Swampland Conjectures and Gravitational Positivity

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Swampland Conjectures and Gravitational Positivity

1. Swampland program
   - quantum gravity constraints on QFT models
   - toward quantum gravity phenomenology

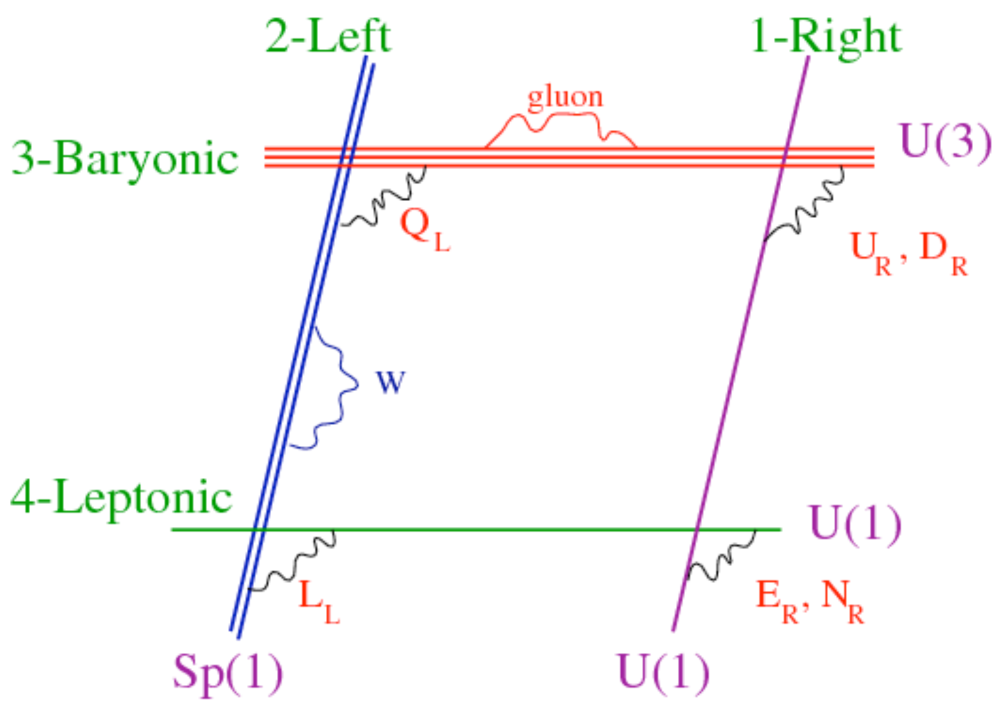
2. Gravitational positivity bounds
   - consistency of gravitational scattering

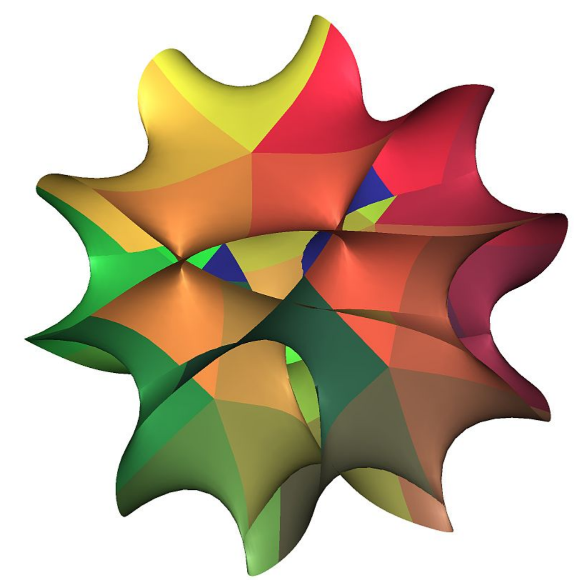
3. Exploring swampland w/gravitational positivity
   - QED and dark photon
   - bound on scalar potential

