Mapping the ghost-free bigravity into braneworld model

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ghost-free bigravity

bigravity : gravitational theory which contain two gravitons interacting each other

$$S = \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}\,[R + V(g,\,\tilde{g})] + \frac{\chi M_{pl}^2}{2}\int d^4x\sqrt{-\tilde{g}}\tilde{R}$$

For general interaction *V*, an extra DoF whose kinetic term has wrong sign appears.

… **Boulware-Deser ghost**

Boulware and Deser (1972)

To avoid BD ghost, *V* should be tuned as

$$V = m^2\sum_{n=0}^{4} c_n \epsilon^{\mu_1\cdots\mu_n}_{\nu_1\cdots\nu_n} K^{\nu_1}_{\mu_1}\cdots K^{\nu_n}_{\mu_n}\ ,\ K^{\mu}_{\nu} = (\sqrt{g^{-1}\tilde{g}})^{\mu}_{\nu}$$

de Rham, Gabadadze, Tolley (2011)
Hassan and Rosen (2012)

➡

- We can construct a realistic cosmological model at low energies.
- The gravitational wave has a characteristic feature.

… two gravitons causes "graviton oscillation" like neutrino oscillation

Problems of ghost-free bigravity

- What is the hidden metric $\tilde{g}$?
- The form of the interaction is derived technically and artificially.
- The cosmological solutions in ghost-free bigravity do not exist or become unstable at high energies.

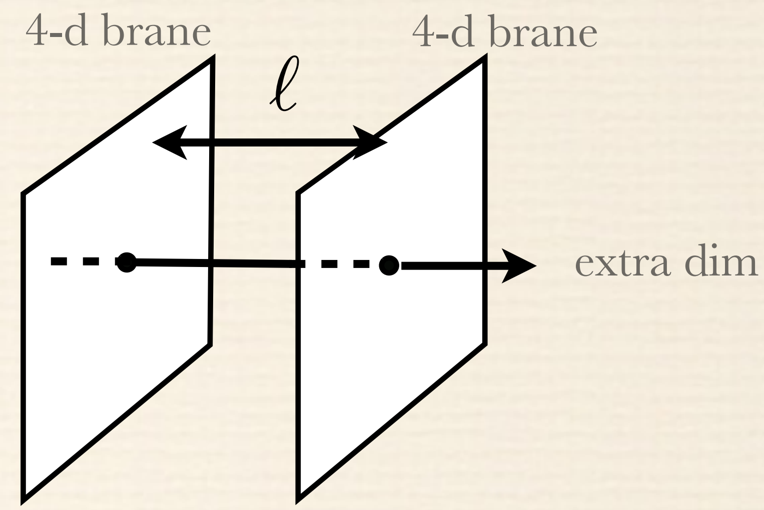
$\rightarrow$ bigravity must be extended to more fundamental theory.

We want to embed ghost-free bigravity to higher dimensional gravity.

How?

Consider 5-dim braneworld model sandwiched by two branes.

$$S = \frac{M_5^3}{2} \int d^5x \sqrt{-g}\, R \; + \text{(boundary term)}$$

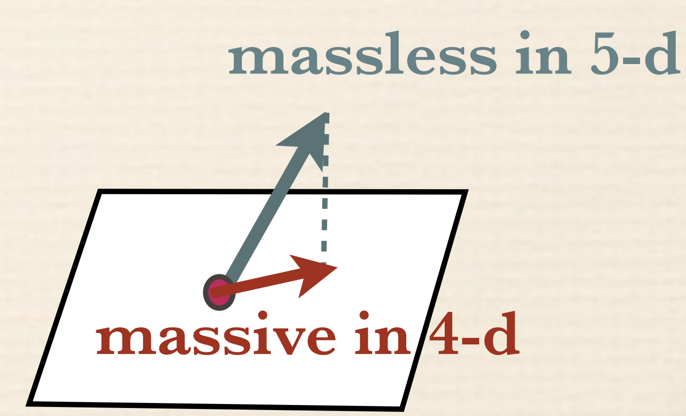


- There is no BD ghost.

- two metrics induced on two branes ⇔ two metrics in bigravity

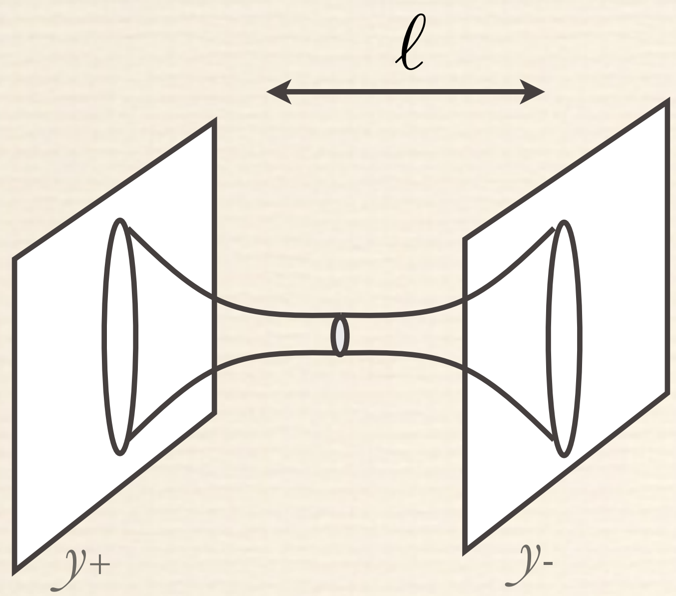
- 5-dim massless graviton → 4-dim massive graviton on the branes.

