Polylog-Overhead Highly Fault-tolerant Measurement-Based Quantum Computation:

Application of Entanglement without Geometrical Constraints

arXiv:2006.05416

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Joint work with K. Fukui, Y. Takeuchi, S. Tani, and M. Koashi

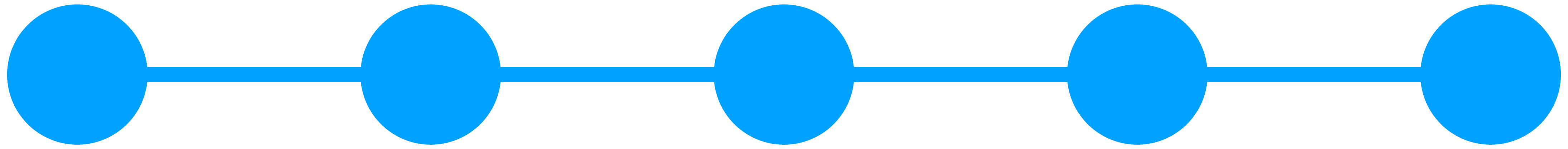
Background

Results

Outlook

Entanglement

Nonlocal property appearing in multipartite quantum states



$$\sum_l \alpha_l \left|\psi_l\right\rangle^{ABCDE}$$

- Playing important roles in analyzing many-body quantum physics

  Analysis of AdS/CFT, topologically ordered states,…

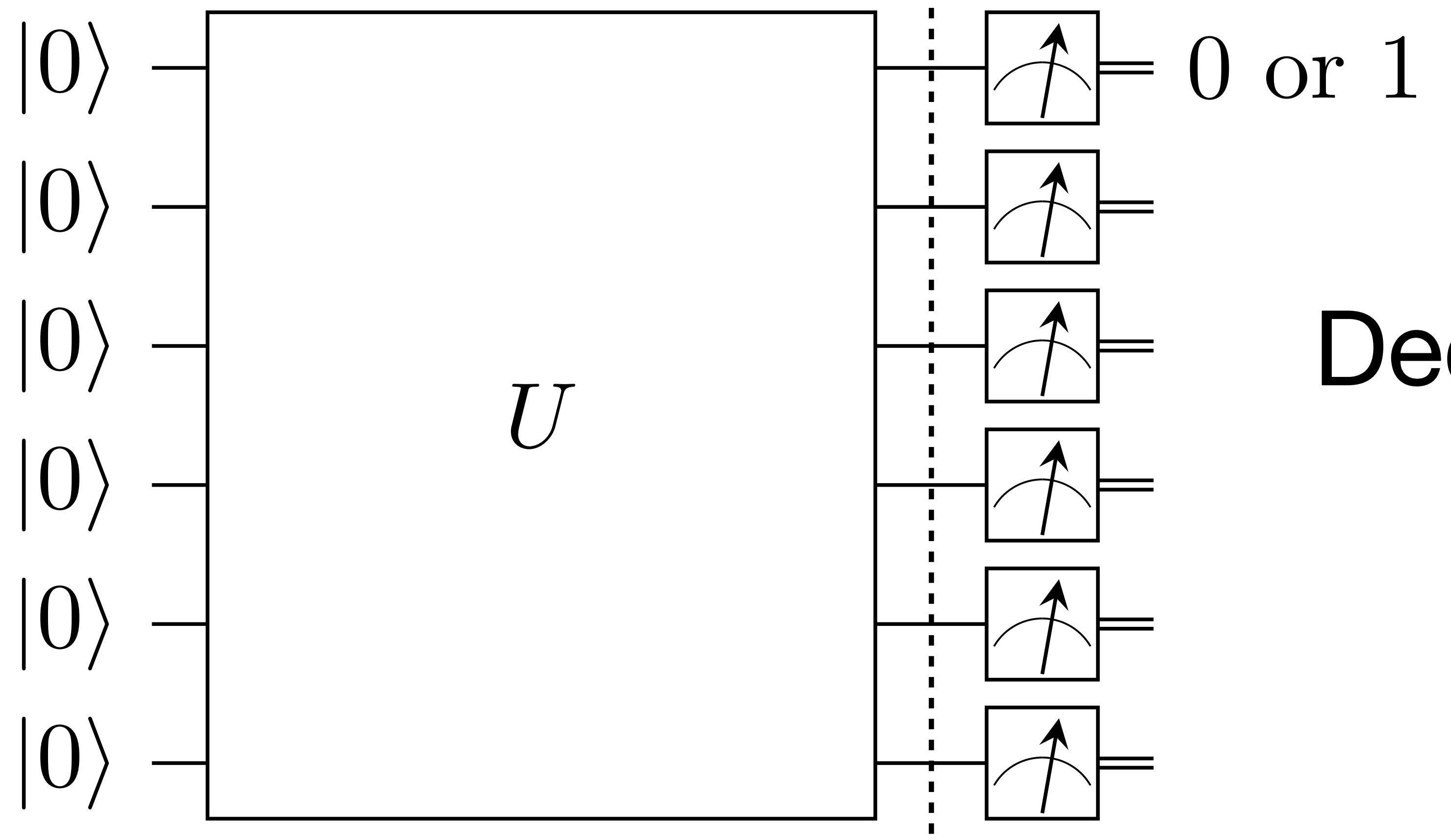
- Also useful for implementing **quantum computation**

  → **measurement-based quantum computation (MBQC)**

This talk: What entanglement structure is “good” for implementing QC?

Quantum Computation

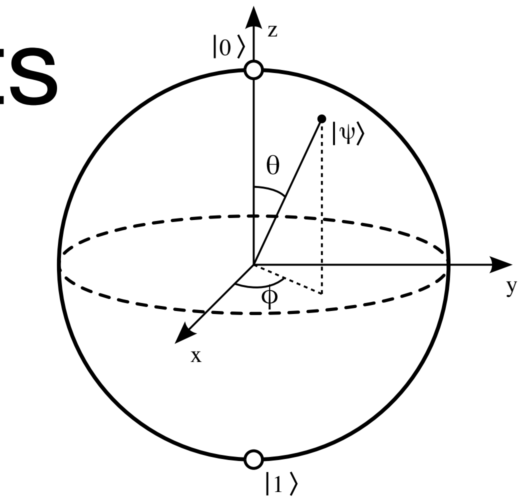
Initialization of qubits + unitary time evolution + measurements



