“Relativistic Cosmology,” YITP in Kyoto, 10th Sep 2014.

Cosmology in generalized Horndeski theories with second-order equations of motion

R. Kase and S. Tsujikawa, Phys.Rev. D90 (2014) 044073

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1. Introduction

Discovery of late-time cosmic acceleration

In 1998, the discovery of late-time cosmic acceleration based on Type Ia supernovae is reported. The source for this acceleration is named dark energy.

The equation of state defined below characterizes dark energy.

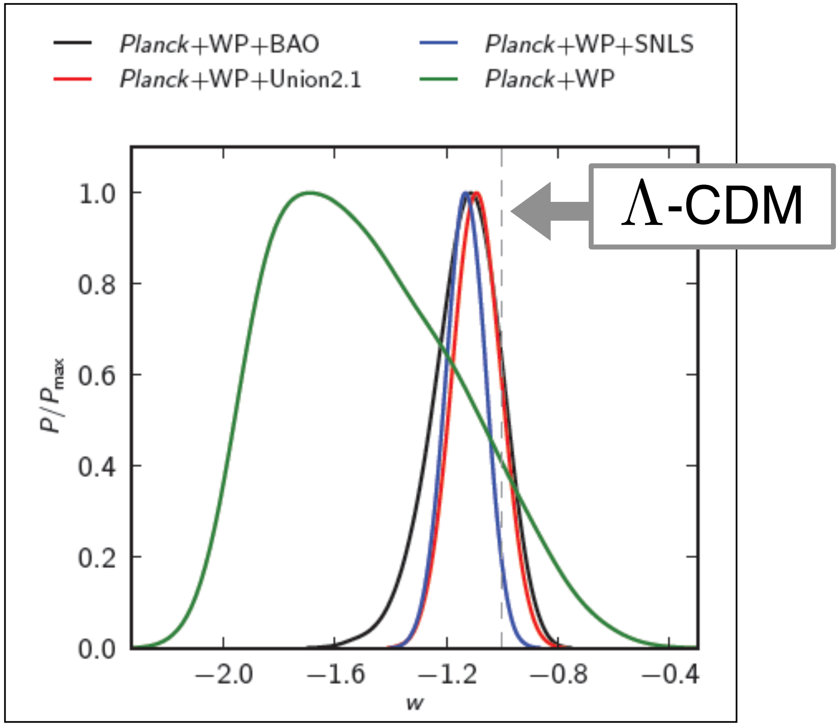
$$w \equiv P/\rho$$

Condition for acceleration :

$$w < -1/3$$

• Planck+WP+SNLS

$$w = -1.13^{+0.13}_{-0.14} \text{ (95\%CL)}$$



Planck collaboration arXiv:1303.5076 [astro-ph.CO]

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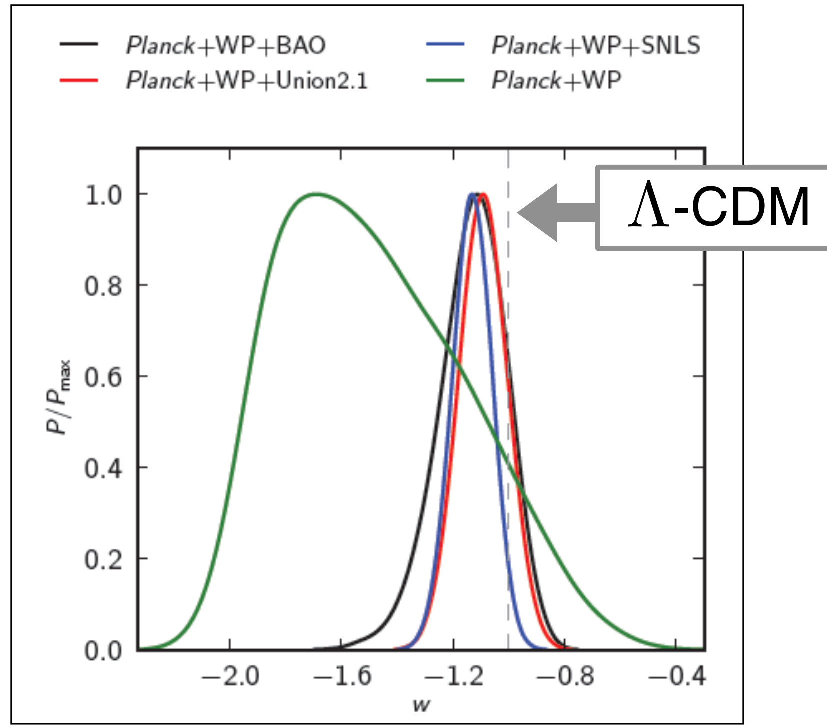


Planck collaboration arXiv:1303.5076 [astro-ph.CO]

Dark energy problem may imply some modification of gravity on large scales.

Unified approach 1: Horndeski Lagrangians

$$L = \sum_{i=2}^{5} L_i$$

$$L_2 = G_2(\chi, Y),$$

$$L_3 = G_3(\chi, Y)\Box\chi,$$

$$L_4 = G_4(\chi, Y)\, R - 2G_{4,Y}(\chi, Y)\left[(\Box\chi)^2 - \chi^{;\mu\nu}\chi_{;\mu\nu}\right],$$

$$L_5 = G_5(\chi, Y)G_{\mu\nu}\chi^{;\mu\nu} + \frac{1}{3}G_{5,Y}(\chi, Y) \times (\text{field derivatives}),$$

“;” covariant derivative

$Y \equiv \chi_{;\mu}\chi^{;\mu}$

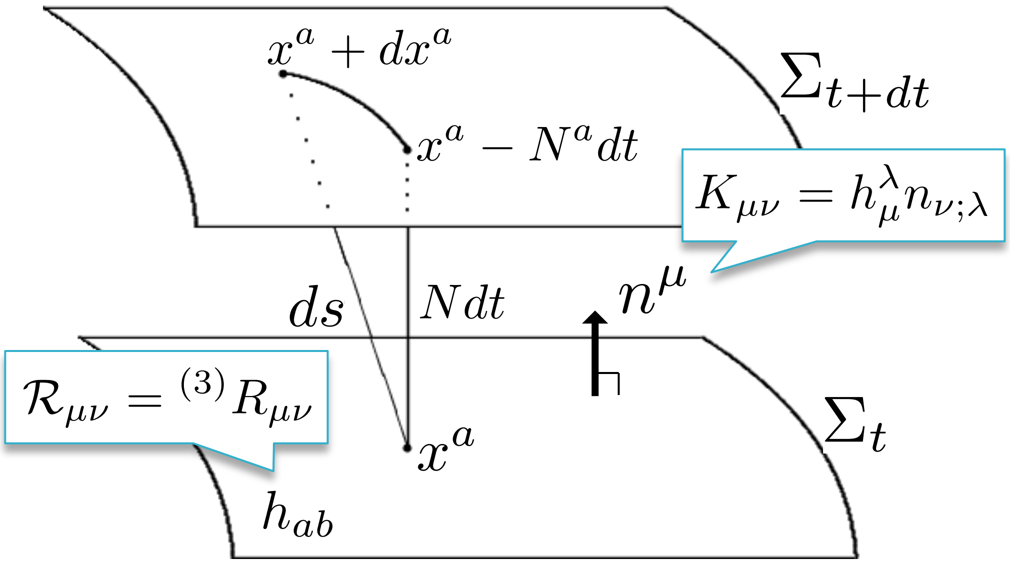
$G_{i,Y} \equiv \partial G_i / \partial Y$

