Extracting boundary CFT data from lattice models with tensor network renormalization

[1] SI, S. Morita, N. Kawashima, arXiv:1905.02351.
[2] SI, S. Morita, N. Kawashima, arXiv:1911.09907.
[3] SI, arXiv:2007.03182.

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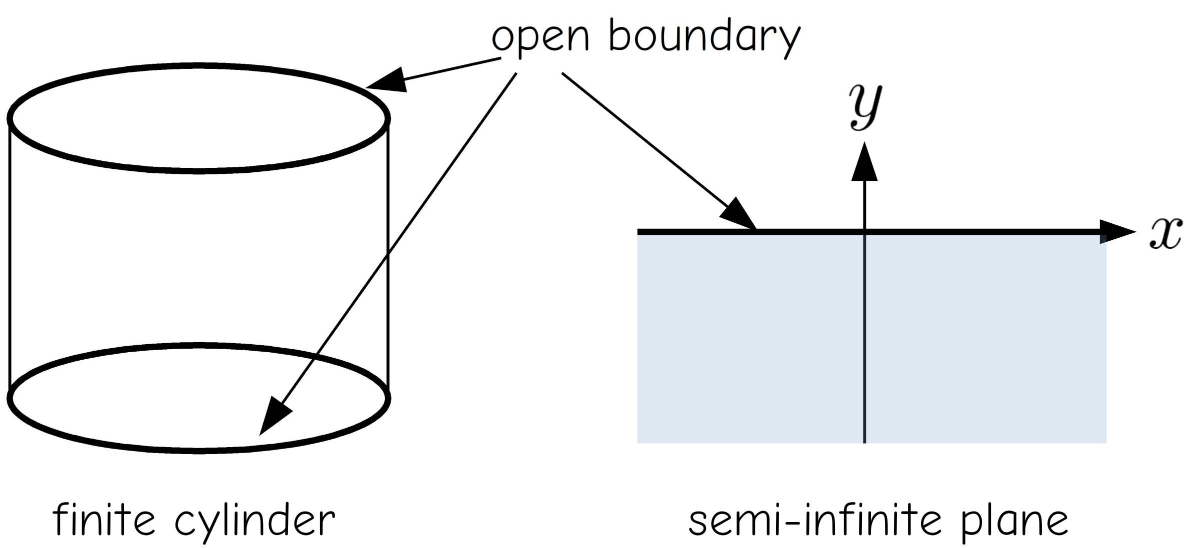
March 5, 2021 @YITP workshop

Boundary CFT (BCFT)

✔ Boundary CFT (BCFT) = CFT with **open boundary** [Cardy (1984)]



Boundary CFT (BCFT)

✔ Boundary CFT (BCFT) = CFT with **open boundary** [Cardy (1984)]

✔ BCFT is important in various regions of physics, such as...

- D-brane, AdS/BCFT
- impurity problem (Kondo effect)
- surface critical phenomena

✔ In this talk, we discuss numerical ways of studying BCFT_2 emergent on lattice models

critical lattice model with open boundary (UV) $\xrightarrow[\text{RG}]{}$ **BCFT** (IR)

Outline

✔ Review: **Tensor renormalization group (TRG)** methods

- TN representation and TRG
- Tensor Network Renormalization (TNR)
- Study CFT emergent on lattice models numerically

✔ Extension to **open-boundary** systems

- Extension to open-boundary systems
- Numerical results of Ising model

✔ Application to **surface critical phenomena**

- Surface critical phenomena of the tricritical Ising model
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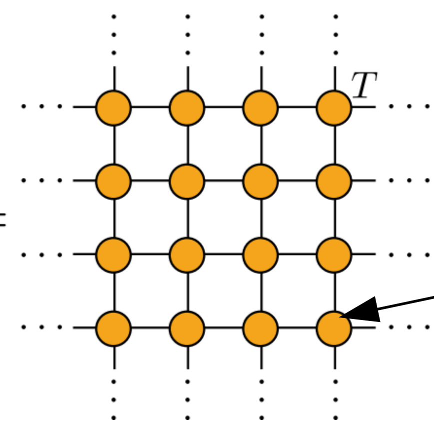
Tensor network representation

✔ **2D tensor network representation:**

Ex1.) **Euclidean path integral** of 1+1d quantum model
Partition function of 2d classical model

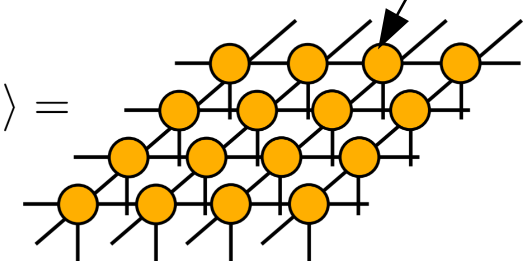
$$Z = \mathrm{tTr} \otimes \left[T_{\alpha\beta\gamma\delta}\right]^N =$$



Local information is implemented:
interaction b/w spins,
etc...

