# Hamiltonian structure of scalar-tensor theories beyond Horndeski

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Based on arXiv:1408.0670 with Chunshan Lin, Ryo Namba and Rio Saitou

# Why alternative gravity theories?



- Experimentally, we do not know gravity at short and large scales.
- Cosmology & Compact objects (e.g. BH) as Blackbody radiation & Hydrogen atom

### A motivation for IR modification

- Gravity at long distances
   Flattening galaxy rotation curves
   extra gravity

   Dimming supernovae
   accelerating universe
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

#### Dark component in the solar system?

**Precession of perihelion observed in 1800's...** 



which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

# Zoo of gravity theories

Horava-Lifshitz projectable/non-projectable with/without U(1) Supertring I, IIA, IIB, hetero O(32), hetero E8xE8



# Checkpoints

- What are the physical d.o.f.?
- How they interact?
- What is the regime of validity?

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- How they interact?
- What is the regime of validity?

If two or more theories give the same answers to the there questions above then they are the same even if they look different.

# EFT of inflation/DE

- Time diffeo is broken by the background but spatial diffeo is preserved
- All terms that respect spatial diffeo must be included in the EFT action
- Derivative & perturbative expansions
- Diffeo can be recovered by introducing NG boson

#### Simplest: ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

Backgrounds characterized by

 $\Rightarrow \left\langle \partial_{\mu} \phi \right\rangle \neq 0 \text{ and timelike}$ 

 Gauge choice:  $\phi(t, \vec{x}) = t$ .  $\pi \equiv \delta \phi = 0$ (Unitary gauge) Residual symmetry:  $\vec{x} \rightarrow \vec{x}'(t, \vec{x})$ 

Write down most general action invariant under this residual symmetry.

(  $\implies$  Action for  $\pi$ : undo unitary gauge!)

Start with flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\partial h_{\mu\nu} = \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}$$

Under residual  $\xi^i$ 

$$\partial h_{00} = 0, \partial h_{0i} = \partial_0 \xi_i, \partial h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ<sup>i</sup>  $\begin{pmatrix} \left(h_{00}\right)^2 & \mathsf{OK} \\ \left(h_{0i}\right)^2 & \end{pmatrix}^2$  $K_{ij}^{2} = \frac{1}{2} \left( \partial_{0} h_{ij} - \partial_{j} h_{0i} - \partial_{i} h_{0j} \right)$ Action for  $\pi$  $\boldsymbol{\xi}^{\mathbf{0}} = \boldsymbol{\pi} \begin{bmatrix} h_{00} \rightarrow h_{00} - 2\partial_0 \boldsymbol{\pi} \\ K_{ii} \rightarrow K_{ii} + \partial_i \partial_i \boldsymbol{\pi} \end{bmatrix}$  $\square \square \square = L_{EH} + M^{4} \left\{ \left( h_{00} - 2\dot{\pi} \right)^{2} - \frac{\alpha_{1}}{M^{2}} \left( K + \vec{\nabla}^{2} \pi \right)^{2} - \frac{\alpha_{2}}{M^{2}} \left( K^{ij} + \vec{\nabla}^{i} \vec{\nabla}^{j} \pi \right) \left( K_{ij} + \vec{\nabla}_{i} \vec{\nabla}_{j} \pi \right) + \cdots \right\}$ 



#### $\implies$ Good low-E effective theory.

- The structure of low-E EFT is determined by the symmetry breaking pattern!
- Would end up with the same EFT, independently from starting Lagrangian.
- Can make robust predictions!

## Extension to FLRW background

Creminelli, Luty, Nicolis, Senatore 2006 Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

- Action invariant under  $x^i \rightarrow x^i(t,x)$
- Ingredients
  - $g_{\mu\nu} g^{\mu\nu} R_{\mu\nu\rho\sigma} \nabla_{\mu}$
- 1<sup>st</sup> derivative of t

$$\begin{array}{ll} \partial_{\mu}t = \delta^{0}_{\mu} & n_{\mu} = \frac{\partial_{\mu}t}{\sqrt{-g^{\mu\nu}\partial_{\mu}t\partial_{\nu}t}} = \frac{\delta^{0}_{\mu}}{\sqrt{-g^{00}}} \\ g^{00} & h_{\mu\nu} = g_{\mu\nu} + n_{\mu}n_{\nu} \end{array}$$

