Super-sample covariance/signal (SSC/SSS)

Masahiro Takada
(Kavli IPMU, U. Tokyo)

@KITP WS, Sep 16 2014

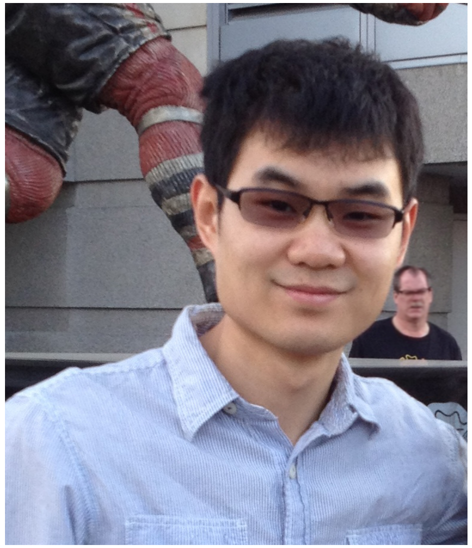
Collaborators

Masahiro Takada & Wayne Hu, PRD 87, 2013

Yin Li, W. Hu & M. Takada, PRD 89, 2014

Yin Li, W. Hu & M. Takada, arXiv:1408.1081

Yin Li, W. Hu & M. Takada+ in prep

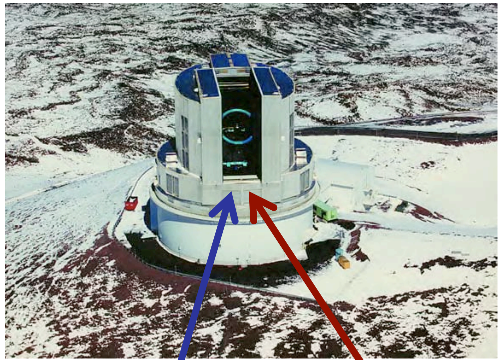

Yin Li

Also based on works with Sarah Bridle, Bhuvnesh Jain, Ryuichi Takahashi, Issha Kayo, Masanori Sato, David Spergel, Emmanuel Schaan, …

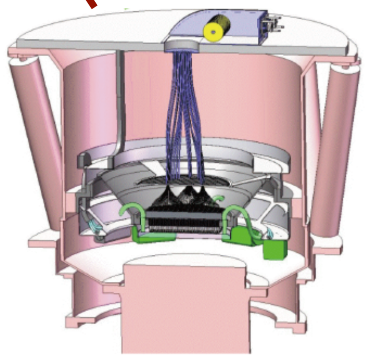
Subaru Telescope

SuMIRe = Subaru Measurement of Images and Redshifts

- IPMU director Hitoshi Murayama funded (~ $32M) by the Cabinet in Mar 2009, as one of the stimulus package programs
- Build *wide-field imaging* camera (Hyper Suprime-Cam) and *wide-field* multi-object spectrograph (Prime Focus Spectrograph) for the Subaru Telescope (8.2m)
- Explore the fate of our Universe: dark matter, dark energy
- Keep the Subaru Telescope a world-leading telescope in the TMT era
- Precise images of 1B galaxies; now started in March 2014
- Measure distances of ~4M galaxies; the first light is in 2018

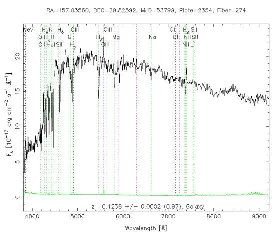


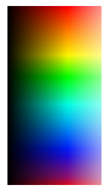
Subaru (NAOJ)



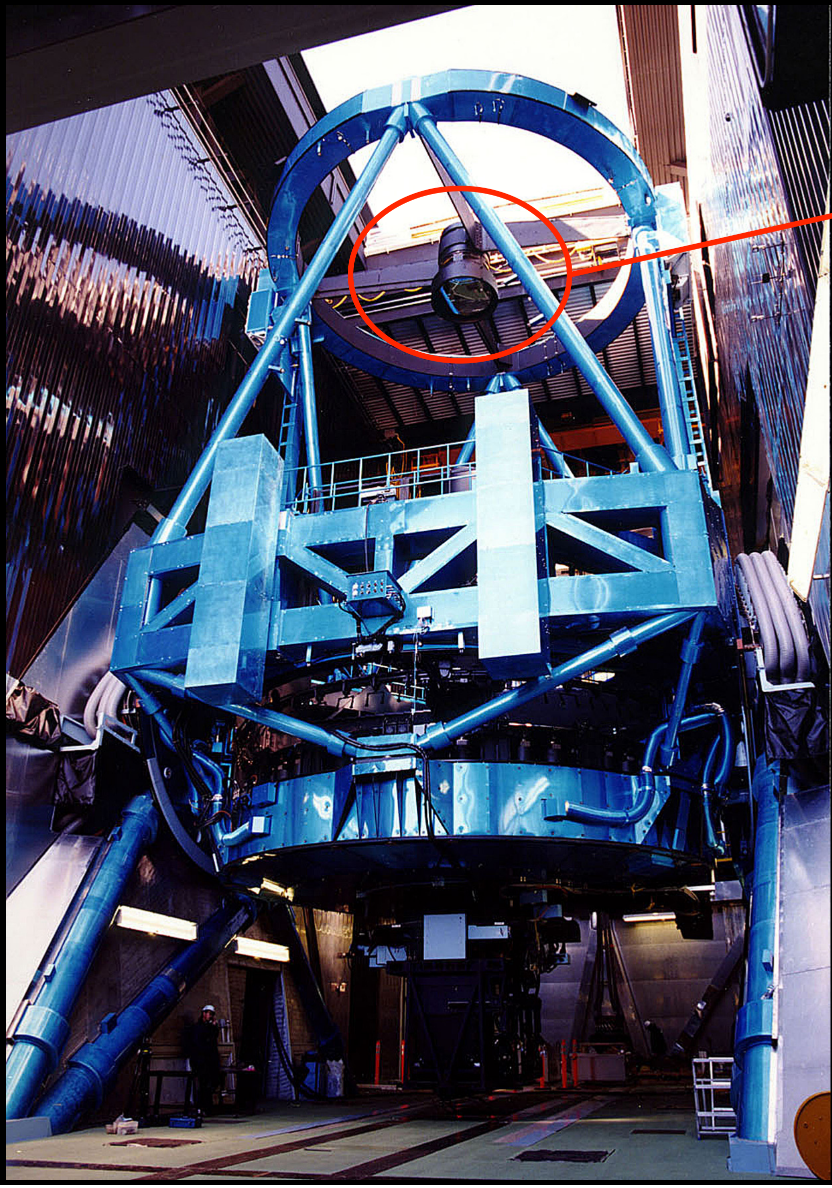
PFS

HSC

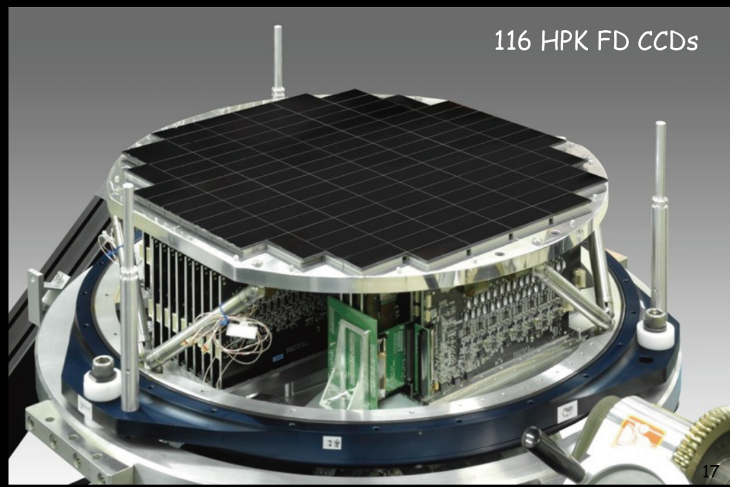




Hyper Suprime-Cam



- largest camera
- 3m high
- weigh 3 ton
- 104 CCDs
(~0.9B pixels)

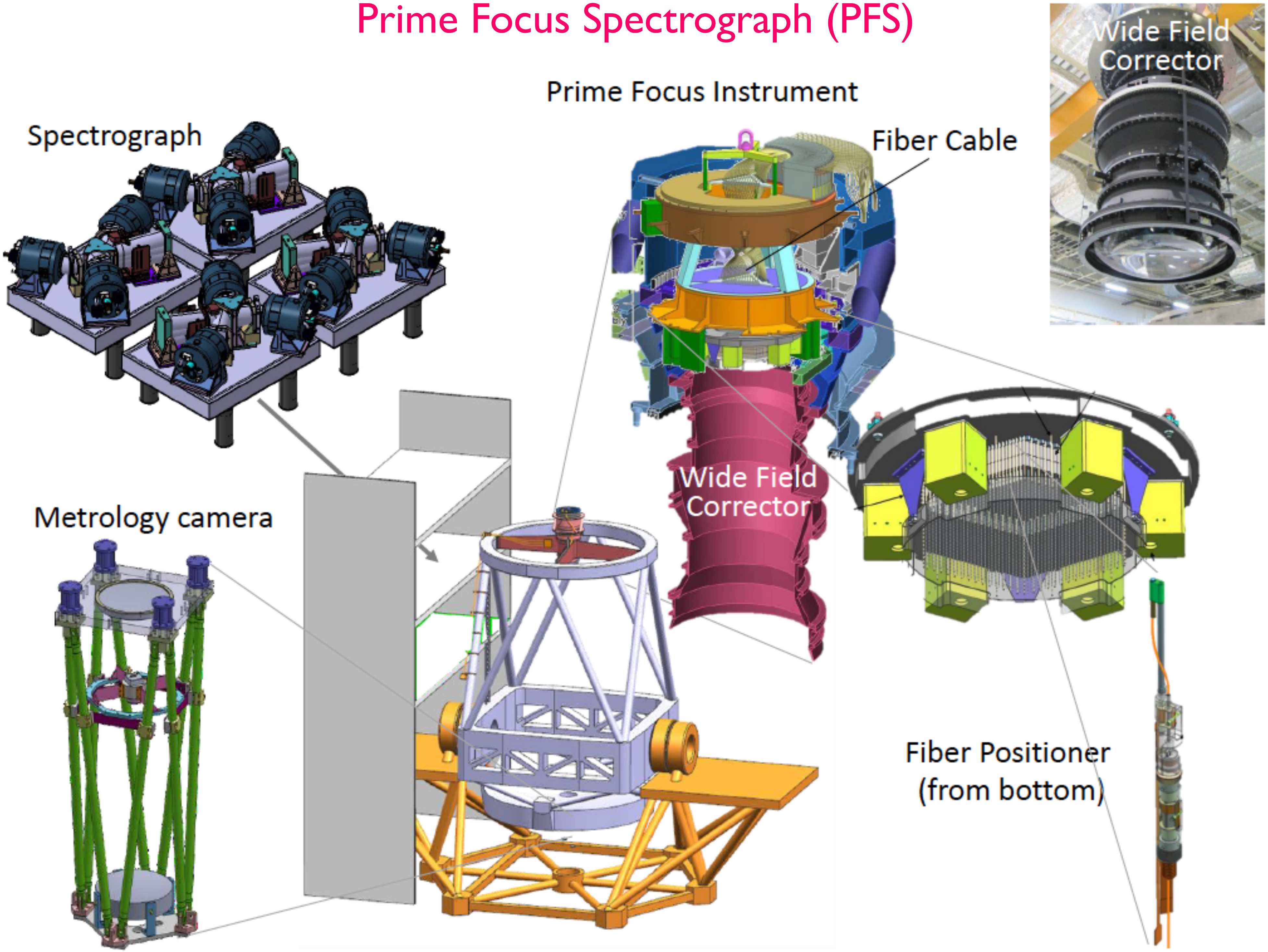


Hyper Suprime-Cam FoV

wi

- **Fas**
- a cos

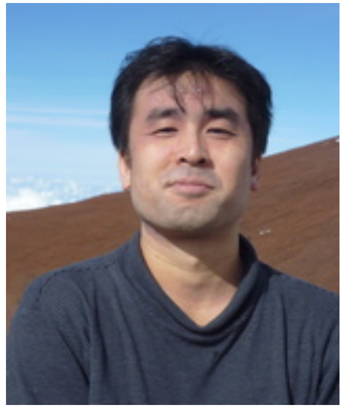
Prime Focus Spectrograph (PFS)



