Recent progress in theoretical physics based on quantum information theory
@YITP March 2, 2021

Holographic domain walls, multi boundary traversable wormholes and Replica wormholes

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Based on arXiv:2011.12962 + work in progress

Introduction

Recently the page curve of evaporating black hole is reproduced from semiclassical gravity computation:

[Almheiri-Engerhardt-Marolf-Maxfield, 19]
[Penington, 19]
[Almheiri-Mahajan-Maldacena-Zhao, 19]

Spacetime wormholes play important roles

[Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini, 19]
[Penington-Stanford-Shenker-Yang, 19]

One of important aspects:
Application of RT/HRT/EW formula for "holographic states"
in non-gravitational system, or entangled with gravitational system

[Harlow, 16] [Hayden-Penington, 18]
[AMMZ, 19]

Today we consider holographic state in free field (or Ising models), based on bra-ket wormholes obtained from traversable wormholes.

Using JT gravity + matter system as domain walls between holographic CFTs and free CFTs.

Traversable wormholes in JT gravity + matter system

Traversable wormhole solution:

ansatz:

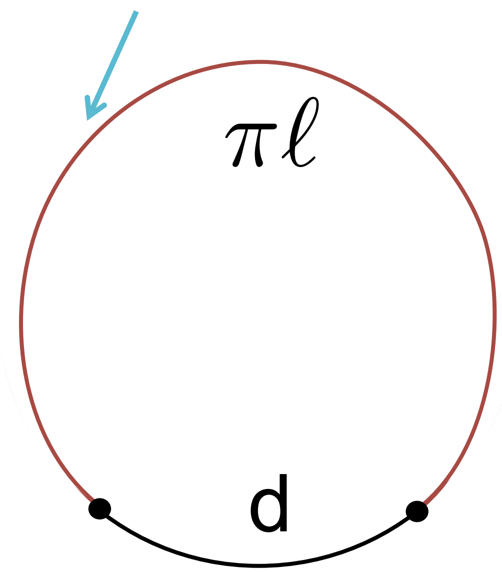
$$ds^2 = \frac{-dt^2 + d\sigma^2}{\ell^2 \sin^2 \frac{\sigma}{\ell}}$$

$$\phi(\sigma) = \frac{2\bar{\phi}_r}{\pi \ell}\left[\frac{\frac{\pi}{2} - \sigma}{\tan \frac{\sigma}{\ell}} + 1\right]$$

$$\langle T^{mat}_{++} \rangle = \frac{c}{48\pi\ell^2} - \frac{\pi^2 c}{12(\pi\ell + d)^2}$$

2d Gravity

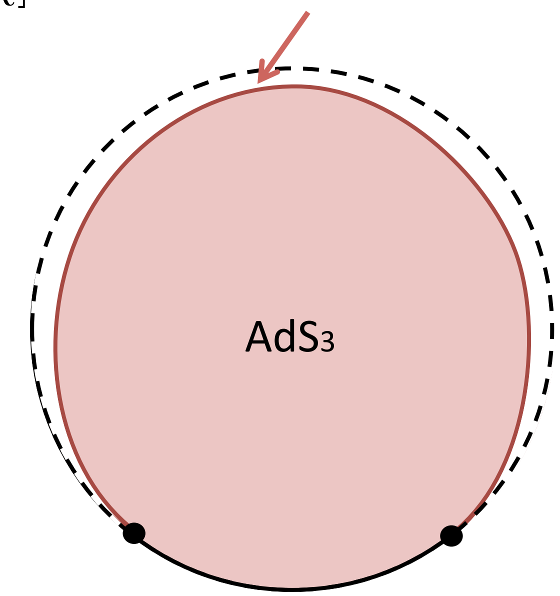
$I_{\rm grav}[g^{(2)}_{ij}, \phi] + I_{\rm CFT}[g^{(2)}_{ij}, \chi]$



CFT2

3d Gravity

Planck brane

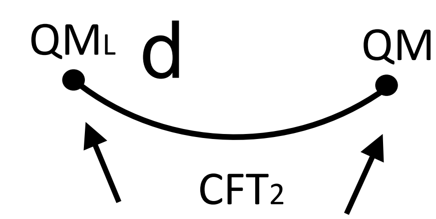


AdS3 Boundary

QM

Ground state in coupled system

$|G\rangle$



CFT with Holographic boundary

ℓ : "wormhole length", dynamically determined by EOM

Alternatively: use variational method (approximate by TFD)

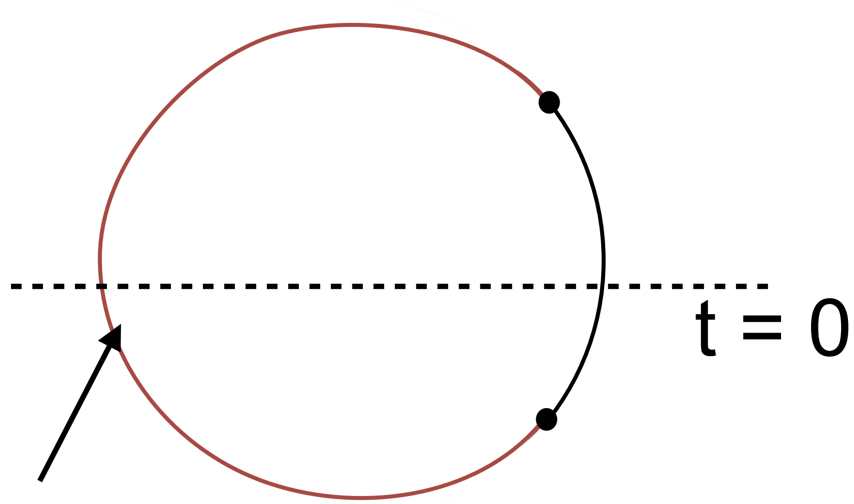
$$E(\ell) = 2 \times \underbrace{\frac{\pi \bar{\phi}_r}{4G_N} T_H^2}_{\text{BH mass}} + \underbrace{\frac{c}{24\ell}}_{\text{Weyl anomaly}} - \underbrace{\frac{c\pi}{6(\pi\ell + d)}}_{\text{Casimir energy}}, \qquad \ell = \frac{1}{2\pi T_H}$$

