Entanglement transitions in random tensor networks and monitored quantum circuits

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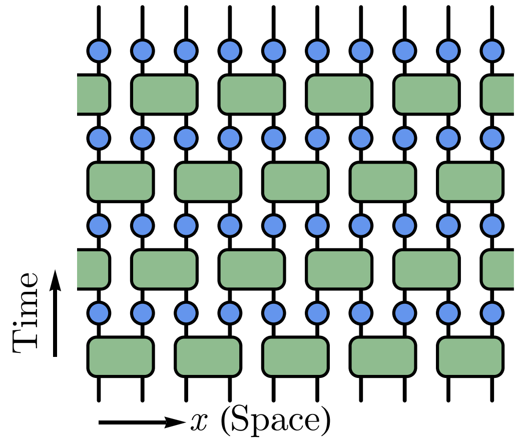
(UMass Amherst)

YITP workshop



RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig, 1807.07082

C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig, 1908.08051

J. Lopez-Piqueres, Brayden Ware & RV, 2003.01138

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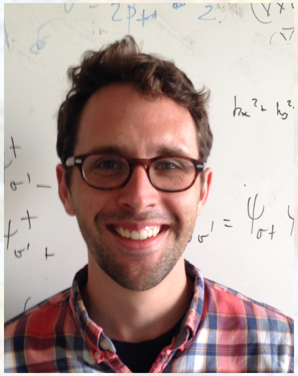
Collaborators



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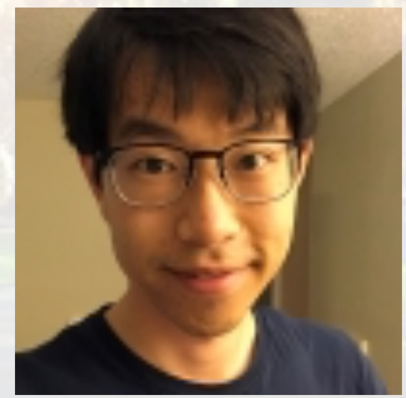

C.-M. Jian
Station Q—> Cornell

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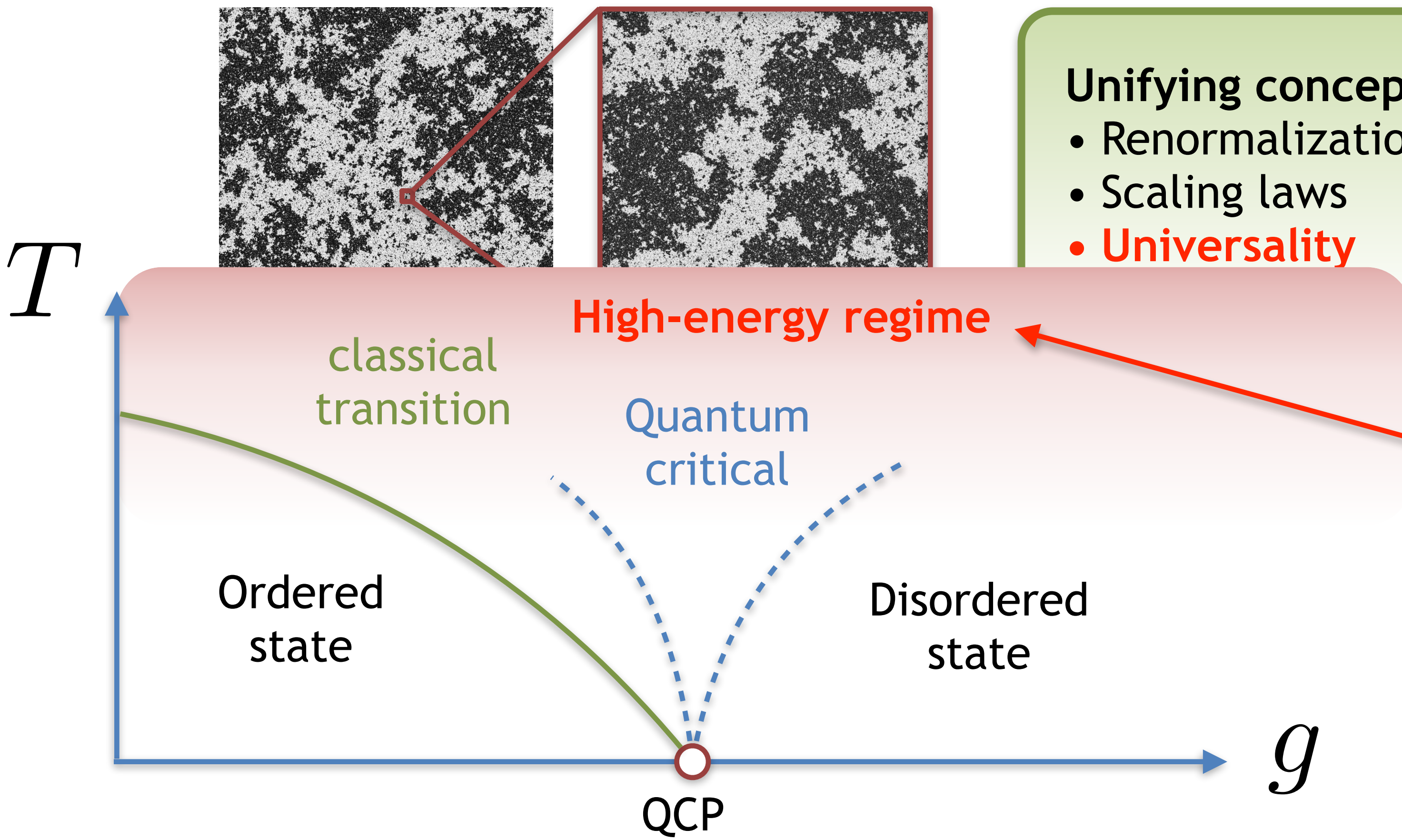
Universality at high energy, far from equilibrium?

