8 Sep. 2014
workshop on “Relativistic cosmology”
@YITP

Non-linear mode-coupling of large-scale structure

in *non-relativistic* cosmology

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What we did (or are doing now)

We measured, for the first time, the mode coupling kernel of large-scale structure (LSS) from cosmological N-body simulations:

Mode coupling kernel

$$\delta P^{\rm nl}(k) = \int d\ln q \, \boxed{K(k,q)} \, \delta P^{\rm lin}(q)$$

Comparing it with perturbation theory (PT), we found

✓ Kernel is generally UV-suppressed, in contrast with PT prediction

✓ Discrepancy with PT prediction appears even at low-k, where PT works very well

➡ May help to understand or improve theoretical treatment of LSS

Large-scale structure (LSS)

Spatial inhomogeneity of <u>mass distribution</u> at 1~10^3 Mpc

dark matter + baryon (galaxies)

- Traditionally probed by galaxy redshift surveys
- Plays a crucial role to pin down the nature of gravity or dark energy through the measurement of

✓ baryon acoustic oscillation : cosmic expansion

DA(z), H(z)

✓ redshift-space distortions : growth of structure

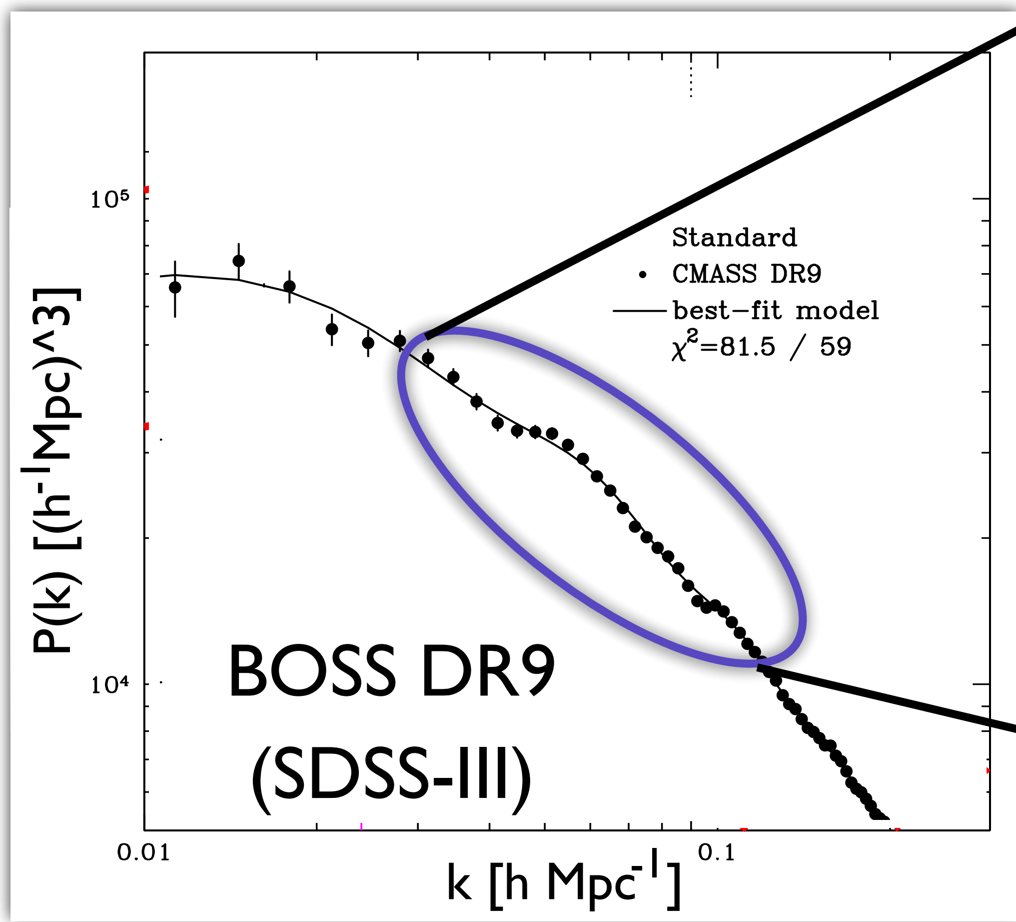
f(z)=dln D+(z)/dln a

imprinted on power spectrum and correlation function

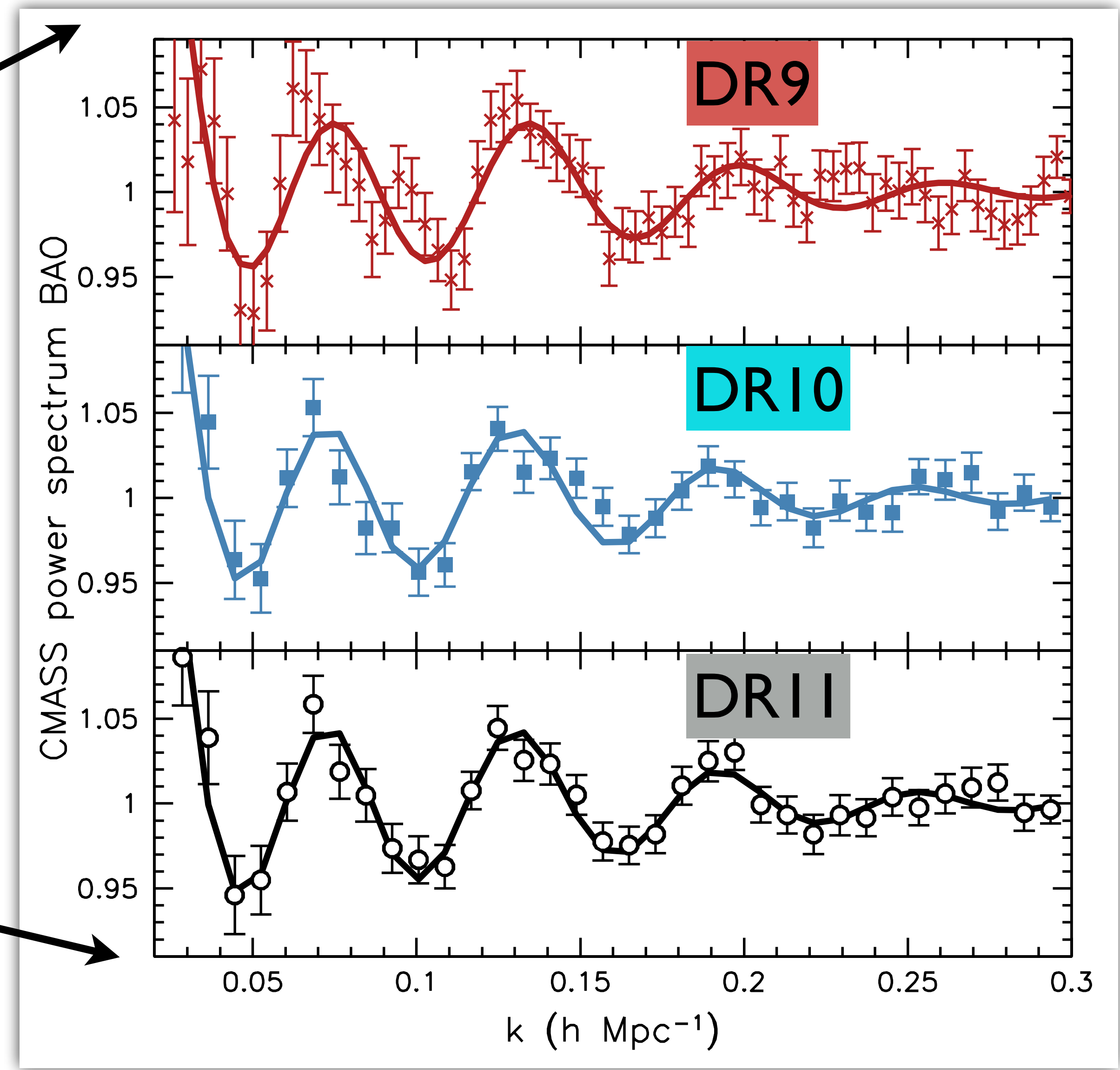
Baryon acoustic oscillations

Wiggle structure seen in the (angle-averaged) power spectrum

Anderson et al. ('12)



