Relativistic Cosmology, YITP, Kyoto
September 9th 2014

Einstein's signature in large-scale structure:

non-Gaussianity, scale-dependent bias and GR

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Wands & Slosar, arXiv:0902.1084

Bruni, Crittenden, Koyama, Maartens, Pitrou & Wands, arXiv:1106.3999

Bruni, Hidalgo, Meures & Wands, arXiv:1307.1478

Bruni, Hidalgo & Wands, arXiv:1405.7006

motivation

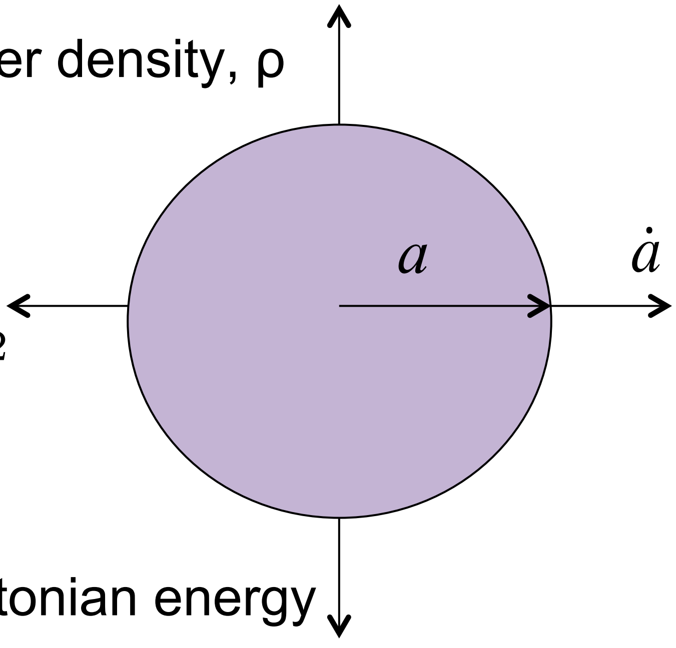
1. cosmic structure on very largest scales provides window onto the primordial density perturbation and hence models of very early universe
   - *primordial (non-)Gaussianity from inflation*

2. can we trust Newtonian solutions for large-scale structure formation on the largest scales?

Newtonian ΛCDM cosmology

homogeneous (FRW) background (Milne, 1930s)

same evolution+continuity equations for matter density, ρ



+ Friedmann constraint:

$$K = \frac{1}{2}\dot{a}^2 - \frac{G(4\pi\rho a^3/3)}{a} - \frac{\Lambda}{6}a^2$$

interpret GR spatial curvature = Newtonian energy

note: cosmological constant Λ = constant vacuum density

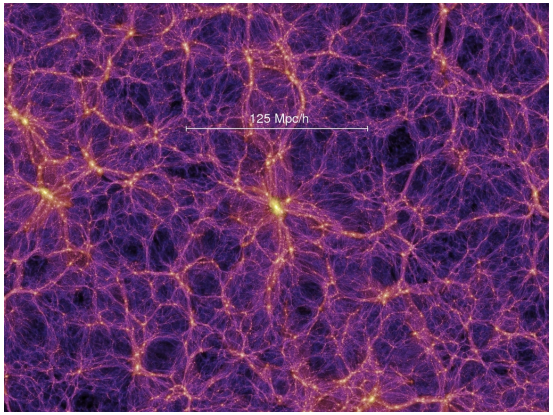
Standard Newtonian+Gaussian universe

Gaussian metric perturbation $\zeta(x)$ on large scales from single-field, slow-roll inflation

Gaussian Newtonian potential $\Phi=(3/5)\zeta$ on large scales using linear perturbation theory

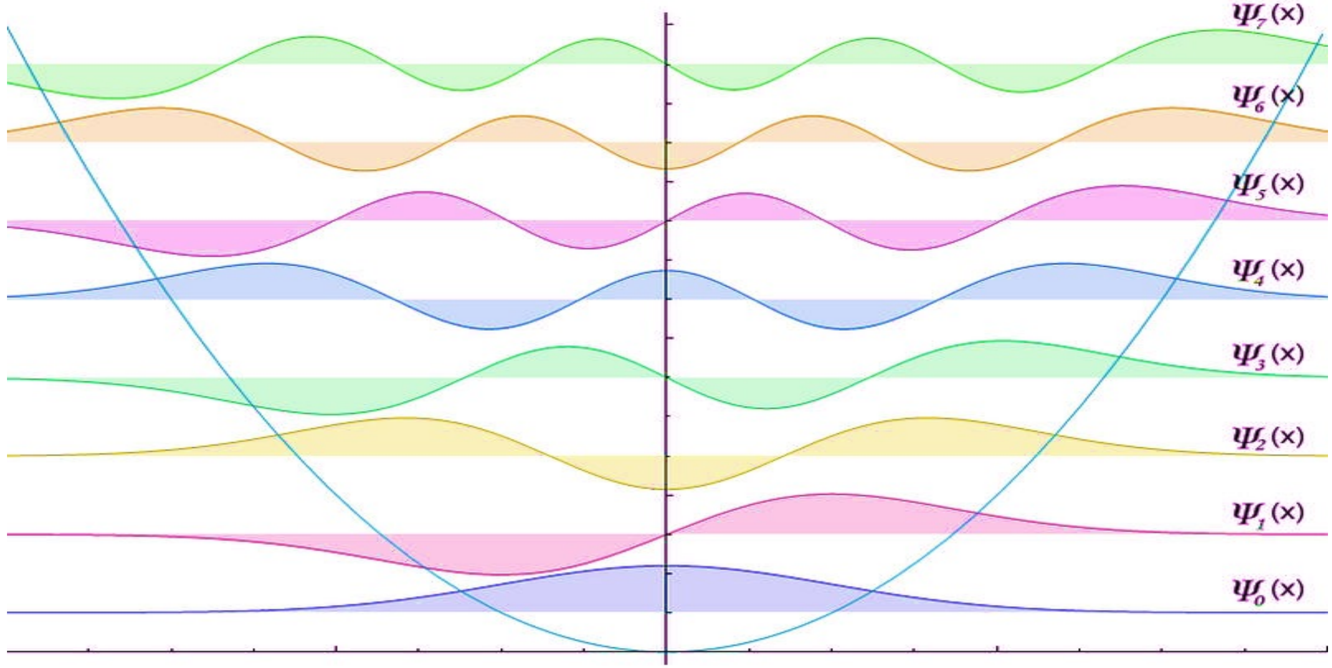
Gaussian initial matter density δ using Poisson equation $\nabla^2\Phi = 4\pi G\delta\rho$