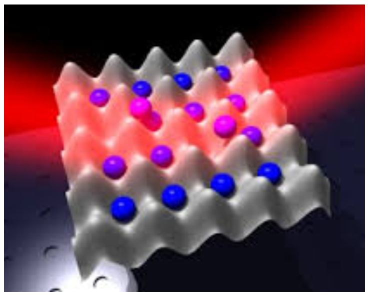
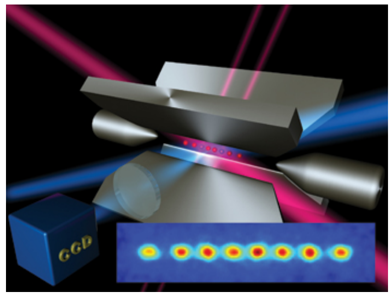
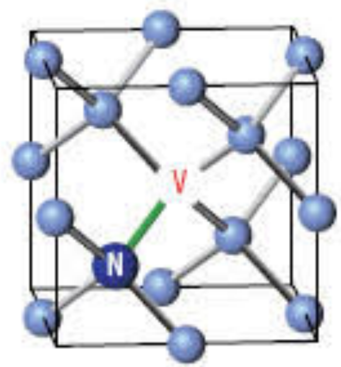
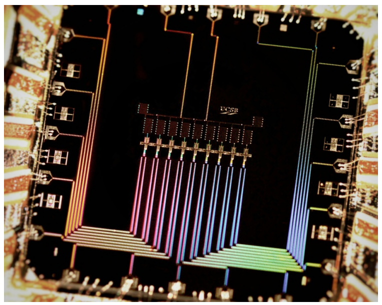


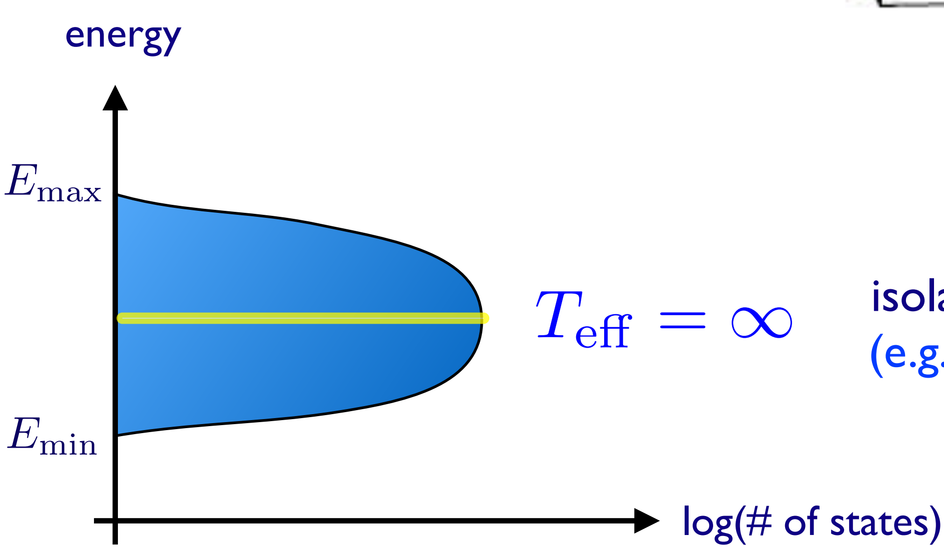
Unifying concepts:
- Renormalization group
- Scaling laws
- Universality

rgy/low

Classical?
Non Universal?

Dynamics of isolated systems: More exotic alternatives?

Ultracold atoms	Trapped Ions	NV Centers	Superconducting Qubits
			

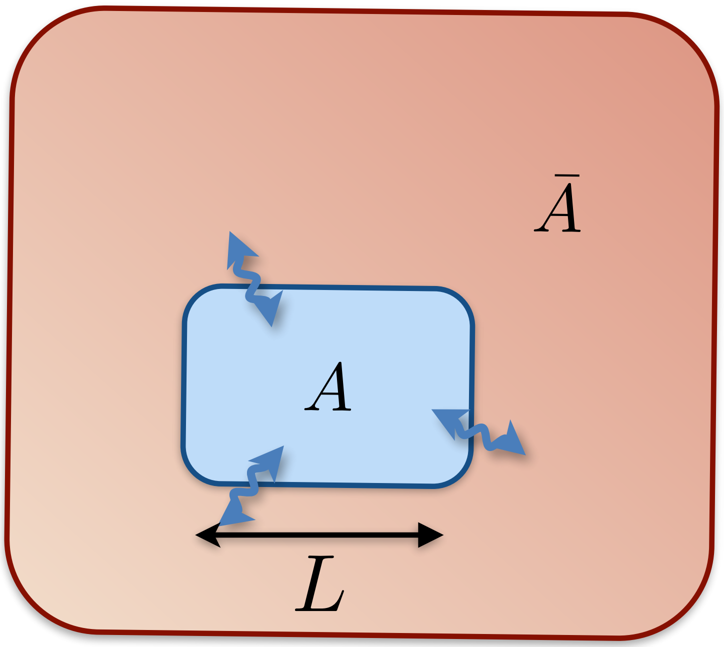


$$i\hbar\partial_t\psi = H\psi$$

isolated system with bounded spectrum
(e.g. fermions on a lattice, spin chains…)

ground state + an extensive number of excitations

Entanglement Dynamics and Thermalization



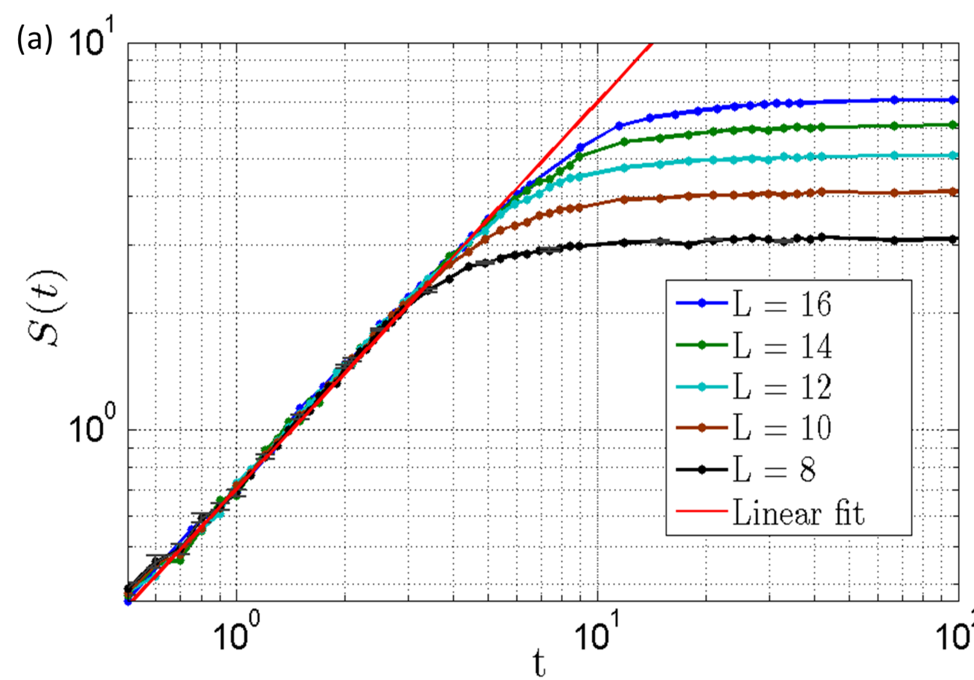
$$\rho_A(t \to \infty) \to \mathrm{e}^{-H/T_{\mathrm{eff}}}$$

Highly (volume law) entangled at long times.