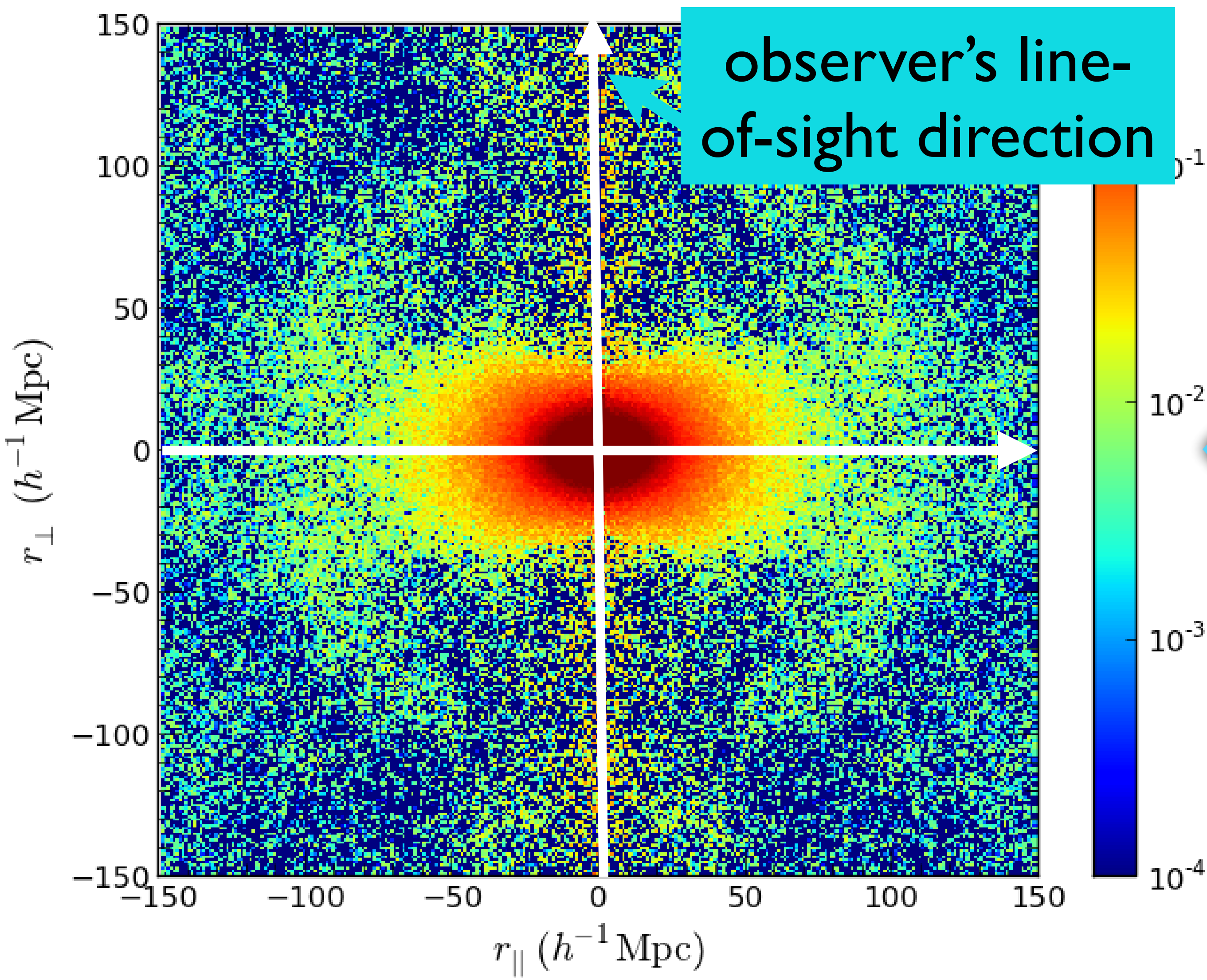
Anderson et al. ('13)



Redshift-space distortions

Anisotropies seen in the correlation function is caused by *redshift-space distortion*

Samushia et al.('13)



real space

$$\vec{\mathbf{s}} = \vec{\mathbf{r}} + \frac{(\vec{\mathbf{v}}\cdot\hat{\mathbf{z}})}{a\,H(z)}\,\hat{\mathbf{z}}\ ;$$

redshift space

$\mathbf{v}$: peculiar velocity

$\hat{\mathbf{z}}$: observer's line-of-sight direction

$\xi(r_\perp, r_\parallel)$

BOSS DR11, CMASS samples

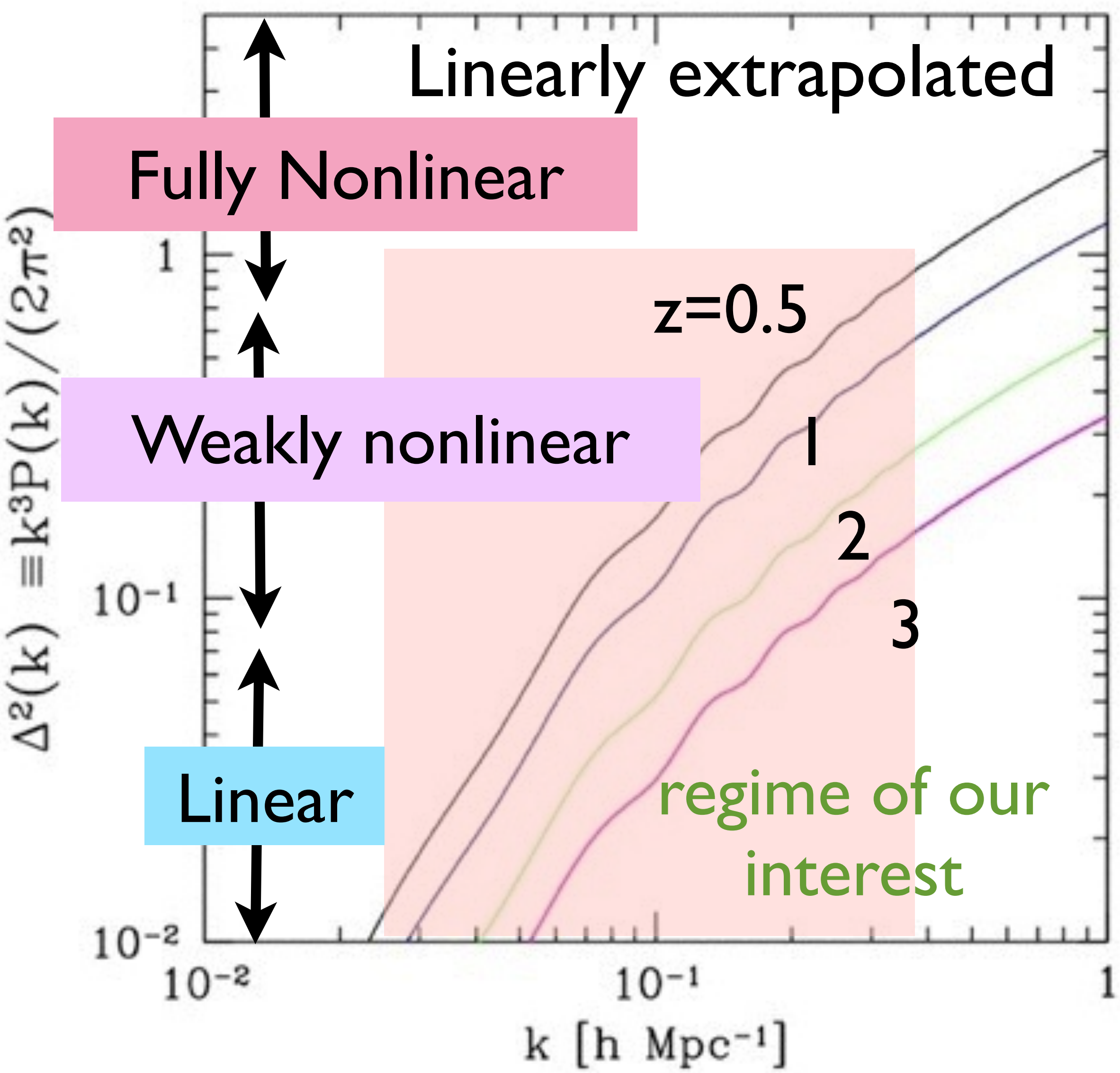
700,000gals @ 0.43<z<0.7

Role of perturbation theory

An accurate template of power spectrum/correlation function is needed for precision measurements

including nonlinear effects on

gravitational evolution/**redshift-space distortions**/**galaxy biasing**



Since what we want to measure basically lie at quasi-linear scales,

perturbation theory treatment can work very well (in principle)

In the rest of my talk,

I will focus on nonlinear gravitational evolution of LSS

Perturbation theory of LSS

LSS ＝pressureless & irrotational fluid

（CDM＋baryon）

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Makino, Sasaki & Suto ('92), ...

Basic eqs.

$$\frac{\partial\delta}{\partial t}+\frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right]=0$$

$$\frac{\partial\vec{v}}{\partial t}+\frac{\dot{a}}{a}\vec{v}+\frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v}=-\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi=4\pi G\,\bar{\rho}_m\,\delta$$

Single-stream approximation of collision less Boltzmann eq.

Bernardeau et al. Phys.Rep.367 ('02) 1

Standard PT

Regarding linear fluctuation $|\delta_0|\ll 1$ as the small expansion parameter : $\delta=\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\cdots$

Power spectrum calculation

power spectrum $\langle \delta(\boldsymbol{k};t)\delta(\boldsymbol{k}';t)\rangle = (2\pi)^3\,\delta_{\rm D}(\boldsymbol{k}+\boldsymbol{k}')\,P(|\boldsymbol{k}|;t)$

