YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

ICG Portsmouth

Relativistic Cosmology
Yukawa Institute for Theoretical Physics 08-19 Sep 2014

16 Sept, 2014

Homogeneous Instantons in Bigravity Theory

Ying-li Zhang (YITP)

Based on:

M. Sasaki, D. Yeom and YZ, in prep

Outline

1. Bigravity Theory
2. Setup of Model
3. Hawking-Moss solutions
4. Summary and future prospects

1. Bigravity Theory

“Can a graviton have mass ?”

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h| \ll 1$$

To the lowest order in h, one finds the Lagrangian:

$$L = L_{\rm EH}(h) + \frac{m_g^2}{2}\left(h_{\mu\nu}h^{\mu\nu} + \alpha h^2\right) \ ,$$

decompose $h_{\mu\nu} = h^{\perp}_{\mu\nu} + \partial_{(\mu}A^{\perp}_{\nu)} + \partial_\mu\partial_\nu\chi,$

where $\partial^\mu h^{\perp}_{\mu\nu} = \partial^\mu A^{\perp}_\mu = 0,$

$$L \supset -\frac{m_g^2}{2}\left[(\partial_\mu\partial_\nu\chi)^2 + \alpha(\Box\chi)^2\right] ,$$

So to avoid higher-order derivatives, we impose

$\alpha = -1$ $\Longrightarrow$ **Fierz-Pauli 1939**

$$\boxed{L = L_{\mathrm{EH}}(h) + \frac{m_g^2}{2}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) ,}$$

- The unique massive gravity theory in linear level without ghost in Minkowski background;
- Diffeomorphism invariance is broken due to mass term.

- **Boulware-Deser ghost** (Boulware & Deser '72)

  If consider non-Minkowski background (e.g. FLRW), there appears a sixth mode which is a ghost

Is it possible to find a ghost-free nonlinear MG?

A breakthrough:

C. de Rham, G. Gabadadze, Phys. Rev. D 82, 044020 (2010);
C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett 106, 231101 (2011);
S. F. Hassan and R. A. Rosen, JHEP 1107, 009 (2011)

$$S_{dRGT} = \frac{M_{\mathrm{P}}^2}{2}\int d^4x\sqrt{-g}R_{\mathrm{g}} + m_{\mathrm{g}}^2 M_{\mathrm{P}}^2 \int d^4x\sqrt{-g}\sum_{n=1}^{4}\alpha_n \mathcal{U}_n(\mathcal{K}) + \int d^4x\sqrt{-g}\mathcal{L}_{\mathrm{m}}$$

$$\mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \left(\sqrt{g^{-1}f}\right)^{\mu}_{\nu}$$

$$\begin{aligned}
\mathcal{U}_1(\mathcal{K}) &= [\mathcal{K}] \equiv \mathcal{K}^{\mu}_{\mu}, \\
\mathcal{U}_2(\mathcal{K}) &= \frac{1}{2!}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right), \\
\mathcal{U}_3(\mathcal{K}) &= \frac{1}{3!}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right), \\
\mathcal{U}_4(\mathcal{K}) &= \frac{1}{4!}\left([\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\right),
\end{aligned}$$

Problems:

No flat FLRW solution;

Nonlinear ghost instability if both metric are FLRW;

Superluminal problem……

$$S_{BG} = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R_g \boxed{+\frac{M_f^2}{2}\int d^4x\sqrt{-f}R_f}$$

Dynamical

$$+ m_g^2 M_P^2 \int d^4x\sqrt{-g}\sum_{n=1}^{4}\alpha_n \mathcal{U}_n(\mathcal{K}) + \int d^4x\sqrt{-g}\mathcal{L}_m$$

$$\mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \left(\sqrt{g^{-1}f}\right)^{\mu}_{\nu}$$

$$\begin{aligned}
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\end{aligned}$$

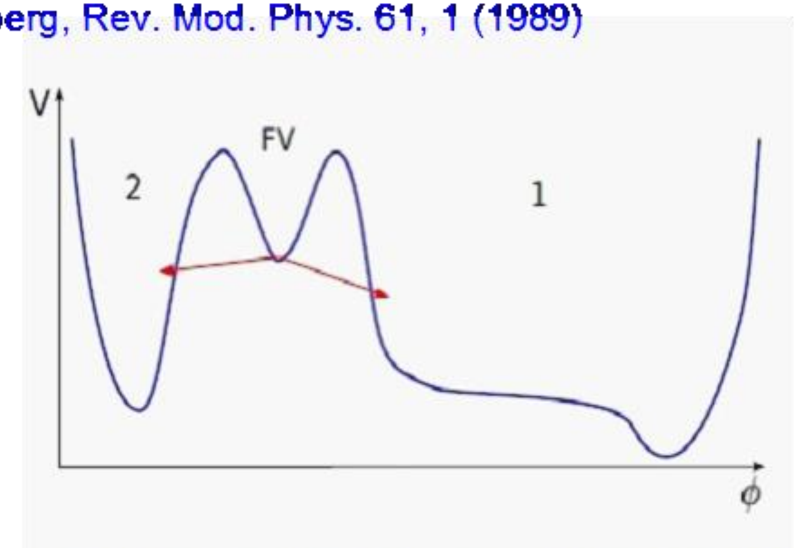
Bigravity: both of the metrics are dynamical