minimize variational energy gives the same ℓ

Bra-ket wormhole interpretation

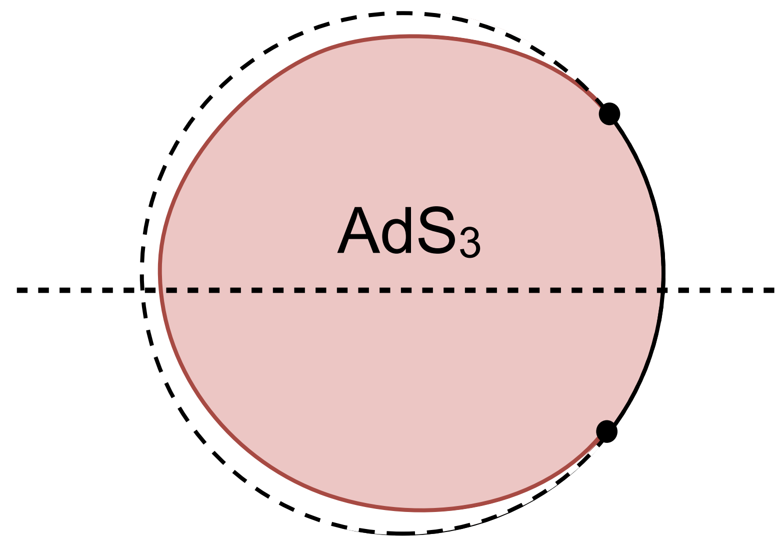
・start from traversable wormhole, exchange (Euclidean) time and space

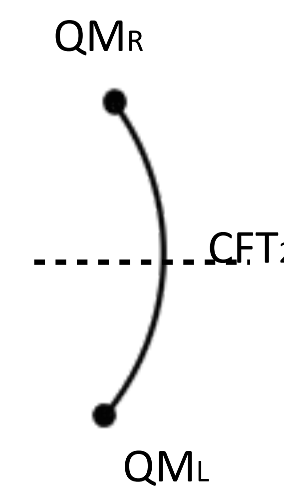
2d gravity | 3d gravity | QM



wormhole connects bra and ket

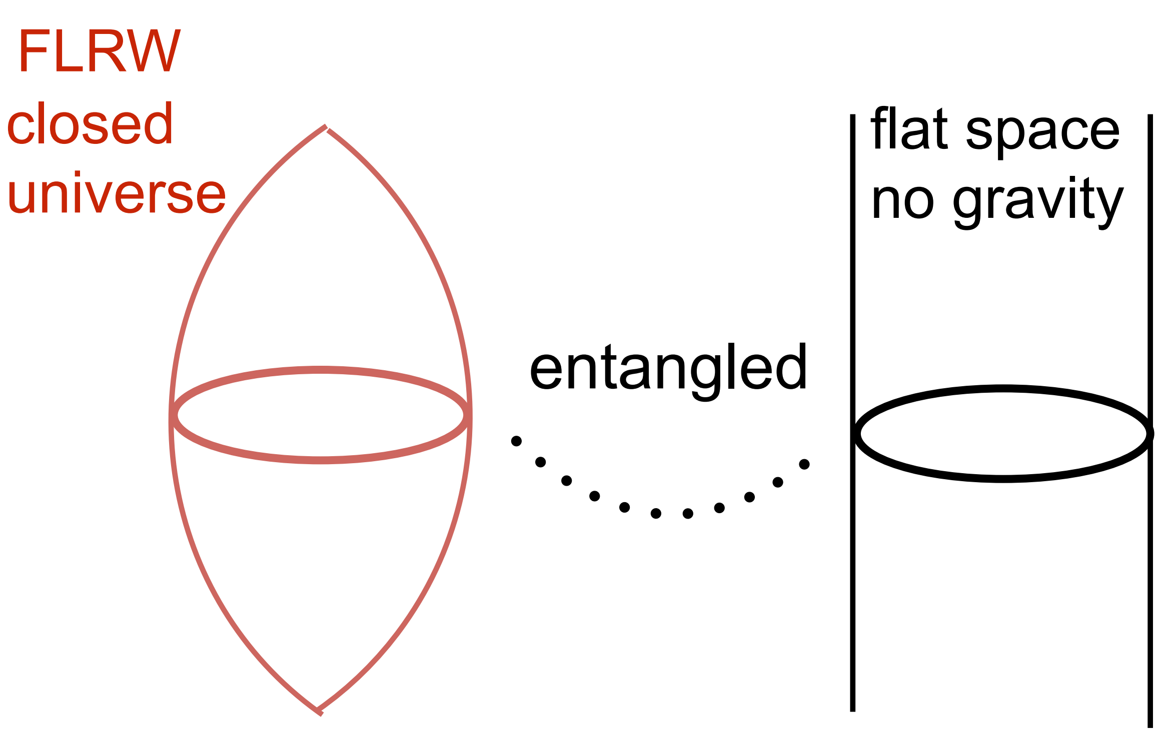
[cf: Page, 86] [Chen-Gorbenko-Maldacena, 20]