Only one massive mode must have small mass
to reproduce bigravity as a low energy effective theory.

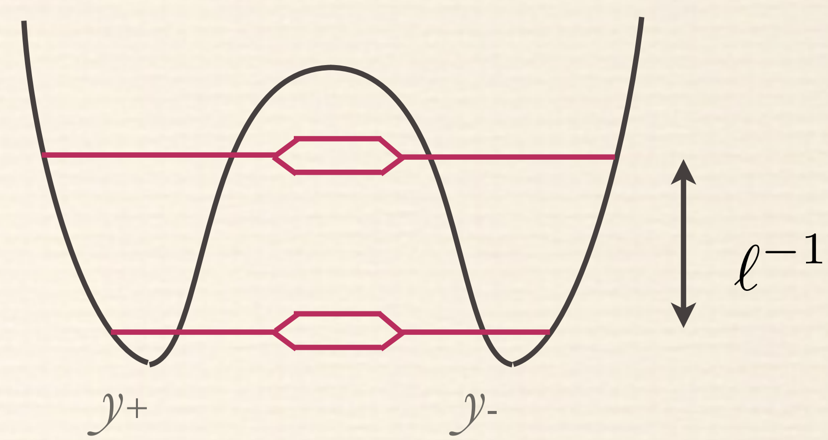


Dvali-Gabadadze-Poratti 2-brane model

4-dim mass spectrum ~ eigenvalue problem in quantum mechanics



effective potential by gravity



high potential barrier

$\rightarrow$ nearly degenerate two small mass

However, such thin throat structure is unstable.

$M_5^3 r_c \int d^4x \sqrt{-g^{(4)}}\, R^{(4)}$ can take its place.

DGP model $$S = \frac{M_5^3}{2} \int d^5x \sqrt{-g} \left({}^5R + \sum_{\pm} \underline{r_c^{(\pm)}} \delta(y - y_\pm)\, {}^4R_{(\pm)} \right)$$

additional length-scale parameter

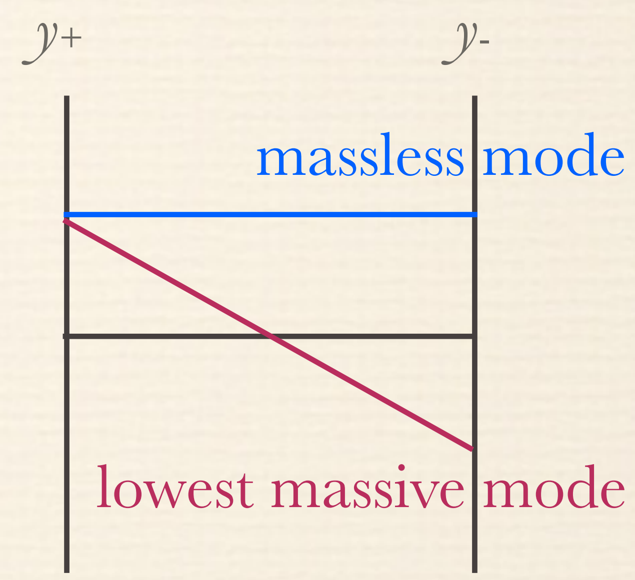
graviton's mass spectrum

bulk equation: $R^{(5)} = 0$

junction condition: $K^{(\pm)}_{\mu\nu} = r_c^{(\pm)} \left(G^{\pm(4)}_{\mu\nu} - \frac{1}{3} G^{\pm(4)} g_{\mu\nu} \right)$

linearization around the de Sitter brane solution
and mode decomposition (mass eigenvalue $m_i^2 \leftrightarrow \Box^{(4)}$)

- a massless mode always exists
- the lowest massive mode



For $\ell \ll r_c$, eigenfunctions become

Then j.c. becomes $g_{\mu\nu}/\ell \simeq r_c \Box^{(4)} g_{\mu\nu} = r_c m_1^2 g_{\mu\nu}$

$$m_1^2 \simeq \frac{1}{r_c \ell} \ll \frac{1}{\ell^2} \simeq m_2^2$$ **: hierarchy**

Stabilization mechanism (Goldberger & Wise)

There is an extra scalar d.o.f. corresponding to the brane separation.

