Relativistic Cosmology seminar, Yukawa Institute
11th September 2014

Interacting vacuum energy
the i-Vacuum

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D Wands, J De-Santiago & Y Wang, arXiv:1203.6776
Y Wang, D Wands, GB Zhao & L Xu, arXiv:1404.5706
V Salvatelli, N Said, M Bruni, A Melchiorri & D Wands, arXiv:1406.7297

ICG Portsmouth

Outline:

- **minimal design for dark energy: *the iVacuum***

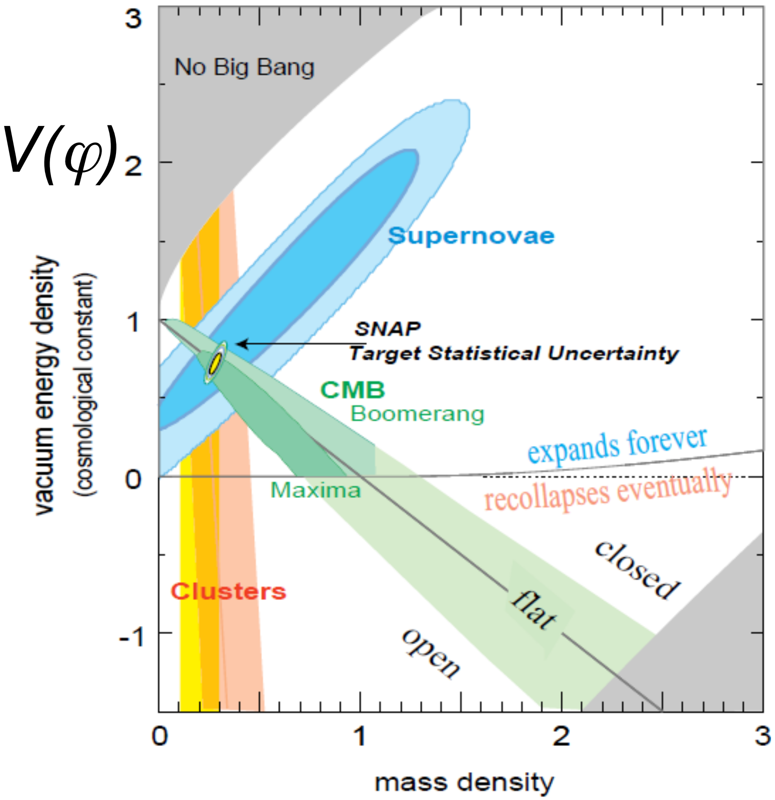
- **dark energy cosmology: interacting vacuum+matter**
  - *homogeneous cosmology, Q(t)*
  - *linear perturbations, covariant Q^μ*
    - *barotropic (adiabatic) perturbations*
    - *geodesic (non-adiabatic) perturbations*

  - ***two examples:***
    - ***decomposed Chaplygin gas cosmology***
    - ***model-independent Q(z)***
      - ***evidence for late-time interaction?***

- **summary**

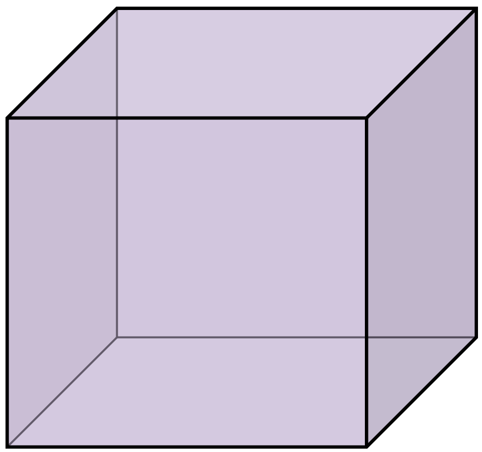
Dark energy models

- **quintessence**
  - self-interacting scalar fields, $V(\varphi)$
- **barotropic fluid**
  - exotic equation of state, $P(\rho)$
  - unified dark matter + energy
- **interacting dark energy, $\Gamma(t)$**
  - coupled quintessence



motivated by astronomical observations, but lacking persuasive physical model

Simplest model



- **vacuum energy, V**
  - energy of empty space
  - undiluted by expansion
  - ***no new degrees of freedom***

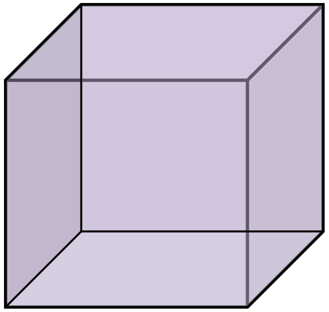
$$\check{T}^{\mu}_{\nu} = -V g^{\mu}_{\nu}$$

perfect fluid $\quad T^{\mu}_{\nu} = P g^{\mu}_{\nu} + (\rho + P) u^{\mu} u_{\nu}$

with $\quad \check{\rho} = -\check{P} = V$

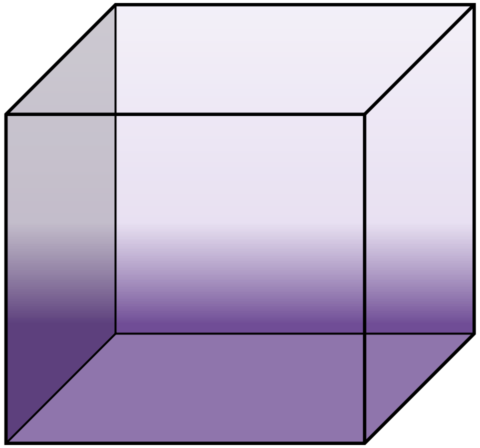
but no particle flow, hence 4-velocity, u, undefined