• 2<sup>nd</sup> derivative of t

$$K_{\mu\nu} \equiv h^{\rho}_{\mu} \nabla_{\rho} n_{\nu}$$

### Unitary gauge action



# NG boson

- Undo unitary gauge  $t \to \tilde{t} = t \pi(\tilde{t}, \vec{x})$   $H(t) \to H(t + \pi), \quad \dot{H}(t) \to \dot{H}(t + \pi),$   $\lambda_i(t) \to \lambda_i(t + \pi), \quad a(t) \to a(t + \pi),$  $\delta^0_\mu \to (1 + \dot{\pi})\delta^0_\mu + \delta^i_\mu \partial_i \pi,$
- NG boson in decoupling (subhorizon) limit

$$\begin{split} I_{\pi} &= M_{Pl}^{2} \int dt d^{3} \vec{x} \, a^{3} \left\{ -\frac{\dot{H}}{c_{s}^{2}} \left( \dot{\pi}^{2} - c_{s}^{2} \frac{(\partial_{i} \pi)^{2}}{a^{2}} \right) \right. \\ &\left. -\dot{H} \left( \frac{1}{c_{s}^{2}} - 1 \right) \left( \frac{c_{3}}{c_{s}^{2}} \dot{\pi}^{3} - \dot{\pi} \frac{(\partial_{i} \pi)^{2}}{a^{2}} \right) + O(\pi^{4}, \tilde{\epsilon}^{2}) + L_{\tilde{\delta}K, \tilde{\delta}R}^{(2)} \right\} \\ &\left. \frac{1}{c_{s}^{2}} = 1 - \frac{4\lambda_{1}}{\dot{H}}, \quad c_{3} = c_{s}^{2} - \frac{8c_{s}^{2}\lambda_{2}}{-\dot{H}} \left( \frac{1}{c_{s}^{2}} - 1 \right)^{-1} \end{split}$$

# **GLPV** theory $ds^{2} = -N^{2}dt^{2} + h_{ij}(dx^{i} + N^{i}dt)(dx^{j} + N^{j}dt)$ $S = \int d^3x dt N \sqrt{h} \sum_{n=2} L_n$ $L_2 = A_2(t, N)$ $L_3 = A_3(t, N)K$ $L_4 = A_4(t, N)K_2 + B_4(t, N)R$ $L_5 = A_5(t, N)K_3 + B_5(t, N)K^{ij}G_{ij}$ $K = K^{i}_{i} \qquad K_{2} = K^{2} - K^{i}_{j}K^{j}_{i}$ $K_3 = K^3 - 3KK^{ij}K_{ij} + 2K^i_{\ i}K^j_{\ k}K^k_{\ i}$

# Questions

- Covariant version of GLPV theory is more general than Horndeski theory
- Horndeski theory is the most general scalar tensor theory in 4 dim with 2<sup>nd</sup> order EOMs
- Theories with higher derivative EOMs usually suffer from Ostrogradski's ghost
- Does GLPV include more than 3 dof?

Hamiltonian analysis for the case with  $A_5 = 0$ Lin, Mukohyama, Namba, Saitou 2014

#### Primary constraints

N<sub>i</sub> and N are non-dynamical

 $\left. \begin{array}{c} \pi_i = 0 \\ \pi_N = 0 \end{array} \right|_{\mathrm{primary constraints}}$ 

• No more primary constraints

$$\pi^{ij} = \frac{\sqrt{h}}{2} \left[ A_3 h^{ij} + 2A_4 (h^{ij} K - K^{ij}) + B_5 G^{ij} \right]$$
$$K_{ij} = -\frac{1}{A_4} \left[ \frac{1}{\sqrt{h}} \left( \pi_{ij} - \frac{1}{2} h_{ij} \pi \right) + \frac{A_3}{4} h_{ij} - \frac{B_5}{2} \left( R_{ij} - \frac{1}{4} R h_{ij} \right) \right]$$

### Secondary constraints

Hamiltonian

$$\mathcal{H} = -N\sqrt{h} \left[ \frac{1}{A_4} \left( \frac{\pi^i{}_j \pi^j{}_i}{h} - \frac{\pi^2}{2h} \right) + \frac{A_3\pi}{2\sqrt{h}A_4} - \frac{3A_3^2}{8A_4} + A_2 + B_4 R - \frac{B_5}{A_4\sqrt{h}} \left( \pi^{ij}R_{ij} - \frac{1}{4}\pi R \right) + \frac{A_3B_5}{8A_4}R + \frac{B_5^2}{4A_4} \left( R^{ij}R_{ij} - \frac{3}{8}R^2 \right) \right]$$

