Reviving Open Inflation

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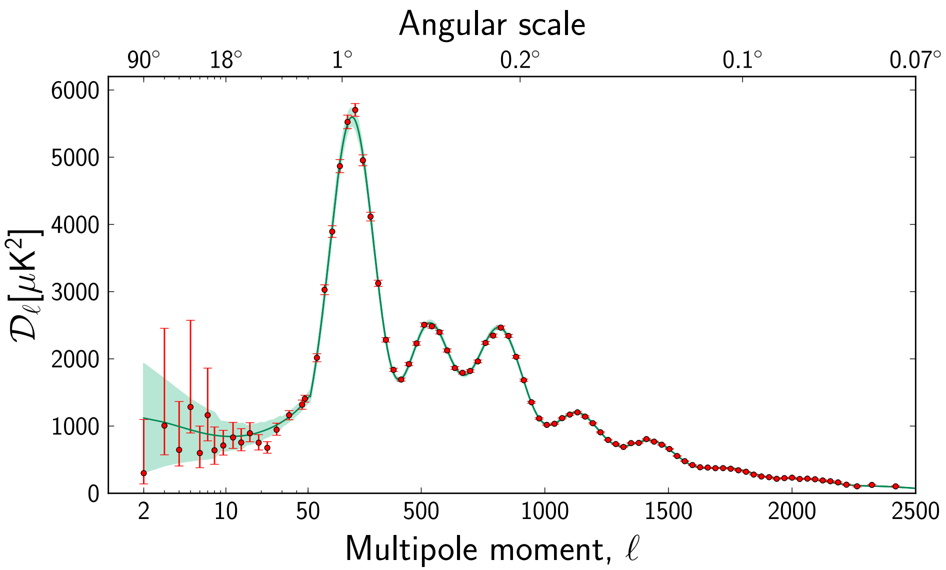
Based on arXiv: 1407.5816 in collaboration with Y-l. Zhang and M. Sasaki

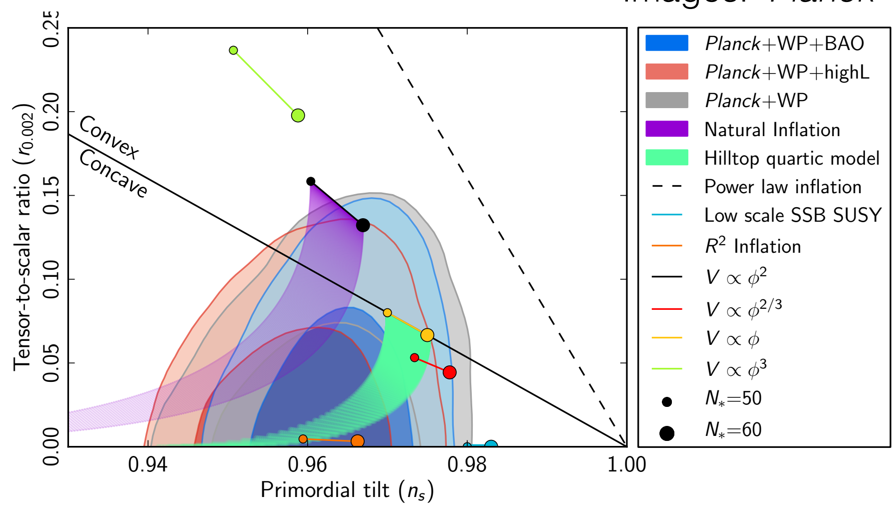
Inflation

An epoch of **inflation** in good **agreement with observations:**

- Solves horizon, flatness and monopole problems
- Predicts:
  - ✓ CMB temperature fluctuations: $\frac{\delta T}{T} \sim \mathcal{O}(10^{-5})$
  - ✓ **5σ deviation from scale invariance:** $n_s = 0.9603 \pm 0.0073$
  - ✓ perturbations close to **Gaussian**: $f_{NL}^{\mathrm{local}} = 2.7 \pm 5.8$
  - ? Primordial **tensor perturbations?**



Images: *Planck*



BICEP2: primordial tensor modes?

- **Tensor perturbations detected** through B-mode signal?

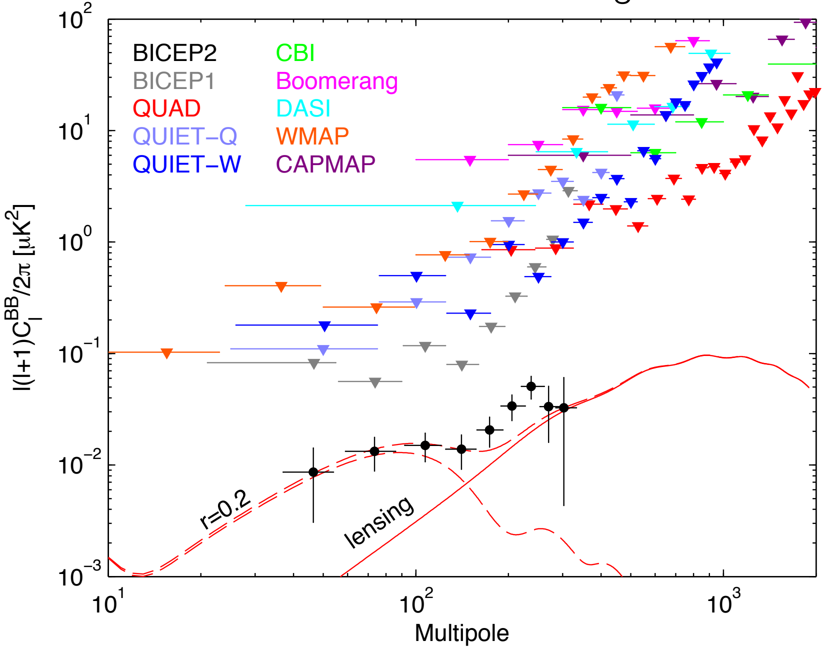
$$r = 0.20^{+0.07}_{-0.05} \quad \text{(68\% confidence)}$$

