2014/09/16 Relativistic Cosmology

Primordial Gravitational Wave in Bimetric Gravity

Yuki Sakakihara (Kyoto University)

This research is collaborated with Jiro Soda (Kobe University)

Introduction

Though observations indicate the existence of a graviton,
we still know few about its features.

does it have its mass? how many species?

Suppose there are two (or more) gravitons,

in order to realize $1/r$ gravitational force
$\rightarrow$ at least, one of them is sufficiently light.

- Two interacting massless gravitons can not exist.
- A massless graviton and a massive graviton can exist.

The theory including two gravitons (= a massless graviton and a massive graviton)

$\Rightarrow$ We can realize such a theory with two metrics interacting each other.

Bimetric gravity

(de Rham et. al., 2011, Hassan and Rosen, 2012)

two metrics
$$\begin{cases} g_{\mu\nu} & : \text{physical metric} \\ f_{\mu\nu} & : \text{the other metric} \end{cases}$$

In order that the theory has stable solutions,
the form of the interaction terms are determined.
(with five theoretical parameters)

minimal bimetric model

$$\boxed{m^2 M_e^2 \int d^4x \, \frac{1}{2} \sqrt{-g} \left(L^\mu_\nu L^\nu_\mu - (L^\mu_\mu)^2 \right)}$$

$$L^\mu_\nu := \delta^\mu_\nu - \sqrt{(g^{\mu\lambda} f_{\lambda\nu})}$$

$$\frac{1}{M_e^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}$$

Bimetric action

$$S = \frac{M_g^2}{2} \int d^4x \sqrt{-g} R[g_{\mu\nu}]$$

kinetic terms of physical metric

$$+ \frac{M_f^2}{2} \int d^4x \sqrt{-f} R[f_{\mu\nu}] + m^2 M_e^2 \int d^4x \, \frac{1}{2} \sqrt{-g} \left(L^\mu_\nu L^\nu_\mu - (L^\mu_\mu)^2 \right)$$

kinetic terms of the other metric

interaction terms of the metrics

$$L^\mu_\nu := \delta^\mu_\nu - \sqrt{(g^{\mu\lambda} f_{\lambda\nu})}$$

$$\frac{1}{M_e^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}$$

Constraints in Bimetric gravity

- the first class constraints

The general coordinate invariance is kept only if the metrics are simultaneously transformed.

$\rightarrow$ 4 constraints

$$L^\mu_\nu := \delta^\mu_\nu - \sqrt{\underline{(g^{\mu\lambda} f_{\lambda\nu})}}$$

- the second class constraints

The form of the interaction terms produces a primary constraint and the time derivative gives a secondary constraint.

$\rightarrow$ 2 constraints

Inflation in bimetric gravity

If the other metric exists,
will anything go well? do some problems arise?
How are the effects on observations?

For example, about inflation

- Can we construct inflating solutions with a inflaton as in the case of GR? → Yes, we can.
- Are they stable solutions? → One branch of the solutions is guaranteed to be stable. (YS et al 2013)
- What is the feature of the gravitational waves generated during inflation?

Inflation in bimetric gravity

If the other metric exists,
will anything go well? do some problems arise?
How are the effects on observations?

For example, about inflation

- Can we construct inflating solutions with a inflaton as in the case of GR? → Yes, we can.
- Are they stable solutions? → One branch of the solutions is guaranteed to be stable. (YS et al 2013)
- What is the feature of the gravitational wave generated during inflation?

Today's topic

Outline

（１） We include a inflaton
and construct homogeneous isotropic inflating solutions.

（２） We impose slow-roll approximation.
(We consider up to the first order of the slow-roll parameter.)

（３） We calculate tensor perturbations
on the homogeneous isotropic solutions.

