Newtonian system in a huge void universe model

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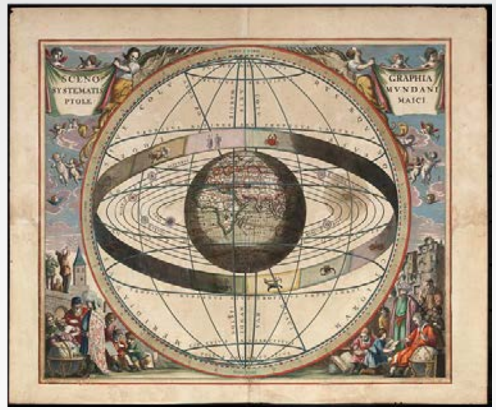
RN, Nakao, Yoo, arXiv:1409.2099 ***today***

Contents

The Copernican Principle:

We are not living at a special position in the universe.



The Copernican Principle & Observed isotropy of CMB

⇒ homogeneous and isotropic FLRW universe model

- Fundamental working hypothesis in modern cosmology
- Technological developments may enable us to test the CP

Effects of the inhomogeneity to observations

The non-Copernican cosmological model

Observed isotropy ⇒ **isotropic (radial) inhomogeneity**

- An alternative model to dark energy Tomita(1999), Célérier(1999), ⋯

  $\Omega_\Lambda = 0.692 \pm 0.010 \pm ? = 0?$

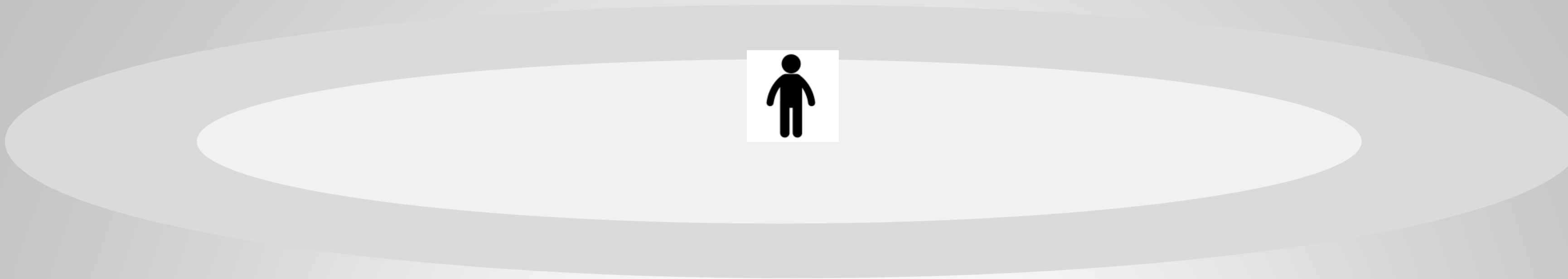
- Effects to the energy condition

  dark energy & inhomogeneity Valkenburg, Kunz, Marra(2013)

  $\Omega_\Lambda = 0.692, \mathrm{w} = \frac{\mathrm{p}}{\rho} = -1.13^{+0.13}_{-0.10} \pm ?$

It is an essential task in modern precision cosmology to eliminate these systematic errors.

A huge void universe model



- The non-Copernican model
- We live close to a center of a huge void
- horizon-scale, highly nonlinear
- Observational tests of the void model
  - ✓ SN Ia distance-redshift relation
  - ✓ CMB acoustic peaks
  - ✓ ⋯
  - ✓ **The large-scale structures**

Contents

The Cosmological Newtonian system

Well developed nonlinear structures; galaxies, clusters.

The Cosmological Newtonian system in the FLRW model:

1. the length scale of the system is much less than the horizon;

$$\ell_{\rm N} \ll H^{-1}$$

2. the relative velocities are much less than the speed of light and the energy densities much larger than the stresses;

$$|\boldsymbol{v}_{\rm N}| \ll c, \qquad p \ll \rho$$

3. the self-gravity of the system is not negligible but very weak;

$$|\Phi_{\rm N}| \ll 1$$

The Newtonian system in the void model

***The Newtonian system in the void model*:**

1. **the length scale of the system is much less than the spacetime curvature radius of the background model;**

   $$\ell_{\mathrm{N}} \ll \mathcal{R}$$

2. $|\boldsymbol{v}_{\mathrm{N}}| \ll c, \qquad p \ll \rho$
3. $|\Phi_{\mathrm{N}}| \ll 1$

In the case of the huge void model, the size of the void is the same order as the cosmological horizon.

$$L_{\mathrm{void}} \sim H^{-1} \sim \mathcal{R}$$

Overview of our approximation scheme

The system of our interest:

A huge void + Newtonian perturbations (galaxies, clusters, etc.).