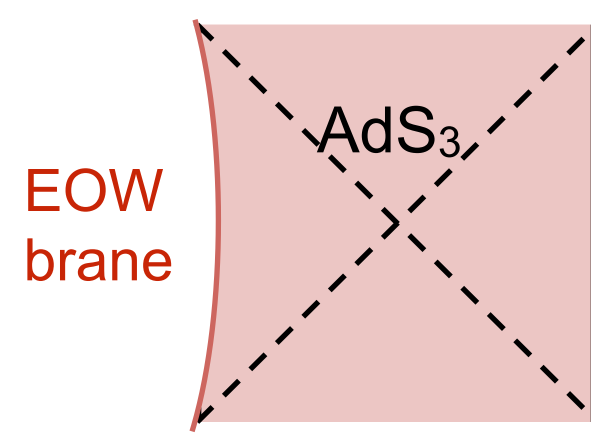


$|\Psi\rangle \in \mathcal{H}_{CFT}$

・wick rotation $\tau = it$ in 2d gravity description



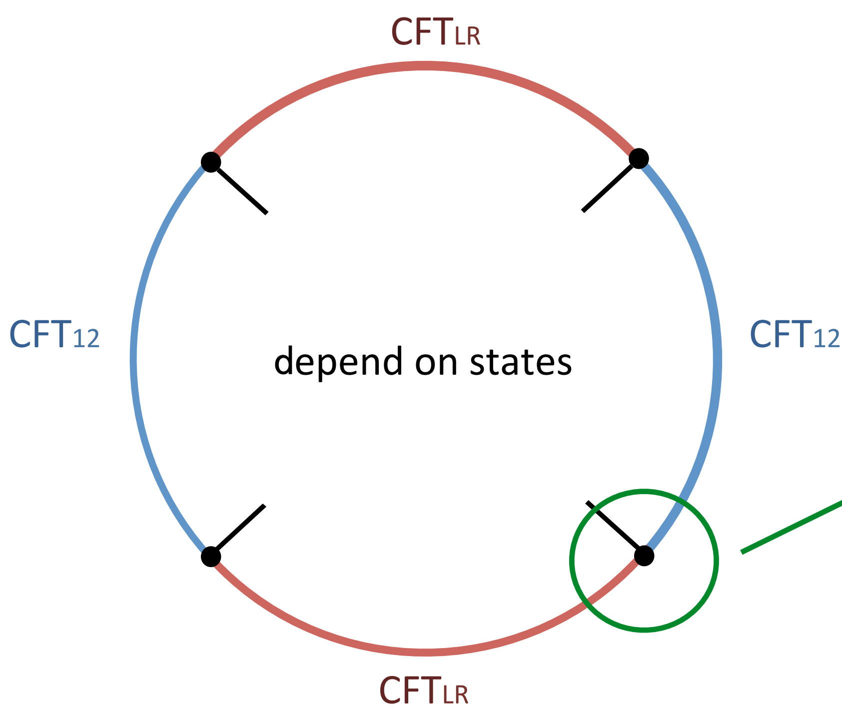
3d gravity description

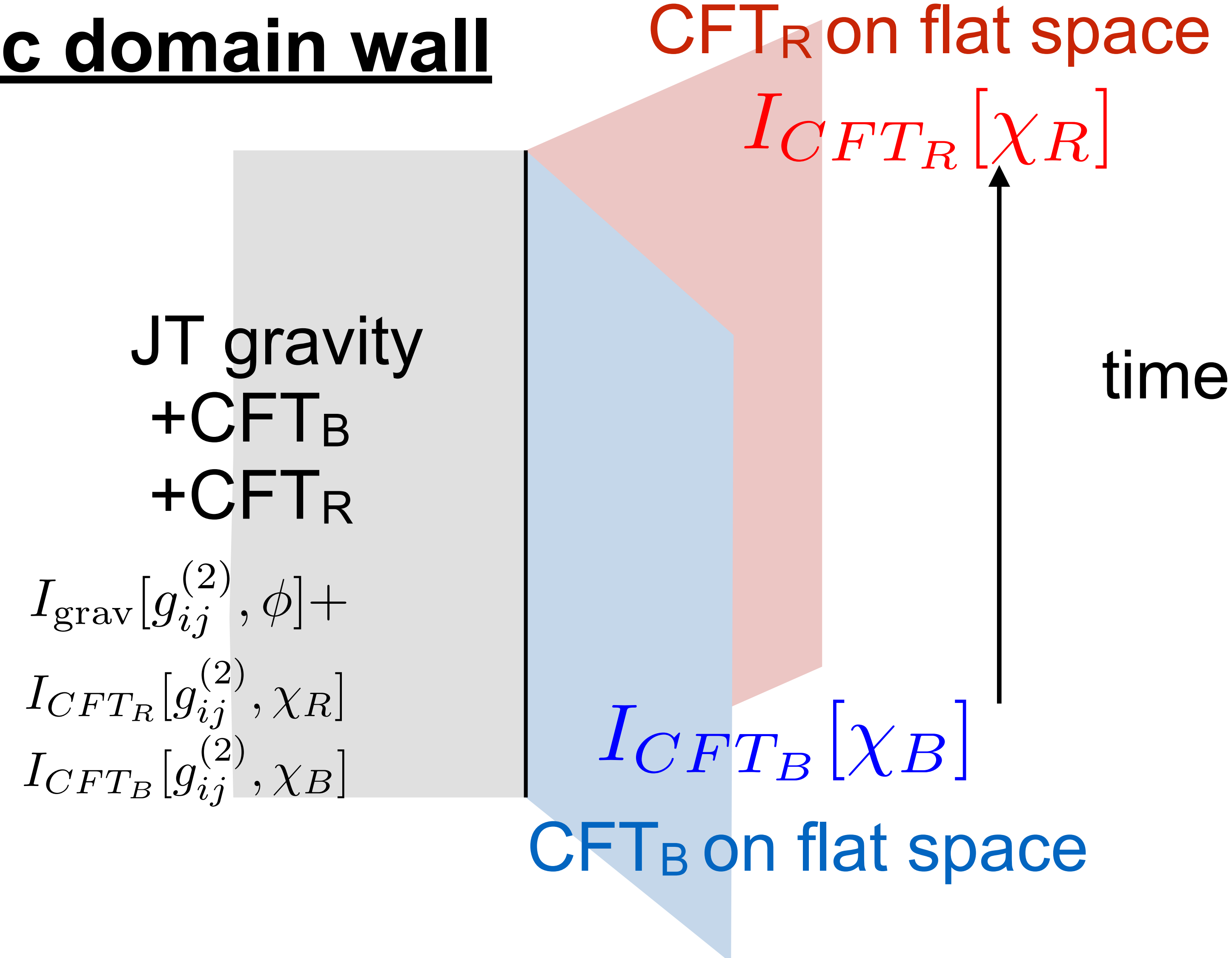


[Gubser , 99]

[cf:Cooper-Rozali-Swingle-Raamsdonk-Waddell-Wakeham, 18]

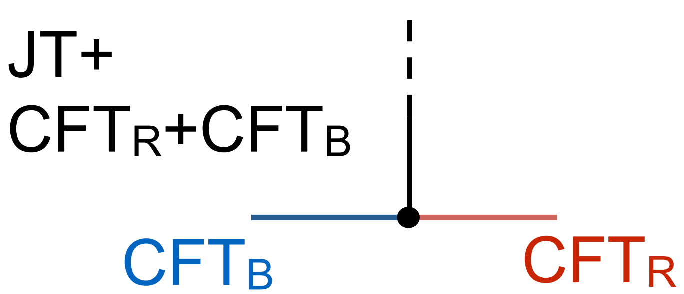
Generalization to Holographic domain wall



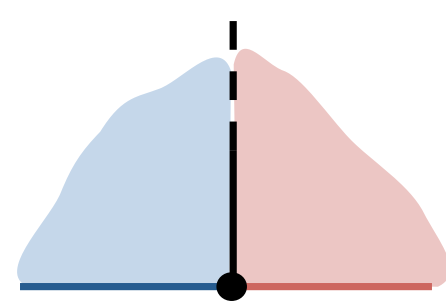


time slice

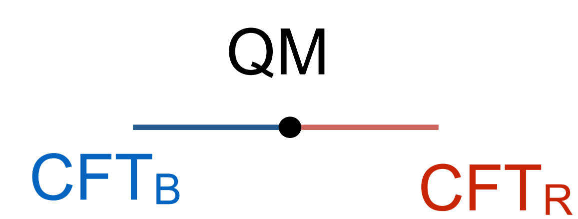
2d gravity



3d gravity



QM



- two CFTs are joined by domain walls with holographic duals

[cf: Chen-Myers-Neuenfeld-Reyes-Sandor ,20]