PFS Collaboration

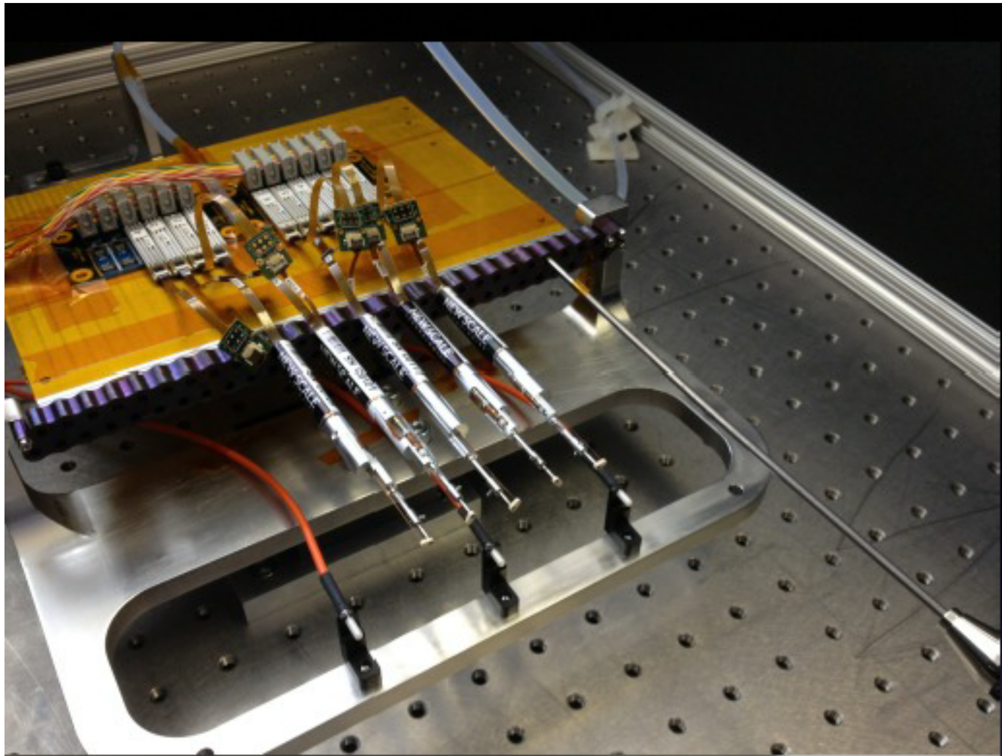
PM H. Sugai

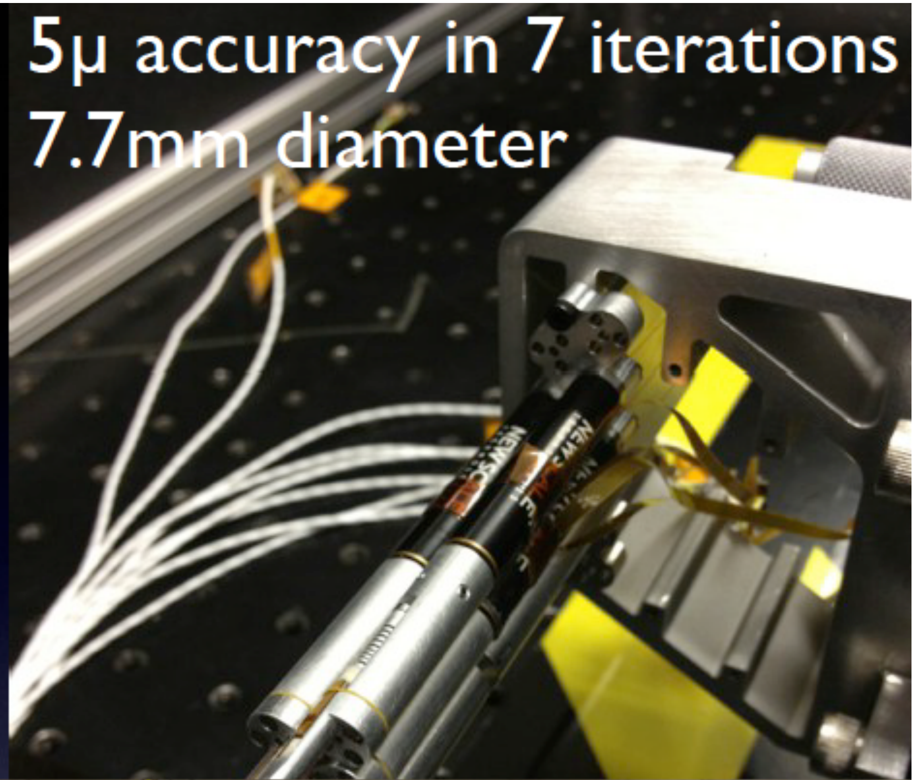
M. Takada: PS & Science WG co-chair



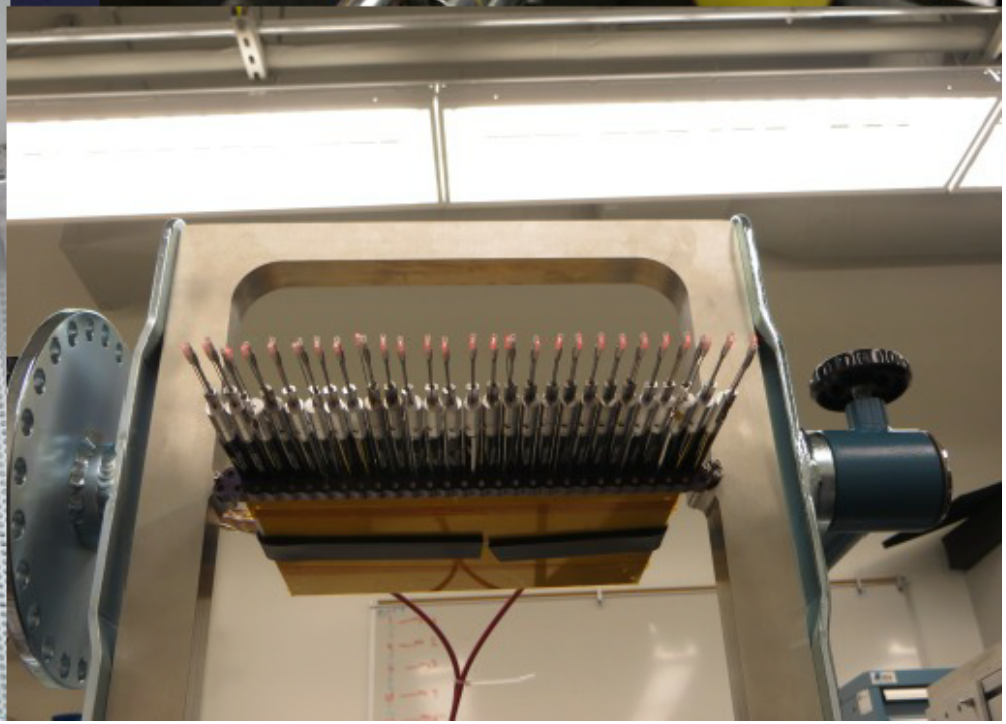
SE N. Tamura

Kavli IPMU plays a leading role in this international collaboration




5μ accuracy in 7 iterations
7.7mm diameter

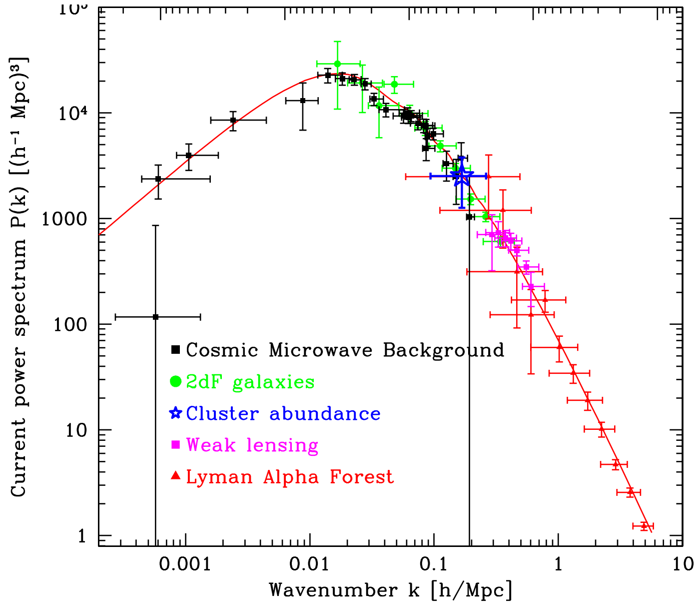

NEW SCALE
SN C0118
SN C0122
SN C0103

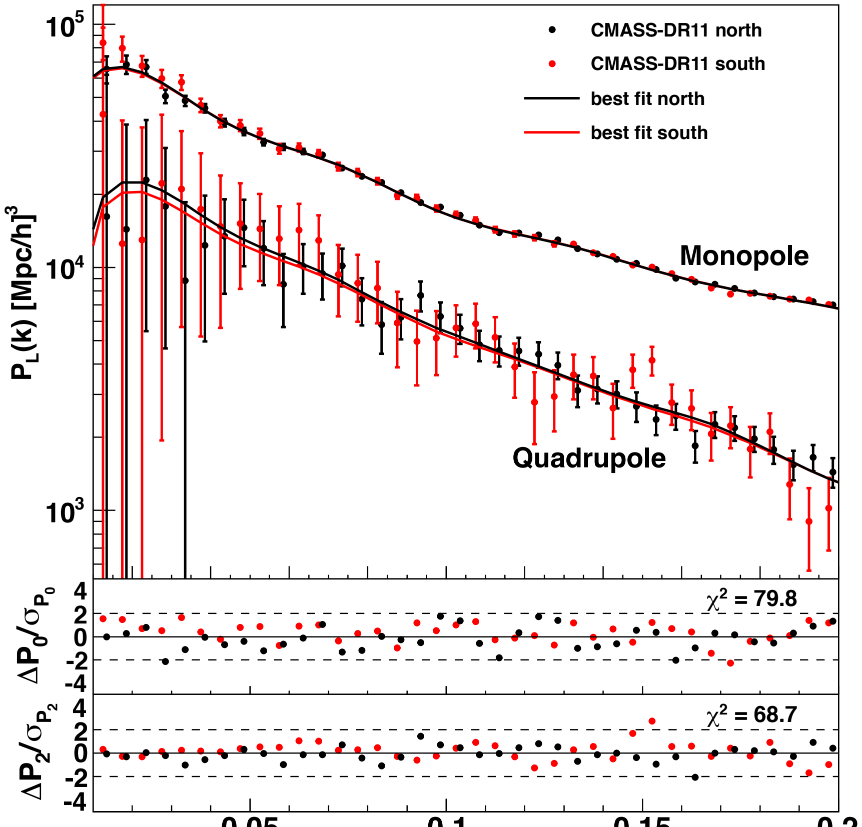



Dark Energy Competition
SDSSIII
BOSS (2009-)
KIDS (2012-)
DESI (2018?-)
A Proposal to NOAO for the
BigBOSS Experiment
at Kitt Peak National Observatory
October 1, 2010
DARK ENERGY
SURVEY
DES (2012-)
Euclid (2020)
LSST
Large Synoptic Survey Telescope
LSST (2022-)
WFIRST (2020?-)
WFIRST
Wide-Field Infrared Survey Telescope

Upcoming wide-area galaxy surveys

- BOSS (done): 10,000 deg^2, 0.3<z<0.6, V~4 (Gpc/h)^3
- eBOSS (2014-): 10,000 deg^2, 0.6<z<1, V~9 (Gpc/h)^3
- Subaru PFS (2018-23): 1,400 deg^2, 0.8<z<2.4, V~9 (Gpc/h)^3
- DESI (2018?): 10,000 deg2, 0.8<z<1.2, V~25 (Gpc/h)^3
- Euclid (2021-26): 15,000 deg^2, 0.8<z<2, V~40 (Gpc/h)^3
- *Will allow an access to VERY long-wavelength modes at low-z, not z~1100*





Beutler, Saito+13

courtesy of color code: Wayne Hu

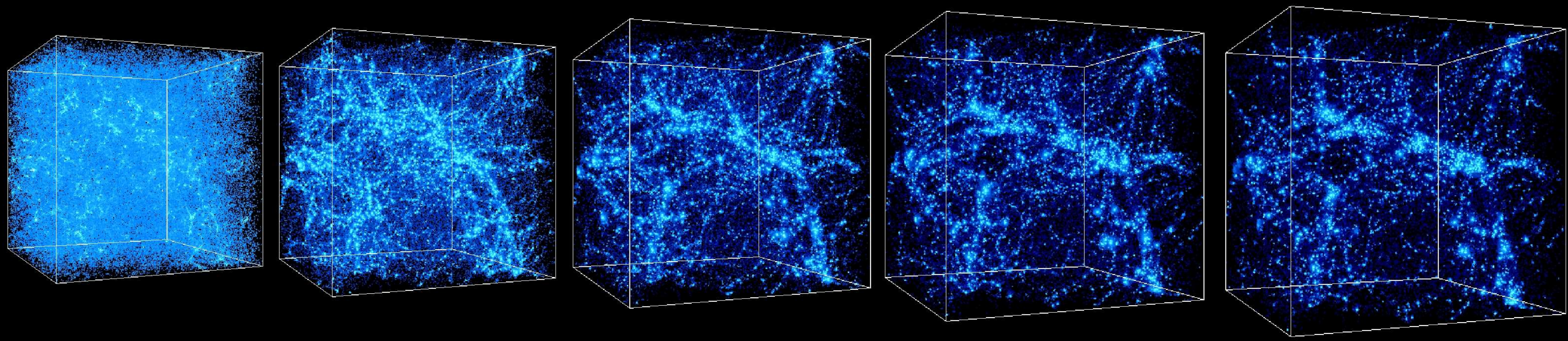
My talk is about cross-correlation between very long-wavelength (unobservable) mode and short-wavelength (observable) modes

unobservable
(beyond survey size)

observable
(inside survey)

- A consequence of gravity in nonlinear structure formation, and therefore this effect is relevant for any large-scale structure probes – *can be very IMPORTANT*

Nonlinear structure formation: mode-coupling



- Due to the nonlinear nature of gravity, LSS fields (DM or galaxy distribution or WL field) becomes non-Gaussian at small scales
- All Fourier modes become correlated with each other; e.g., the perturbation theory of structure formation predicts

$$\tilde{\delta}^{(2)}(\vec{k}) = \int \frac{d^3\vec{q}}{(2\pi)^3} F_2(\vec{q}, \vec{k}-\vec{q})\tilde{\delta}^{(1)}(\vec{q})\tilde{\delta}^{(1)}(\vec{k}-\vec{q})$$

Nonlinear structure formation

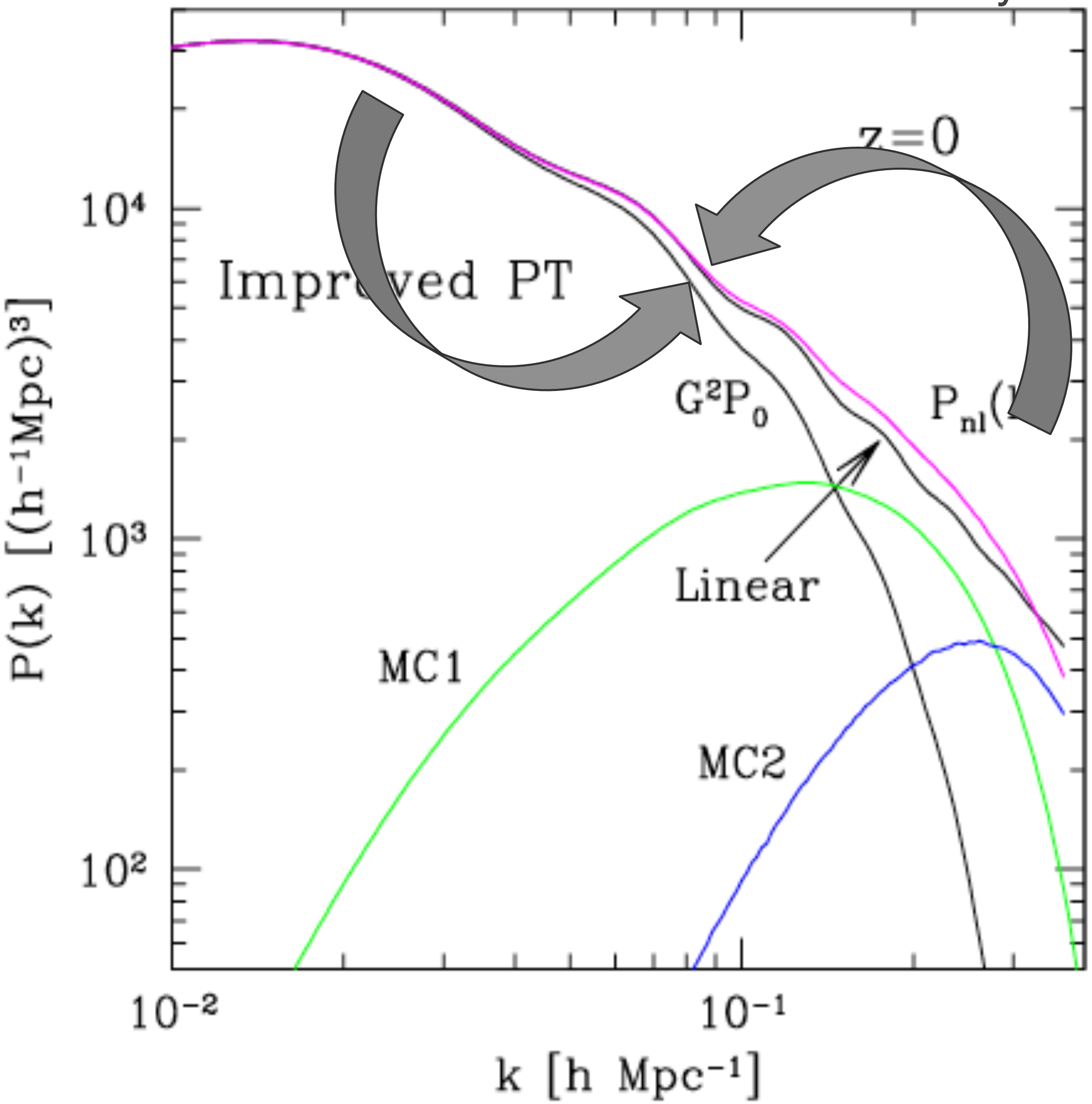
- Neutrino mass, test of gravity, halo bias, …

Taruya +

- Nonlinear mode coupling around k~0.1 – O(1) h/Mpc
  - From small q to large k seems OK
  - From large q to small k: ????