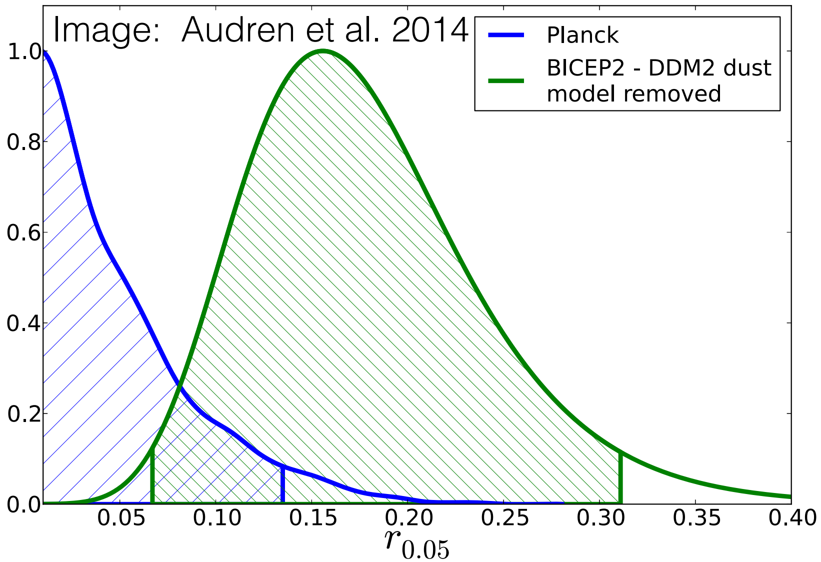
$$r = 0.16^{+0.06}_{-0.05} \quad \text{(removing dust)}$$

- In **tension with Planck?**

$$r < 0.11 \, (95\% \, CL)$$

Image: BICEP2





- **Not so bad:** (Audren et al. 2014)

Planck: $k_* = 0.002 \, \text{Mpc}^{-1} \; n_t = -r/8$

BICEP2: $k_* = 0.05 \, \text{Mpc}^{-1} \; n_t = 0$

For $n_t = 0$ Planck+WMAP give:

$$r_{0.05} < 0.135 \, (95\% \, CL)$$

Anomalies

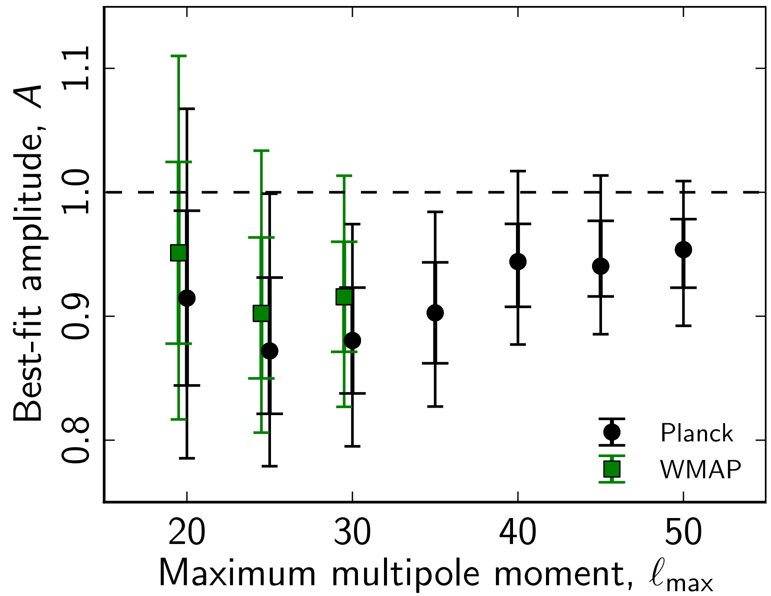
Still much uncertainty - **what is the exact nature of inflation?**
Could **anomalies** represent important **clues?**

- e.g. **5-10% power deficit** on **large scales** $(l \leq 40)$
  statistical significance 2.5—3σ

$$C_l(A, n) = A C_l^{\text{fid}} \left(\frac{l}{l_0}\right)^n$$

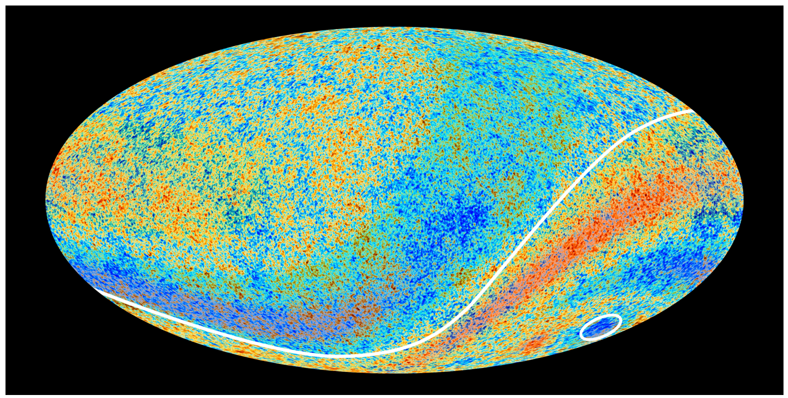
$$l_0 = \frac{(2 + l_{\text{max}})}{2}$$

Images: *Planck*



Other anomalies include:

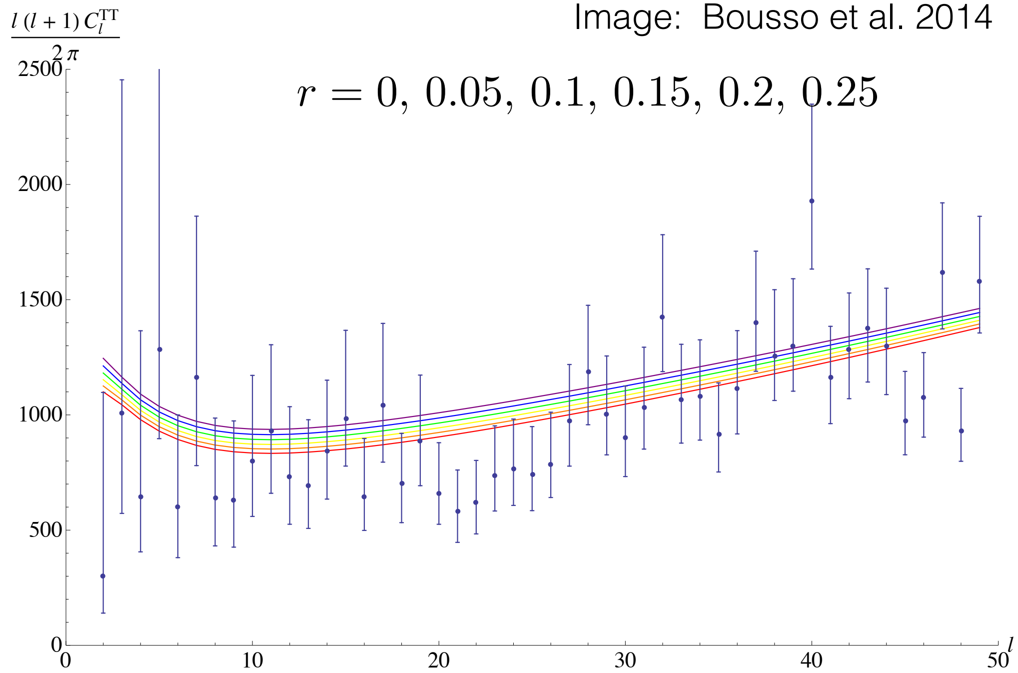
- Hemispheric/Dipole Asymmetry
  also **evidence for open inflation?**
  Kanno et al. 2013
- Cold spot