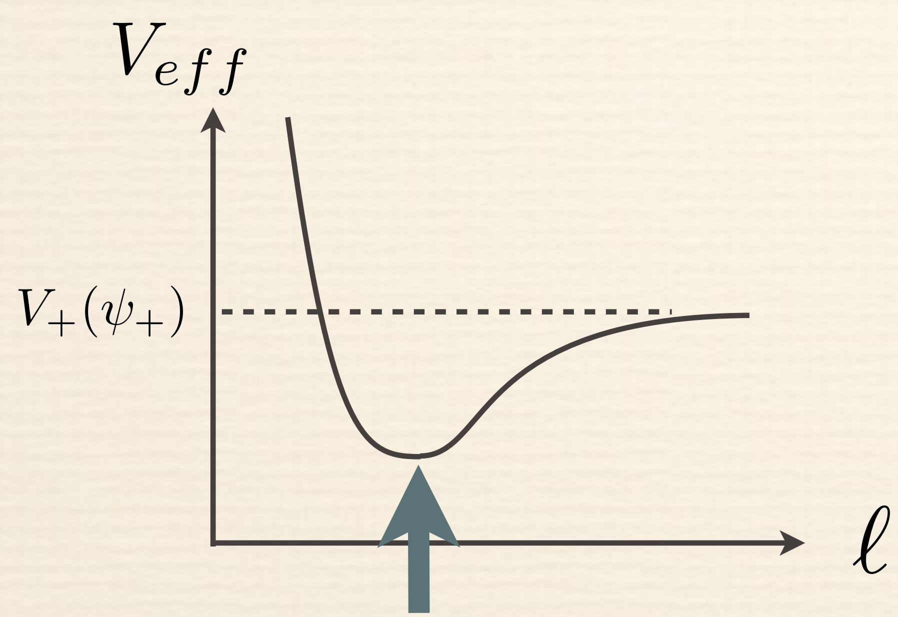
… We should remove it to reproduce bigravity !

We introduce a **stabilization scalar field** to fix the brane separation.

$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - V_B(\psi) - \sum_{\sigma=\pm}\underline{V_{(\sigma)}(\psi)\delta(y-y_\sigma)}\right)$$

$\psi(y_\pm)$: fixed

$\partial_y\psi \to \infty \quad \text{as} \quad \ell \to 0$



The distance between two branes are stabilized.

mass spectrum (scalar mode)

stabilization mechanism $\longrightarrow$ **no massless mode**

If stabilization is weak: $\left|\dfrac{\partial_y \mathcal{H}}{\mathcal{H}^2}\right| \sim \dfrac{(\partial_y \psi)^2}{M_5^3 \mathcal{H}^2} \ll 1$,

the lowest mass becomes $$\mu^2 \approx \frac{2\displaystyle\int_{y_+}^{y_-} \frac{dy}{a^2} + \sum_\sigma \frac{2r_c^{(\sigma)}}{a_\sigma^2} \frac{1}{1-\sigma 2 r_c^{(\sigma)} \mathcal{H}_\sigma}}{\displaystyle\int_{y_+}^{y_-} \frac{dy}{a^4(-\mathcal{H}')}}$$

$a(y)$: scale factor

$\mathcal{H}$: 5-d curvature scale

- stronger stabilization ($\boxed{\text{large } |\mathcal{H}'|}$) $\longrightarrow$ **large μ^2**
- $\boxed{1 \mp 2r_c^{(\pm)} \mathcal{H}_\pm < 0}$ make μ^2 negative : tachyonic instability

$\longrightarrow$ corresponds to the **self accelerating branch**

: K.Izumi et al. (2007)

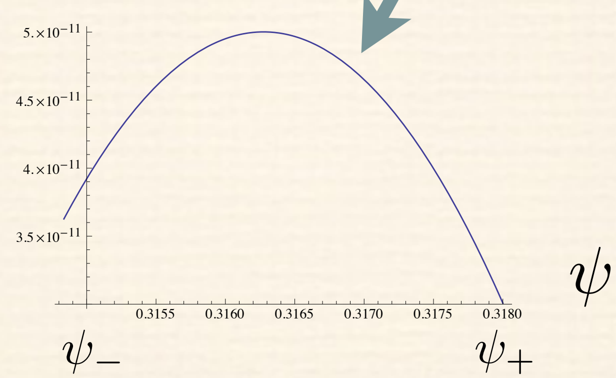
A model which reproduces bigravity

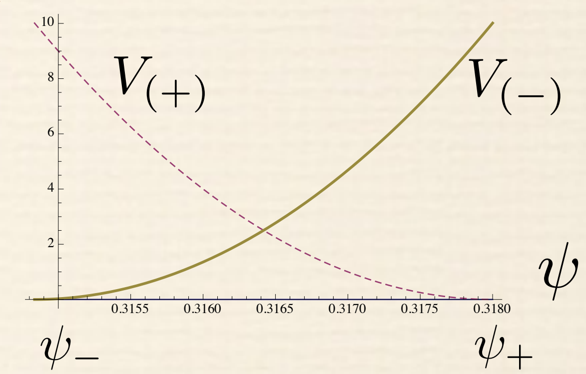
parameters $M_5 = 1.00$ (**brane separation l**)

$r_c^{(\pm)} = 1.00 \times 10^5\,, \quad \ell = 1.00$ << (**strength of induced gravity $r_c^{(\pm)}$**)