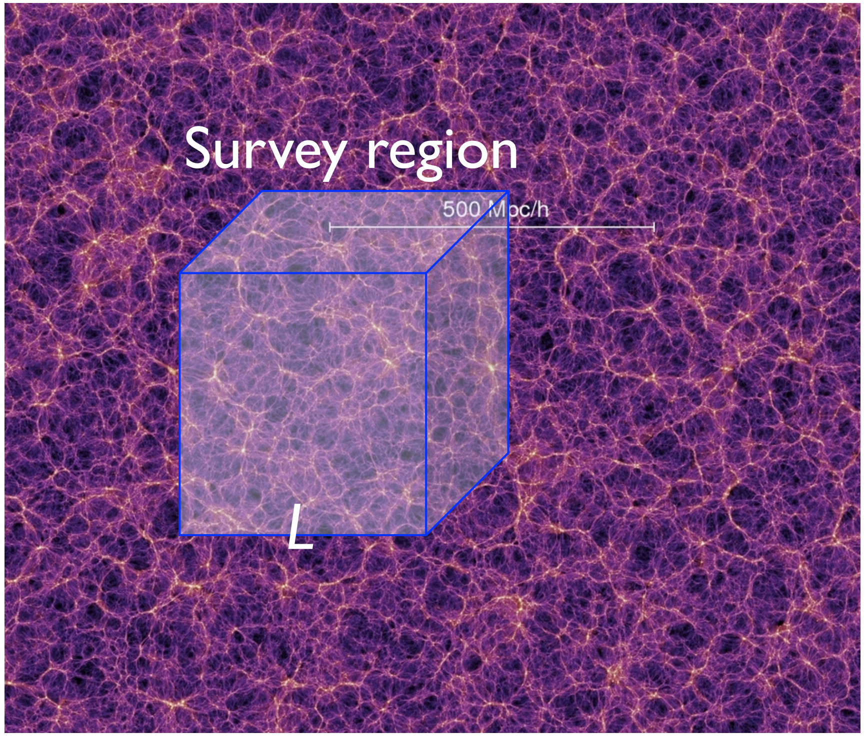
$$\epsilon_{\delta<} = \int_0^k \frac{d^3\vec{k}'}{(2\pi)^3} P^L(k')$$

$$\epsilon_{s>} = k^2 \int_k^{\infty} \frac{d^3\vec{k}'}{(2\pi)^3} \frac{P^L(k')}{k'^2}$$



Super-survey (sample) modes

MT & Hu 13



- The observed field is given as

$$\delta_W(\mathbf{x}) = W(\mathbf{x})\delta(\mathbf{x})$$

$$W(\mathbf{x}) = 1 \text{ if } \mathbf{x} \in \mathbf{S}$$

$$\text{otherwise } W(\mathbf{x}) = 0$$

- The Fourier-transformed field is

$$\tilde{\delta}_{W,\mathbf{k}} = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \tilde{W}_{\mathbf{k}-\mathbf{q}} \tilde{\delta}_{\mathbf{q}}$$

  – The width of W(k) is ~1/L
  – In this way, we can explicitly include contributions of modes outside a survey region

- The background density mode within a survey region

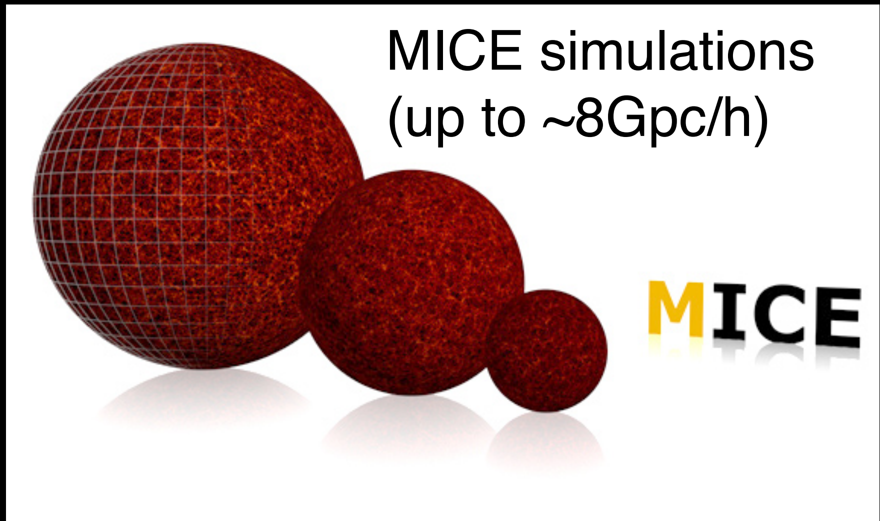
$$\bar{\delta}_b = \frac{1}{V_S} \int d^3\mathbf{x} W(\mathbf{x}) \delta(\mathbf{x})$$

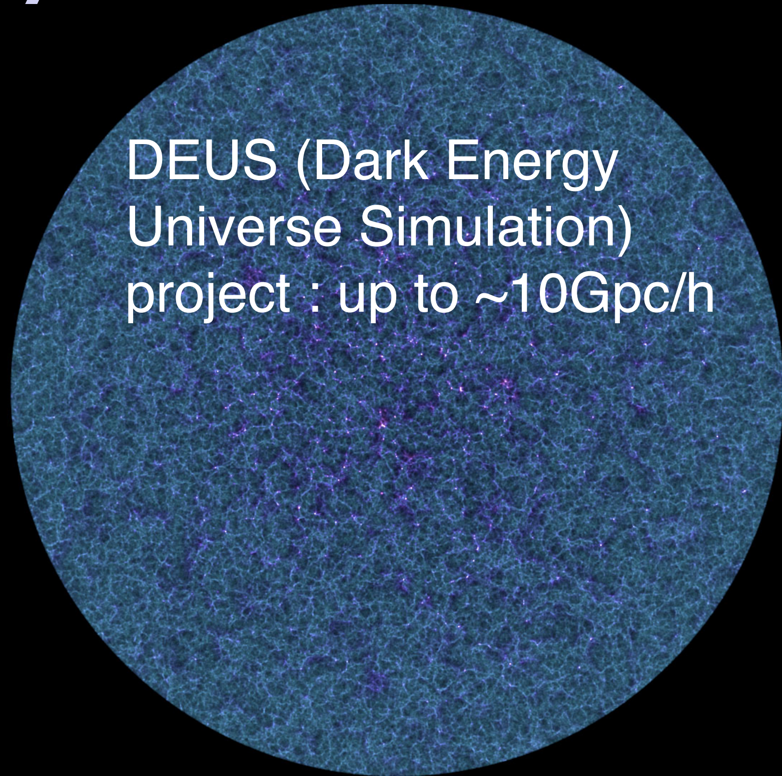
generally non-zero on realization basis

$$\langle \bar{\delta}_b \rangle_{\rm ens} = 0$$

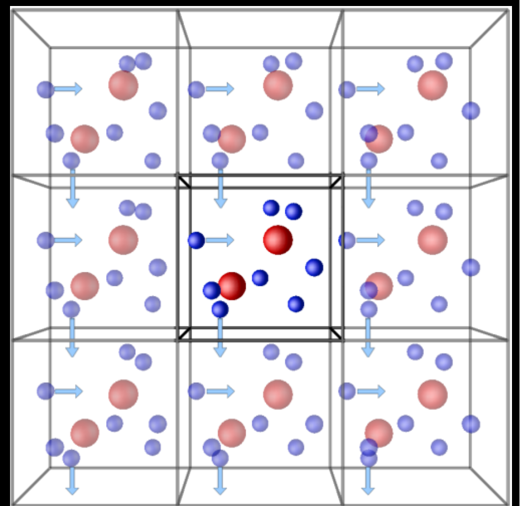
$$\sigma_b^2 = \langle \bar{\delta}_b^2 \rangle \neq 0$$

Limitations of N-body simulations?

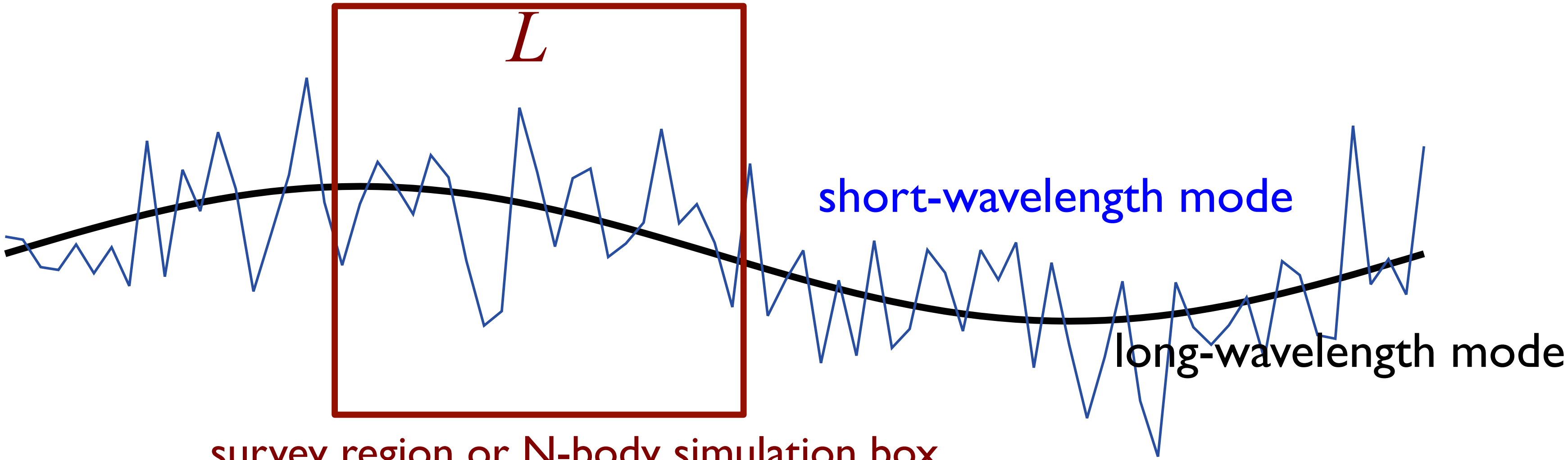




- N-body sim. now 40 yrs history
- *Employ periodic boundary conditions*
- How large volume do we need?
- If we run a very large-box simulation, most of the computation time is for the linear or quasi-nonlinear dynamics? Is this against the aim of N-body simulations?
- How to include a super-box mode (DC mode)?
- Occasionally some papers have discussed the effect of DC mode (e.g., Pen 99; Sirko 05), but has not really implemented



Super-survey or -box modes

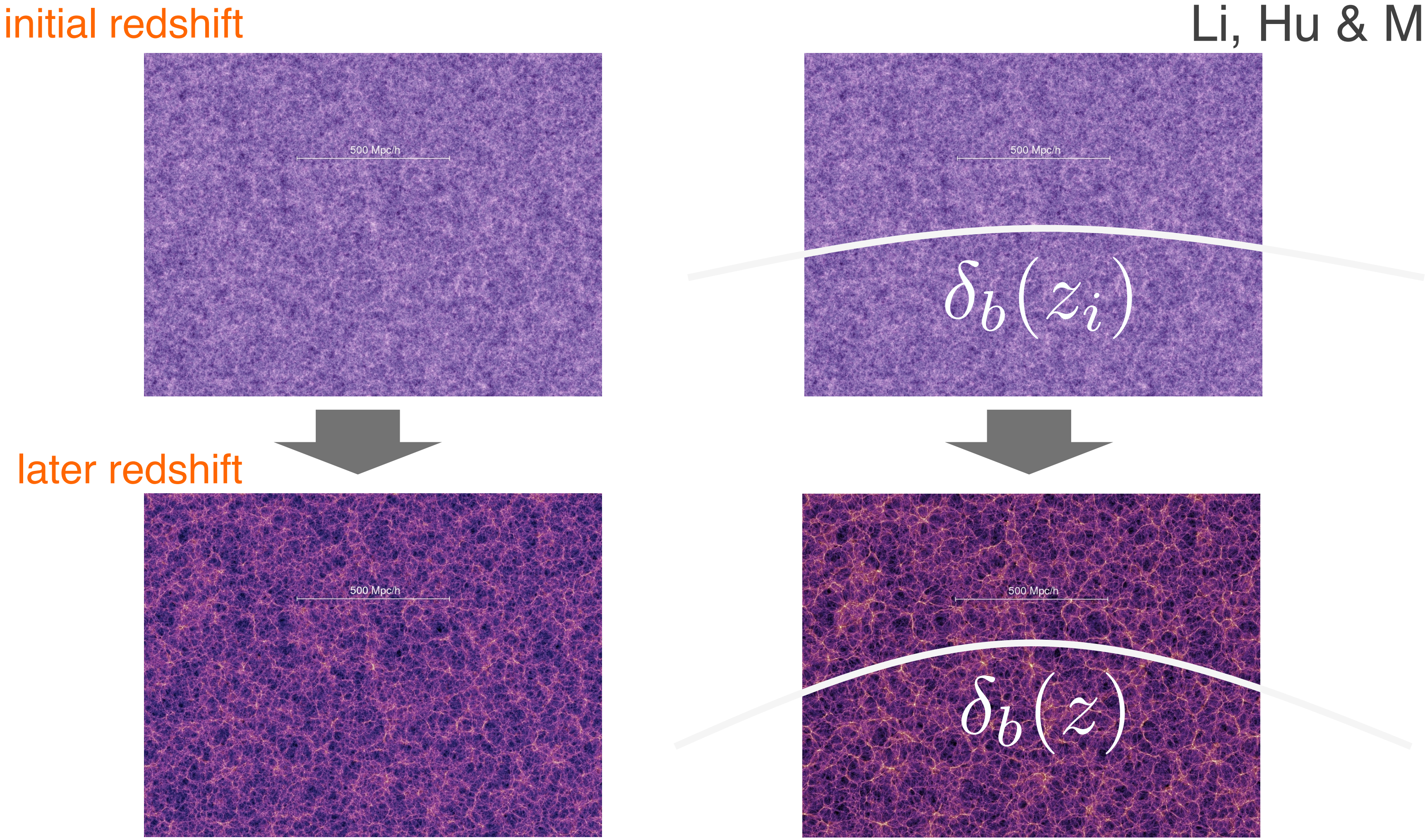


Long-wavelength modes can be expanded around the survey region

$$\begin{aligned}\Phi_L(\mathbf{x}) &\simeq \bar{\Phi}_L + \nabla_i\Phi_L(\mathbf{x})L_i + \frac{1}{2}\nabla_i\nabla_j\Phi_L L_i L_j + \cdots \\ &= \bar{\Phi}_L + \frac{1}{2}(\Delta\Phi_L)\frac{1}{3}L^2 + \nabla_i\Phi_L(\mathbf{x})L_i + \frac{1}{2}\tau_{ij}L_iL_j + \cdots \\ &= \bar{\Phi}_L + \underbrace{2\pi G\bar{\rho}\bar{\delta}_b\frac{1}{3}L^2}_{\text{mean density modulation}} + \underbrace{\nabla_i\Phi_L(\mathbf{x})L_i}_{\text{gradient field}} + \underbrace{\frac{1}{2}\tau_{ij}L_iL_j}_{\text{tidal field}} + \cdots\end{aligned}$$

Separate universe simulation

Li, Hu & MT 14



- How can we include the super-box (DC) mode in a simulation?
- We know that the DC mode grows according to the linear growth rate
  - For a sufficiently high redshift such as the initial redshift employed in a simulation (say z~50 or 100), the amplitude is very small and the effect is negligible

Separate universe simulation (contd.)

- Full GR can solve the dynamics of all-wavelength modes

$$G_{\mu\nu}[g_{\alpha\beta}] = 8\pi G T_{\mu\nu}(\rho)$$

- Usually employ a decomposition of background and perturbations

$$g_{\alpha\beta}(\mathbf{x}, t) = \bar{g}_{\alpha\beta}[a(t)] + \delta g_{\alpha\beta}(\mathbf{x}, t)$$
$$\rho(\mathbf{x}, t) = \bar{\rho}(t) + \delta\rho(\mathbf{x}, t)$$

- *Separate universe technique*: the mean density modulation is absorbed into background quantities

$$\bar{\rho}_W(t) = \bar{\rho}(t)\left[1 + \bar{\delta}_b(t)\right]$$
$$\bar{\rho}a^3 = \bar{\rho}_W a_W^3 \longrightarrow a_W \simeq a\left[1 - \frac{1}{3}\bar{\delta}_b(t)\right]$$

Separate universe simulation (contd.)