We introduce small parameters:

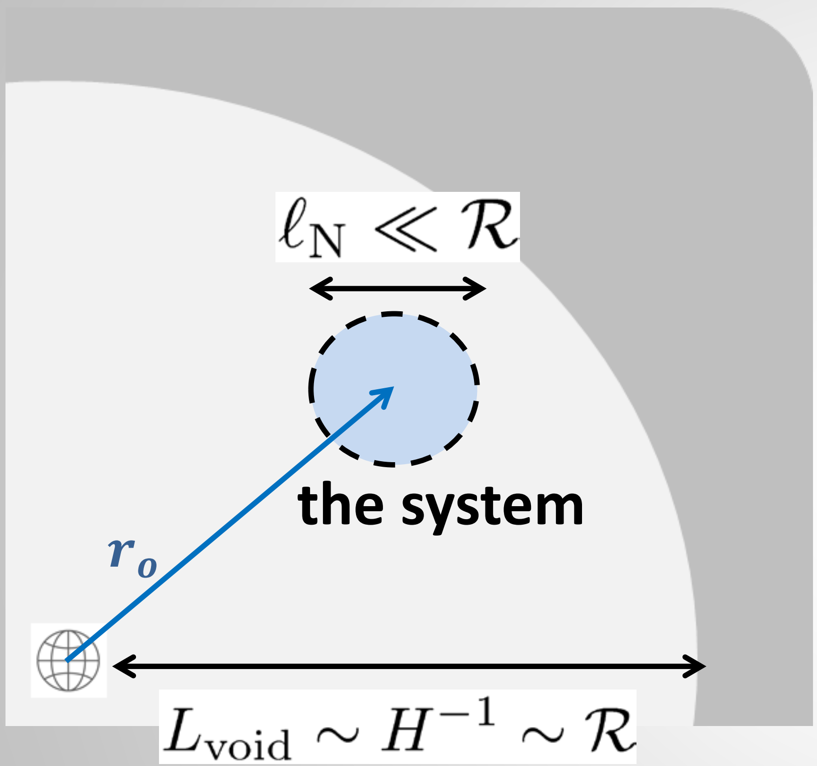
$$\epsilon := \frac{|\boldsymbol{v}_\mathrm{N}|}{c}, \qquad \kappa := \frac{\ell_\mathrm{N}}{\mathcal{R}}.$$

Then, we derive basic equations for the Newtonian system in the void model as follows.

1. We treat the background void universe model in the system in a perturbative manner by using κ.
2. We add the Newtonian perturbations to the void model, and expand them by using ϵ and κ.