S.F.Hassan and R.A.Rosen JHEP. 1202 126 (2012)

How about the Cosmological Constant Problem?

A possible resolution: Landscape of Vacua

S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)



S. Coleman and F. de Luccia, Phys.Rev. D21, 3305, (1980)

- the field can (and will) tunnel from a metastable minimum to a lower one;

- this process is driven by instanton.

In dRGT case: YZ, Ryo Saito and Misao Sasaki, JCAP 02 (2013) 029 [1210.6224]; Misao Sasaki, Dong-han Yeom and YZ, CQG 30 (2013) 232001 [1307.5948]; YZ, Ryo Saito, Dong-han Yeom and Misao Sasaki, JCAP 02 (2014) 022 [1312.0709].

It is natural and interesting to extend the analysis to Bigravity Theory

2. Setup of Model

Action

$$S = \frac{1}{2}\int d^4x \left[\sqrt{-g}M_{\mathrm{P}}^2\left(R_{\mathrm{g}} + \boxed{\lambda_{\mathrm{g}}}\right) + \sqrt{-f}M_{\mathrm{f}}^2\left(R_{\mathrm{f}} + \boxed{\lambda_{\mathrm{f}}}\right)\right]$$
$$+ m_{\mathrm{g}}^2 M_{\mathrm{e}}^2 \int d^4x\sqrt{-g}\sum_{n=1}^{4}\alpha_n \mathcal{U}_n(\mathcal{K}) + \int d^4x \left[\sqrt{-g}\mathcal{L}_{\mathrm{mg}} + \sqrt{-f}\mathcal{L}_{\mathrm{mf}}\right]$$

$$\boxed{M_{\mathrm{e}} = \left(M_{\mathrm{P}}^{-2} + M_{\mathrm{f}}^{-2}\right)^{-1/2}}$$

Interaction between $g_{\mu\nu}$ and $f_{\mu\nu}$

S.F.Hassan and R.A.Rosen JHEP. 1202 126 (2012)

Euclidean Approach

- Wick rotation $\tau = it, \quad S_E = iS$
- tunneling probability per unit time per unit volume

$$\boxed{\begin{aligned}&\Gamma/V = Ce^{-B},\\ &B = S_E[\underbrace{g_{\mu\nu,B},\phi_B}_{\text{bounce solution}}] - S_E[\underbrace{g_{\mu\nu,F},\phi_F}_{\text{'false vacuum'}}],\end{aligned}}$$

- bounce solutions are explored by assuming an O(4) symmetry

$$ds^2_{\rm g,E} = N^2(\tau)d\tau^2 + a^2(\tau)d\Omega_3^2$$

$$ds^2_{\rm f,E} = N_{\rm f}^2(\tau)d\tau^2 + b^2(\tau)d\Omega_3^2$$

- Inserting ansatz into action and varying with respect to N and N_f : (Friedman eqs.)

$$\frac{\dot{a}^2}{N^2a^2} = \frac{K}{a^2} - \frac{1}{3M_{\rm P}^2}\left[-\frac{\dot{\phi}_{\rm g}^2}{2N^2} + \Lambda_{\rm g}\left(X,\phi_{\rm g}\right)\right]$$

$$\frac{\dot{b}^2}{N_{\rm f}^2b^2} = \frac{K_{\rm f}}{b^2} - \frac{1}{3M_{\rm f}^2}\left[-\frac{\dot{\phi}_{\rm f}^2}{2N_{\rm f}^2} + \Lambda_{\rm f}\left(X,\phi_{\rm f}\right)\right]$$

$$X \equiv b/a$$

$$\Lambda_{\rm g} \equiv M_{\rm P}^2\lambda_{\rm g}^{\rm eff} + m_g^2M_e^2\sum_{n=0}^{3}A_nX^n$$

$$\Lambda_{\rm f} \equiv M_{\rm f}^2\lambda_{\rm f}^{\rm eff} + m_g^2M_e^2\sum_{n=0}^{3}B_nX^{n-3}$$

$$\lambda_{\rm g/f}^{\rm eff} \equiv \lambda_{\rm g/f} + \frac{V_{\rm g/f}}{M_{\rm P/f}^2}$$

