Non-perturbative Dynamics of Cosmological Fields

1. Gravitational Aspects of Oscillons/Solitons
2. Stochastic Particle Production In Cosmology

Mustafa A. Amin

RICE

May 9, 2019

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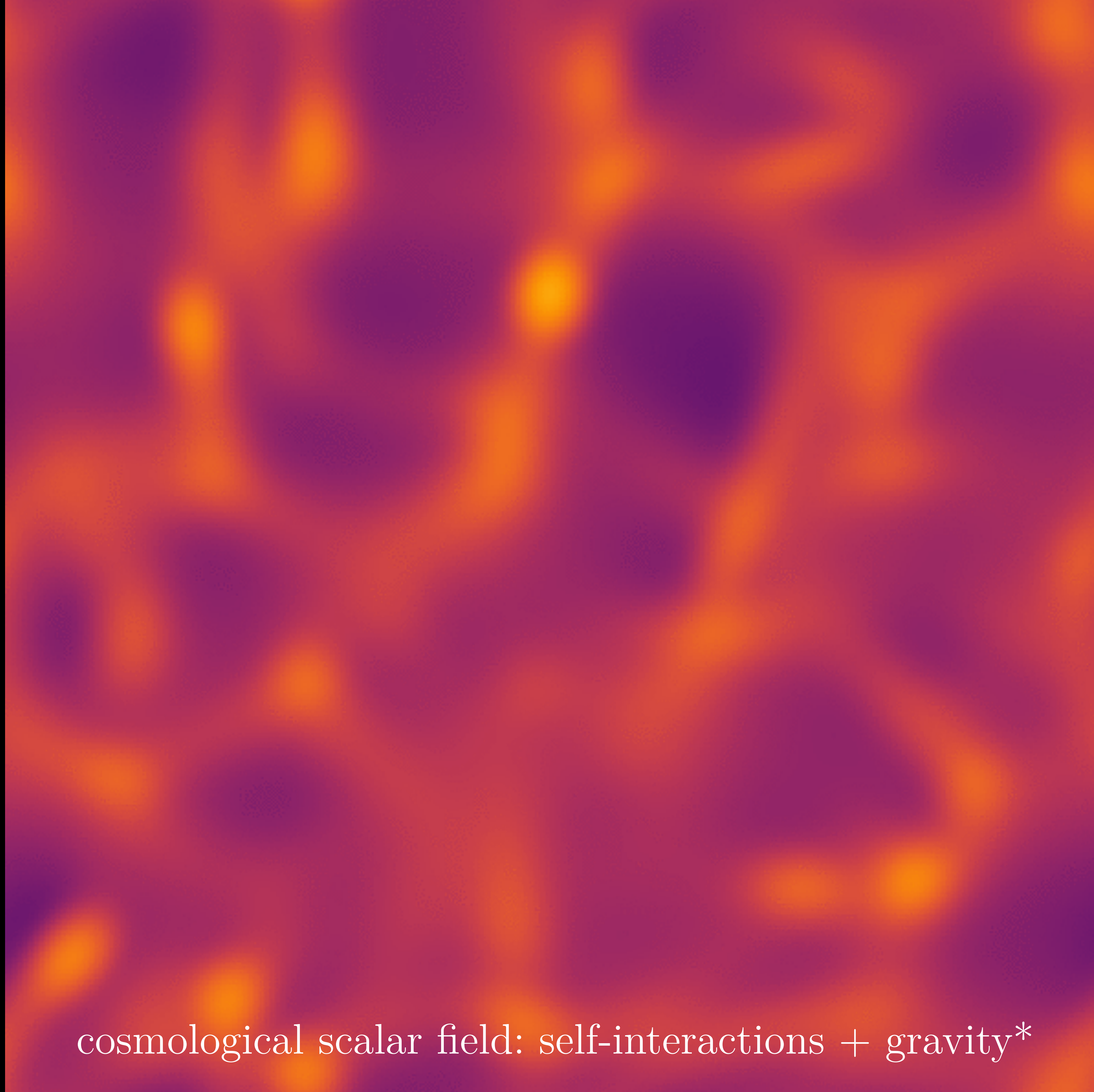
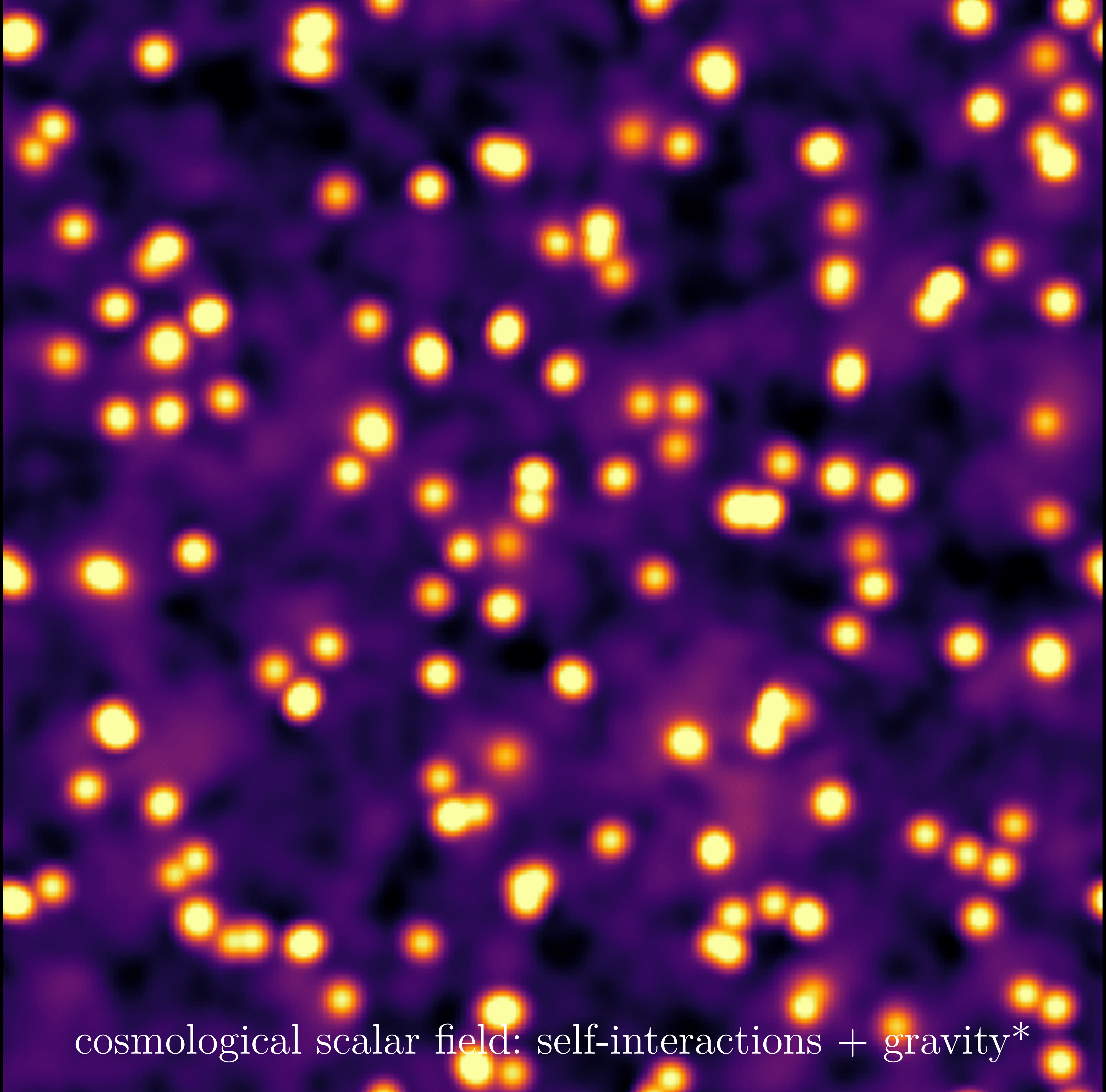
cosmological scalar field: self-interactions + gravity*