Contents

The Fermi-normal coordinates

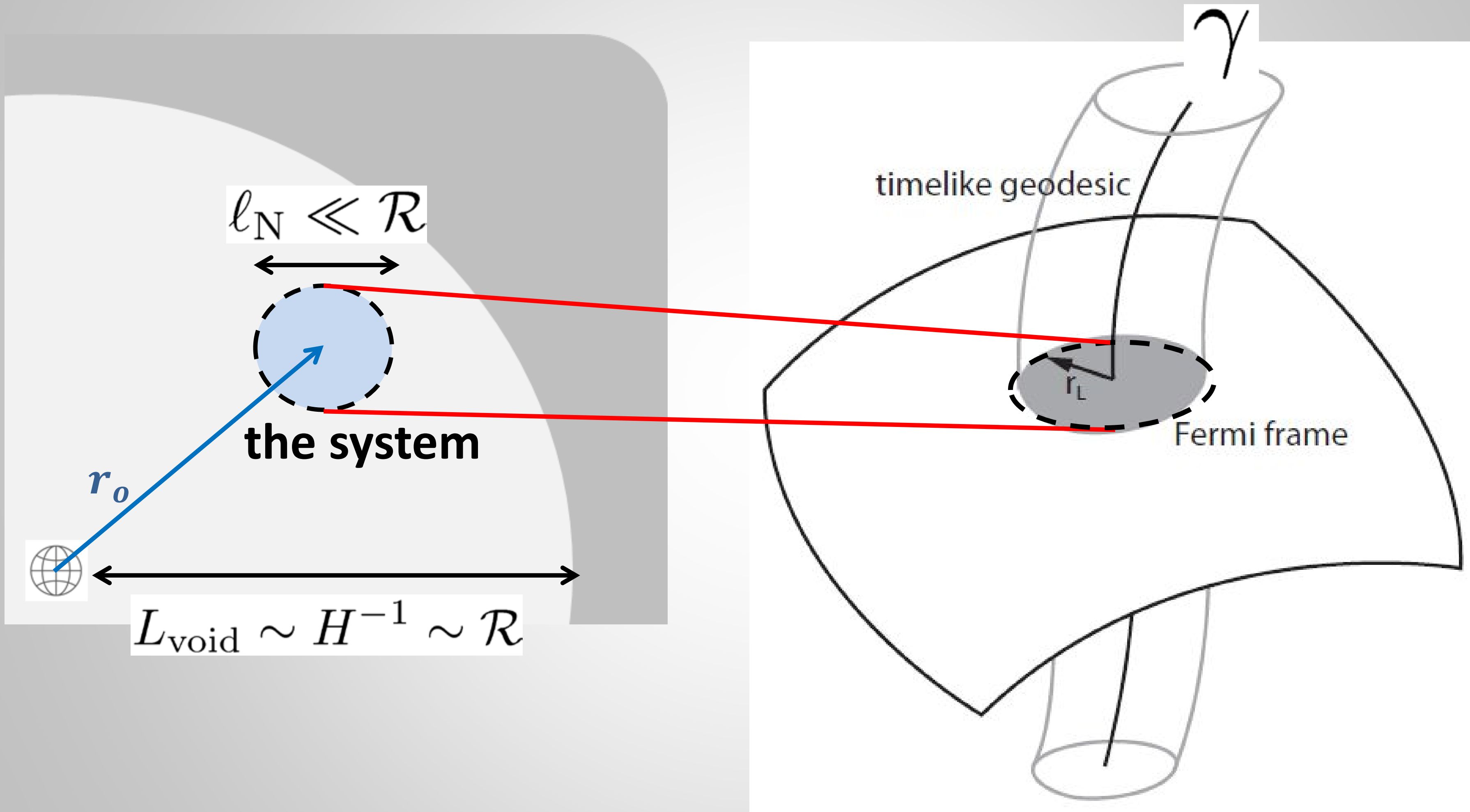


The curvature is almost spatially constant within the system.

⇩

We treat the background model in a perturbative manner by using the Fermi-normal coordinates.

The Fermi-normal coordinates



From Baldauf et al. JCAP 1110, 031 (2011)

Lemaître-Tolman-Bondi(LTB) spacetimes

the void universe model (dust, spherical) ⇒ LTB spacetimes

metric & stress energy tensor

$$
\begin{aligned}
ds^2 &= -dt'^2 + a_{||}^2(t',r')\frac{dr'^2}{1-k(r')} + a_{\perp}^2(t',r')r'^2(d\theta'^2 + \sin^2\theta' d\phi'^2), \\
T^{\mu\nu} &= \rho'(t',r')u^{\mu'}u^{\nu'}; \quad u^{\mu} = (1,0,0,0).
\end{aligned}
$$

- **The isometries in LTB are less than those in FLRW**

Radial-Hubble & Transverse-Hubble

$$
H_{||}(t',r') = \frac{\partial_t a_{||}}{a_{||}}, \quad H_{\perp}(t',r') = \frac{\partial_t a_{\perp}}{a_{\perp}}
$$

The Fermi-normal coordinates in the void model

the Fermi-normal coordinates $x^\mu = (x^0, x^i)$

metric
$$
\begin{aligned}
g_{00} &= -1 - \hat{R}_{0i0j} x^i x^j + \mathcal{O}\left(|\boldsymbol{x}|^3\right), \\
g_{0i} &= -\frac{2}{3} \hat{R}_{0jik} x^j x^k + \mathcal{O}\left(|\boldsymbol{x}|^3\right), \\
g_{ij} &= \delta_{ij} - \frac{1}{3} \hat{R}_{ikjl} x^k x^l + \mathcal{O}\left(|\boldsymbol{x}|^3\right),
\end{aligned}
$$

$$
\text{where} \quad \hat{R}_{\mu\nu\rho\sigma}(x^0) := e^{\alpha'}_{(\mu)} e^{\beta'}_{(\nu)} e^{\gamma'}_{(\rho)} e^{\delta'}_{(\sigma)} R_{\alpha'\beta'\gamma'\delta'}\big|_\gamma = \mathcal{O}\left(\mathcal{R}^{-2}\right),
$$

- ✓ the origin; $x^i = 0 \Leftrightarrow (r', \theta', \phi') = (r_o{}', \theta_o{}', \phi_o{}')$
- ✓ In the domain of our interest, $|\boldsymbol{x}| = \mathcal{O}(\ell_{\mathrm{N}}) \ll \mathcal{R}$

We can write the metric as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa^2 h^{\mathrm{B}}_{\mu\nu} + \mathcal{O}(\kappa^3).$

$$
\text{where} \quad \kappa := \frac{\ell_{\mathrm{N}}}{\mathcal{R}}.
$$

The Fermi-normal coordinates in the void model

the energy density

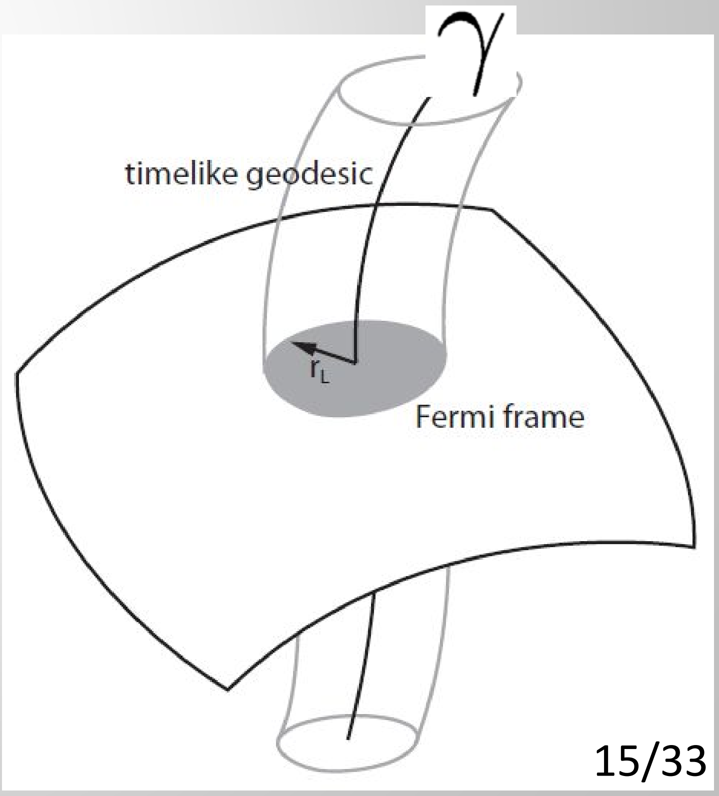
$$\rho(x^\mu) = \underline{\rho_{\rm B}(x^0)} + \kappa \left[\left(\frac{\sqrt{1-k(r')}}{\partial_{r'} R(t',r')} \right) \partial_{r'} \rho'(t',r') \right] \Bigg|_\gamma x^1 + \mathcal{O}\left(\kappa^2 \rho_{\rm B}\right),$$