Homogeneous vacuum



- **8πG V = Λ = constant**
  - empirical value is cosmological constant problem

Inhomogeneous vacuum



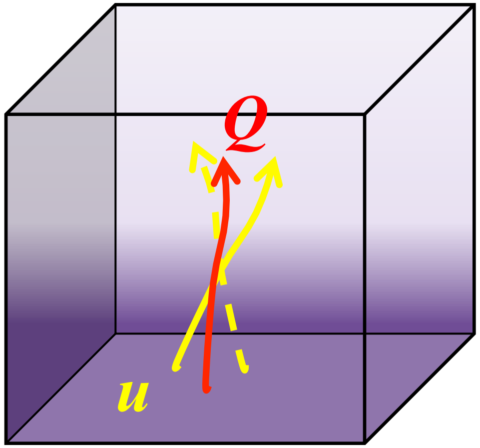
- **interacting vacuum:**

$$\nabla_\mu \check{T}^\mu_\nu = \nabla_\mu \left(-V g^\mu_\nu\right)$$
$$= -\nabla_\nu V$$
$$\equiv Q_\nu \qquad \text{= \textit{energy flow}}$$

- **conservation of total (matter + vacuum) energy:**

$$\nabla_\mu G_{\mu\nu} = 8\pi G_N \nabla_\mu \left(T^\mu_\nu + \check{T}^\mu_\nu\right) = 0 \quad \Rightarrow \quad \nabla_\mu T^\mu_\nu = -Q_\nu$$

4-velocity



- **perfect fluid**

$$T^\mu_\nu = P g^\mu_\nu + (\rho + P) u^\mu u_\nu$$

$$\Rightarrow \quad T^\mu_\nu u^\nu = -\rho u^\mu$$

- **vacuum**

$$\check{T}^\mu_\nu = -V g^\mu_\nu$$

$$\Rightarrow \quad \check{T}^\mu_\nu u^\nu = -V u^\mu \quad \forall \ u^\mu$$

  - all observers see same vacuum energy so 4-velocity undefined
  - **but *energy flow* defines irrotational potential flow**

$$\boxed{Q_\mu = -\nabla_\mu V}$$

FLRW vacuum cosmology:

homogeneous 3D space $\Rightarrow$ $V = V(t)$

- Friedmann equation

$$H^2 = \frac{8\pi G_N}{3}(\rho + V) - \frac{K}{a^2}$$

- Continuity equations for matter + vacuum

$$\begin{aligned} \dot{\rho} + 3H(\rho + P) &= -Q\,, \\ \dot{V} &= Q\,. \end{aligned}$$

- e.g.,

  Freese et al (1987); Berman (1991); Pavon (1991); Chen & Wu (1992); Carvalho et al (1992); Al-Rawaf & Taha (1996); Shapiro & Sola (2002); Sola (2011); …

- Freedom to choose any $V(t)$
  - *more a description than an explanation? like $V(\varphi)$?*

Dark energy cosmology

- any dark energy cosmology (with $\rho_{de}+P_{de}\geq 0$) can be decomposed into interacting vacuum + fluid

for example:

- interacting vacuum + scalar field (quintessence):

$$\rho_{de} = \frac{1}{2}\dot{\varphi}^2 + V, \quad P_{de} = \frac{1}{2}\dot{\varphi}^2 - V, \quad Q = V'(\varphi)\dot{\varphi}$$

$$\boxed{V(\varphi) \quad \Rightarrow \quad Q_\mu = V'(\varphi)\nabla_\mu \varphi}$$

- interacting vacuum + matter: Wands, De-Santiago & Wang (2012)

$$\rho_{de} = \rho_m + V, \quad P_{de} = -V, \quad Q = \dot{V}$$

- identical at background level
  distinguish by evolution of perturbations

Linear perturbations

inhomogeneous 3D space

$$ds^2 = -(1+2\phi)dt^2 + 2a\partial_i B dt dx^i + a^2\left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right] dx^i dx^j$$

- Matter:

$$\rho \to \rho(t) + \delta\rho(t, x^i)\,, \quad P \to P(t) + \delta P(t, x^i)\,, \quad u_\mu \to u_\mu = [-1-\phi\,, \partial_i\theta]$$

- Vacuum:

$$V \to V(t) + \delta V(t, x^i)$$

matter 3-momentum

- Interaction: (see Kodama & Sasaki 1984)

$$\boxed{Q_\nu \to (Q+\delta Q)u_\nu + f_\nu = [-Q(1+\phi) - \delta Q\,, \partial_i(f+Q\theta)]}$$

vacuum-matter momentum transfer

- need physical (covariant) interaction to determine energy-momentum transfer 4-vector Q_ν
  - *same FRW cosmologies may have different perturbations*

perturbed equations of motion

inhomogeneous 3D space ⇒ $V(t,x^i) = V(t) + \delta V(t,x^i)$

- matter+vacuum energy conservation:

$$\begin{aligned} \dot{\delta\rho} + 3H(\delta\rho + \delta P) - 3(\rho + P)\dot{\psi} + (\rho + P)\frac{\nabla^2}{a^2}\left(\theta + a^2\dot{E} - aB\right) &= -\delta Q - Q\phi\,, \\ \delta\dot{V} &= \delta Q + Q\phi\,. \end{aligned}$$

- matter+vacuum momentum conservation:

$$\begin{aligned} (\rho + P)\dot{\theta} - 3c_s^2 H(\rho + P)\theta + (\rho + P)\phi + \delta P &= -f + c_s^2 Q\theta\,, \\ -\delta V &= f + Q\theta\,. \end{aligned}$$

vanishing vacuum momentum requires vacuum pressure gradient balanced by force on vacuum

$$\nabla_i\left(-V\right) = \nabla_i\left(f + Q\theta\right)$$

Gauge-invariant perturbations

- vacuum perturbation on hypersurfaces orthogonal to energy transfer vanishes identically:

$$\Delta\check{\rho}_{\rm com} = \delta V + \left(f + \dot{V}\theta\right) = 0$$

  - *∃ hypersurfaces on which vacuum is homogeneous*

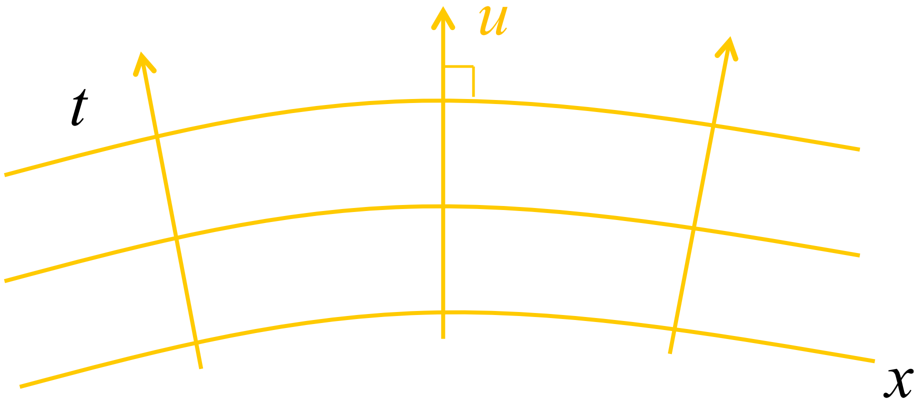
- comoving matter density: $\delta\rho_{\rm com} = \delta\rho + \dot{\rho}\theta$