Secondary constraints

# $\mathcal{H}_ipprox 0$ is not first class

Usual algebra

$$\begin{split} \left\{ \bar{\mathcal{H}}[f], \bar{\mathcal{H}}[g] \right\}_{\mathrm{P}} &\approx \bar{\mathcal{H}}\left[ [f,g] \right] \approx 0 \quad \text{for } \forall f^{i}, \ \forall g^{i} \\ \bar{\mathcal{H}}[f] &\equiv \int d^{3}x f^{i}(x) \mathcal{H}_{i}(x) \qquad [f,g]^{i} \equiv f^{j} \partial_{j} g^{i} - g^{j} \partial_{j} f^{i} \end{split}$$
• However,

$$\begin{split} \left\{\bar{\mathcal{H}}[f],\bar{\mathcal{C}}[\varphi]\right\}_{\mathrm{P}} &\approx -\int d^3x \frac{\partial^2 \mathcal{H}}{\partial N^2} \varphi f^i \partial_i N \\ \text{does not vanish weakly...} \end{split}$$

- This is actually expected, since the scalar dof should contribute to  $\mathsf{T}_{0i}$  .

#### $\mathcal{H}_i^{\text{tot}} = \mathcal{H}_i + \pi_N \partial_i N$ is first class $\left\{\bar{\mathcal{H}}^{\text{tot}}[f], \bar{\pi}_N[\varphi]\right\}_{\text{P}} \approx \bar{\pi}_N[f\partial\varphi] \approx 0 ,$ $\left\{ \bar{\mathcal{H}}^{\text{tot}}[f], \bar{\mathcal{C}}[\varphi] \right\}_{\mathbf{P}} \approx \bar{\mathcal{C}}[f\partial\varphi] \approx 0 ,$ $\left\{ \bar{\mathcal{H}}^{\text{tot}}[f], \bar{\mathcal{H}}^{\text{tot}}[g] \right\}_{\text{P}} \approx \bar{\mathcal{H}}^{\text{tot}}\left[[f, g]\right] \approx 0$ $\left\{\pi_i(x), \mathcal{H}_i^{\text{tot}}(y)\right\}_{\mathbf{p}} = 0$ $\bar{\mathcal{H}}^{\text{tot}}[f] \equiv \int d^3x f^i(x) \mathcal{H}_i^{\text{tot}}(x) \quad \bar{\pi}_N[\varphi] \equiv \int d^3x \varphi(x) \pi_N(x)$ • $\pi_i \approx 0$ is also first class $\{\pi_i(x),\pi_j(y)\}_{\mathbf{P}}=0, \{\pi_i(x),\pi_N(y)\}_{\mathbf{P}}=0,\$ $\{\pi_i(x), \mathcal{C}(y)\}_{\mathbf{P}} = 0$

• No further secondary constraints  $\{\bar{\mathcal{H}}^{\text{tot}}[f], H\}_{\text{P}} \approx \bar{\mathcal{H}}[[f, N]] + \bar{\pi}_{N}[f\partial\lambda_{N}] \approx 0$   $\pi_N \approx 0$  &  $\mathcal{C} \approx 0$  are second class The determinant  $\det \left( \begin{array}{c} \{\pi_N(x), \pi_N(y)\}_{\mathrm{P}} & \{\pi_N(x), \mathcal{C}(y)\}_{\mathrm{P}} \\ \{\mathcal{C}(x), \pi_N(y)\}_{\mathrm{P}} & \{\mathcal{C}(x), \mathcal{C}(y)\}_{\mathrm{P}} \end{array} \right)$ does not vanish since  $\begin{cases} \pi_N(x), \pi_N(y) \rbrace_{\mathrm{P}} = 0 \\ \{ \mathcal{C}(x), \pi_N(y) \rbrace_{\mathrm{P}} \end{cases} = -\frac{\partial^2 \mathcal{H}}{\partial N^2} \delta^3(x-y)$  The consistency conditions  $\{\pi_N(x), H_{\text{tot}}\}_{\mathrm{P}} \approx 0 \quad \frac{\partial}{\partial t} \mathcal{C}(x) + \{\mathcal{C}(x), H_{\text{tot}}\}_{\mathrm{P}} \approx 0$ just determines  $\lambda_N$  &  $\lambda_C$ .  $H_{\text{tot}} = \int d^3x \left[ \mathcal{H} + N^i \mathcal{H}_i + n^i \mathcal{H}_i^{\text{tot}} + \lambda^i \pi_i + \lambda_N \pi_N + \lambda_{\mathcal{C}} \mathcal{C} \right]$ 