where $\rho_{\rm B}(x^0) := \rho'(t',r')|_\gamma$.

the 4-velocity

$$\begin{aligned} u^0(x^\mu) &= 1 + \mathcal{O}(\kappa^2), \\ u^1(x^\mu) &= \underline{\kappa H_{||}^{\rm B}(x^0) x^1} + \mathcal{O}(\kappa^2), \\ u^2(x^\mu) &= \underline{\kappa H_{\perp}^{\rm B}(x^0) x^2} + \mathcal{O}(\kappa^2), \\ u^3(x^\mu) &= \underline{\kappa H_{\perp}^{\rm B}(x^0) x^3} + \mathcal{O}(\kappa^2), \end{aligned}$$

$$\begin{aligned} H_{||}^{\rm B}(x^0) &:= H_{||}(t',r')|_\gamma , \\ H_{\perp}^{\rm B}(x^0) &:= H_{\perp}(t',r')|_\gamma . \end{aligned}$$



The background equations in the Newtonian system

At the leading order of κ, we have the following relation:

$$\begin{cases} \partial_0 \rho_{\rm B} + \partial_j \left(\rho_{\rm B} v_{\rm B}^j \right) = 0. \\ \partial_0 v_{\rm B}^i + v_{\rm B}^j \partial_j v_{\rm B}^i = -\partial_i \Phi_{\rm B}. \\ \nabla^2 \Phi_{\rm B} = 4\pi \rho_{\rm B}. \end{cases}$$

where $\Phi_{\rm B}(x^\mu) := -\frac{1}{2} h_{00}^{\rm B}(x^\mu).$

- The tidal force produced by the void causes the anisotropic volume expansion.
- The radial inhomogeneities of the energy density and the volume expansion do not appear at the leading order.

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A huge void *+ Newtonian perturbations*

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + \kappa^2\ h^{\mathrm{B}}_{\mu\nu} + \mathcal{O}(\kappa^3) \underline{+ h^{\mathrm{N}}_{\mu\nu}} \\
\rho &= \rho_{\mathrm{B}} + \mathcal{O}\left(\kappa\rho_{\mathrm{B}}\right) \underline{+ \rho_{\mathrm{N}}} \\
v^i &= \kappa\ v^i_{\mathrm{B}} + \mathcal{O}(\kappa^2) \underline{+ v^i_{\mathrm{N}}}
\end{aligned}
$$

Ordering of the magnitude of perturbations

There are two time-scales in the system;

$$\mathcal{T} := \frac{\mathcal{R}}{c} \quad \text{and} \quad t_{\mathrm{N}} := \frac{\ell_{\mathrm{N}}}{v_{\mathrm{N}}}.$$

By their definition, we have

$$\frac{\mathcal{T}}{t_{\mathrm{N}}} = \frac{\epsilon}{\kappa}. \quad \text{where} \quad \epsilon := \frac{|\boldsymbol{v}_{\mathrm{N}}|}{c}, \quad \kappa := \frac{\ell_{\mathrm{N}}}{\mathcal{R}}.$$

For the Newtonian perturbation ψ_N, we have

$$\frac{\partial \psi_{\mathrm{N}}}{\partial x^0} = \begin{cases} \mathcal{O}\left(\epsilon \, \frac{\partial \psi_{\mathrm{N}}}{\partial x^i}\right) & \text{for} \quad \epsilon \gg \kappa \quad (\mathcal{T} \gg t_{\mathrm{N}}), \\ \mathcal{O}\left(\kappa \, \frac{\partial \psi_{\mathrm{N}}}{\partial x^i}\right) & \text{for} \quad \epsilon \ll \kappa \quad (\mathcal{T} \ll t_{\mathrm{N}}), \end{cases}.$$

Classifying into three cases; $\epsilon \gg \kappa, \epsilon \simeq \kappa, \epsilon \ll \kappa$

where $\quad \epsilon := \frac{|\boldsymbol{v}_{\mathrm{N}}|}{c}, \quad \kappa := \frac{\ell_{\mathrm{N}}}{\mathcal{R}}.$