Combination of $\alpha_1 \sim \alpha_4$

- with respect to a and b

$$\ddot{a} = \frac{\dot{a}\dot{N}}{N} - \frac{aN^2}{3M_{\rm P}^2}\left(\frac{\dot{\phi}_{\rm g}^2}{N^2} + \Lambda_{\rm g}\right) + \frac{m_g^2M_e^2aN}{6M_{\rm P}^2}\sum_{n=0}^{3}\left[nNA_n + (n-3)\,N_{\rm f}B_n\right]X^n$$

$$\ddot{b} = \frac{\dot{b}\dot{N}_{\rm f}}{N_{\rm f}} - \frac{bN_{\rm f}^2}{3M_{\rm f}^2}\left(\frac{\dot{\phi}_{\rm f}^2}{N_{\rm f}^2} + \Lambda_{\rm f}\right) - \frac{m_g^2M_e^2bN_{\rm f}}{6M_{\rm f}^2}\sum_{n=0}^{3}\left[nNA_n + (n-3)\,N_{\rm f}B_n\right]X^{n-3}$$

- Taking derivative of Friedman eqs with respect to τ and combining with the second derivative eqs and scalar eqs:

$$\boxed{\left(\frac{\dot{b}}{\dot{a}} - N_{\rm f}\right) \sum_{n=0}^{3} (3-n) B_n \left(\frac{b}{a}\right)^n = 0}$$

Branch I

Branch II $\Rightarrow$ $\boxed{\dfrac{b}{a} = X_{\pm} = const}$

$$\boxed{\begin{aligned} &A_3 X^4 + X^3 \left\{ A_2 - \frac{M_{\rm P}^2}{M_{\rm f}^2} \left[B_3 + \frac{1}{m_g^2 M_e^2} \left(M_{\rm f}^2 \lambda_{\rm f}^{\rm eff}(\phi_{\rm f}) - \frac{\dot{\phi}_{\rm f}^2}{2N_{\rm f}^2} \right) \right] \right\} + X^2 \left(A_1 - 3A_3 \frac{M_{\rm P}^2}{M_{\rm f}^2} \right) \\ &+ X \left[A_0 - A_2 \frac{M_{\rm P}^2}{M_{\rm f}^2} + \frac{1}{m_g^2 M_e^2} \left(M_{\rm P}^2 \lambda_{\rm g}^{\rm eff}(\phi_{\rm g}) - \frac{\dot{\phi}_{\rm g}^2}{2} \right) \right] - \frac{A_1}{3} \frac{M_{\rm P}^2}{M_{\rm f}^2} = 0 \end{aligned}}$$

- Taking derivative of Friedman eqs with respect to τ and combining with the second derivative eqs and scalar eqs:

$$\left(\frac{\dot b}{\dot a} - N_{\rm f}\right)\sum_{n=0}^{3}(3-n)B_n\left(\frac{b}{a}\right)^n = 0$$

Branch I

Branch II

$$\frac{b}{a} = X_\pm = const$$

$$A_3X^4 + X^3\left\{A_2 - \frac{M_{\rm P}^2}{M_{\rm f}^2}\left[B_3 + \frac{1}{m_g^2M_e^2}\left(M_{\rm f}^2\lambda_{\rm f}^{\rm eff}(\phi_{\rm f}) - \cancel{\frac{\dot\phi_{\rm f}^2}{2N_{\rm f}^2}}\right)\right]\right\} + X^2\left(A_1 - 3A_3\frac{M_{\rm P}^2}{M_{\rm f}^2}\right)$$

$$+X\left[A_0 - A_2\frac{M_{\rm P}^2}{M_{\rm f}^2} + \frac{1}{m_g^2M_e^2}\left(M_{\rm P}^2\lambda_{\rm g}^{\rm eff}(\phi_{\rm g}) - \cancel{\frac{\dot\phi_{\rm g}^2}{2}}\right)\right] - \frac{A_1}{3}\frac{M_{\rm P}^2}{M_{\rm f}^2} = 0$$

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$$\left(\frac{\dot b}{\dot a} - N_{\rm f}\right)\sum_{n=0}^{3}(3-n)B_n\left(\frac{b}{a}\right)^n = 0$$