Bimetric gravity + inflaton

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}] + \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\varphi - V[\varphi]\right)$$

kinetic terms of physical metric scalar field (inflaton)

$$+\frac{M_f^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}] + m^2 M_e^2\int d^4x\,\frac{1}{2}\sqrt{-g}\left(L^\mu_\nu L^\nu_\mu - (L^\mu_\mu)^2\right)$$

kinetic terms of the other metric interaction terms of the metrics

$$L^\mu_\nu := \delta^\mu_\nu - \sqrt{(g^{\mu\lambda}f_{\lambda\nu})}$$

$$\frac{1}{M_e^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}$$

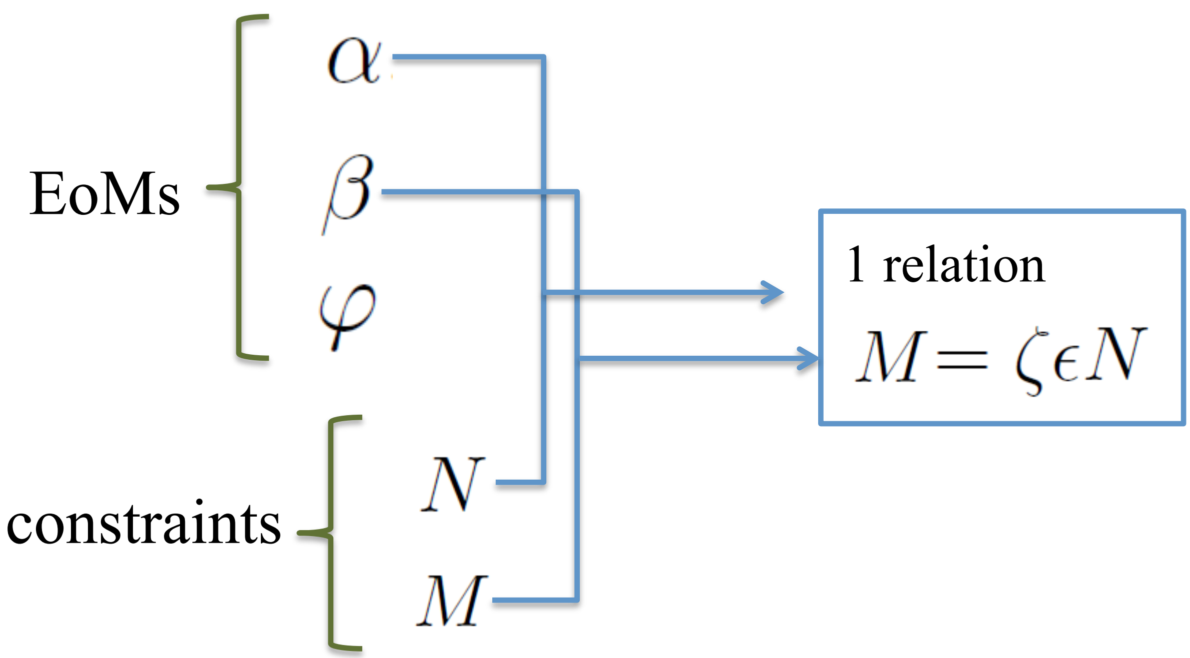
homogeneous isotopic solutions

substitute
the homogeneous isotropic ansatz

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + e^{2\alpha(t)}(dx^2 + dy^2 + dz^2)$$