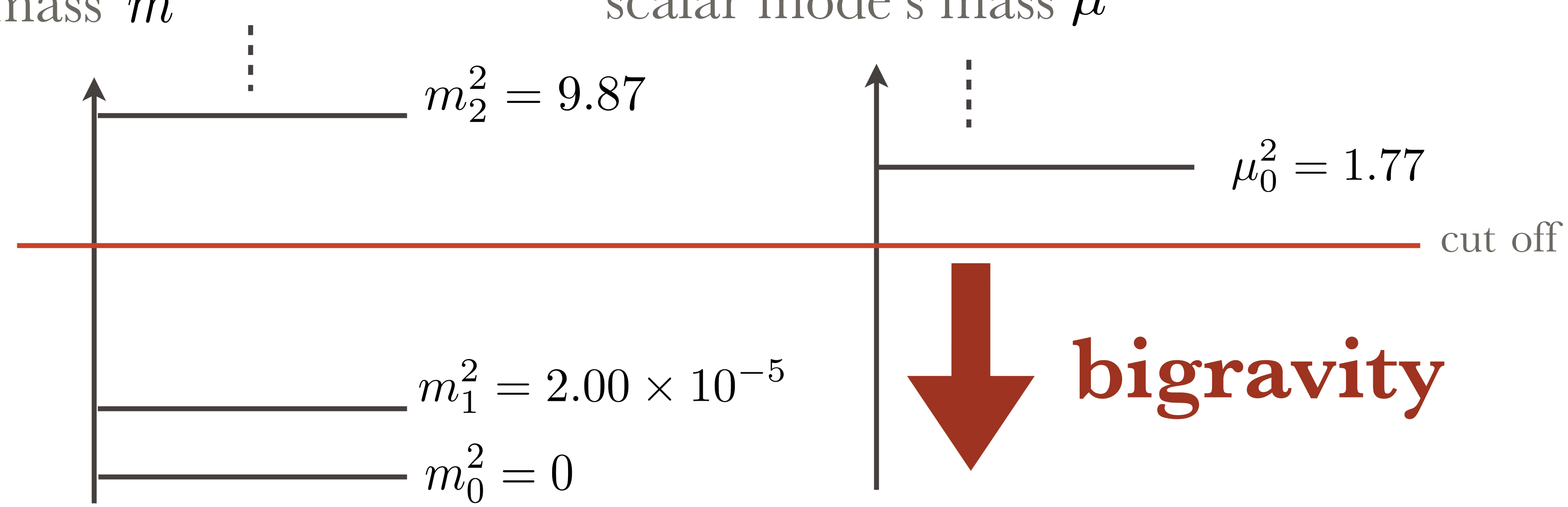
potential of scalar field

$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - \underline{V_B(\psi)} - \sum_{\sigma=\pm}\underline{V_{(\sigma)}(\psi)}\delta(y-y_\sigma)\right)$$





graviton's mass m^2 scalar mode's mass μ^2



bigravity

Correspondence between DGP 2-brane model and ghost-free bigravity

Truncating the scalar modes and taking the limit $\ell/r_c^{(\pm)} \to 0$,

	DGP 2-brane model	ghost-free bigravity
variables	$h_{\mu\nu}(y_\pm) = h^{(0)}_{\mu\nu} u_0(y_\pm) + h^{(1)}_{\mu\nu} u_1(y_\pm)$	$h_{\mu\nu}\ ,\ \tilde{h}_{\mu\nu}$
parameters	$r_c^{(\pm)},\ \ell\ \to\ m_1^2\,,\ u_0(y_\pm)\,,\ u_1(y_\pm)$	$M_{pl},\ \chi,\ m^2,\ c_n$ $\to m_{eff},\ \chi\omega^2$

ω : ratio of scale factors of two metrics

DGP 2-brane model is identical to ghost-free bigravity at the linear level.

$$h_{\mu\nu}(y_+) \leftrightarrow h_{\mu\nu}$$
$$h_{\mu\nu}(y_-) \leftrightarrow \tilde{h}_{\mu\nu}$$
$$m_1^2 \leftrightarrow m_{eff}^2$$
$$r_c^{(+)}/r_c^{(-)} \leftrightarrow \chi\omega^2$$

ghosts in DGP model

H : 4-dim comoving curvature scale

the regularity on +brane imposes

$$2\left(\underbrace{\sum_i \frac{u_i^2(y_+)}{m_i^2 - 2H^2}}\right) + \frac{1}{H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\frac{2\kappa^2}{3H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\underbrace{\sum_i \frac{v_i^2(y_+)}{\mu_i^2 + 4H^2}}\right) + \mathcal{H}_+\right) = 0$$

diverges as $m^2 \to 2H^2$: Higuchi bound

diverges as $\mu^2 \to -4H^2$

: critical mass that scalar ghost appears

$2r_c\mathcal{H}_+ - 1 > 0$: **self-accelerating branch**

$\mu_i^2 + 4H^2 \to \mp\epsilon$ means $m_i^2 - 2H^2 \to \pm\epsilon$

➡ **ghost never disappears** K.Izumi et. al. (2007)