Newtonian N-body simulations



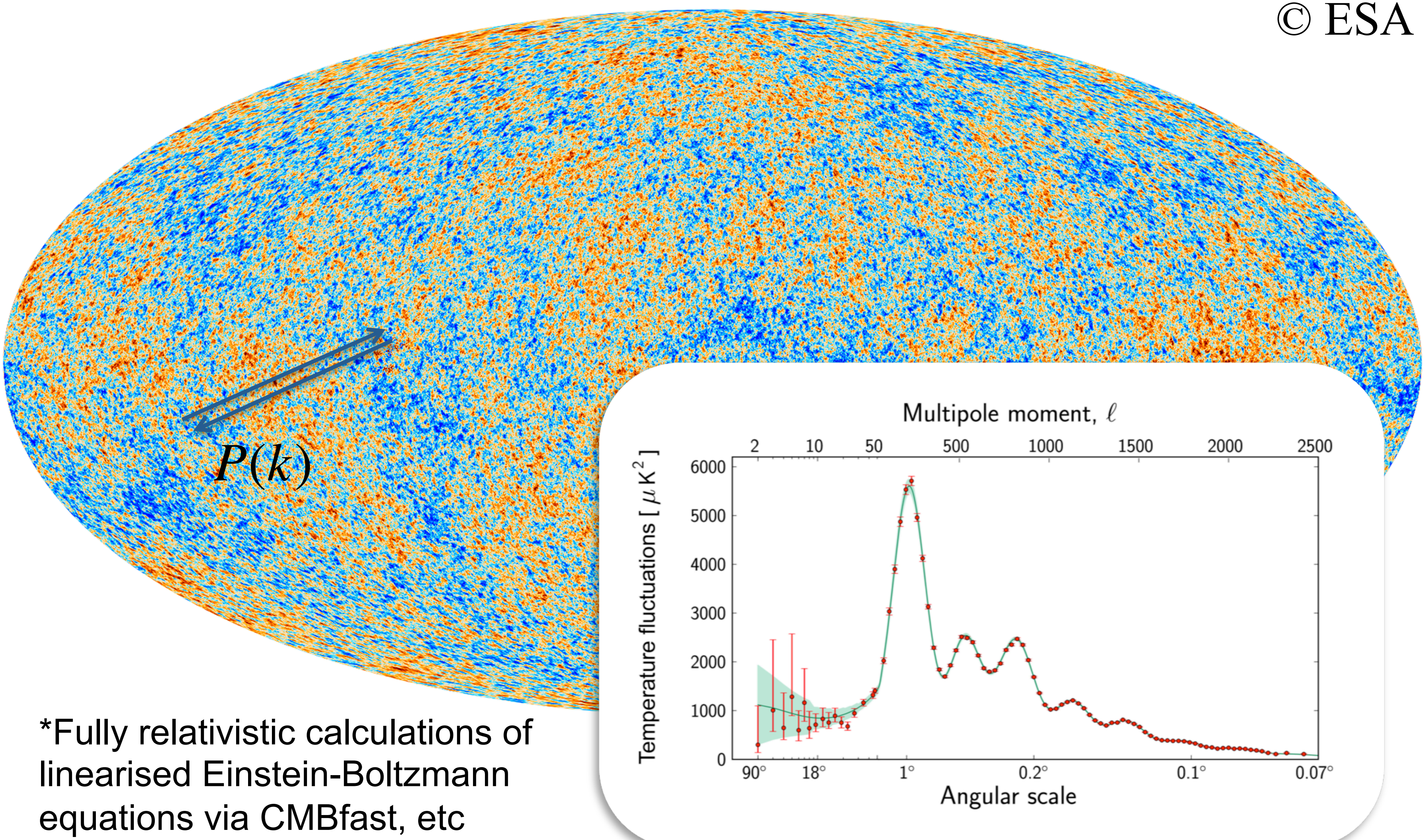
Primordial Gaussianity from inflation

- **Quantum field fluctuations during inflation**
  - ***ground state of simple harmonic oscillator***
  - ***almost free field in almost de Sitter space***
  - ***almost scale-invariant and almost Gaussian***



Wikipedia: AllenMcC

Planck2013 - ΛCDM model of primordial cosmology



*Fully relativistic calculations of linearised Einstein-Boltzmann equations via CMBfast, etc

Primordial non-Gaussianity from inflation

$$\zeta = N(\phi(x,t_i)) - \bar{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I(x) + \ldots$$

Starobinsky 85; Salopek & Bond 90; Sasaki & Stewart 96; Lyth & Rodriguez 05

- **Power spectra probe background *dynamics* (H, ε, ...)**

$$\left\langle \zeta_{k_1} \zeta_{k_2} \right\rangle = (2\pi)^3 P_\zeta(k) \delta^3(k_1 + k_2) \quad , \quad P_\zeta(k) \propto k^{n-4}$$

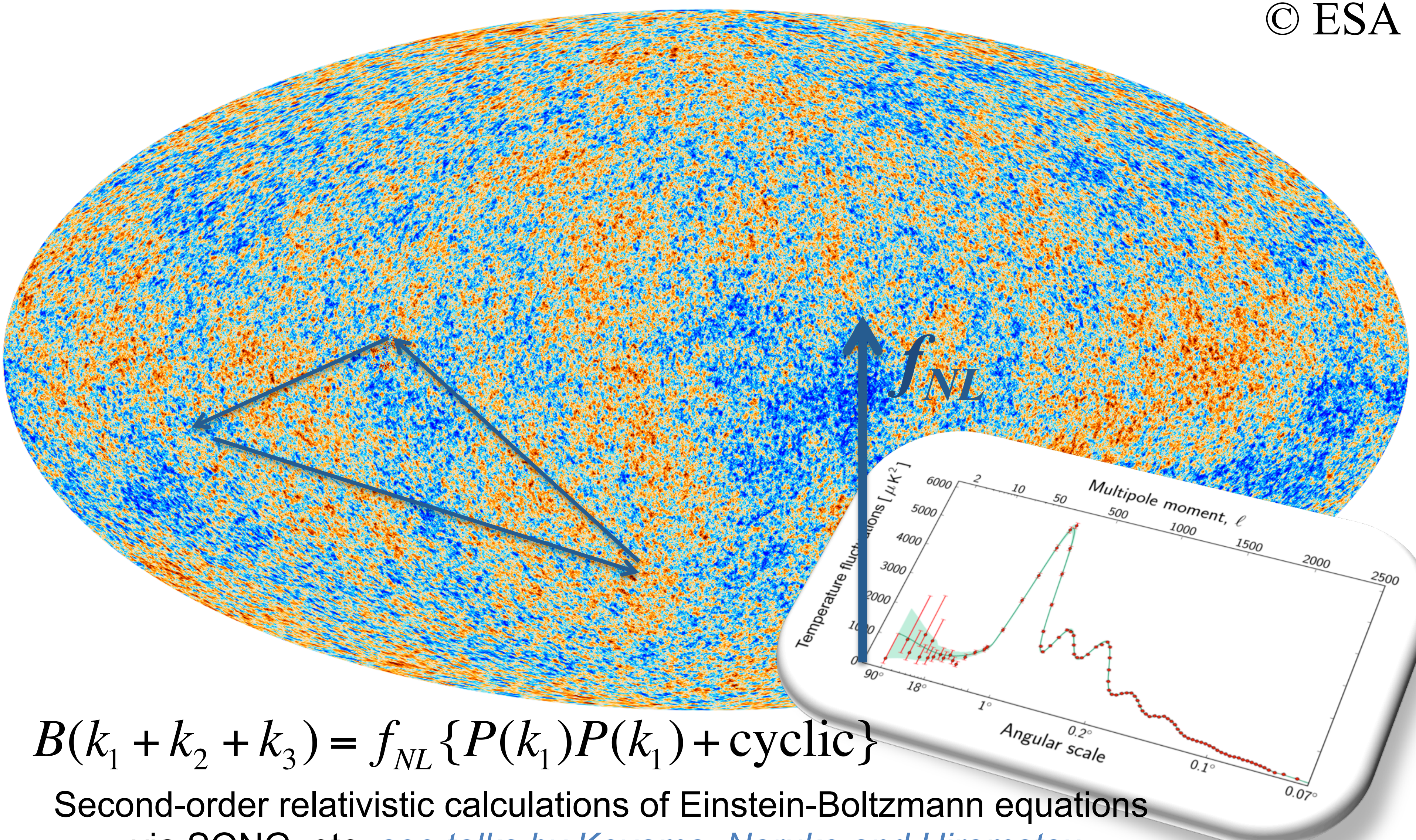
  - ***2-pt function contains all the information in a Gaussian random field***
  - ***Strong evidence now for deviations from scale-invariance: n-1 = -0.04***
  - ***First detection of primordial gravitational waves claimed by BICEP2: r = 0.2***