$$f_{\mu\nu}dx^\mu dx^\nu = -M^2(t)dt^2 + e^{2\beta(t)}(dx^2 + dy^2 + dz^2)$$

→ variational principle

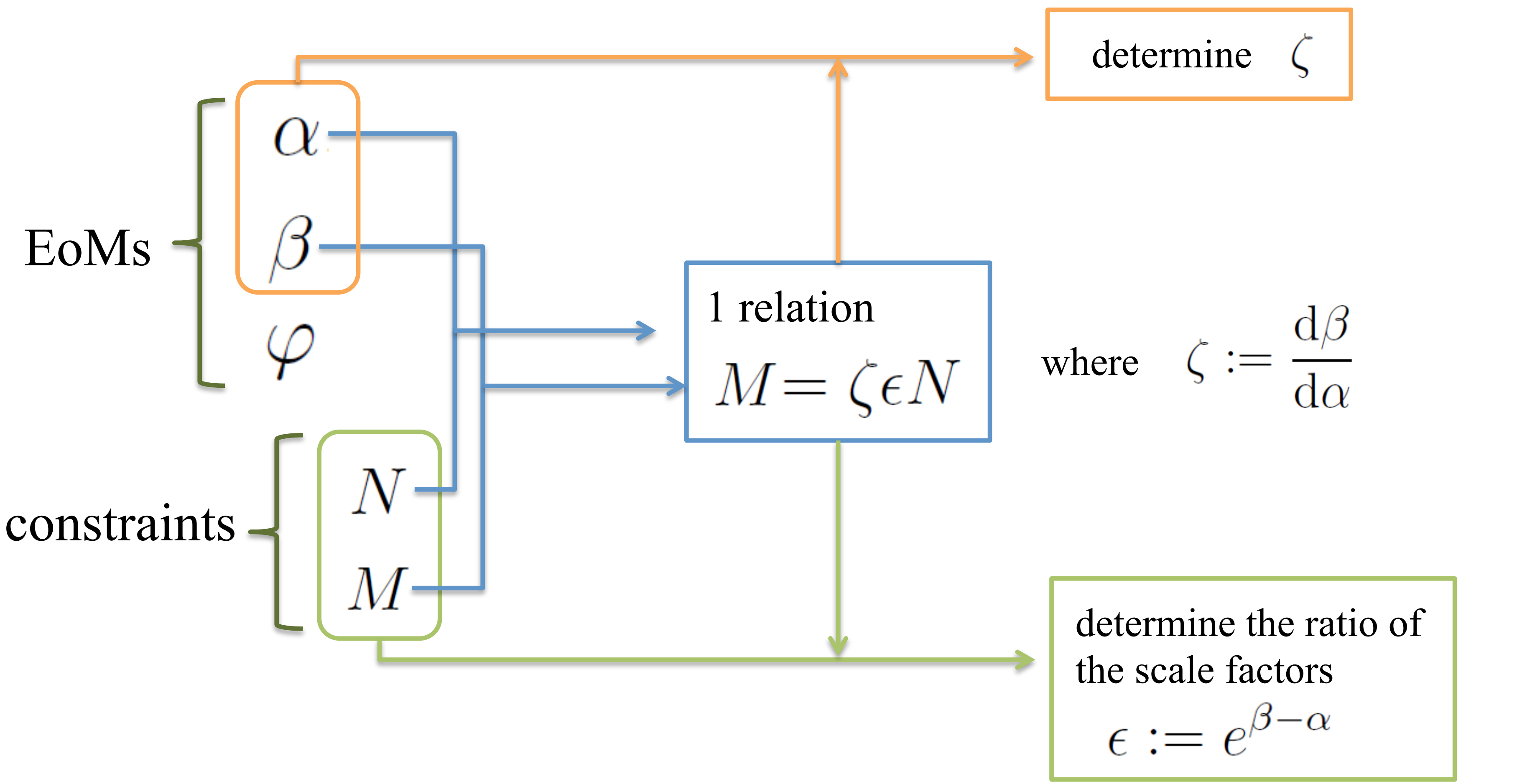


where $\zeta := \dfrac{\mathrm{d}\beta}{\mathrm{d}\alpha}$

homogeneous isotopic solutions

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + e^{2\alpha(t)}(dx^2+dy^2+dz^2)$$

$$f_{\mu\nu}dx^\mu dx^\nu = -M^2(t)dt^2 + e^{2\beta(t)}(dx^2+dy^2+dz^2)$$



homogeneous isotopic solutions

$$\tan a := M_g/M_f$$
$$H := \dot{\alpha}$$

$$\dot{H} = m^2 \cos^2 a \left(\frac{3}{2} - \epsilon\right)(\zeta - 1)\epsilon - \frac{1}{2M_g^2}\dot{\varphi}^2$$ the EoM of α

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\mathrm{d}V}{\mathrm{d}\varphi} = 0$$ the EoM of φ

$$H^2 = m^2 \cos^2 a(-2 + 3\epsilon - \epsilon^2) + \frac{1}{3M_g^2}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]$$ Hamiltonian constraint

$$m^2 \sin^2 a \frac{1-\epsilon}{\epsilon} = m^2 \cos^2 a(-2 + 3\epsilon - \epsilon^2) + \frac{1}{3M_g^2}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]$$ determines ϵ

$$\zeta = 1 + \frac{\dot{\varphi}^2}{M_g^2(m_{\text{eff}}^2 - 2H^2)}$$ determines ζ

where $m_{\text{eff}}^2(\epsilon) := m^2\left[\cos^2 a\epsilon + \sin^2 a\frac{1}{\epsilon}\right](3 - 2\epsilon)$

homogeneous isotopic solutions

$$\tan a := M_g/M_f$$
$$H := \dot{\alpha}$$

$$\dot{H} = m^2 \cos^2 a \left(\frac{3}{2} - \epsilon\right)(\zeta - 1)\epsilon - \frac{1}{2M_g^2}\dot{\varphi}^2$$ the EoM of α