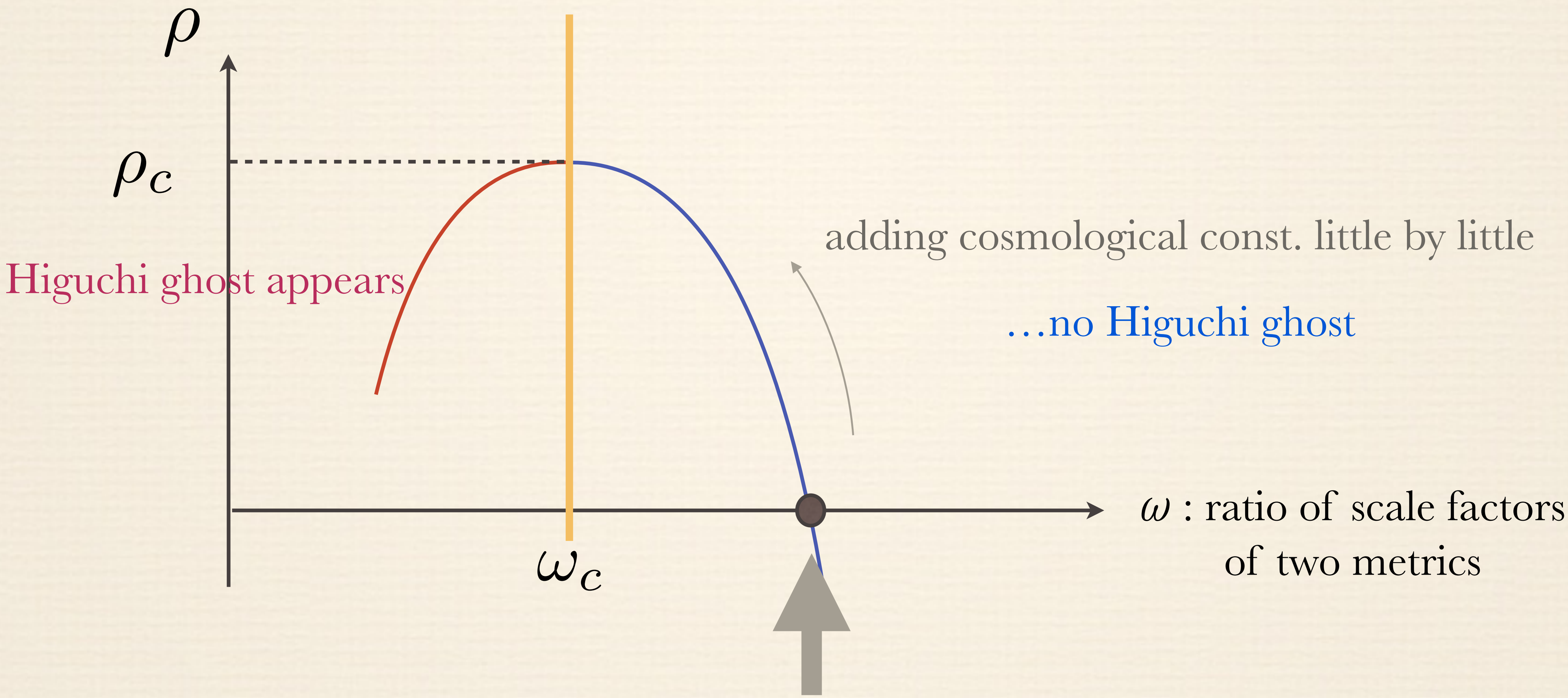
$2r_c\mathcal{H}_+ - 1 < 0$: **normal branch**

The same identity prohibits m_i^2 & μ_i^2 from crossing their critical masses

➡ **no ghost**

Higuchi ghost in ghost-free bigravity

de Sitter solution does not exist above this critical density, and Higuchi ghost appears after crossing the critical ω.



choose the branch connected to the flat vacuum spacetime with positive graviton mass^2

Breakdown of the stabilization in DGP model

Considering to add cosmological const. δH on the brane,
the junction condition imposes

$$\pm\delta\mathcal{H}_{\pm} = r_c^{(\pm)} a^{-2} \delta H^2$$

On the other hand, $|\mathcal{H}| \lesssim \dfrac{1}{r_c^{\pm}}$ must be satisfied to avoid the scalar-mode instability.

$\delta H^2 \gtrsim \dfrac{1}{r_c^{(\pm)2}}$ **cause instability and break the brane stabilization.**

This stabilization mechanism is too weak to sustain the spacetime structure which reproduces bigravity.

We should invent some other mechanism to stabilize two branes more strongly.

Breakdown of the correspondence between DGP 2-brane model and ghost-free bigravity

When we consider to increase the energy scale in the ghost-free branch,

DGP 2-brane model ⟷ **ghost-free bigravity**

at relatively low energy,
we cannot choose normal branch
and scalar-mode ghost appears