Ex.2) **PEPS** wave function

$$|\Psi\rangle = \mathrm{tTr} \otimes \left[T_{s_i}^{\alpha\beta\gamma\delta}\right]^N |s_1, s_2, \cdots, s_N\rangle =$$

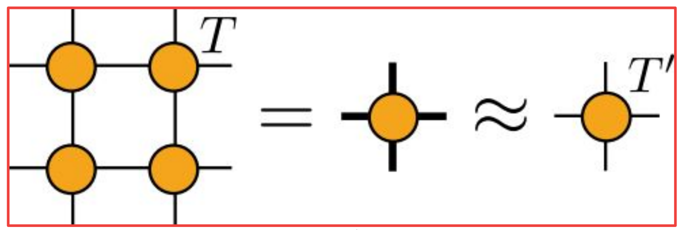


Can we extract many-body information from local tensor?

Tensor renormalization group

✔ Tensor renormalization group (**TRG**) [Levin, Nave (2007)]

- Kadanoff's real-space RG on tensor network



✔ Under RG, tensors converge to **fixed points** (FP)

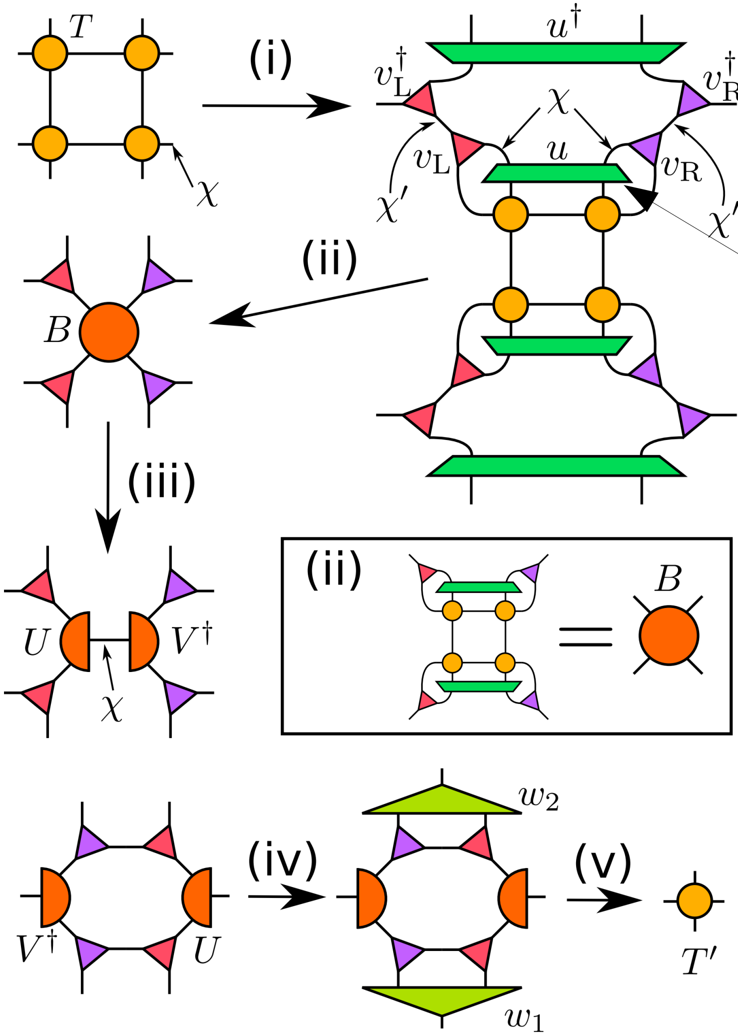
How to perform this approximation?

- In trivial phase, the FP tensor is trivial
- At criticality, the FP tensor carries information of CFT

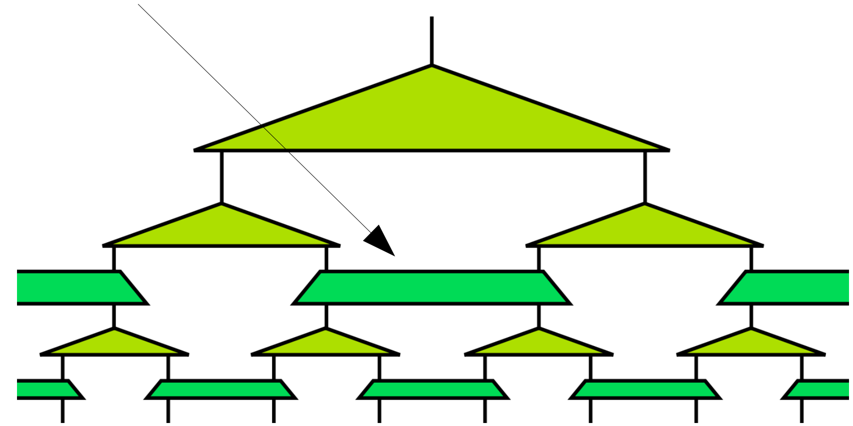
[Gu and Wen (2009)]