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\mathrm{d}V}{\mathrm{d}\varphi} = 0$$ the EoM of φ

$$H^2 = m^2 \cos^2 a(-2 + 3\epsilon - \epsilon^2) + \frac{1}{3M_g^2}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]$$ Hamiltonian constraint

$$m^2 \sin^2 a \frac{1-\epsilon}{\epsilon} = m^2 \cos^2 a(-2 + 3\epsilon - \epsilon^2) + \frac{1}{3M_g^2}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]$$ determines ϵ

$$\zeta = 1 + \frac{\dot{\varphi}^2}{M_g^2(m_{\mathrm{eff}}^2 - 2H^2)}$$ determines ζ

There are three branches, but only one branch is reasonable.

$\epsilon > 0$ and $H^2 > 0$

Stability: This branch satisfies Higuchi bound in de Sitter limit. (YS et al 2013)

$$m_{\mathrm{eff}}^2 > 2H^2 \qquad \text{where} \qquad m_{\mathrm{eff}}^2(\epsilon) := m^2\left[\cos^2 a\epsilon + \sin^2 a\frac{1}{\epsilon}\right](3 - 2\epsilon)$$

Slow-roll approximation

Slow-roll parameter $s := -\frac{\dot{H}}{H^2}$ $\quad s \ll 1$

We neglect $\mathcal{O}(s^2)$, $\dot{s}$

$$\dot{\varphi}^2 = 2M_g^2 s H^2 \left(1 + \frac{\epsilon^2(3-2\epsilon)}{\tan^2 a}\right)$$

$$\zeta = 1 + \frac{\dot{\varphi}^2}{M_g^2(m_{\rm eff}^2 - 2H^2)}$$

$$= 1 + 2s(1-\epsilon)$$

$\rightarrow$ $\boxed{\delta\zeta = \mathcal{O}(s)}$ where $\delta\zeta := \zeta - 1$

ζ is equal to 1 in the slow-roll limit

From the definition of ζ , $\boxed{\delta\zeta} = \frac{\mathrm{d}\beta}{\mathrm{d}\alpha} - 1 = \boxed{\frac{\dot{\epsilon}/\epsilon}{H}}$ $\quad \epsilon$ is time dependent.

Tensor perturbation

$$\delta g_{ij} = q_{ij} \ , \quad \delta f_{ij} = p_{ij}$$ satisfy TT conditions: $$q^i{}_{j|i} = 0 \ , \quad q^i{}_i = 0 \ , \qquad p^i{}_{j|i} = 0 \ , \quad p^i{}_i = 0$$

flavor eigen state (g and f)

$$\delta^2 \mathcal{L} = \frac{M_g^2}{4} e^{3\alpha} \Bigg[\underbrace{\frac{1}{2}\dot{q}^2 - \frac{1}{2}\frac{k^2}{e^{2\alpha}} q^2}_{\text{g-metric terms}} + \underbrace{\frac{\epsilon^2}{\tan^2 a}\left(\frac{1}{2\zeta}\dot{p}^2 - \frac{1}{2}\frac{\zeta k^2}{e^{2\beta}} p^2\right)}_{\text{f-metric terms}}$$

$$\underbrace{+ m^2 \cos^2 a \left(-\frac{3}{2}\epsilon + \frac{1}{2}\epsilon^2 + \frac{1}{2}\zeta\epsilon^2 \right) (p - q)^2}_{\text{mixing(interaction) terms}} \Bigg]$$

Cross terms remain in the slow-roll(de Sitter) limit.

It is difficult to obtain solutions analytically.