- **Higher-order correlations probe *interactions***

$$\left\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \right\rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3)$$

  - ***physics+gravity ➔ non-linearity ➔ non-Gaussianity***

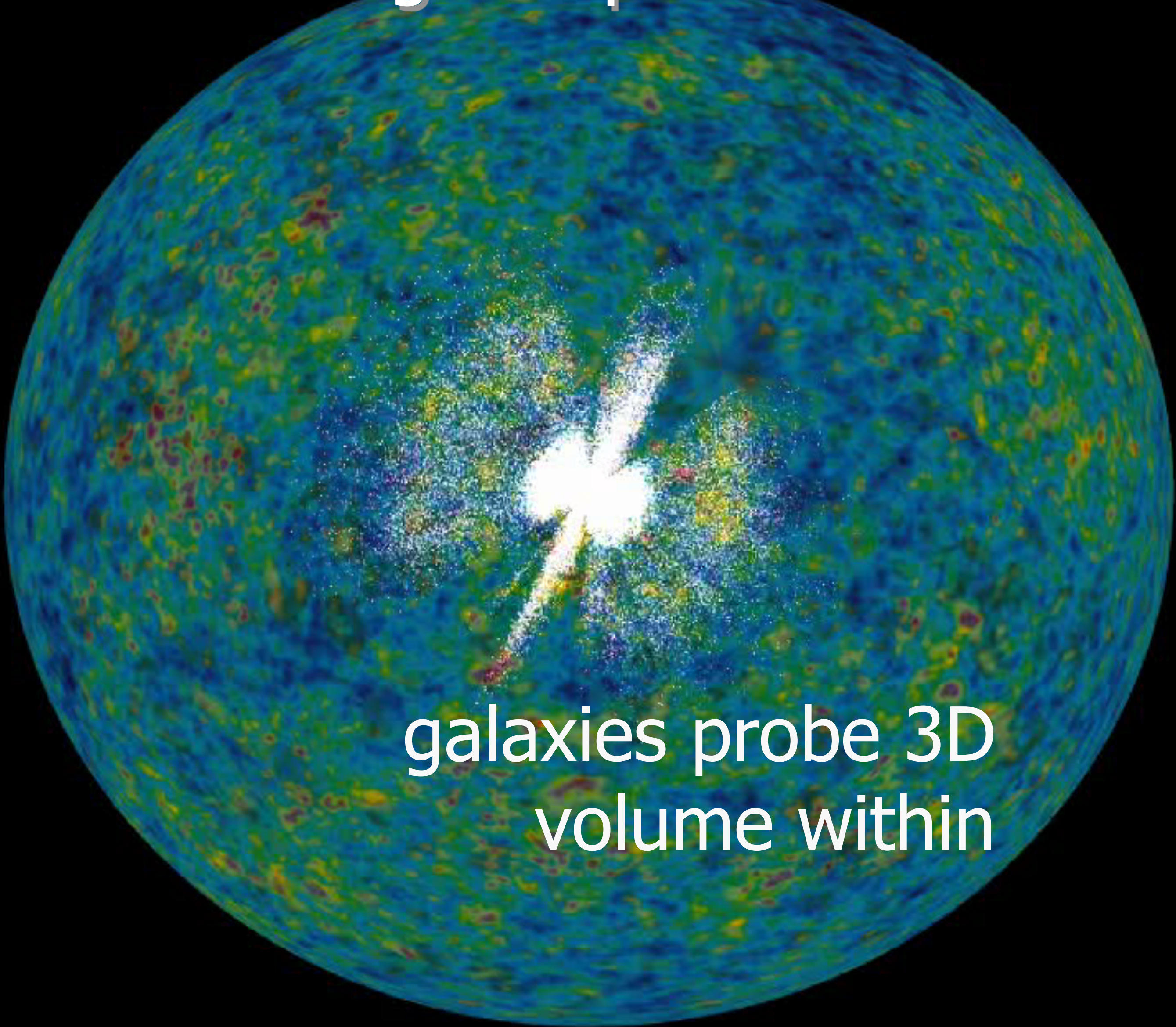
more information in higher-order correlators…



$$B(k_1 + k_2 + k_3) = f_{NL}\{P(k_1)P(k_1) + \text{cyclic}\}$$

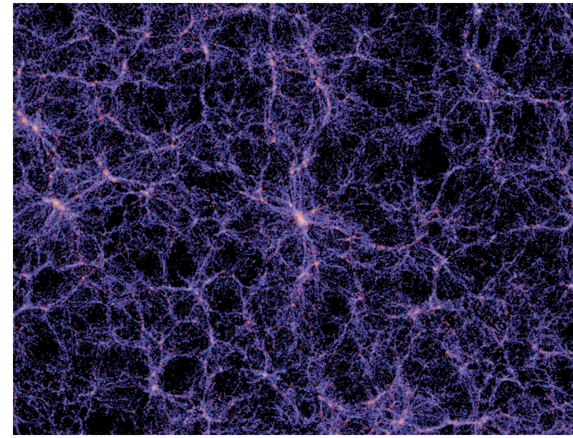
Second-order relativistic calculations of Einstein-Boltzmann equations via SONG, etc, *see talks by Koyama, Naruko and Hiramatsu*

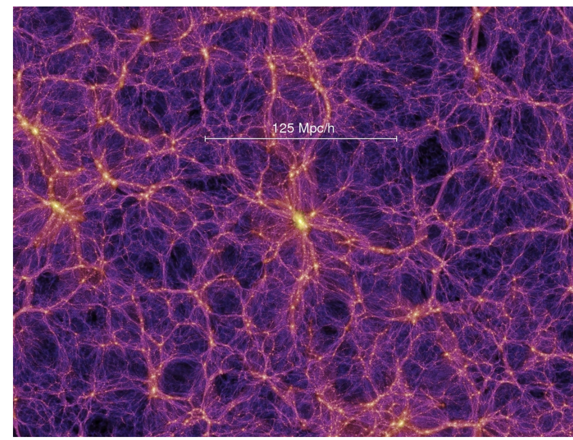
CMB last scattering 2D sphere



Large scale structure

Millenium simulation





Galaxy distribution on large scales:

$$\delta_g(x,t) \equiv \frac{\delta n_g}{n_g}$$

***biased* tracer of underlying matter dark matter + baryons):**

$$\delta_m(x,t) \equiv \frac{\delta \rho_m}{\rho_m}$$

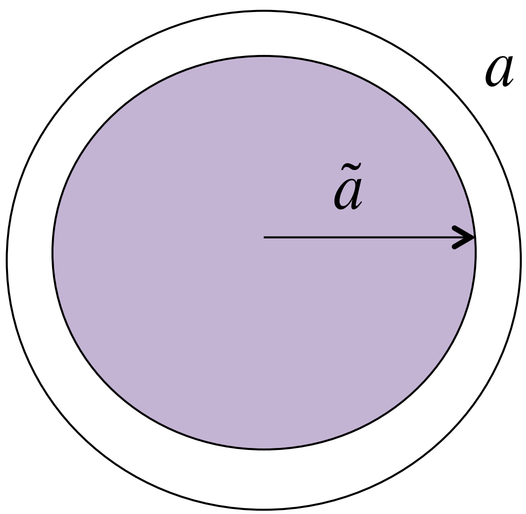
Assume galaxies form in matter overdensities (halos) that have collapsed under their own gravity

$$\delta_g(x,t) \equiv b_g(t)\ \delta_m(x,t)$$

Spherical collapse model

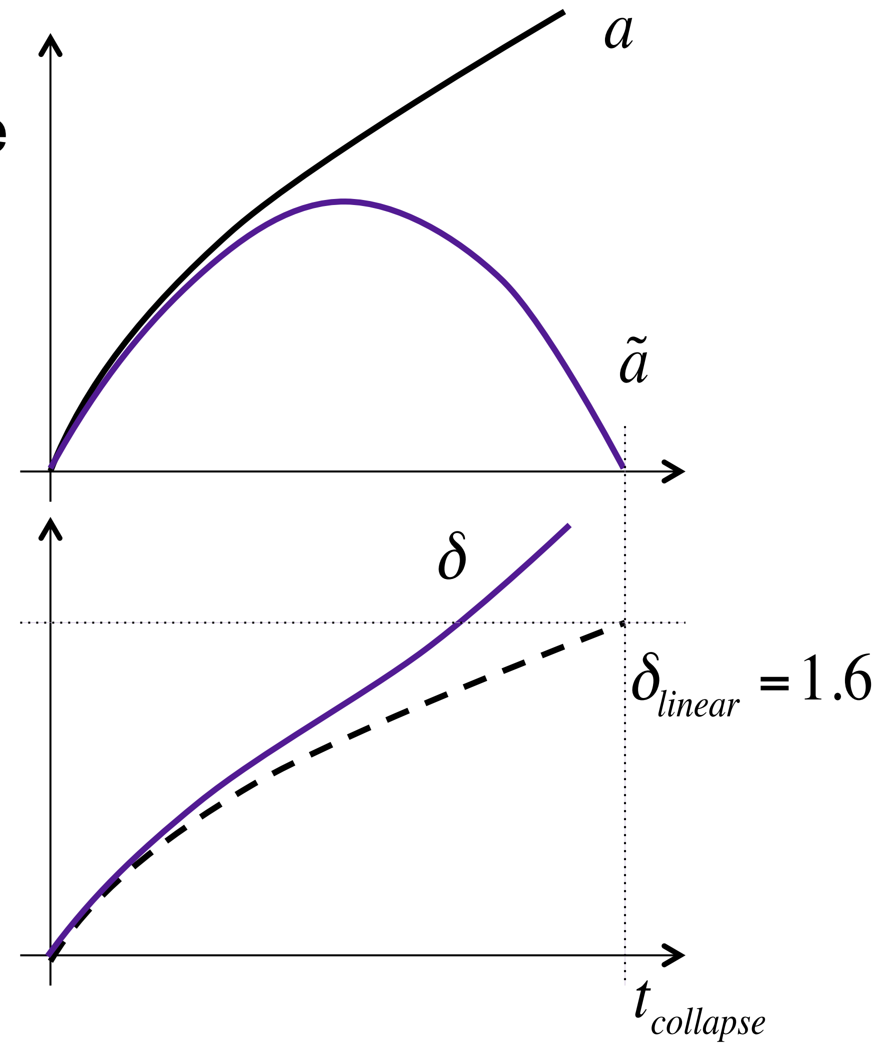
GR dynamics = Newtonian

in comoving-synchronous gauge

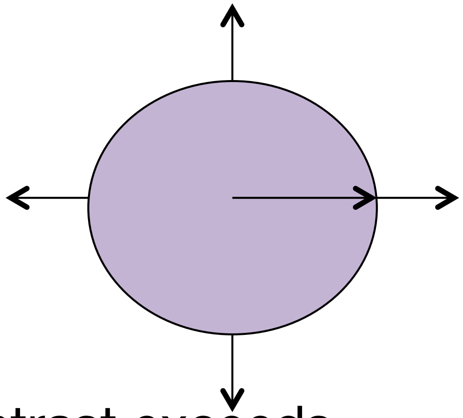


exterior: $ds^2 = -dt^2 + a^2 d\Omega^2_{k=0}$

interior: $ds^2 = -dt^2 + \tilde{a}^2 d\Omega^2_{k>0}$



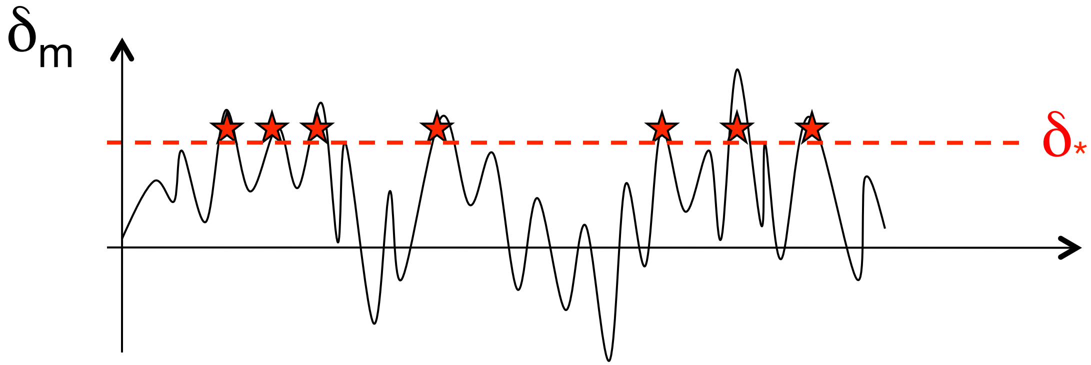
Spherical collapse model



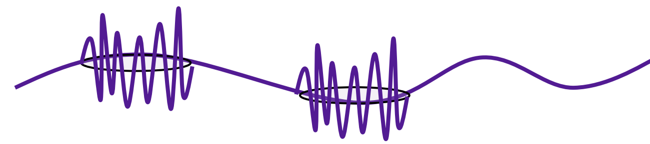
exact parametric solution for spherical collapse

works same in Newtonian gravity and in GR

small-scale collapse of peaks when linear density contrast exceeds threshold: $\delta_m > \delta_* \approx 1.6$