### Number of degrees of freedom

- 20-dim phase space  $(N, N^i, h_{ij}, \pi_N, \pi_i, \pi^{ij})$
- 6 first class constraints each of them reduce phase space dim by 2
- 2 second class constraints each of them reduce phase space dim by 1
- Physical phase space dim:  $20 6 \times 2 2 = 6$
- This corresponds to 3 degrees of freedom
- 2 in tensor sector + 1 in scalar sector

# Gauge fixing

- Gauge fixing function for  $\pi_i \approx 0 \& \mathcal{H}_i^{\text{tot}} \approx 0$   $\mathcal{G}^i(x) \approx 0 \qquad \mathcal{F}^i(x) \approx 0$  $\det \begin{pmatrix} \{\pi_N(x), \pi_N(y)\}_{\text{P}} & \{\pi_N(x), \mathcal{C}(y)\}_{\text{P}} \\ \{\mathcal{C}(x), \pi_N(y)\}_{\text{P}} & \{\mathcal{C}(x), \mathcal{C}(y)\}_{\text{P}} \end{pmatrix} \neq 0$
- We end up with 14 second class constraints in 20-dim phase space → 6-dim → 3 dof
- Total Hamiltonian

 $\left| H_{\text{tot}}' \right| = \int d^3x \left[ \mathcal{H} + N^i \mathcal{H}_i + n^i \mathcal{H}_i^{\text{tot}} + \lambda^i \pi_i + \lambda_i^{\mathcal{G}} \mathcal{G}^i + \lambda_i^{\mathcal{F}} \mathcal{F}^i + \lambda_N \pi_N + \lambda_{\mathcal{C}} \mathcal{C} \right]$ 

# Simple gauge fixing

• Gauge fixing function for  $\pi_i \approx 0$  &  $\mathcal{H}_i^{\mathrm{tot}} \approx 0$ 

$$\mathcal{G}^i = N^i \qquad \mathcal{F}^i = \mathcal{F}^i(N, h_{ij}, \pi_N, \pi^{kl}; t)$$

$$\det\left(\left\{\mathcal{H}_{i}^{\mathrm{tot}}(x),\mathcal{F}^{j}(y)\right\}_{\mathrm{P}}\right)\neq\mathbf{0}$$

- $(N^{i}, \pi_{i})$  fully eliminated  $\{\mathcal{G}^{i}(x), H'_{\text{tot}}\}_{P} \approx 0$  $\frac{\partial}{\partial t}\mathcal{F}^{i}(x) + \{\mathcal{F}^{i}(x), H'_{\text{tot}}\}_{P} \approx 0$   $\lambda_{i}^{\mathcal{G}} = -\mathcal{H}_{i}$
- Instead of N<sup>i</sup>, n<sup>i</sup> plays the role of the shift

$$H'_{\text{tot}} = \int d^3x \left[ \mathcal{H} + n^i \mathcal{H}_i^{\text{tot}} + \lambda_i^{\mathcal{F}} \mathcal{F}^i + \lambda_N \pi_N + \lambda_{\mathcal{C}} \mathcal{C} \right]$$

# Conclusion

- We performed the Hamiltonian analysis for GLPV theory with A<sub>5</sub>=0 and confirmed that the number of degrees of freedom is 3.
- How general is this conclusion for theories in which the absence of higher time derivatives is guaranteed only in the unitary gauge?
- How to prove the same statement without fixing the gauge? (in progress with Ryo&Rio)
- Applications? (inflation, dark energy, Vainshtein, emergent time, etc.)

preliminary results on New matter coupling in massive gravity 8 new quasidilaton theory

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#### Effective metric

- Recent proposal of an effective metric in massive gravity de Rham, Heisenberg, Ribeiro 2014  $g_{\mu\nu}^{\text{eff}} = \alpha^2 g_{\mu\nu} + 2\alpha\beta g_{\mu\rho} \left(\sqrt{g^{-1}f}\right)_{\ \nu}^{
  ho} + \beta^2 f_{\mu\nu}$
- Claim: BD ghost shows up only above the cutoff scale of the theory and thus can (and should) be integrated out, i.e. we don't worry about it
- BD ghost does not show up in linear perturbations around FLRW background. Gumrukcuoglu, Heisenberg, Mukohyama arXiv:1409???