String Theory Landscape

string theory = a framework to generate QFT models

which incorporate quantum gravity appropriately





infinitely many QFT models can be generated

by changing brane configurations, shapes of extra dimensions etc,

→ string theory landscape

Landscape vs Swampland

Q. Is every QFT model realized in the string theory landscape?

1. ∃ minimum length in string theory

   → we cannot set extra dimensions arbitrary small (cf. string duality)

   → bounds on model parameters (ex. axion decay const. $f \lesssim M_{\rm Pl}$)

   [Banks-Dine-Fox-Gorbatov '03, ⋯]

2. AdS/CFT

   - consistency requirements from the dual CFT
   - holographic quantum information, conformal bootstrap, etc

   ex. no global symmetry in QG [Banks-Dixon '88, Banks-Seiberg '10, Harlow-Ooguri '18, ⋯]

3. thought experiments on BH evaporation [cf. various talks in this workshop]

nontrivial stringy/QG constraints on QFT models → swampland [Vafa '06]

We would like to use such QG constraints for phenomenology

No Global Symmetry in Quantum Gravity

\# option 1: the symmetry is gauged

charged particles are coupled to gauge bosons

no global symmetry just says gauge coupling $g \neq 0$

if $g = 10^{-100}$ is allowed, phenomenologically useless

\# option 2: the symmetry is broken at some scale

ex. no global symmetry just prohibits exactly flat potentials

if very very weak symmetry breaking is allowed,

phenomenologically useless (cf. Nakata-san's talk)

For pheno, we need more quantitative QG constraints!

In the swampland program [a review: Palti ’19],

various quantitative QG constraints are proposed

ex. Weak Gravity Conjecture, Distance Conjecture, ⋯

But, most of them are still at the level of conjectures.

Our motivation:

Can we derive such quantitative QG constraints

from consistency of gravitational scattering amplitudes?

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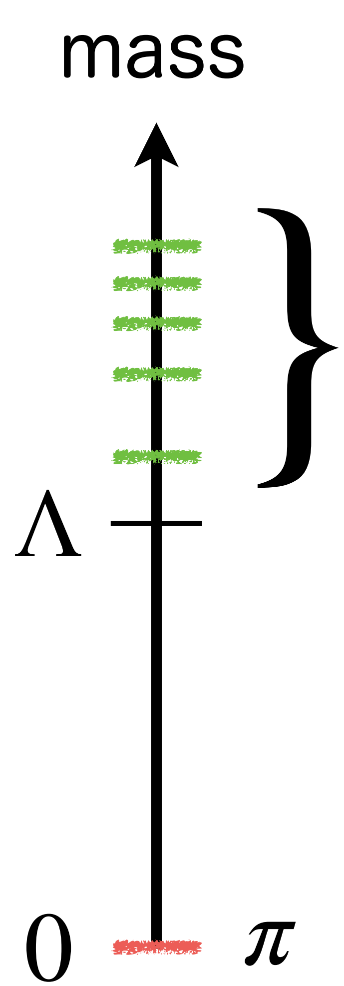
Basic idea of positivity: UV constraints on IR EFT

Positivity Bounds (w/o gravity) [Adams et al '06]



IR EFT of π :

$$S = \int d^4x \left[-\frac{1}{2}(\partial_\mu \pi)^2 + \frac{\alpha}{\Lambda^4}(\partial_\mu \pi \partial^\mu \pi)^2 + \cdots \right]$$