- Quintessence and K−essence $G_2 = G_2(\chi, Y)\,, \quad G_3 = 0\,, \quad G_4 = M_{\rm pl}^2/2\,, \quad G_5 = 0\,.$
- $f(R)$ gravity and Brans−Dicke gravity $G_3 = 0\,, \quad G_4 = F(\chi)\,, \quad G_5 = 0\,.$
- Galileon $G_2 = c_2 Y\,, \quad G_3 = c_3/M^3\,, \quad G_4 = M_{\rm pl}^2/2 + c_4 Y^2/M^6\,, \quad G_5 = c_5 Y^2/M^9\,.$

Horndeski Lagrangians describe the most general scalar−tensor theory with **second-order** equations of motion on the **general** background.

- Unified approach 2:
Effective Field theory (EFT) on the cosmological background

J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, JCAP 1308, 025 (2013)



$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

Under the unitary gauge $(\delta\chi = 0)$,

$$\chi = \chi(t)$$

constant time hypersurfaces

$\Updownarrow$

uniform χ hypersurfaces

$$n^\mu = -\chi_{;}{}^{\mu}/\sqrt{-Y}$$

A scalar field χ associated with the modification of gravity is absorbed into the constant time hypersurfaces.

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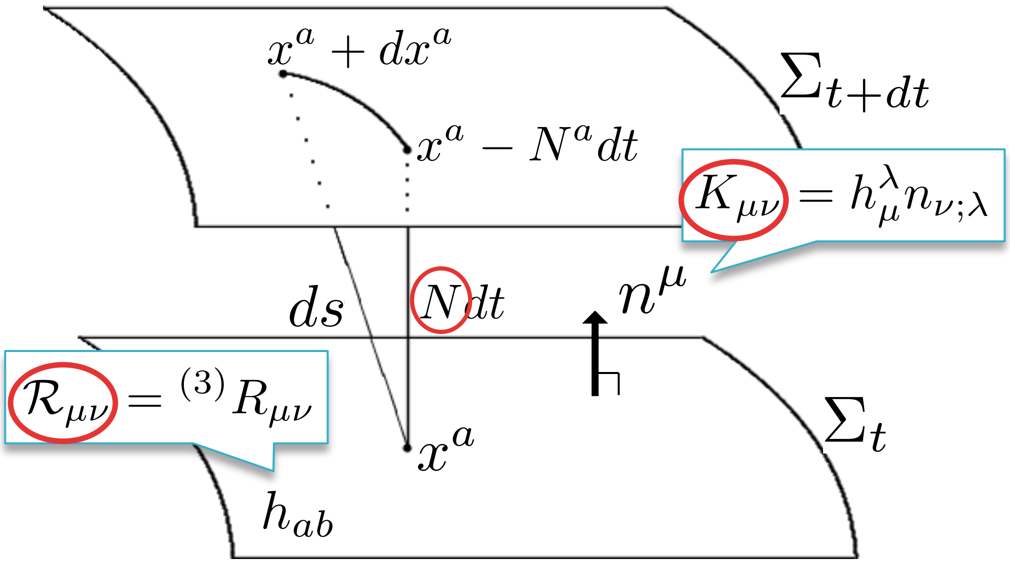

$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

Under the unitary gauge $(\delta\chi = 0)$,

$$\chi = \chi(t)$$

constant time hypersurfaces

$\Updownarrow$

uniform χ hypersurfaces

$$n^\mu = -\chi_;{}^\mu / \sqrt{-Y}$$

$$S = \int d^4x \sqrt{-g} L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t)$$

$$K \equiv K^\mu{}_\mu \,, \quad \mathcal{S} \equiv K_{\mu\nu} K^{\mu\nu} \,, \quad \mathcal{R} \equiv \mathcal{R}^\mu{}_\mu \,, \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} \,, \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu} K^{\mu\nu} \,.$$

Horndeski Lagrangians in the EFT language

$$\boxed{n^\mu = -\chi_{;}{}^{\mu}/\sqrt{-Y}}$$

$$G_2(\chi, Y) \rightarrow G_2(N, t) \quad (Y = -\dot{\chi}^2/N^2)$$

$$G_3(\chi, X)\Box\chi \rightarrow 2(-Y)^{3/2} F_{3,Y} K - Y F_{3,\chi} \quad (G_3 = F_3 + 2Y F_{3,Y})$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$\boxed{\begin{aligned} L = {} & A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R} \\ & + A_5(N,t)K_3 + B_5(N,t)\left(\mathcal{U} - K\mathcal{R}/2\right) , \\ & \left(K_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S}) + O(3)\right) \end{aligned}}$$

The Horndeski theory is a subclass of the EFT of modified gravity.

$$\boxed{S = \int d^4x \sqrt{-g} L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t)}$$

$$K \equiv K^\mu{}_\mu , \quad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu} , \quad \mathcal{R} \equiv \mathcal{R}^\mu{}_\mu , \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} , \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu} .$$

Horndeski Lagrangians in the EFT language

$$L = A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R}$$
$$+ A_5(N,t)K_3 + B_5(N,t)(\mathcal{U} - K\mathcal{R}/2) \;,$$
$$\left(K_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S}) + O(3)\right)$$

$$L_4 = G_4(\chi, Y)R - 2G_{4,Y}(\chi, Y)\left[(\Box\chi)^2 - \chi^{;\mu\nu}\chi_{;\mu\nu}\right] \;,$$
$$L_5 = G_5(\chi, Y)G_{\mu\nu}\chi^{;\mu\nu} + \frac{1}{3}G_{5,Y}(\chi, Y) \times (\text{field derivatives}) \;,$$

In the case of Horndeski theories there are the following relations:

$$\boxed{A_4 = 2YB_{4Y} - B_4 \,, \quad A_5 = -YB_{5Y}/3 \,.}$$

Generalized Horndeski theories

$$L = A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R}$$
$$+ A_5(N,t)K_3 + B_5(N,t)\left(\mathcal{U} - K\mathcal{R}/2\right) ,$$
$$\left(K_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S}) + O(3)\right)$$

Gleyzes, Langlois, Piazza, and Vernizzi (GLPV) generalized Horndeski theory in such a way that the coefficients A_4 , A_5 , B_4 , B_5 are not necessarily related to each other.