Unitary

$(\mathbb{C}^2)^{\otimes N} = \mathbb{C}^{2^N}$

Qubits

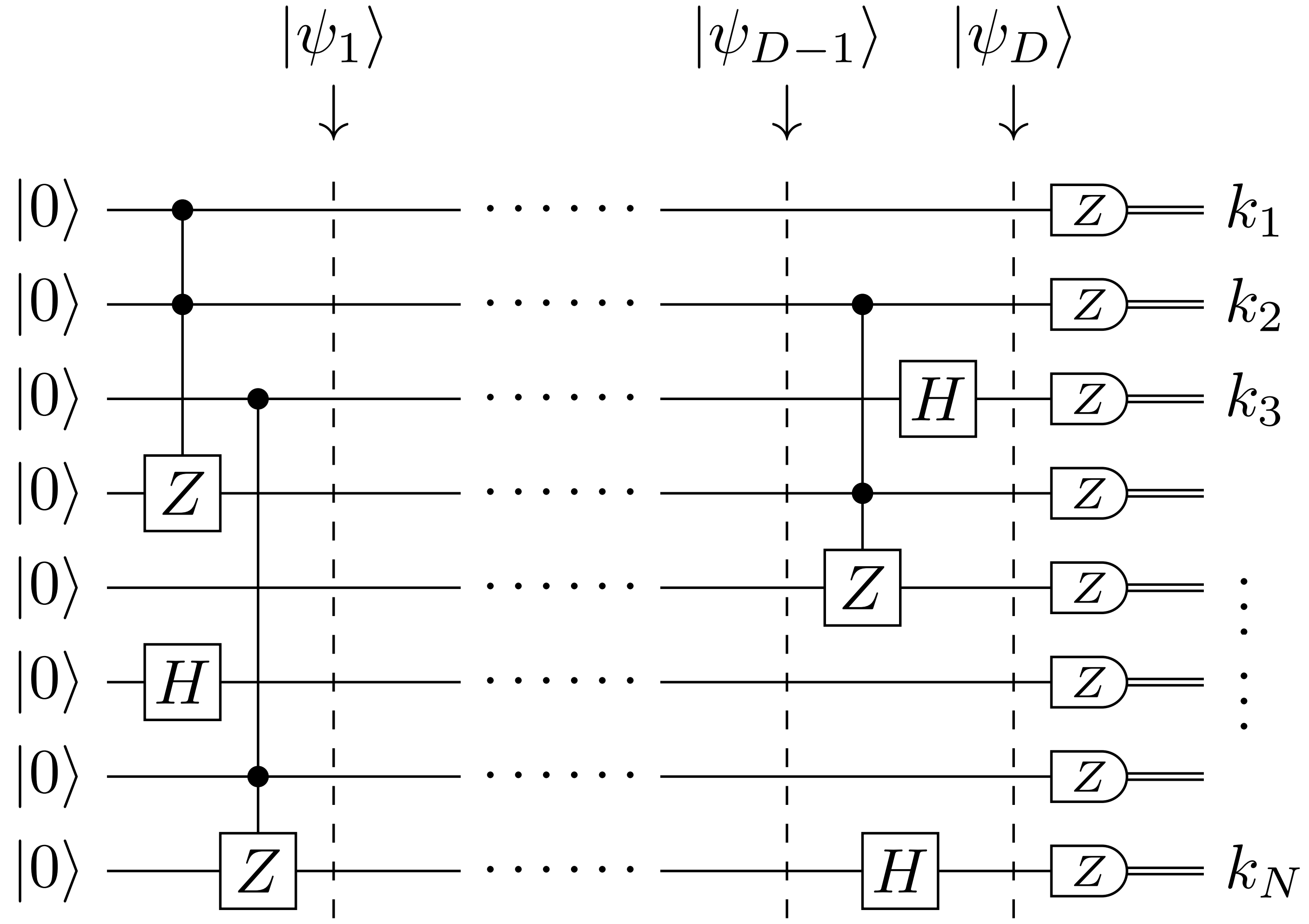


Measurement
Probability = square of amplitude in a basis

Entangled

$U\,|0\rangle^{\otimes N}$

Decomposition



Gate set

E.g. $\{H, CCZ\}$

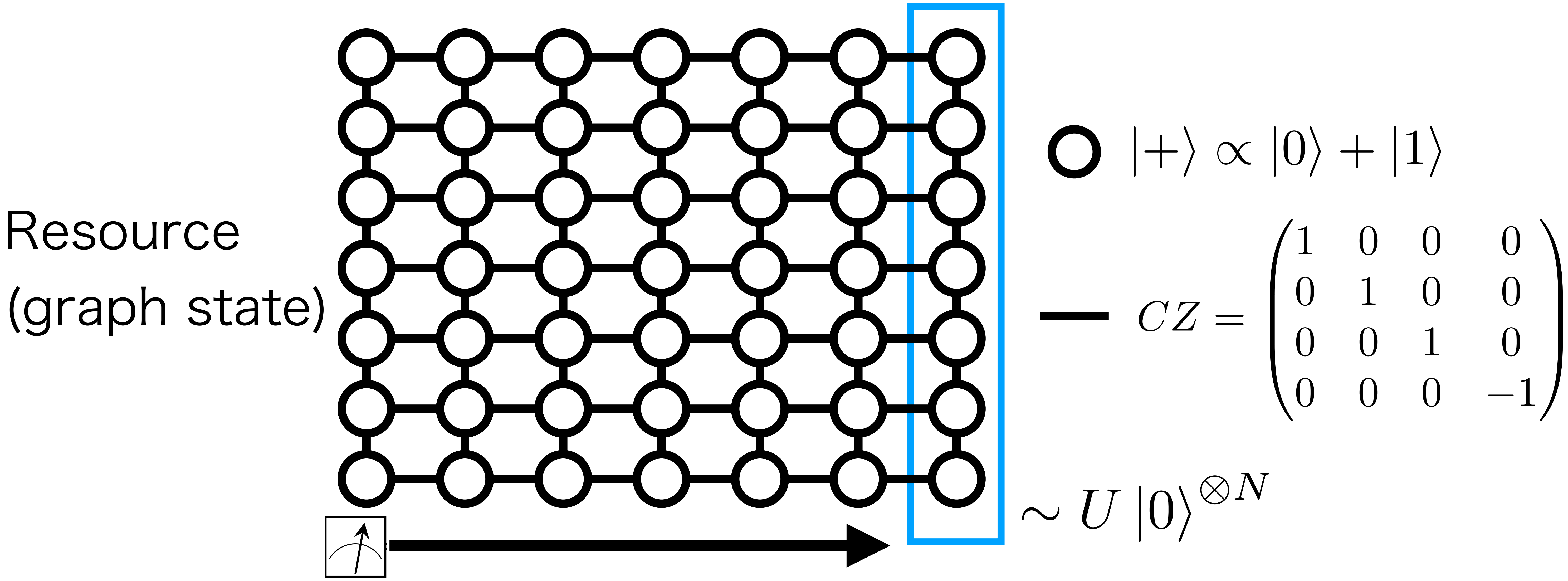
$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