Examples of those three cases:

- $\epsilon \gg \kappa$

  The solar system; $\epsilon \simeq 10^{-4} \gg \kappa \simeq 10^{-15}$

- $\epsilon \simeq \kappa$

  the cluster of galaxies; $\epsilon \simeq \kappa \simeq 10^{-3}$

- $\epsilon \ll \kappa$

  the BAO; $\epsilon \simeq 10^{-3} \ll \kappa \simeq 10^{-2}$

In the case of $\epsilon \gg \kappa$

Basic equations governing the Newtonian perturbations at the leading order:

$$\begin{aligned}
\partial_0 \rho_{\rm N} + \partial_i \left(\rho_{\rm N} v^i_{\rm N} \right) &= 0, \\
\partial_0 v^i_{\rm N} + v^j_{\rm N} \partial_j v^i_{\rm N} &= -\partial^i \Phi_{\rm N}, \\
\nabla^2 \Phi_{\rm N} &= 4\pi \rho_{\rm N}.
\end{aligned}$$

where $\Phi_{\rm N} := -h^{\rm N}_{00}/2$

- The effects of the background void model do not appear up to the leading order.

In the case of $\epsilon \simeq \kappa$

Basic equations at the leading order:

$$
\begin{aligned}
\partial_0 \rho_{\mathrm{N}} + \partial_i \left(\rho_{\mathrm{N}} v_{\mathrm{N}}^i\right) + \partial_i \left(\rho_{\mathrm{N}} v_{\mathrm{B}}^i\right) + \partial_i \left(\rho_{\mathrm{B}} v_{\mathrm{N}}^i\right) &= 0, \\
\partial_0 v_{\mathrm{N}}^i + v_{\mathrm{N}}^j \partial_j v_{\mathrm{N}}^i + v_{\mathrm{N}}^j \partial_j v_{\mathrm{B}}^i + v_{\mathrm{B}}^j \partial_j v_{\mathrm{N}}^i &= -\partial^i \Phi_{\mathrm{N}}, \\
\nabla^2 \Phi_{\mathrm{N}} &= 4\pi \rho_{\mathrm{N}}.
\end{aligned}
$$

- The effects of the background void model appear through $\boldsymbol{\rho_B}$ **and** $\boldsymbol{v_B^i}$**.**
- Non-linear terms of the Newtonian perturbations exist, and the equations may be studied by **the N-body numerical simulations.**

In the case of $\epsilon \ll \kappa$

Basic equations at the leading order:

$$\begin{aligned} \partial_0 \rho_{\mathrm{N}} + \partial_i \left(\rho_{\mathrm{N}} v^i_{\mathrm{B}}\right) + \partial_i \left(\rho_{\mathrm{B}} v^i_{\mathrm{N}}\right) &= 0, \\ \partial_0 v^i_{\mathrm{N}} + v^j_{\mathrm{N}} \partial_j v^i_{\mathrm{B}} + v^j_{\mathrm{B}} \partial_j v^i_{\mathrm{N}} &= -\partial^i \Phi_{\mathrm{N}}, \\ \nabla^2 \Phi_{\mathrm{N}} &= 4\pi \rho_{\mathrm{N}}. \end{aligned}$$

- The effects of the background void model appear through $\boldsymbol{\rho_B}$ **and** $\boldsymbol{v_B^i}$**.**
- Non-linear terms of the Newtonian perturbations **do not** exist, and we can easily solve the equations numerically.
- We call the system ***the Newtonian linear perturbations.***

Contents

rewriting the basic equations

- expansion, shear, vorticity :

$$\partial_j v_i = \frac{1}{3}\Theta\delta_{ij} + \sigma_{\langle ij\rangle} + \omega_{[ij]}$$

- The Lagrangian coordinates q^μ for the background :

$$\frac{\partial}{\partial q^0} = \partial_0 + v_{\mathrm{B}}^j \partial_j \quad \text{and} \quad \frac{\partial}{\partial q^i} = \partial_i.$$

- The Fourier transform for perturbations :

$$\delta_{\mathrm{N}}(q^0, q^i) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{ik_j q^j}\,\tilde{\delta}_{\mathrm{N}}(q^0, k^i).$$