J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, arXiv:1404.6495

$$A_4 \, , \; A_5 \, , \; B_4 \, , \; B_5 : \text{arbitrary functions}$$

Generalized Horndeski theories

$$L = A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R}$$
$$+ A_5(N,t)K_3 + B_5(N,t)\left(\mathcal{U} - K\mathcal{R}/2\right) \ ,$$
$$\left(K_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S}) + O(3)\right)$$

Although GLPV theories go beyond the Horndeski domain, the number of physical degrees of freedom is same as Horndeski theories.

→ **Mukohyama-san's talk!**

It is not trivial whether the screening mechanism works sufficiently or not compared to the Horndeski theory.

→ **Kobayashi-san's talk!**

Once we go beyond the Horndeski domain, non-trivial modifications to all the propagation speeds of DE/matter fields.

L. Gergely and S. Tsujikawa Phys. Rev. D89 (2014) 064059

2. Background EOMs and GLPV theories

$$S = \int d^4x \sqrt{-g} \left[L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t) + \sum_{I=1}^{N-1} P^{(I)}(X_I) \right]$$

$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

BG: $\bar{N} = 1 \,, \quad \bar{N}^i = 0 \,, \quad \bar{h}_{ij} = a^2(t) \delta_{ij} \,.$

$\sum_{I=1}^{N-1} P^{(I)}(X_I)$: in order to model perfect fluids of matter components

Expanding the Lagrangian up to linear order as

e.g. $L_N = \partial L / \partial N$

$$L = \bar{L} + L_N \delta N + L_K \delta K + L_{\mathcal{S}} \delta \mathcal{S} + L_{\mathcal{R}} \delta \mathcal{R} + L_{\mathcal{Z}} \delta \mathcal{Z} + L_{\mathcal{U}} \delta \mathcal{U} + \mathcal{O}(2)$$

using $K_{ij} = \left(\partial_t h_{ij} - \nabla_i N_j - \nabla_j N_i \right) / (2N)$ with 3D covariant derivative ∇_i

$$S = \int d^4x \sqrt{-\bar{g}} \left[\mathcal{E}^N \delta N + \mathcal{E}^h \delta \sqrt{h} \right]$$

2. Background EOMs and GLPV theories

$$S=\int d^4x\sqrt{-g}\left[L(N,K,\mathcal{S},\mathcal{R},\mathcal{Z},\mathcal{U};t)+\sum_{I=1}^{N-1}P^{(I)}(X_I)\right]$$

$$ds^2=-N^2dt^2+h_{ab}(dx^a+N^a dt)(dx^b+N^b dt)$$

BG: $\bar{N}=1\,,\quad \bar{N}^i=0\,,\quad \bar{h}_{ij}=a^2(t)\delta_{ij}\,.$

Then the BG EOMs follow as

$$\mathcal{E}^N=0\rightarrow \bar{L}+L_N-3H\mathcal{F}=\rho_M$$

$$\mathcal{E}^h=0\rightarrow \bar{L}-\dot{\mathcal{F}}-3H\mathcal{F}=-P_M$$

$$\left(\mathcal{F}\equiv L_K+2HL_{\mathcal{S}}\,,\quad \rho_M=\sum_{I=1}^{N-1}\left[2X_IP^{(I)}_{X_I}-P^{(I)}\right]\,,\quad P_M=\sum_{I=1}^{N-1}P^{(I)}\right)$$

2. Background EOMs and GLPV theories

$$S = \int d^4x\sqrt{-g}\left[L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t) + \sum_{I=1}^{N-1} P^{(I)}(X_I)\right]$$

$$ds^2 = -N^2dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

BG: $\bar{N} = 1\,, \quad \bar{N}^i = 0\,, \quad \bar{h}_{ij} = a^2(t)\delta_{ij}\,.$

Applying to the GLPV theories,

$$L = A_2 + A_3K + A_4(K^2 - \mathcal{S}) + B_4\mathcal{R} + A_5K_3 + B_5\left(\mathcal{U} - K\mathcal{R}/2\right)\,,$$

$$A_2 - 6H^2A_4 - 12H^3A_5 + 2\dot{\chi}^2\left(A_{2Y} + 3HA_{3Y} + 6H^2A_{4Y} + 6H^3A_{5Y}\right) = \rho_M\,,$$

$$A_2 - 6H^2A_4 - 12H^3A_5 - \dot{A}_3 - 4\dot{H}A_4 - 4H\dot{A}_4 - 12H\dot{H}A_5 - 6H^2\dot{A}_5 = -P_M\,.$$

$$\left(\mathcal{F} \equiv L_K + 2HL_{\mathcal{S}}\,, \quad \rho_M = \sum_{I=1}^{N-1}\left[2X_IP^{(I)}_{X_I} - P^{(I)}\right], \quad P_M = \sum_{I=1}^{N-1}P^{(I)}\right)$$

3. Cosmological perturbations

$$S = \int d^4x \sqrt{-g} \left[L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t) + \sum_{I=1}^{N-1} P^{(I)}(X_I) \right]$$

$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

$$N = 1 + \delta N \,, \quad N_i = \partial_i \psi \equiv \partial \psi / \partial x^i \,,$$

$$h_{ij} = a^2(t) e^{2\zeta} \hat{h}_{ij} \,, \quad \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \gamma_{il}\gamma_{lj}/2 \,, \quad \det \hat{h} = 1 \,.$$

Expanding the Lagrangian up to second order as

$$L = \bar{L} + L_N \delta N + L_K \delta K + L_{\mathcal{S}} \delta \mathcal{S} + L_{\mathcal{R}} \delta \mathcal{R} + L_{\mathcal{Z}} \delta \mathcal{Z} + L_{\mathcal{U}} \delta \mathcal{U}$$
$$+ \frac{1}{2} \left(\delta N \frac{\partial}{\partial N} + \delta K \frac{\partial}{\partial K} + \delta \mathcal{S} \frac{\partial}{\partial \mathcal{S}} + \delta \mathcal{R} \frac{\partial}{\partial \mathcal{R}} + \delta \mathcal{U} \frac{\partial}{\partial \mathcal{U}} \right)^2 L + O(3) \,,$$

using $K_{ij} = (\partial_t h_{ij} - \nabla_i N_j - \nabla_j N_i) / (2N)$ and integrating by parts...