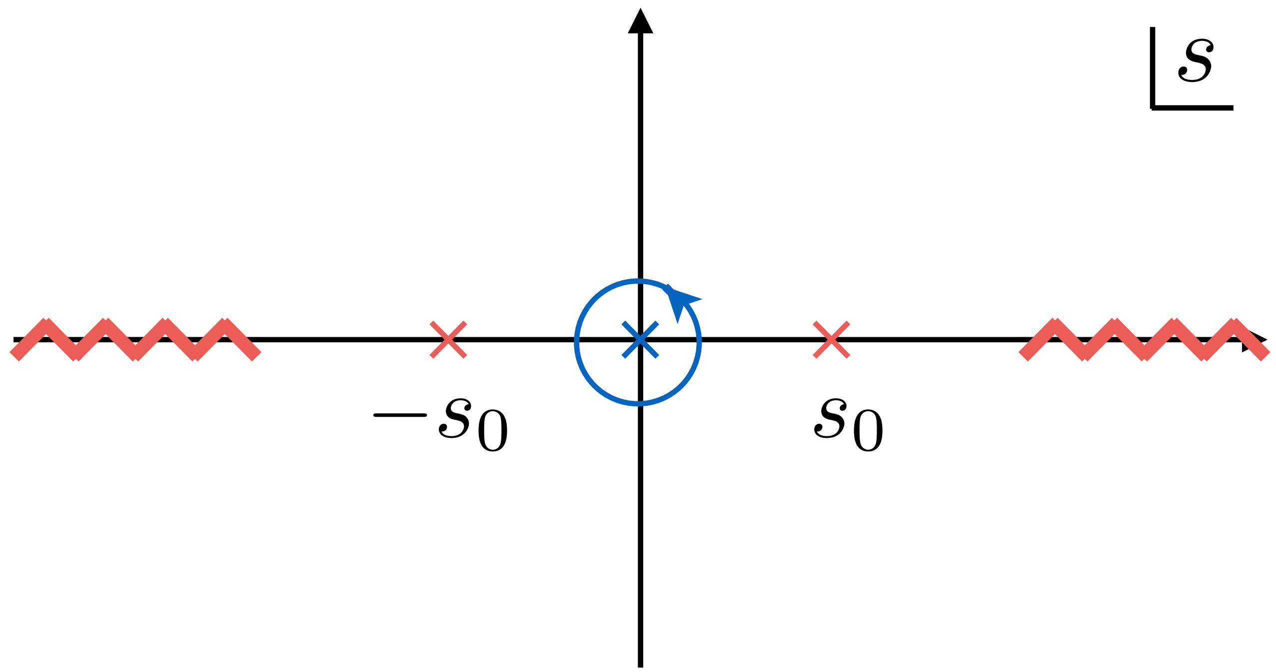
ex. a shift symmetric scalar π coupled to heavy particles

- info of heavy particles are encoded into α and higher orders
- unitarity and analyticity of scattering amplitudes imply $\alpha > 0$

[see the next slide for derivation]

Positivity Bounds (w/o gravity) [Adams et al '06]

Consider $\pi\pi \to \pi\pi$ scattering in the forward limit



analyticity of forward amplitude $M(s)$

IR expansion of the amplitude:

$$M(s) = \sum_{n=1}^{\infty} a_{2n}\, s^{2n}$$

- we show that $a_n > 0$
- in particular, $a_2 = \dfrac{4\alpha}{\Lambda^4} > 0$

a_n can be evaluated as follows:

$$a_{2n} = \oint_{C_0} \frac{ds}{2\pi i} \frac{M(s)}{s^{2n+1}}$$

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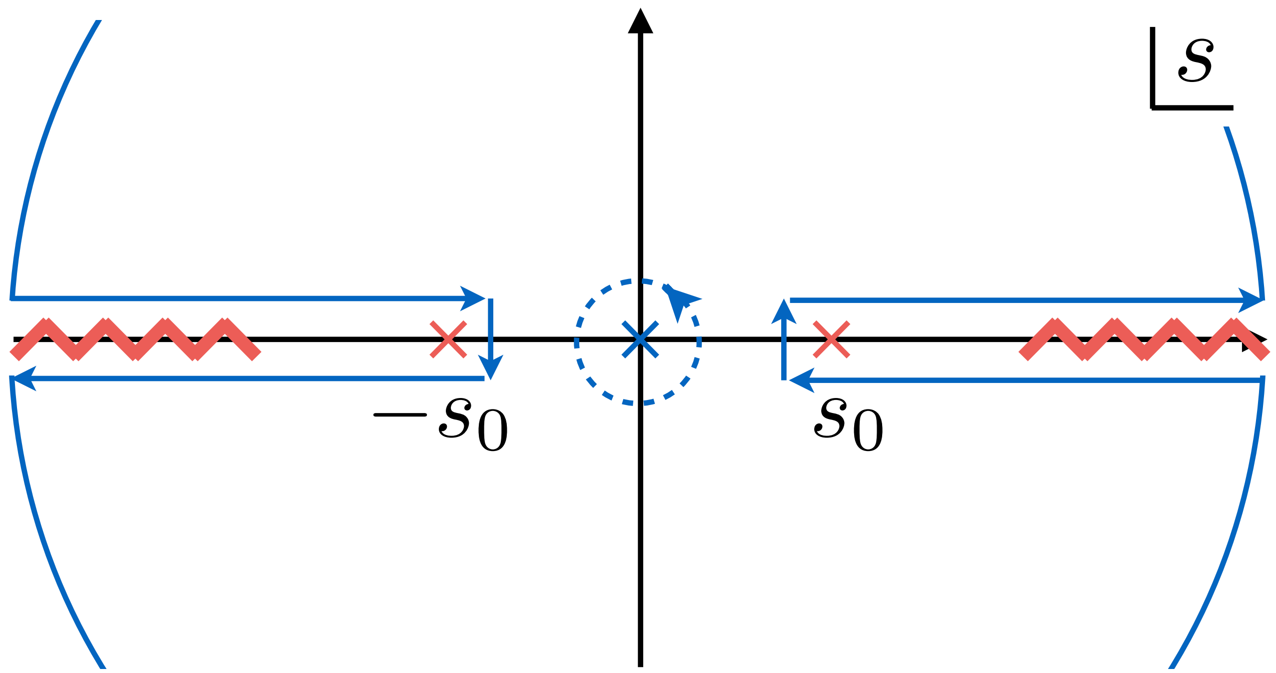

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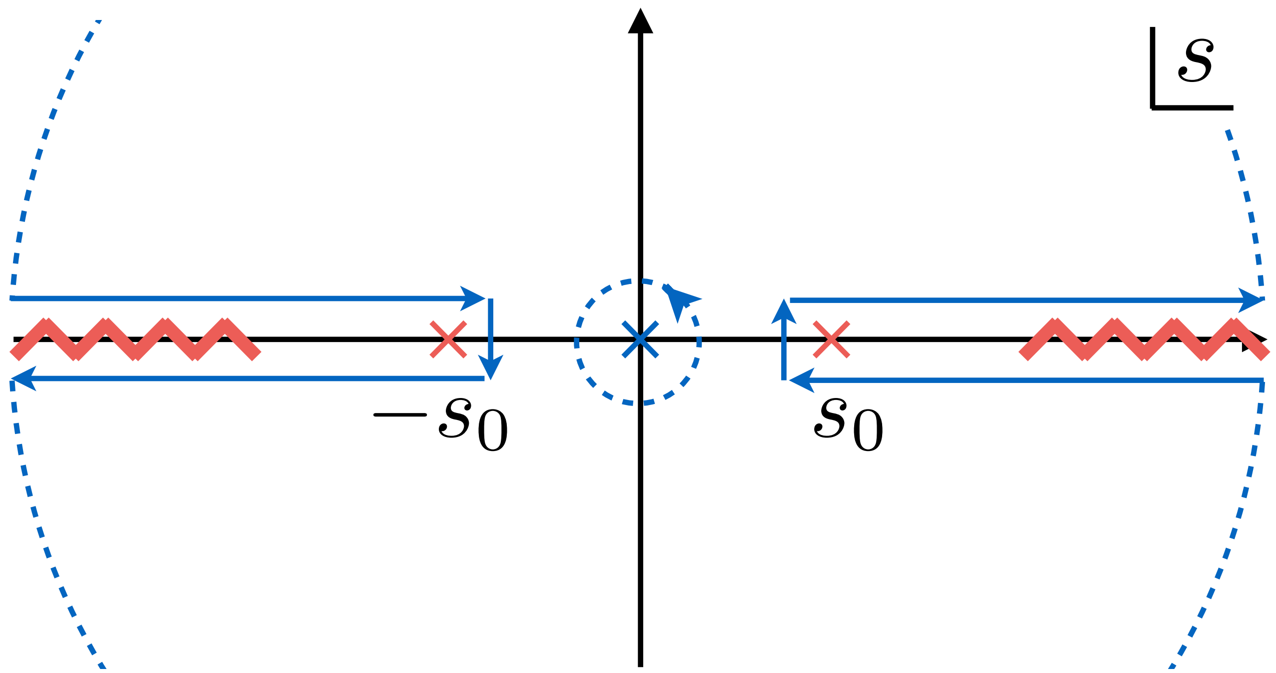