The Newtonian linear perturbations

$$
\begin{aligned}
\frac{\partial \tilde{\delta}_{\rm N}}{\partial q^0} &= -\tilde{\Theta}_{\rm N}, \\
\frac{\partial \tilde{\Theta}_{\rm N}}{\partial q^0} &= -\frac{2}{3}\Theta_{\rm B}\tilde{\Theta}_{\rm N} - 2\delta^{ij}\delta^{kl}\sigma^{\rm B}_{ij}\tilde{\sigma}^{\rm N}_{ji} - 4\pi\rho_{\rm B}\tilde{\delta}_{\rm N}, \\
\frac{\partial \tilde{\sigma}^{\rm N}_{ij}}{\partial q^0} &= -\frac{2}{3}\Theta_{\rm B}\tilde{\sigma}^{\rm N}_{ij} - \frac{2}{3}\tilde{\Theta}_{\rm N}\sigma^{\rm B}_{ij} - 2\left(\delta^{kl}\sigma^{\rm B}_{k(i}\tilde{\sigma}^{\rm N}_{j)l} - \frac{1}{3}\delta_{ij}\delta^{kl}\delta^{mn}\sigma^{\rm B}_{km}\tilde{\sigma}^{\rm N}_{ln}\right) \\
&+ \left(k_i k_j - \frac{1}{3}k^2\delta_{ij}\right)\tilde{\Phi}_{\rm N}, \\
\frac{\partial \tilde{\omega}^{\rm N}_{ij}}{\partial q^0} &= -\frac{2}{3}\Theta_{\rm B}\tilde{\omega}^{\rm N}_{ij} + 2\delta^{kl}\sigma^{\rm B}_{k[i}\tilde{\omega}^{\rm N}_{j]l}, \\
-k^2\tilde{\Phi}_{\rm N} &= 4\pi\rho_{\rm B}\tilde{\delta}_{\rm N},
\end{aligned}
$$

- The ordinary differential equations with respect to q^0.
- **Fourier modes are decoupled with the other modes.**

We note that linear perturbation equations in LTB cannot be reduced to a decoupled set of ordinary differential equations.

Evolution of density contrast

$$\frac{\partial \tilde{\delta}_{\rm N}}{\partial q^0} = -\tilde{\Theta}_{\rm N},$$

$$\frac{\partial \tilde{\Theta}_{\rm N}}{\partial q^0} = -\frac{2}{3}\Theta_{\rm B}\tilde{\Theta}_{\rm N} - \underline{6\sigma_{\rm B}}\tilde{\sigma}^{\rm N}_{11} - 4\pi\rho_{\rm B}\tilde{\delta}_{\rm N},$$

$$\frac{\partial \tilde{\sigma}^{\rm N}_{11}}{\partial q^0} = -\frac{4}{3}\sigma_{\rm B}\tilde{\Theta}_{\rm N} - \frac{2}{3}\Theta_{\rm B}\tilde{\sigma}^{\rm N}_{11} - \underline{2\sigma_{\rm B}}\tilde{\sigma}^{\rm N}_{11} - 4\pi\rho_{\rm B}\left(\mu^2 - \frac{1}{3}\right)\tilde{\delta}_{\rm N},$$

where $\Theta_{\rm B}(x^0) = H^{\rm B}_{\parallel}(x^0) + 2H^{\rm B}_{\perp}(x^0)$, $\underline{\sigma_{\rm B}(x^0) := \frac{1}{3}\left[H^{\rm B}_{\parallel}(x^0) - H^{\rm B}_{\perp}(x^0)\right]}$,

$$\mu := k_1/k$$

Due to the existence of **the background shear**, the density contrast couples with the shear perturbations.

The growth factor

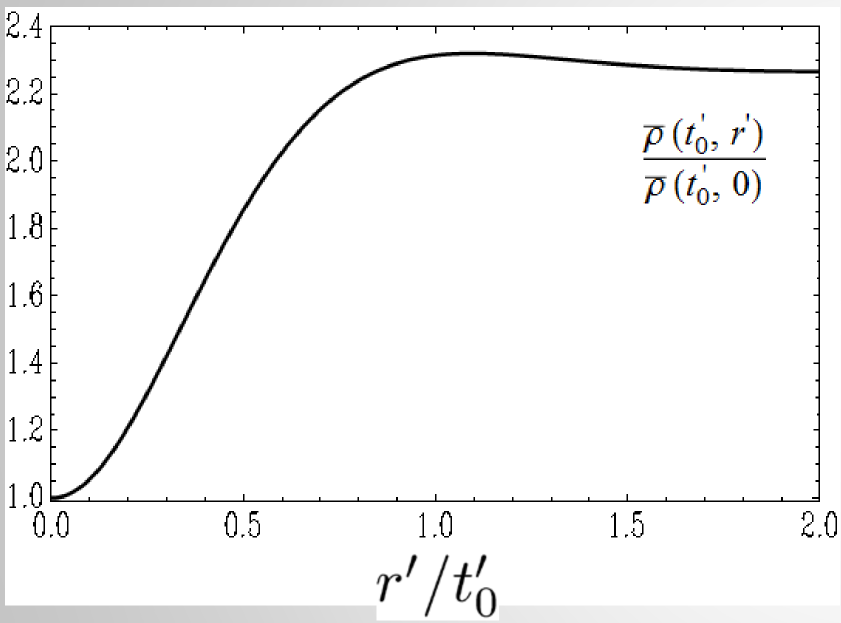
The growth factor depends on the direction of the wave vector and the radial position of the system.