Branch I

Provided with $\dot\phi_{\rm g} \simeq \dot\phi_{\rm f} \simeq 0$, there exists $b/a = X = const$ if $A_1 A_3 > 0$

$$A_3 X^4 + X^3\left\{A_2 - \frac{M_{\rm P}^2}{M_{\rm f}^2}\left[B_3 + \frac{1}{m_g^2 M_e^2}\left(M_{\rm f}^2\lambda_{\rm f}^{\rm eff}(\phi_{\rm f}) - \cancel{\frac{\dot\phi_{\rm f}^2}{2N_{\rm f}^2}}\right)\right]\right\} + X^2\left(A_1 - 3A_3\frac{M_{\rm P}^2}{M_{\rm f}^2}\right)$$

$$+X\left[A_0 - A_2\frac{M_{\rm P}^2}{M_{\rm f}^2} + \frac{1}{m_g^2 M_e^2}\left(M_{\rm P}^2\lambda_{\rm g}^{\rm eff}(\phi_{\rm g}) - \cancel{\frac{\dot\phi_{\rm g}^2}{2}}\right)\right] - \frac{A_1}{3}\frac{M_{\rm P}^2}{M_{\rm f}^2} = 0$$

3. Hawking-Moss(HM) solutions

- HM solutions can be found at the local maximum of the potentials

$$a(\tau) = \sqrt{\frac{3M_{\mathrm{P}}^2}{\Lambda_{\mathrm{g}}}} \sin\left(\sqrt{\frac{\Lambda_{\mathrm{g}}}{3M_{\mathrm{P}}^2}}\tau\right)$$

$$b(\tau) = \sqrt{\frac{3M_{\mathrm{f}}^2}{\Lambda_{\mathrm{f}}}} \sin\left(\sqrt{\frac{\Lambda_{\mathrm{f}}}{3M_{\mathrm{f}}^2}} f(\tau)\right) \qquad \boxed{\dot{f}(\tau) \equiv N_{\mathrm{f}}}$$

- Inserting HM solutions into Euclidean action, one can estimate the tunneling probability. In the following, we consider the two branches and their reduction to dRGT massive gravity theory.

✦ Branch I

$$\boxed{N_{\mathrm{f}} \equiv \dot{f} = \frac{\dot{b}}{\dot{a}} = \frac{b}{a} = X = \frac{M_{\mathrm{f}}}{M_{\mathrm{P}}}\sqrt{\frac{\Lambda_{\mathrm{g}}}{\Lambda_{\mathrm{f}}}}}$$

implies that the bubble expansion in the fiducial metric side synchronizes with the one in the physical side

$$\begin{cases} a(\tau) = \sqrt{\dfrac{3M_{\mathrm{P}}^2}{\Lambda_{\mathrm{g}}}} \sin\left(\sqrt{\dfrac{\Lambda_{\mathrm{g}}}{3M_{\mathrm{P}}^2}}\;\tau\right), \\ b(\tau) = \sqrt{\dfrac{3M_{\mathrm{f}}^2}{\Lambda_{\mathrm{f}}}} \sin\left(\sqrt{\dfrac{\Lambda_{\mathrm{g}}}{3M_{\mathrm{P}}^2}}\;\tau\right). \end{cases}$$

- on-shell action:

$$\boxed{S_{\mathrm{E,\ HM}}^{\mathrm{branch-I}} = -24\pi^2 \left(\frac{M_{\mathrm{P}}^4}{\Lambda_{\mathrm{g,HM}}} + \frac{M_{\mathrm{f}}^4}{\Lambda_{\mathrm{f,HM}}} \right)}$$

looks like two copies of GR for dS

reason: (1). $b/a = X = const$ implies that the interaction term is constant;

(2). synchronization of two bubble expansion.

- on-shell action:

$$\boxed{S_{\mathrm{E,\ HM}}^{\mathrm{branch-I}} = -24\pi^2 \left(\frac{M_{\mathrm{P}}^4}{\Lambda_{\mathrm{g,HM}}} + \frac{M_{\mathrm{f}}^4}{\Lambda_{\mathrm{f,HM}}} \right)}$$

looks like two copies of GR for dS

reason: (1). $b/a = X = const$ implies that the interaction term is constant;

(2). synchronization of two bubble expansion.

$$\boxed{\text{dRGT limit:} \quad s \equiv M_{\mathrm{P}}^2 / M_{\mathrm{f}}^2 \ll 1}$$

Worry: $S_{\mathrm{E,\ HM}}^{\mathrm{branch-I}} \longrightarrow -\infty$ normalization or real divergence?