r increases anomaly significance

- Planck + WMAP suggest **power deficit** on large scales,
  even with r = 0
- Non-zero **tensor modes** — as suggested by BICEP2 — would **contribute to** C_l^{TT} **on large scales** $l \lesssim 100$
- If signal contains contribution from *r*:
  the **scalar contribution must be even more suppressed**

Image: Bousso et al. 2014



Alleviating the tension

- A **non-standard tensor spectrum?**
  Planck gives *r* at $l \approx 30$, whilst BICEP2 at $l \approx 60$
  $\Rightarrow$ Large, **positive** n_t **preferred, but inflation predicts** $n_t \leq 0$

- **Suppression of the scalar spectrum:**

  - Non-zero **running:** $\alpha_s = dn_s/d\ln k$
    $\Rightarrow$ **Require** $|\alpha_s| \sim \mathcal{O}(10^{-2})$ **, but inflation gives** $|\alpha_s| \sim \mathcal{O}(\epsilon^2)$

  - **Anti-correlated isocurvature modes:**

    $$C_l^{TT}\big|_{\mathrm{SW}} \propto \mathcal{P}_\zeta + 4\mathcal{P}_{\zeta S} + 4\mathcal{P}_S$$

    negative contribution

  - A **fast-roll phase** at the beginning of inflation:

    $$\mathcal{P}_\zeta \propto \frac{H^4}{\dot{\phi}^2}\bigg|_{k=aH} \Rightarrow$$

    enhanced $\dot{\phi}$ leads to suppression

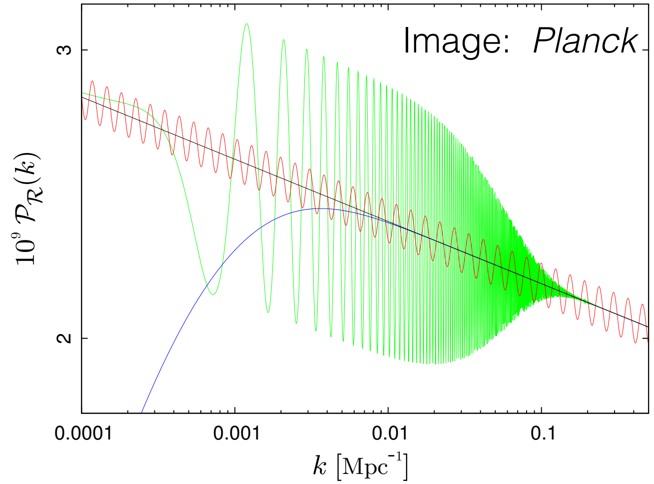
    Occurs in Open Inflation, but also have **additional effects from tunnelling**

Evidence for non-power-law spectrum

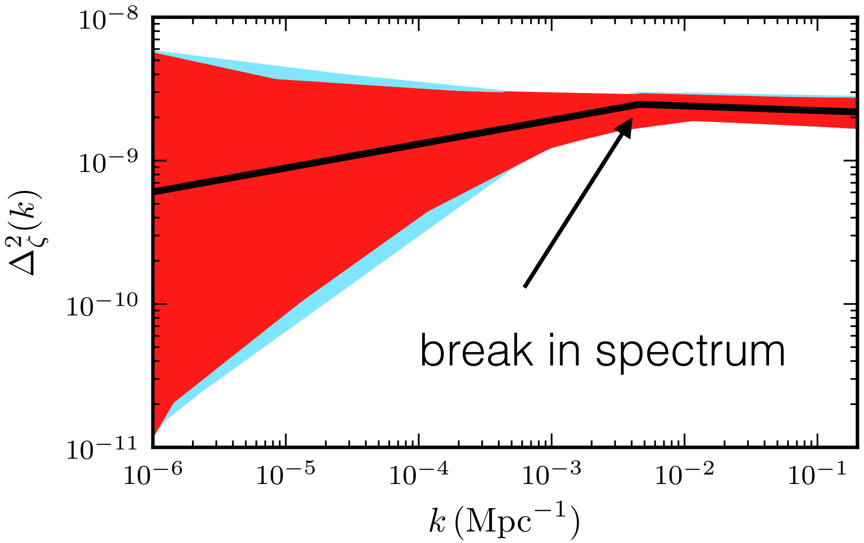
- Even before detection of r a **cut-off spectrum** was **favoured**

$$\mathcal{P}_\zeta = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \left[1 - \exp\left\{-\left(\frac{k}{k_c}\right)^{\lambda_c}\right\}\right]$$

Model	$-2\Delta \ln \mathcal{L}_{\max}$	$\ln B_{0X}$	Parameter	Best fit value
Cutoff	−2.9	0.3	$\ln\left(k_c/\mathrm{Mpc}^{-1}\right)$ λ_c	−8.493 0.474