- The Hubble expansion rate is modified as

$$H_W(t) \simeq H(t) - \frac{1}{3}\dot{\bar{\delta}}_b(t) \qquad \text{cf. } \bar{\delta}_b \propto D(t)$$

- The comoving wavelength in SU is also modified as

$$\lambda^{\rm phy} = \lambda_W^{\rm phy} \quad \rightarrow \quad a\lambda^{\rm co} = a_W \lambda_W^{\rm co}$$

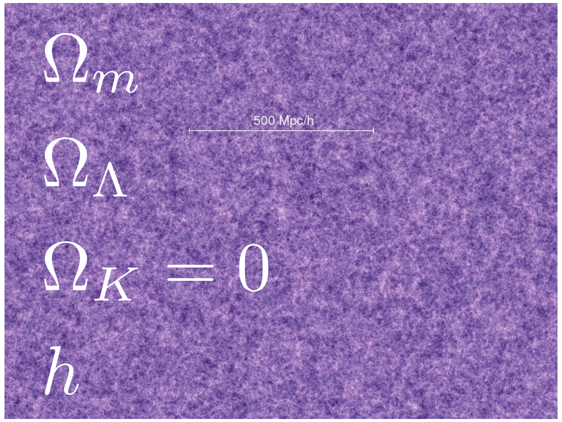
$$\rightarrow \quad k_W \simeq k\left[1 - \frac{1}{3}\bar{\delta}_b(t)\right]$$

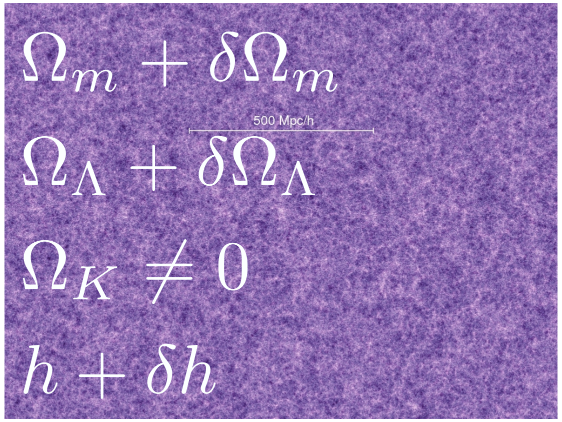
The super-survey mode causes a shift in the location of BAO peaks

Separate universe simulation (contd.)

The effect of such a super-survey (here DC) mode can be treated by changing the background cosmological model (an effective curvature parameter) (also, Frenk+ 88; Sirko 05; Gnedin+09; Baldauf et al. 12)

initial redshift





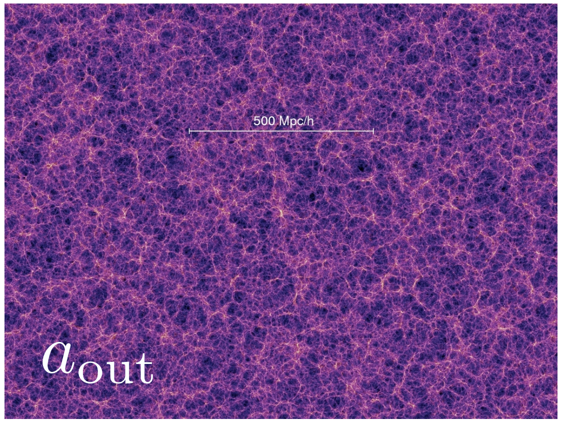
$$\bar{\rho}_{m,W} = \bar{\rho}_m \left(1 + \delta_b(z)\right)$$

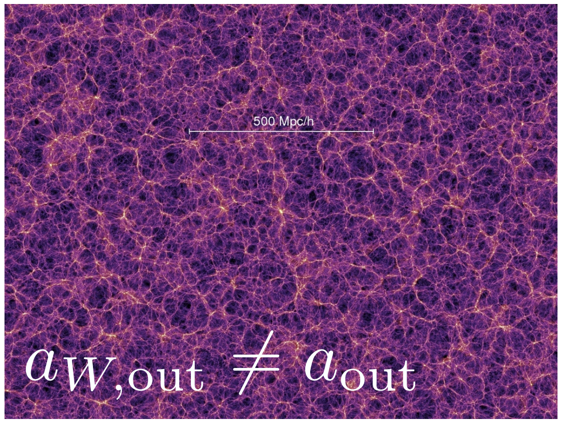
$$a_W \approx a\left(1 - \frac{\delta_b}{3}\right)$$

$$\frac{\delta h}{h} \approx -\frac{5\Omega_m}{6}\frac{\delta_b}{D}$$

$$\frac{\delta\Omega_m}{\Omega_m} = \frac{\delta\Omega_\Lambda}{\Omega_\Lambda} \approx -2\frac{\delta h}{h}$$

later redshift



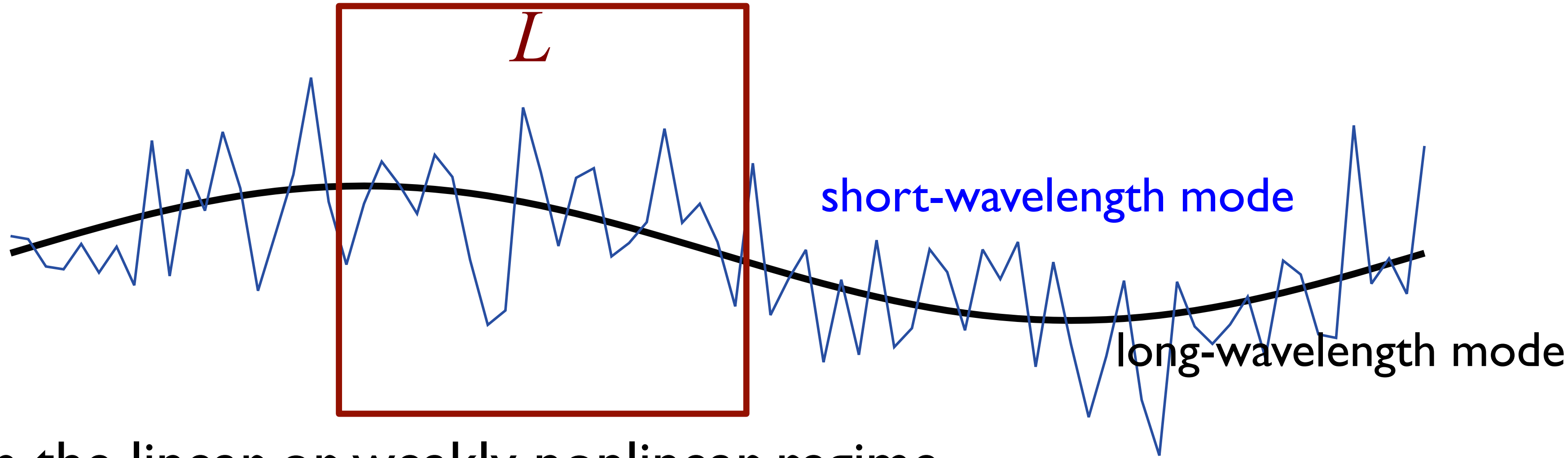


The two simulations look identical at sufficiently high redshift

We can use the same seeds of the initial density fluctuations (which help to reduce the stochasticity)

Li, Hu & MT 14

Effects of super-survey modes on the NL dynamics of short-wavelength modes



- In the linear or weakly nonlinear regime

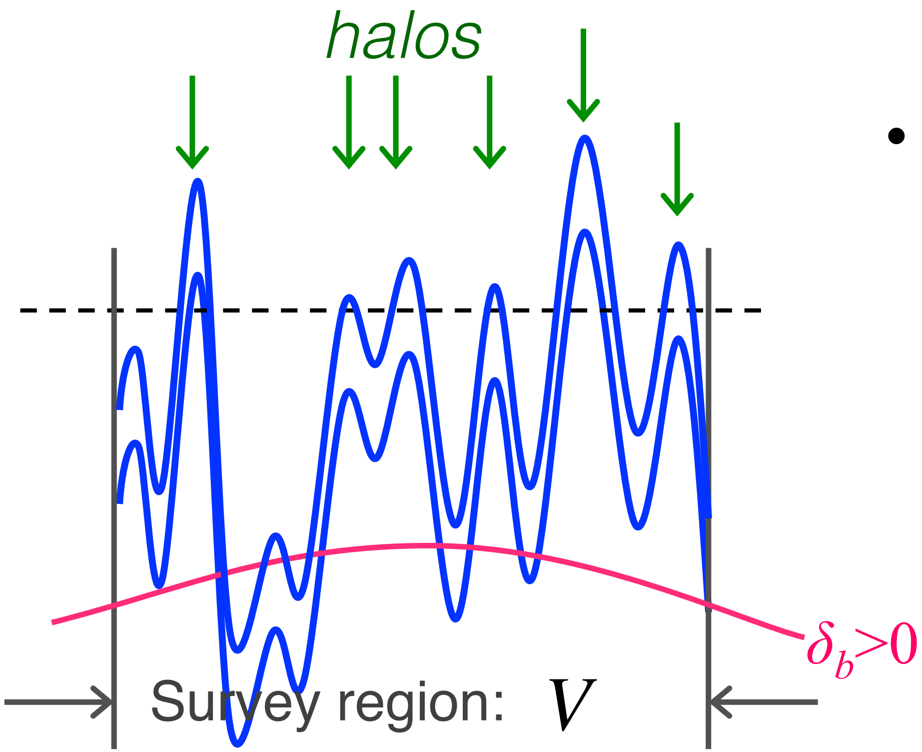
$$\ddot{\delta}_s + 2H_W\dot{\delta}_s - 4\pi G\bar{\rho}_W\delta_s = 0$$

$$\ddot{\delta}_s + 2H\dot{\delta}_s - 4\pi G\bar{\rho}\delta_s = \frac{2}{3}\dot{\bar{\delta}}_b\dot{\delta}_s + 4\pi G\bar{\rho}\bar{\delta}_b\delta_s$$

$$\Rightarrow \delta_s \propto D(t)\left[1 + \frac{13}{21}\bar{\delta}_b\right]$$

All short-wavelength modes are affected (also see P.Valageas 14)

Effects of super-survey mode on NL scales



- Physical picture of the SSC effect
  - Suppose that a survey region is embedded in a large-scale *overdensity* region
  - Growth of all the small-scale fluctuations is accelerated
  - The power spectrum we can observe tends to have *greater* amplitudes than the ensemble average

Halo bias theory predicts

$$\left.\frac{dn_h}{dM}\right|_{\bar{\delta}_b} \simeq \left.\frac{dn_h}{dM}\right|_{\bar{\delta}_b=0} \left[1 + b(M)\bar{\delta}_b(t)\right]$$

Power spectrum response

- *Power spectrum response*: the response of power spectrum at each *k* bin to the super-survey mode

$$P(k;\delta_b) \simeq P(k;\delta_b=0) + \left.\frac{\partial P}{\partial \delta_b}\right|_{\delta_b=0} \delta_b$$

Power spectrum response (assuming the linear delta_b)

- Different LSS probes have different response
  - Weak lensing shear: $\gamma \sim \partial_i \partial_j \Phi \sim \bar{\rho}\delta$
  - Galaxy clustering: $\delta_g \equiv \frac{\delta n_g}{\bar{n}_{W,g}} \sim \frac{\delta}{1+\delta_b}$

- *Reponses of the power spectra wrt "global" or "local" mean*

$$P(k) = (1+\delta_b)^2 P_W(k) \rightarrow \frac{\partial \ln P(k)}{\partial \delta_b} = 2 + \frac{\partial \ln P_W(k)}{\partial \delta_b}$$

"Growth" and "Dilation" effects in Power spectrum response

- The power spectrum response has two contributions

$$
\begin{aligned}
\left.\frac{d\ln\Delta^2(k_W,\delta_b)}{d\delta_b}\right|_k &= \left.\frac{\partial\ln\Delta^2_W(k_W,\delta_b)}{\partial\delta_b}\right|_{k_W} + \frac{\partial\ln\Delta^2_W(k_W,\delta_b)}{\partial\ln k_W}\frac{\partial\ln k_W}{\partial\delta_b} \\
&\simeq \left.\frac{\partial\ln\Delta^2_W(k_W,\delta_b)}{\partial\delta_b}\right|_{k_W} - \frac{1}{3}\frac{\partial\ln\Delta^2_W(k_W,\delta_b)}{\partial\ln k_W} \\
&\simeq \left.\frac{\partial\ln\Delta^2_W(k_W,\delta_b)}{\partial\delta_b}\right|_{k_W} - \frac{1}{3}\frac{\partial\ln\Delta^2}{\partial\ln k}
\end{aligned}
$$