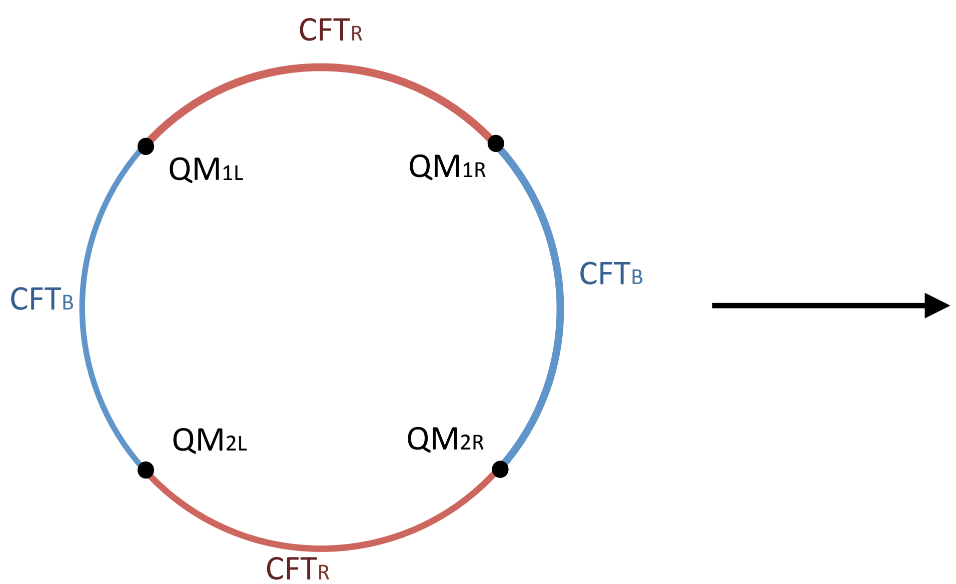
[Ooguri-Takayanagi, 20]

2 traversable Wormhole solution with Holographic matters:

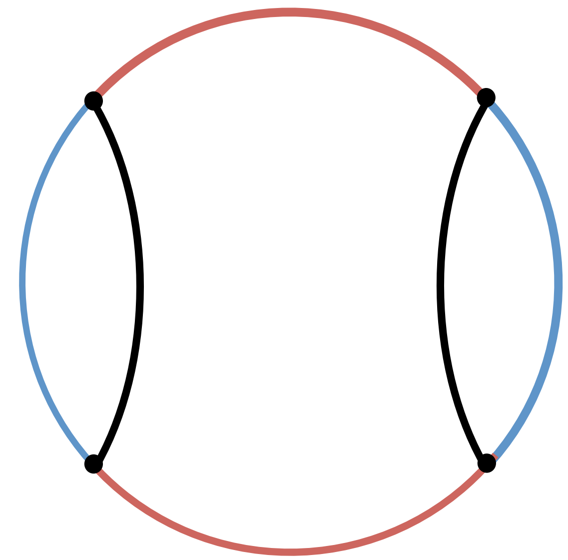
- start from four domain walls

QM

(time slice)

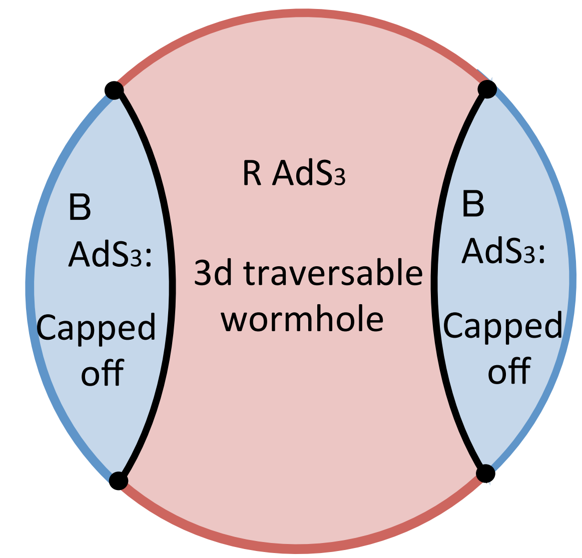


2d gravity



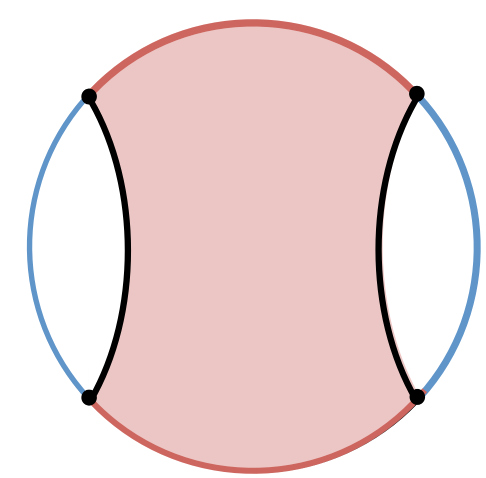
3d gravity

Both of $\mathrm{CFT_B}$ and $\mathrm{CFT_R}$ are holographic



dominant for $c_R < c_B$

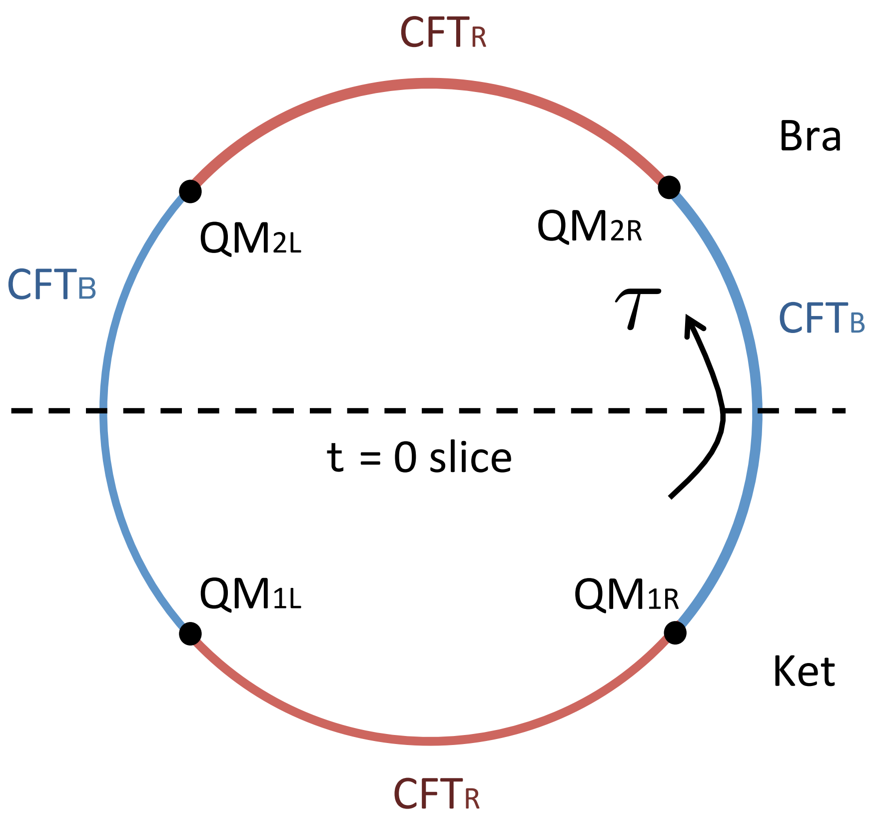
JT Gravity + Holographic CFT + other CFT
is similar to Randall-Sundram II
(4d gravity + 5d gravity (= 4d holographic CFT)
+ Standard model)