- **Evidence increased after BICEP2**



Abazajian et al. 2014

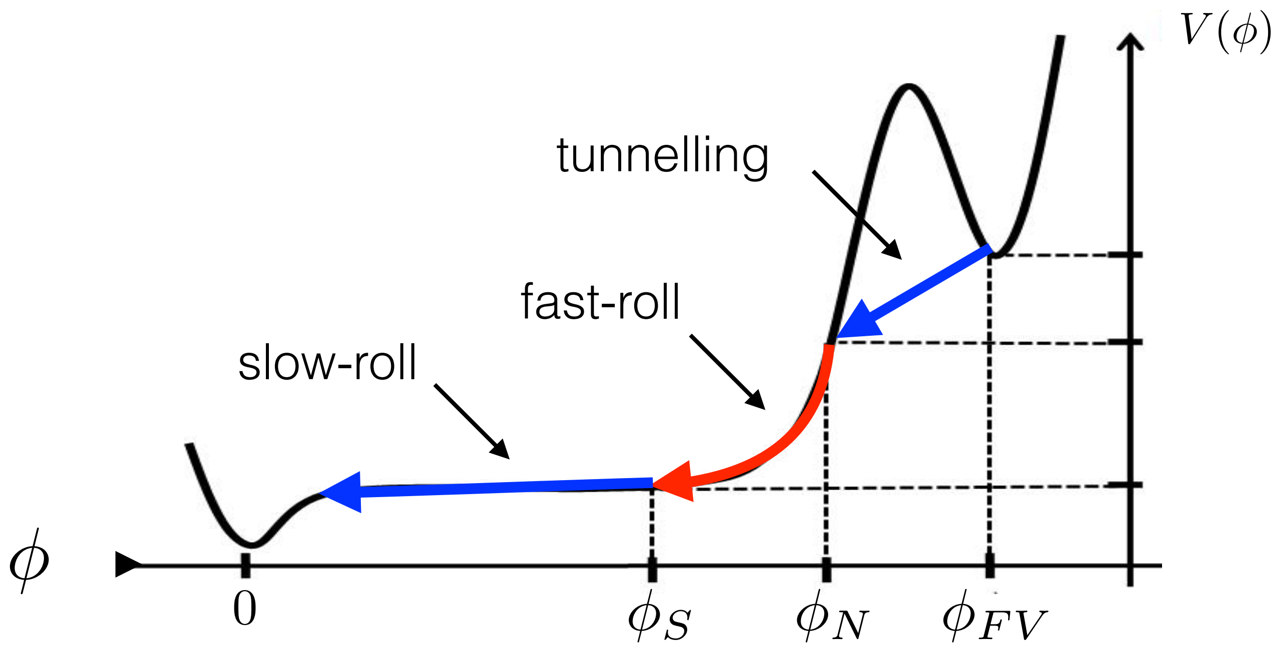
Model	$\Delta \log Z_{\mathrm{Broad}}$	$\Delta \log Z_{\mathrm{Informative}}$	$2\Delta \log \mathcal{L}_{\max}$
No Knots	—	—	—
1 Knot	1.6	3.1	6.2
Model	$\Delta \log Z_{\mathrm{Broad}}$	$\Delta \log Z_{\mathrm{Informative}}$	$2\Delta \log \mathcal{L}_{\max}$
ΛCDM + r	—	—	—
Running	1.1	—	3.8

broken spectrum preferred

i.e. only large scales affected

Open Inflation: a good candidate

- String theory predicts a **landscape of vacua**
- Our **universe** may have **emerged after false-vacuum decay**



- Two key features:
  1. **Universe after tunnelling is open**
  2. **Steep potential** near barrier

$\Rightarrow$ If slow-roll phase short enough, e.g. N ~ 60, expect to see **signatures of spatial curvature and steep potential?**

Coleman de Luccia instanton

- False vacuum decay determined using **instanton method:**

  Transition rate: $\frac{\Gamma}{V} \simeq A e^{-B/\hbar}$ $\quad B = S_E(\phi) - S_E(\phi_{FV})$

  $$S_E(\phi) = \text{Euclidean action}$$

- $O(4)$ **symmetry** is assumed to **minimise $S_E(\phi)$**
  (has been proven in absence of gravity) Coleman et al. '78

  $$ds_E^2 = d\xi^2 + a_E^2(\xi)(d\chi_E^2 + \sin^2\chi_E d\Omega_2^2) \qquad \phi = \phi(\xi)$$

- Minimising the action gives Euclidean equations of motion:

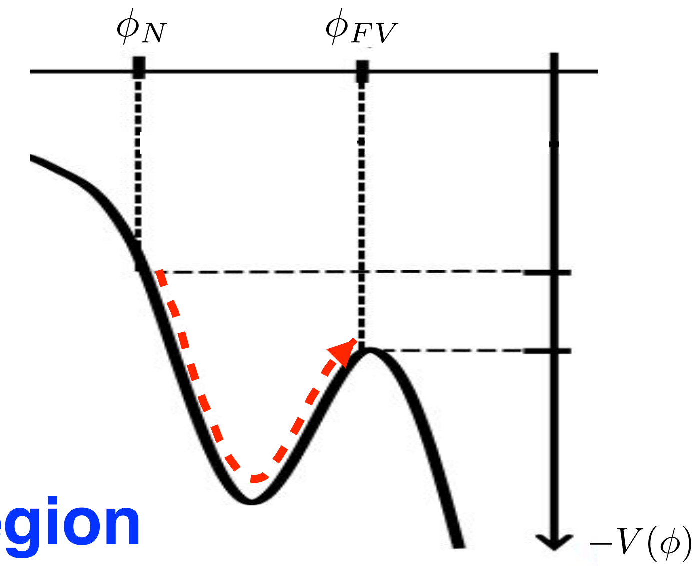
  $$H_E^2 = \frac{1}{3}\left(\frac{\dot{\phi}^2}{2} - V\right) + \frac{1}{a_E^2} \qquad \ddot{\phi} + 3H_E\dot{\phi} - V_\phi = 0$$

$\Rightarrow$ **interpret as dynamics in potential -V**

Coleman de Luccia instanton

Bounce solution:

- BC $\xi = 0$: $a_E = 0 = \dot{\phi}$
  $\phi = \phi_N, \; \dot{a}_E = 1$
- BC $\xi = \xi_F \geq 0$: $a_E = 0 = \dot{\phi}$
  $\phi = \phi_{FV}, \; \dot{a}_E = -1$
- **Require $|V_{\phi\phi}| > H^2$ in tunnelling region**



Analytically continue to Lorentzian signature:

$$ds^2 = -dt^2 + a^2(t)(d\chi^2 + \sinh^2\chi d\Omega^2)$$

$$H^2 = \frac{1}{3}\left(\frac{\dot{\phi}^2}{2} + V\right) + \frac{1}{a^2} \qquad \ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$$