ghost instability never appears

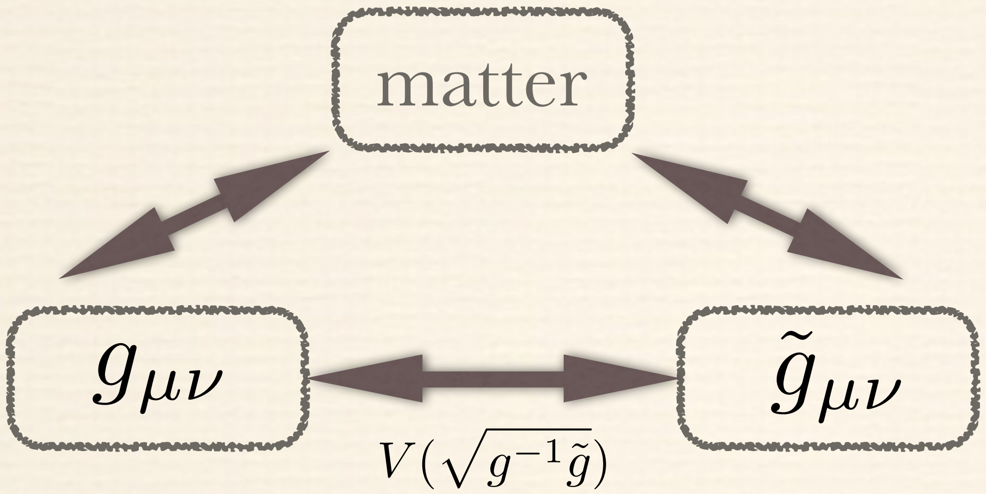
…The correspondence between two models breaks at high energies.

Is there any brane model which reproduces ghost-free bigravity?

doubly coupled matter

It is natural to consider **doubly coupled matter** in ghost-free bigravity by introducing **5-dim matter field** in the brane model.

However,



coupling through the matter generally detunes the structure of the interaction.

$\rightarrow$ **BD ghost revives**

Yamashita, De Felice and Tanaka (2014)

The model of doubly coupled matter is strongly restricted by the condition to avoid the BD ghost.

Yamashita, De Felice and Tanaka (2014)

de Rham, Heisenberg and Riberto (2014)

Noller and Melville (2014)

Hassan, Kocic and Schmidt-May (2014)

…inconsistent with the viewpoint of braneworld model

Summary

- We want to derive the ghost-free bigravity from some more fundamental theory which is valid at high energies … higher dimensional gravity
- We obtain the ghost-free bigravity as 4-dim effective theory of DGP 2-brane model with a stabilization scalar field.
- The mechanism how ghosts appear in two models are clearly different, and the scalar ghost in the brane model, which is absent in ghost-free bigravity, breaks the correspondence between two models at a relatively low energy.
- This idea suggests that it is natural to consider doubly coupled matter in the ghost-free bigravity, however, doubly coupled matter brings a BD ghost.

Future works

- Seeking for a way to make the stabilization mechanism stronger

  … Introducing **Gauss-Bonnet term**, the coupling between the stabilization scalar field and the graviton becomes weak and the stabilization becomes effectively strong?

- Investigation of doubly coupled matter in terms of the brane model

  … We will investigate how 5-dim matter couples to two gravitons in the 4-dim effective theory, perturbatively.

  Calculating 3rd order vertex may help to find the new matter coupling?

bigravity and Boulware-Deser ghost

bigravity … gravitational theory
which contain two gravitons interacting each other

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}\left[R^{(g)} + \underline{2m^2V(g,\,f)}\right] + \cancel{\frac{M_f^2}{2}\int d^4x\sqrt{-f}R^{(f)}}$$

fix f

The interaction term breaks general covariance for g

→ $\underbrace{\text{GR (helicity-2) + 4 gauge breaking (helicity-1, helicity-0,}}_{\text{massive graviton}}$ $\boxed{\text{helicity-0}}$)

This mode's kinetic term has opposite sign!!

Boulware-Deser ghost

Boulware and Deser (1972)

In order to obtain healthy bigravity, we have to tune the interaction form so that the ghost mode is killed by constraints.

ghost-free bigravity

Choosing the form of the interaction as

$$V = \sum_{n=0}^{4} c_n \epsilon^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} K^{\nu_1}_{\mu_1} \ldots K^{\nu_n}_{\mu_n} \qquad K^{\nu}_{\mu} = \sqrt{g^{\nu\rho} f_{\rho\mu}}$$

de Rham, Gabadadze, Tolley (2011)

ADM decomposition

$$N^{-2} = -g^{00}\,, \quad N_i = g_{0i}\,, \quad \gamma_{ij} = g_{ij}\,,$$
$$L^{-2} = -f^{00}\,, \quad L_i = f_{0i}\,, \quad {}^3f_{ij} = f_{ij}\,.$$

define new shift-like vector n^i
and rewrite N^i with n^i

Then Hamiltonian becomes linear in N, L, L^i.

$$H = N\mathcal{C} + LC^L + L^i C^L_i\,.$$

conjugate momentum

$\mathcal{C}$, C^L, C^L_i are functions of $\{\gamma_{ij},\, \pi^{ij},\, {}^3f_{ij},\, p^{ij}\}$

This Hamiltonian constraint and its consistency relation kill BD ghost !

Hassan and Rosen (2012)

Also there are 4 gauge dofs $\Rightarrow$ one massless graviton and one massive graviton

Correspondence between ghost-free bigravity and DGP 2-brane model

We showed that the behavior of these two models is identical
in the very low energy regime.