- comoving vacuum density may be non-zero

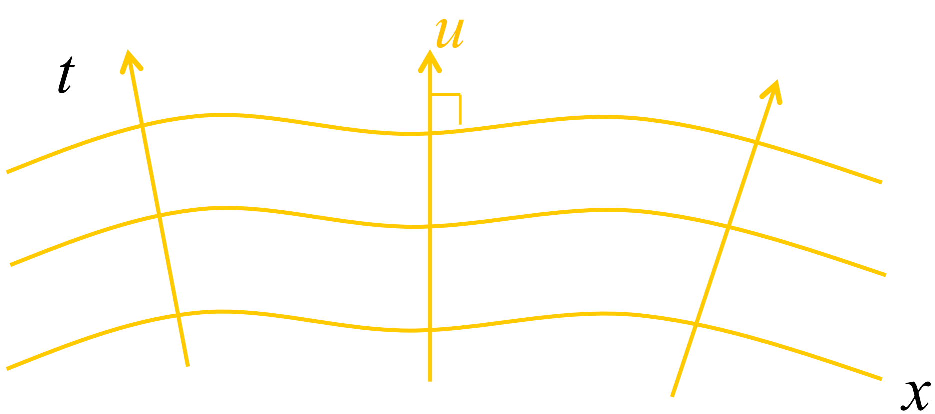
$$\delta\check{\rho}_{\rm com} = \delta V + \dot{V}\theta = -f$$

- *e.g., Poisson equation:*

$$\nabla^2\Phi = 4\pi G_N\left(\delta\rho_{\rm com} + \delta\check{\rho}_{\rm com}\right)$$

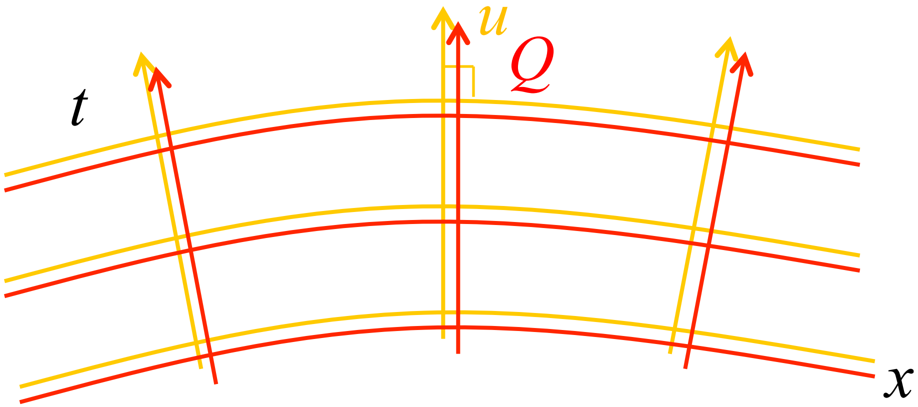


FRW cosmology

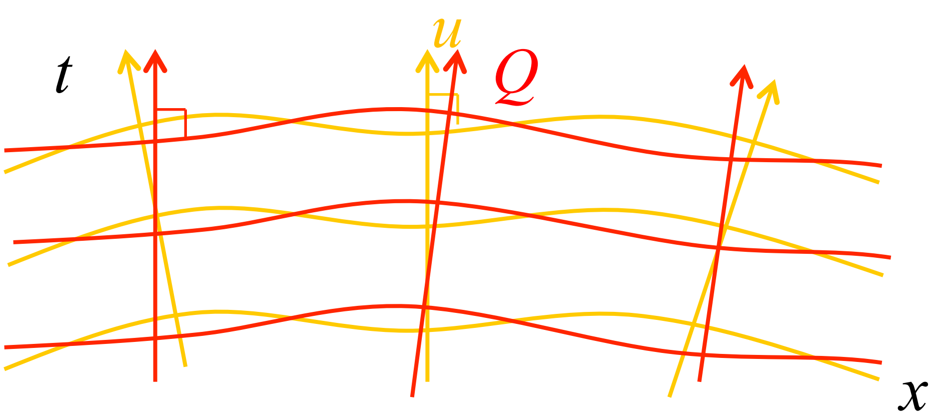


FRW cosmology
+ linear perturbations

comoving-orthogonal
coordinates *(t,x)*



FRW cosmology



FRW cosmology
+ linear perturbations

comoving-orthogonal
coordinates (t,x)

vacuum energy uniform on spaces orthogonal to energy flow

Gauge-invariant perturbations (II)

- curvature perturbation on uniform-matter hypersurfaces:

$$\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho$$

- curvature perturbation on uniform-*vacuum* hypersurfaces:

$$\check{\zeta} = -\psi - \frac{H}{\dot{V}}\delta V$$

- relative vacuum-matter perturbation:

$$\check{S} = 3\left(\check{\zeta} - \zeta\right)$$

  - *e.g., non-adiabatic vacuum pressure perturbation:*

$$\delta\check{P}_{\rm nad} = \frac{(1+c_s^2)Q[Q+3H(\rho+P)]}{9H^2(\rho+P)}\check{S}$$

Dark energy cosmology

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for example:

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- interacting vacuum + matter: Wands, De-Santiago & Wang (2012)

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- identical at background level
  distinguish by evolution of perturbations

e.g., generalised Chaplygin gas

Kamenshchik, Moschella and Pasquier (2001); Bento Bertolami & Sen (2002)