Tensor network renormalization

✔ Tensor network renormalization (TNR) [Evenbly and Vidal (2015)]

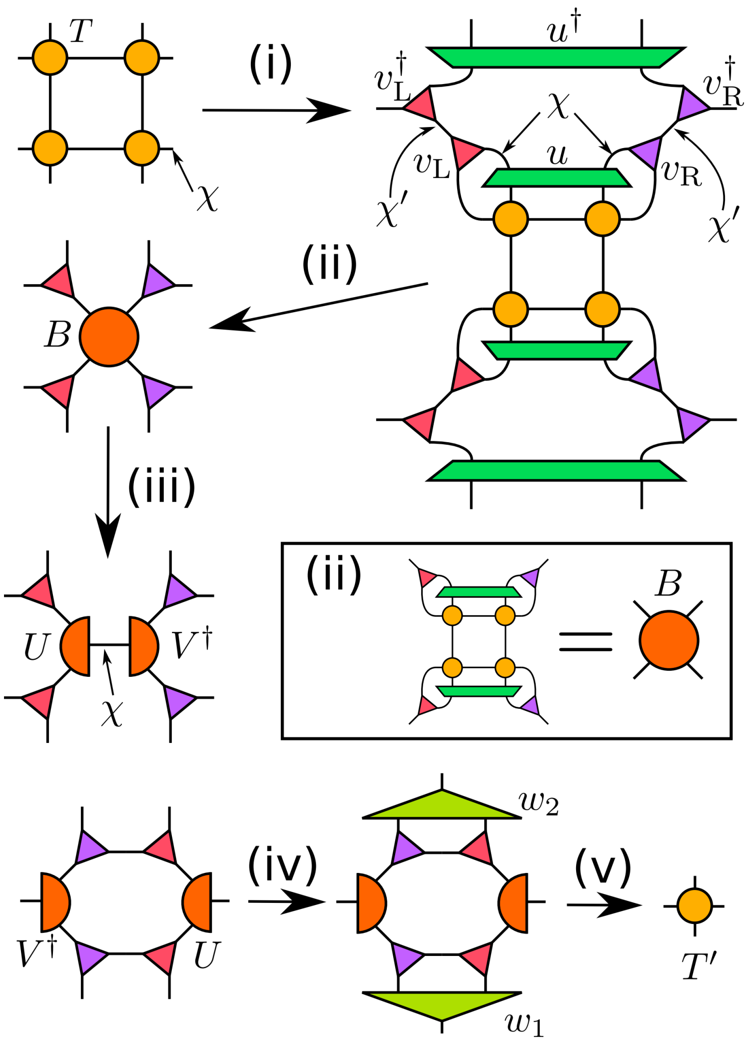


disentangler in MERA [Vidal (2007)]