These equations **describe our open universe after tunnelling**

Initial conditions: $a = 0 = \dot{\phi}, \; \phi = \phi_N, \; \dot{a} = 1$

Dynamics after bubble nucleation

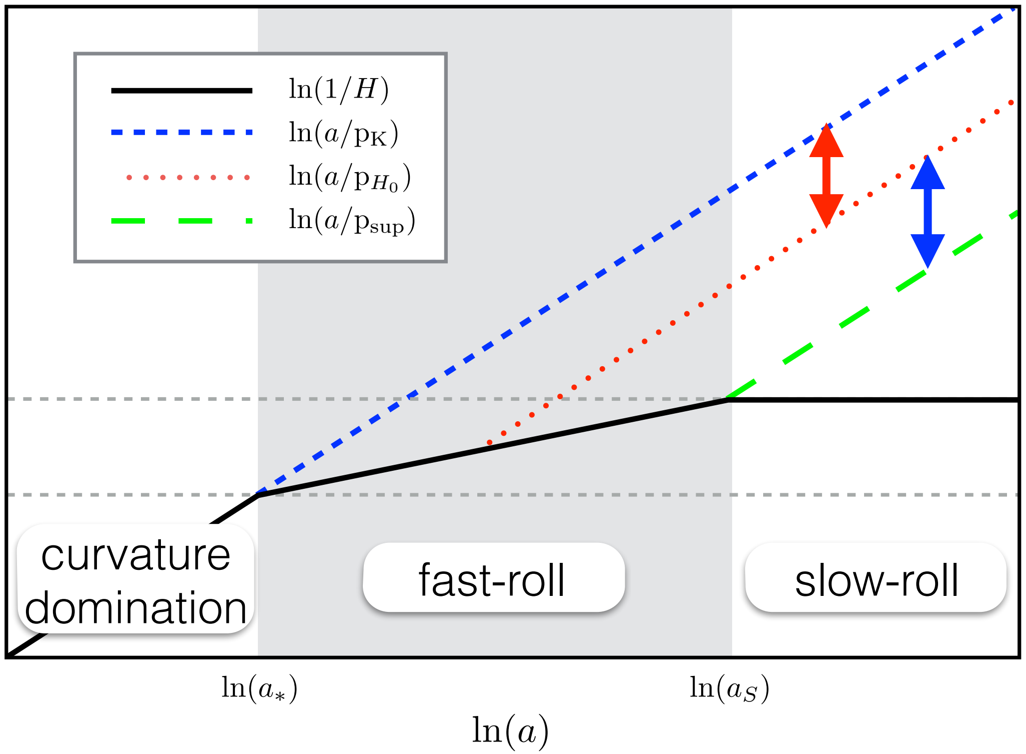
- Given initial conditions expect **three stages:**

1. **Curvature domination:** $H = \frac{1}{a},\ a = t,\ \dot{\phi} = -\frac{V_\phi t}{4}$

   Large Hubble friction $\Rightarrow$ field slowly rolling

2. **Fast-roll phase** after transition to potential domination due to steepness of potential near tunnelling barrier

3. **Slow-roll inflation**



- **Observational constraints:**

A Spatial curvature of universe

$$\ln\left(\frac{p_{H_0}}{p_K}\right) = \ln\left(\frac{1}{\sqrt{\Omega_K}}\right) \gtrsim \ln(10)$$

B Scale of onset of suppression e.g. Abazajian et al.:

$$p_{\rm knot}/p_{H_0} \simeq 20$$

Additional suppression

- In open inflation:
  fast-rolling not the only source of suppression

- Additional suppression reflects **memory of the tunnelling phase**

Spectra given as: $$\mathcal{P}_{\mathcal{R}} = \frac{p^3}{2\pi^2}|\mathcal{R}_c^p|^2 \qquad \mathcal{P}_T = \frac{p^3}{2\pi^2}|Up|^2$$

where: $$U_p'' + 2\mathcal{H}U_p' + (p^2+1)U_p = 0$$

$$\mathcal{R}_c^{p\prime\prime} + 2A(\eta,p)\mathcal{R}_c^{p\prime} + B(\eta,p)\mathcal{R}_c^p = 0$$

non-standard

- Use **fitting functions** based on analytic results of Garriga et al.

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_{t=t_{\mathcal{R},p}} \frac{\cosh(\pi p)+\cos(\delta_p)}{\sinh(\pi p)}\frac{p^2}{c_1^2+p^2},$$

$$\mathcal{P}_T = 4\left(\frac{H}{2\pi}\right)^2_{t=t_{T,p}} \frac{\cosh(\pi p)-1}{\sinh(\pi p)}\frac{p^2}{c_2^2+p^2}$$

- p-dependent suppression factor
- modified horizon crossing condition

Additional suppression

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_{t=t_{\mathcal{R},p}} \frac{\cosh(\pi p)+\cos(\delta_p)}{\sinh(\pi p)} \frac{p^2}{c_1^2+p^2},$$

$$\mathcal{P}_T = 4\left(\frac{H}{2\pi}\right)^2_{t=t_{T,p}} \frac{\cosh(\pi p)-1}{\sinh(\pi p)} \frac{p^2}{c_2^2+p^2}$$

- $\mathcal{R}_c^p$ **modified h.c. condition:** $p^2+4 = a^2H^2\left(1+\frac{\ddot{\phi}}{\dot{\phi}H}\right)^2 - \left(1+2\frac{\ddot{\phi}}{\dot{\phi}H}\right)$

  $\Rightarrow$ **large wavelength modes freeze later**

  $\Rightarrow$ their **amplitudes** are thus **suppressed**

- *p*-dependent suppression factor:

Bubble wall effects: $\delta_p \begin{cases} p \gg 1 \;\rightarrow\; \text{irrelevant} \\ p \ll 1 \,:\, \delta_p - \pi \propto p \;\rightarrow\; \text{take}\, \delta_p = \pi \end{cases}$

$$\Rightarrow \quad \frac{\cosh(\pi p)-1}{\sinh(\pi p)} \frac{p^2}{c_{1,2}^2+p^2} \begin{cases} p \gg 1 \;\rightarrow\; 1 \\ p \ll 1 \;\rightarrow\; \pi p^3/(2c_{1,2}^2) \end{cases}$$

Examples:

- Consider two **toy models** from Linde et al. ’99.