3. Cosmological perturbations

$$S = \int d^4x\sqrt{-g}\left[L(N,K,\mathcal{S},\mathcal{R},\mathcal{Z},\mathcal{U};t) + \sum_{I=1}^{N-1} P^{(I)}(X_I)\right]$$

$$ds^2 = -N^2dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

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$$h_{ij} = a^2(t)e^{2\zeta}\hat{h}_{ij}\,, \quad \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \gamma_{il}\gamma_{lj}/2\,, \quad \det\hat{h} = 1\,.$$

$$\begin{aligned}\mathcal{L}_2 = a^3\Bigg\{&\frac{1}{2}(2L_N + L_{NN} - 6H\mathcal{W} + 12H^2L_{\mathcal{S}})\delta N^2 + \left[\mathcal{W}\left(3\dot{\zeta} - \frac{\partial^2\psi}{a^2}\right) - 4(\mathcal{D}+\mathcal{E})\frac{\partial^2\zeta}{a^2}\right]\delta N\\ &+ 4L_{\mathcal{S}}\dot{\zeta}\frac{\partial^2\psi}{a^2} - 6L_{\mathcal{S}}\dot{\zeta}^2 + 2\mathcal{E}\frac{(\partial\zeta)^2}{a^2} + \sum_{I=1}^{N-1}\Bigg[(2\dot{\phi}_I^2P^{(I)}_{X_IX_I} - P^{(I)}_{X_I})(\dot{\phi}_I^2\delta N^2 - 2\dot{\phi}_I\dot{\delta\phi}_I\delta N + \dot{\delta\phi}_I^2)\\ &- 6\dot{\phi}_IP^{(I)}_{X_I}\zeta\dot{\delta\phi}_I - 2\dot{\phi}_IP^{(I)}_{X_I}\delta\phi_I\frac{\partial^2\psi}{a^2} + P^{(I)}_{X_I}\frac{(\partial\delta\phi_I)^2}{a^2}\Bigg]\Bigg\},\end{aligned}$$

Varying this Lagrangian density with respect to δN and $\partial^2\psi$, we obtain the Hamiltonian and momentum constraints. Inserting them back...

3. Cosmological perturbations

$$\mathcal{L}_2 = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{1}{a^2} \partial_j \vec{\mathcal{X}}^t \boldsymbol{G} \partial_j \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) ,$$

$$\vec{\mathcal{X}}^t = (\zeta, \delta\phi_1/M_{\rm pl}, \cdots, \delta\phi_{N-1}/M_{\rm pl}) .$$

- No-ghost conditions

In order to avoid ghost instabilities, the matrix $\boldsymbol{K}$ must be positive definite.

Under the tensor no-ghost condition, $L_{\mathcal{S}} > 0$, it follows that...

$$K_{11} = \frac{2L_{\mathcal{S}}}{\mathcal{W}^2} \left(g_2 + \frac{8L_{\mathcal{S}}}{M_{\rm pl}^2} \sum_{I=2}^{N} \dot{\phi}_{I-1}^2 K_{II} \right) ,$$

$$K_{II} = \left[2\dot{\phi}_{I-1}^2 P^{(I-1)}_{X_{I-1}X_{I-1}} - P^{(I-1)}_{X_{I-1}} \right] M_{\rm pl}^2 ,$$

$$K_{1I} = K_{I1} = -\frac{4L_{\mathcal{S}}\dot{\phi}_{I-1}}{M_{\rm pl}\mathcal{W}} K_{II} ,$$

$$g_2 = 4L_{\mathcal{S}}(2L_N + L_{NN}) + 3(L_{KN} + 2HL_{\mathcal{S}N})^2 > 0 ,$$

$$2\dot{\phi}_I^2 P^{(I)}_{X_I X_I} - P^{(I)}_{X_I} > 0 \qquad (I = 1, 2, \cdots, N-1).$$

3. Cosmological perturbations

$$\mathcal{L}_2 = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{1}{a^2} \partial_j \vec{\mathcal{X}}^t \boldsymbol{G} \partial_j \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) ,$$

$$\vec{\mathcal{X}}^t = \left(\zeta, \delta\phi_1 / M_{\rm pl}, \cdots, \delta\phi_{N-1} / M_{\rm pl} \right) .$$

- Propagation speeds

In the limit of a large wave number, $\det\left(\omega^2 \boldsymbol{K} - k^2 \boldsymbol{G}/a^2\right) = 0\,.$

Introducing the scalar sound speed $\omega^2 = c_s^2\, k^2/a^2$, it follows that

$$\prod_{I=1}^{N} \left(c_s^2 - c_{s{\rm H}I}^2\right) = -\frac{8L_{\mathcal{S}}}{g_2} \left(\frac{\mathcal{C}_3 \mathcal{W}}{16 L_{\mathcal{S}}^2} + 1 \right) \times \sum_{I=2}^{N} \left[\dot{\phi}_{I-1}^2 P_{X_{I-1}}^{(I-1)} \left\{ 2c_s^2 + c_{s{\rm H}I}^2 \left(\frac{\mathcal{C}_3 \mathcal{W}}{16 L_{\mathcal{S}}^2} - 1 \right) \right\} \prod_{J \neq I, J \geq 2}^{N} \left(c_s^2 - c_{s{\rm H}J}^2\right) \right] ,$$

where,

$$c_{s{\rm H}1}^2 = \frac{\mathcal{W}^2}{2L_{\mathcal{S}} g_2} \left[G_{11} + \frac{16 L_{\mathcal{S}}^2}{\mathcal{W}^2} \sum_{I=2}^{N} \dot{\phi}_{I-1}^2 P_{X_{I-1}}^{(I-1)} \right] , \quad c_{s{\rm H}I}^2 = \frac{P_{X_{I-1}}^{(I-1)}}{P_{X_{I-1}}^{(I-1)} - 2\dot{\phi}_{I-1}^2 P_{X_{I-1} X_{I-1}}^{(I-1)}} .$$

3. Cosmological perturbations

$$\mathcal{L}_2 = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{1}{a^2} \partial_j \vec{\mathcal{X}}^t \boldsymbol{G} \partial_j \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) ,$$

$$\vec{\mathcal{X}}^t = (\zeta, \delta\phi_1/M_{\rm pl}, \cdots, \delta\phi_{N-1}/M_{\rm pl}) .$$

- Propagation speeds

In the limit of a large wave number, $\det \left(\omega^2 \boldsymbol{K} - k^2 \boldsymbol{G}/a^2 \right) = 0$.