Linear growth vs time

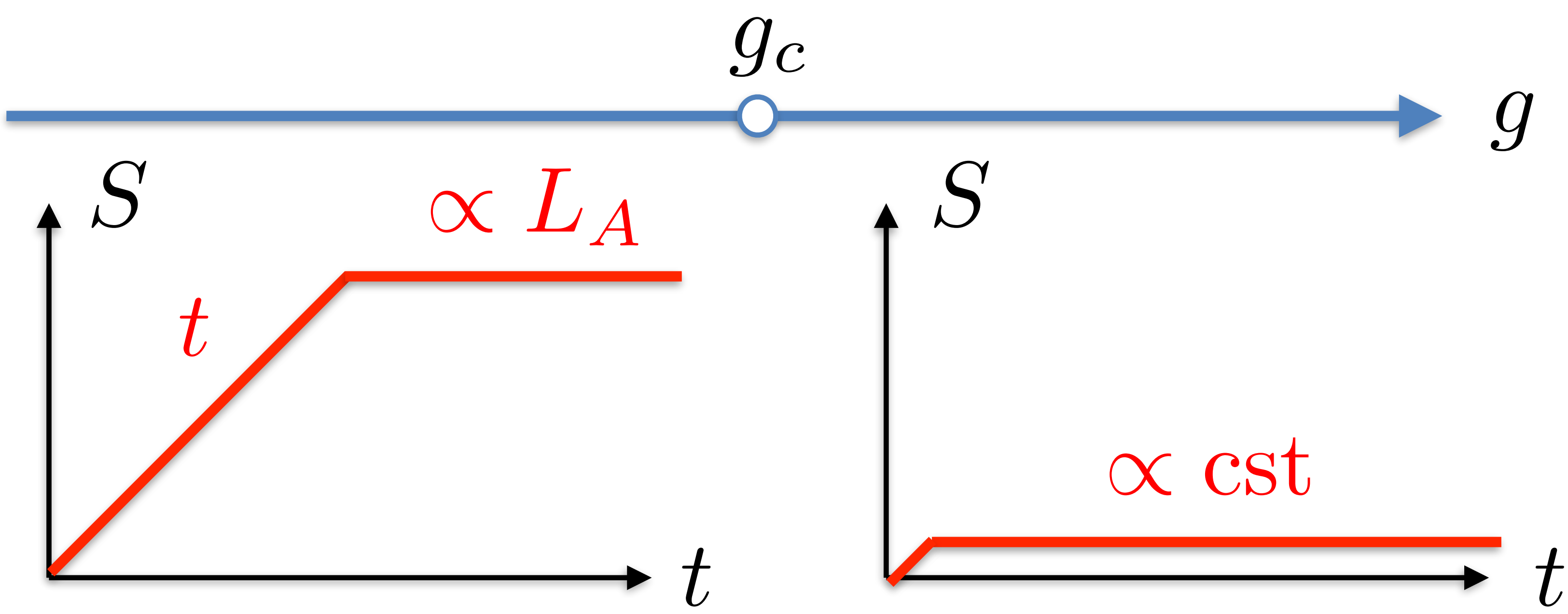
$$S_A(t) \sim t$$

Cardy & Calabrese '05
Kim & Huse '13

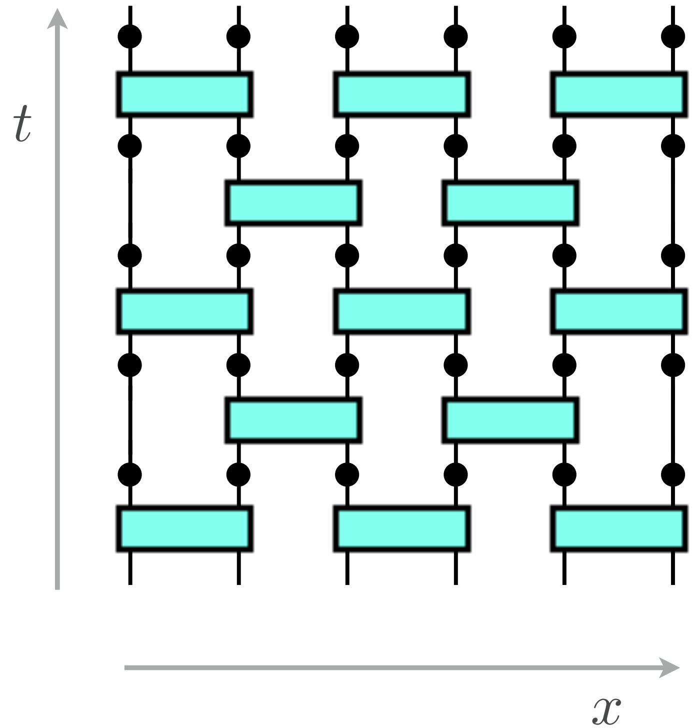


Dynamical and eigenstate "entanglement transitions"

- Can we slow down entanglement growth?
- Entanglement "phase transitions"? In eigenstates or dynamics