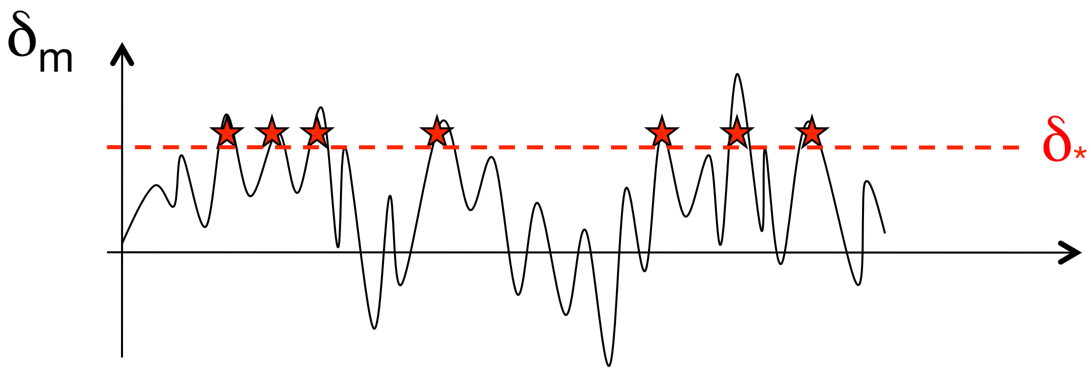
Peak-background split



$$\delta_m = \delta_{short} + \delta_{long}$$

large-scale fluctuations δ_{long} raise local background density

small-scale collapse of peaks where linear density contrast exceeds threshold: $\delta_m > \delta_* \approx 1.6$



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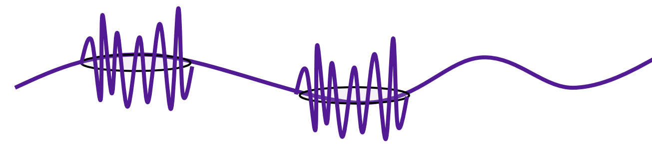

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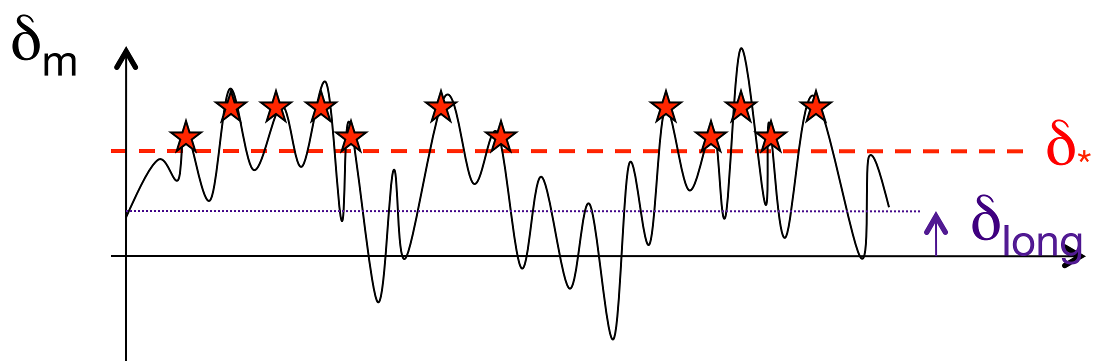


- large-scale modes lower effective threshold for collapse
- enhancing number of peaks above threshold

linear bias: $\boldsymbol{\delta_g = b\ \delta_m}$

$b \rightarrow b_G$ = constant on large scales *for a Gaussian density*

Scale-dependent bias from non-Gaussianity

Dalal et al, arXiv:0710.4560

Local model of non-Gaussianity:

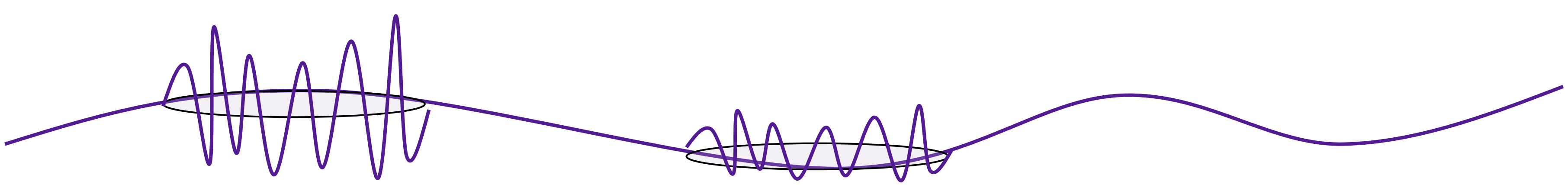
$$\Phi(x) = \phi_G(x) + f_{NL}\left(\phi_G{}^2(x) - \left\langle \phi_G{}^2 \right\rangle\right)$$

peak-background (small-scale – large-scale) split:

$$\phi_G(x) = \phi_s(x) + \phi_l(x)$$

$$\Rightarrow \quad \Phi(x) = \left(1 + 2 f_{NL}\phi_l(x)\right)\phi_s(x) + \phi_l(x)$$
$$+ f_{NL}\left(\phi_s{}^2(x) + \phi_l{}^2(x) - \left\langle \phi_s{}^2 \right\rangle - \left\langle \phi_l{}^2 \right\rangle\right)$$

- large-scale modes add to local background density $+\,\delta_{long}$
  and modulate amplitude on small scales x $f_{NL}\,\phi_l$



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$$+ f_{NL}\left(\phi_s{}^2(x) + \phi_l{}^2(x) - \left\langle \phi_s{}^2 \right\rangle - \left\langle \phi_l{}^2 \right\rangle\right)$$

- large-scale modes enhance power on small scales $\propto f_{NL}\,\phi_l$
- relate potential to density via Poisson equation:

$$\nabla^2 \Phi = 4\pi G_N \delta\rho \quad \Rightarrow \quad \phi_l = \frac{2}{3}\left(\frac{aH}{k}\right)^2 \delta_l$$