Introducing the scalar sound speed $\omega^2 = c_s^2 k^2/a^2$, it follows that

$$\prod_{I=1}^{N} (c_s^2 - c_{s{\rm H}I}^2) = -\frac{8L_{\mathcal{S}}}{g_2} \left(\frac{\mathcal{C}_3 \mathcal{W}}{16 L_{\mathcal{S}}^2} + 1 \right) \times \sum_{I=2}^{N} \left[\dot{\phi}_{I-1}^2 P_{X_{I-1}}^{(I-1)} \left\{ 2c_s^2 + c_{s{\rm H}I}^2 \left(\frac{\mathcal{C}_3 \mathcal{W}}{16 L_{\mathcal{S}}^2} - 1 \right) \right\} \prod_{J \neq I, J \geq 2}^{N} (c_s^2 - c_{s{\rm H}J}^2) \right] ,$$

In Horndeski theories, this term is 0.

where,

$$c_{s{\rm H}1}^2 = \frac{\mathcal{W}^2}{2L_{\mathcal{S}} g_2} \left[G_{11} + \frac{16 L_{\mathcal{S}}^2}{\mathcal{W}^2} \sum_{I=2}^{N} \dot{\phi}_{I-1}^2 P_{X_{I-1}}^{(I-1)} \right] , \quad c_{s{\rm H}I}^2 = \frac{P_{X_{I-1}}^{(I-1)}}{P_{X_{I-1}}^{(I-1)} - 2\dot{\phi}_{I-1}^2 P_{X_{I-1}X_{I-1}}^{(I-1)}} .$$

3. Cosmological perturbations

$$\mathcal{L}_2 = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{1}{a^2} \partial_j \vec{\mathcal{X}}^t \boldsymbol{G} \partial_j \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) ,$$

$$\vec{\mathcal{X}}^t = (\zeta, \delta\phi_1/M_{\rm pl}, \cdots, \delta\phi_{N-1}/M_{\rm pl}) \, .$$

- Propagation speeds

In the limit of a large wave number, $\det \left(\omega^2 \boldsymbol{K} - k^2 \boldsymbol{G}/a^2 \right) = 0 \, .$

Introducing the scalar sound speed $\omega^2 = c_s^2 \, k^2/a^2$, it follows that

$$\prod_{I=1}^{N} (c_s^2 - c_{s{\rm H}I}^2) = -\frac{8L_{\mathcal{S}}}{g_2} \left(\frac{\mathcal{C}_3 \mathcal{W}}{16 L_{\mathcal{S}}^2} + 1 \right) \times$$

Outside the Horndeski domain, it's not 0. $\boxed{\text{N-th degree equation}}$

$$\sum_{I=2}^{N} \left[\dot{\phi}_{I-1}^2 P_{X_{I-1}}^{(I-1)} \left\{ 2c_s^2 + c_{s{\rm H}I}^2 \left(\frac{\mathcal{C}_3 \mathcal{W}}{16 L_{\mathcal{S}}^2} - 1 \right) \right\} \prod_{J \neq I, J \geq 2}^{N} (c_s^2 - c_{s{\rm H}J}^2) \right] ,$$

where,

$$c_{s{\rm H}1}^2 = \frac{\mathcal{W}^2}{2 L_{\mathcal{S}} g_2} \left[G_{11} + \frac{16 L_{\mathcal{S}}^2}{\mathcal{W}^2} \sum_{I=2}^{N} \dot{\phi}_{I-1}^2 P_{X_{I-1}}^{(I-1)} \right] , \quad c_{s{\rm H}I}^2 = \frac{P_{X_{I-1}}^{(I-1)}}{P_{X_{I-1}}^{(I-1)} - 2\dot{\phi}_{I-1}^2 P_{X_{I-1} X_{I-1}}^{(I-1)}} \, .$$

3. Cosmological perturbations

$$\mathcal{L}_2 = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{1}{a^2} \partial_j \vec{\mathcal{X}}^t \boldsymbol{G} \partial_j \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) ,$$

$$\vec{\mathcal{X}}^t = \left(\zeta, \delta\phi_1 / M_{\rm pl}, \cdots, \delta\phi_{N-1} / M_{\rm pl} \right) .$$

- Propagation speeds

In the limit of a large wave number, $\det \left(\omega^2 \boldsymbol{K} - k^2 \boldsymbol{G} / a^2 \right) = 0$.

Introducing the scalar sound speed $\omega^2 = c_s^2 k^2 / a^2$, it follows that

If the deviation from the Horndeski domain is small...

$$\mathcal{C}_3 = \mathcal{C}_3^{\rm H} \left(1 + \delta\mathcal{C}_3 \right) , \quad c_{s1}^2 = c_{s{\rm H}1}^2 + \delta c_{s1}^2 , \quad c_{sI}^2 = c_{s{\rm H}I}^2 + \delta c_{sI}^2 \quad (I = 2, 3, \cdots, N) .$$

$$\delta c_{s1}^2 \simeq \sum_{I=2}^{N} \xi_{I-1} \, \delta\mathcal{C}_3 \qquad \left(\xi_I \equiv \frac{16 L_S \dot{\phi}_I^2 P_{X_I}^{(I)}}{g_2} \right) ,$$

$$\delta c_{sI}^2 \simeq -\frac{c_{s{\rm H}I}^2}{2(c_{s{\rm H}I}^2 - c_{s{\rm H}1}^2 - \delta c_{s1}^2)} \xi_{I-1} \delta\mathcal{C}_3^2 \quad (I = 2, 3, \cdots, N) .$$

3. Cosmological perturbations

$$\mathcal{L}_2 = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{1}{a^2} \partial_j \vec{\mathcal{X}}^t \boldsymbol{G} \partial_j \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) ,$$

$$\vec{\mathcal{X}}^t = (\zeta, \delta\phi_1/M_{\rm pl}, \cdots, \delta\phi_{N-1}/M_{\rm pl}) \,.$$

- Propagation speeds

In the limit of a large wave number, $\det\left(\omega^2 \boldsymbol{K} - k^2 \boldsymbol{G}/a^2\right) = 0\,.$
Introducing the scalar sound speed $\omega^2 = c_s^2\, k^2/a^2$, it follows that

As we will see later $\xi_{I-1} \gg 1\,,\ \delta\mathcal{C}_3 \ll 1$ realize for the covariantized version of Galileon model.

$$\delta c_{s1}^2 \simeq \sum_{I=2}^{N} \xi_{I-1}\, \delta\mathcal{C}_3 \sim \mathcal{O}(1)\,,$$

$$\delta c_{sI}^2 \simeq -\frac{c_{sHI}^2}{2(c_{sHI}^2 - c_{sH1}^2 - \delta c_{s1}^2)} \xi_{I-1} \delta\mathcal{C}_3^2 \ll 1\,.$$

4. Application to Galileon theories

$$L = A_2 + A_3 K + A_4(K^2 - \mathcal{S}) + B_4 \mathcal{R} + A_5 K_3 + B_5 \left(\mathcal{U} - K\mathcal{R}/2\right) \,,$$

(A) Covariant Galileon

A. Nicolis, R. Rattazzi, E. Tricherini (2009)

C. Deffayet, G. Esposito-Farese, A. Vikman (2009)

$$A_2 = \frac{c_2}{2} Y \,, \quad A_3 = \frac{c_3}{3M^3}(-Y)^{3/2} \,, \quad A_4 = -\frac{M_{\rm pl}^2}{2} - \frac{3c_4}{4M^6} Y^2 \,, \quad A_5 = \frac{c_5}{2M^9}(-Y)^{5/2} \,,$$
$$B_4 = \frac{M_{\rm pl}^2}{2} - \frac{c_4}{4M^6} Y^2 \,, \quad B_5 = -\frac{3c_5}{5M^9}(-Y)^{5/2} \,.$$