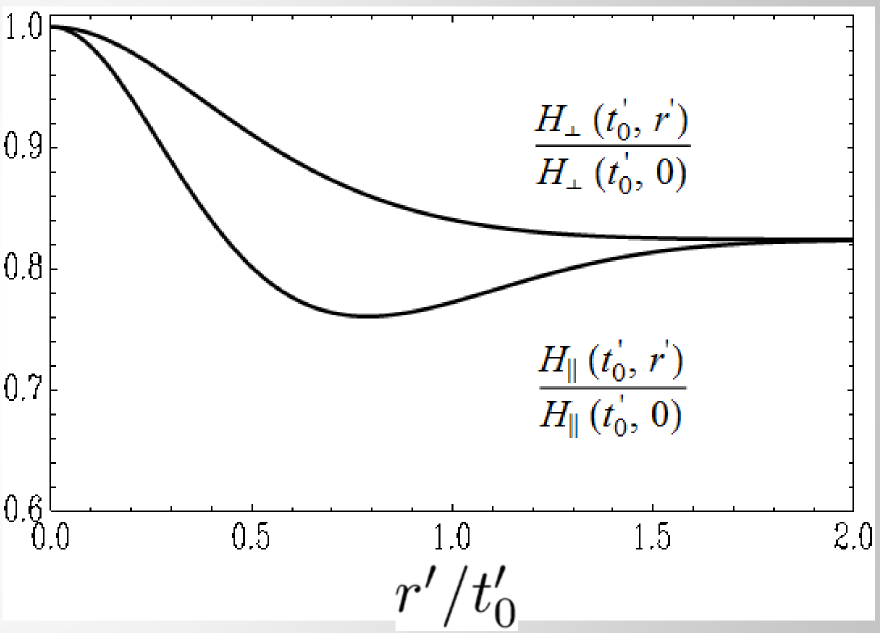
$$\tilde{\delta}_{\mathrm{N}}(q^0, k^i; r'_{\mathrm{o}}) \;=\; \underline{D(q^0, \mu; r'_{\mathrm{o}})}\delta(k^i).$$