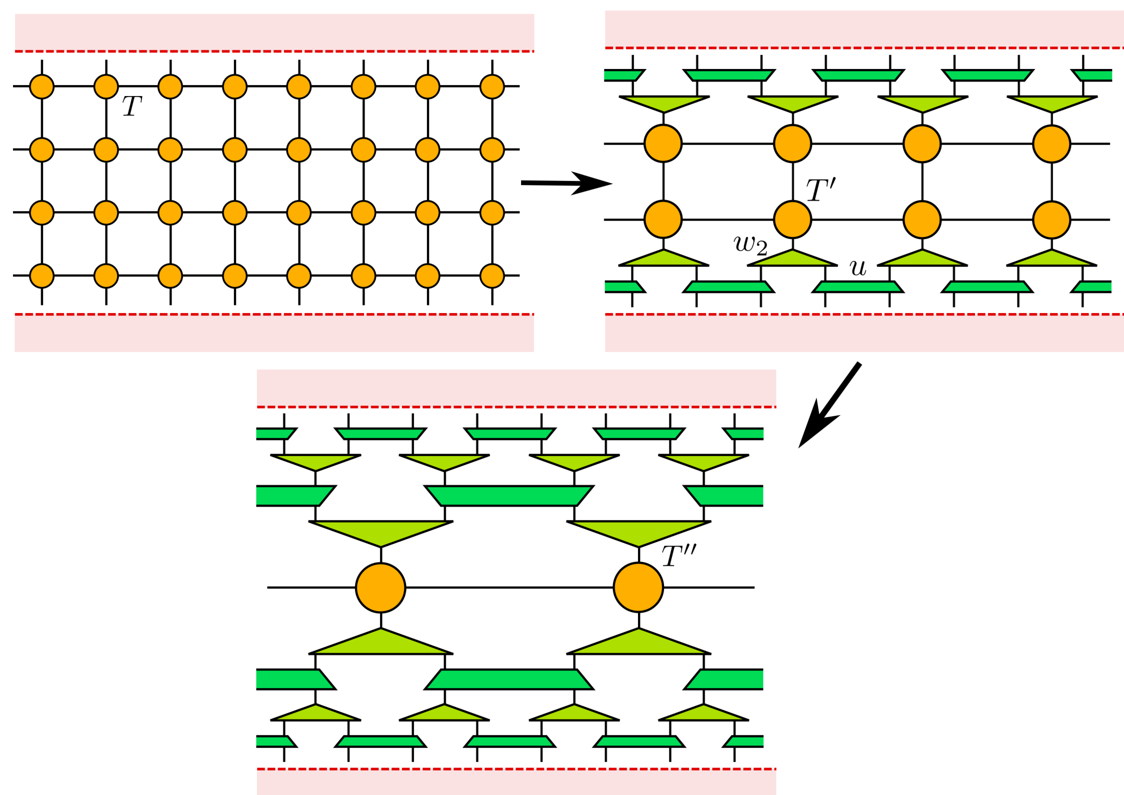
Tensor network renormalization

✔ Tensor network renormalization (TNR) [Evenbly and Vidal (2015)]



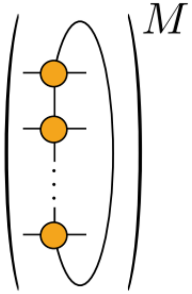
TNR generates MERA!
→ efficiently simulate critical systems

[Evenbly and Vidal (2016)]

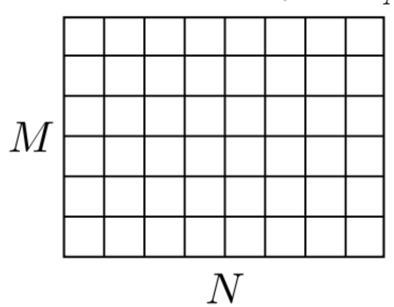


Fixed point tensor and CFT

✔ Extract CFT spectrum from FP tensor

$Z = \mathrm{tTr}$ 

N x M torus $(\tau = i\frac{M}{N})$



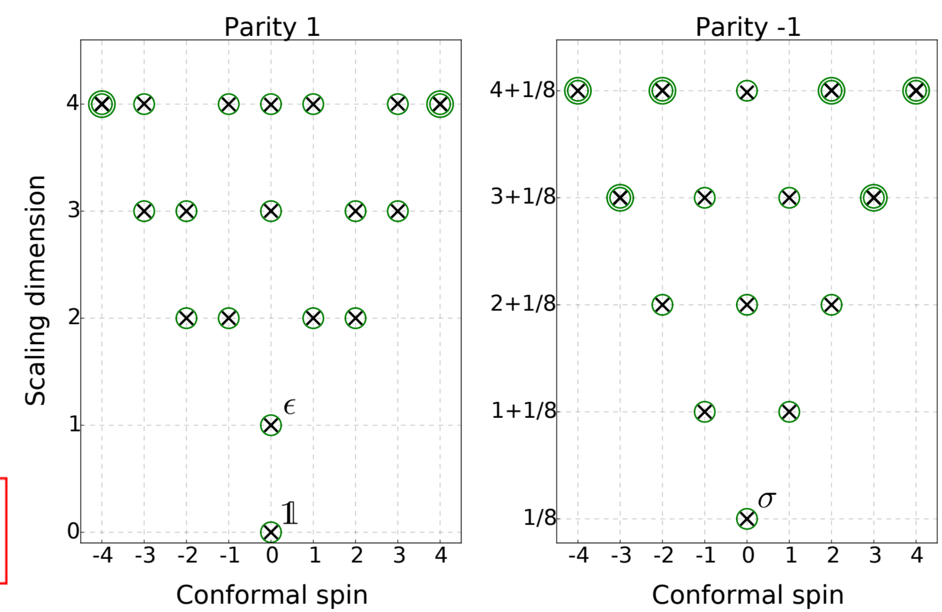
$$= \mathrm{Tr}\exp\left[-\frac{2\pi M}{N}\left(L_0 + \bar{L}_0 - \frac{c}{12}\right) + MNf + \mathcal{O}(M/N^{\gamma})\right] \quad (\gamma > 1)$$

$$\Rightarrow \boxed{\ln \lambda_k = -\frac{2\pi}{N}\left(\Delta_k - \frac{c}{12}\right) + Nf + \mathcal{O}(N^{-\gamma})}$$

eigenvalues of the transfer matrix

scaling dimensions

Ex.) 2d critical Ising model



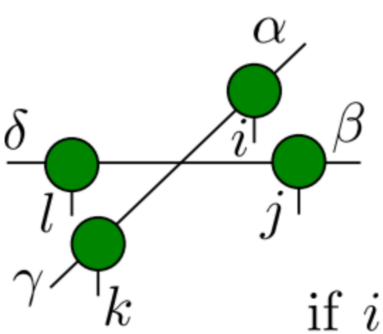
[Hauru et al. (2016)]

Able to compute accurate spectrum!