Average over initial fluctuation

For Gaussian initial condition for δ_0

$P^{(mn)} \simeq \langle \delta^{(m)}\delta^{(n)}\rangle$

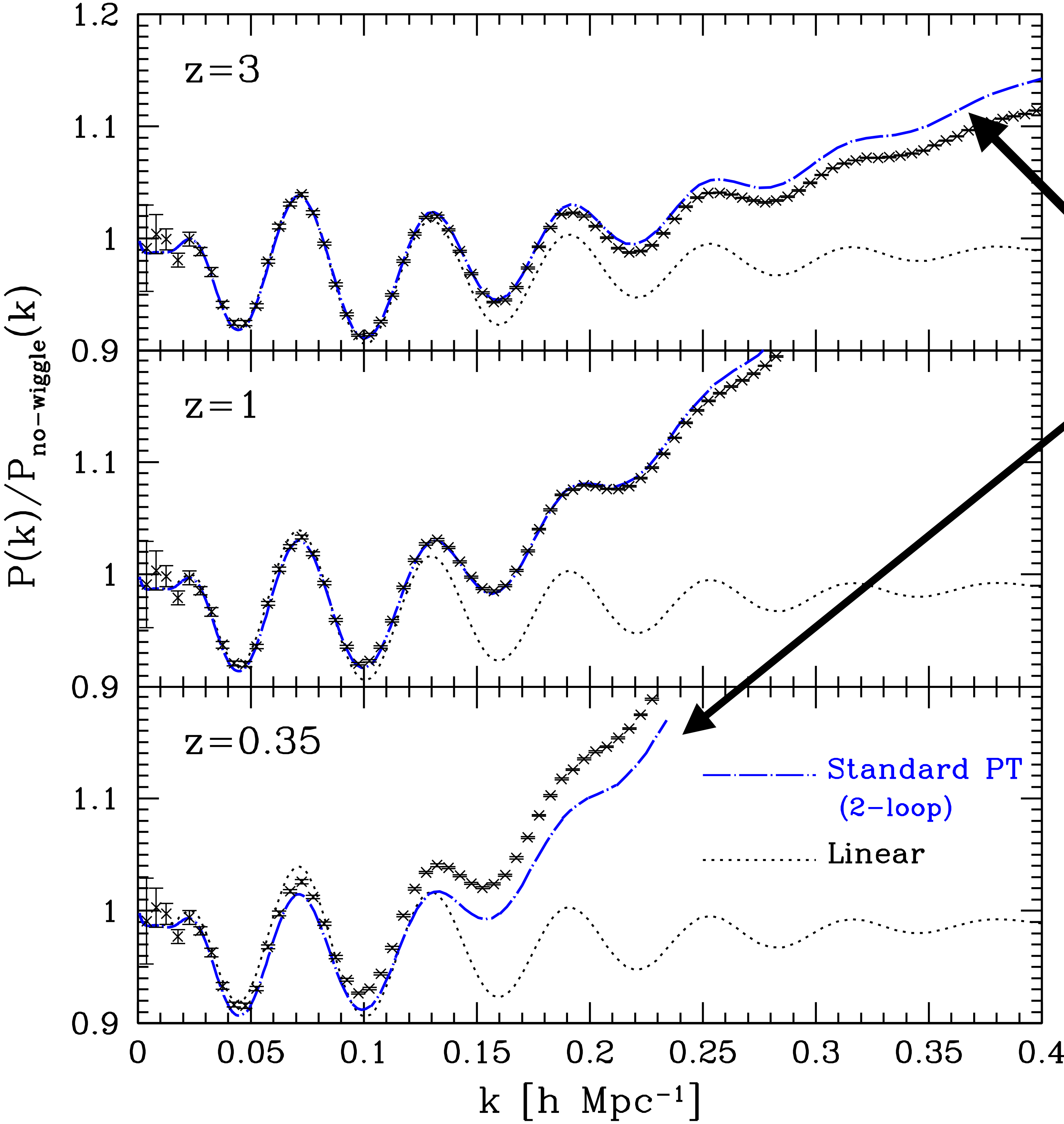
$$P(k) = \underbrace{P^{(11)}(k)}_{\text{Linear (tree)}} + \underbrace{\left(\boxed{P^{(22)}(k)} + P^{(13)}(k)\right)}_{\text{1-loop}} + \underbrace{\left(P^{(33)}(k) + \boxed{P^{(24)}(k)} + P^{(15)}(k)\right)}_{\text{2-loop}} + \cdots$$

$$P^{(22)}(k) = 2\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\,\mathcal{F}^{(2)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\mathcal{F}^{(2)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\,P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{q}|),$$

$$P^{(24)}(k) = 24\int \frac{d^3\boldsymbol{p}\,d^3\boldsymbol{q}}{(2\pi)^6}\,\mathcal{F}^{(4)}(\boldsymbol{p},\boldsymbol{q},-\boldsymbol{q},\boldsymbol{k}-\boldsymbol{p})\mathcal{F}^{(2)}(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p})\,\underbrace{P_0(p)}_{\text{linear power spectrum}}P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{p}|),$$

$\mathcal{F}^{(n)}$: kernel of n-th order PT solution （non-linear mode coupling）

Standard PT vs simulations



Standard PT results up to 2-loop order

overshoot at high-z

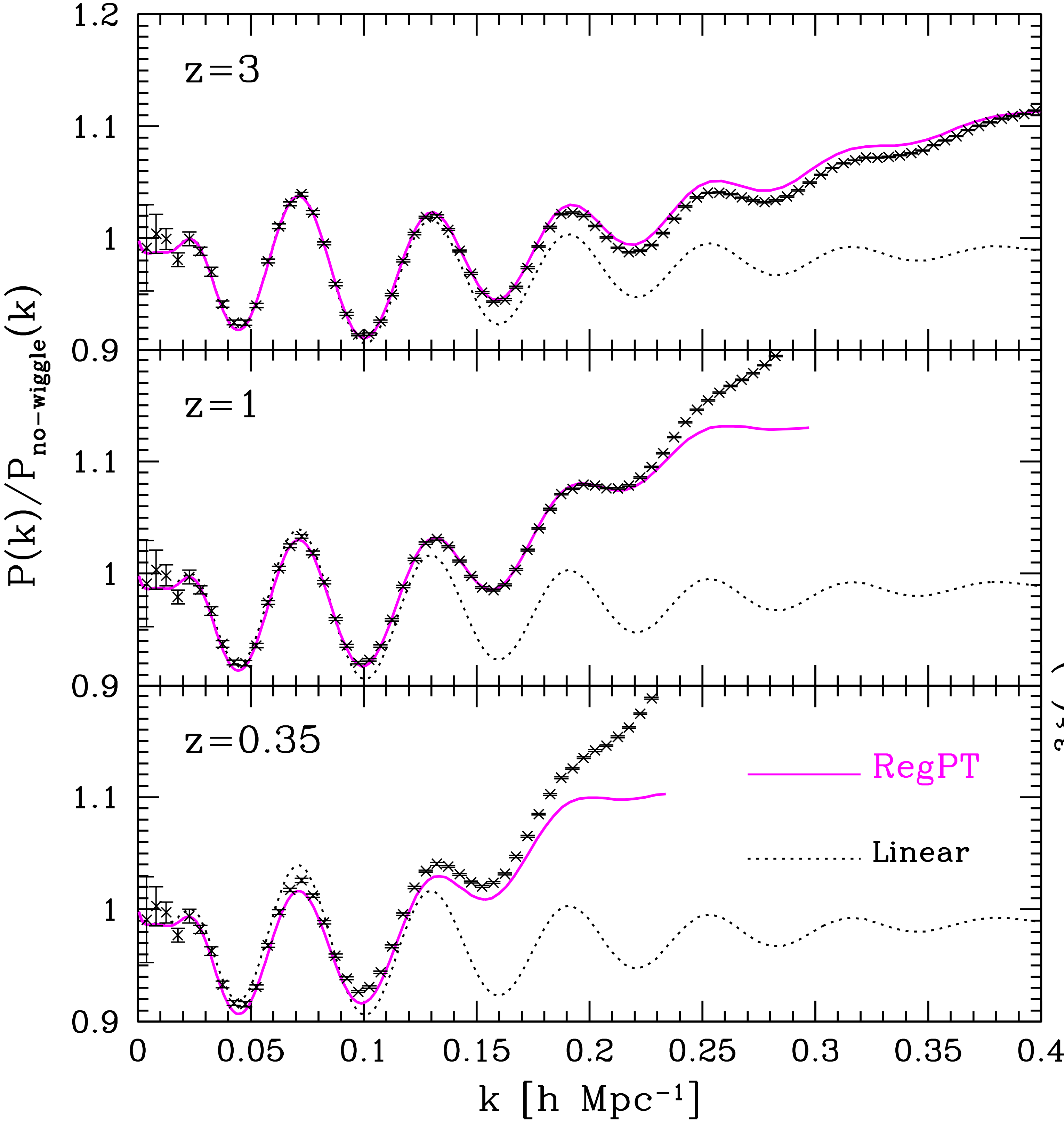
underestimate at low-z

Higher-loop corrections do not always help to improve the prediction

→ reorganize PT expansion

(e.g., Multi-point propagators
Bernardau et al. ('08,'10, '11)
AT et al. ('12, '14))