Model 1:

$$V(\phi) = \frac{1}{2}m^2\phi^2\left(1+\frac{\alpha^2}{\beta^2+(\phi-\nu)^2}\right)$$

$$\phi_N = 17.14$$

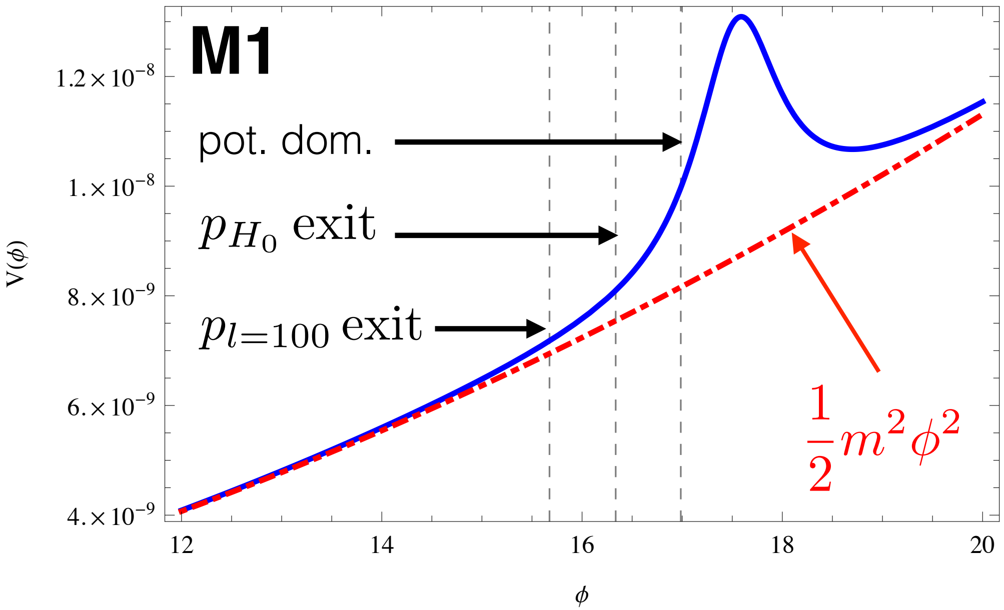
$\nu = 3.5 \times \sqrt{8\pi}$
$\beta^2 = 2\alpha^2$
$\beta = 0.1 \times \sqrt{8\pi}$
$m = 1.5 \times 10^{-6} \times \sqrt{8\pi}$

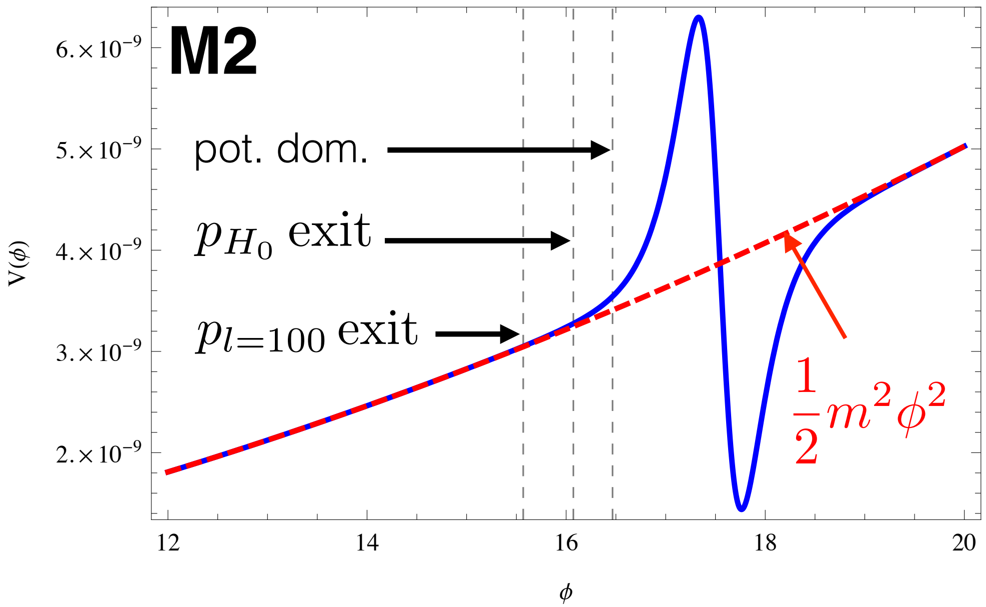
Model 2:

$$V(\phi) = \frac{m^2}{2}\left(\phi^2 - B^2\frac{\sinh\left[A(\phi-\nu)\right]}{\cosh^2\left[A(\phi-\nu)\right]}\right)$$

$$\phi_N = 16.55$$

$m = 10^{-6} \times \sqrt{8\pi}$
$A = 20/\sqrt{8\pi}$
$B = 4 \times \sqrt{8\pi}$
$\nu = 3.5 \times \sqrt{8\pi}$





- **M2 “sharper” - expect suppression to be more localised**

Background evolution: Model 1

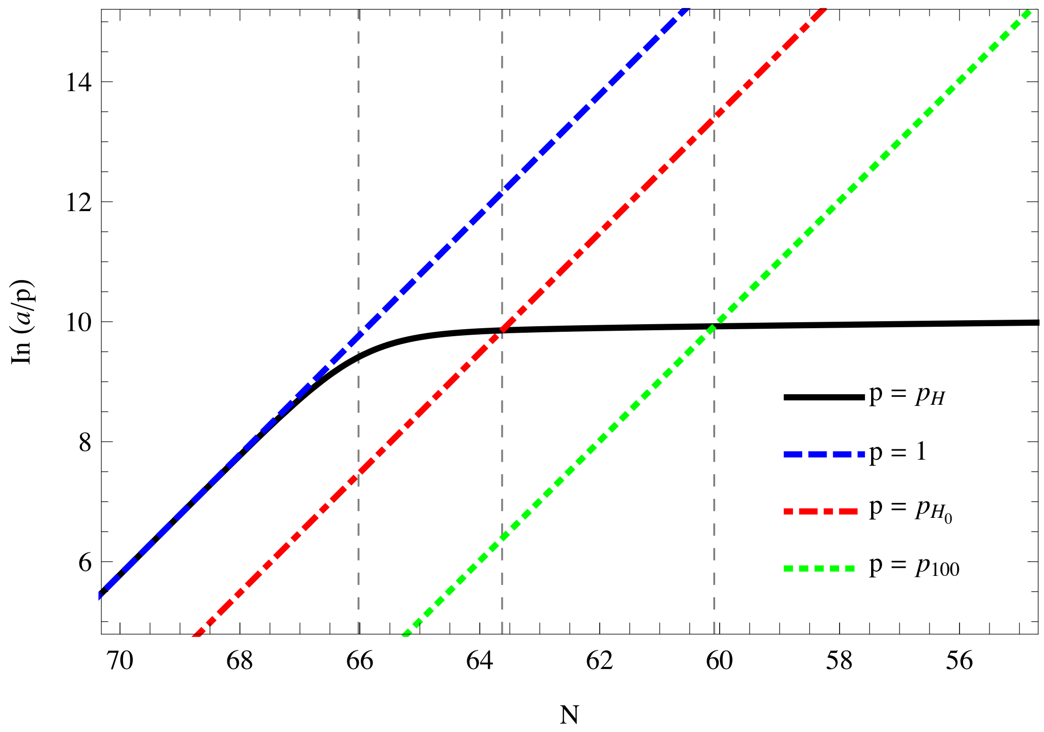
Plot 1: Hubble evolution

From L to R, vertical lines =

- Potential—Curvature equality
- Horizon exit of scale p_{H_0} as determined **assuming**