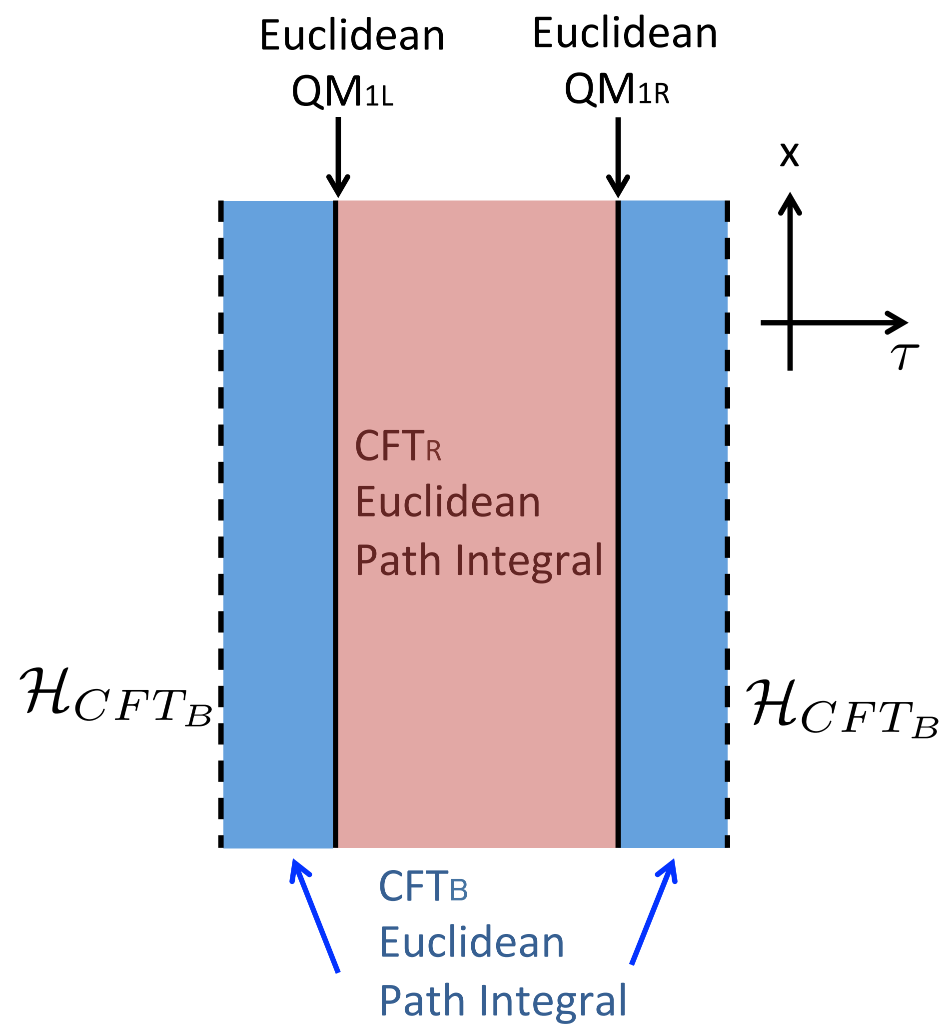


Only $\mathrm{CFT_R}$ is holographic

Wick rotation:

QM description



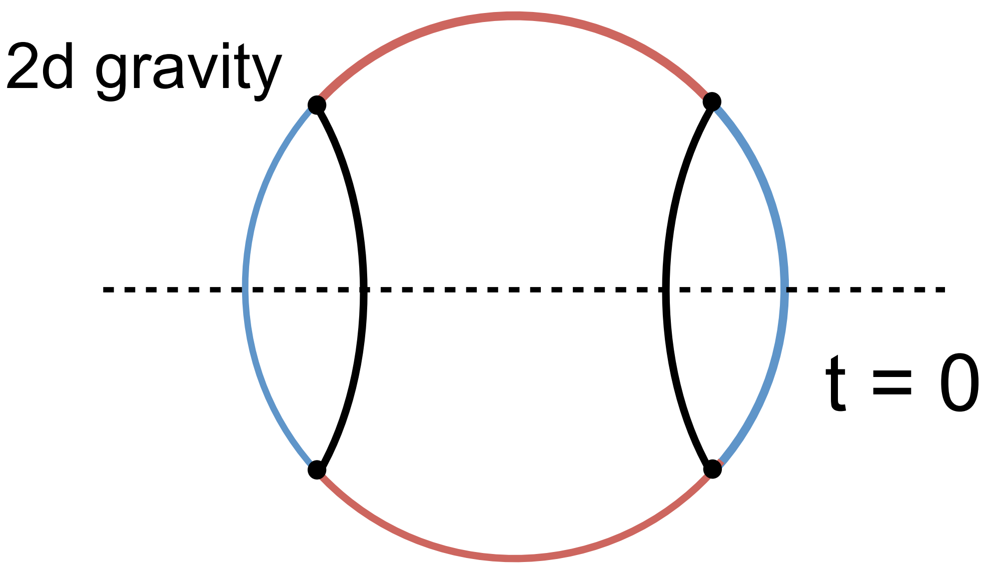


The path integral prepare a state $|\Psi\rangle \in \mathcal{H}_{CFT_B} \otimes \mathcal{H}_{CFT_B}$

- We can also add bulk particles by inserting some operators in CFT_R
- Similar to problem in quantum quenches in QFT [cf:Calabrese-Cardy, 05]