Measurement-induced transition

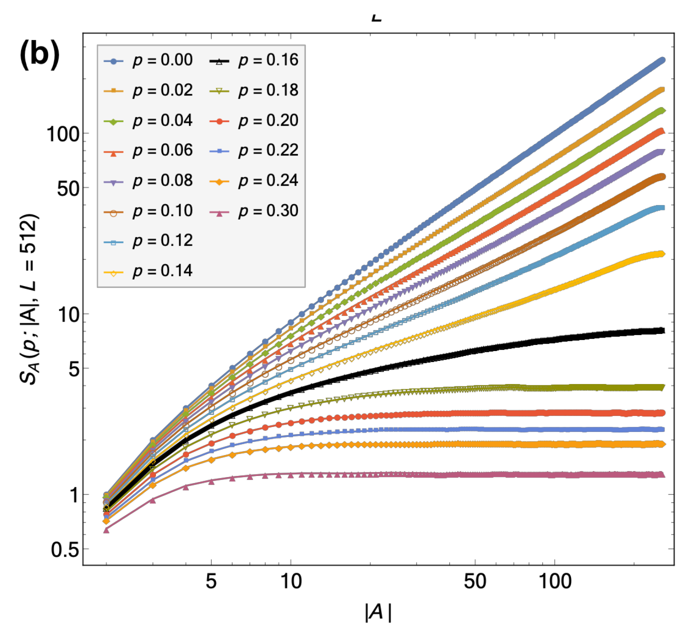


- Chaotic dynamics: Random unitary circuits (Haar or Clifford)

  Nahum, Vijay & Haah '17, …

- Local projective measurements with probability "p"

Skinner, Ruhman & Nahum '19
Li, Chen, Fisher '19

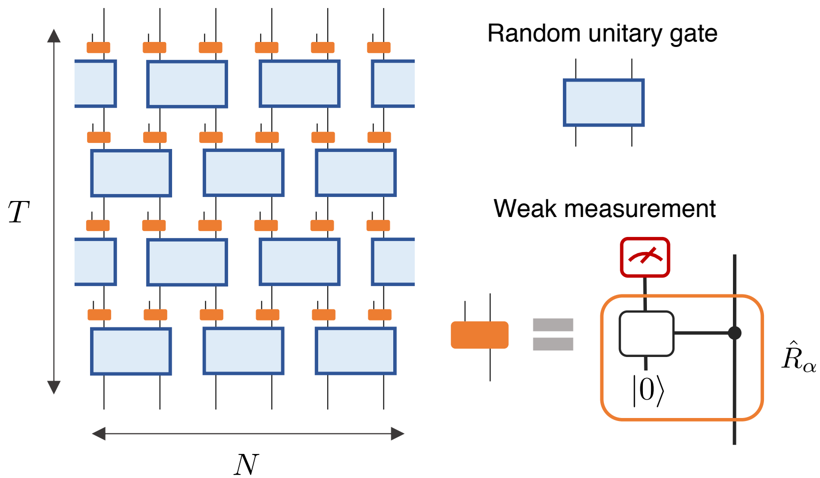


Competition between scrambling/chaotic dynamics and disentangling measurements

Related to quantum error correction problem:

Gullans and Huse, Bao, Choi, Altman, …

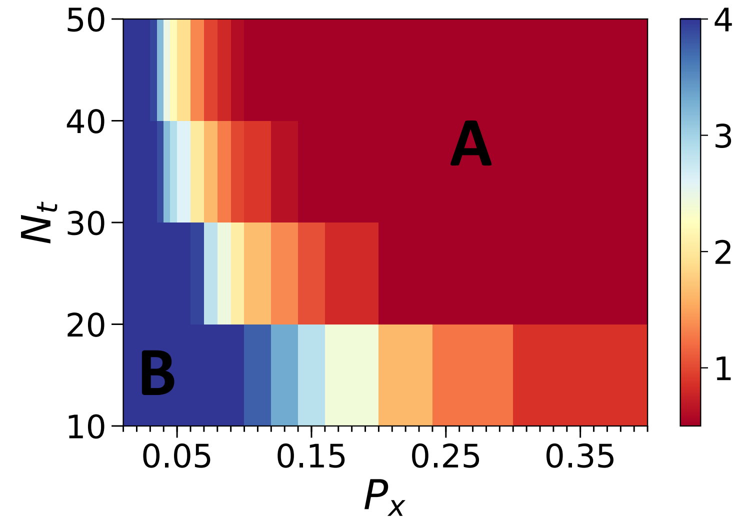
Many recent results/variants



Theory of the transition

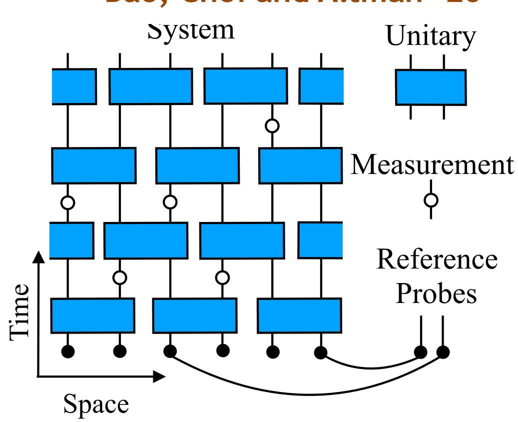
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig '20