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optical theorem

$$a_{2n} = \oint_{C_0} \frac{ds}{2\pi i}\frac{M(s)}{s^{2n+1}} = \cancel{\oint_{C_\infty} \frac{ds}{2\pi i}\frac{M(s)}{s^{2n+1}}} + \frac{2}{\pi}\int_{s_0}^{\infty} \frac{ds}{s^{2n+1}} \mathrm{Im}\, M(s) > 0$$

- assumed $|M(s)| < s^2$ for $|s| \to \infty$ (cf. Froissart bound)

In this way,

unitarity & analyticity of UV theory imply $\alpha > 0$ in IR EFT

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quantum gravity

QFT models

Gravitational Positivity Bounds

forward limit of $\pi\pi \to \pi\pi$ scattering in the presence of gravity

IR expansion: $$M(s) = -\frac{2s^2}{M_{\rm Pl}^2 t} + \sum_{n=1}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t)$$

- the first term is from t-channel graviton exchange
- $a_2 > 0$ does not follow from the previous argument anymore

approximate positivity [Hamada-TN-Shiu '18, Tokuda-Aoki-Hirano '20, ⋯]

- intuitively, positivity should hold if gravity is subdominant
- if we assume weakly coupled UV completion of gravity,

$$a_2 > \mathcal{O}(1) \cdot \frac{1}{M_{\rm Pl}^2 M_s^2}$$ (M_s: mass of higher spin Regge states)

- sign and value of $\mathcal{O}(1)$ depend on details of Regge trajectories

[see Tokuda-Aoki-Hirano '20 for details]

Gravitational positivity bound:

for weakly coupled UV completion of gravity,

$a_2 > \mathcal{O}(1) \cdot \dfrac{1}{M_{\rm Pl}^2 M_s^2}$ (M_s: mass of higher spin Regge states)

IR expansion: $M(s) = -\dfrac{2s^2}{M_{\rm Pl}^2 t} + \sum_{n=1}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t)$