(B) *Covariantized* Galileon

The covariantized version of the Minkowski Galileon obtained by replacing partial derivatives with covariant derivatives (outside the Horndeski domain). Adding the Einstein-Hilbert term it follows that

$$A_2 = \frac{c_2}{2} Y \,, \quad A_3 = \frac{c_3}{3M^3}(-Y)^{3/2} \,, \quad A_4 = -\frac{M_{\rm pl}^2}{2} - \frac{3c_4}{4M^6} Y^2 \,, \quad A_5 = \frac{c_5}{2M^9}(-Y)^{5/2} \,,$$
$$B_4 = \frac{M_{\rm pl}^2}{2} \,, \quad B_5 = 0 \,.$$

4. Application to Galileon theories

$$L = A_2 + A_3 K + A_4(K^2 - \mathcal{S}) + B_4\mathcal{R} + A_5 K_3 + B_5\left(\mathcal{U} - K\mathcal{R}/2\right) ,$$

(A) Covariant Galileon

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$$\boxed{B_4 = \frac{M_{\rm pl}^2}{2} - \frac{c_4}{4M^6}Y^2 , \quad B_5 = -\frac{3c_5}{5M^9}(-Y)^{5/2} .}$$

Since $\bar{\mathcal{R}}_{\mu\nu} = 0$, these two models cannot be distinguished at the BG level.

(B) *Covariantized* Galileon

The covariantized version of the Minkowski Galileon obtained by replacing partial derivatives with covariant derivatives (outside the Horndeski domain). Adding the Einstein-Hilbert term it follows that

$$A_2 = \frac{c_2}{2}Y , \quad A_3 = \frac{c_3}{3M^3}(-Y)^{3/2} , \quad A_4 = -\frac{M_{\rm pl}^2}{2} - \frac{3c_4}{4M^6}Y^2 , \quad A_5 = \frac{c_5}{2M^9}(-Y)^{5/2} ,$$

$$\boxed{B_4 = \frac{M_{\rm pl}^2}{2} , \quad B_5 = 0 .}$$

4. Application to Galileon theories

- BG EOMs

$$3M_{\rm pl}^2 H^2 = \rho_{\rm DE} + \rho_M\,,$$
$$3M_{\rm pl}^2 H^2 + 2M_{\rm pl}^2 \dot{H} = -P_{\rm DE} - P_M\,,$$

$$\rho_{\rm DE} = -\frac{1}{2}c_2\dot{\chi}^2 + \frac{3c_3 H\dot{\chi}^3}{M^3} - \frac{45c_4 H^2\dot{\chi}^4}{2M^6} + \frac{21c_5 H^3\dot{\chi}^5}{M^9}\,,$$
$$P_{\rm DE} = -\frac{1}{2}c_2\dot{\chi}^2 - \frac{c_3\dot{\chi}^2\ddot{\chi}}{M^3} + \frac{3c_4\dot{\chi}^3}{2M^6}\left[8H\ddot{\chi} + (3H^2 + 2\dot{H})\dot{\chi}\right] - \frac{3c_5 H\dot{\chi}^4}{M^9}\left[5H\ddot{\chi} + 2(H^2 + \dot{H})\dot{\chi}\right]\,.$$

- Matter components

Matter component

$$P^{(1)}(X_1) = b_1 X_1^2\,,$$
$$P^{(2)}(X_2) = b_2(X_2 - X_0)^2\,,$$

R. J. Scherrer (2004)

$$w^{(1)} = 1/3\,,$$ **radiation**

$$w^{(2)} = (X_2 - X_0)/(3X_2 - X_0)\,.$$

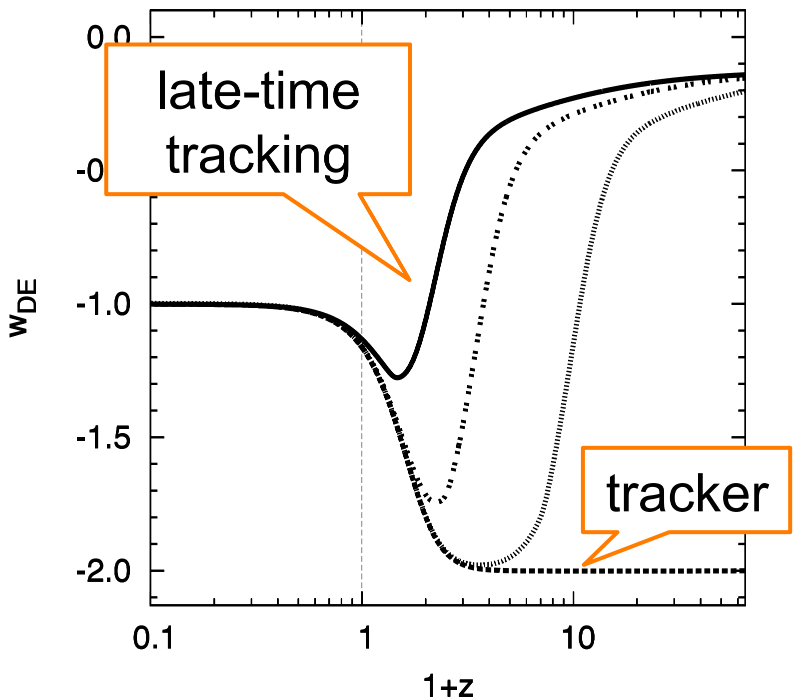
non-relativistic matter for $X_2 \simeq X_0$

4. Application to Galileon theories

- BG evolution

$$r_1 \equiv \frac{\dot{\chi}_{\rm dS} H_{\rm dS}}{\dot{\chi} H}\,, \quad r_2 \equiv \frac{H}{H_{\rm dS}} \left(\frac{\dot{\chi}}{\dot{\chi}_{\rm dS}} \right)^5 ,$$

- There is the dS point at $r_1 = r_2 = 1$.
- The tracker solution $(r_1 = 1)$ is in tension with the observational data.
- The late-time tracking solution $(r_1^{\rm ini} \ll 1)$ is consistent with the observational data.