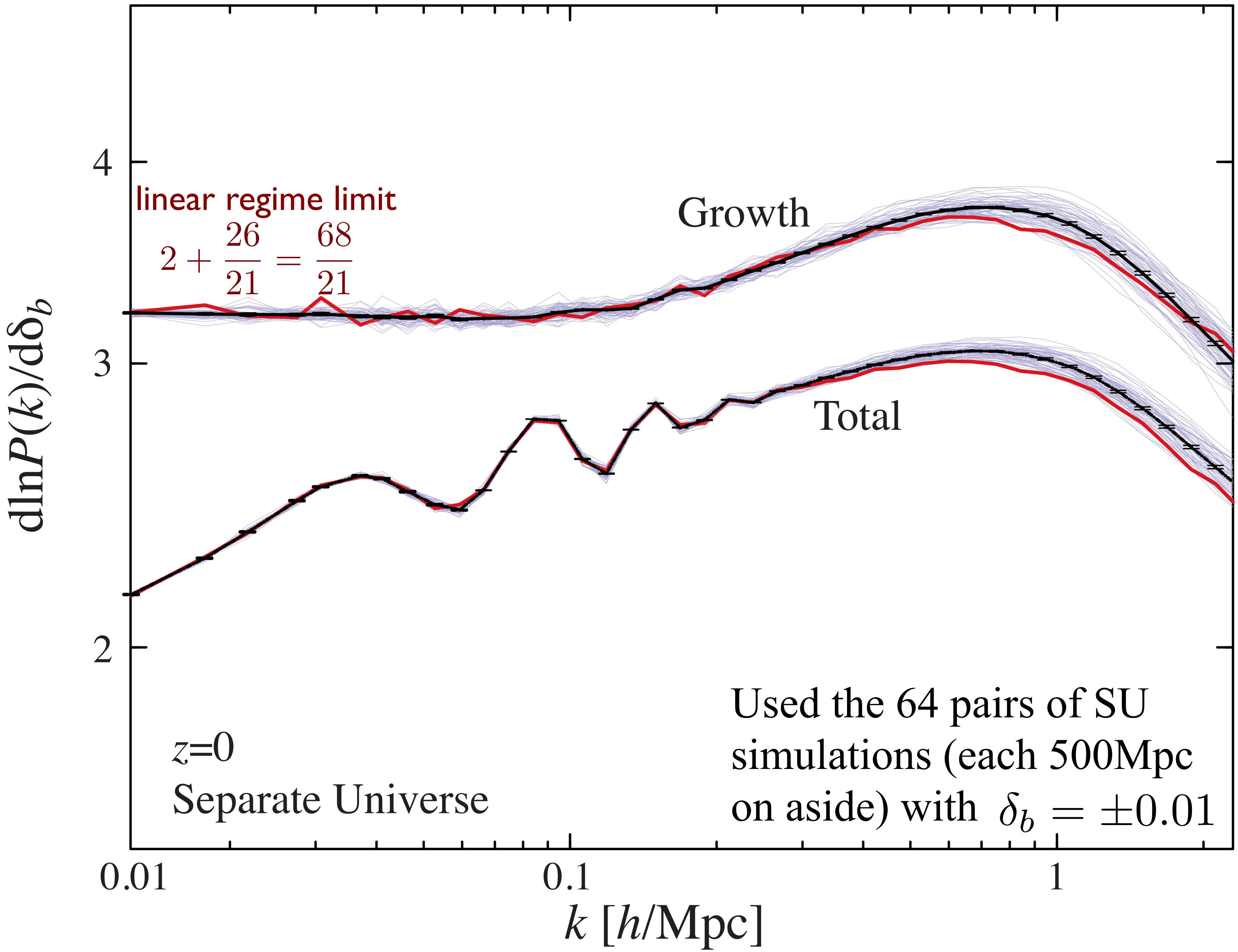
Growth effect

enhancement/suppression in the growth of short-wavelength modes due to delta_b

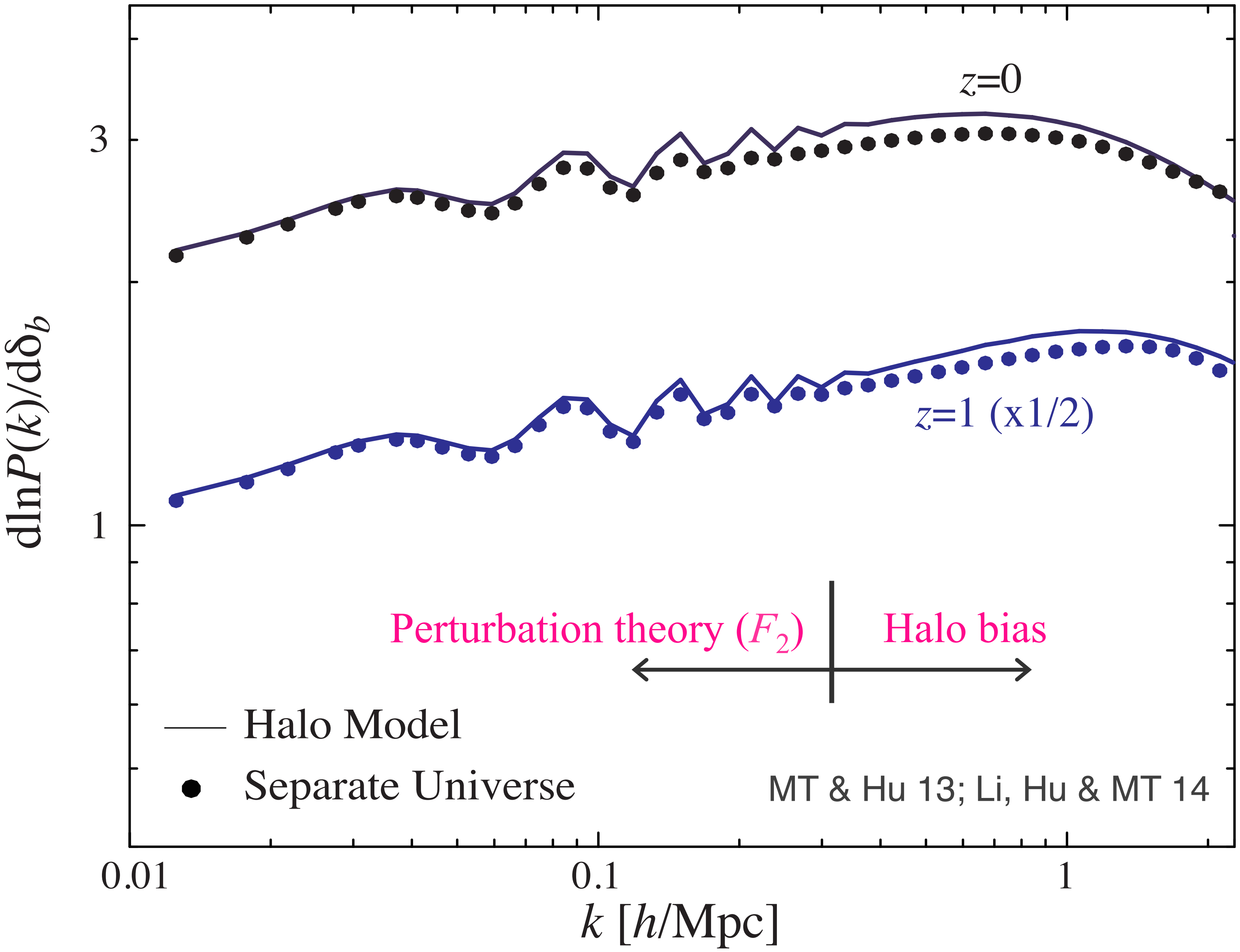
Dilation effect

More contraction/expansion of comoving volume due to delta_b

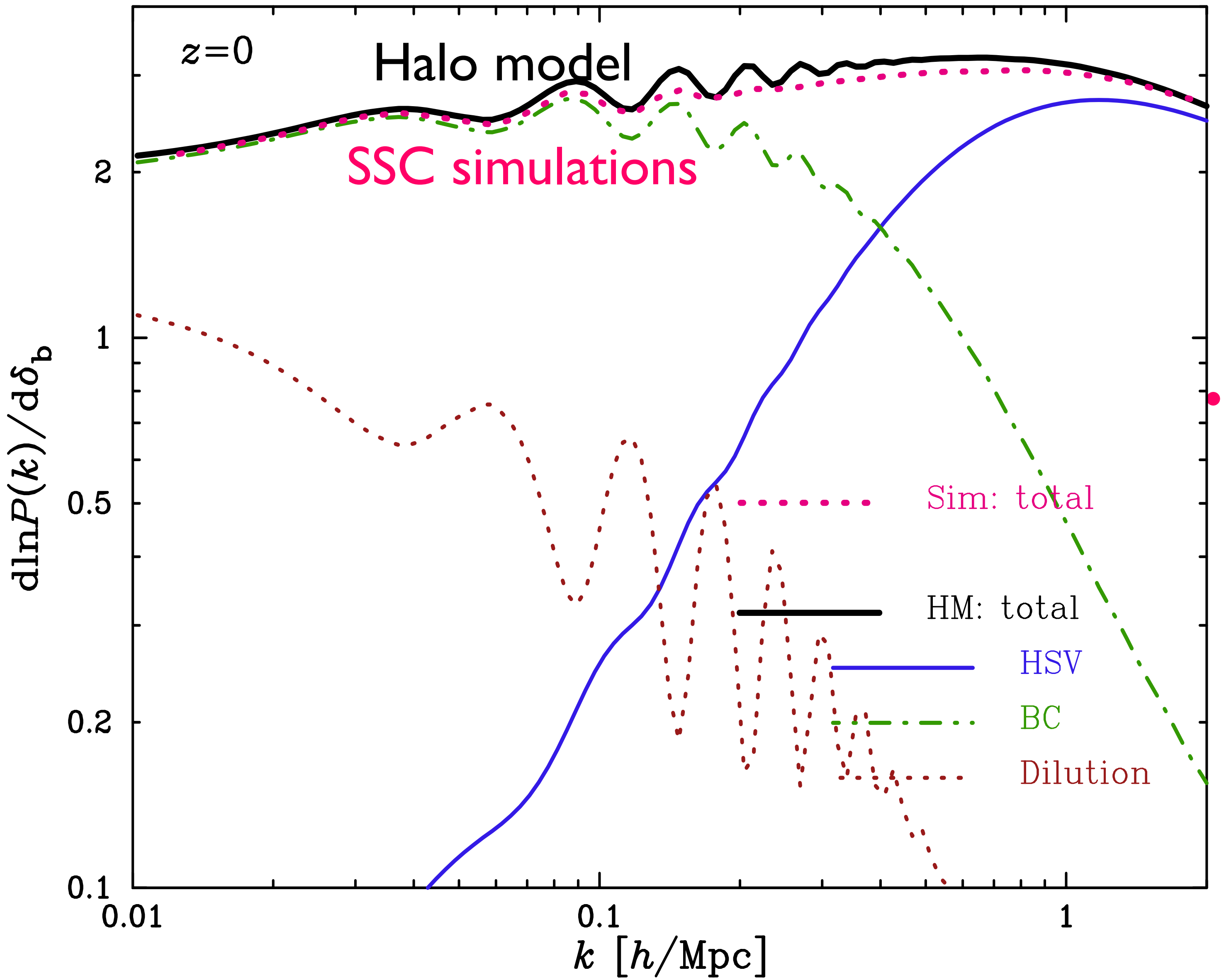
Results of SU simulation



SU simulation vs. Halo model



SU simulations vs. Halo model



- *The halo model fairly well reproduces the simulation results!*

Super-Sample Covariance (SSC)

MT & Hu 13

- The new formula for the power spectrum covariance (also see Hamilton et al. 2006)

$$\mathrm{Cov}[P(k), P(k')] = \frac{2}{N_{\rm mode}} P(k)^2 \delta^K_{kk'} + \frac{1}{V_s}\bar{T}(k,k')$$

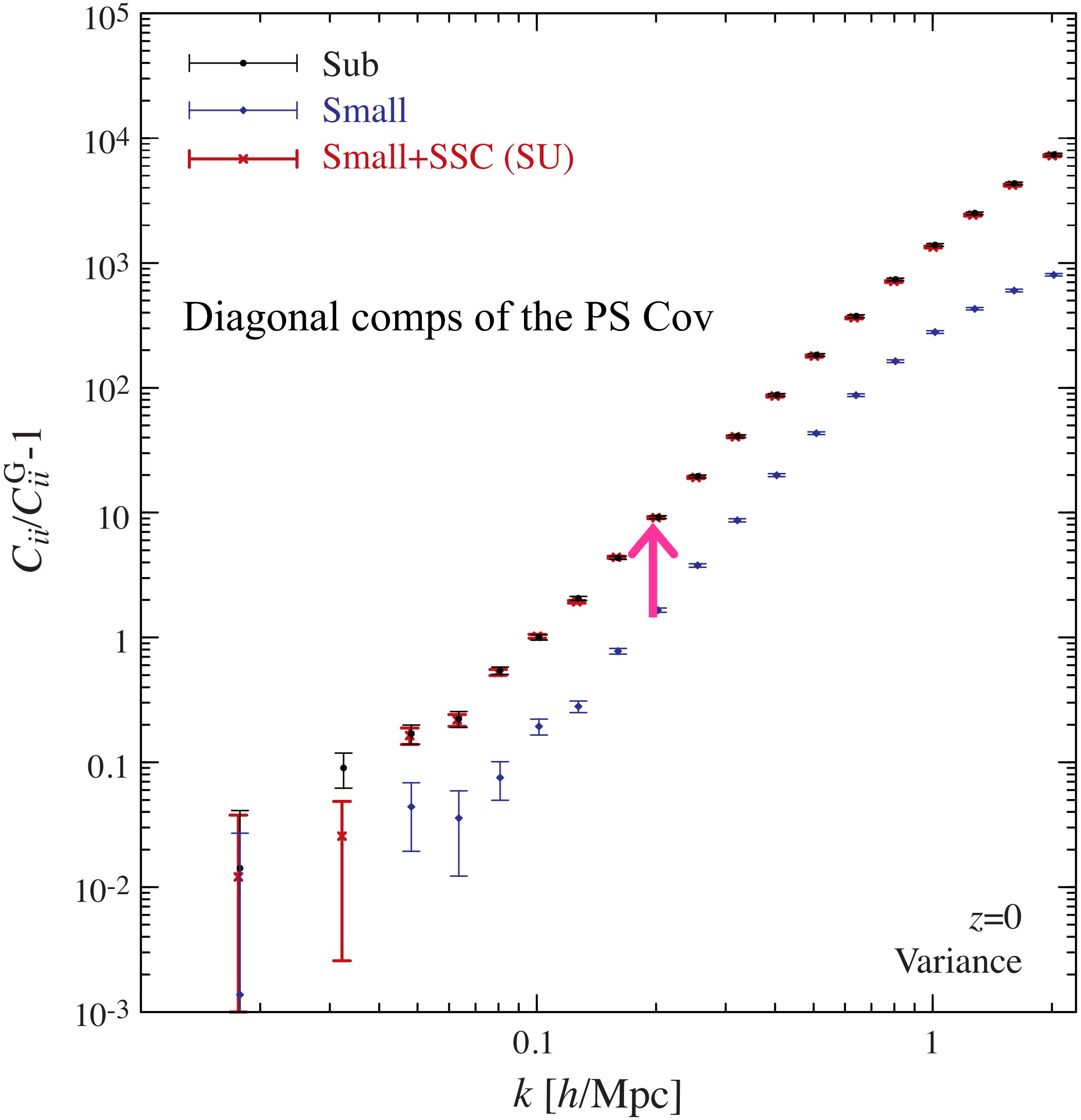
$$+\sigma_b^2 \frac{\partial P(k)}{\partial \delta_b}\frac{\partial P(k')}{\partial \delta_b}$$

New term: super-sample covariance

Here σ_b^2 is the rms of the long-wavelength density modes for the survey volume

$$\sigma_b^2 \equiv \frac{1}{V_W^2}\int \frac{d^3\vec{q}}{(2\pi)^3}|\tilde{W}(\vec{q})|^2 P^L(q)$$

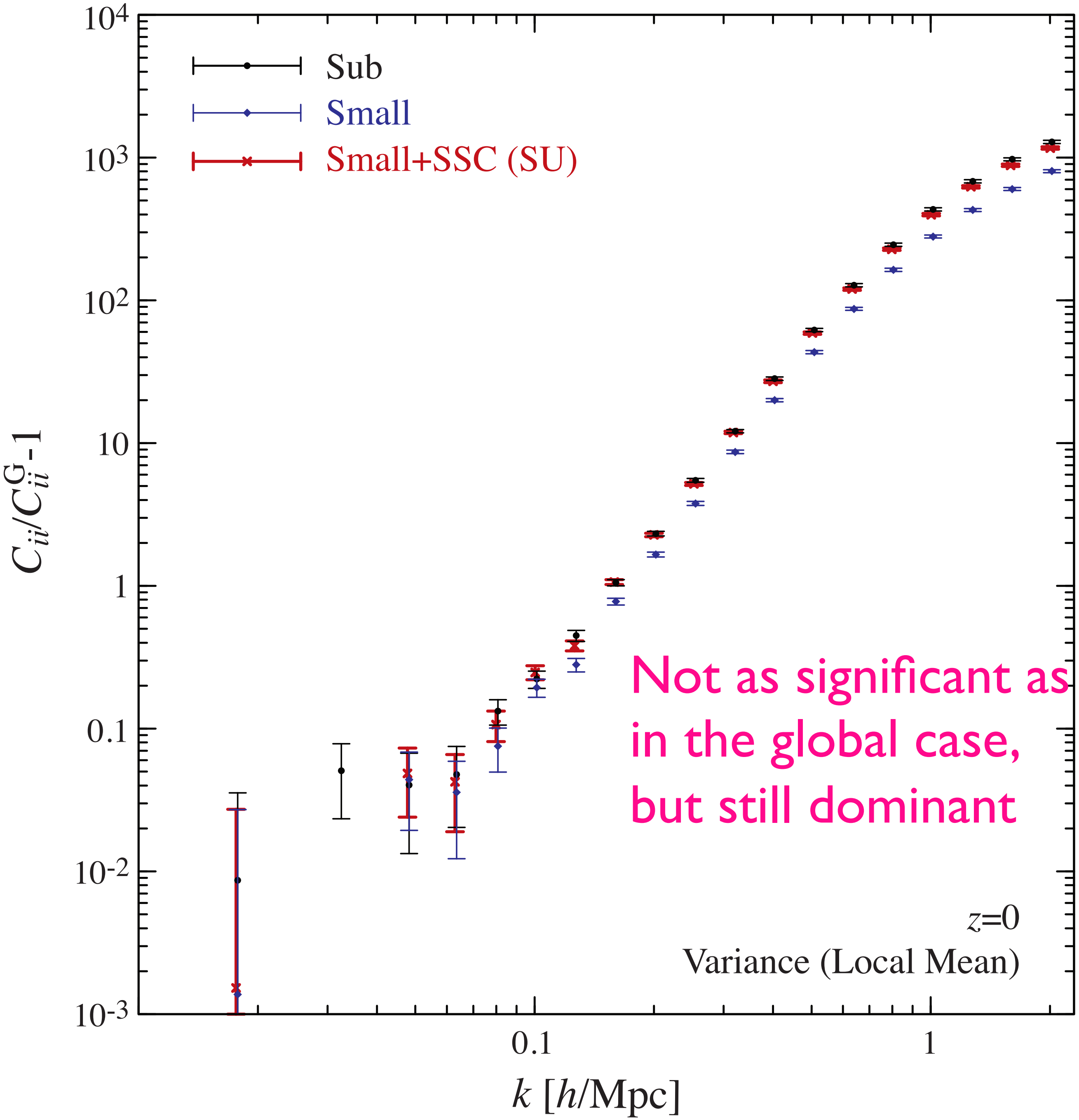
Comparison with large-vol. simulations



- Each of the SU simulations: 500 Mpc/h on a side
- 7 large-volume simulations (4Gpc/h)^3
  - Subdivided the simulations into 3584 sub-volumes of (500Mpc/h)^3