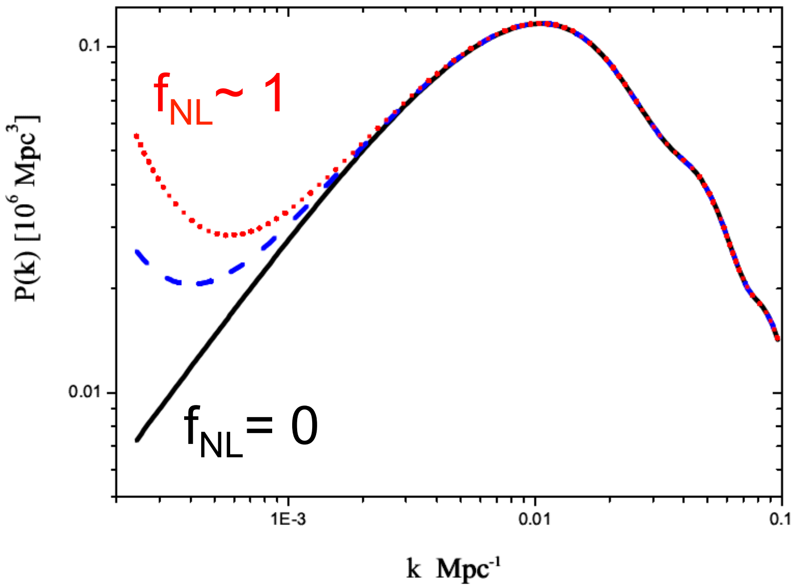
$\Rightarrow$ **scale-dependent bias:**

diverges on large scales

$$b \rightarrow b_G + 2 f_{NL}\left(\frac{aH}{k}\right)^2 (b_G - 1)$$

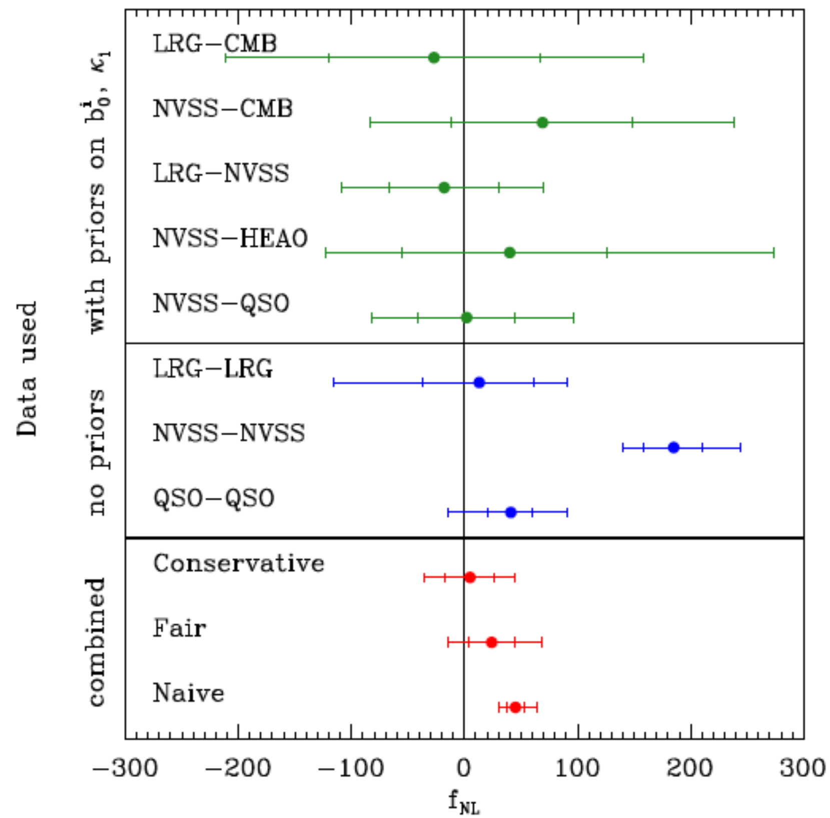
f_{NL} constraints from galaxy power spectra

$$P_g(k) = b^2 P_m(k)$$



Giannantonio et al, arXiv:1303.1349

-36 < f_{NL} < +45 (conservative 95% c.l.)



Future constraints:

ESA Euclid satellite (Amendola et al)
$|f_{NL}| < 5$

SKA (Square Km Array) (Santos et al)
$|f_{NL}| < 0.1$?

Galaxy bias *in General Relativity?*

peak-background split in GR

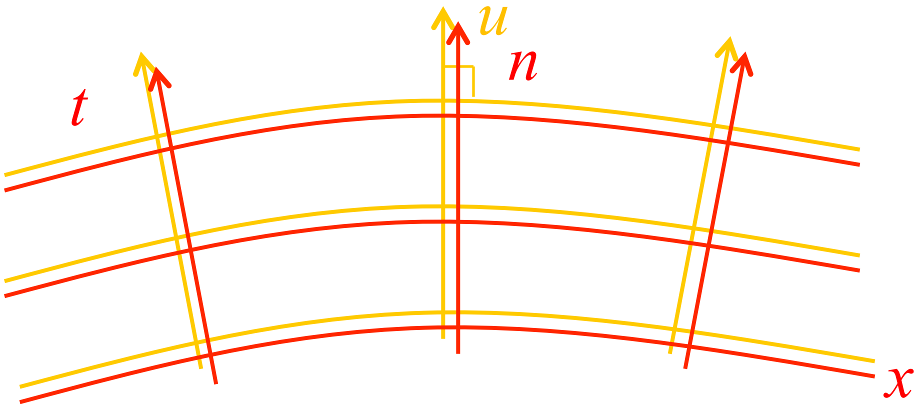
- small-scale ($R \ll H^{-1}$) peak collapse
  - described by Newtonian gravity

- large-scale background needs GR ($R \approx H^{-1}$)
  - density perturbation is gauge dependent

$$t \to t + \delta t, \quad \delta_m \to \tilde{\delta}_m = \delta_m + 3H\delta t, \quad \delta_g \to \tilde{\delta}_g = \delta_g + 3H\delta t$$

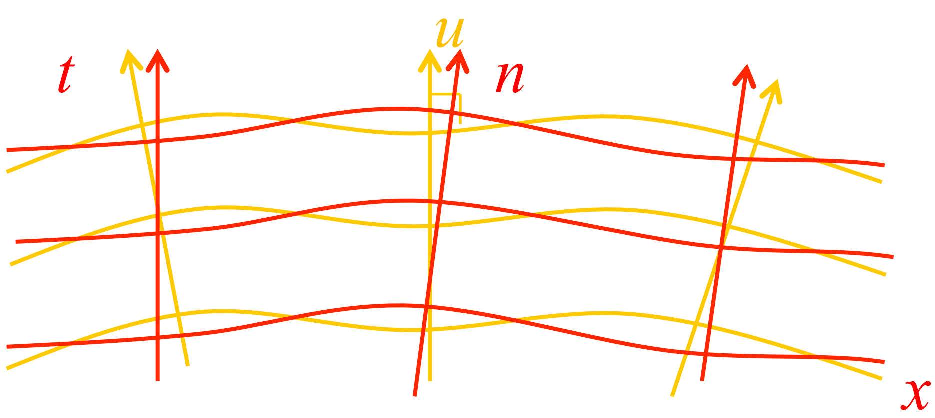
$\Rightarrow$ ***bias is a gauge-dependent quantity***

$$\delta_g = b\delta_m \Rightarrow \quad \tilde{\delta}_g = b\tilde{\delta}_m - 3H(b-1)\delta t$$



FRW cosmology

no unique choice of time (slicing) and space coordinates (threading) in an inhomogeneous universe



FRW cosmology
+ perturbations

comoving-orthogonal coordinates *(t,x)*

What is correct gauge to define bias?

peak-background split works in GR with right variables

(Wands & Slosar, 2009; Bruni, Crittenden, Koyama, Maartens, Pitrou & Wands 2011)

- Newtonian potential = GR longitudinal gauge metric: $\Phi = \psi^{(N)}$

- GR Poisson equation:

  relates Newtonian potential to density perturbation in comoving-synchronous gauge:

$$\delta_m^{(c)} = \frac{2}{3}\left(\frac{aH}{k}\right)^2 \Phi$$

- GR spherical collapse:

  local collapse criterion applies to density perturbation in comoving-synchronous gauge: $\delta_m^{(c)} > \delta_* \approx 1.6$