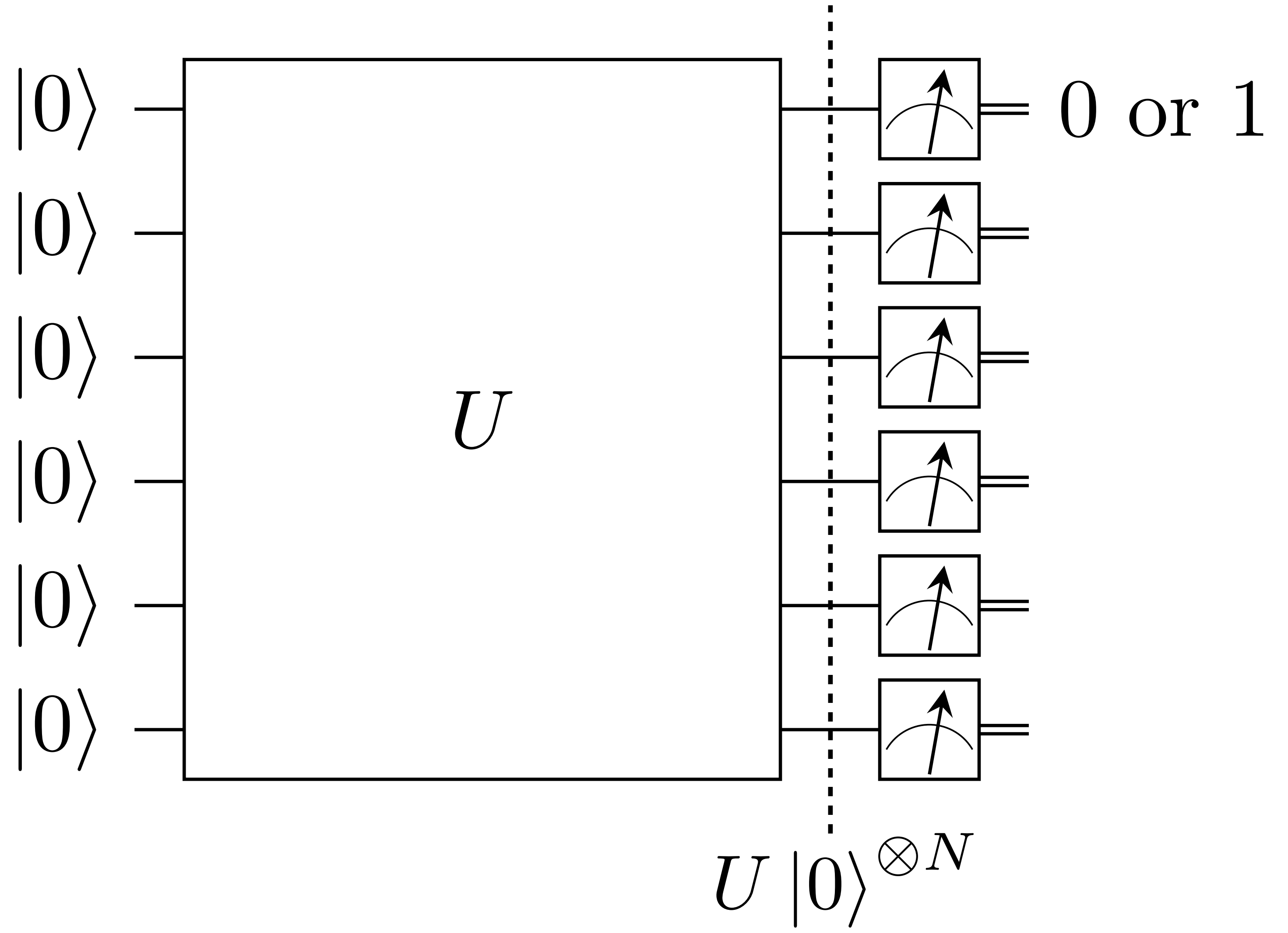
$$CCZ = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

Cost of implementing quantum computation = O(number of gates)

Measurement-Based Quantum Computation

Entanglement & Measurement to Implement Quantum Computation





1. Prepare a fixed entangled state **independent of what to compute**

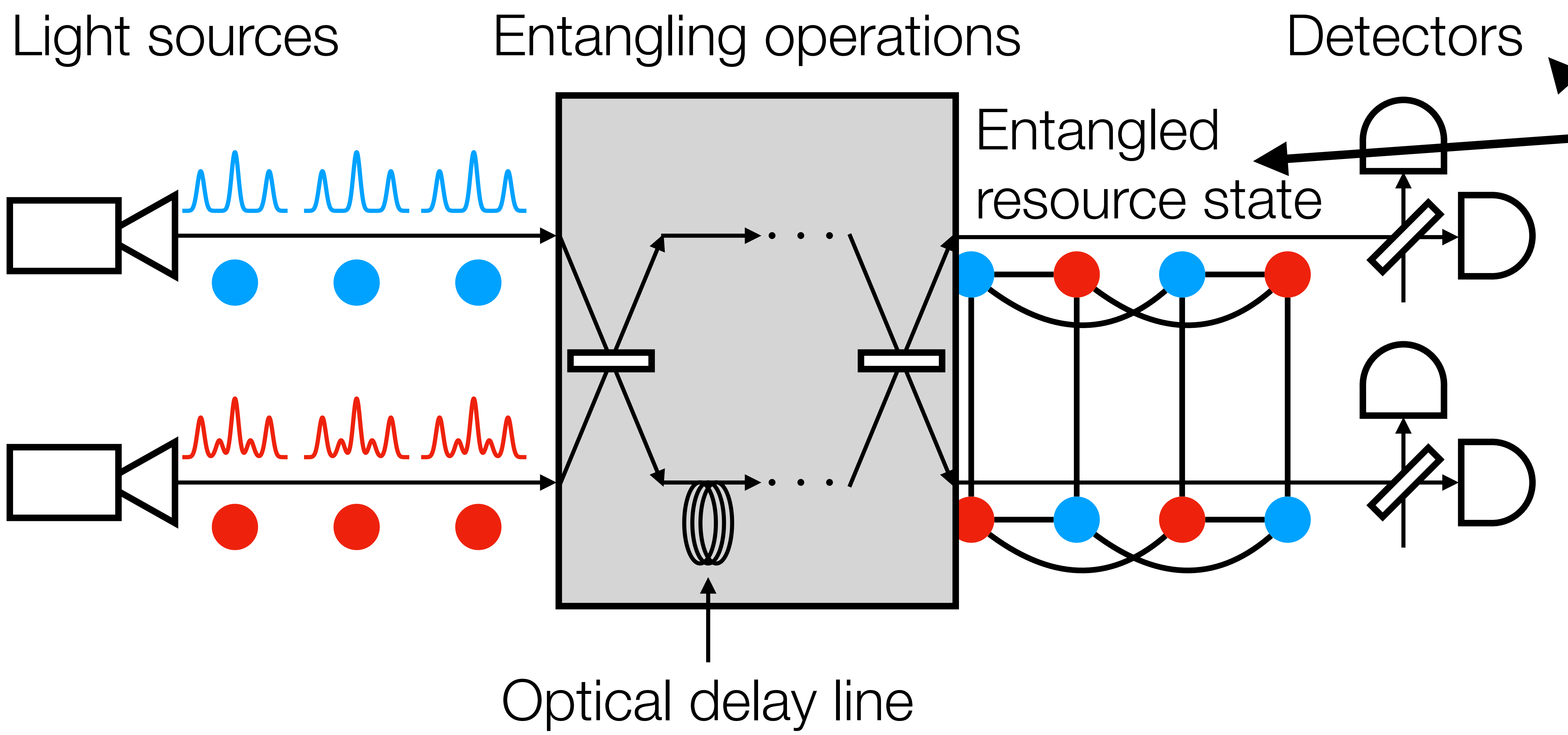
   Resource state: **graph state** on a 2D/3D lattice, ground states of some 2D local Hamiltonians (AKLT state etc.),…

2. Perform measurements **adaptively** to conduct QC as desired

   (in measurement bases conditioned on previous measurement outcomes)

Motivation of this work: Photonic MBQC

Technology realizing entanglement without geometrical constraints



Independent of what to compute → scalable

Switch measurement bases to conduct computation

- \+ Large-scale entanglement **over 1,200,000 subsystems**
- \+ Fast Gaussian operations & homodyne detection **at 40-ns intervals**
- \+ Freely **moving in space**: crucial for low overhead