#### Quasidiaton D'Amico, Gabadadze, Hui, Pirtskhalava, 2012

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or
   (ii) extended theories
- Quasidilaton: scalar  $\sigma$  with global symmetry:  $\sigma \to \sigma + \sigma_0 \quad \phi^a \to e^{-\sigma_0/M_{\rm Pl}} \phi^a$
- Action

$$S = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[ R - 2\Lambda - \frac{\omega}{M_{\rm Pl}^2} \partial_\mu \sigma \partial^\mu \sigma + 2m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right]$$
$$\mathcal{K}^{\mu}_{\ \nu} = \delta^{\mu}_{\ \nu} - e^{\sigma/M_{\rm Pl}} \left( \sqrt{g^{-1}f} \right)^{\mu}_{\ \nu} \qquad f_{\mu\nu} = \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

• Scaling solution = self-accelerating de Sitter (H = const > 0 with  $\Lambda$  = 0)

#### **Extension of quasidilaton**

arXiv: 1306.5502 [hep-th] /w A. De Felice

- Self-accelerating solution in the original quasidilaton theory has ghost instability
   [Gumrukcuoglu, Hinterbichler, Lin, Mukohyama, Trodden 2013; D'Amico, Gabadadze, Hui, Pirtskhalava 2013]
- Simple extension:  $f_{\mu\nu} \rightarrow \tilde{f}_{\mu\nu}$  $\tilde{f}_{\mu\nu} \equiv f_{\mu\nu} - \frac{\alpha_{\sigma}}{M_{\text{Pl}}^2 m_g^2} e^{-2\sigma/M_{\text{Pl}}} \partial_{\mu}\sigma \partial_{\nu}\sigma$ • Self-accel solution is stable within 5 d.o.f. if

$$\begin{aligned} 0 < \omega < 6 \qquad X^2 < \frac{\alpha_{\sigma} H^2}{m_g^2} < r^2 X^2 \qquad X \equiv \frac{e^{\bar{\sigma}/M_{\rm Pl}}}{a} \\ M_{\rm GW}^2 \equiv \frac{(r-1)X^3 m_g^2}{X-1} + \frac{\omega H^2 (rX+r-2)}{(X-1)(r-1)} > 0 \qquad r \equiv \frac{n}{N} a \end{aligned}$$

New quasidilaton theory? Mukohyama, arXiv: 14?????  $I_{\rm newQD}[g_{\mu\nu}, f_{\mu\nu}, \sigma] = M_{\rm Pl}^2 m_g^2 \int d^4x \sqrt{-g} \left[ \mathcal{L}_2(\bar{\mathcal{K}}) + \alpha_3 \mathcal{L}_3(\bar{\mathcal{K}}) + \alpha_4 \mathcal{L}_4(\bar{\mathcal{K}}) \right]$  $-\frac{\omega}{2} \int d^4x \sqrt{-g_{\rm eff}} g_{\rm eff}^{\mu\nu} \partial_{\mu}\sigma \partial_{\nu}\sigma$  $g_{\mu\nu}^{\text{eff}} = g_{\mu\nu} + 2\beta e^{\sigma/M_{\text{Pl}}} g_{\mu\rho} \left(\sqrt{g^{-1}f}\right)^{\rho} + \beta^2 e^{2\sigma/M_{\text{Pl}}} f_{\mu\nu}$ 

- Quasidilaton kinetic term is now defined on the effective metric  $\rightarrow$  new parameter  $\beta$
- Self-accerating de Sitter solution is stable in a range of parameters with  $\alpha_{\sigma}$  = 0 if  $\beta$  is non-zero

#### No conclusion yet...

- New matter coupling opens up new possibilities
- How heavy is the would-be BD ghost?
- UV sensitivity of quasidilaton theory? Can it be ameliorated by the new coupling?
- • •
- many questions