- exotic dark energy with barotropic equation of state:

$$P_{\mathrm{gCg}} = -A\rho_{\mathrm{gCg}}^{-\alpha}$$

  - *two constants (A and dimensionless α)*

  - unified dark matter + dark energy model

$$\begin{aligned}\rho_{\mathrm{gCg}} &= \left(A + Ba^{-3(1+\alpha)}\right)^{1/(1+\alpha)} \\ &\to B^{1/(1+\alpha)}a^{-3} \quad \text{as } a \to 0 \\ &\to A^{1/(1+\alpha)} \quad \text{as } a \to +\infty\end{aligned}$$

  - can be related to generalised higher-dimensional DBI scalar field

e.g., decomposed Chaplygin gas

Bento, Bertolami and Sen (2004)

$$u_m^\mu = u^\mu \;,\quad \rho_m = \rho_{\rm de} + P_{\rm de}\;,\qquad V = -P_{\rm de}$$

- FRW interaction can be written as

$$Q = 3\alpha H \left(\frac{\rho_m V}{\rho_m + V} \right)$$

  - *model has one dimensionless parameter,* α
  - *A appears as an integration constant*

$$A = (\rho_m + V)^\alpha V$$

- decomposed model allows two independent perturbations
  - *matter perturbations:* $\zeta_m = -\psi - \dfrac{H}{\dot\rho_m}\delta\rho_m$
  - *vacuum perturbations:* $\check\zeta = -\psi - \dfrac{H}{\dot V}\delta V$

Two models for perturbations:

- Barotropic (adiabatic) model: $V=V(\rho_m)$:

$$\check{\zeta} = \zeta_m \quad \Rightarrow \quad \check{S} = 0$$

  - *adiabatic sound speed* $$c_s^2 = \frac{\dot{P}_{\rm gCg}}{\dot{\rho}_{\rm gCg}} = \frac{\alpha V}{\rho_m + V}$$
  - *comoving vacuum perturbation* $$f = -\delta\check{\rho}_{\rm com} = -\frac{\dot{V}}{\dot{\rho}_m}\delta\rho_{\rm com}$$

- Geodesic (non-adiabatic): energy transfer along 4-velocity, $Q_\nu = Q\, u_\nu$

$$\check{\zeta} \neq \zeta_m \quad \Rightarrow \quad \check{S} \neq 0$$

  - *zero momentum transfer:* $f = -\delta\check{\rho}_{\rm com} = 0$
  - *zero sound speed:* $\delta\check{P}_{\rm com} = 0 \quad \Rightarrow \quad c_s^2 = \left(\frac{\delta P}{\delta\rho}\right)_{\rm com} = 0$

Barotropic model matter power spectrum:

“The end of unified dark matter?”

Havard Sandvik, Max Tegmark ,
Matias Zaldarriaga, Ioav Waga
astro-ph/0212114

$$c^2 = -\alpha w$$

$$-0.00000081 < \alpha < 0.0000079$$

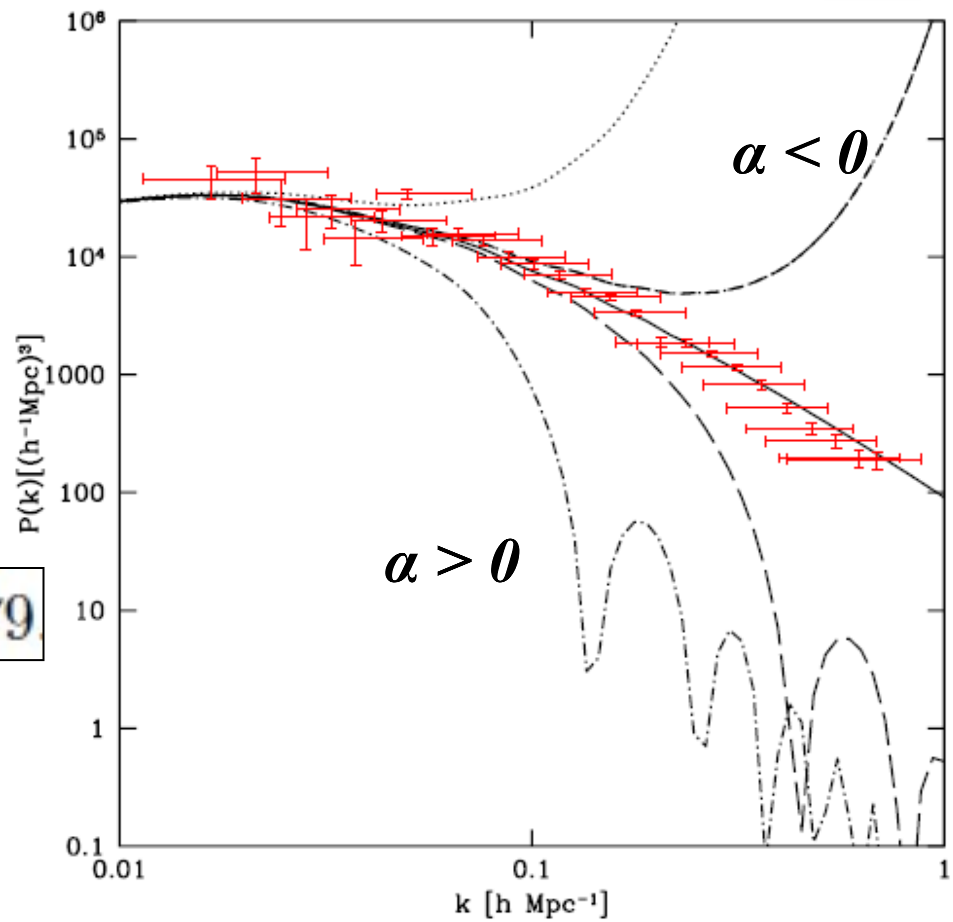


FIG. 1. UDM solution for perturbations as function of wavenumber, k. From top to bottom, the curves are GCG models with $\alpha = -10^{-4}$, -10^{-5}, 0 (ΛCDM), 10^{-5} and 10^{-4}, respectively. The data points are the power spectrum of the 2df galaxy redshift survey.