Resumed PT vs simulations

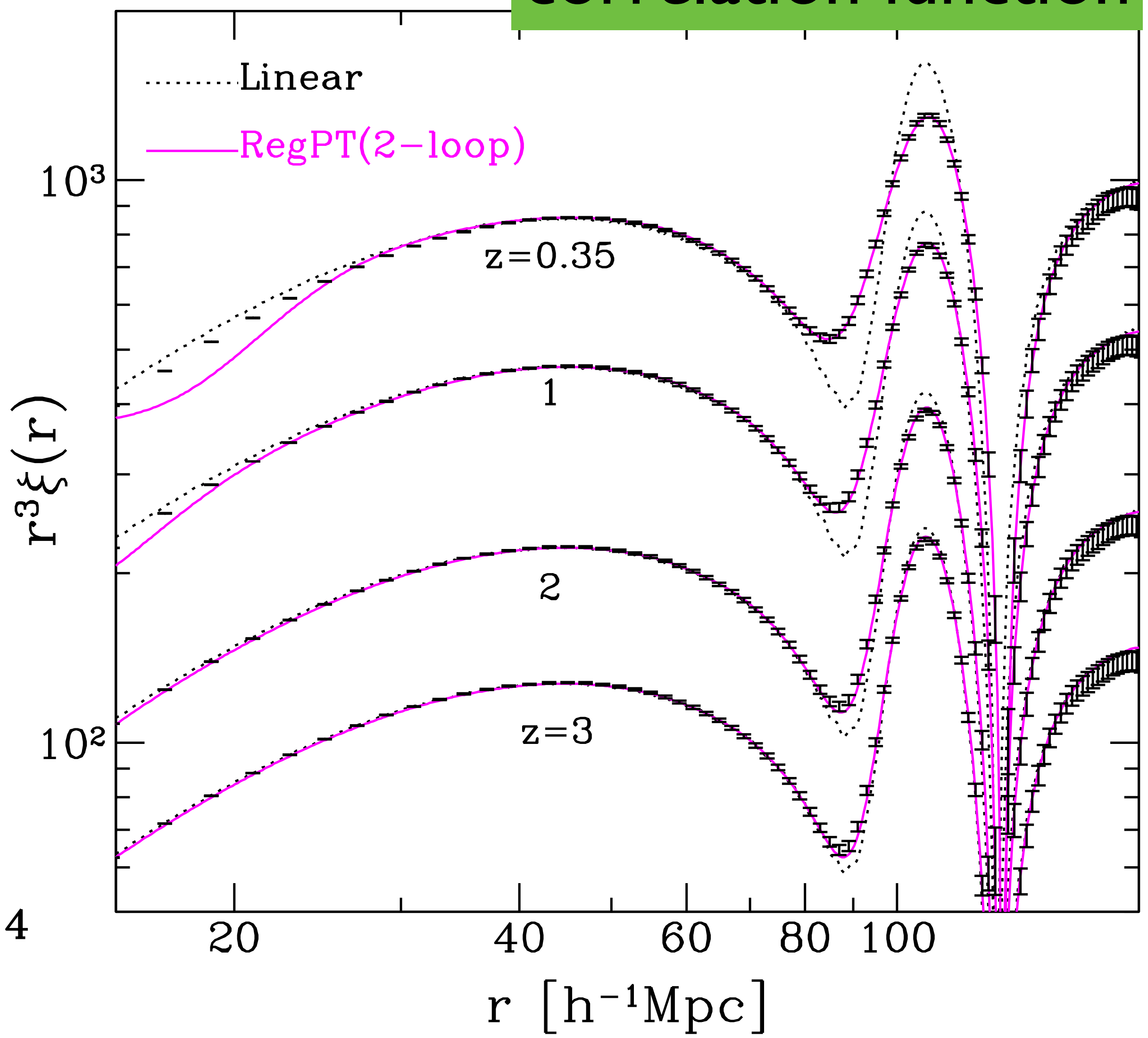


RegPT 2-loop

regularized multi-point propagator expansion

(AT et al. '12)

correlation function



Curse of UV divergence

Improved PT： consistently reproduce standard PT at low-k

But,

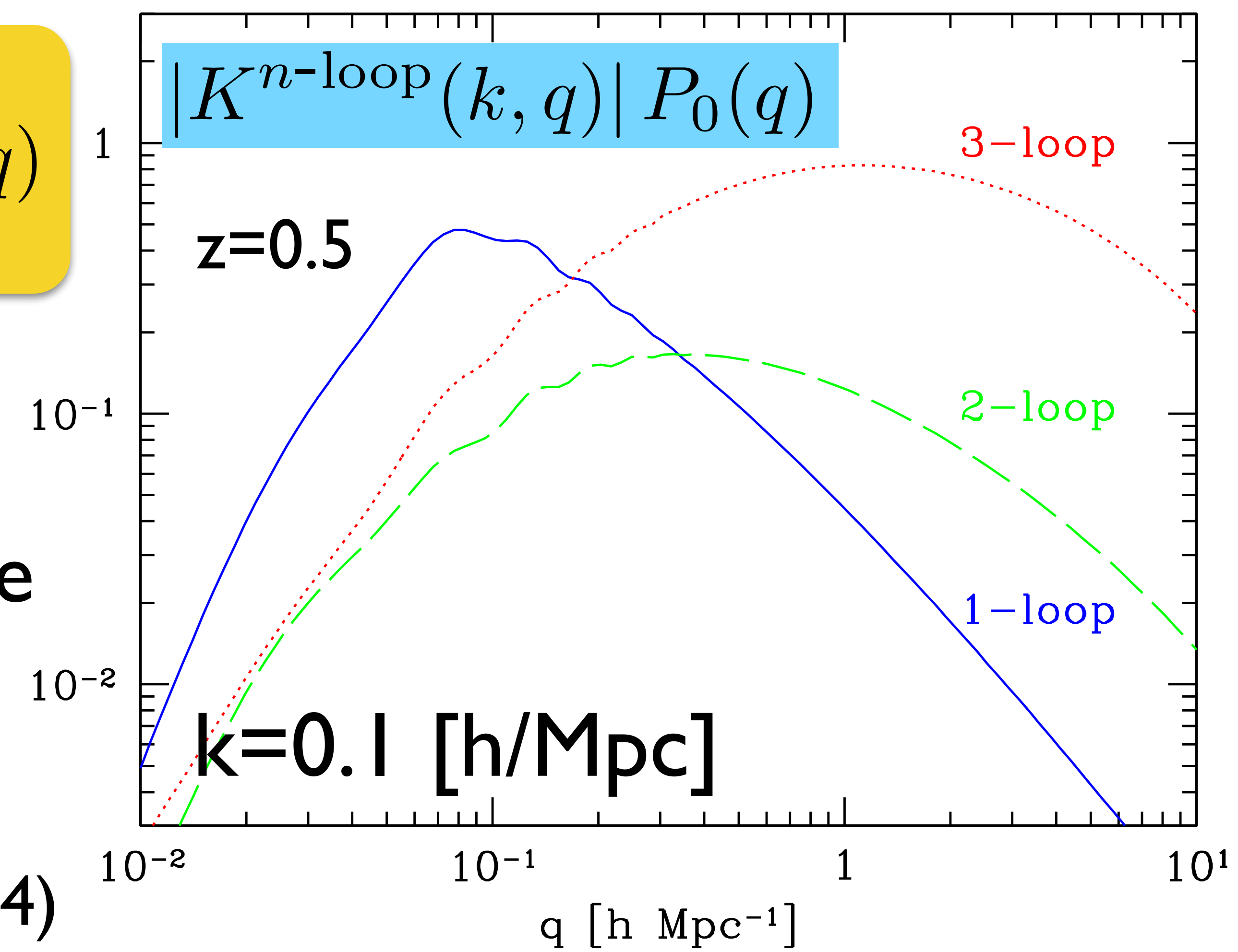
Q Is standard PT calculation really trustable at low-k ?

Bad UV behavior of mode-coupling kernel

mode-coupling kernel

$$P^{n\text{-loop}}(k) \propto \int d\ln q\, K^{n\text{-loop}}(k,q)\, P_0(q)$$

A part of power spectrum correction

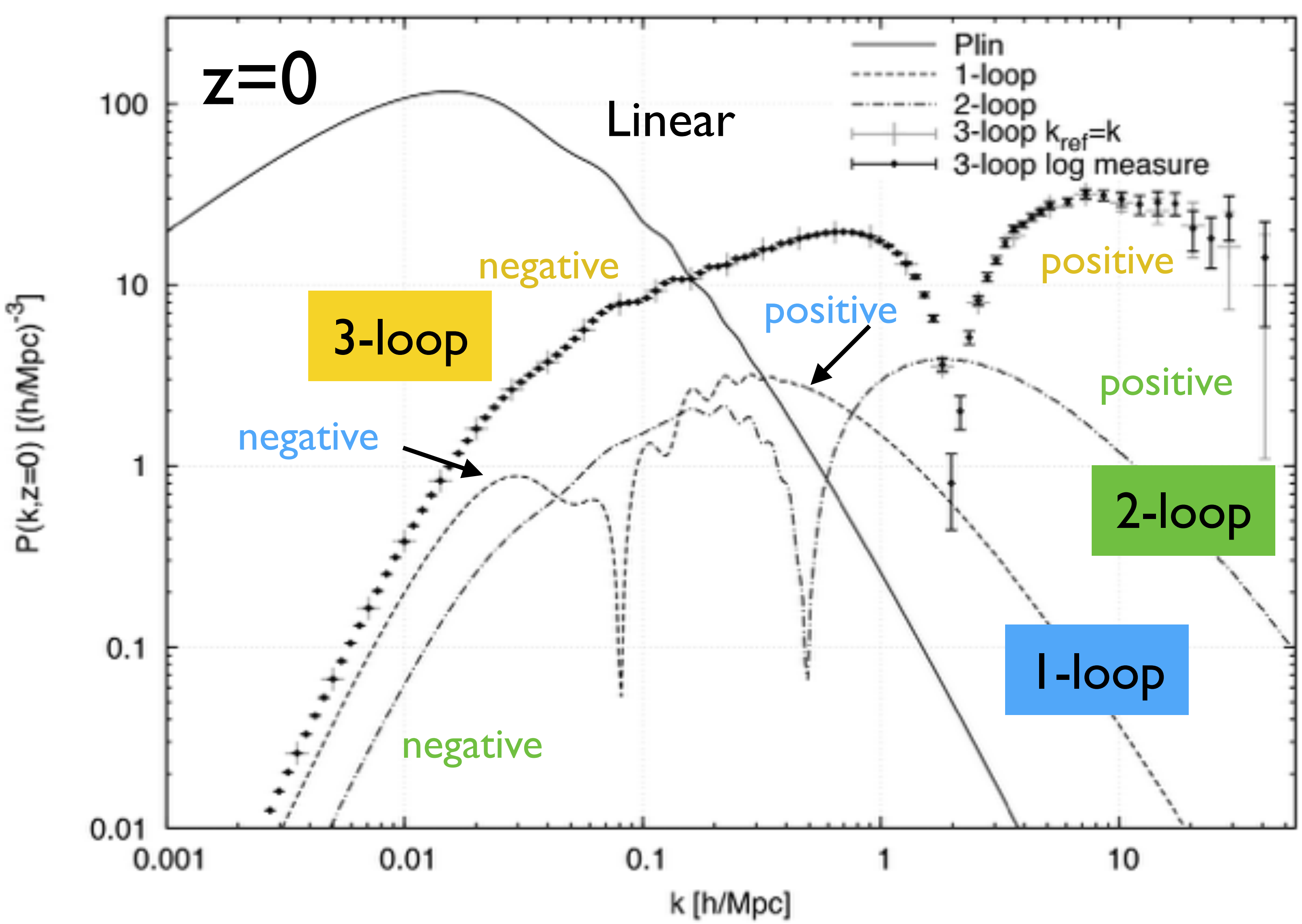


PT result at low-k gets a significantly large contribution at very high-*q*, where PT treatment cannot be applied