Phase transition of toric code

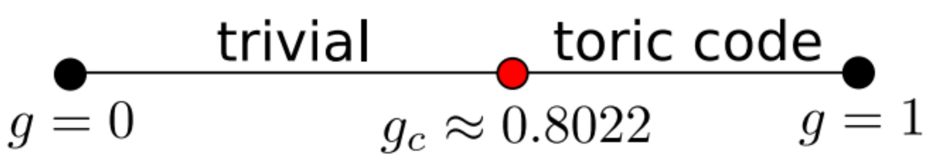
✔ TNR can also renormalize PEPS wave function:

Ex.) Z_2 toric code <-> trivial phase in 2d



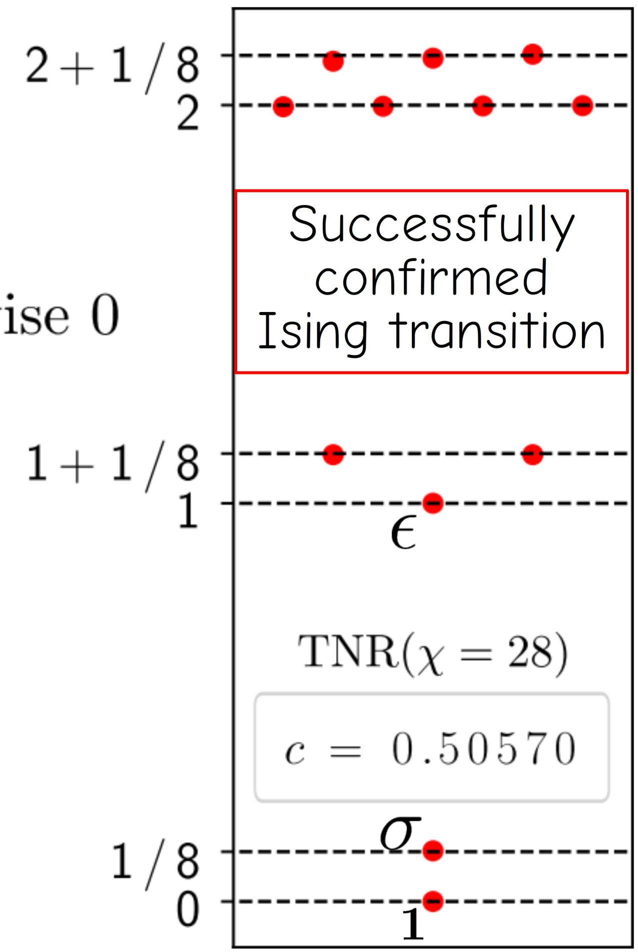
$$= \delta_{i\alpha}\delta_{j\beta}\delta_{k\gamma}\delta_{l\delta}g^{i+j+k+l}$$

$$\text{if } i+j+k+l = 0 \text{ mod } 2, \text{ otherwise } 0$$



H. He, H. Moradi, and X.-G. Wen, PRB **90**, 205114 (2014).

Renormalize $\langle\Psi|\Psi\rangle$, and
diagonalize the transfer matirx



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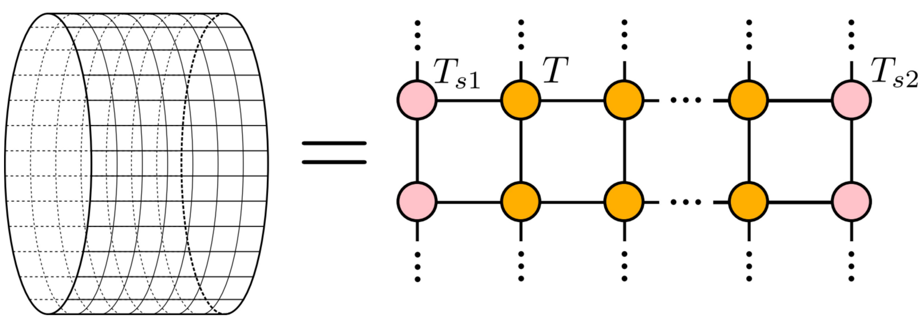
TNR for open-boundary system