Requiring **different design principle** from matter-based implementation

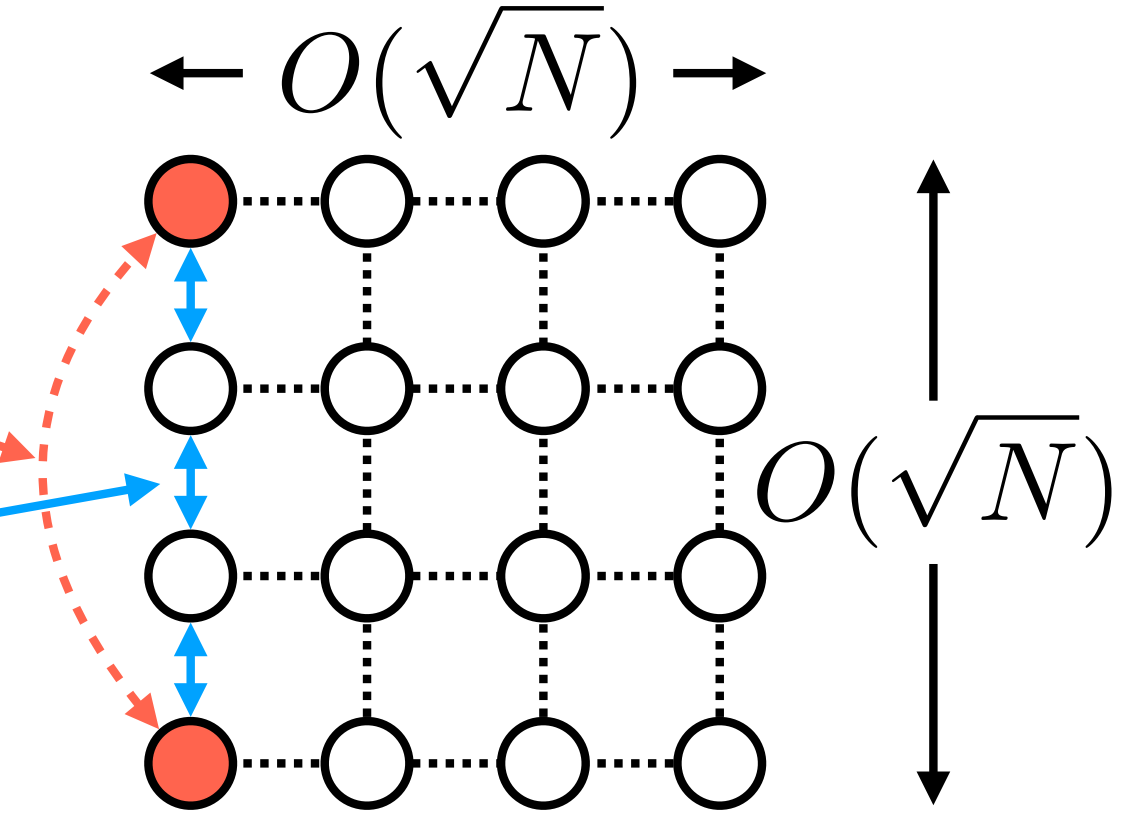
Takeda & Furusawa, APL Photonics 4, 060902 (2019).

Problem: Overhead in implementing QC

Example: geometrical constraints → canceling out useful poly speedup

Geometrically nonlocal gate to be implemented

$O(\sqrt{N})$ gates required for implementation per gate

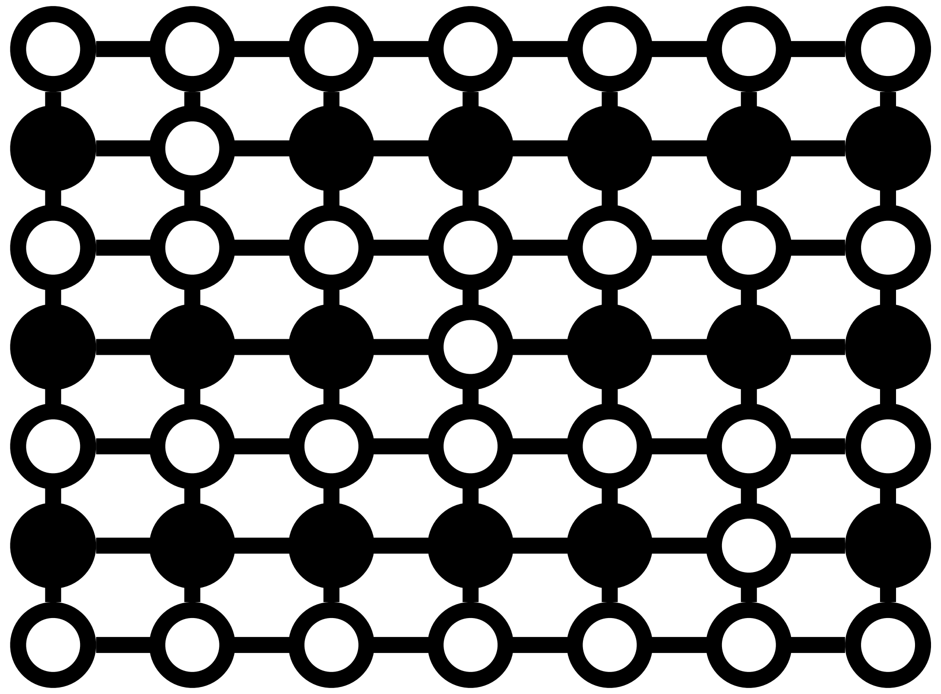


Overhead → $O(\sqrt{N})$

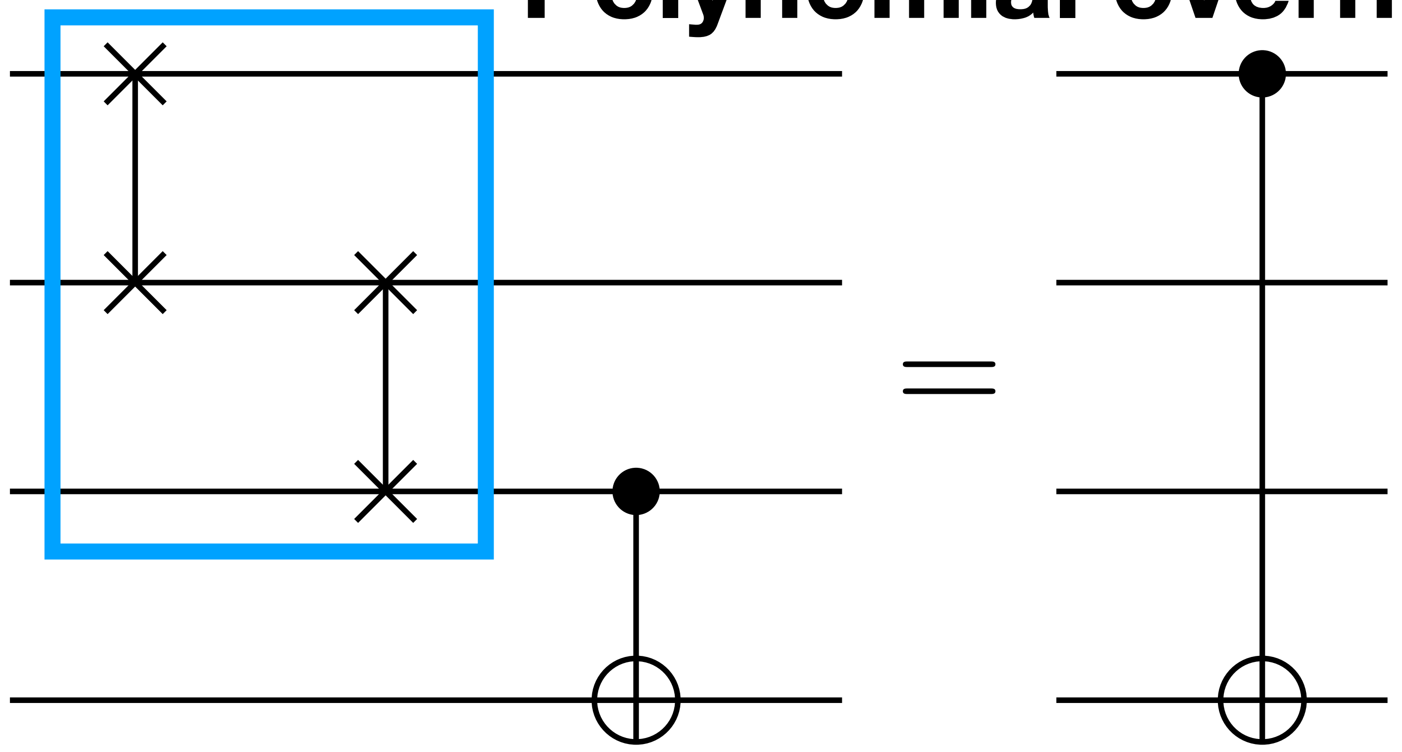
Definition of "polynomial"
$O(N^\alpha),\ \alpha > 0$

Existing MBQC protocols

Cluster state



Polynomial overhead



Is there a better **entanglement structure for reducing overhead**?