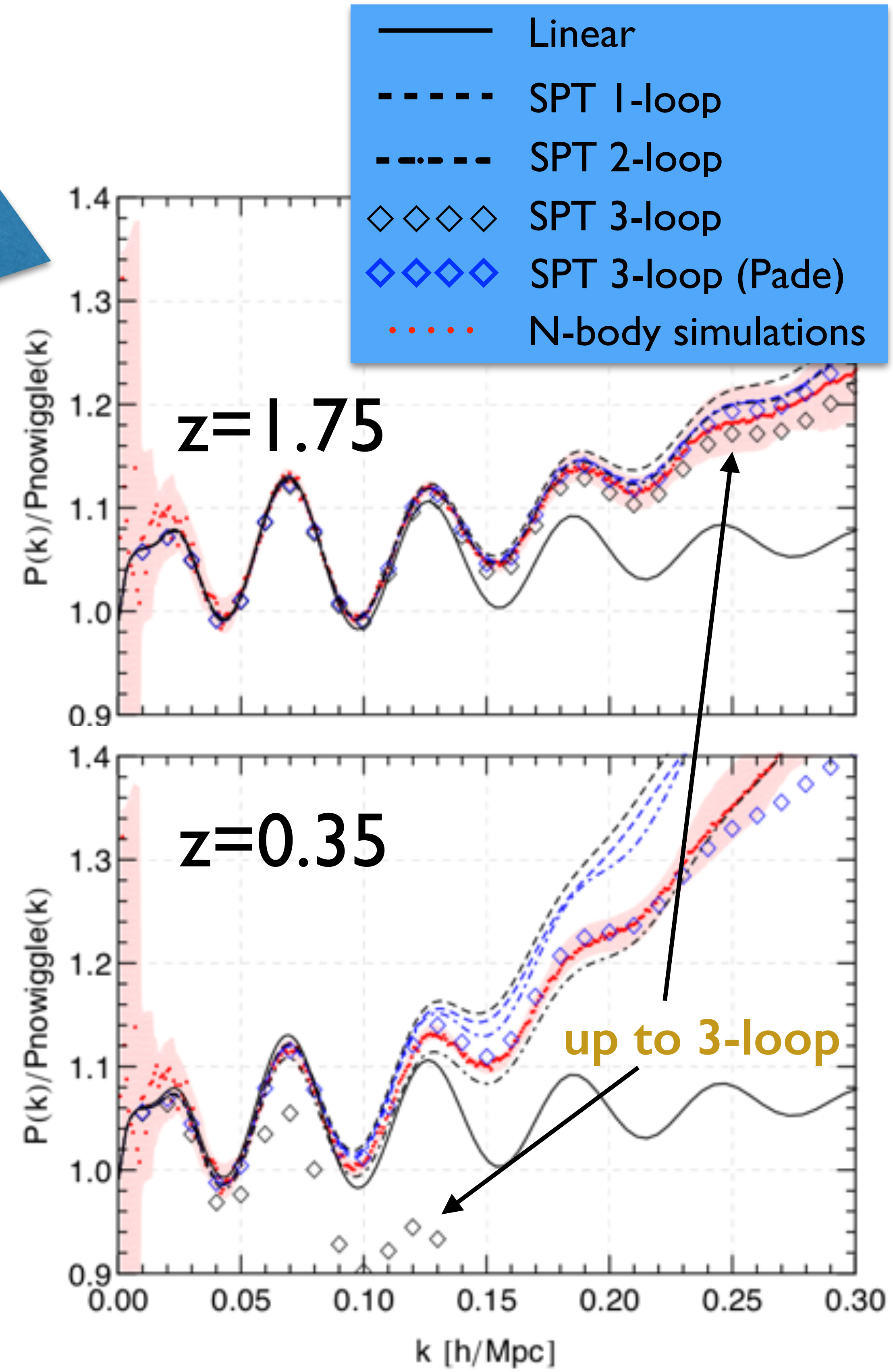
Bernardeau, AT & Nishimichi ('14)

Standard PT up to 3-loop order

Blas et al. JCAP 01 ('14) 010



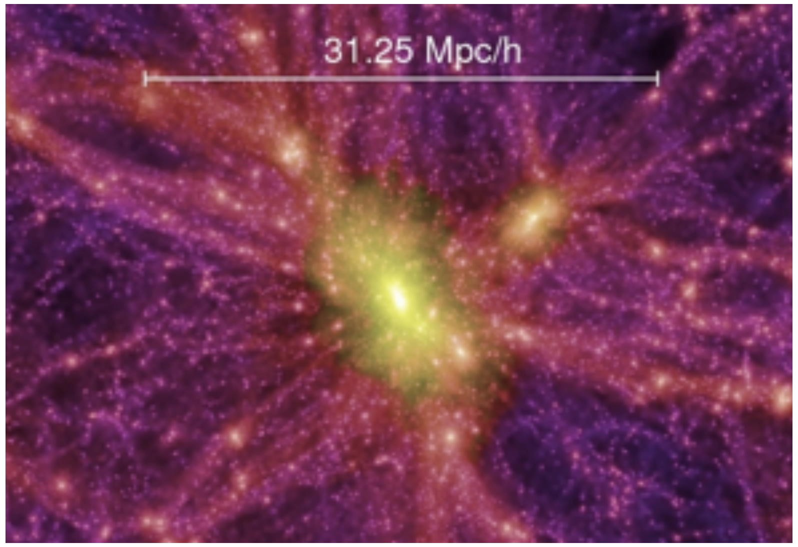
ratio of power spectrum



Need an effective theory ?

Due to the UV divergence,

- Break down of PT calculation even at $k \to 0$?
- Need to reconsider PT formulation ?



Fluid treatment needs to be regularized or reformulated, taking account of small-scale physics (halo formation or virialization)

Effective field theory of large-scale structure (EFTofLSS)

Baumann et al. ('12), Carrasco, Herzberg & Senatore ('12), Carrasco et al. ('13ab), Porto, Senatore & Zaldarriaga ('14), …

Phenomenologically introduce viscousity & anisotropic stress to characterize deviations from pressureless & irrotational fluid

…these are calibrated only with N-body simulation

Question

Quantitatively, how much the UV contribution can affect the power spectrum at low-k ?

We numerically measure the *mode-coupling kernel* from N-body simulations, and compare it with PT calculation

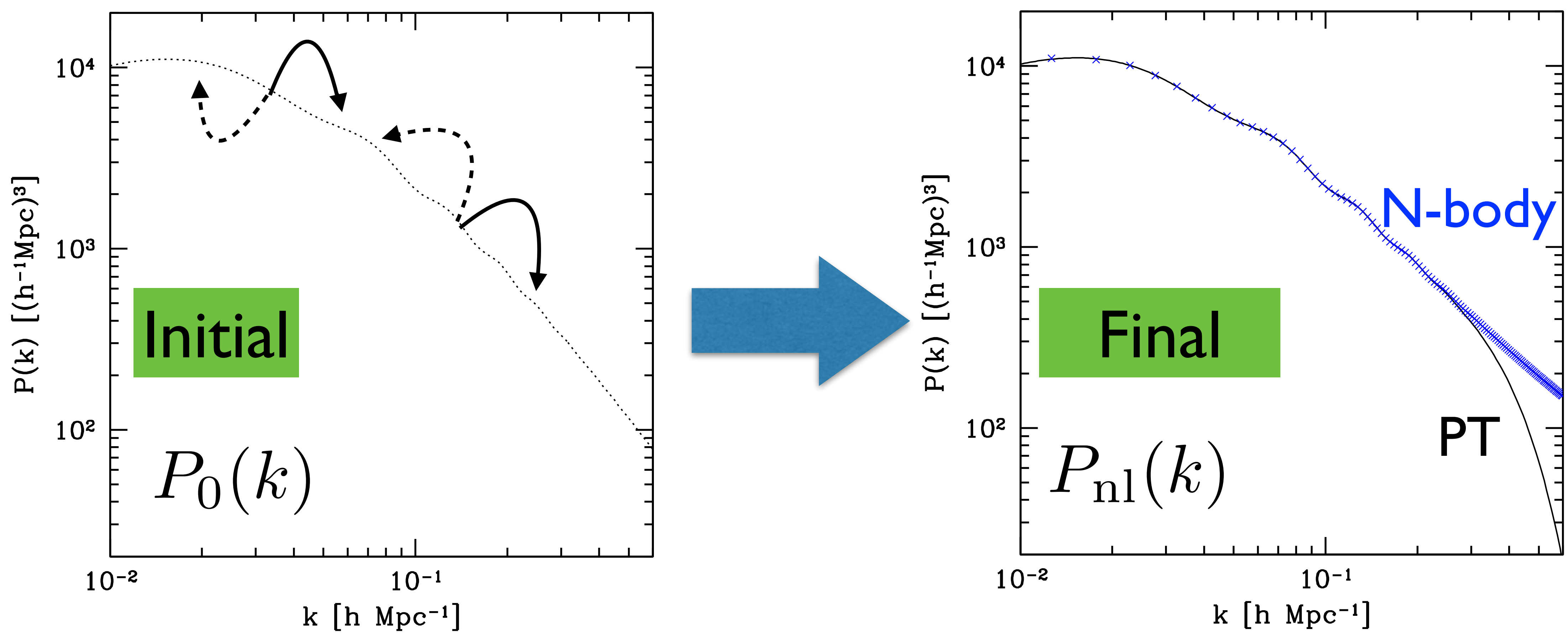
But,

the mode-coupling kernel I mentioned so far is somewhat ambiguous, and needs to be properly defined

→ We shall start defining mode-coupling kernel, which is suited to measure N-body simulation, and can be computed with PT

Mode-coupling kernel

How the power of each Fourier mode in initial power spectrum is mapped into each mode of final power spectrum through the non-linear gravitational evolution ?