Geodesic (non-adiabatic) model:

Energy flow is along dark matter velocity, $Q_\nu = Q\, u_\nu$

- No momentum exchange in the dark matter rest frame

  ⇒ matter follows geodesics $\quad f = -\delta\check{\rho}_{\rm com} = 0$

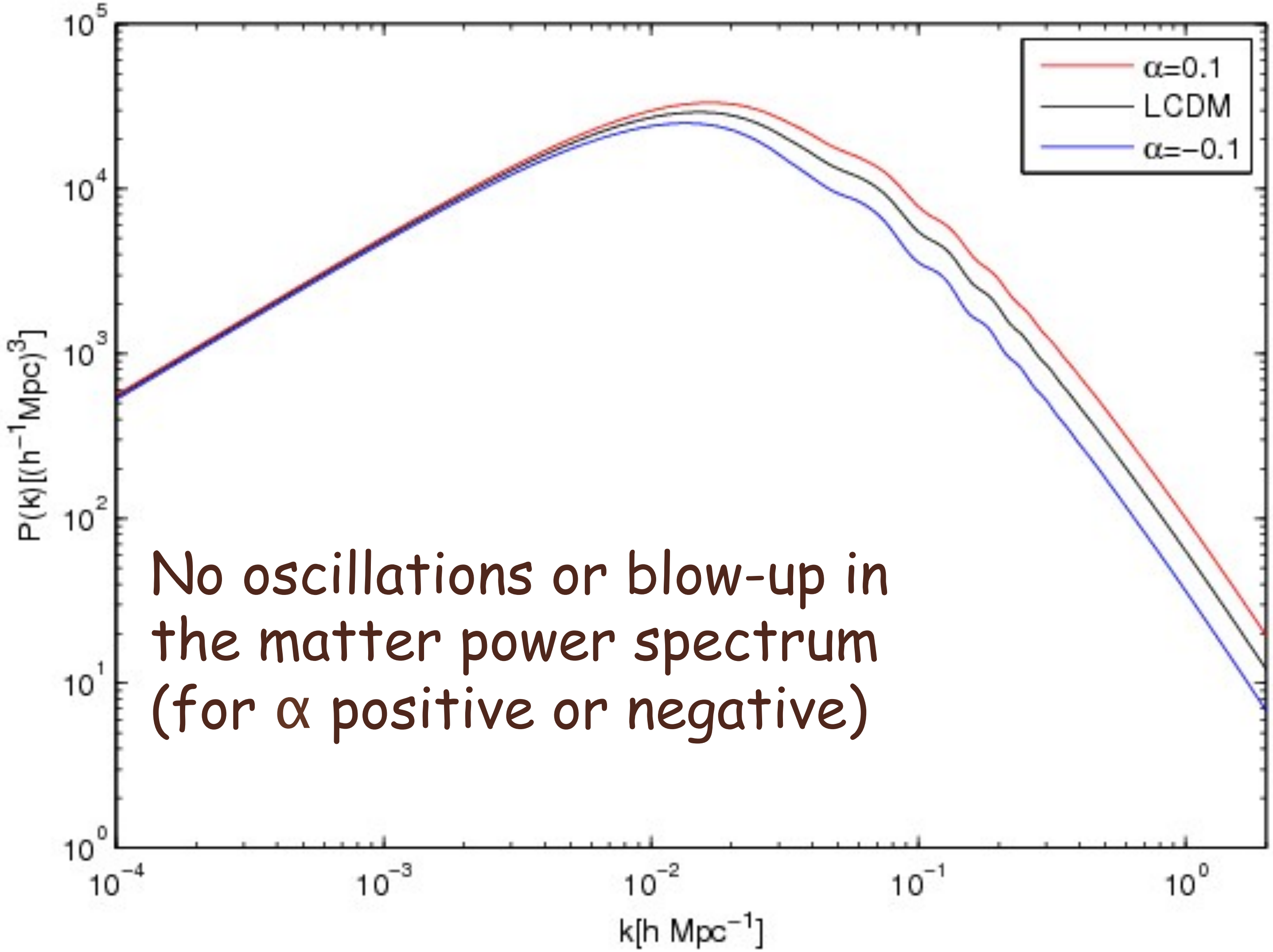
  ⇒ zero sound speed $\quad \delta\check{P}_{\rm com} = 0$

  ⇒ note: matter velocity *irrotational* (like a scalar field)

$$u_\mu \propto \nabla_\mu V$$

See also Dust of dark energy, Lim, Sawicki & Vikman, arXiv:1003.5751
Creminelli, d'Amico, Norena, Senatore & Vernizzi, arXiv:0911.2701
Irrotational dark matter, Sawicki, Marra, Valkenburg, arXiv:1307.6150
Mimetic dark matter, Chamseddine, Mukhanov & Vikman, arXiv:1403.3961

Geodesic model matter power spectrum, P(k)



No oscillations or blow-up in the matter power spectrum (for α positive or negative)

Model-independent constraints on geodesic interaction:

Salvatelli, Said, Bruni, Melchiorri & Wands, arViv:1406.7297

$$Q^\mu(z) = -q_V(z) H V u^\mu$$

single dimensionless parameter, q_V, allowed to vary with redshift

in principle decompose into many independent redshift bins

in practice choose 4 redshift bins constrained by CMB, SN, +redshift space distortions

$$q_V(z) = \begin{cases} q_1 & z > 2.5 & \text{CMB} \\ q_2 & 0.9 < z < 2.5 & \text{hi z SN} \\ q_3 & 0.3 < z < 0.9 & \text{RSD} \\ q_4 & 0 < z < 0.3 & \text{lo z SN} \end{cases}$$