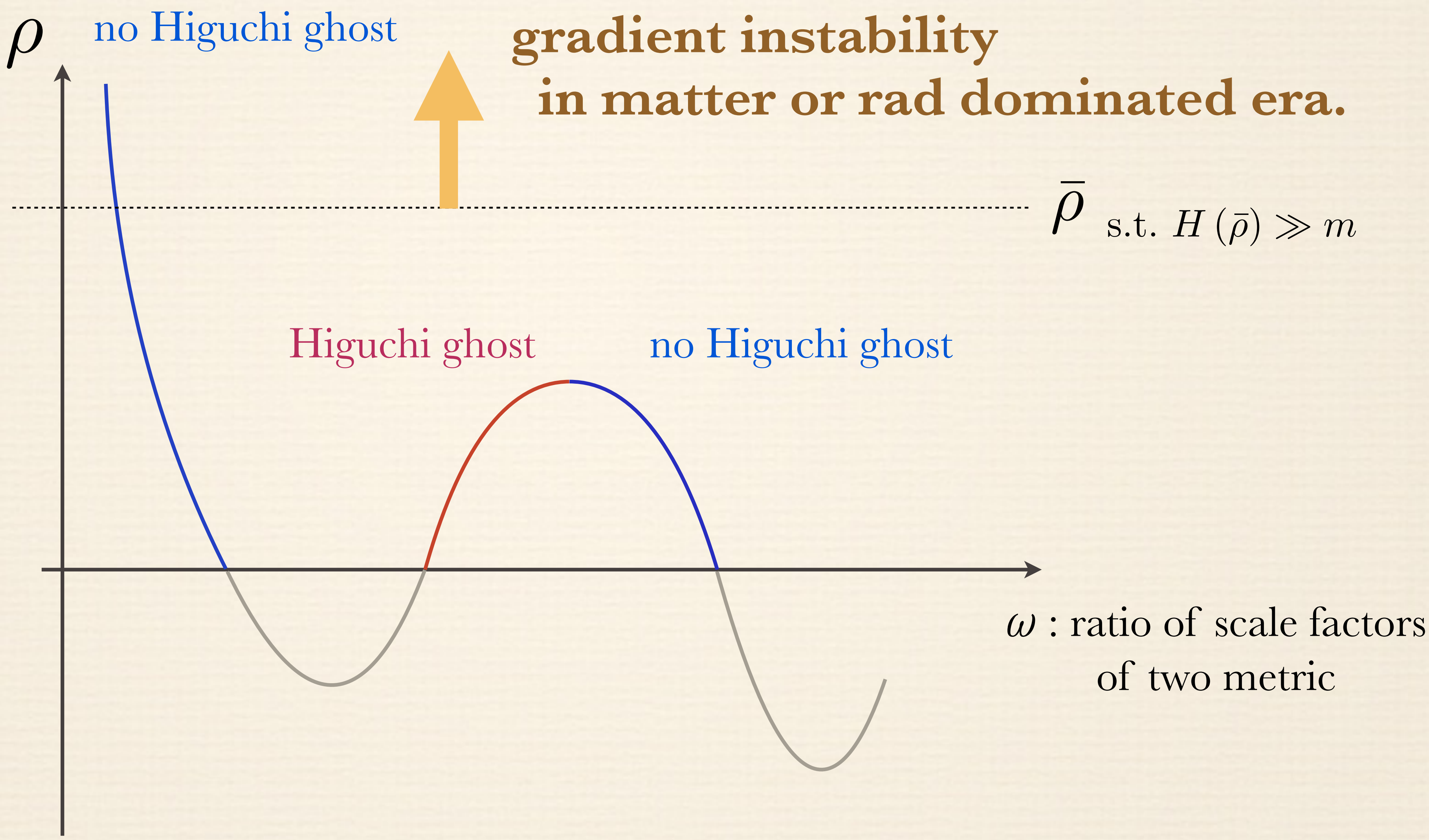
Yamashita and Tanaka (2014)

ghost-free bigravity		**DGP 2-brane model**
two metrics	⟷	two metric induced on the two branes
graviton's mass	⟷	the mass of the lowest massive mode

⬇

It is natural to consider **doubly coupled matter** in ghost-free bigravity by introducing **5-dim matter field** in brane model.

Cosmological solution in ghost-free bigravity



Higuchi ghost in dRGT bigravity

In dRGT model, equation for the de Sitter solution insists

$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

ω : ratio of scale factor of two metric

effective mass for massive graviton

$$m_{eff}^2 = m^2(1+(\chi\omega^2)^{-1})\Gamma(\omega) = -\frac{m^2\omega}{3}\underline{f'(\omega)} + 2H^2$$

this sign determines the ghost appearance

$$\Gamma(\omega) \equiv c_1\omega + 4c_2\omega^2 + 6c_3\omega^3$$

For flat vacuum solution, $H \to 0$ as $\omega \to \omega_0$ where $\rho_m(\omega_0) \to 0$,