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Results

Polylog-overhead highly fault-tolerant MBQC for photonic systems [1]

- **Protocol for photonic MBQC that achieves poly-logarithmic overhead**

  ← New entanglement structure for **low-overhead** qubit permutation

  ← **Optimized for photonic architectures**: univercal MBQC only by measurements that are easy to implement.

- **Highly fault-tolerant photonic MBQC protocol with polylog overhead**

  ← Combining Gottesman-Kitaev-Preskill (GKP) code with 7-qubit code (Not in this talk)

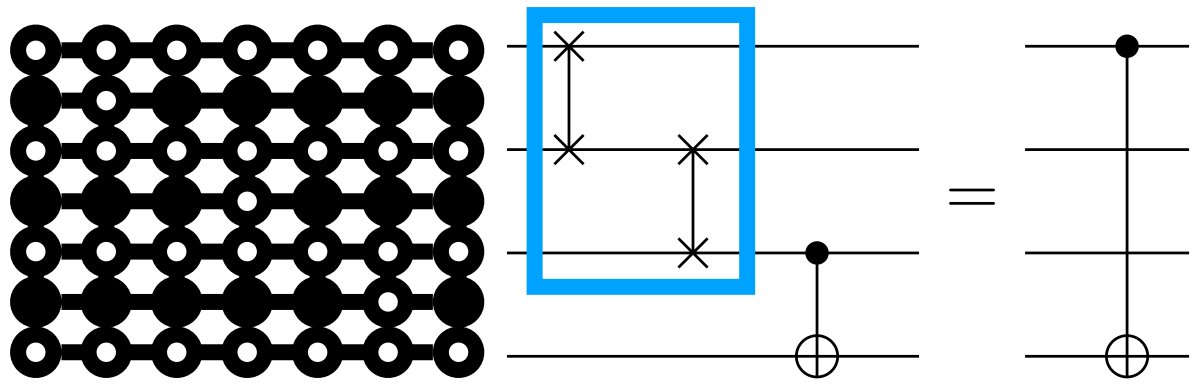
Open a new way toward realization of speedups including those polynomial

[1] H. Yamasaki, K. Fukui, Y. Takeuchi, S. Tani, M. Koashi, arXiv:2006.05416

Implementation of quantum computation

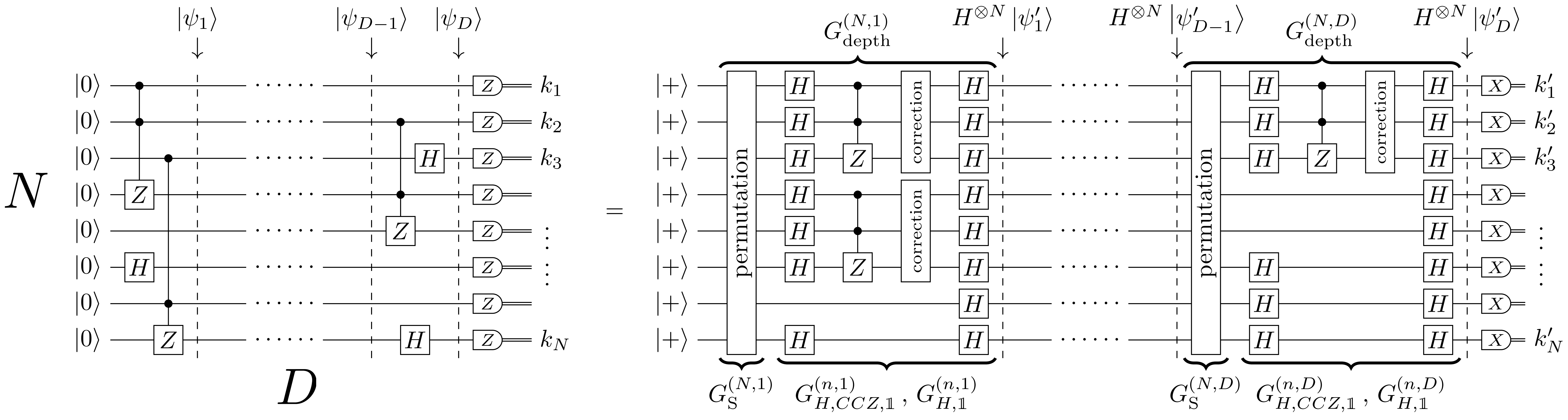
= Sampling from the same distribution as a given quantum circuit

What MBQC implements within entanglement structure



Each gate to be implemented

Given circuit



Equivalent circuit used for MBQC

$O(DN)$ gates composed of $\{H, CCZ\}$ (computationally universal gate set)

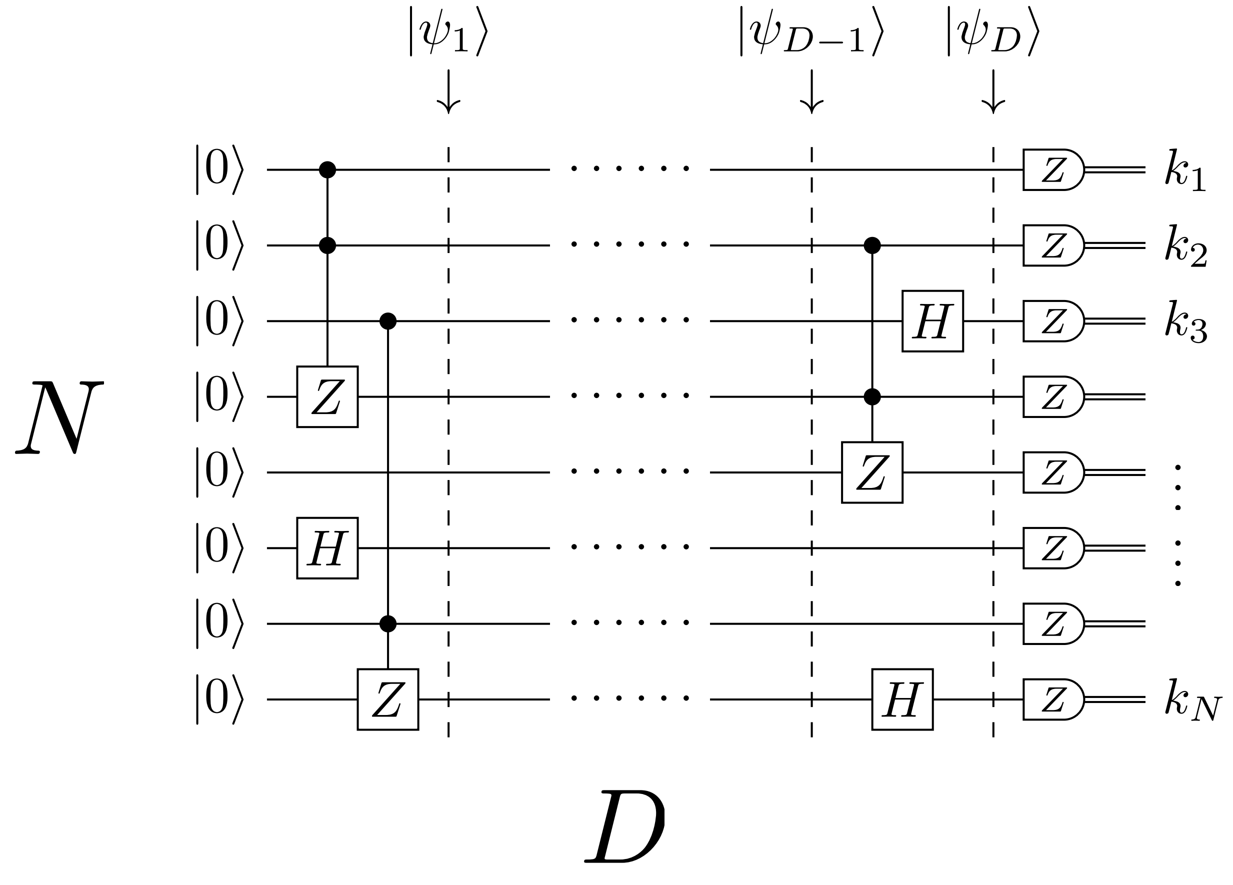
$t(D, N)$ gates: including overhead in implementation