and require $q_i < 0$ to avoid any negative matter density

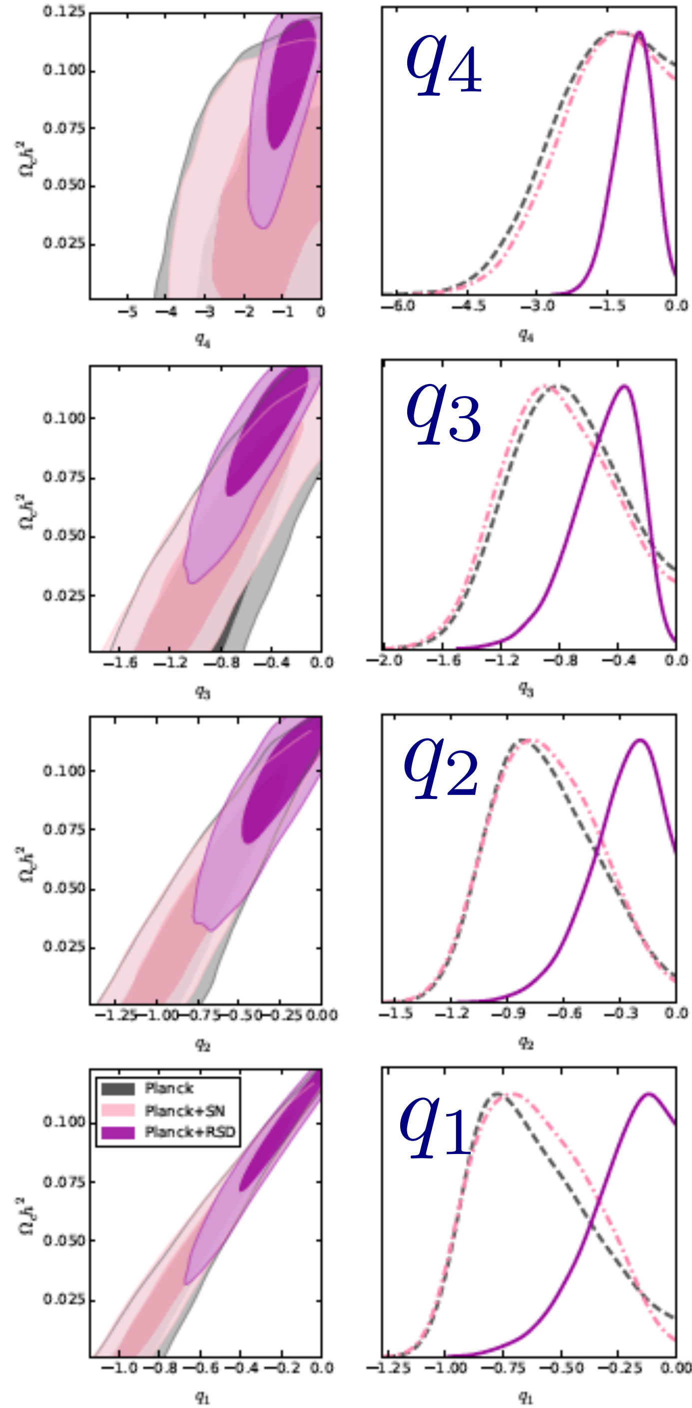
Observational constraints:

Salvatelli, Said, Bruni, Melchiorri & Wands, arViv:1406.7297

$$Q(z) = -q_V(z) H V$$

main degeneracy (shown right) is with present matter density $\Omega_m h^2$ broken by including redshift-space distortions (RSD)

	Planck	Planck+SN	Planck+RSD
$100\Omega_b h^2$	2.203 ± 0.029	2.203 ± 0.029	2.217 ± 0.028
$\Omega_c h^2$	< 0.060	$0.049^{+0.018}_{-0.044}$	$0.0918^{+0.026}_{-0.010}$
$100\theta_{MC}$	$1.0463^{+0.0032}_{-0.0024}$	$1.0460^{+0.0023}_{-0.0028}$	$1.04302^{+0.00095}_{-0.00183}$
τ	$0.087^{+0.012}_{-0.014}$	$0.086^{+0.012}_{-0.014}$	$0.086^{+0.012}_{-0.013}$
n_s	0.9597 ± 0.0078	0.9599 ± 0.0078	$0.9638^{+0.0071}_{-0.0078}$
$\ln(10^{10} A_s)$	$3.084^{+0.024}_{-0.026}$	$3.082^{+0.024}_{-0.027}$	3.078 ± 0.024
q_1	$-0.62^{+0.18}_{-0.31}$	$-0.61^{+0.21}_{-0.29}$	> -0.29
q_2	$-0.70^{+0.24}_{-0.33}$	$-0.69^{+0.26}_{-0.31}$	$-0.291^{+0.255}_{-0.098}$
q_3	$-0.76^{+0.37}_{-0.40}$	$-0.80^{+0.36}_{-0.42}$	$-0.49^{+0.28}_{-0.16}$
q_4	> -2.12	$-1.58^{+1.51}_{-0.506}$	$-0.92^{+0.48}_{-0.34}$



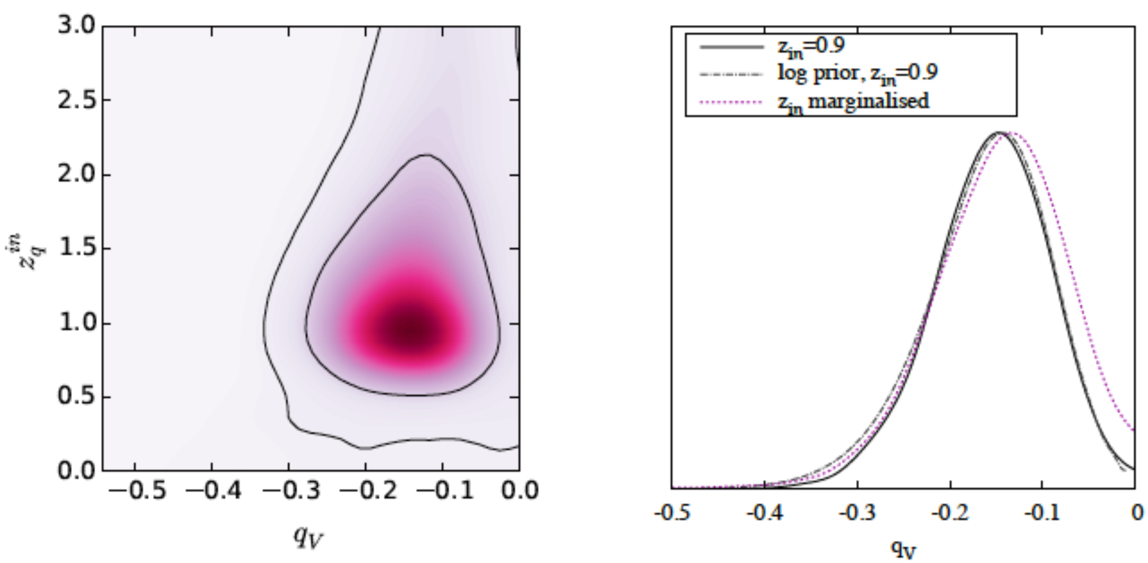
A late-time interaction?