- The SU results are in remarkably nice agreement with the sub-volume results
- The SSC dominates the PS cov. at all the scales of k>0.1 h/Mpc

The SSC effect for PS of local mean



Not as significant as in the global case, but still dominant

Super-sample signal

Li, Hu & MT arXiv:1408.1081

- The super-survey effect can be *realized as an additional signal*, instead of additional source to the sample variance – open up a window of *constraining very large-scale modes*

$$\hat{P}(k;\delta_b) = \underbrace{\hat{P}(k;\delta_b=0)}\left[1+\underbrace{\frac{\partial \ln P(k)}{\partial \delta_b}}\underbrace{\delta_b}\right]$$

Power spectrum of sub-survey modes, measured from the survey region

Power spectrum response given as a function of cosmological model

Super-survey mode

- The SS effects is in *the same way* as a change in cosmological paras
- Can find *a minimum variance estimator* of the super survey mode

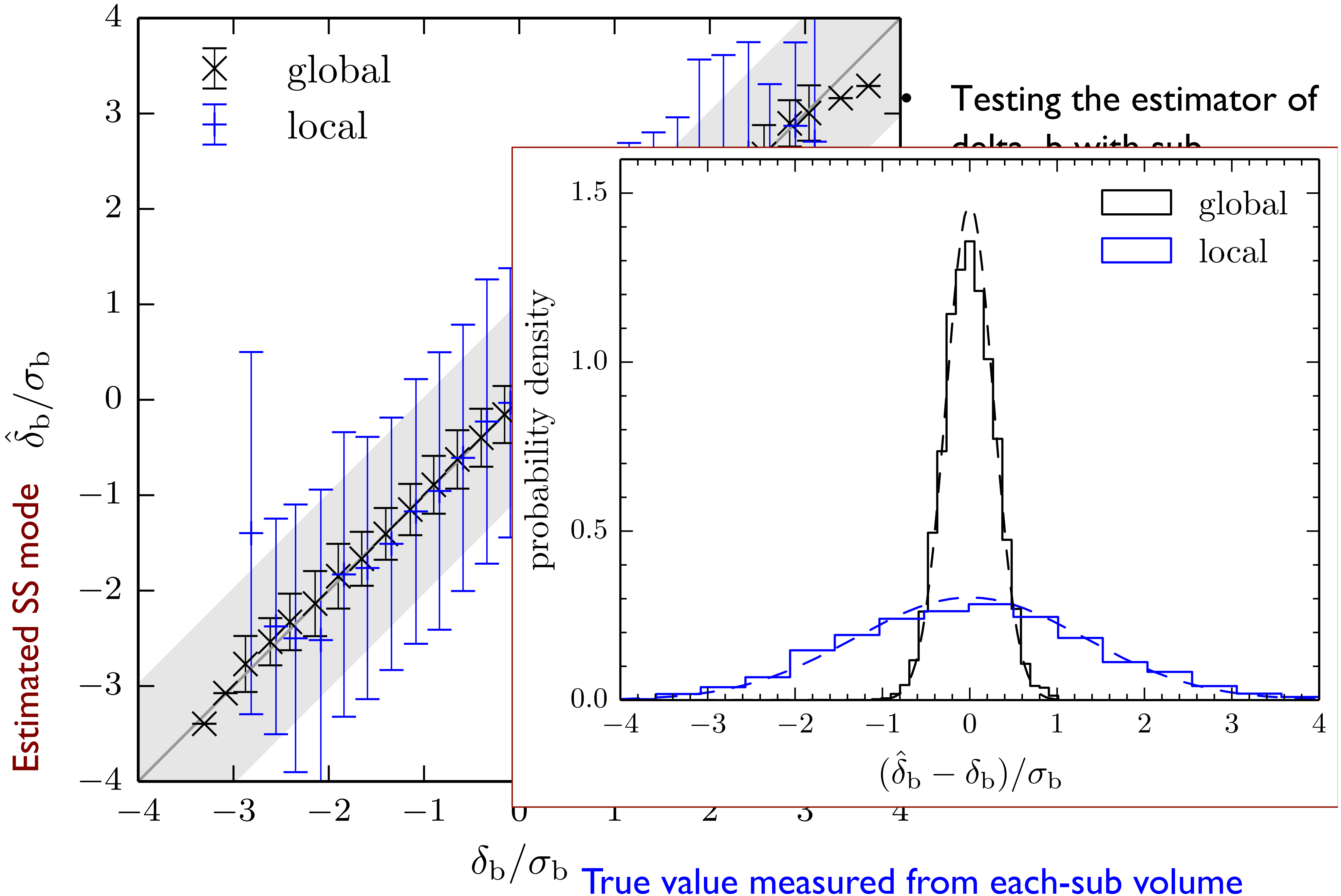
$$\hat{\delta}_b = \sum_{k_i;k_i\in k_{\rm max}} \underbrace{w_i}\left[\underbrace{\hat{P}(k_i)} - \underbrace{P(k_i;\delta_b=0,p_\mu)}\right]$$

weight

Power spectrum measured

Model including the super-survey mode

Test with sub-volume simulations

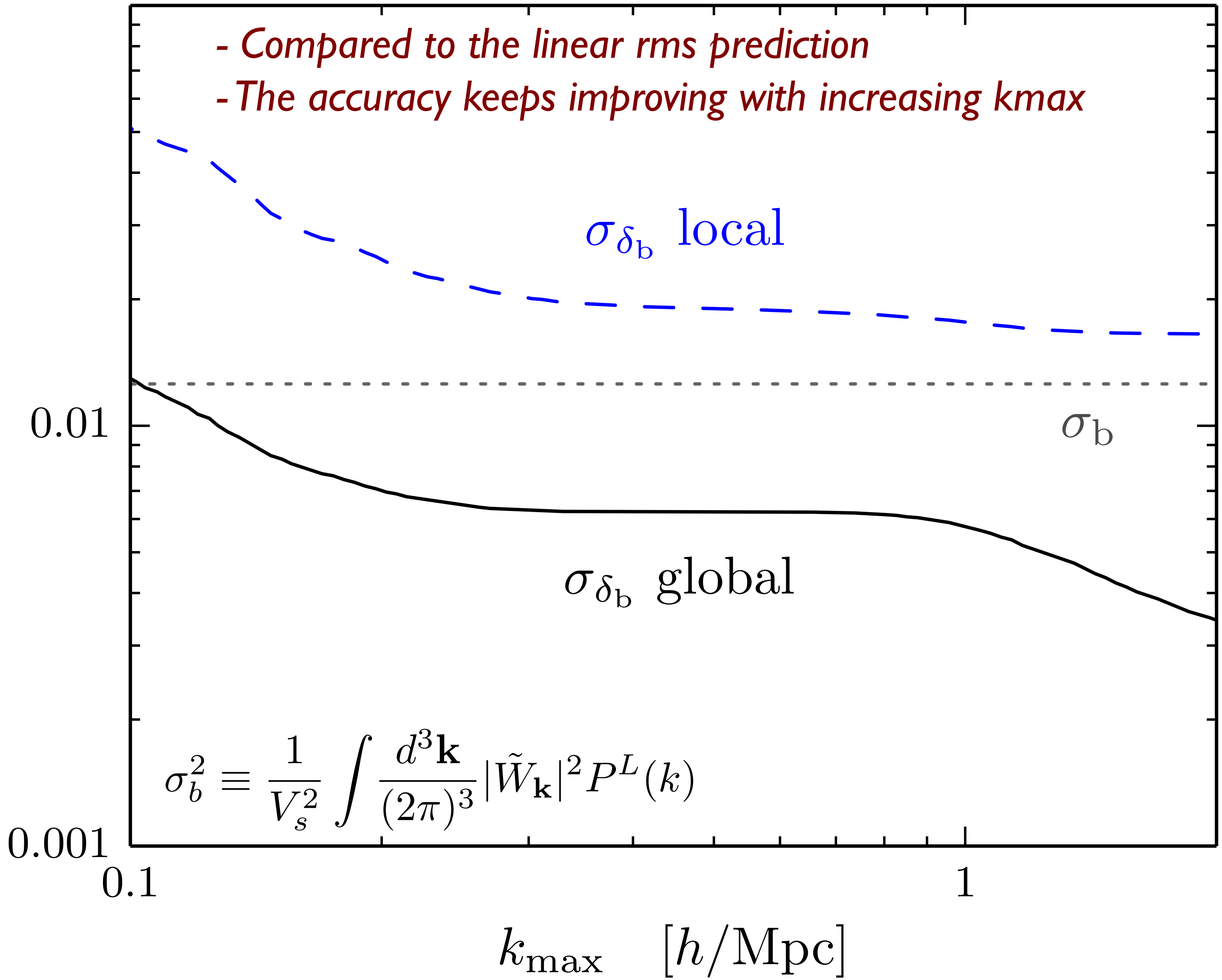


True value measured from each-sub volume

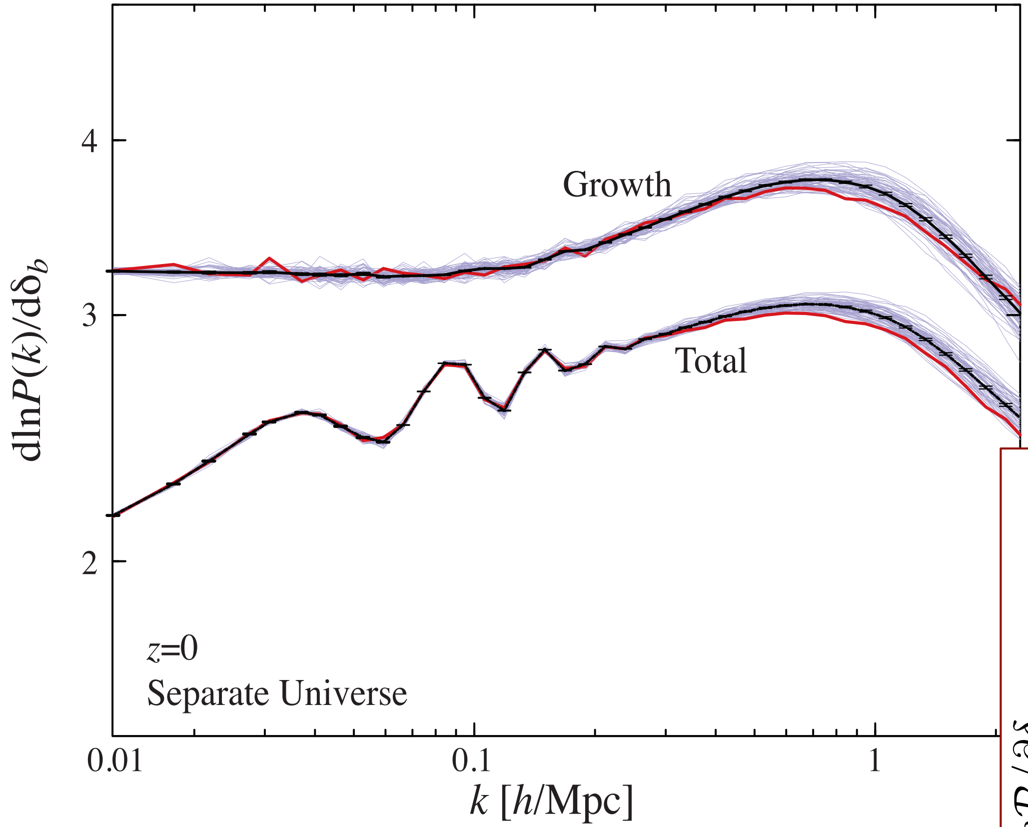
- Testing the estimator of delta_b with sub

Accuracy of constraining super-sample signal

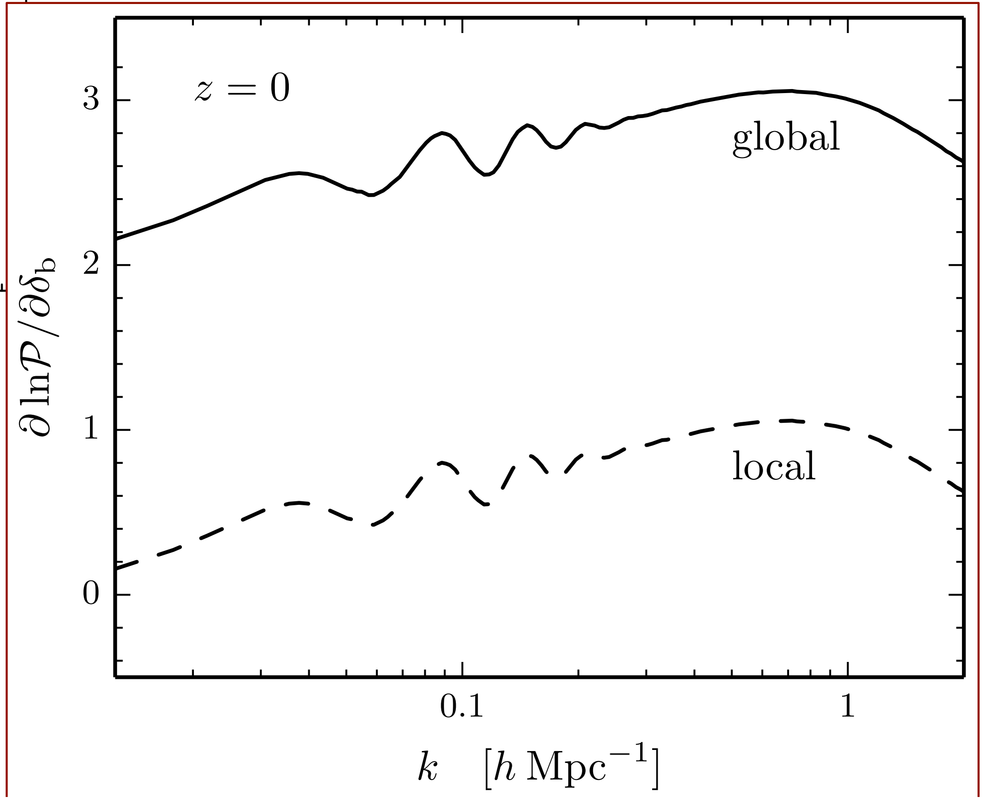
- Included the local or global matter power spectrum information up to k_{max}