Complexity of MBQC includes **preparation, measurements, & classical process**

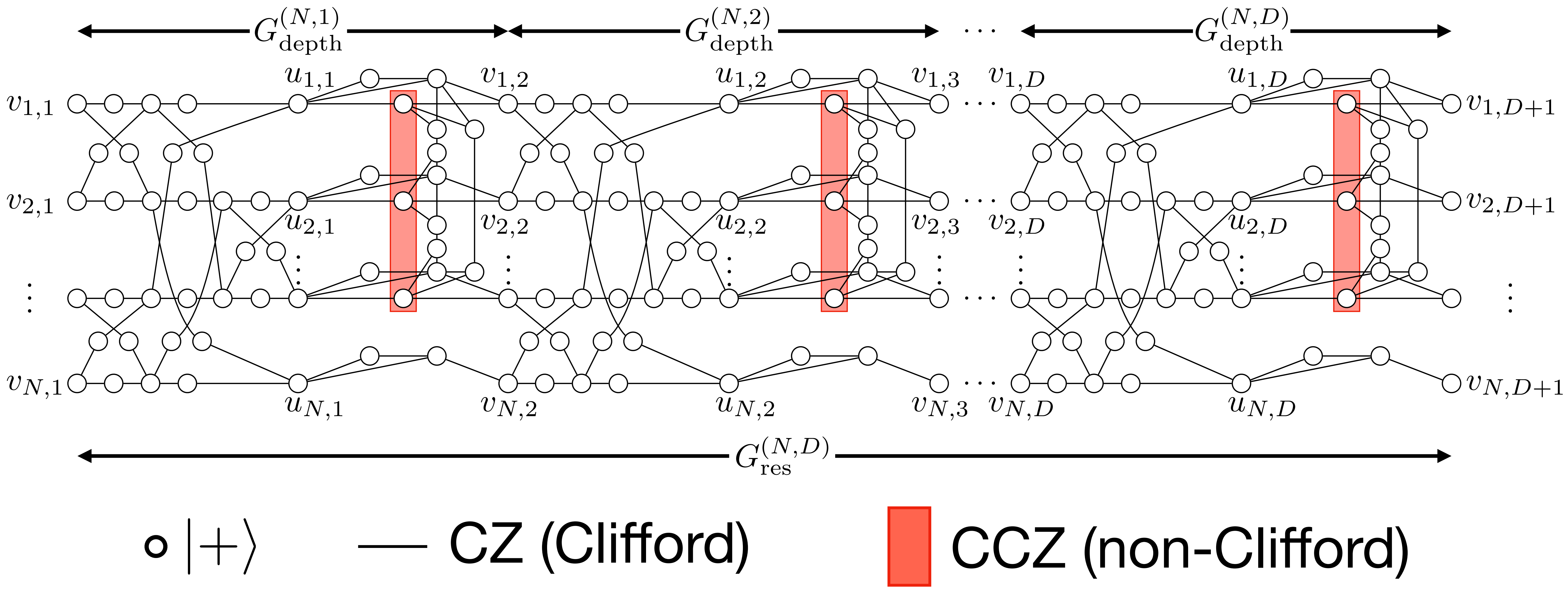
Overhead: $t(D,N)/DN$ → we bound this $O(\mathrm{polylog}(DN))$

Features of our MBQC protocol

Optimized for photonic fault-tolerant quantum computation [1]