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Strategy

1. Suppose that we have a UV complete QFT model w/o gravity

2. If we couple it to gravity, the model is no more UV complete

3. We can ask if the model w/gravity is UV completable

4. If we assume weakly coupled UV completion,

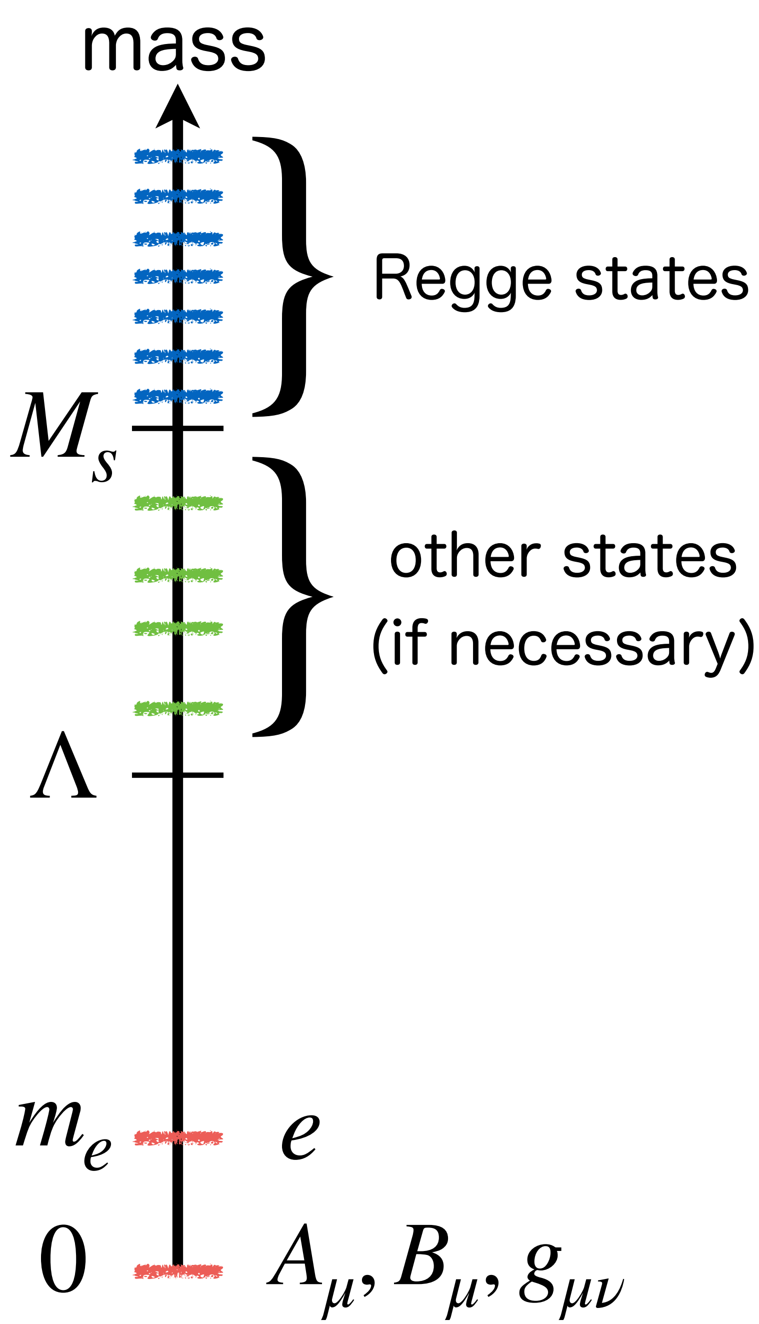
   we can use gravitational positivity as a criterion

   → provides swampland conditions

application 1: QED and dark photon

[Andriolo-Junghans-TN-Shiu '18 + a bit more]

A Toy Model for Dark Physics



<u>our world (QED)</u>

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu}^2 - \bar{\psi}(\not{D} + m)\psi \right]$$

gravity: $S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R + \cdots \right]$

<u>dark sector (pure Maxwell)</u>

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} H_{\mu\nu}^2 \right] \quad (H = dB)$$

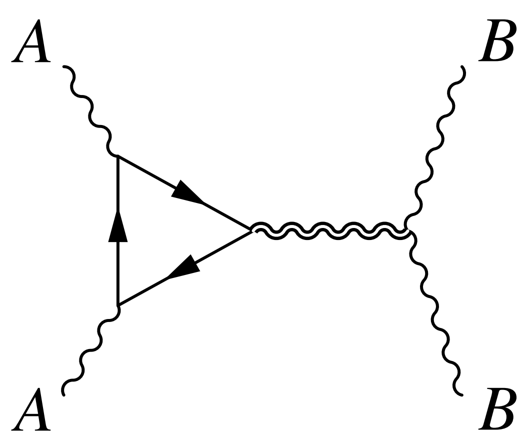
Q. Is this UV completable? Consistent w/gravitational positivity?