Mass eigen state

Rotation $$\begin{pmatrix} q \\ p \end{pmatrix} = \frac{1}{(\kappa^2+\epsilon^2)^{1/2}} \begin{pmatrix} \kappa & -\epsilon \\ \kappa & \kappa^2/\epsilon \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ where $\kappa = \zeta^{1/2} \tan a$

$$\delta^2 \mathcal{L} = \frac{M_g^2}{4} e^{3\alpha} \Big[\underbrace{\frac{1}{2}\dot{x}^2 - \frac{1}{2}\frac{k^2}{e^{2\alpha}} x^2}_{\text{Massless part}} + \underbrace{\frac{1}{2}\dot{y}^2 - \frac{1}{2}\tilde{m_{\rm eff}}^2 y^2 - \frac{1}{2}\frac{k^2}{e^{2\alpha}} y^2}_{\text{Massive part}}$$

$$\boxed{\begin{aligned} &+\delta\zeta\left(3H^2 - \frac{2k^2}{e^{2\alpha}}\right)\frac{1}{2}\frac{\epsilon^2}{\kappa^2+\epsilon^2}x^2 + \delta\zeta\left(3H^2 - \frac{2k^2}{e^{2\alpha}}\right)\frac{1}{2}\frac{\kappa^2}{\kappa^2+\epsilon^2}y^2 \\ &+ \delta\zeta\frac{2\kappa\epsilon}{\kappa^2+\epsilon^2}\left(-H\dot{x}y - \frac{k^2}{e^{2\alpha}}xy\right)\end{aligned}}\Big]$$

Terms disappearing in the slow-roll limit

→ **diagonal in the slow-roll limit**

We can obtain analytic solutions in the slow-roll limit and construct higher-order solutions in the slow-roll parameter successively.

Calculation of Tensor Spectra

(1) Scale transformation into canonical variables (x, y) → (X, Y)

$$
\begin{aligned}
\mathcal{H} &= \pi_X X' + \pi_Y Y' - \delta^2\mathcal{L} \\
&= \int d^3k\, \frac{1}{2}\pi_X^2 + \frac{1}{2}\left[k^2\left(1+\frac{4s\epsilon^2(1-\epsilon)}{\kappa^2+\epsilon^2}\right) - \frac{2+3s\left(1+\frac{2\epsilon^2(1-\epsilon)}{\kappa^2+\epsilon^2}\right)}{\eta^2}\right]X^2 \\
&\quad + \frac{1}{2}\pi_Y^2 + \frac{1}{2}\left[k^2\left(1+\frac{4s\kappa^2(1-\epsilon)}{\kappa^2+\epsilon^2}\right) - \frac{2+3s\left(1+\frac{2\kappa^2(1-\epsilon)}{\kappa^2+\epsilon^2}\right) - \tilde{m}_{\mathrm{eff}}^2/H^2}{\eta^2}\right]Y^2 \\
&\quad + s\frac{2\kappa\epsilon(1-\epsilon)}{\kappa^2+\epsilon^2}\left[\left(2k^2-\frac{3}{\eta^2}\right)XY + \frac{1}{\eta}(X\pi_Y - \pi_X Y)\right]
\end{aligned}
$$

non-interacting Hamiltonian (including only the terms like XX, YY)

interacting Hamiltonian (including only the terms like XY、 order s)

(2) Construction of homogeneous solutions X_0 , Y_0

(3) Introduction of the effect up to the first order of the slow-roll parameter using the interaction picture

(Since the interacting Hamiltonian includes XY terms, the effect of the first order of the slow-roll parameter contributes only to the XY correlation.)

$$\langle XX\rangle = \langle X_0(x)X_0(x)\rangle + i\int_{-\infty}^{\eta} d\eta'(\cancel{\langle H_{\mathrm{int}}(\eta')X_0(x)X_0(x)\rangle} - \cancel{\langle X_0(x)X_0(x)H_{\mathrm{int}}(\eta')\rangle})$$

$$\langle XY\rangle = \cancel{\langle X_0(x)Y_0(x)\rangle} + i\int_{-\infty}^{\eta} d\eta'(\langle H_{\mathrm{int}}(\eta')X_0(x)Y_0(x)\rangle - \langle X_0(x)Y_0(x)H_{\mathrm{int}}(\eta')\rangle)$$

$$\langle YY\rangle = \langle Y_0(x)Y_0(x)\rangle + i\int_{-\infty}^{\eta} d\eta'(\cancel{\langle H_{\mathrm{int}}(\eta')Y_0(x)Y_0(x)\rangle} - \cancel{\langle Y_0(x)Y_0(x)H_{\mathrm{int}}(\eta')\rangle})$$

Tensor Spectra in the mass eigen state

Subscripts 0 mean the values in de Sitter limit.