S. Nesseris, A. De Felice and S. Tsujikawa, Phys. Rev. D 82, 124054 (2010)

4. Application to Galileon theories

- Evolution of the propagation speed along the late-time tracking

(A) Covariant Galileon

$$c_{s1}^2 = \begin{cases} \dfrac{1}{40}(\Omega_r + 1) & [(\mathrm{i})\ r_1 \ll 1,\ r_2 \ll 1]\,, \\[2ex] \dfrac{8+10\alpha-9\beta+\Omega_r(2+3\alpha-3\beta)}{3(2-3\alpha+6\beta)} & [(\mathrm{ii})\ r_1 = 1,\ r_2 \ll 1]\,, \\[2ex] \dfrac{(\alpha-2\beta)(4+15\alpha^2-48\alpha\beta+36\beta^2)}{2(2+3\alpha-6\beta)(2-3\alpha+6\beta)} & [(\mathrm{iii})\ r_1 = 1,\ r_2 = 1]\,. \end{cases}$$

Under the no-ghost conditions,

$$L_S > 0 \qquad\qquad \beta > 0$$

$$g_2 > 0 \quad \Rightarrow \quad -2 < 3(\alpha - 2\beta) < 2$$

$$\boxed{\begin{aligned} c_2 x_{\mathrm{dS}}^2 &= 6 + 9\alpha - 12\beta \\ c_3 x_{\mathrm{dS}}^3 &= 2 + 9\alpha - 9\beta \end{aligned}}$$

the above propagation speed of sound is positive in any regime.

A. De Felice and S. Tsujikawa, Phys. Rev. Lett. 105, 111301 (2010)

4. Application to Galileon theories

- Evolution of the propagation speed along the late-time tracking

(B) *Covariantized* Galileon

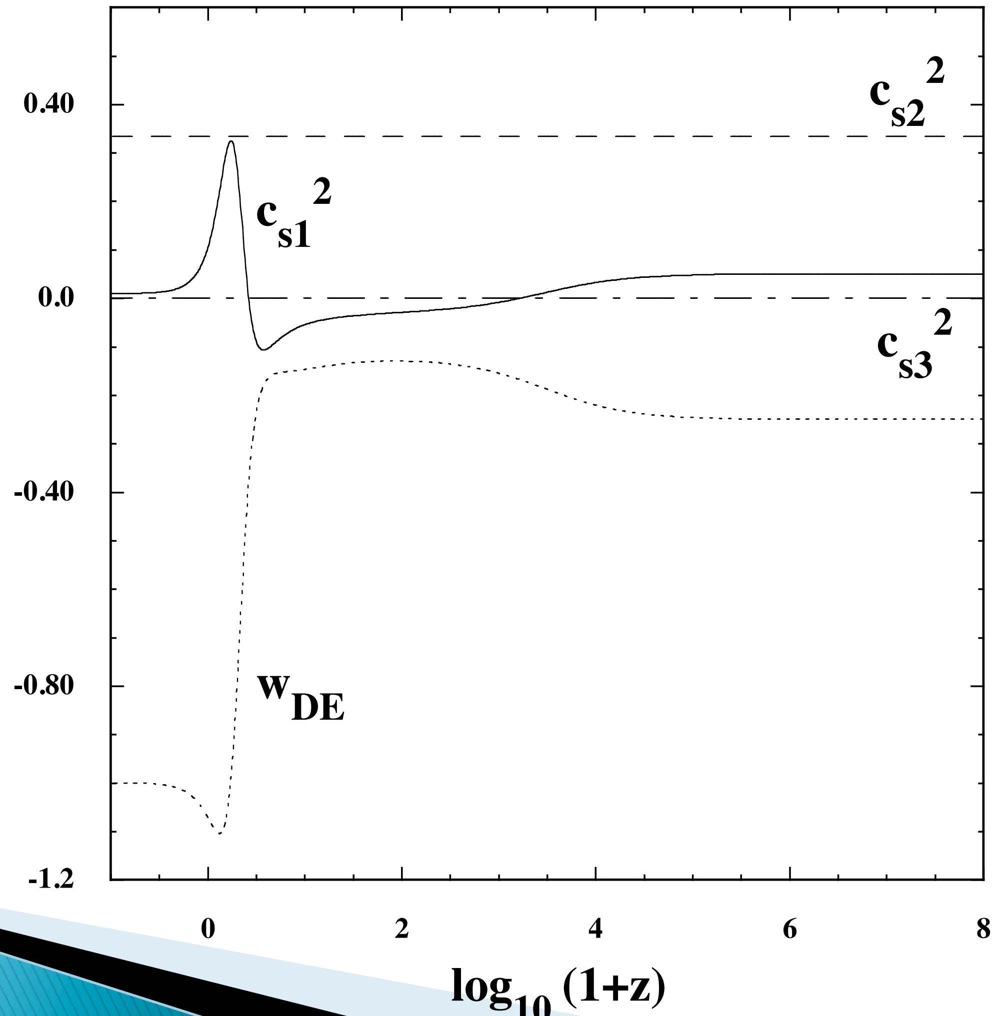
$$c_{s1}^2 = \begin{cases} \dfrac{1}{40}(3\Omega_r - 1) & [\text{(i)}\ r_1 \ll 1,\ r_2 \ll 1]\,, \\ \dfrac{16 - 15(\alpha - 2\beta) + \Omega_r(4 - 3\alpha + 6\beta)}{6(2 - 3\alpha + 6\beta)} & [\text{(ii)}\ r_1 = 1,\ r_2 \ll 1]\,, \\ \dfrac{\alpha - 2\beta}{2 + 3\alpha - 6\beta} & [\text{(iii)}\ r_1 = 1,\ r_2 = 1]\,. \end{cases}$$

In the regime (i), the propagation speed become negative during the matter dominated epoch!!

4. Application to Galileon theories

- Evolution of the propagation speed along the late-time tracking

(B) *Covariantized* Galileon



5. EFT including Horava-Lifshitz gravity

In order to include the Horava gravity and its extension in the EFT framework, we need to add extra terms to the EFT Lagrangian as a essential building block.

Gao's talk!

X. Gao arXiv:1406.0822

- Projectable Horava-Lifshitz gravity $(\delta N = 0)$

$$L = \frac{M_{\rm pl}^2}{2}\left[\mathcal{S} - \lambda K^2 + \mathcal{R} - M_{\rm pl}^{-2}\left(g_2 \mathcal{R}^2 + g_3 \mathcal{Z}\right) - M_{\rm pl}^{-4}\left(g_4 \mathcal{Z}_1 + g_5 \mathcal{Z}_2\right)\right]$$

$$\left(\mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\,, \quad \mathcal{Z}_1 \equiv \nabla_i \mathcal{R} \nabla^i \mathcal{R}\,, \quad \mathcal{Z}_2 \equiv \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk}\right)$$

The terms $\mathcal{Z}_1$, $\mathcal{Z}_2$ allow the $z = 3$ scaling characterized by the transformation $t \to c^3 t$ and $x^i \to c x^i$.

The theory is power-counting renormalizable.

However, in this theory, the no-ghost condition and the condition to avoid a Laplacian instability cannot be satisfied at the same time. Moreover there is the strong coupling problem in the deep IR regime.

5. EFT including Horava-Lifshitz gravity

In order to include the Horava gravity and its extension in the EFT framework, we need to add extra terms to the EFT Lagrangian as a essential building block.

Gao's talk!