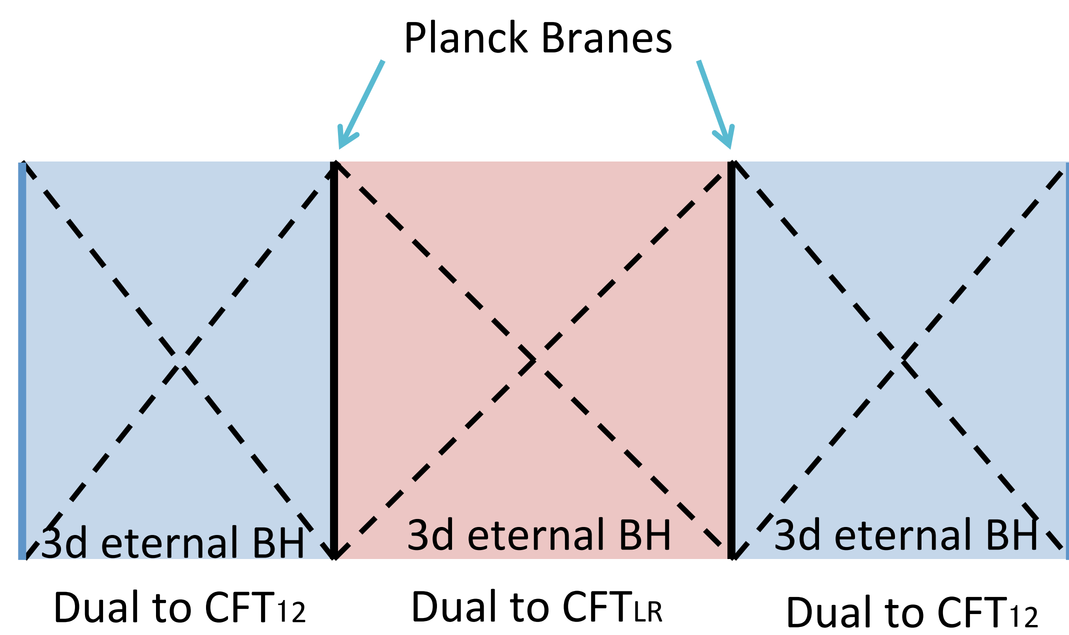
Lorentzian time continuation

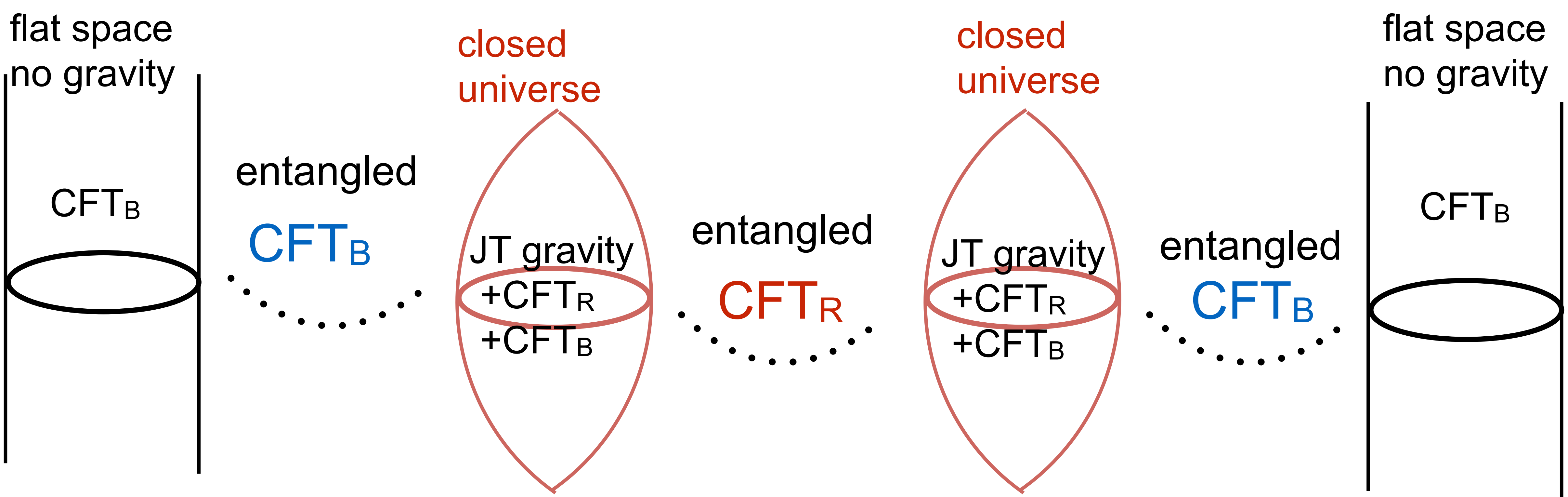
bra-ket wormhole state



dominant saddle for $c_R < c_B$

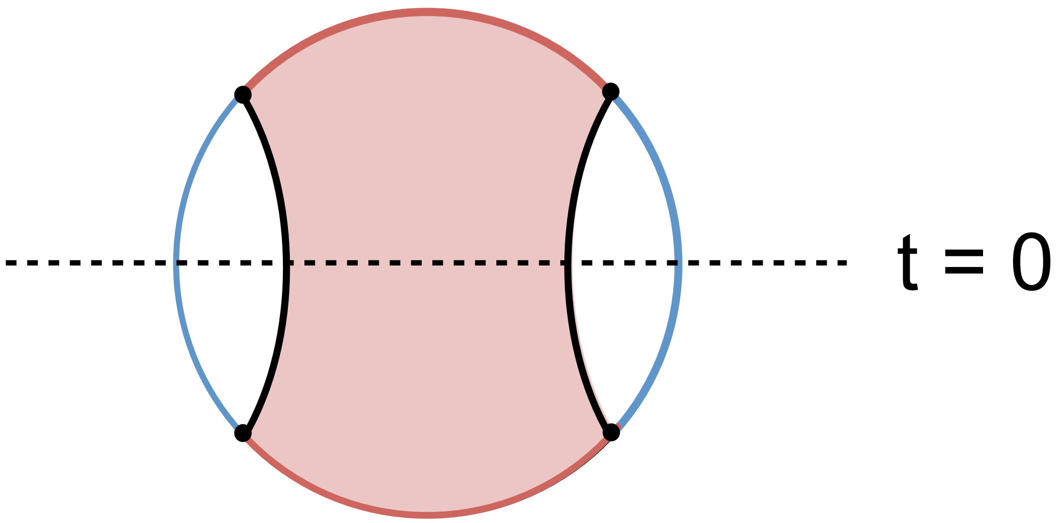
3d Gravity

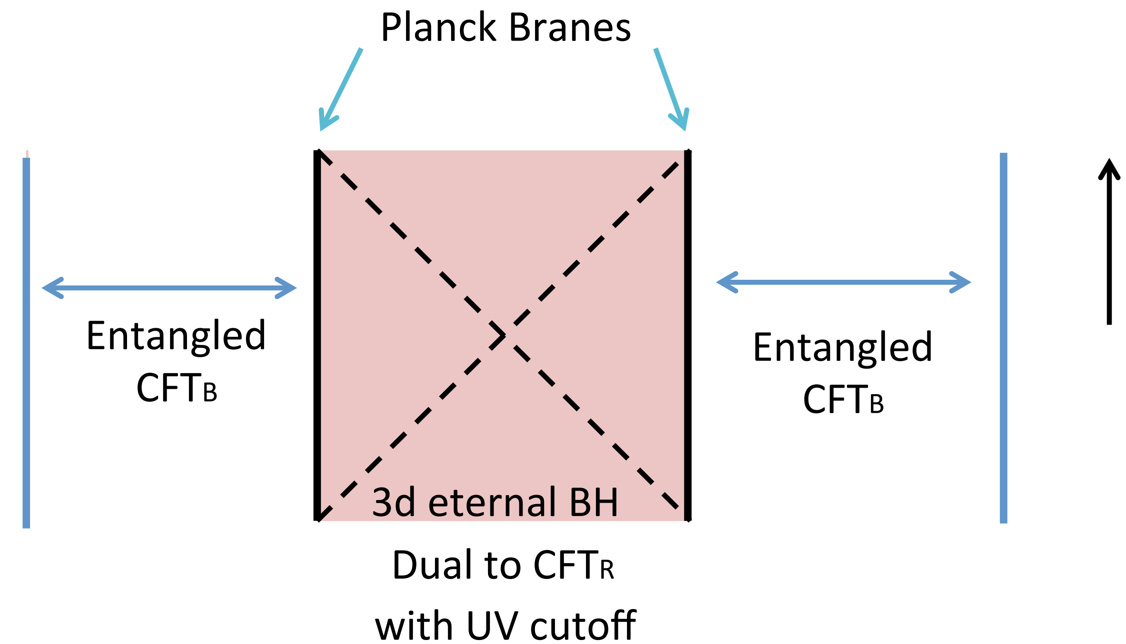




Embed holographic states to free field Hilbert spaces:

"partially doubly holographic"





[cf:"Holo-ween" Simidzija-Raamsdonk, 20]

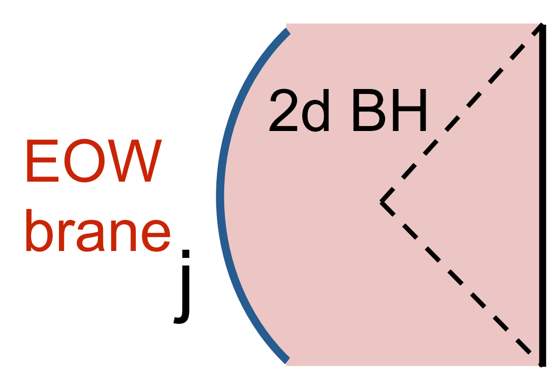
states in $\mathcal{H}_{CFT_B} \otimes \mathcal{H}_{CFT_B}$

- Take $\mathrm{CFT_B}$ to be bunch of free fields (or Ising CFTs)