null interaction excluded at 95% c.l. in bins 3 and 4 (that is z < 0.9)

improves best fit chi-squared by -7

almost equally well fit by simple step-function switching on at $z<z_{in}$ =0.9

$$q_V(z) = \begin{cases} 0 & z > 0.9 \\ q_{34} & z < 0.9 \end{cases}$$

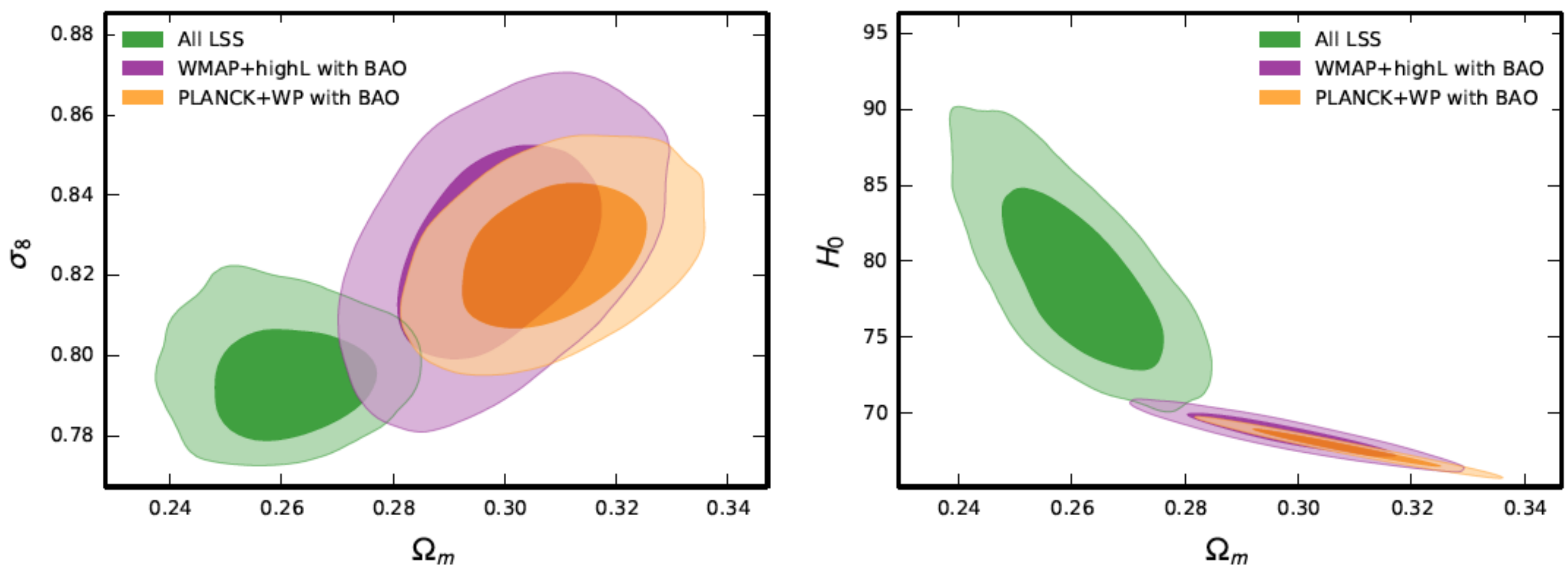


could resolve tension between Planck and growth of structure (σ_8)?

Tension between CMB+LSS

Battye, Charnock and Moss, arXiv:1409.2769

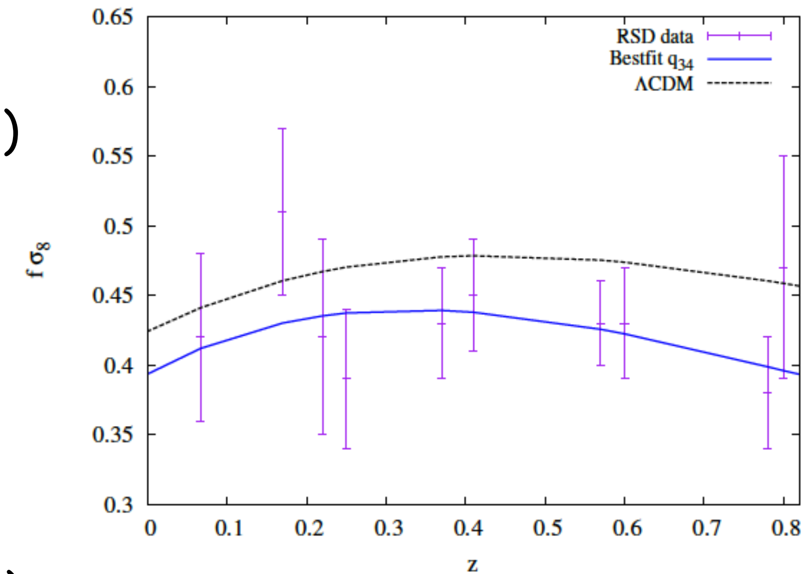
tension between WMAP/Planck vs LSS (RSD, SZ clusters and lensing)