Bao, Choi and Altman '20



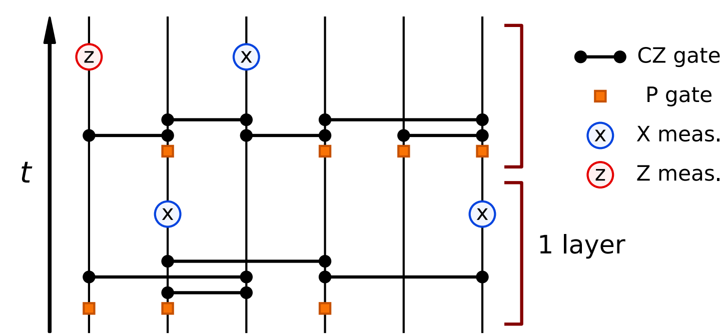
Bose Hubbard model

Tang and Zhu '20



Different observables

Gullans and Huse '20

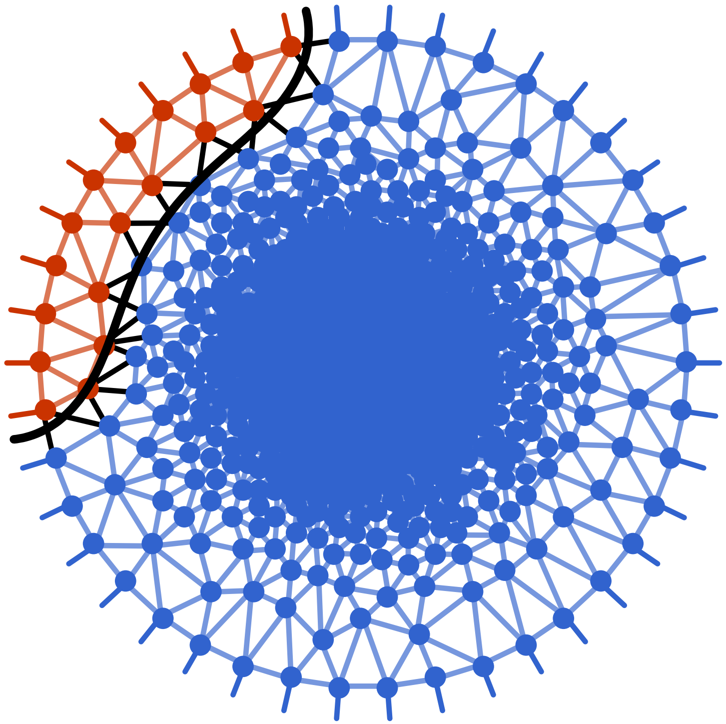


Measurement-only dynamics

Ippoliti, …, Khemani '20

Related example: Holographic Random Tensor Networks

RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig '19



- 1D Wavefunction at boundary of random PEPS
- Random Gaussian tensors, fixed bond dimension

Turns out to be closely related to the measurement transition!

Infinite bond dimension:
Hayden, Nezami, Qi, Thomas, Walker, Yang '16

Measurements: numerical results

- Second order transition in all Renyi entropies

$$S_n = \frac{1}{1-n} \log \mathrm{tr} \rho_A^n$$

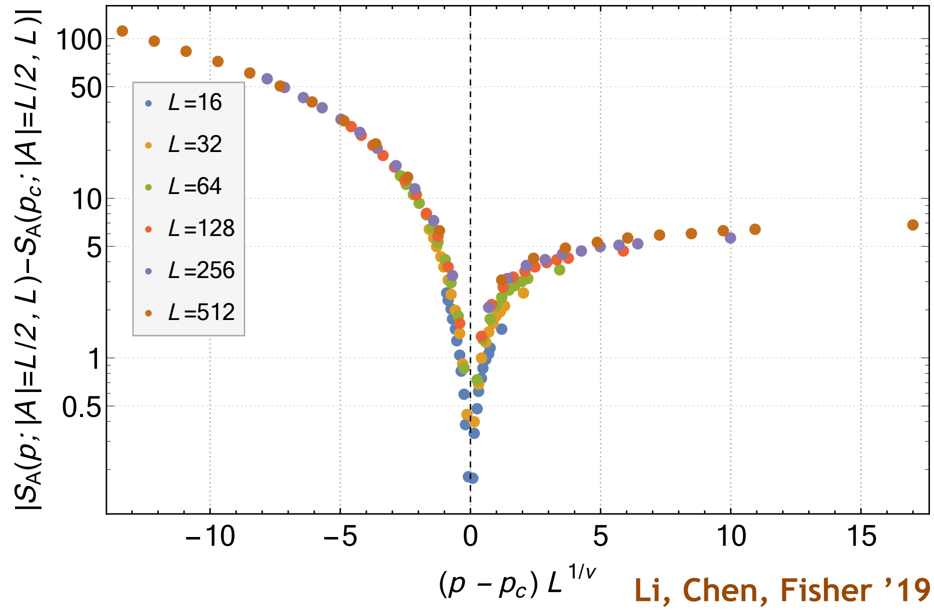
- Critical exponents (universality!)

$$\nu \simeq 1.3 \quad z = 1$$

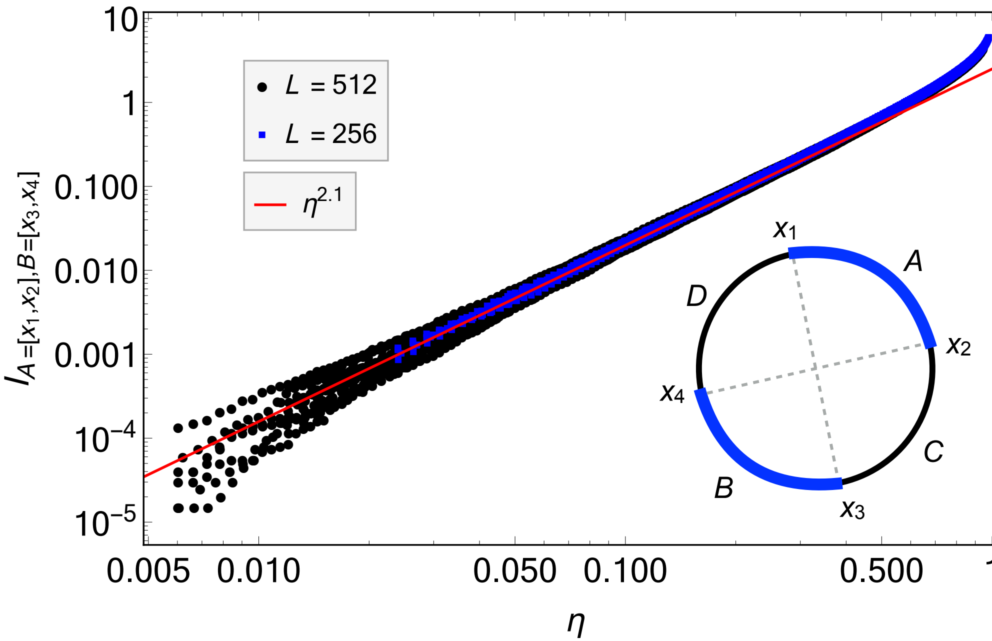
- conformal invariance (!)

Li, Chen, Ludwig, Fisher ’19

$$I = S_A + S_B - S_{AB}$$
$$= f(\eta)$$



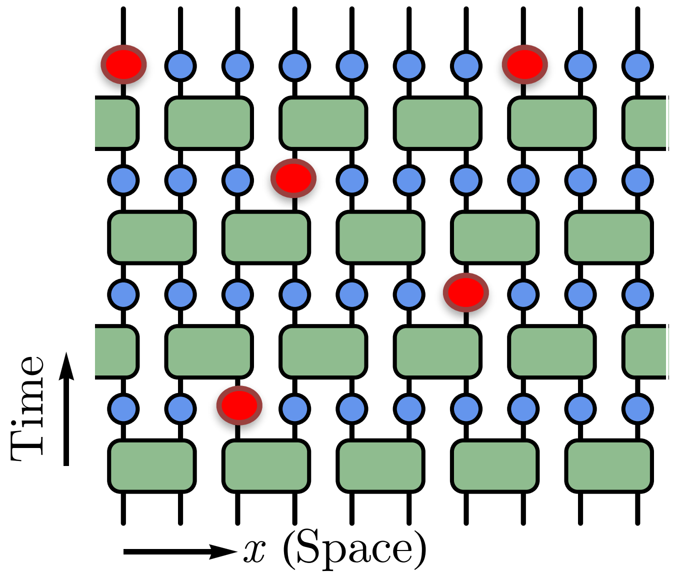
Li, Chen, Fisher ’19



Replica trick and stat mech model

- How can we analyze this transition? Seems like a hard, non-linear problem…
- Need to compute entanglement entropy, averaged over Haar gates, measurement locations, and measurement outcomes.