  $\rightarrow$ realize holographic states in free fields Hilbert sp.

cf: JT gravity with auxiliary system

[Penington-Stanford-Shenker-Yang 19]

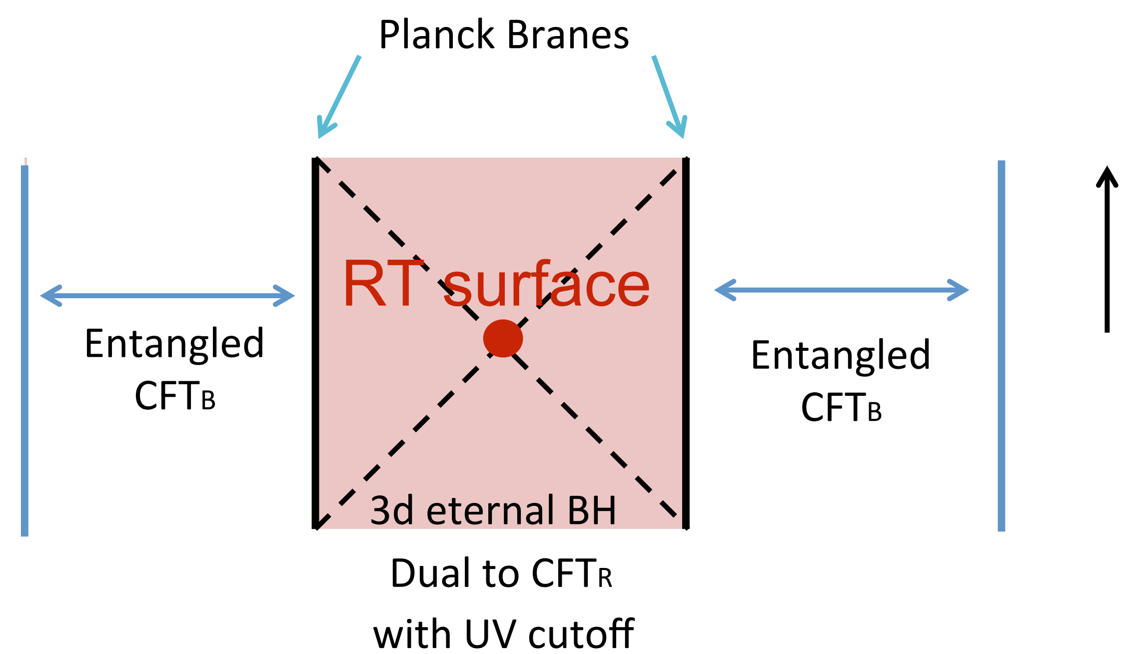


$= |\psi_j\rangle_B$

$$\rho_R \rightarrow |\rho_R\rangle = \sum \langle\psi_i|\psi_j\rangle_B \, |i\rangle_R \otimes |j\rangle_R \in \mathcal{H}_{aux} \otimes \mathcal{H}_{aux}$$

Entanglement entropy and Replica wormholes:

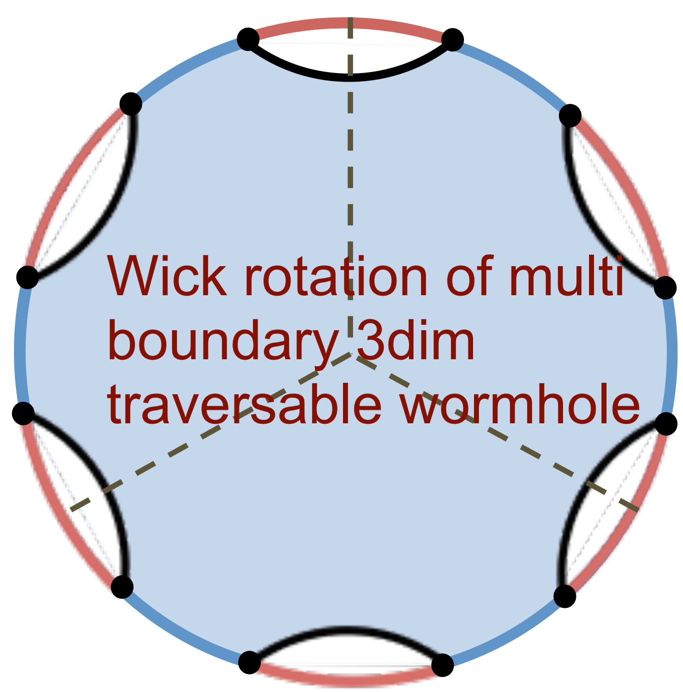
Entanglement entropy between two side is calculated using RT formula for states in non-holographic CFTs