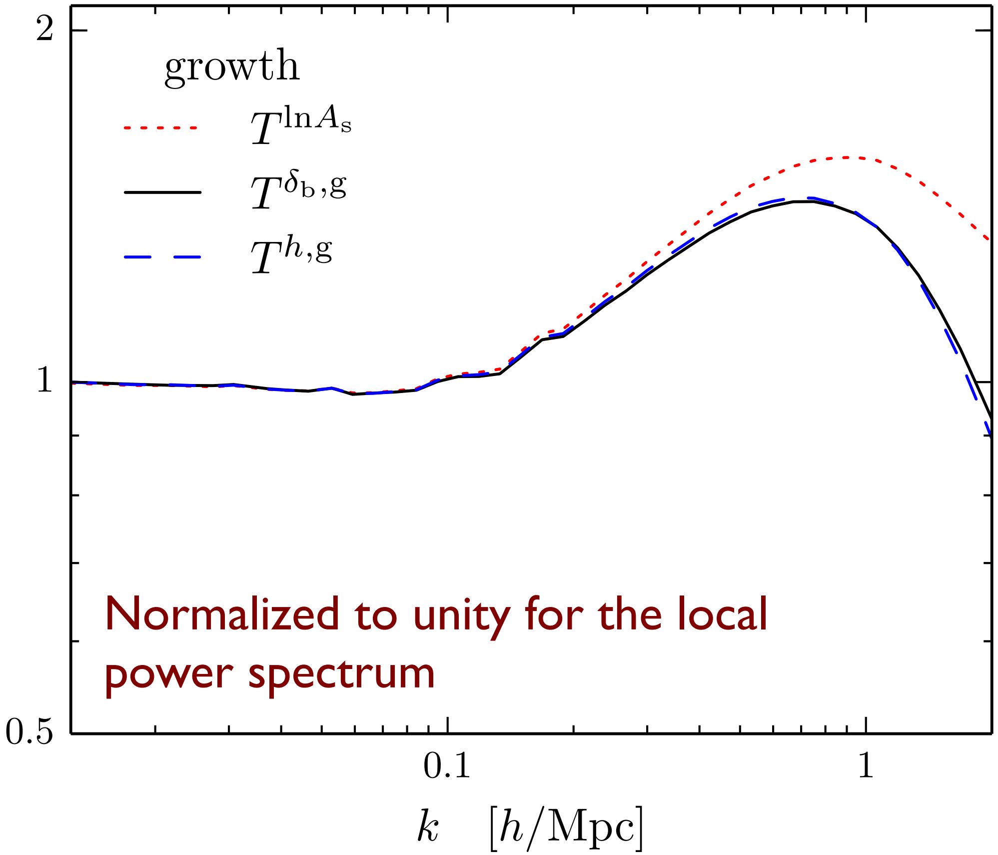
Degeneracy with cosmological parameters



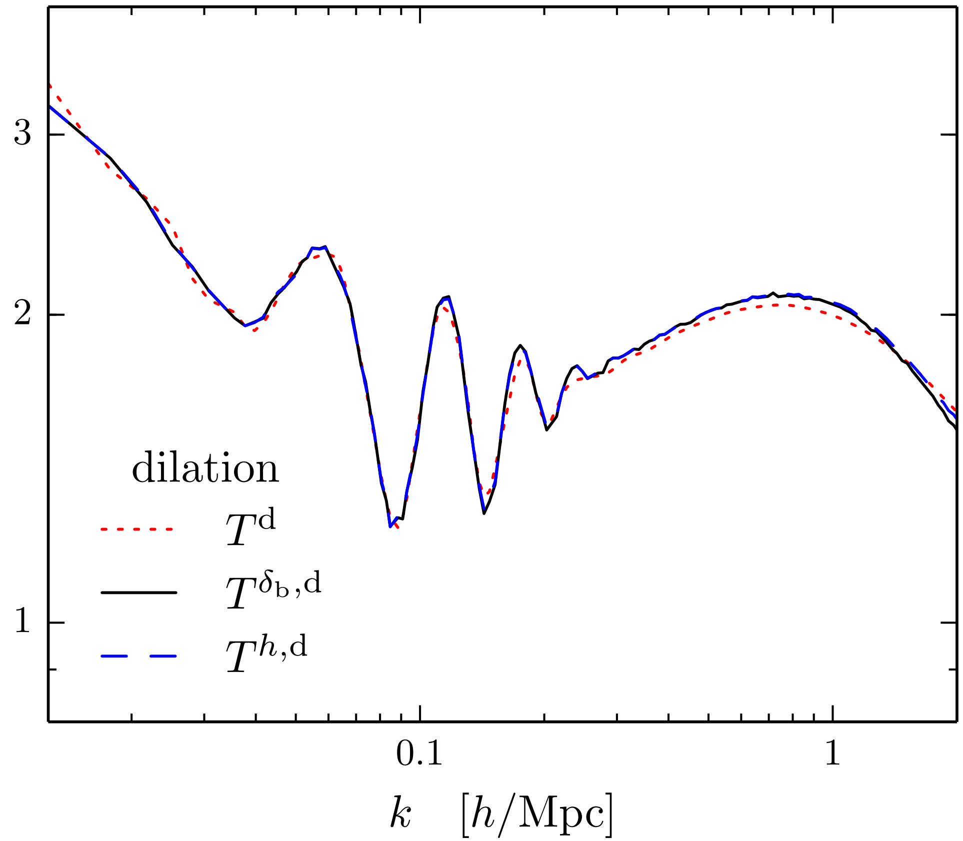
- The effect of SS mode on the power spectrum is degenerate with cosmological parameters
- To what extent?



Degeneracy (contd.)

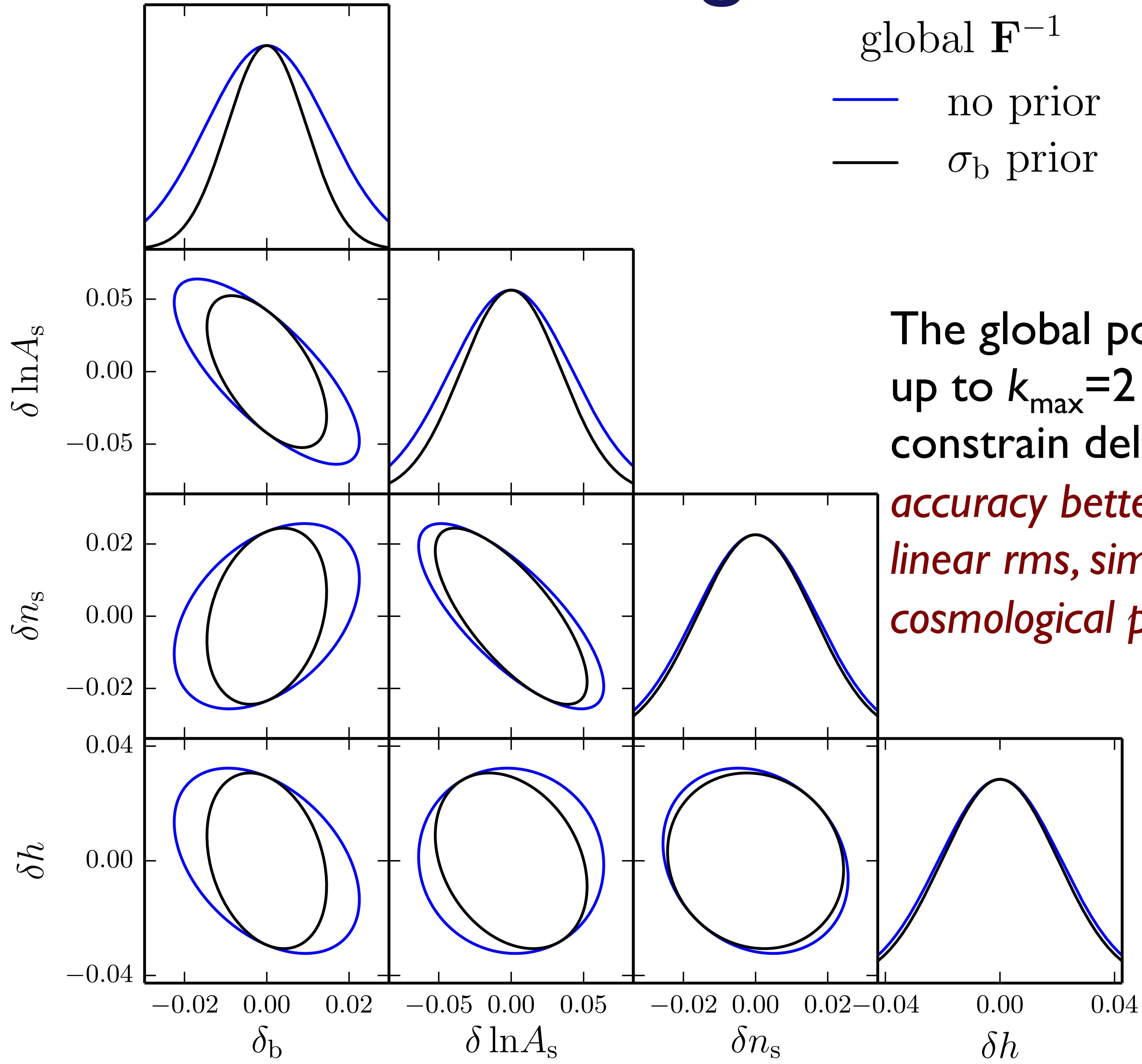


- Considered As, h and n_s
- The effects of A_s and h are decomposed into the effects of "growth" and "dilation"
- The effect of ns is different



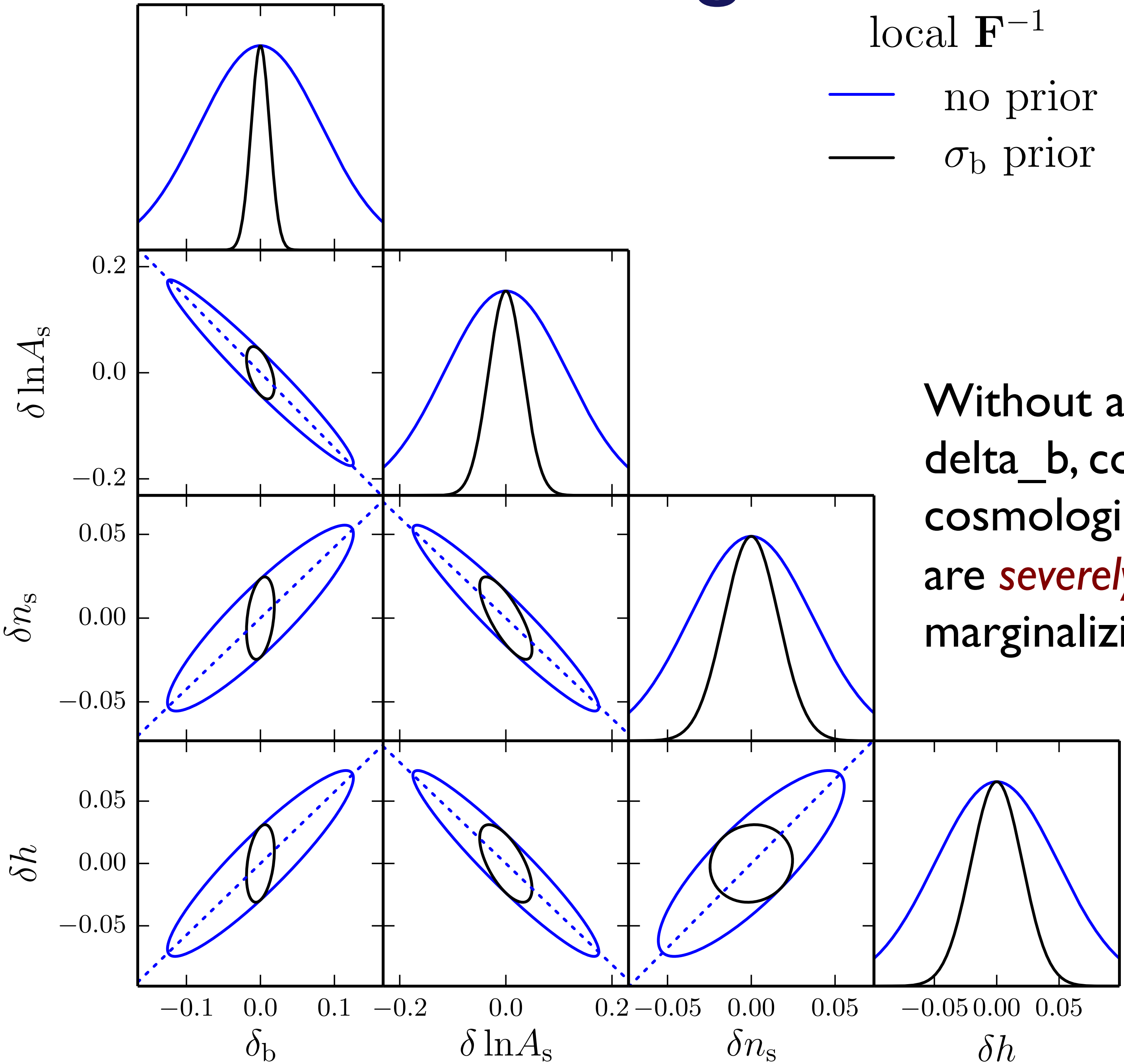
- Here we fixed $\Omega_m h^2$; so if h is changed, Ω_m is changed
- The change in h causes both the growth and dilation effects

Result: the global case



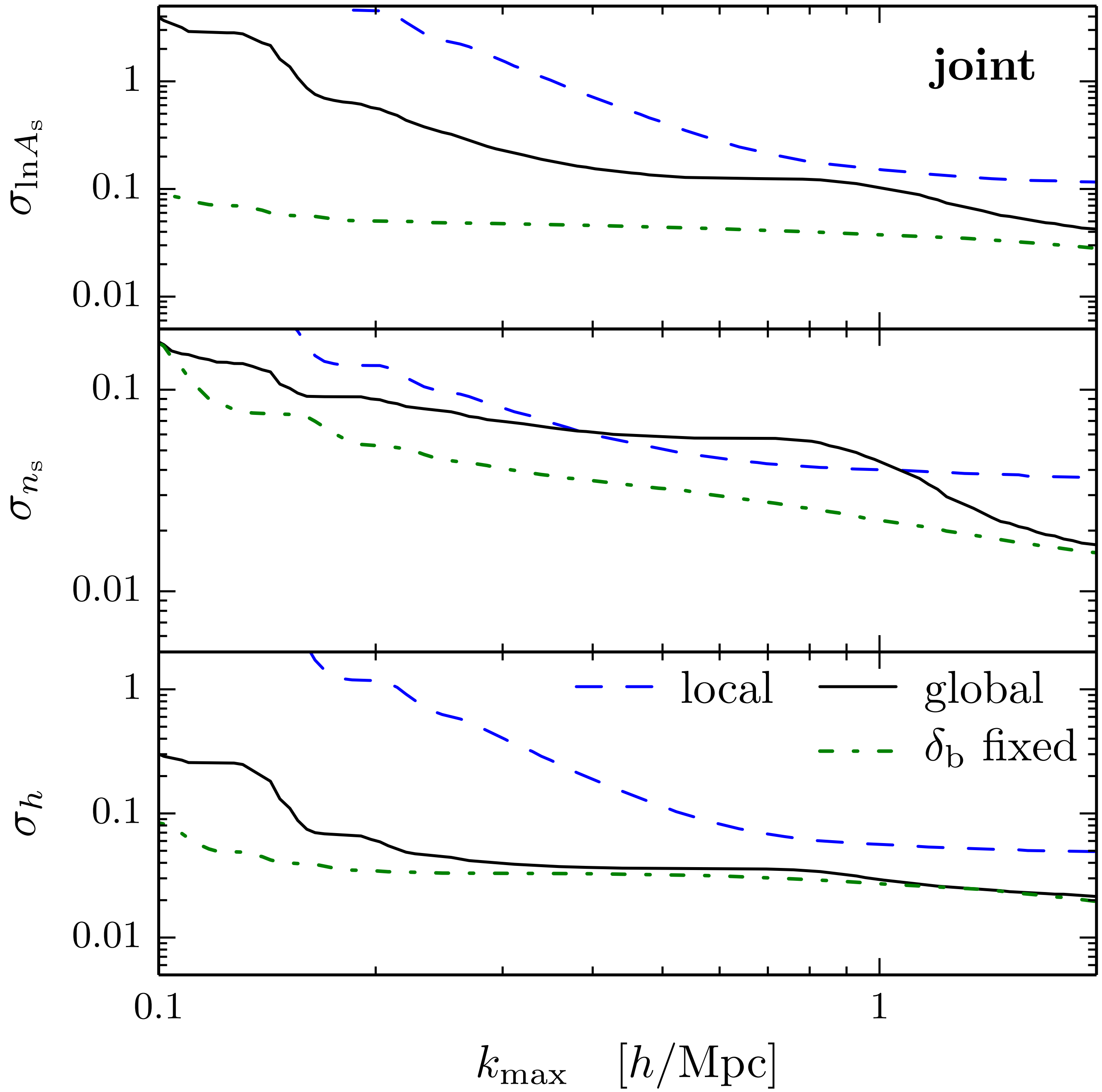
The global power spectrum up to k_{max}=2 h/Mpc can constrain delta_b to *an accuracy better than the linear rms, simultaneously with cosmological parameters*