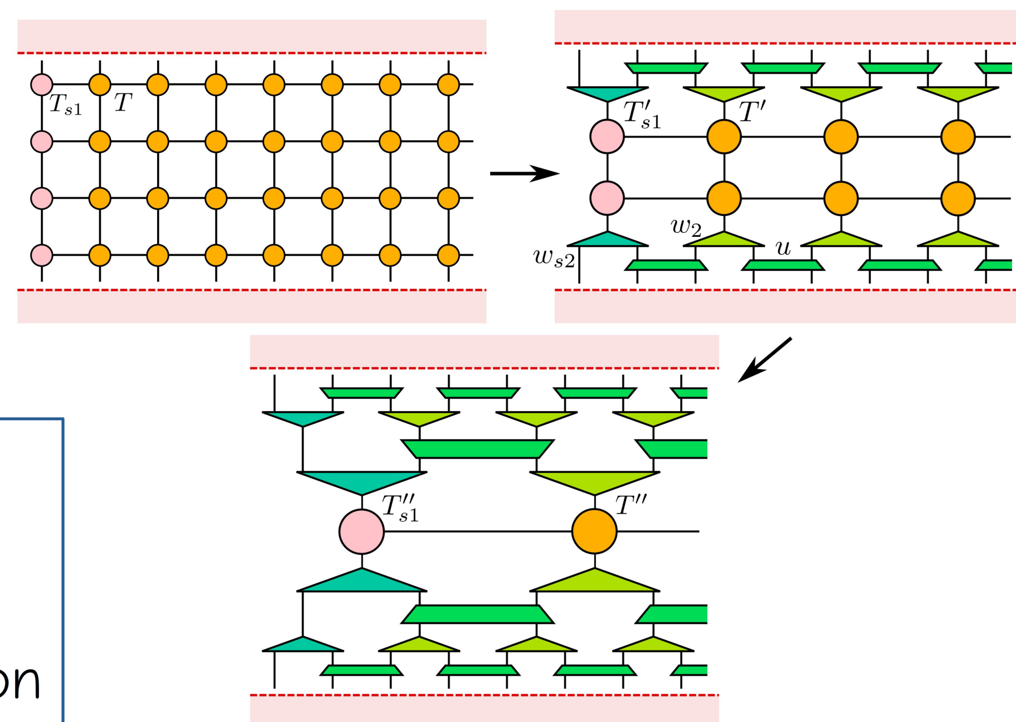
✔ Generalize TNR for open-boundary systems: [SI et al. (2020)]

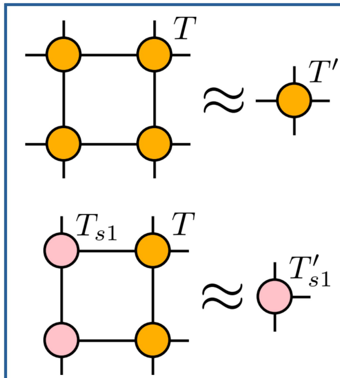
TN on a finite cylinder:

- T (bulk tensor)
- T_{s1} , T_{s2} (boundary tensor)



Boundary TNR yields boundary MERA
→ efficiently deal with critical system





Bulk RG is the same as ordinary TNR

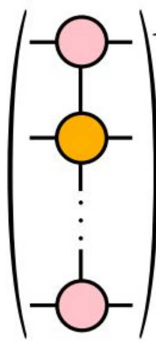
+

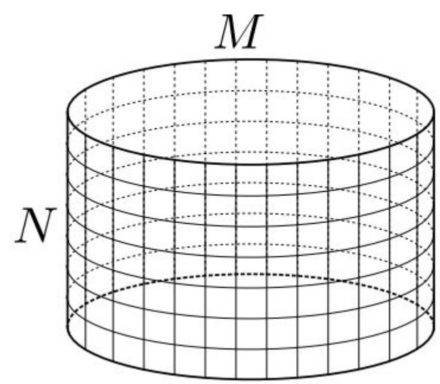
Consider renormalization of boundary tensors

BCFT spectrum from TNR

✔ The partition function on a finite cylinder at criticality:

$$Z = \mathrm{tTr}\left(\right)^M$$





$$= \mathrm{Tr}\ \exp\left[-\frac{\pi M}{N}\left(L_0 - \frac{c}{24}\right) + MNf + Mf_s + S_{AL} + \mathcal{O}(M/N^\gamma)\right]$$

$(\gamma > 1)$

eigenvalues of the transfer matrix

$$\Rightarrow \boxed{\ln \lambda_k = -\frac{\pi}{N}\left(\Delta_k - \frac{c}{24}\right) + Nf + f_s + \mathcal{O}(N^{-\gamma})}$$