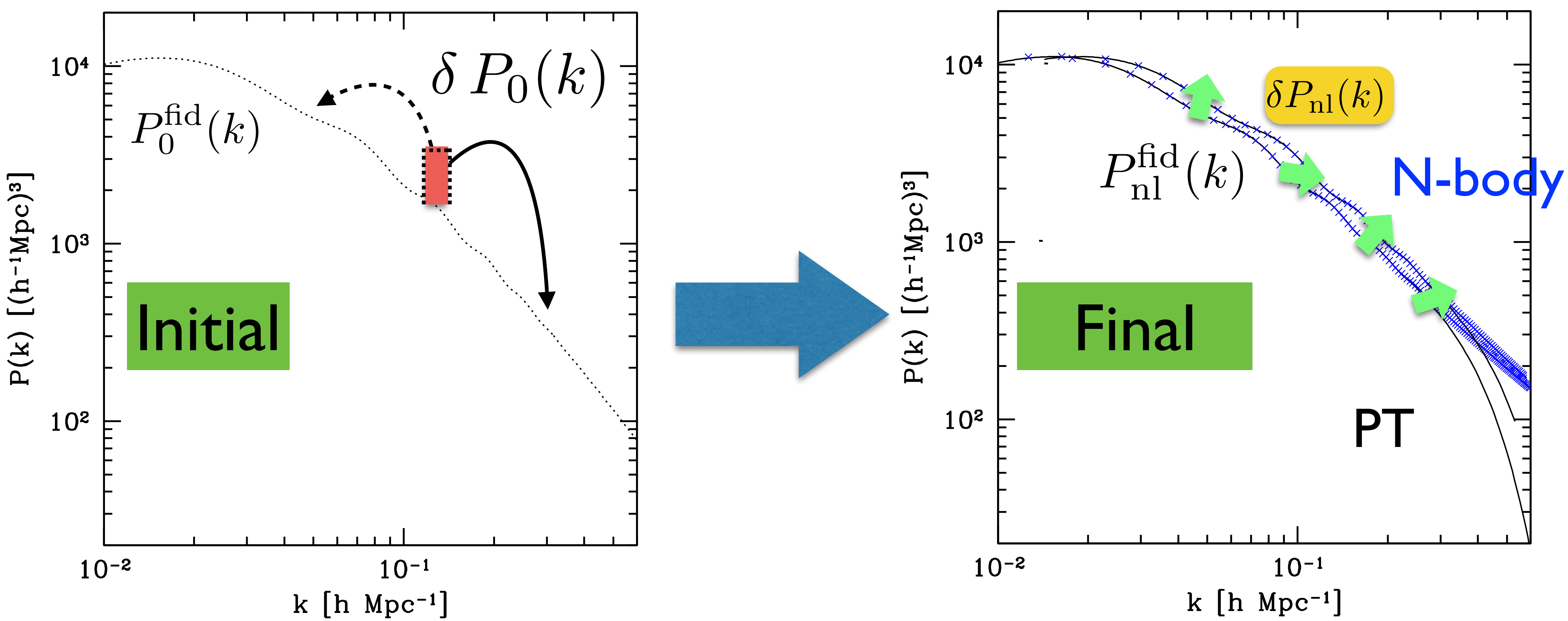


$$P_{\rm nl}(k) = \int d\ln q \; J(k,q) \; P_0(q)$$

this is hard to measure

Mode-coupling kernel

How the small disturbance added in initial power spectrum can contribute to each Fourier mode in final power spectrum through the non-linear gravitational evolution ?



$$P_0(k) \to P_0^{\rm fid}(k) + \delta\, P_0(k)$$

Measure from N-body simulation

$$P_{\rm nl}(k) = P_{\rm nl}^{\rm fid}(k) + \delta P_{\rm nl}(k)\ ; \quad \delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$

Measurement of kernel

Definition in terms of functional derivative :

$$K(k,q) = q\,\frac{\delta P_{\rm nl}(k)}{\delta P_0(q)}$$

Estimator for mode-coupling kernel (discretized):

$$\widehat{K}(k_i,q_j)\,P_0(q_j) \equiv \frac{P_{\rm nl}^{+}(k_i) - P_{\rm nl}^{-}(k_i)}{\Delta \ln P_0\,\Delta \ln q} \quad ; \quad \Delta \ln q = \ln q_{j+1} - \ln q_j$$

$P_{\rm nl}^{\pm}(k)$: Final output of non-linear power spectrum, for which a small perturbation $P_{0,j}^{\pm}(k)$ is added in initial power spectrum, $P_0(k)$

$$\ln\left[\frac{P_{0,j}^{\pm}(q)}{\ln P_0(q)}\right] = \begin{cases} \pm\frac{1}{2}\Delta \ln P_0 & ; \quad q_j \leq q < q_{j+1} \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Measurement of kernel

- initial power spectrum $P_0(k)$ ： ΛCDM by wmap5
- initial perturbation ($\Delta \ln P_0$) ： 1% of $P_0(k)$
- divide k=0.006~0.12 [h/Mpc] into logarithmic15 (or 13)-bins ：

initial k-bin ： $q_1 = 0.006\, h\, \mathrm{Mpc}^{-1}$ (or $q_1 = 0.012\, h\, \mathrm{Mpc}^{-1}$)

width of k-bin ： $\Delta \ln q = \ln(\sqrt{2})$

Run many simulations…

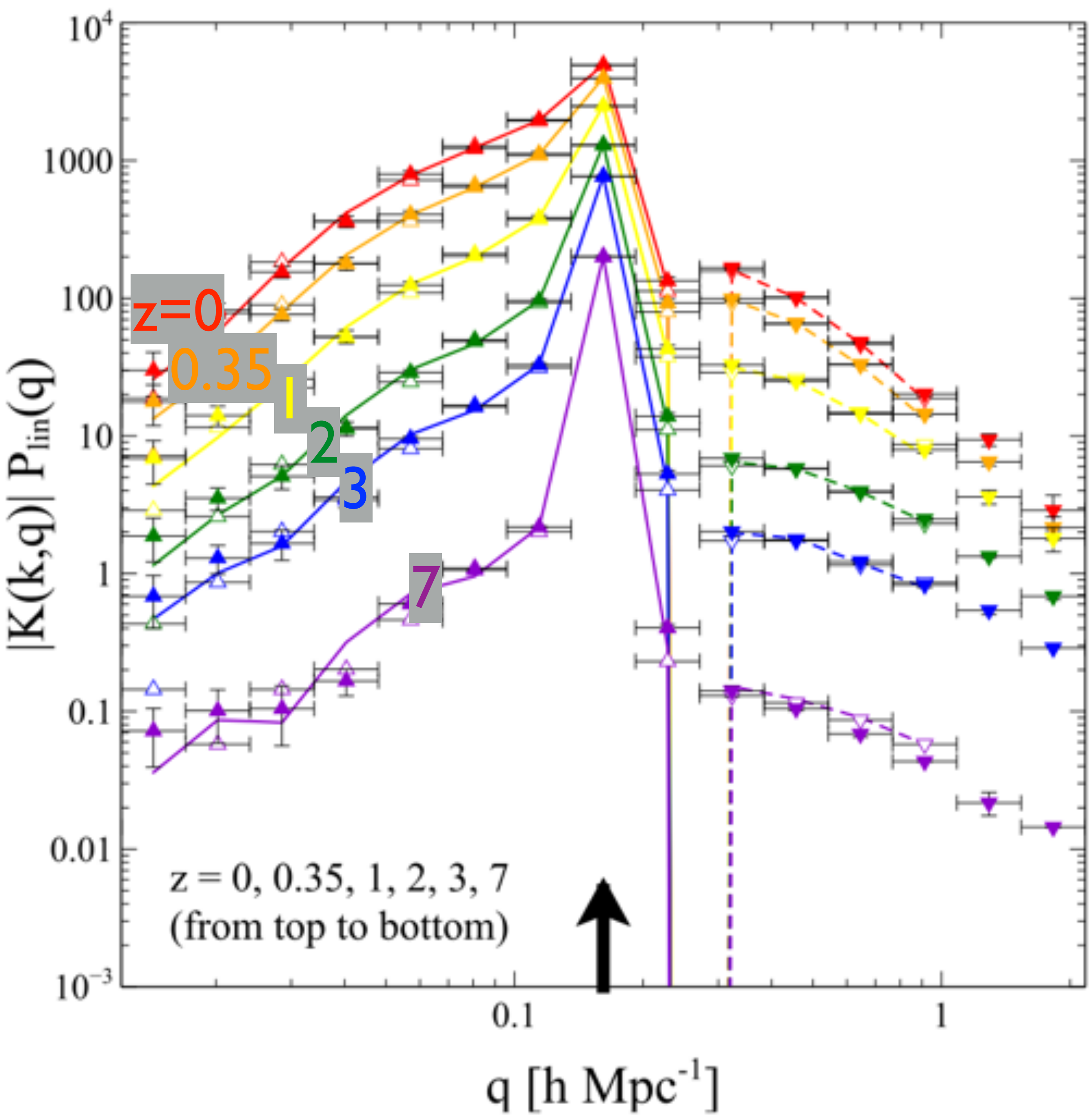
T.Nishimishi

TABLE I: Simulation parameters. Box sizes are in unit of h^{-1}Mpc.