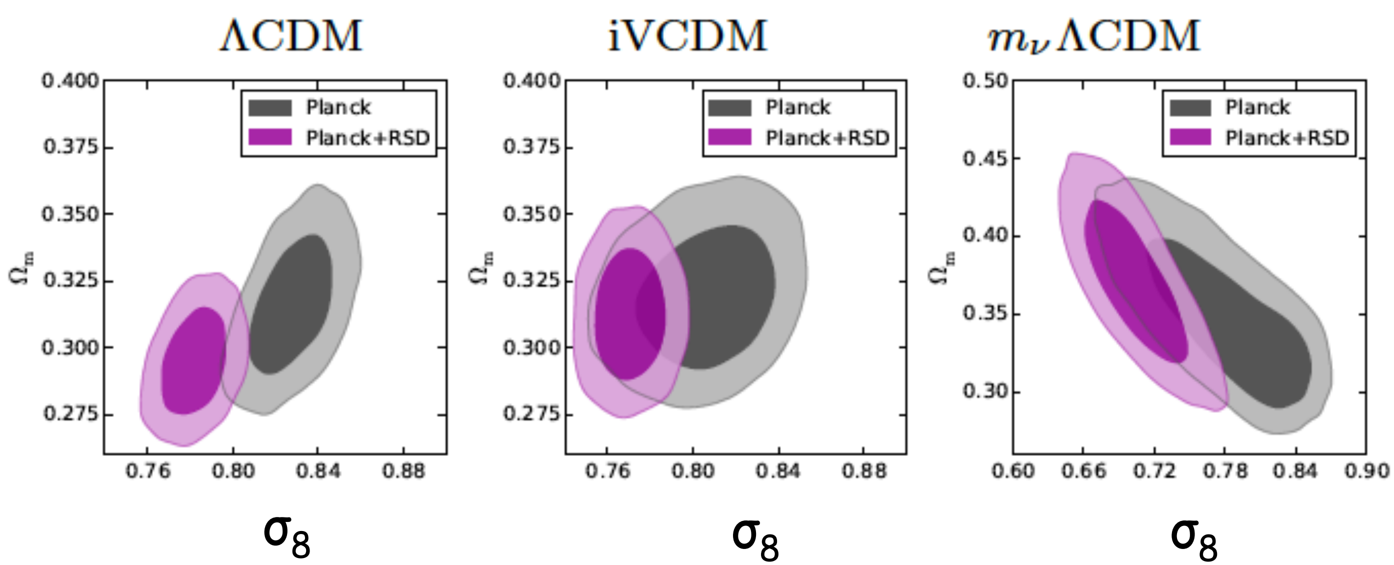
see also Battye & Moss; Hamann & Hasenkamp; Wyman et al; Leistedt, Peiris & Verde 2014

Redshift-space distortions vs CMB

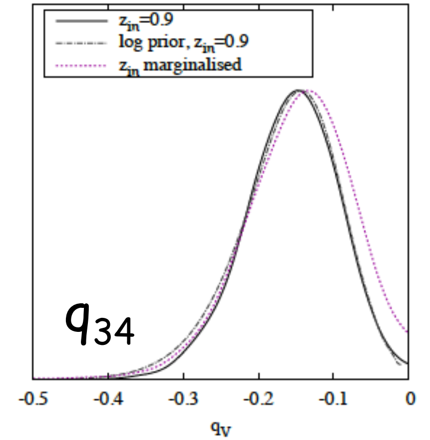
Redshift-space distortions due to linear growth of matter perturbations (-> peculiar velocities)



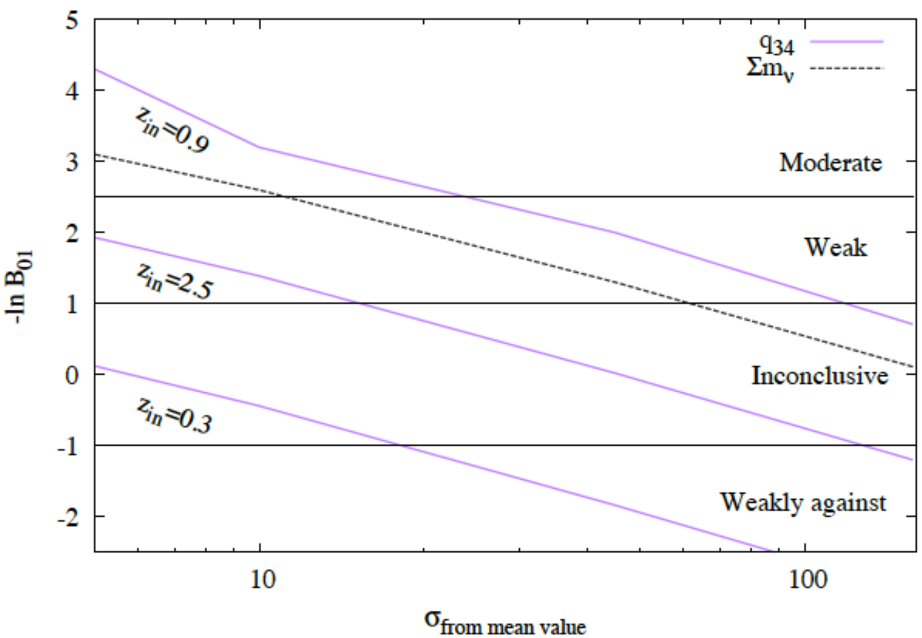
tension between Planck and RSD (σ_8)



Evidence for a late-time interaction?



Bayes evidence for a one-parameter extension of base (ΛCDM) model



but dependent on theory prior range allowed, shown here in terms of standard deviation from mean value, since q_{34} is a phenomenological parameter

still need better theoretical perspectives:

- **simplified model of more general interacting DE?**

- **why/how does the vacuum interact?**
  - integrating out dynamical degrees of freedom?
  - singular limit of k-essence, P(X,phi)
    - Dust-dark energy (Lim et al)
    - Mimetic dark energy (Mukhanov et al)

- **broken Lorentz-invariance**
  - time-dependent vacuum defines preferred foliation of spacetime
  - time-dependent zeta for adiabatic matter perturbations
  - simple model of modified gravity (c.f. Horava-Lifshitz)

summary:

- **vacuum energy is simplest model for acceleration**
  - vacuum+matter: no new degrees of freedom

- **inhomogeneous vacuum** implies **energy-momentum transfer**

- **any dark energy cosmology** can be decomposed into **interacting vacuum+fluid** (like scalar field quintessence)

- **spatially inhomogeneous perturbations** require **covariant model for interactions**

- **model-independent constraints**
  - *evidence for a late-time interaction?*