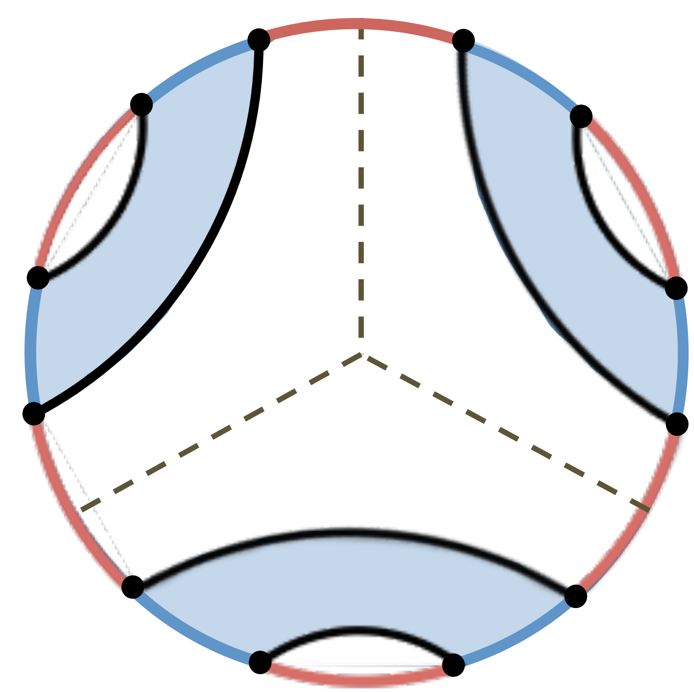
Replica wormhole justify this calculation

CFTs with 4n domain walls

n=3 Renyi Entropy



Replica wormhole



No wormhole

Lewkowycz-Maldacena derivation and holographic Renyi entropy:

[cf:Lewkowycz-Maldacena, 13]

Partition function: $Z_n \approx e^{-E_n L}$ for large L

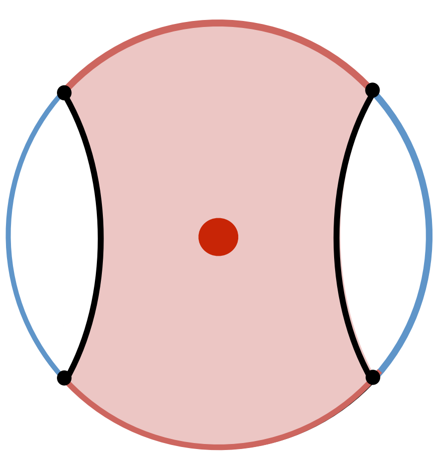
$\rightarrow$ Calculation reduces to that of energy in traversable wormholes !

$$E_n = E(\ell_n, n) \qquad \left.\frac{\partial E(\ell, n)}{\partial \ell}\right|_{\ell=\ell_n} = 0$$

$$E(\ell, n) = n\left(\frac{\bar{\phi}_r}{4\pi G_N \ell^2} + \frac{c_R + c_B}{12\ell} - \frac{c_R \pi}{3(\pi\ell + d_R)} - \frac{c_B \pi}{12 n^2 (\pi\ell + d_B)}\right)$$

($\ell_n = \frac{6\bar{\phi}_r}{\pi G_N} \frac{1}{3c_R - (1 - \frac{1}{n^2})c_B}$ for $d_R, d_B \ll \ell$)

$$E(\ell_n, n) - nE(\ell_1, 1) = \underbrace{[E(\ell_n, n) - E(\ell_1, n)]}_{O((n-1)^2)} + \underbrace{[E(\ell_1, n) - nE(\ell_1, 1)]}_{\frac{c_B}{24}\left(n - \frac{1}{n}\right)\frac{\pi}{\pi\ell_1 + d_B}}$$



= dimension of twist operator !

Lewkowycz-Maldacena derivation and holographic Renyi entropy:

Energy par a Replica: $\hat{E}(\ell, n) = \dfrac{E(\ell, n)}{n}$

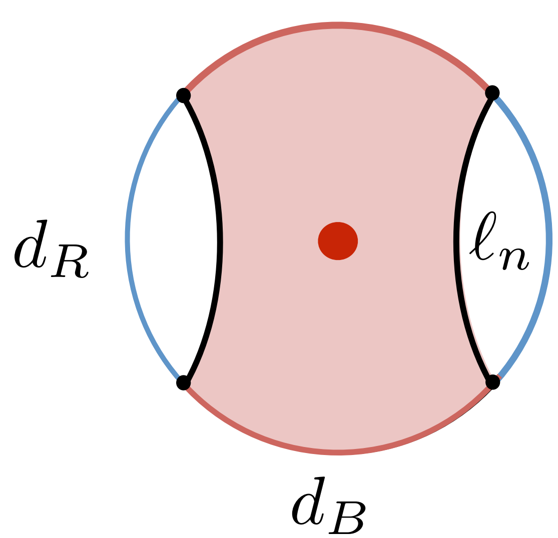
$$n^2 \frac{d}{dn} \hat{E}(\ell(n), n) = n^2 \underbrace{\frac{\partial \hat{E}}{\partial \ell}}_{=0} \frac{d\ell_n}{dn} + n^2 \underbrace{\frac{\partial \hat{E}}{\partial n}}_{= \frac{c_B}{24} \frac{1}{\pi \ell_n + d_B} \frac{\partial}{\partial n}\left(1 - \frac{1}{n^2}\right)}$$

by saddle eq.

:derivative of twist op dim

$$= \frac{c_B}{12} \frac{1}{n(\pi \ell_n + d_B)}$$

→ Similar result w/ holographic Renyi Entropy [Dong, 16] holds.



Analysis of other saddles are still work in progress…

Conclusion/Future works

- We discussed relations between traversable WH and bra-ket WH.

- Using JT gravity + matters as holographic defects on CFTs. They can be used to embed holographic 2d CFT state into Free fields Hilbert spaces.

- Even in free CFTs, entanglement entropy is calculated by RT formula for holographic states

- These are justified by Replica wormholes. Replica wormholes in this situation are wick rotation of traversable wormholes.

- It is interesting to consider other entanglement measures in this setup

- Tensor networks/ de Sitter version etc…