal{N}^2_{A|B_1B_2} \geq \mathcal{N}^2_{A|B_1} + \mathcal{N}^2_{A|B_2}$$



This inequality does not bound strength of correlation between B1 and B2 as a function of correlation between A and (B1B2)

A new monogamy inequality: *S. Camalt 2017*

$$\tilde{E}_{\max} \geq \tilde{E}(B_1 : B_2) + E(A : B_1B_2)$$



For maximally entangled system B1B2, $\tilde{E}(B_1 : B_2) = \tilde{E}_{\max}$ and this inequality says $E(A : B_1B_2) = 0$

relation usually used to illustrate entanglement monogamy