where $\mu := k_1/k$

a huge void model

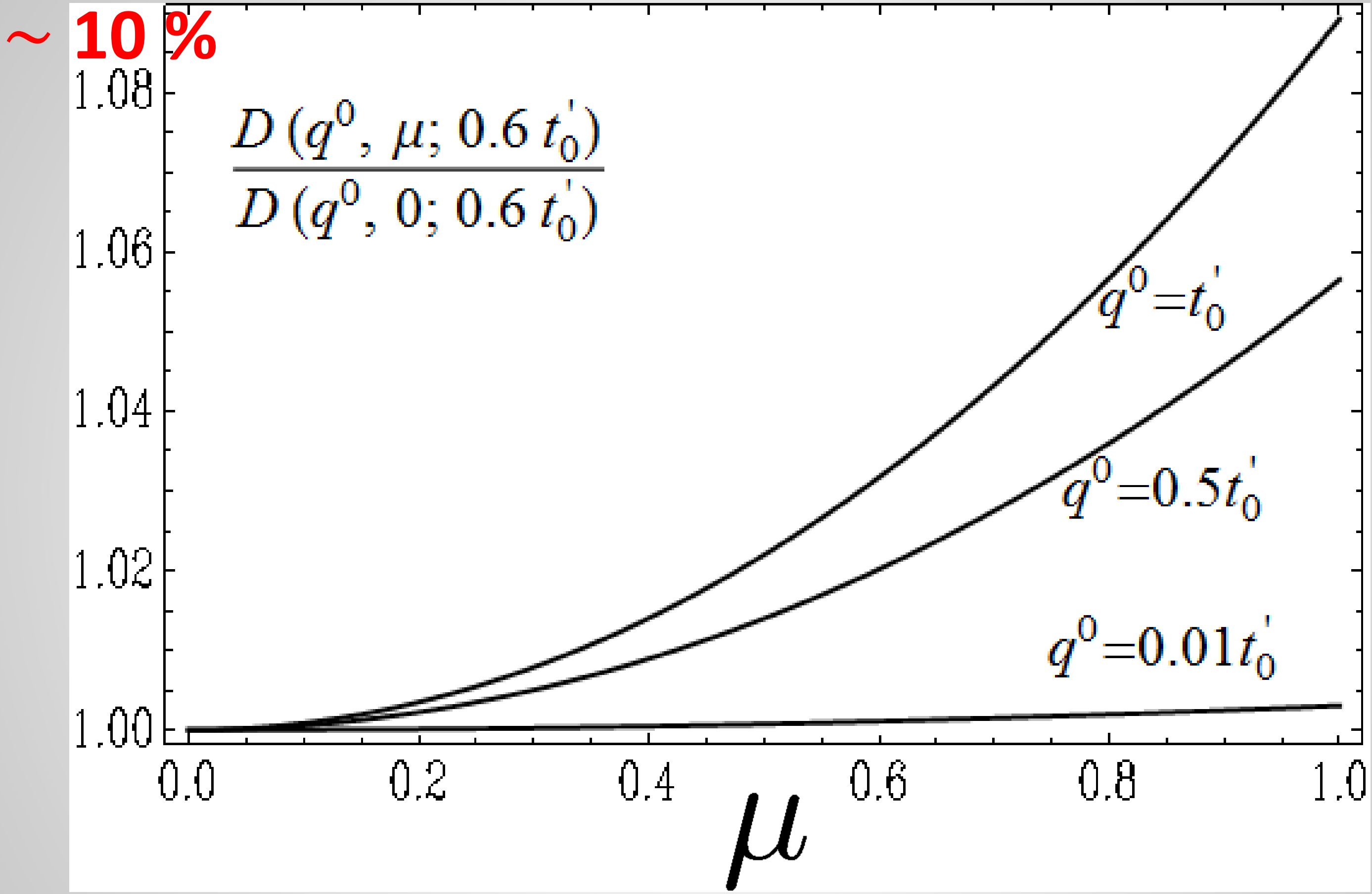
The energy density & Hubble functions





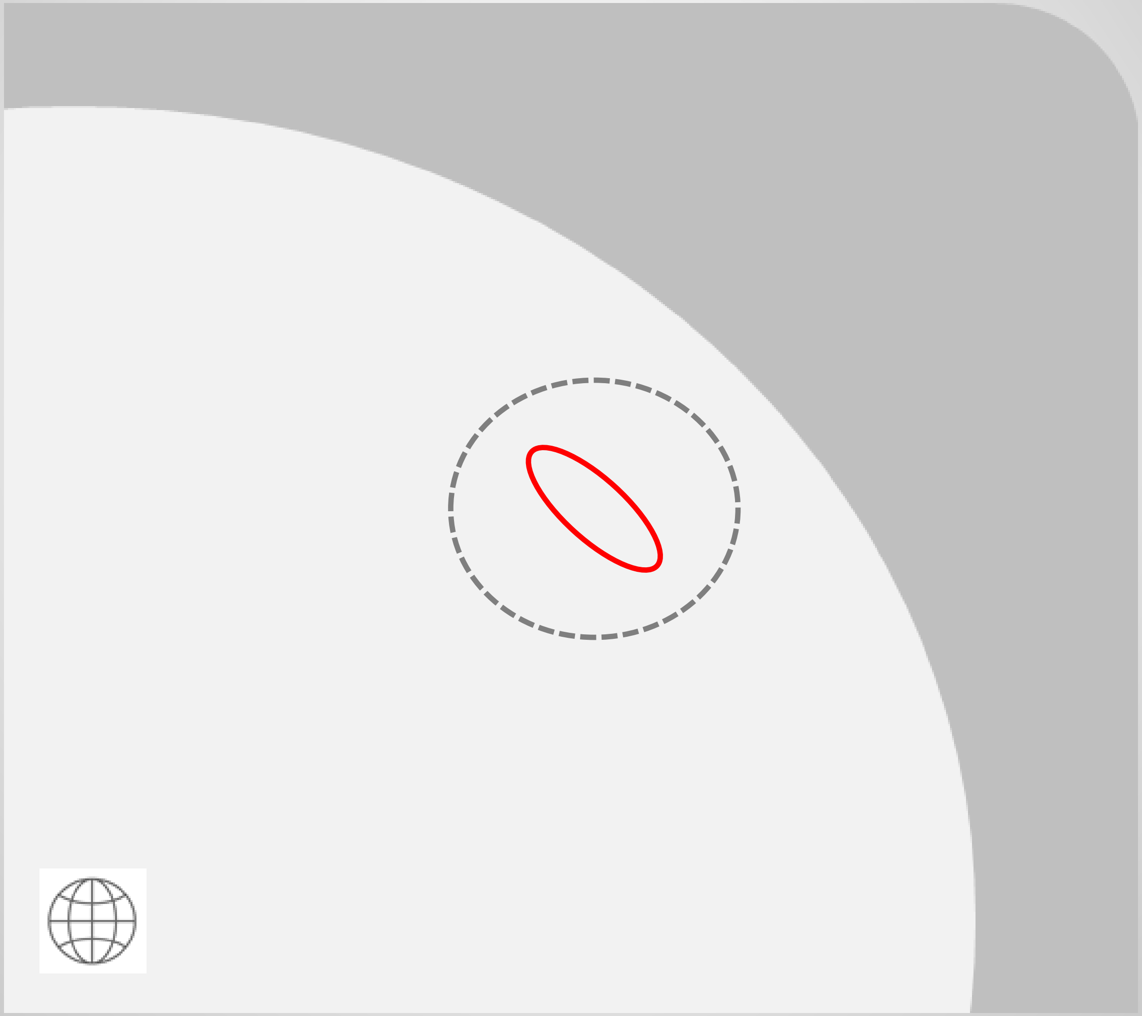
$$\Omega_M^{\rm in} = 0.3, \quad \Omega_M^{\rm out} = 1.0$$

Growth factors $D(q^0, \mu, r_o{}')$ at $r_o{}' = 0.6\ t_0'$



The anisotropy of the growth factor is about 10 % at the present time.

Evolution of density contrast in the void model



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Summary

- We derived the basic equations of the Newtonian system in the huge void model.
- We solved the linear Newtonian perturbations, and showed that the growth of perturbations in the void model significantly differ from that in the FLRW.

Future work

- Comparing with observational results on the Redshift Space Distortions. $P_\ell(\mu)$ with $\ell = 0,2,4,6,\cdots$
- N-body numerical simulation in the huge void model.

Thank you for your attention.