In the first order of the slow-roll parameter, …

$$\langle xx \rangle = \left(\frac{H_0}{\pi M_g}\right)^2 (-\eta)^{3+2s-2\nu_X} \left(\frac{k}{2}\right)^{3-2\nu_X} \left(\frac{\Gamma(\nu_X)}{\Gamma(\frac{3}{2})}\right)^2 \underline{\left(1+\frac{4s\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)^{-\nu_X}} H_0^{2s} \qquad \sim \text{const.}$$

variation of the propagation speed

where $\quad \nu_X = \frac{3}{2} + s\left(1 + \underline{\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}}\right)$

variation of the mass value

$$\langle xy \rangle \propto \frac{1}{e^{3\alpha}} \qquad \to 0$$

$$\delta^2 \mathcal{L} \sim \delta\zeta \left(\boxed{3H^2}\boxed{-\frac{2k^2}{e^{2\alpha}}}\right)\frac{1}{2}\frac{\epsilon^2}{\kappa^2+\epsilon^2}x^2$$

$$\langle yy \rangle \propto \frac{1}{e^{3\alpha}} \qquad \to 0$$

Tensor Spectra in the flavor eigen state

From the relation $\begin{pmatrix} q \\ p \end{pmatrix} = \frac{1}{(\kappa^2+\epsilon^2)^{1/2}} \begin{pmatrix} \kappa & -\epsilon \\ \kappa & \kappa^2/\epsilon \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$,

$$\langle qq \rangle = \langle qp \rangle = \langle pp \rangle = \frac{\kappa^2}{\kappa^2+\epsilon^2}\langle xx \rangle + \mathcal{O}(e^{-3\alpha})$$

$$= \underline{\frac{\kappa^2}{\kappa^2+\epsilon^2}}\left(\frac{H_0}{\pi M_g}\right)^2 H_0^{2s}\,\underline{(-\eta)^{-2s\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}}}\left(\frac{k}{2}\right)^{\underline{-2s\left(1+\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)}}\left[\frac{\Gamma\left(\frac{3}{2}+s\left(1+\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)\right)}{\Gamma\left(\frac{3}{2}\right)}\right]^2\underline{\left(1-\frac{6s\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)}$$

Features

- Tensor amplitudes are suppressed due to the mixing in flavor eigen states.
- The amplitudes do not conserve on the super-horizon and amplified in the order of the slow-roll parameter.

competitive

- spectral index $n_T = -2s\left(1+\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)$
- The amplitudes are suppressed due to the variation of the propagation speed in mass eigen states.