scaling dimensions

* In BCFT, there is only a single copy of the Virasoro algebra

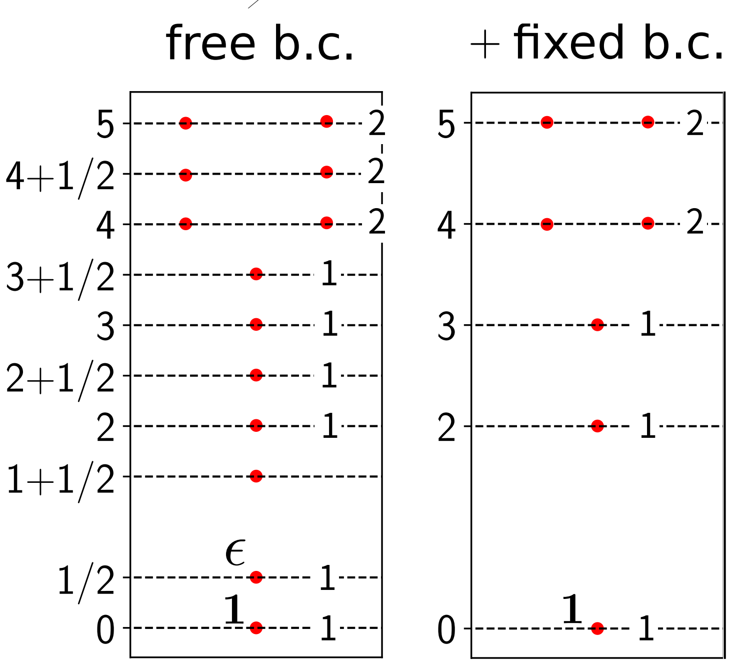
Application to 2d Ising BCFT

✔ In BCFT, <u>the operator content depends on the b.c.</u>

ex.) Though the primary fields for Ising CFT are {**1**, **ε**, **σ**},

$$Z_{\mathrm{free,free}} = \chi_{\mathbf{1}} + \chi_{\epsilon}$$

$$Z_{+,+} = Z_{-,-} = \chi_{\mathbf{1}}$$



Spectrum are consistent with BCFT conjecture!

[SI, S. Morita, N. Kawashima, (2020)]

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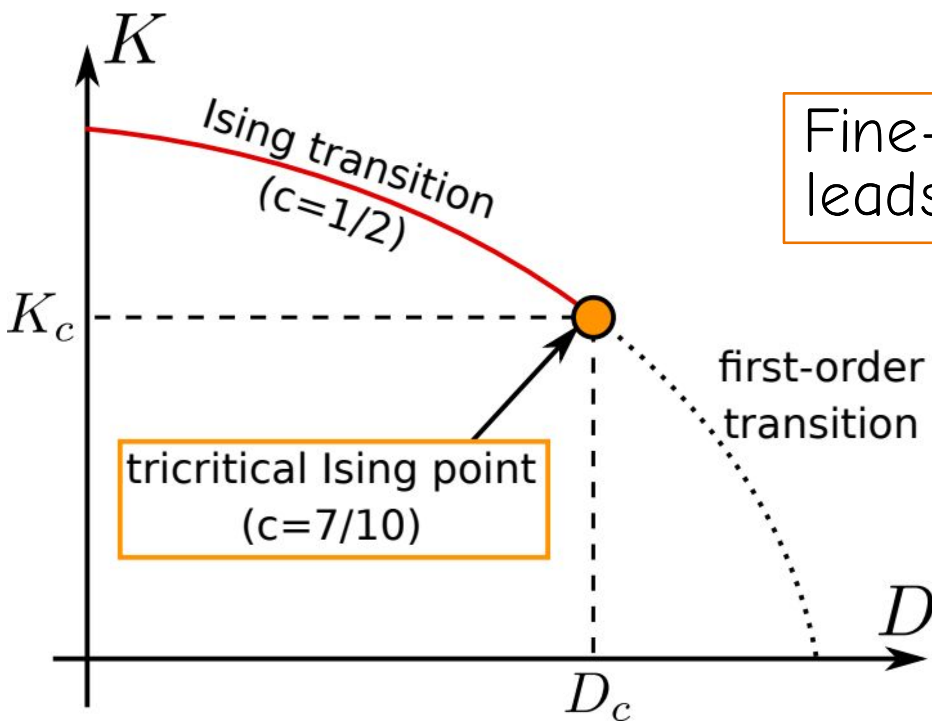
Tricritical Ising model

✔ 2d classical dilute Ising (Blume-Capel) model

[Blume (1966); Capel (1966)]

$$\beta\mathcal{H} = -K\sum_{\langle ij\rangle}\sigma_i\sigma_j - D\sum_i(\sigma_i)^2 \quad \text{where } \sigma_i = -1, 0, +1$$

- this model possesses a tricritical point with c=7/10

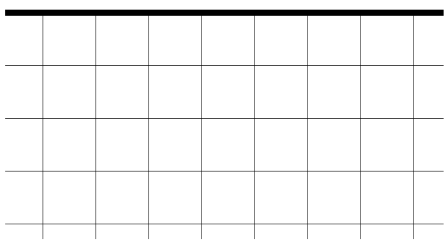


Fine-tuning parameters
leads to another universality class

Phase transition on 1d edges

- ✔ No phase transition in 1d classical systems (Peierls' argument)
- ✔ Then, how about 1d "edges" attached to 2d bulk?
- ✔ If the bulk is fine-tuned at tricritical Ising point, the 1d edge exhibit a phase transition

[Affleck (2000)]



semi-infinite plane

2d dilute Ising model (Blume-Capel model)

$$\beta\mathcal{H} = -K_c \sum_{\langle ij\rangle \in \text{bulk}} \sigma_i\sigma_j - D_c \sum_i \sigma_i^2 - K_s \sum_{\langle ij\rangle \in \text{edge}} \sigma_i\sigma_j \quad \text{where } \sigma = 0, \pm 1$$

1d classical chain is not interesting, but 1d classical surface can be interesting!