For each circuit: quantum trajectories (with different measurement outcomes) are weighted by $p_m = \mathrm{tr}\rho$

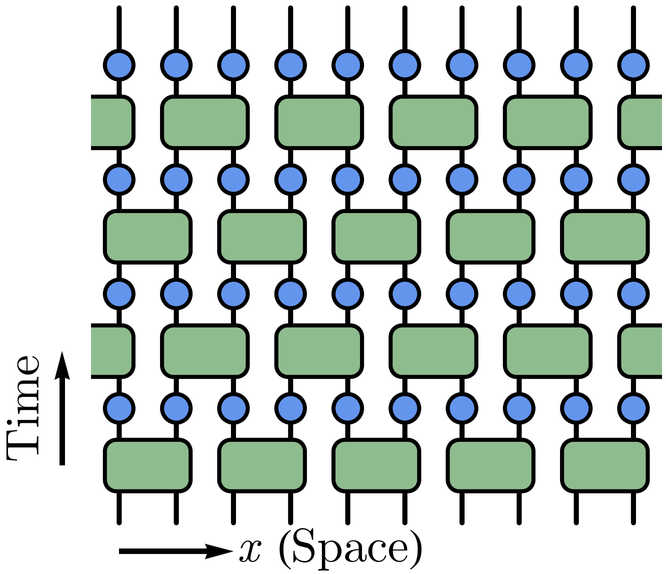


$=$ Projective measurement

$$\sum_m p_m = 1 \quad \text{(Born rule)}$$

$$S_n = \left\langle \sum_m (\mathrm{tr}\rho) \frac{1}{1-n} \log \frac{\mathrm{tr}\rho_A^n}{(\mathrm{tr}\rho)^n} \right\rangle_{\text{circuits}}$$

Replica trick and stat mech model

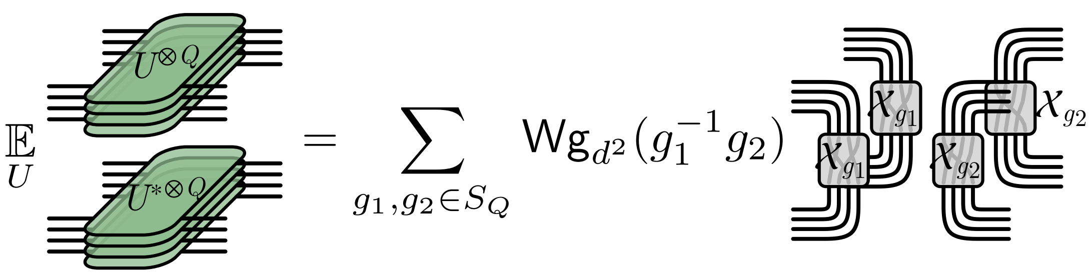


Replica trick:

$$\boxed{\log \mathrm{tr}\rho_A^n = \lim_{m\to 0} \frac{\partial}{\partial m} (\mathrm{tr}\rho_A^n)^m}$$

For n and m integers, average of $(\mathrm{tr}\rho_A^n)^m$

can be mapped onto the partition function of a 2D classical stat mech model defined on the circuit!



spins =

$$g_i \in S_{Q=nm+1}$$

Replica trick and stat mech model

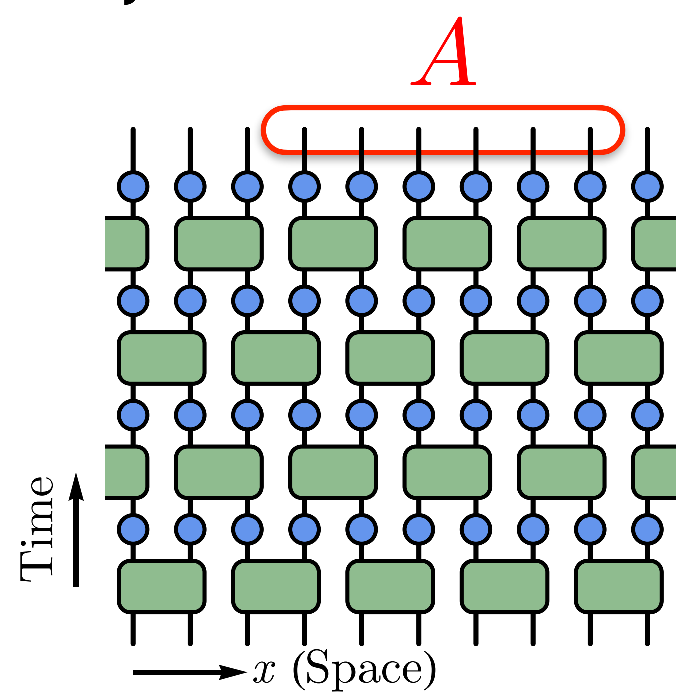
Partition function:

$$\boxed{Z_0 = \left\langle \sum_m (\mathrm{tr}\rho)^Q \right\rangle_{\text{circuits}} = \sum_{\{g_i \in S_Q\}} \mathrm{e}^{-\mathcal{H}}}$$