name	box	particles	start-z	bins	runs	total
L9-N9	512	512^3	31	15	4	120
L9-N8	512	256^3	15	13	4	104
L10-N9	1024	512^3	31	15	1	30

Measured results of kernel

mode-coupling kernel measured at k=0.162 [h/Mpc]



$$K(k,q) = q\,\frac{\delta P_{\rm nl}(k)}{\delta P_0(q)}$$

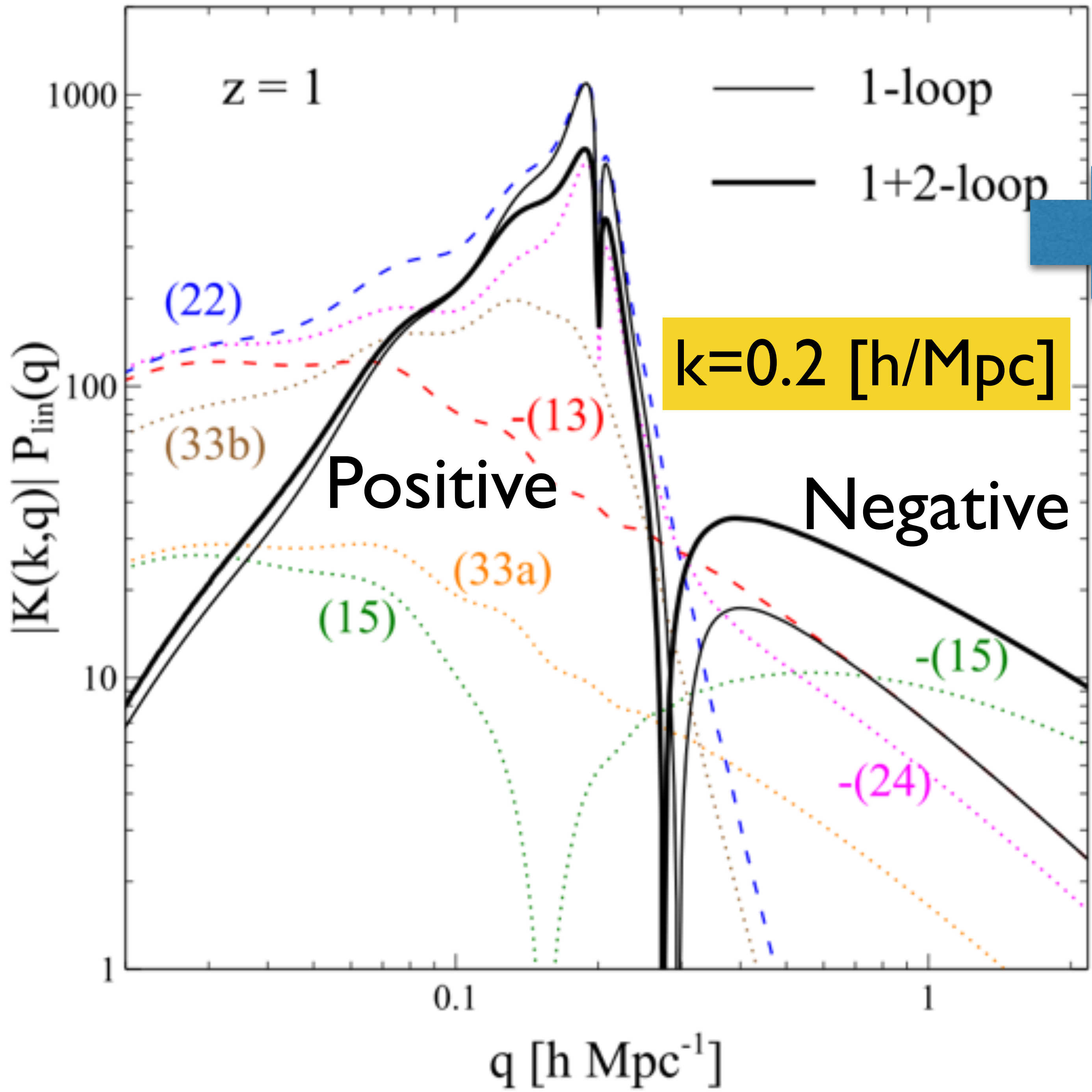
▲ or —— : positive

▼ or ------ : negative

FIG. 1: Kernel function measured from simulations. We plot $|K(k,q)|P_{\rm lin}(q)$ as a function of initial wavenumber q for a fixed value of final wavenumber k indicated by the vertical arrow in the panels. Filled (open) symbols show the measurement from L9-N9 (L10-N9), while lines depict L9-N8. Positive values are shown by upper triangle or solid line, while lower triangles and dashed line show negative contribution.

PT result of kernel

Note—. delta-function contribution removed



For a proper comparison with N-body results, we take a weighted average in each k-bin

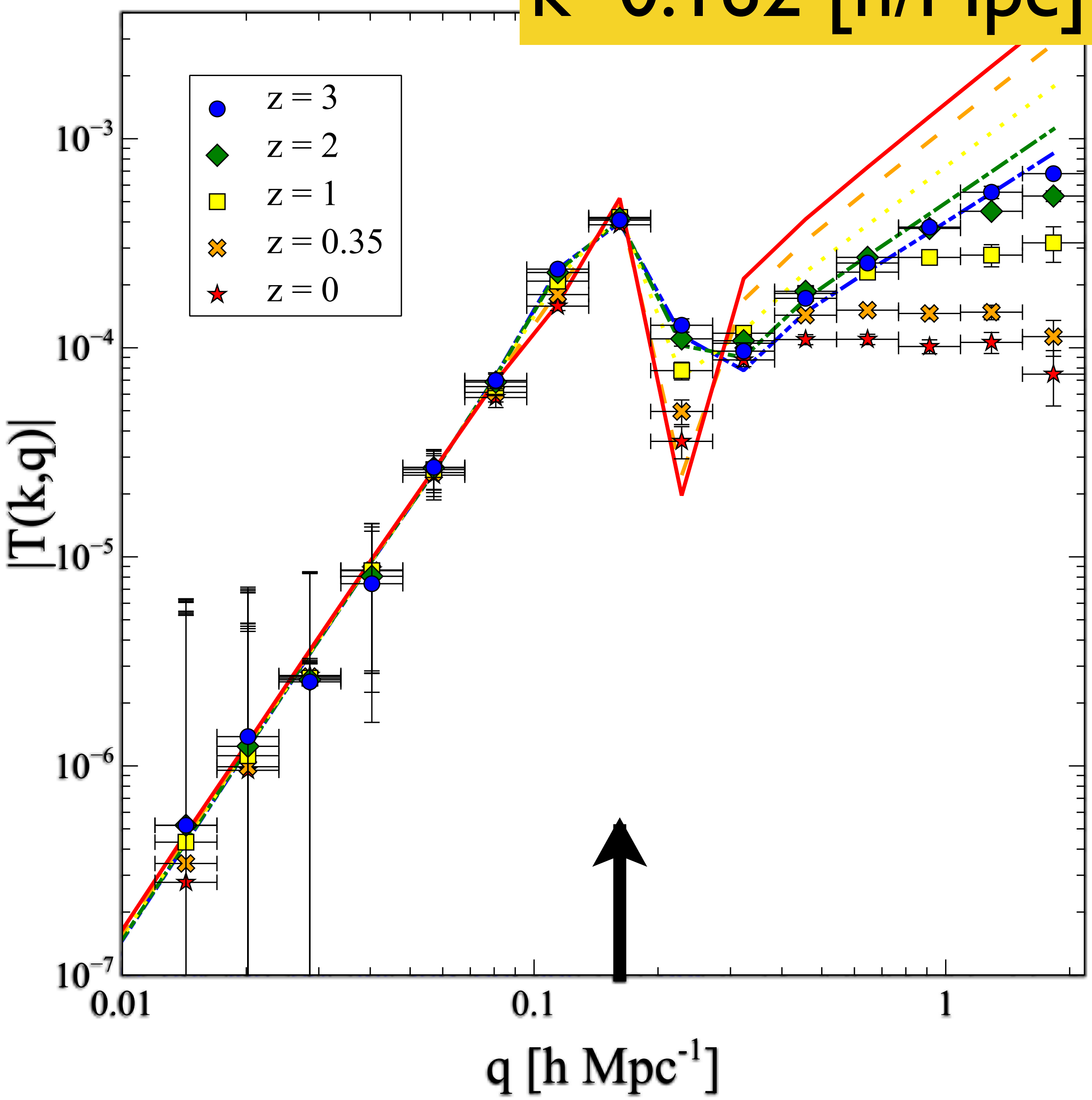
Taking also account of the delta-function contribution

FIG. 2: Kernel function as predicted by PT calculations up to one- (thin solid) and two-loop (thick solid) order computed for $k = 0.2h$/Mpc at $z = 1$. Dashed (dotted) lines show each of the one- (two-)loop contributions with the legend (ij) showing the perturbative order of the calculation. The legend has a negative sign when the kernel is negative. Note that we ignore terms proportional to the Dirac delta at $k = q$, which is meaningful only when we take a certain binning scheme.