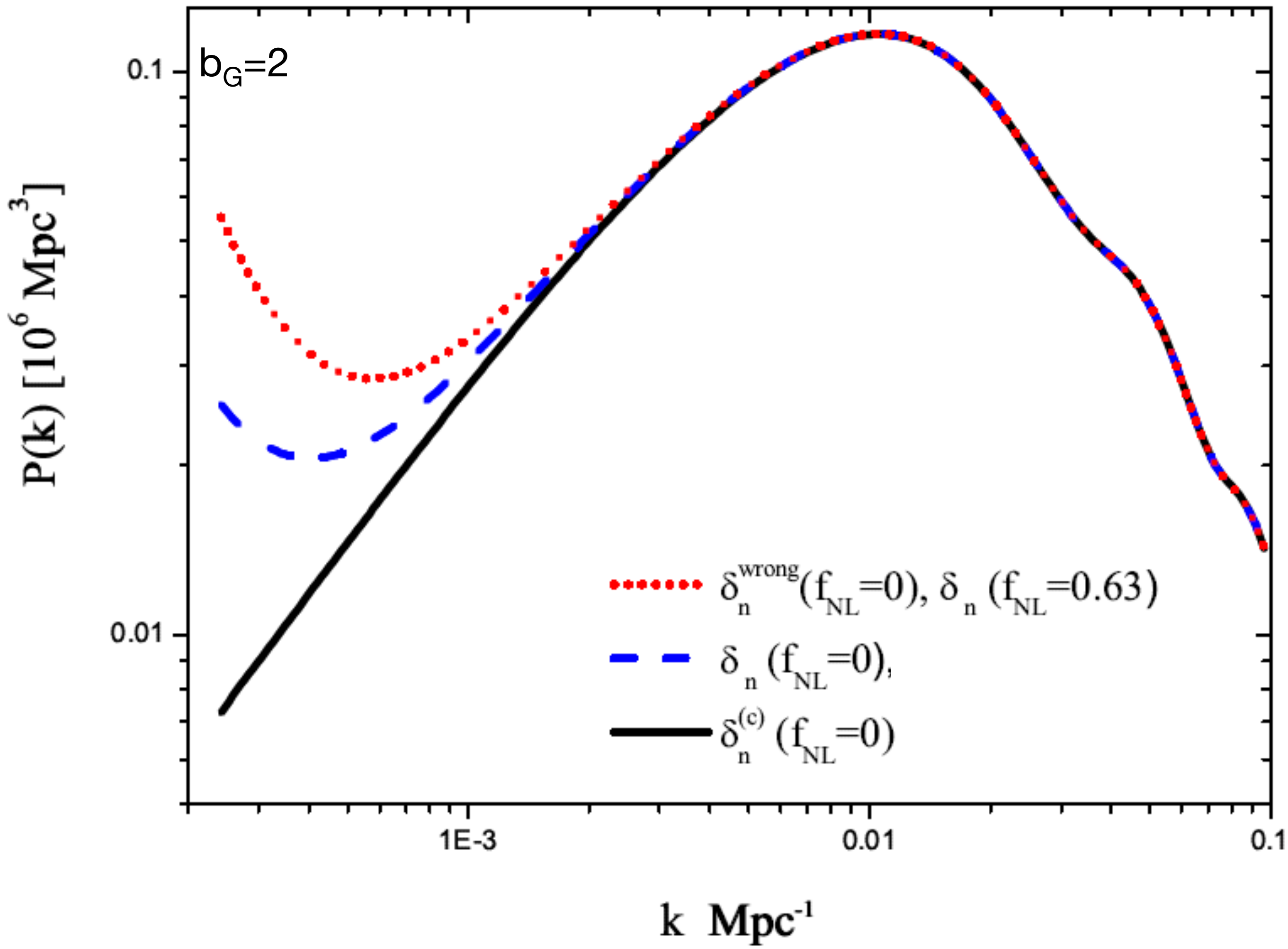
⇒***local bias defined in the comoving-synchronous gauge***

$$\delta_g^{(c)} = b\,\delta_m^{(c)}$$

see also Baldauf, Seljak, Senatore & Zaldarriaga, arXiv:1106.5507

Galaxy power spectrum at z=1

Bruni, Crittenden, Koyama, Maartens, Pitrou & Wands, arXiv:1106.3999



Non-linear equations in GR

Bruni, Hidalgo, Meures & Wands (2013); Bruni, Hidalgo & Wands (2014)

Matter density, ρ, expansion, Θ, and shear, σ, for irrotational flow in ΛCDM

- evolution equations in comoving-synchronous gauge [Ellis, 1971]

$$\dot{\rho} + \Theta\rho = 0$$

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\sigma^2 + 4\pi G\rho - \Lambda = 0$$

coincides with Newtownian equations

- constraint relates density and expansion to spatial curvature, $^{(3)}R$

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + {}^{(3)}R = 16\pi G\rho + 2\Lambda$$

only reduces to Poission equation $\Delta\Phi \sim {}^{(3)}R \sim \delta\rho$ at linear order

Schematic second-order solutions in GR

Tomita (1975)… Bartolo, Matarrese and Riotto (2005); Bruni, Hidalgo, Meures & Wands (2013) …

Matter density contrast, δ, obeys second-order differential equation:

- first-order linearly growing mode:

$$L\{\delta^{(1)}\} = 0$$

$$\Rightarrow \delta^{(1)} = C_1(\vec{x})D_+(t)$$

$$\mathrm{constraint} : C_1 \sim \nabla^2\zeta_1$$

- second-order:

$$L\{\delta^{(2)}\} = Q\{(\delta^{(1)})^2\}$$

$$\Rightarrow \delta^{(2)} = C_2(\vec{x})D_+(t) + P_2(\vec{x})D_{2+}(t,\vec{x})$$

$$\mathrm{constraint} : C_2 \sim \nabla^2\zeta_2 + \zeta_1\nabla^2\zeta_1 + (\nabla\zeta_1)^2$$

  - *particular solution: $P_2(x) \sim (C_1(x))^2$, is the usual "Newtonian" solution*
  - *homogeneous solution: $C_2(x)$, set by primordial non-Gaussianity and intrinsic non-linear GR constraint*

What are the non-linear initial conditions in GR?

Bruni, Hidalgo & Wands (2014)

Matter density, ρ, and expansion, Θ, in ΛCDM

- *GR constraint relates density and expansion to spatial curvature*

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + {}^{(3)}R = 16\pi G\rho + 2\Lambda$$

Non-linear perturbations about FRW:
$$\Theta(t,x^i) = 3H(t) + \theta(t,x^i)\,,$$
$$\rho(t,x^i) = \bar{\rho}(t)\left[1+\delta(t,x^i)\right]$$

At early times use large-scale limit – *gradient expansion*

$$\delta \sim \theta \sim \sigma \sim {}^{(3)}R \sim \nabla^2$$

- GR constraint becomes

$$\frac{{}^{(3)}R}{4} + H\theta = 4\pi G\bar{\rho}\delta + \mathcal{O}(\nabla^4)$$

spatial curvature is a non-linear function of the metric perturbation

Non-linear density from Gaussian $\zeta(x)$

Spatial metric on large scales is conformally flat:

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(t,x^i)}\,\delta_{kj}dx^k dx^j$$

hence simple non-linear form for spatial (Ricci) curvature, $R = {}^{(3)}Ra^2$:

$$R = \exp(-2\zeta)[-4\nabla^2\zeta - 2(\nabla\zeta)^2]$$

determines amplitude of growing mode / large-scale density perturbations

$$\delta_m \propto R(x)D_+(t)$$

Non-Gaussian structure from Gaussian $\zeta(x)$

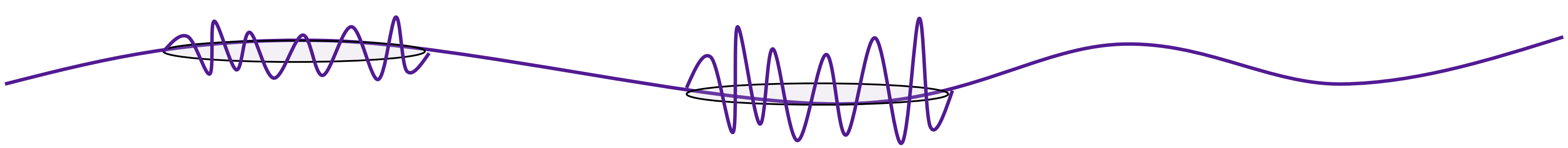
Peak-background split: $\zeta = \zeta_s + \zeta_\ell$