Gravitational Positivity

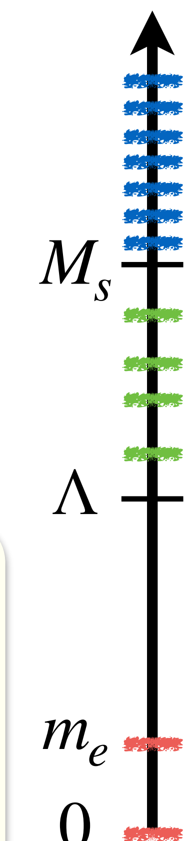
\# positivity of $AB \to AB$ scattering in the presence of gravity

$$a_2 = -\frac{11}{2880\pi^2}\frac{e^2}{m_e^2 M_{\rm Pl}^2} + \frac{\alpha}{\Lambda^4} > \mathcal{O}(1)\cdot\frac{1}{M_{\rm Pl}^2 M_s^2}$$

(α : coupling of heavy states)



negative contribution!



- If we decouple gravity, we can safely take the limit $\Lambda \to \infty$

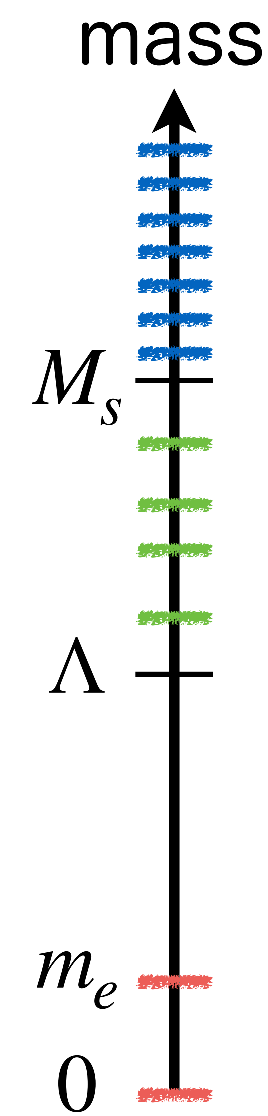
  (recall that the original QFT model w/o gravity is UV complete)

- If there are no heavy states other than gravitational Regge states,

  a natural order estimate is $\dfrac{\alpha}{\Lambda^4} \sim \dfrac{1}{M_{\rm Pl}^2 M_s^2}$

  $\to$ the model is in the swampland unless $M_s \lesssim \dfrac{m_e}{e}$!!!

How to get out of the swampland?



\# option 1: turn on tiny electron-dark photon coupling $\tilde{e}$

$$a_2 \simeq \frac{11}{2880\pi^2}\frac{e^2}{m_e^2 M_{\rm Pl}^2}\left(\frac{2\,\tilde{e}^2 M_{\rm Pl}^2}{m_e^2} - 1\right) > \mathcal{O}(1)\cdot\frac{1}{M_{\rm Pl}^2 M_s^2}$$

$\rightarrow \tilde{e}^2 > \dfrac{m_e^2}{2M_{\rm Pl}^2}$: hidden electric force > gravity (weak gravity)

\# option 2: introduce heavy states mediating two sectors

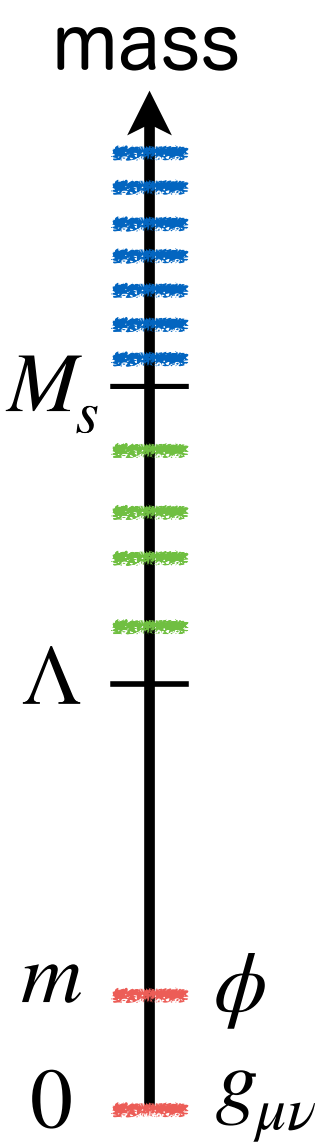
s.t. $\dfrac{\alpha}{\Lambda^4} > \dfrac{11}{2880\pi^2}\dfrac{e^2}{m_e^2 M_{\rm Pl}^2} \rightarrow \Lambda \lesssim \alpha^{1/4}\sqrt{\dfrac{m_e M_{\rm Pl}}{e}}$ (α : coupling)