Hypergraph state: **periodic**

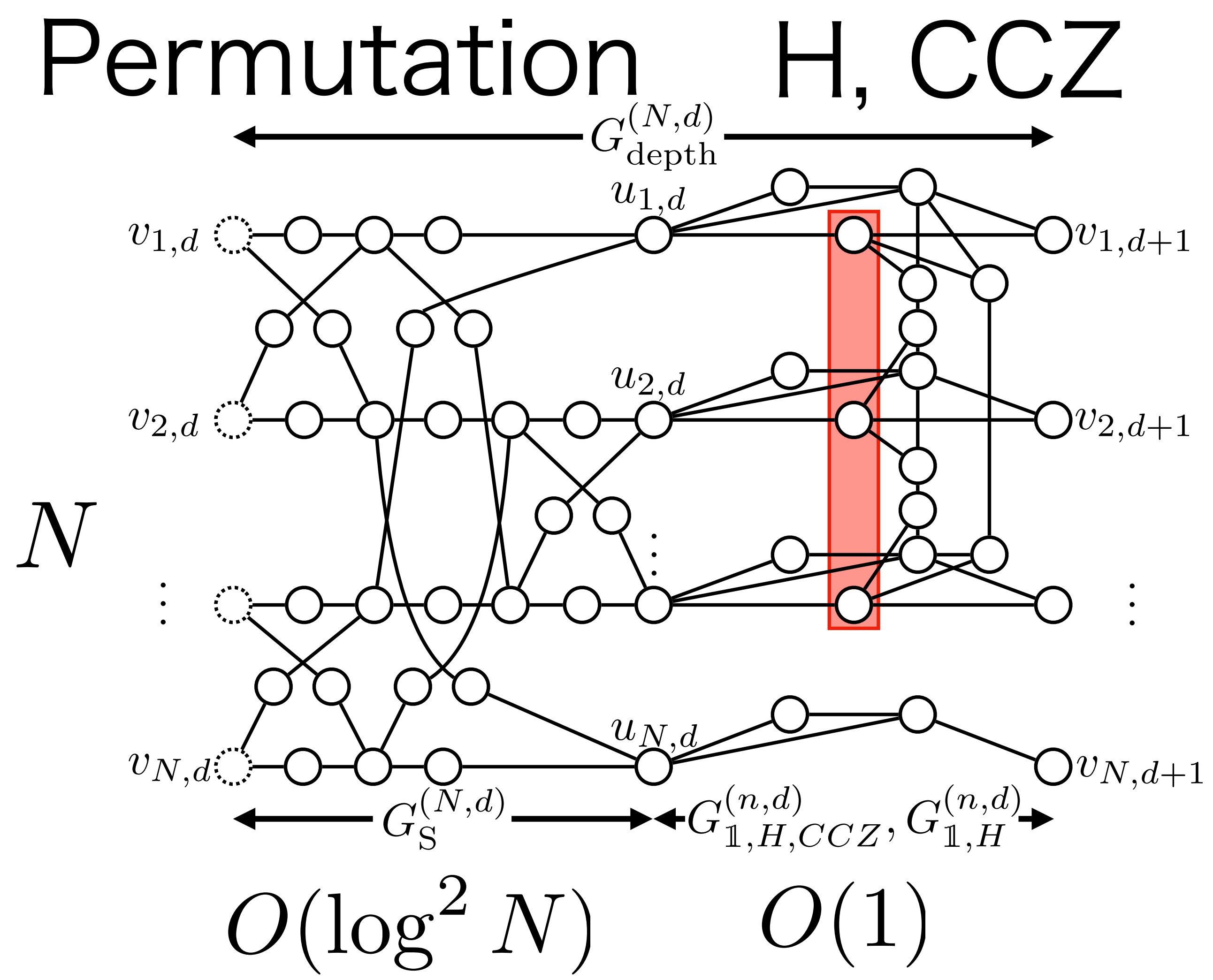


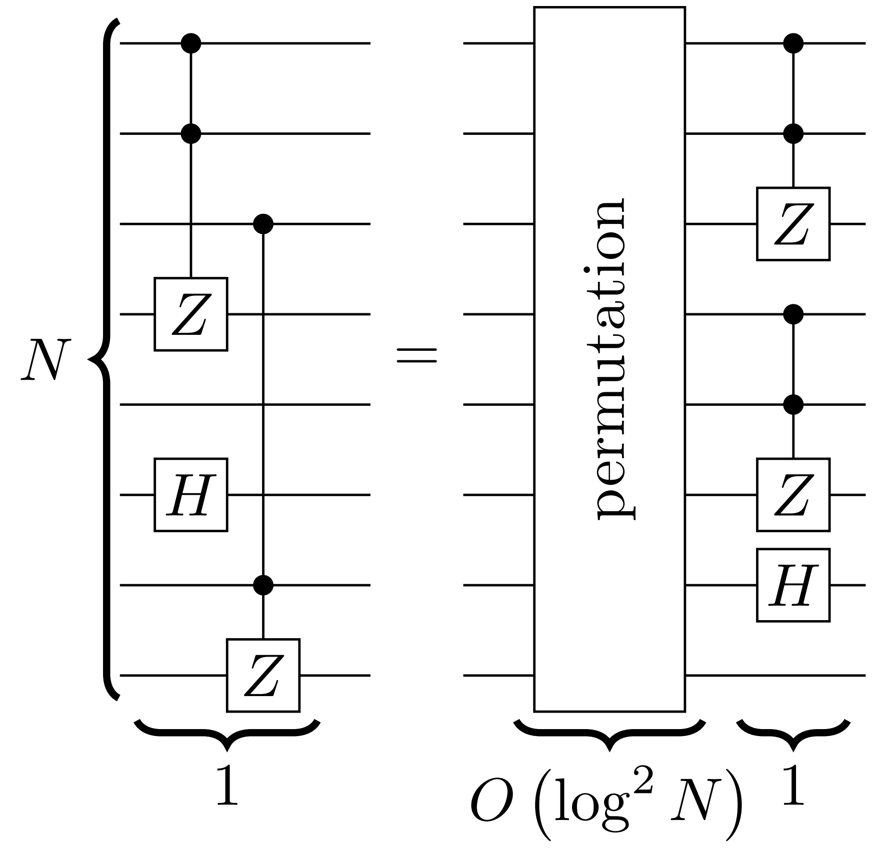
- Polylog overhead by **overcoming geometrical constraints:** new perspective
- Universal only by measurements in X & Z basis = **suitable for error correction**
- **Saving implementation cost in optical experiments** = small number of qubits

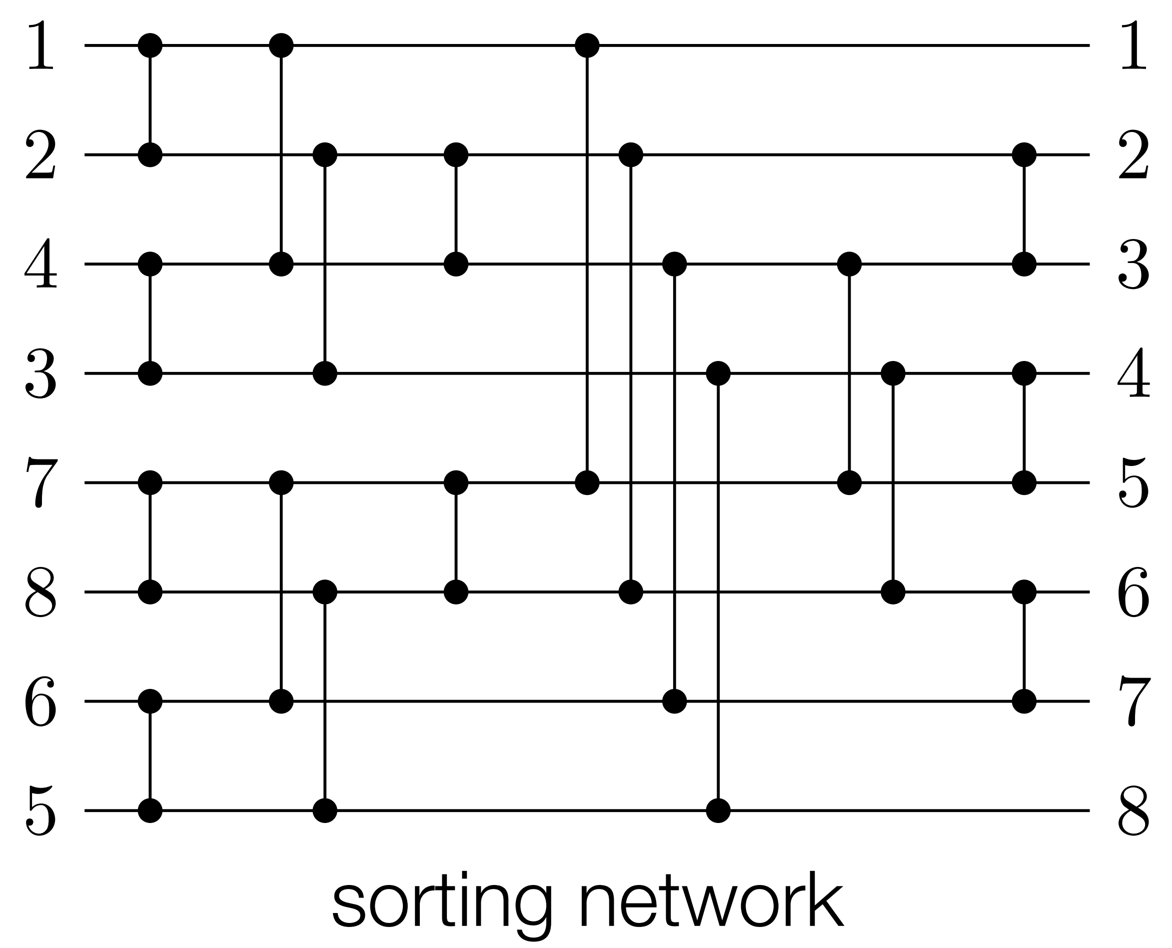
[1] H. Yamasaki, K. Fukui, Y. Takeuchi, S. Tani, M. Koashi, arXiv:2006.05416