Result: the global case



Without a prior of delta_b, constraints on cosmological parameters are *severely degraded* by marginalizing delta_b

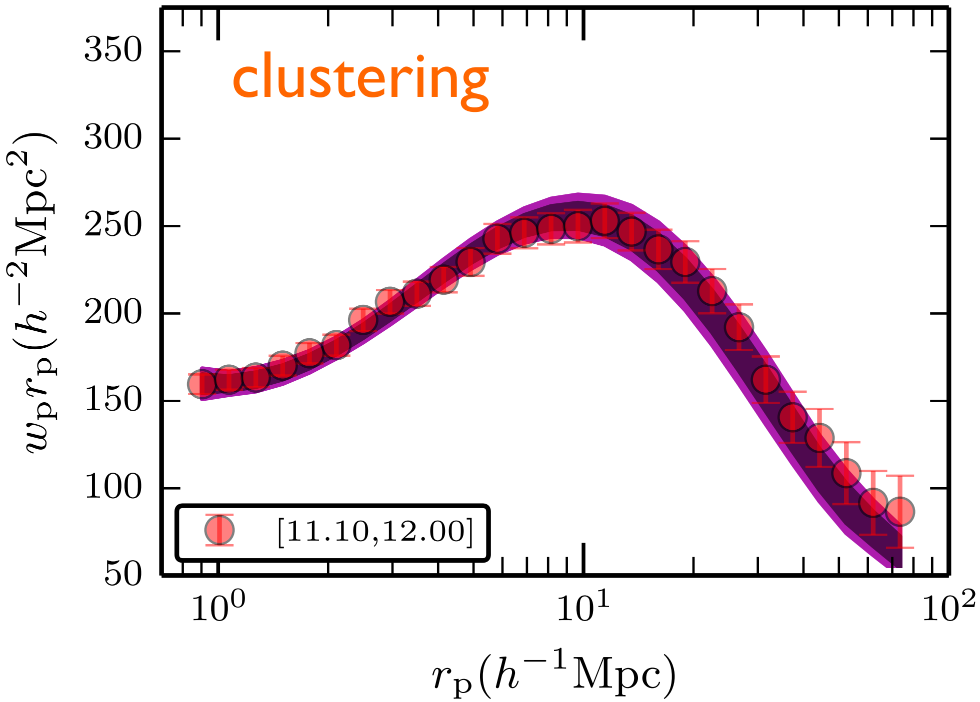
Marginalized errors (contd.)

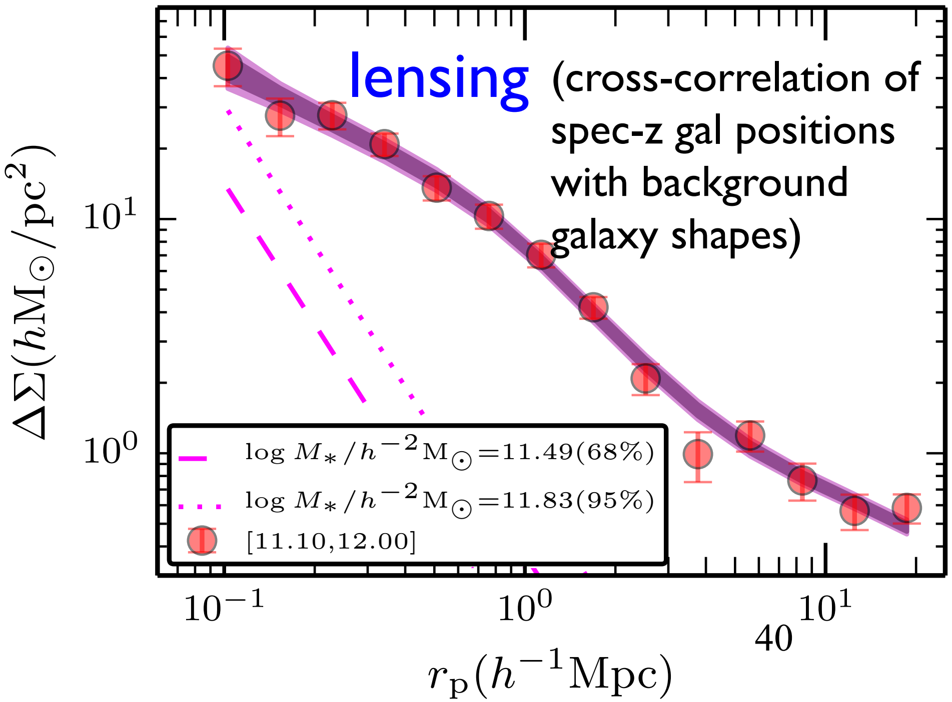


Combined probes: Lensing (imaging) + Clustering (spec-z)

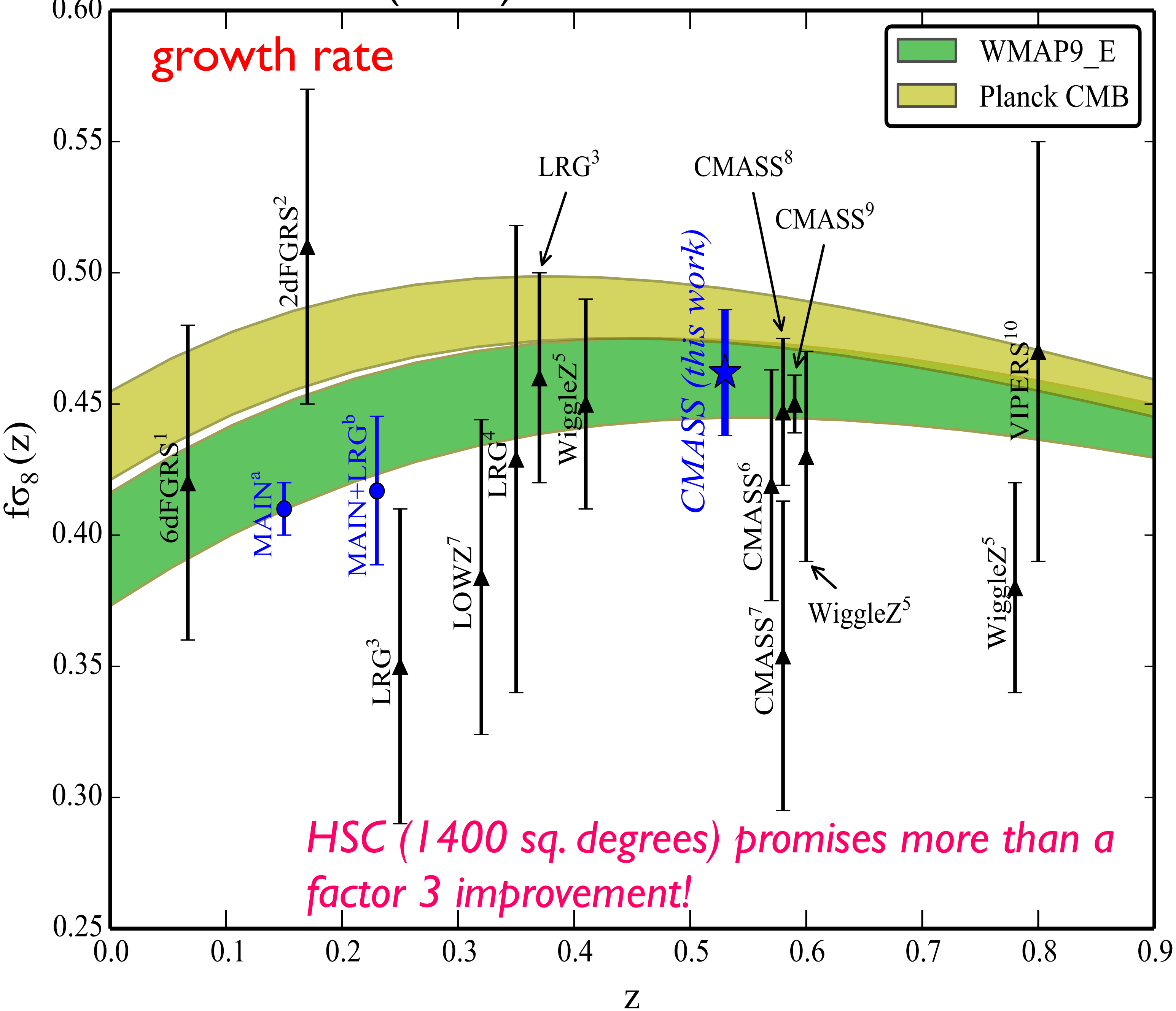

Surhud More

- Lensing: directly measure the DM distribution, but projected
- Clustering: 3D mapping of galaxy distribution; a much higher S/N, but galaxy bias uncertainty
- More, Miyatake, Mandelbaum, MT, Spergel, et al. (2014): CFHTLenS (3.6m imaging, *only ~120 sq. deg*) + BOSS (2.5m spec-z, 10000 sq. deg)



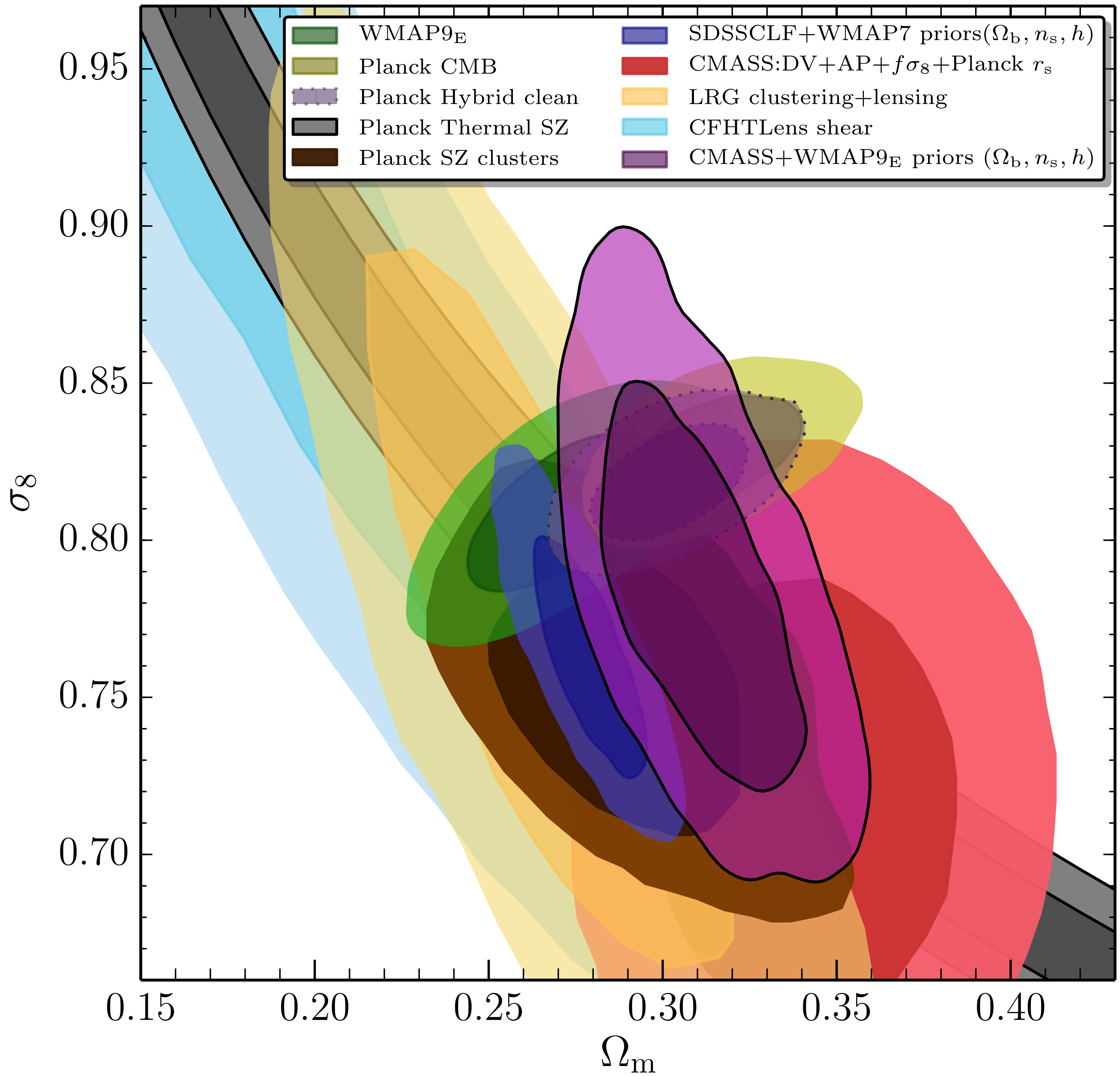


More et al. (2014): arXiv:1407.1856



RSD
[1] Beutler et al. (2012)
[2] Percival et al. (2004)
[3] Samushia et al. (2012)
[4] Chuang & Wang (2013)
[5] Blake et al. (2011)
[6] Beutler et al. (2013)
[7] Chuang et al. (2013)
[8] Samushia et al. (2014)
[9] Reid et al. (2014)
[10] de la Torre et al. (2013)

Clustering+Lensing
[a] Cacciato et al. (2013)
[b] Mandelbaum et al. (2013)



WMAP9$_{\mathrm{E}}$
Planck CMB
Planck Hybrid clean
Planck Thermal SZ
Planck SZ clusters
SDSSCLF+WMAP7 priors($\Omega_{\mathrm{b}}, n_{\mathrm{s}}, h$)
CMASS:DV+AP+$f\sigma_8$+Planck r_{s}
LRG clustering+lensing
CFHTLens shear
CMASS+WMAP9$_{\mathrm{E}}$ priors ($\Omega_{\mathrm{b}}, n_{\mathrm{s}}, h$)
σ_8
0.95
0.90
0.85
0.80
0.75
0.70
0.15
0.20
0.25
0.30
0.35
0.40
Ω_{m}

Summary

- SuMIRe: wide-area imaging and spectroscopic surveys for the *same region of the sky*, with *the same* telescope (Subaru)
- *Super-survey (SS) mode* causes a *significant sample variance* in the power spectrum measurement from the finite-volume survey
  - *"Growth" effect*: the SS mode causes a change in the growth of short-wavelength modes via nonlinear mode coupling
  - *"Dilation" effect*: the SS mode causes a change in the observed comoving scale in the survey region – e.g., a shift in BAO peak location
- *Separate universe simulation technique* is very powerful
- The SS effect can be realized as *an additional signal*, instead of an additional source to the sample variance
- *It opens up a new window of constraining the very long-wavelength modes beyond the matter-radiation equality – GR effect?*