Entanglement entropy = free energy cost of boundary domain wall:

$$S_n = \lim_{m\to 0} \frac{1}{n-1} \frac{\partial}{\partial m} (F_A - F_0)$$

Z_A has boundary fields acting on A



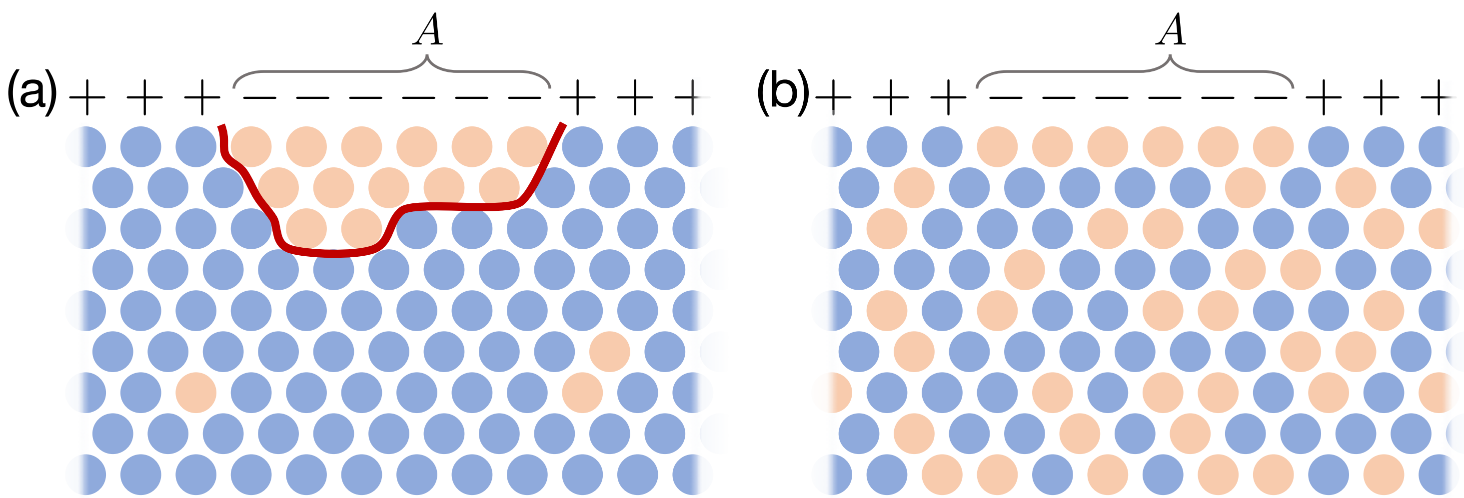
Classical domain wall picture

RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig '19
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig '20
Bao, Choi & Altman '20

Figure from Bao, Choi and Altman

Ferromagnetic (volume-law) phase

Paramagnetic (area-law) phase



"Minimal cut", RT formula

p

$$F_A - F_0 \sim L_A$$

$$F_A - F_0 \sim \mathrm{cst}$$

$$S_A^{(n)} = \lim_{m\to 0} \frac{1}{m(n-1)} (F_A - F_0)$$

Entanglement ~ Free energy cost of domain wall

Consequences for scaling

- Entanglement transition —> simple ordering transition in 2D

- Naturally explains scaling properties near criticality:

$$F_A - F_0 = -\log\langle \phi_{\mathrm{BCC}}(L_A)\phi_{\mathrm{BCC}}(0)\rangle$$

two-point function of a "boundary condition changing operator"

$$\boxed{S_n = \frac{2}{n-1}\frac{\partial \Delta}{\partial m}\log L_A + f_n\left(\frac{L_A}{\xi}\right) + \dots}$$

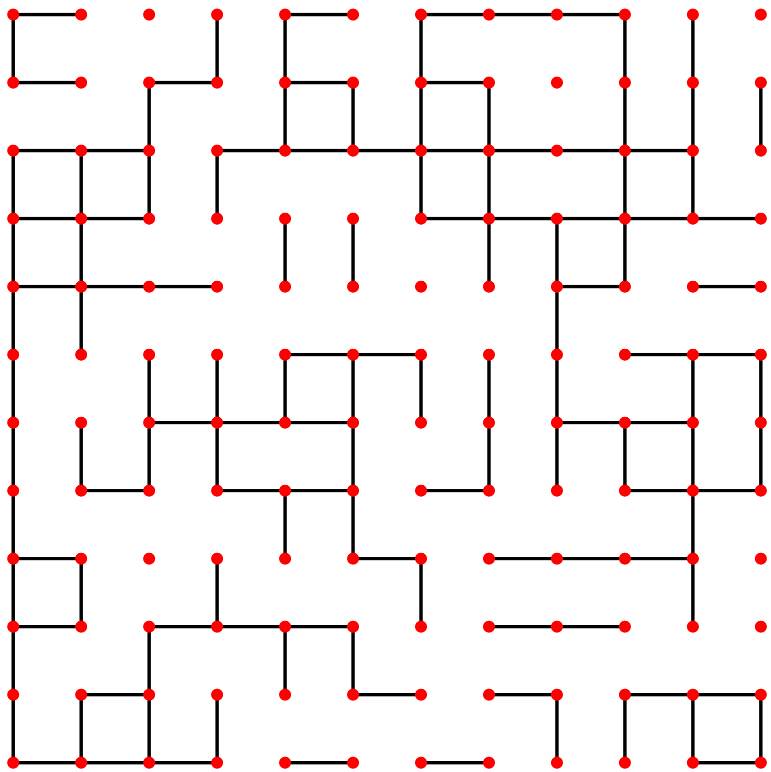
logarithmic critical behavior

- Mutual-information = 4-point function. Explains conformal invariance at criticality

Large on-site Hilbert space: Percolation

Especially simple for infinite on-site Hilbert space:

$$Z_0 = \sum_{\{g_i \in S_Q\}} \prod_{<ij>} \left(p + (1-p)\delta_{g_i,g_j}\right)$$



$Q!$ **-state Potts model**

Replica limit: $Q \to 1$

bond percolation

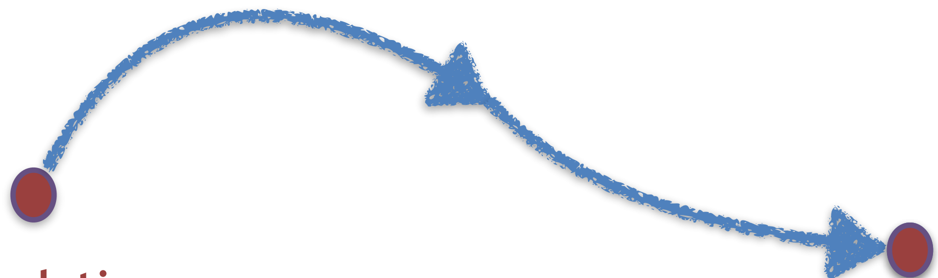
$$\xi \sim |p - p_c|^{-4/3}$$

$$\bar{S}_{n,A} = \frac{1}{3}\log L_A + \ldots,$$

Finite d universality class?