$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

negative when $\Gamma > 0$ i.e. $m_{eff}^2 > 0$

→ **no Higuchi ghost**

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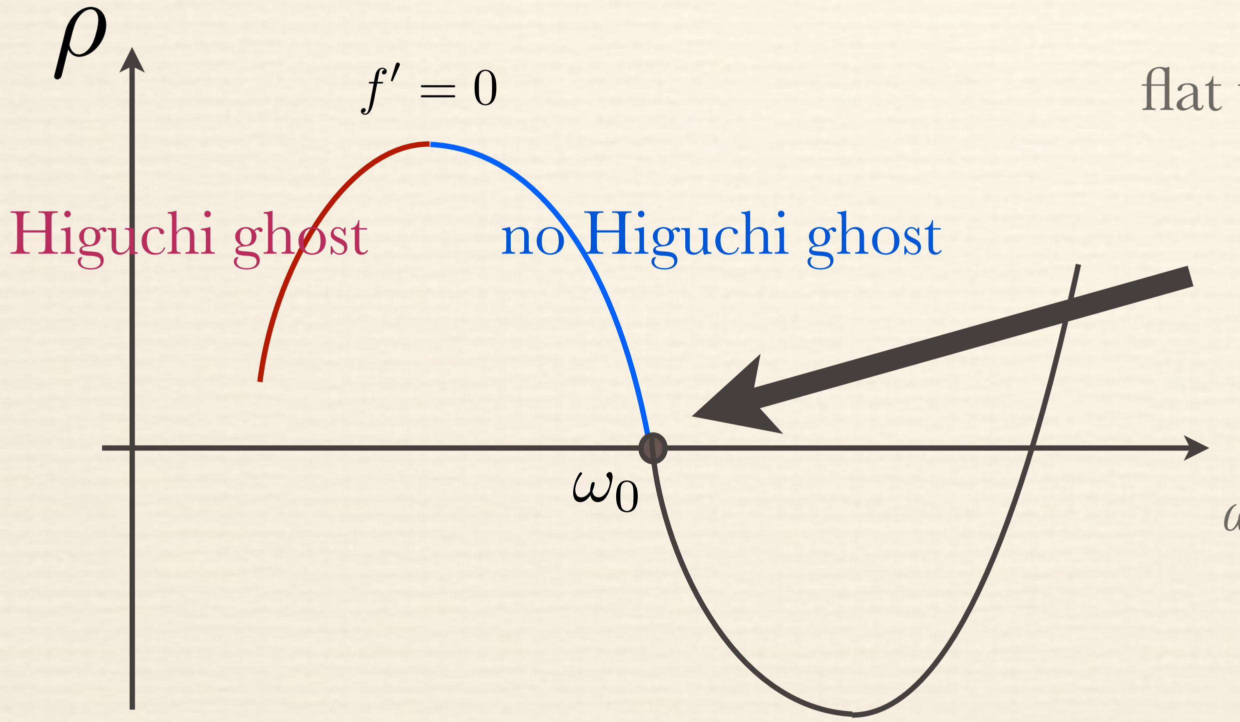

flat vacuum $H = 0, \rho = 0$

$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

...negative when $\Gamma > 0 \Leftrightarrow m_{eff}^2 > 0$

Seeking for models with doubly coupled matter which have no BD ghost

Introduce a k-essence scalar field

$$\mathcal{L}_m = \sqrt{-g}\, P(X,\phi) + \sqrt{-f}\, \tilde{P}(\tilde{X},\phi)$$

$$X = -\frac{1}{2}\, g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\,, \quad \tilde{X} = -\frac{1}{2}\, f^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$$

Consider perturbation around FLRW and Bianchi-1 type anisotropic spacetime, and evaluate the determinant of kinetic matrix A.

- $\det A \neq 0$ → An extra d.o.f. exists.
- $\det A < 0$ in Minkowski limit of the healthy branch → This d.o.f. becomes a ghost.

Result

→ **BD ghost appears unless** $\tilde{P} = \tilde{P}(\phi)$ or $P = P(\phi)$

Seeking for models with doubly coupled matter which have no BD ghost

We also obtain another ghost-free model with doubly coupled matter as below:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_g^2 R^{(g)}}{2} + 2m^2 M_{\rm eff}^2 \sum_n c_n e_n\left(\sqrt{g^{\mu\nu}(f_{\mu\nu} + \alpha\partial_\mu\phi\,\partial_\nu\phi)}\right)\right] + \int d^4x\sqrt{-f}\left[\frac{M_f^2 R^{(f)}}{2} - \frac{1}{2} f^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right]$$

…after field definition $f_{\mu\nu} \to \tilde{f}_{\mu\nu} - \alpha\partial_\mu\phi\,\partial_\nu\phi$,

matter does not couple to *g* and no higher time derivative term arises.

The model of doubly coupled matter is considerably restricted.

… inconsistent with the intuition of braneworld models.

The attempt to extend bigravity to brane models is impossible?

In the case in which there is only one scalar mode in 4-dim effective theory, its coupling to the brane-induced metrics is also restricted ?

matter which couples to one effective metric

In the case in which a matter field minimally couples to one effective metric $g^{\text{eff}}_{\mu\nu}$ s.t.

$$\sqrt{\det g_{\text{eff}}} = \sum_{n=0}^{4} \alpha_n \epsilon^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} K^{\nu_1}_{\mu_1} \ldots K^{\nu_n}_{\mu_n} \,,\; K^{\nu}_{\mu} = \sqrt{g^{\nu\rho}\tilde{g}_{\rho\mu}}$$

BD ghost disappears.

de Rham, Heisenberg and Ribeiro (2014)

Hassan, Kocic and Schmidt-May (2014)