- A way to check: introducing a third metric $h_{\mu\nu}$

$$S' = S - S_h \, ,$$

$$S_h = \int d^4x\sqrt{-h}\left[\frac{M_{\rm h}^2}{2}\left(R_{\rm h}+\lambda_{\rm h}\right) - h^{\mu\nu}\partial_\mu\phi_{\rm h}\partial_\nu\phi_{\rm h} + V_{\rm h}(\phi_{\rm h})\right]$$

$$\Longrightarrow \quad S'^{\,\rm branch-I}_{\rm E,\ HM} = -24\pi^2\left(\frac{M_{\rm P}^4}{\Lambda_{\rm g,HM}} + \frac{M_{\rm f}^4}{\Lambda_{\rm f,HM}} - \frac{M_{\rm h}^2}{\lambda_{\rm h}}\right)$$

provided that $M_{\rm h}^2/\lambda_{\rm h} = M_{\rm f}^2/\lambda_{\rm f}$,

$$S'^{\,\rm branch-I}_{\rm E,\ HM} \longrightarrow S^{\rm branch-I}_{\rm E,\ HM,\ dRGT} = -24\pi^2\frac{M_{\rm P}^4}{\Lambda_{\rm g,HM}}$$

so it smoothly reduces to dRGT case

Branch II

No synchronization

on-shell action:

$$S_{\mathrm{E,\ HM}}^{\mathrm{branch-II}} = -24\pi^2 \left\{ \frac{M_{\mathrm{P}}^4}{\Lambda_{\mathrm{g},\pm}} + \frac{M_{\mathrm{f}}^4}{\Lambda_{\mathrm{f},\pm}} \left[1 - \left(1 - \frac{X_{\pm}^2 M_{\mathrm{P}}^2 \Lambda_{\mathrm{f},\pm}}{M_{\mathrm{f}}^2 \Lambda_{\mathrm{g},\pm}} \right)^{\frac{3}{2}} \right] \right\}$$

dRGT limit

$$S_{\mathrm{E,\ HM}}^{\mathrm{branch-II}} \longrightarrow S_{\mathrm{E,\ HM,\ dRGT}}^{\mathrm{branch-II}} - \frac{24\pi^2 M_{\mathrm{P}}^2}{\lambda_{\mathrm{f}}^{\mathrm{eff}} s} \left[1 - \left(1 - \alpha_{\mathrm{HM}}^2 \right)^{\frac{3}{2}} \right]$$

$$\alpha_{\mathrm{HM}}^2 \equiv \frac{M_{\mathrm{P}}^2 X_{\pm}^2 \lambda_{\mathrm{f}}^{\mathrm{eff}}(\phi_{\mathrm{HM}})}{\Lambda_{\mathrm{g},\pm}}$$

Branch II

No synchronization

on-shell action:

$$S_{\mathrm{E,\ HM}}^{\mathrm{branch-II}} = -24\pi^2 \left\{ \frac{M_{\mathrm{P}}^4}{\Lambda_{\mathrm{g},\pm}} + \frac{M_{\mathrm{f}}^4}{\Lambda_{\mathrm{f},\pm}} \left[1 - \left(1 - \frac{X_{\pm}^2 M_{\mathrm{P}}^2 \Lambda_{\mathrm{f},\pm}}{M_{\mathrm{f}}^2 \Lambda_{\mathrm{g},\pm}} \right)^{\frac{3}{2}} \right] \right\}$$

dRGT limit

$$S_{\mathrm{E,\ HM}}^{\mathrm{branch-II}} \longrightarrow S_{\mathrm{E,\ HM,\ dRGT}}^{\mathrm{branch-II}} - \frac{24\pi^2 M_{\mathrm{P}}^2}{\lambda_{\mathrm{f}}^{\mathrm{eff}} s} \left[1 \boxed{- \left(1 - \alpha_{\mathrm{HM}}^2 \right)^{\frac{3}{2}}} \right]$$

Cannot be normalized !

$$\alpha_{\mathrm{HM}}^2 \equiv \frac{M_{\mathrm{P}}^2 X_{\pm}^2 \lambda_{\mathrm{f}}^{\mathrm{eff}}(\phi_{\mathrm{f,HM}})}{\Lambda_{\mathrm{g},\pm}}$$

Exponentially suppressed in dRGT limit.

Summary and future propects

- We calculated the HM instantons for the bigravity theory;
- Branch I behaves like two copies of GR sectors. Upon a normalization factor, it smoothly reduces to dRGT case;
- In branch II, bubble expansions on both sides does not synchronize with each other. Its dRGT limit is exponentially suppressed;
- It is natural to extend the analysis to Coleman-deLuccia instantons.