Infinite Hilbert space limit has accidental enlarged symmetry:

$$S_{Q!} \to S_Q \times S_Q$$



Percolation

$$c = 0$$

Finite Hilbert space transition

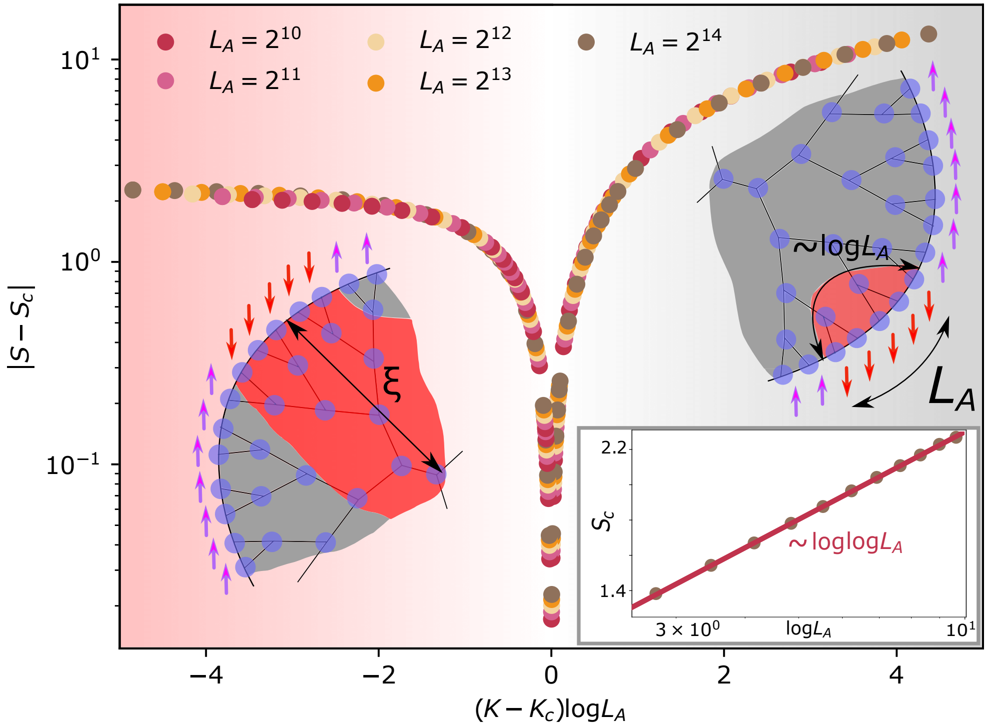
$$c = 0$$

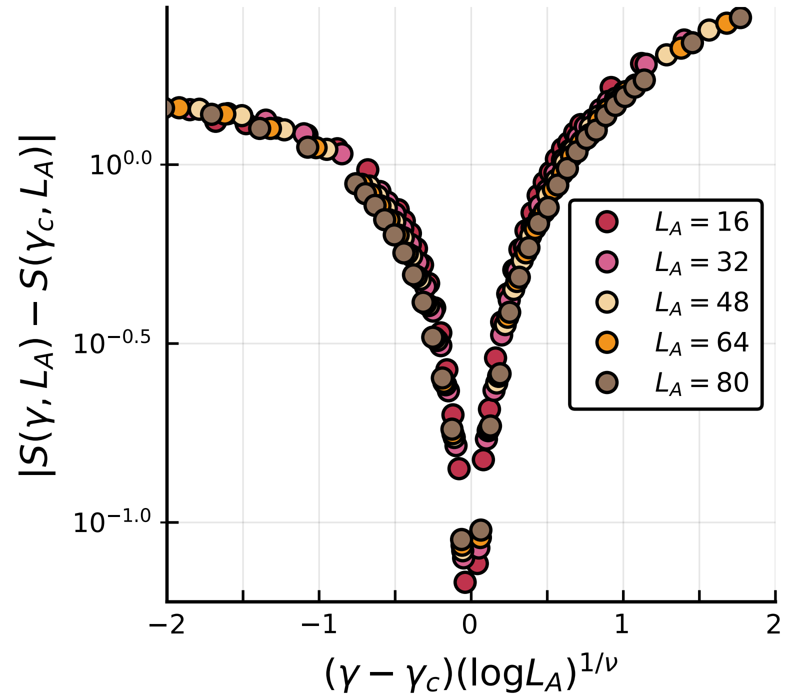
CFT = IR fixed point of percolation + relevant perturbation

- **Field Theory**: $\mathcal{L} = \sum_{a=1}^{Q!} \frac{1}{2}(\partial \phi_a)^2 + \frac{m^2}{2} \sum_a \phi_a^2 + g \sum_a \phi_a^3 + \sum_{a,b \in S_Q} W(a^{-1}b)\phi_a \phi_b$

With $Q \to 1$

"Mean-field limit": tree tensor networks





Replica limit analytically tractable

exact mean-field-like exponents

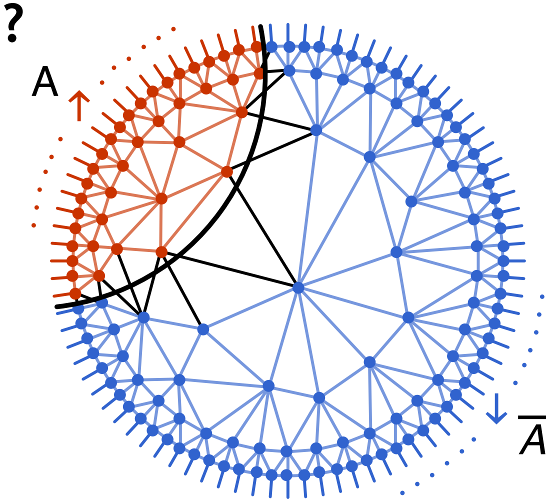
$$S \sim \begin{cases} \frac{\log L_A}{\xi} + \alpha \log\log L_A, & K \to K_c^+ \\ \alpha \log\log L_A, & K = K_c, \\ \alpha \log \xi, & K \to K_c^- \end{cases}$$

J. Lopez-Piqueres, Brayden Ware & RV, arxiv:2003.01138

Though story seems more complicated: Nahum, Roy, Skinner & Ruhman '20

Conclusion

- New class of "entanglement transitions"
- Exact mapping onto classical stat mech model
- Analytic handle on field theory description of such entanglement transitions (c=0 LCFTs are nasty…)
- Classification? Universality class? Experiments?



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