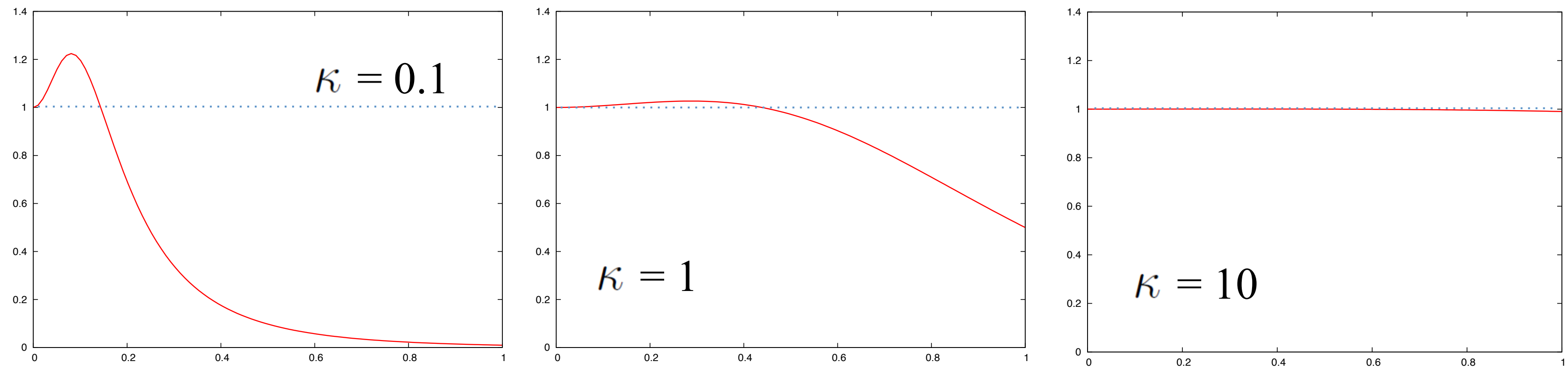
Amplitude and Spectral index

slow-roll parameter = 0.01

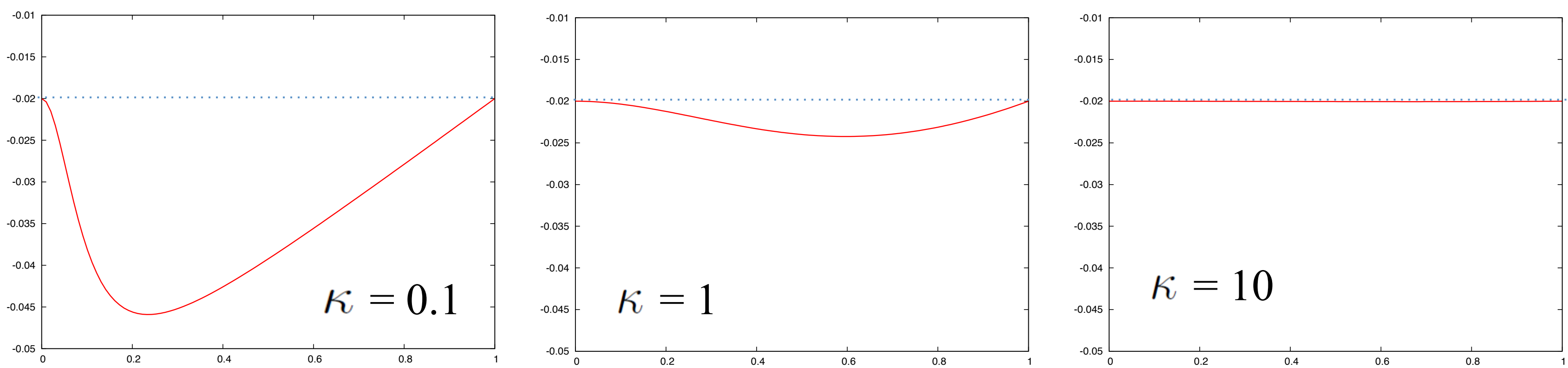
near Massive Gravity ⟷ near General Relativity

$M_f \;>>\; M_g$ $\qquad$ $M_f \;=\; M_g$ $\qquad$ $M_f \;<<\; M_g$

Amplitude



Spectral index



x-axis: ϵ_0 in the range of (0, 1)

which is determined by the ratio of the bare mass and the inflation energy scale.

$\epsilon_0 \sim 0$ for $\quad m^2 \ll V/3M_g^2$

a finite value for $\quad m^2 \sim V/3M_g^2$

$\epsilon_0 \sim 1$ for $\quad m^2 \gg V/3M_g^2$

Note

If we consider $m^2 \ll V/3M_g^2$ situation,

this solution will suffer gradient instability in the radiation dominant era. (de Felice et al, 2014)

Since we have thought only about a minimal bimetric model,

the extension of this discussion to more general model may circumvent this instability.

Future work

- How about the scalar tensor ratio?

  ➡ Calculation of scalar perturbations

- Perturbations do not conserve on the super-horizon in the order of the slow-roll parameter.

  ➡ We need to solve the growth history of the perturbations after inflation.

  For instance, how is the behavior in the radiation dominant era?

- The mass term of the other metric vary depending on the scalar field value.

$$\zeta = 1 + \frac{\dot{\varphi}^2}{M_g^2[m_{\rm eff}^2 - 2H^2]}$$

  ➡ The scalar field oscillates in the reheating era.

  ➡ Parametric resonance may enhance the tensor amplitudes through the interaction terms.