D. Blas, O. Pujolas and S. Sibiryakov, (2010)

X. Gao arXiv:1406.0822

- Non-projectable Horava-Lifshitz gravity $(\delta N \neq 0)$

  In the non-projectable extended version of the Horava gravity, the acceleration vector $a_\nu = n^\lambda n_{\nu;\lambda} = \nabla_\nu \ln N$ does not vanish. In this case one can consider the Lagrangian

$$L_{\mathcal{V}_3} = -\frac{1}{2M_{\rm pl}^2}\left(g_4 \mathcal{Z}_1 + g_5 \mathcal{Z}_2 + \eta_4 \alpha_4 + \eta_5 \alpha_5 + \cdots\right),$$

$$L_{\mathcal{V}_2} = -\frac{1}{2}\left(g_2 \mathcal{R}^2 + g_3 \mathcal{Z} + \eta_2 \alpha_2 + \eta_3 \alpha_3 + \cdots\right),$$

$$L_{\mathcal{V}_1} = \frac{M_{\rm pl}^2}{2}\left(\mathcal{R} + \eta_1 \alpha_1\right),$$

$$\alpha_1 \equiv a_i a^i, \quad \alpha_2 \equiv a_i \Delta a^i, \quad \alpha_3 \equiv \mathcal{R} \nabla_i a^i,$$
$$\alpha_4 \equiv a_i \Delta^2 a^i, \quad \alpha_5 \equiv \Delta \mathcal{R} \nabla_i a^i,$$

$\boxed{L_{\mathcal{V}_3},\ L_{\mathcal{V}_2},\ L_{\mathcal{V}_1} \text{ are invariant under } z = 3,\ 2,\ 1 \text{ rescaling, respectively.}}$

5. EFT including Horava-Lifshitz gravity

$$S = \int d^4x\sqrt{-g}\,L\left(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \cdots, \alpha_5; t\right)$$

RK and S. Tsujikawa, arXiv:1409.1984

$$K \equiv K^{\mu}{}_{\mu}\,, \quad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu}\,, \quad \mathcal{R} \equiv \mathcal{R}^{\mu}{}_{\mu}\,, \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\,,$$

$$\mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}\,, \mathcal{Z}_1 \equiv \nabla_i\mathcal{R}\nabla^i\mathcal{R}\,, \quad \mathcal{Z}_2 \equiv \nabla_i\mathcal{R}_{jk}\nabla^i\mathcal{R}^{jk}\,,$$

$$\alpha_1 \equiv a_i a^i\,, \quad \alpha_2 \equiv a_i\Delta a^i\,, \quad \alpha_3 \equiv \mathcal{R}\nabla_i a^i\,, \quad \alpha_4 \equiv a_i\Delta^2 a^i\,, \quad \alpha_5 \equiv \Delta\mathcal{R}\nabla_i a^i\,,$$

Other terms such, e.g. $\mathcal{R}^j_i\mathcal{R}^k_j\mathcal{R}^i_k$, can be taken into account, but they are irrelevant to scalar linear perturbations on the flat FLRW background.

5. EFT including Horava-Lifshitz gravity

$$S = \int d^4x \sqrt{-g}\, L\left(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \cdots, \alpha_5; t\right)$$

RK and S. Tsujikawa, arXiv:1409.1984

Expanding the EFT Lagrangian up to second order, we derived

- BG EOMs,
- EOMs for linear perturbations,
- stability conditions.

We also applied our general results to

- Horndeski theories,
- GLPV theories,
- Projectable Horava-Lifshitz gravity,
- Non-projectable Horava-Lifshitz gravity.

5. EFT including Horava-Lifshitz gravity

$$S = \int d^4x\sqrt{-g}\,L\,(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \cdots, \alpha_5; t)$$

RK and S. Tsujikawa, arXiv:1409.1984

- Projectable Horava-Lifshitz gravity $(\delta N = 0)$

$$L = \frac{M_{\rm pl}^2}{2}\left[\mathcal{S} - \lambda K^2 + \mathcal{R} - M_{\rm pl}^{-2}\left(g_2\mathcal{R}^2 + g_3\mathcal{Z}\right) - M_{\rm pl}^{-4}\left(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2\right)\right]$$

$$\mathcal{L}_2 = M_{\rm pl}^2 a^3\left(\frac{3\lambda - 1}{\lambda - 1}\dot{\zeta}^2 - \zeta\mathcal{O}\zeta\right)$$

$$\mathcal{O} \equiv \Delta + \frac{\Delta^2}{M_2^2} - \frac{\Delta^3}{M_3^4}\,, \quad \Delta \equiv \nabla^i\nabla_i\,, \quad M_2^2 \equiv M_{\rm pl}^2(8g_2 + 3g_3)^{-1}\,, \quad M_3^4 \equiv M_{\rm pl}^4(8g_4 + 3g_5)^{-1}\,.$$

which coincides with the results in
K. Koyama and F. Arroja, JHEP 1003, 061 (2010),
S. Mukohyama, Class. Quant. Grav. 27, 223101 (2010).

5. EFT including Horava-Lifshitz gravity

$$S = \int d^4x\sqrt{-g}\,L\left(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \cdots, \alpha_5; t\right)$$

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- Non-projectable Horava-Lifshitz gravity $(\delta N \neq 0)$

$$L = \frac{M_{\rm pl}^2}{2}\left[\mathcal{S} - \lambda K^2 + \mathcal{R} + \eta_1\alpha_1 - M_{\rm pl}^{-2}\left(g_2\mathcal{R}^2 + g_3\mathcal{Z} + \eta_2\alpha_2 + \eta_3\alpha_3\right) - M_{\rm pl}^{-4}\left(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2 + \eta_4\alpha_4 + \eta_5\alpha_5\right)\right]$$

In the IR regime, on the Minkowski BG,

$$\mathcal{L}_2 = M_{\rm pl}^2\frac{3\lambda - 1}{\lambda - 1}\left[\dot{\zeta}^2 - c_s^2(\partial\zeta)^2\right] \qquad \left(c_s^2 = \frac{\lambda - 1}{3\lambda - 1}\,\frac{2 - \eta_1}{\eta_1}\right)$$

which coincides with the results in D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. 104, 181302 (2010)

5. Conclusions

- We studied the cosmology of an extended version of Horndeski theories with second-order equations of motion on the flat FLRW background in the presence of multiple scalar fields associated with matter fluids.

- Expanding the action up to second order, we derived the background equations of motion, equations of motion for linear perturbations and stability conditions.

- Although the Background dynamics in Horndeski theories and GLPV theories are same, they can be distinguished at the perturbation level.

- The theories beyond Horndeski induce non-trivial modifications to all the propagation speeds of N scalar fields.

- We applied our general results to the covariant Galileon (a class of Horndeski theories) and the *covariantized* Galileon model (a class of GLPV theories).
  We showed that the propagation speed of the dark energy field become negative during the matter dominated epoch in the latter model.