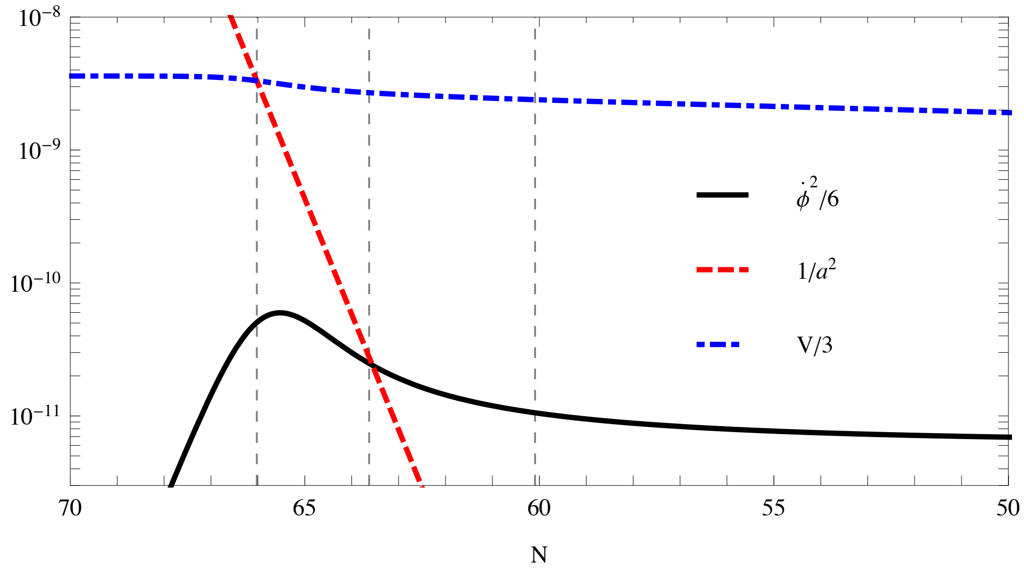
$$\mathbf{\Omega_K = 0.01}$$

- Horizon exit of scale $p_{l=100}$

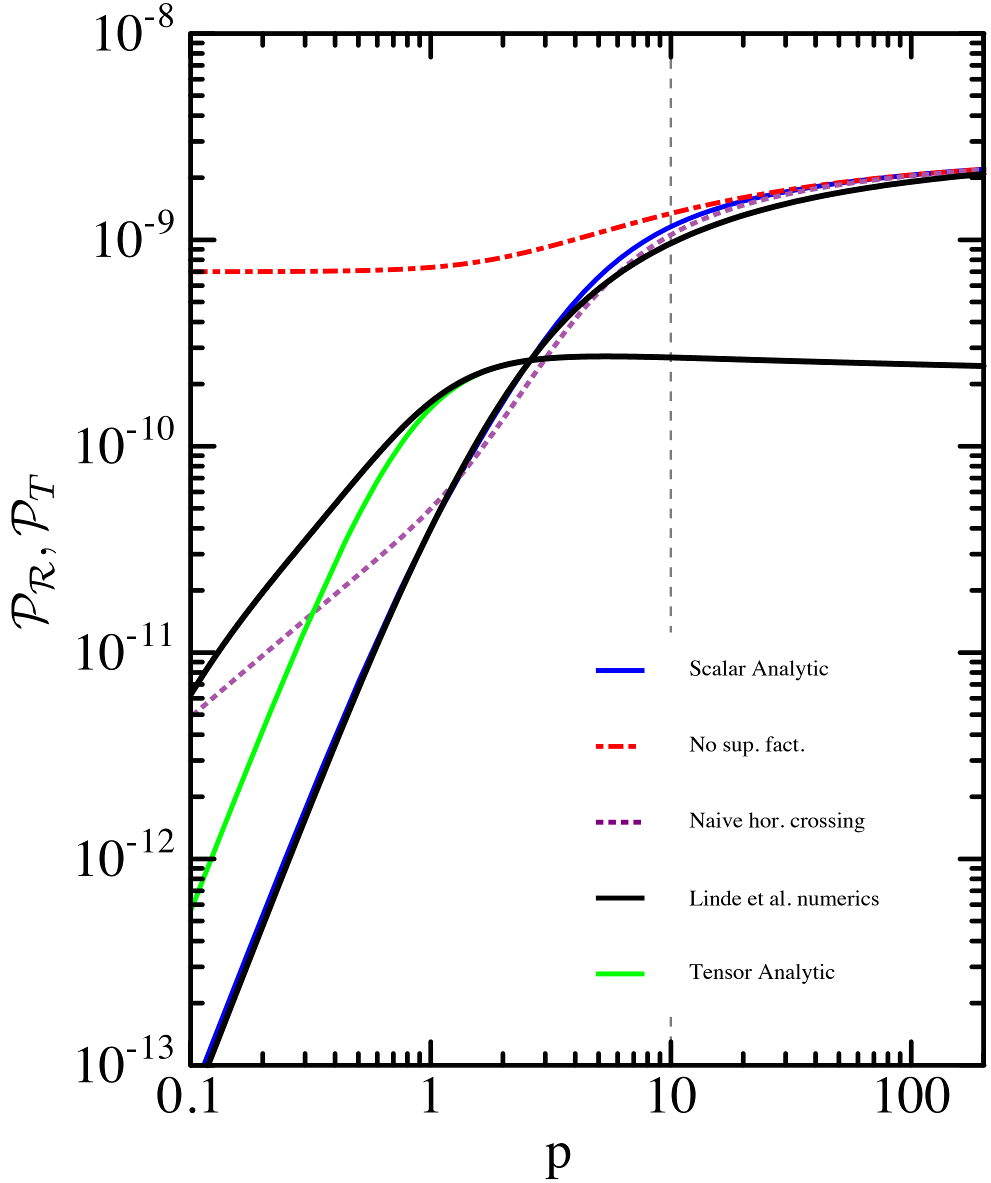


Plot 2: Hubble components

- **Kinetic term** always subdominant, but **enhanced at beginning of potential domination**
- Kinetic—Curvature equality coincides with p_{H_0} exit in M1



Power spectra: Model 1



Recall fitting formulae:

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_{t=t_{\mathcal{R},p}} \frac{\cosh(\pi p) + \cos(\delta_p)}{\sinh(\pi p)} \frac{p^2}{c_1^2 + p^2},$$

$$\mathcal{P}_T = 4\left(\frac{H}{2\pi}\right)^2_{t=t_{T,p}} \frac{\cosh(\pi p) - 1}{\sinh(\pi p)} \frac{p^2}{c_2^2 + p^2}$$

we use $c_1 = 4$ and $c_2 = 1$

assuming $\Omega_K = 0.01,\ p_{H_0} = 10$

- ***p*-dep. supp. factor important**

$\Rightarrow$ **fast-roll not only source of suppression**

- **fast-roll doesn't affect tensors**
- qualitatively similar for M2

Suppression in Models 1 & 2

Transition from blue- to red-tilt at $p_{\rm red}$

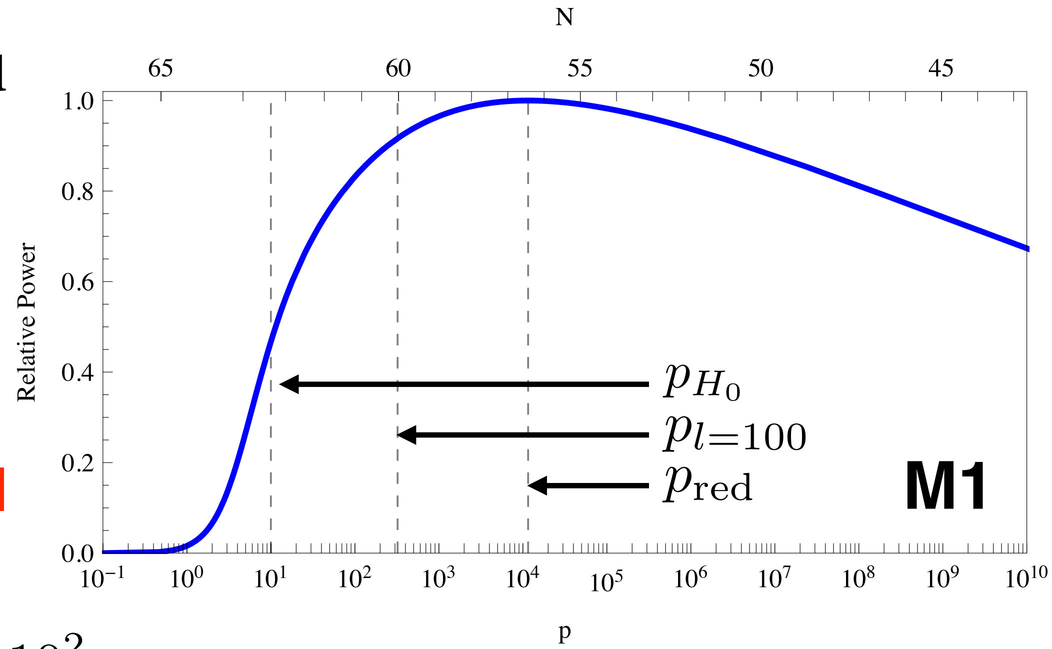
Plot: $\mathcal{P}_\mathcal{R}(p)/\mathcal{P}_\mathcal{R}(p_{\rm red})$

Model 1:

- Curvature—potential equality occurs at N = 66

$\Rightarrow$ **~ 10 e-foldings of fast-roll**

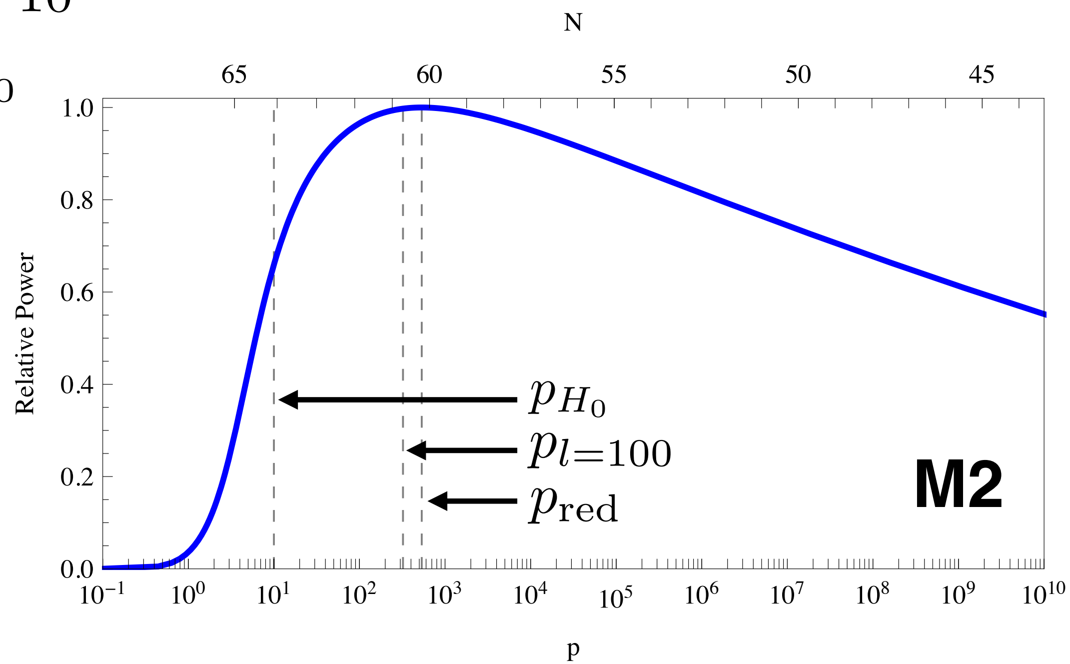
- Even **for unobservable curvature** $\Omega_K \lesssim 10^{-4} \to p_{H_0} \gtrsim 10^2$ **get suppression** for $p \sim p_{H_0}$



Model 2:

- **Can satisfy constraints on Ω_K and give suppression**
- ~ 6 e-foldings of fast-roll

$\Rightarrow$ **suppression on smaller range of scales**



Conclusions

- Planck and WMAP hint at a **deficit in CMB power on large scales**
- This **tension is worsened if the results of BICEP2 are confirmed**
- **Open Inflation** models offer a **viable explanation** for the deficit
- The **source of suppression** in Open Inflation is **two-fold:**

    1. **Fast-rolling** of the inflaton after tunnelling
    2. Additional **effects due to the tunnelling**

- Have studied **two toy models** that are **qualitatively viable**
- A more **quantitative analysis required**