Long-wavelength mode modulates spatial curvature (rescales background)

$$R = \exp(-2\zeta_\ell)R_s + \mathcal{O}(\nabla\zeta_\ell)$$

and hence growing mode / large-scale density perturbation:

$$\delta_m = \exp(-2\zeta_\ell)\delta_s + \mathcal{O}(\nabla\zeta_\ell)$$



large scale ζ_l modulates smaller scale δ_s

Compare GR curvature R(x) and non-Gaussian $\Phi(x)$

GR density perturbations from Gaussian $\zeta(x)$

$$\delta_m = \exp(-2\zeta_\ell)\delta_s + \mathcal{O}(\nabla\zeta_\ell)$$
$$= (1 - 2\zeta_\ell + 2\zeta_\ell^2 - \frac{4}{3}\zeta_\ell^3 + \ldots)\delta_s + \mathcal{O}(\nabla\zeta_\ell)$$

Newtonian density perturbations from non-Gaussian $\Phi(x)$

$$\delta_m = (1 + 2f_{NL}\phi_\ell + 3g_{NL}\phi_\ell^2 + 4h_{NL}\phi_\ell^3 + \ldots)\delta_s + \mathcal{O}(\nabla\phi_\ell)$$

compare term by term using linear relation: $\phi_\ell = (3/5)\zeta_\ell$

Einstein's signature in large-scale structure:

$$f_{NL}^{GR} = -\frac{5}{3} \quad , \quad g_{NL}^{GR} = -\frac{50}{3} \quad , \quad h_{NL}^{GR} = -\frac{125}{81} \quad \ldots$$

Bartolo et al; Verde & Matarrese;
Bruni, Hidalgo & Wands (2014)

alternative viewpoints:

- **“even to the second order perturbations, equations for the relativistic irrotational flow… coincide exactly with the previously known Newtonian equations”**

  Hwang & Noh gr-qc/0412128

  - fluid flow evolution are same
  - but there are non-linear constraints in GR
    - there is no simple Poisson equation relating density to metric potential beyond first order
    - GR corrections to non-linear growing mode at second- and higher-order from a given primordial metric perturbation, ζ, e.g. from inflation

alternative viewpoint:

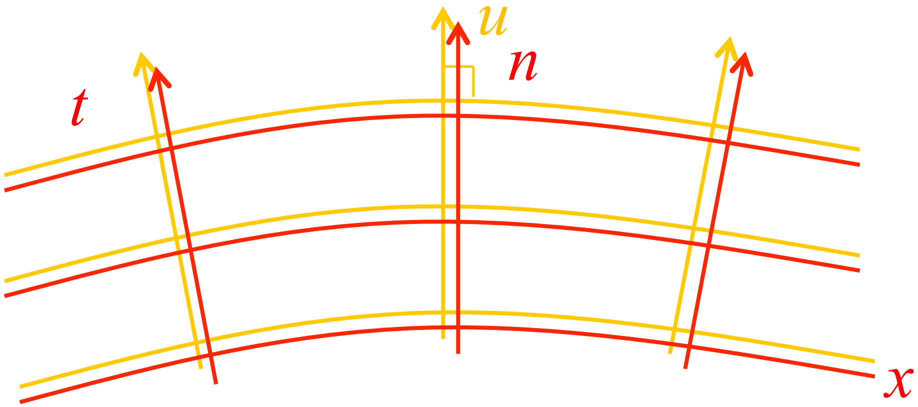
- **"synchronous gauge is an inappropriate coordinate choice when handling the growth of the large-scale structure"** *Hwang et al, arXiv:1408.4656*

  - comoving-synchronous gauge ($v = 0$) = Newtonian *Lagrangian* frame
  - total matter ($v_E \neq 0$) = Newtonian *Eulerian* frame
  - both have time-slicing orthogonal to matter 4-velocity, hence same density at first order
  - frame-dependent density at second order *(exactly as in Newtonian theory)*

$$\delta_E^{(2)} = \delta^{(2)} - 2\partial_i \delta^{(1)} \int \partial^i v_E \, d\tau$$

Bruni, Hidalgo, Meures & Wands, arXiv:1307.1478

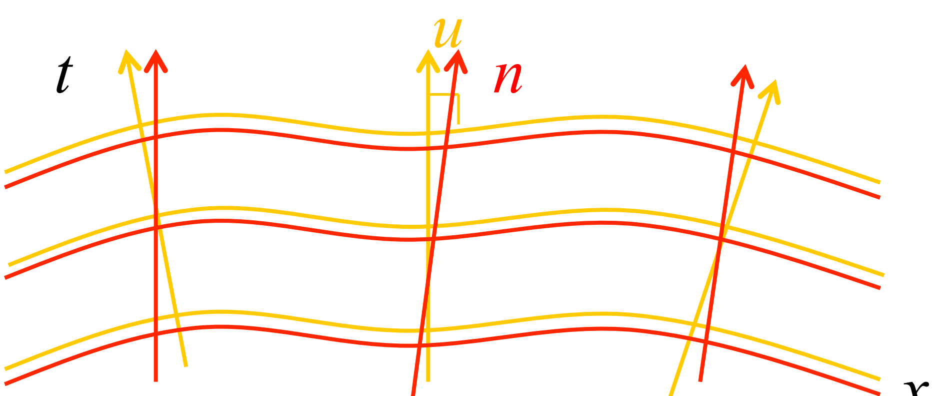
  - local Eulerian bias ≠ local Lagrangian bias *Matsubara, arXiv:1102.4619*



FRW cosmology
- u = matter 4-velocity

equals
- n = constant spatial coordinates

time-slicing orthogonal to matter 4-velocity
but alternative choices of spatial coordinates ("threading")



comoving-orthogonal (Lagrangian) spatial coordinates

conformal (Eulerian) spatial coordinates

Conclusions

- **Large-scale structure** probes **primordial density perturbation** and hence the **very early universe**
  - **primordial non-Gaussianity** can give rise to **scale-dependent bias**
  - future LSS observations might detect $f_{NL}=O(0.1)$?
- **Newtonian cosmology works remarkably well in ΛCDM**
  - but requires care for correct interpretation within GR
- **Galaxy bias** is a **gauge-dependent quantity**
  - use comoving-synchronous gauge for local Lagrangian bias in GR
- **GR vs Newtonian growth of structure**
  - ***non-linear constraints*** **in GR ->** ***non-Gaussian initial density***
  - observations also introduce non-linearities (e.g., lensing)
  - observables are independent of gauge (but calculations are not)
- **GR initial conditions give intrinsic non-Gaussianity**
  - $f_{NL} = -5/3,\ g_{NL} = -50/3,\ h_{NL} = -125/81\ldots$