PT vs N-body simulation

Normalized kernel $T(k,q) = K(k,q)/P^{\rm lin}(k)$

k=0.162 [h/Mpc]



Lines：Standard PT 2-loop

q<k ： reproduce simulation well

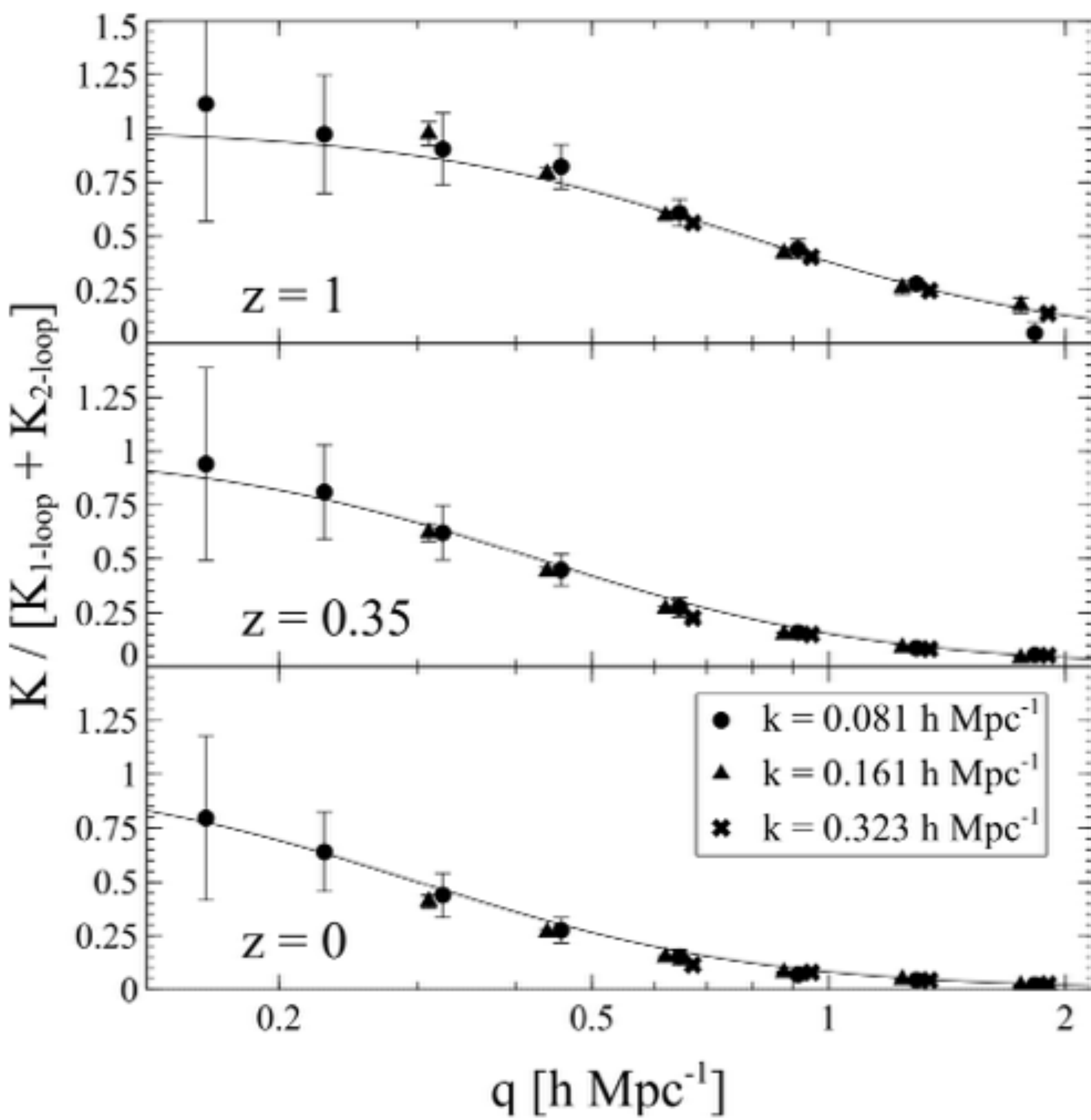
q>k ：discrepancy is manifest (particularly large at low-z)

That is,

UV contribution is suppressed

Characterizing UV suppression

UV suppression is seen at various k & q



ratio of measured kernel to PT prediction

Fitting formula

$$q_0(z) = 0.3/D_+^2(z)\ [h\,\mathrm{Mpc}^{-1}]$$

$$K_{\rm eff}(k,q) = \left[K^{1\text{-loop}}(k,q) + K^{1\text{-loop}}(k,q)\right]\frac{1}{1+(q/q_0)^2}$$

$K^{1\text{-loop}},\ K^{1\text{-loop}}$ Standard PT kernel

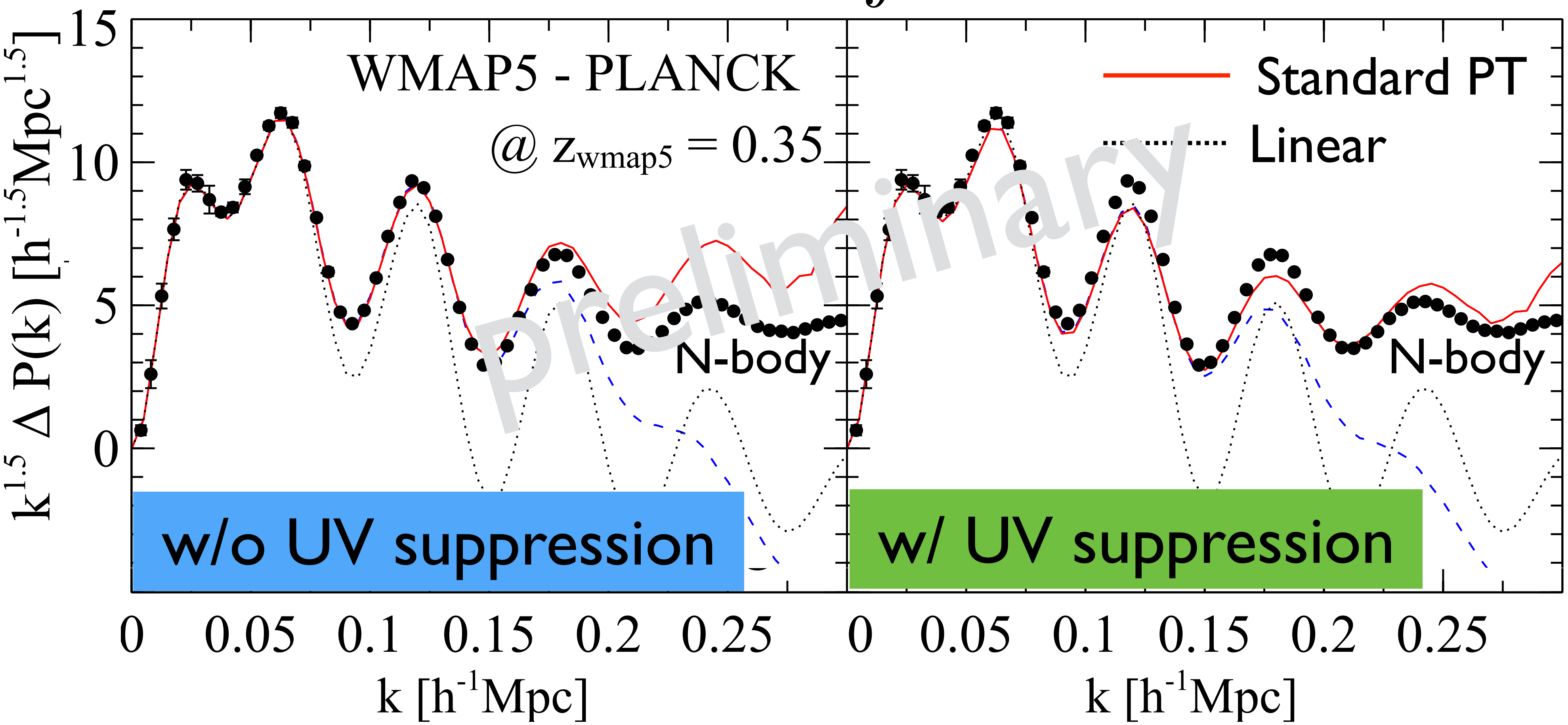
Role of UV suppression

Taking account of the UV suppression, how well standard PT prediction can be improved ?

Here, we consider $\delta P_0(k)$ as the difference between *Planck* and *WMAP5* and compute the nonlinear power spectrum difference $\delta P_{\rm nl}(k)$

$$\delta P_{\rm nl}(k) = \int d\ln q \, \underbrace{K(k,q)}_{\text{WMAP5}} \, \delta P_0(q)$$

WMAP5−Planck



It qualitatively explains N-body results, but not perfect…

Summary & discussion

Measurement of mode-coupling kernel of large-scale structure (LSS) :

$$K(k,q) = q\,\frac{\delta P_{\rm nl}(k)}{\delta P_0(q)}$$

Unlike the standard PT results,

- There appears UV suppression in N-body simulation at k<<q
- Discrepancy can be seen even at low-k, where standard PT can reproduce the N-body result quite well

Physical origin A connection with small-scale physics (formation and merging processes of dark matter halos)

Implication Check the validity and limitation for EFTofLSS

→ A step toward an improved prescription of LSS