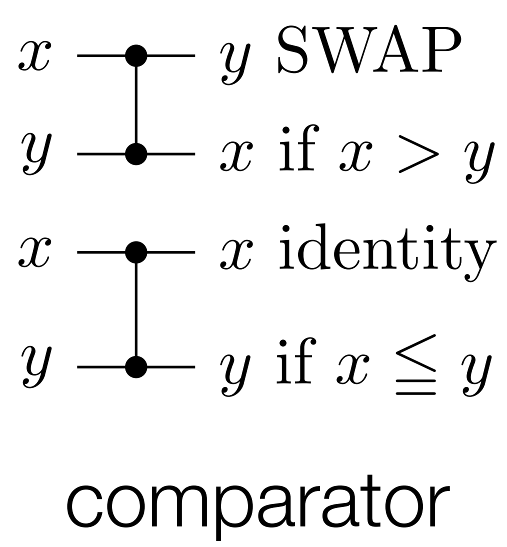
Sketch of Idea: Low-overhead qubit permutation

Introducing the idea of sorting network to entanglement structure

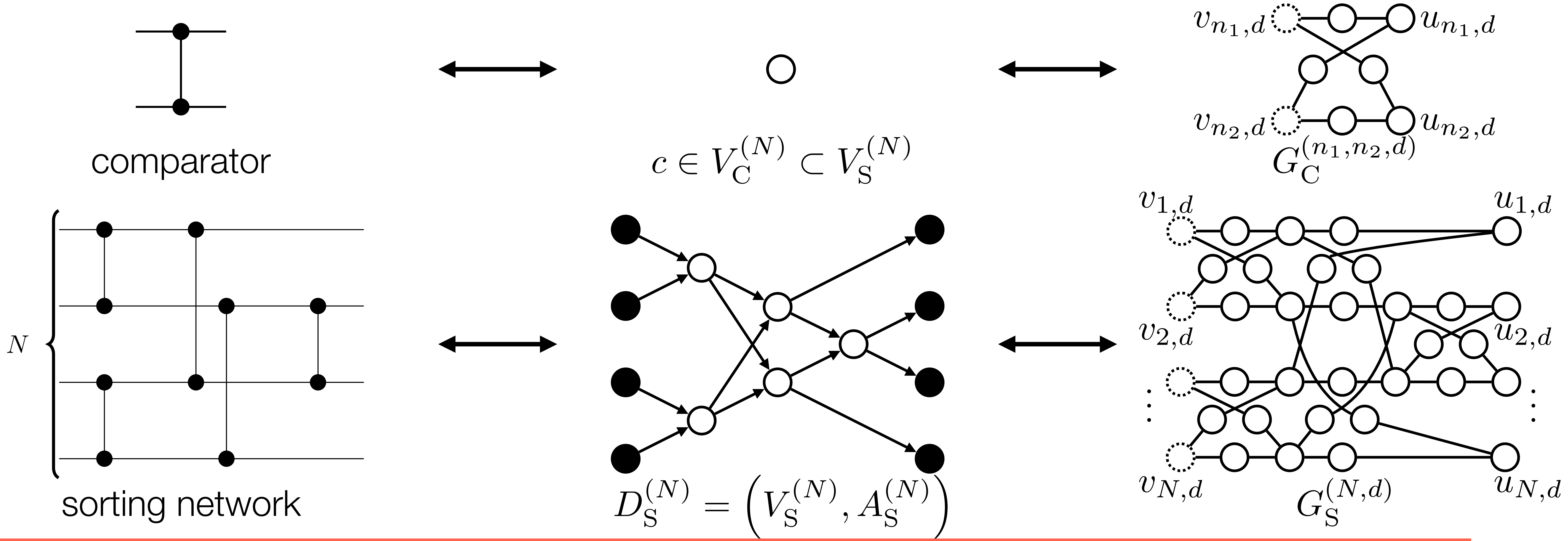








Practical sorting network with $O(\log^2 N)$ depth



Overcoming geometrical constraint at polylog overhead cost

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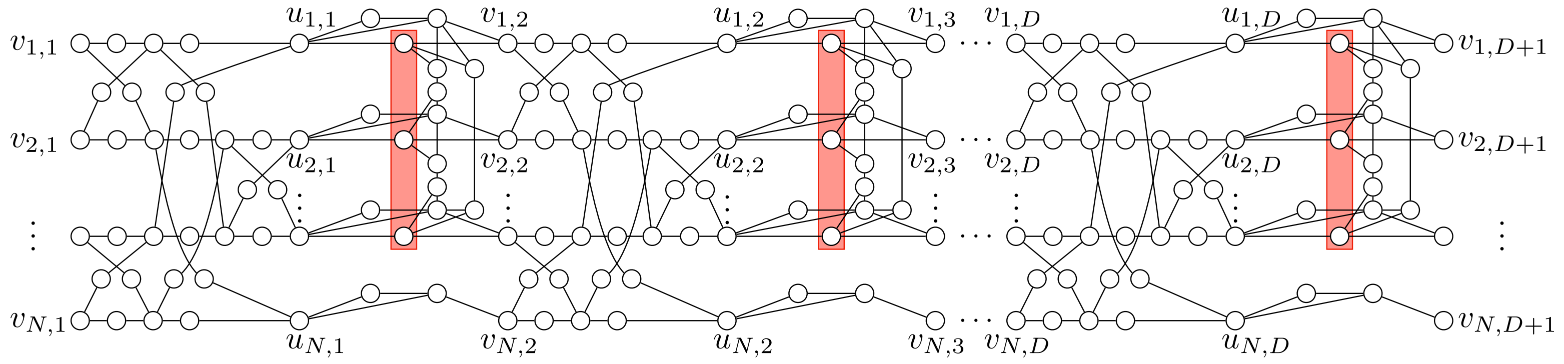
1. New perspective on seeking interesting entanglement structures

- Technologies for entanglement without geometrical constraints become available
- **Design entanglement arbitrarily to find out what we can experience within the law of quantum mechanics**.



2. Useful for realizing polynomial speedup

- Recommendation systems, NP-hard combinatorial optimization: **useful**
- Quantum machine learning (QML) without sparsity/low-rank assumptions

← except our recent work [2]: Exp speedup in QML without sparsity & low rank

3. Acceleration in various applications of MBQC and photonic QIP

- Polylog-overhead blind quantum computation/verification of quantum computation

Server (quantum)

Client (measurement + classical)

[2] H. Yamasaki, S. Subramanian, S. Sonoda, M. Koashi, NeurIPS 2020, arXiv:2004.10756

Summary

Polylog-overhead highly fault-tolerant MBQC for photonic systems

- **<u>Protocol for photonic MBQC that achieves poly-logarithmic overhead</u>**

  ← New entanglement structure for **low-overhead** qubit permutation

  ← **Optimized for photonic architectures**: univercal MBQC only by Pauli-X and Z measurements, **suitable for fault-tolerant MBQC with photonics**

- **<u>Highly fault-tolerant photonic MBQC protocol with polylog overhead</u>**

  ← Combining 7-qubit code with Gottesman-Kitaev-Preskill code (Not in this talk)

Open a new way toward realization of speedups including those polynomial

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