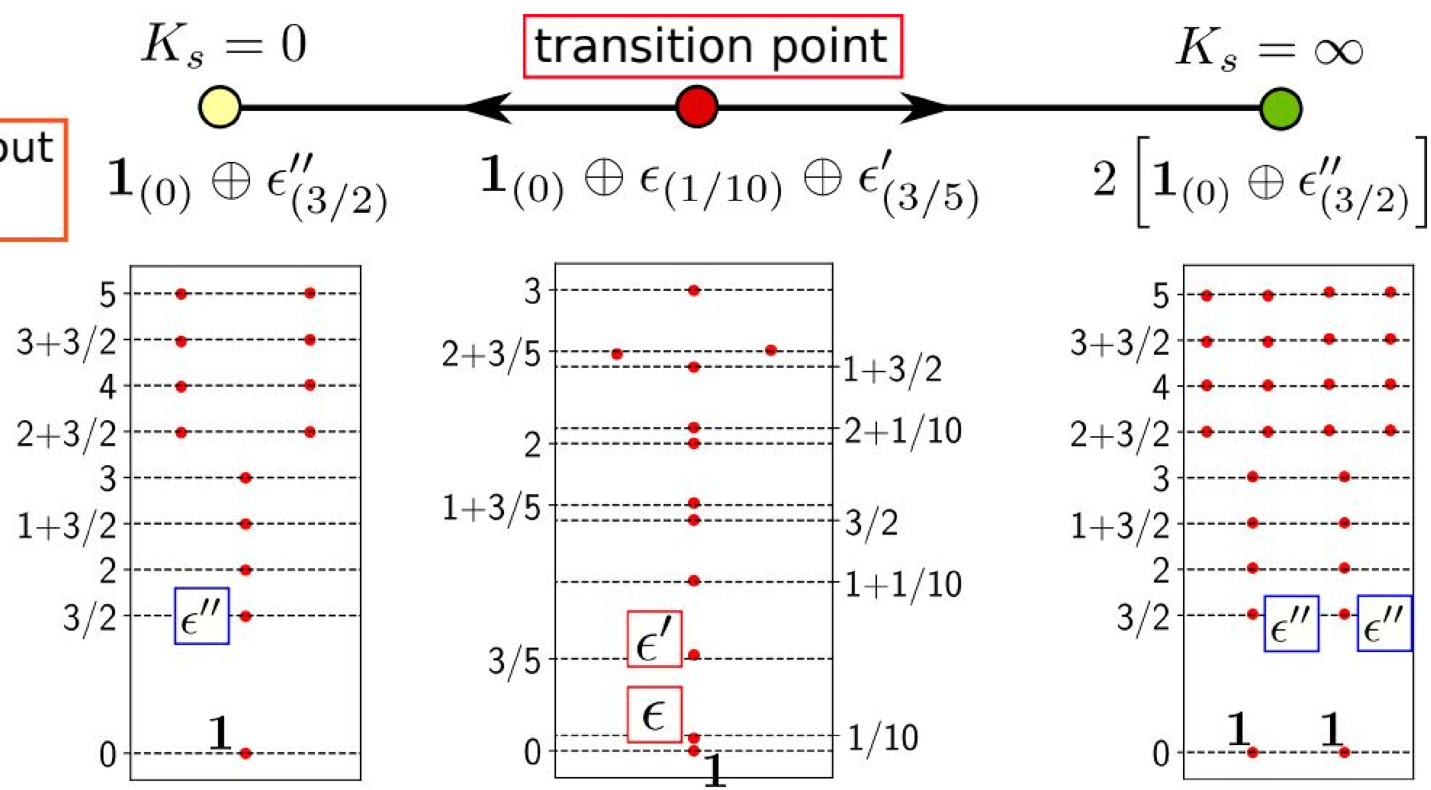
$K_s = 0$ free b.c. ordered b.c. $K_s = \infty$

Tri-critical Ising model

✔ Surface transition in tri-critical Ising model:

[Affleck (2000)]

BCFT conjecture about operator content



Spectrum are consistent with BCFT conjectures

[SI et al. (2020)]

TNR can be helpful to study surface critical phenomena through BCFT!

Take-home messages

✔ **TNR** is the numerical algorithm inspired by MERA, by which one can efficiently extract **2d CFT** data from

- 2d classical lattice models
- 1+1d quantum lattice models
- PEPS wave function of 2d quantum lattice models

✔ We extend TNR for **open-boundary system**, by which we can compute accurate **BCFT** spectrum from lattices

✔ Interesting future issues:

- other BCFT data (boundary/bulk-boundary OPE coefficients)?
- higher dimensions?

[1] arXiv:1905.02351; Phys. Rev. B 100, 35449 (2019).
[2] arXiv:1911.09907; Phys. Rev. B 101, 155418 (2020).
[3] arXiv:2007.03182; to appear in J. Stat. Phys.