- positivity requires non-gravitational interactions mediating the two sectors
- more comprehensive study is given in [Aoki-TN-Tokuda-Tran in progress] especially in the connection with Tower Weak Gravity Conjecture

application 2: bounds on scalar potential

[TN-Tokuda to appear]

Bounds on scalar potential



$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + \cdots\right]$$

in this talk, I impose the Z_2 symmetry $\phi \to -\phi$ for simplicity

[see our paper for more general cases]

the s^2 coefficient at 1 loop

$$a_2 = \frac{\lambda^2}{16\pi^2 m^4} - \frac{\lambda}{24\pi^2 M_{\rm Pl}^2 m^2} + \frac{\alpha}{\Lambda^4}$$ (α : coupling of ϕ & heavy states)

trivially satisfies the gravitational bound

a stronger bound

2 loop

$$\tilde{a}_2 \simeq \frac{\lambda^2}{16\pi^2\Lambda^4} - \frac{10-\pi^2}{4608\pi^4}\frac{\lambda^2}{M_{\rm Pl}^2 m^2} + \frac{\alpha}{\Lambda^4} > \mathcal{O}(1)\cdot\frac{1}{M_{\rm Pl}^2 M_s^2}$$

using the assumption that heavy states are above Λ

$$\tilde{a}_2 \simeq \frac{\lambda^2}{16\pi^2\Lambda^4} - \frac{10-\pi^2}{4608\pi^4}\frac{\lambda^2}{M_{\rm Pl}^2 m^2} + \frac{\alpha}{\Lambda^4} > \mathcal{O}(1)\cdot\frac{1}{M_{\rm Pl}^2 M_s^2}$$

when r.h.s. is subdominant,

we obtain a lower bound $m > \frac{1}{150}\frac{\Lambda^2}{M_{\rm Pl}} \times \sqrt{\frac{\lambda^2}{\lambda^2 + 16\pi^2\alpha}}$.

Therefore, we cannot set the mass arbitrary small,

even though it is allowed in QFT at least if we allow fine-tuning

Summary and Prospects

Summary and Prospects

1. Swampland program
   - quantum gravity constraints on QFT models

2. Gravitational positivity bounds
   - consistency of gravitational scattering
   - in weakly coupled UV completion,

$$M(s) = -\frac{2s^2}{M_{\mathrm{Pl}}^2 t} + \sum_{n=1}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t) \text{ (IR expansion)}$$

$$a_2 > \mathcal{O}(1) \cdot \frac{1}{M_{\mathrm{Pl}}^2 M_s^2} \text{ (} M_s \text{ : mass of higher spin Regge states)}$$

Q. Can we constrain the $\mathcal{O}(1)$ factor further?

- it is non-negative in know string theory examples
- bounds from holography, energy conditions etc?

Summary and Prospects

3. Exploring swampland w/gravitational positivity

- QED and dark photon

  positivity requires non-gravitational coupling to hidden sector

  $\rightarrow$ more realistic models? implications for DM models?

- bound on scalar potential

  positivity implies that mass cannot be set arbitrary small

  $\rightarrow$ implications for SM, inflation, dark energy, neutrino etc?

- other phenomenological applications?

Thank you!