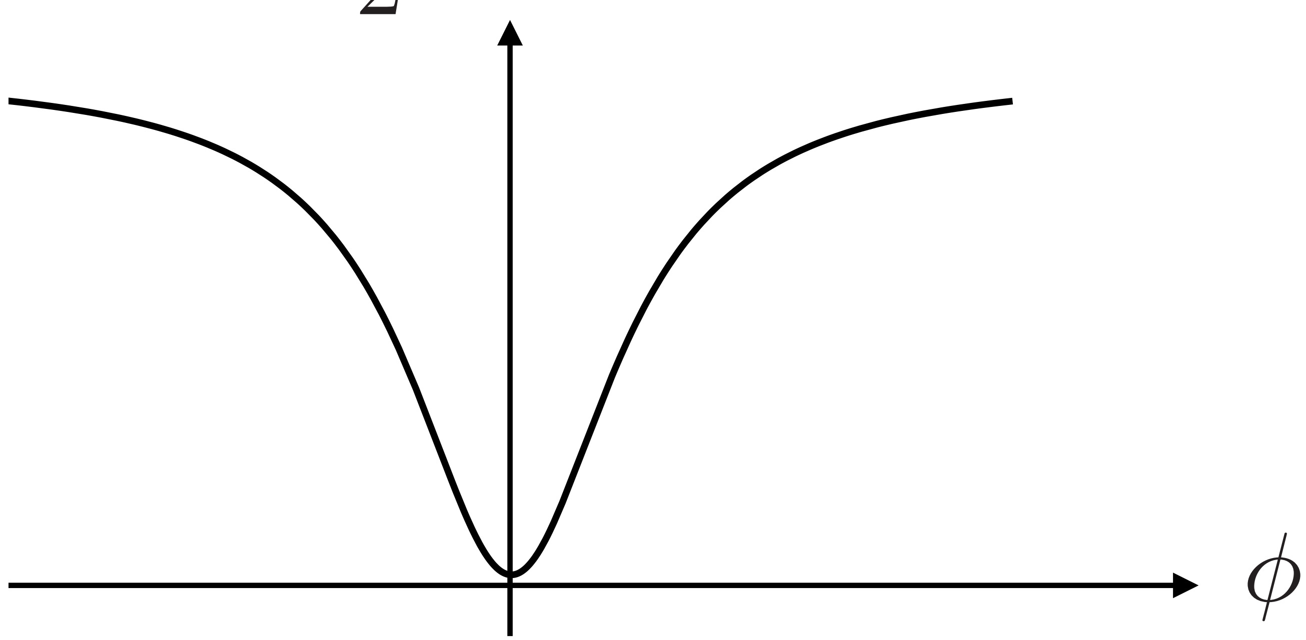

cosmological scalar field: self-interactions + gravity*


cosmological scalar field: self-interactions + gravity*

cosmological scalar fields attractive self-interaction + gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_{\rm nl}(\phi)$$



previous movie was non-relativistic*

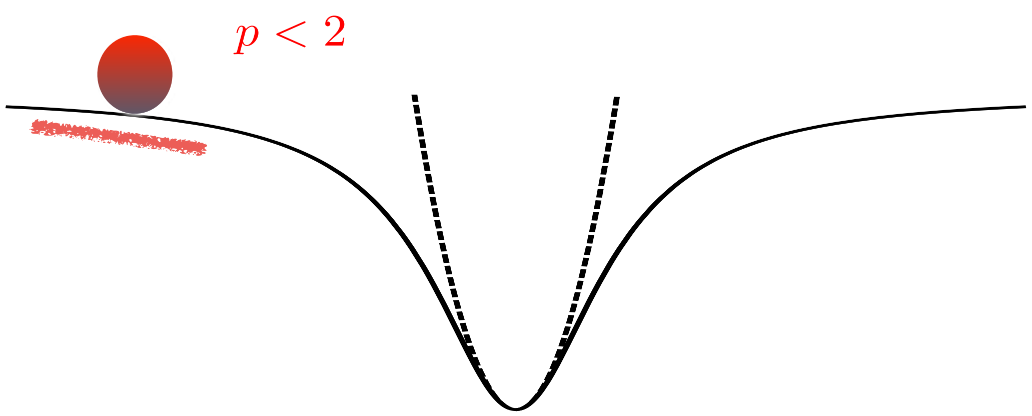
plan for the talk (part 1)

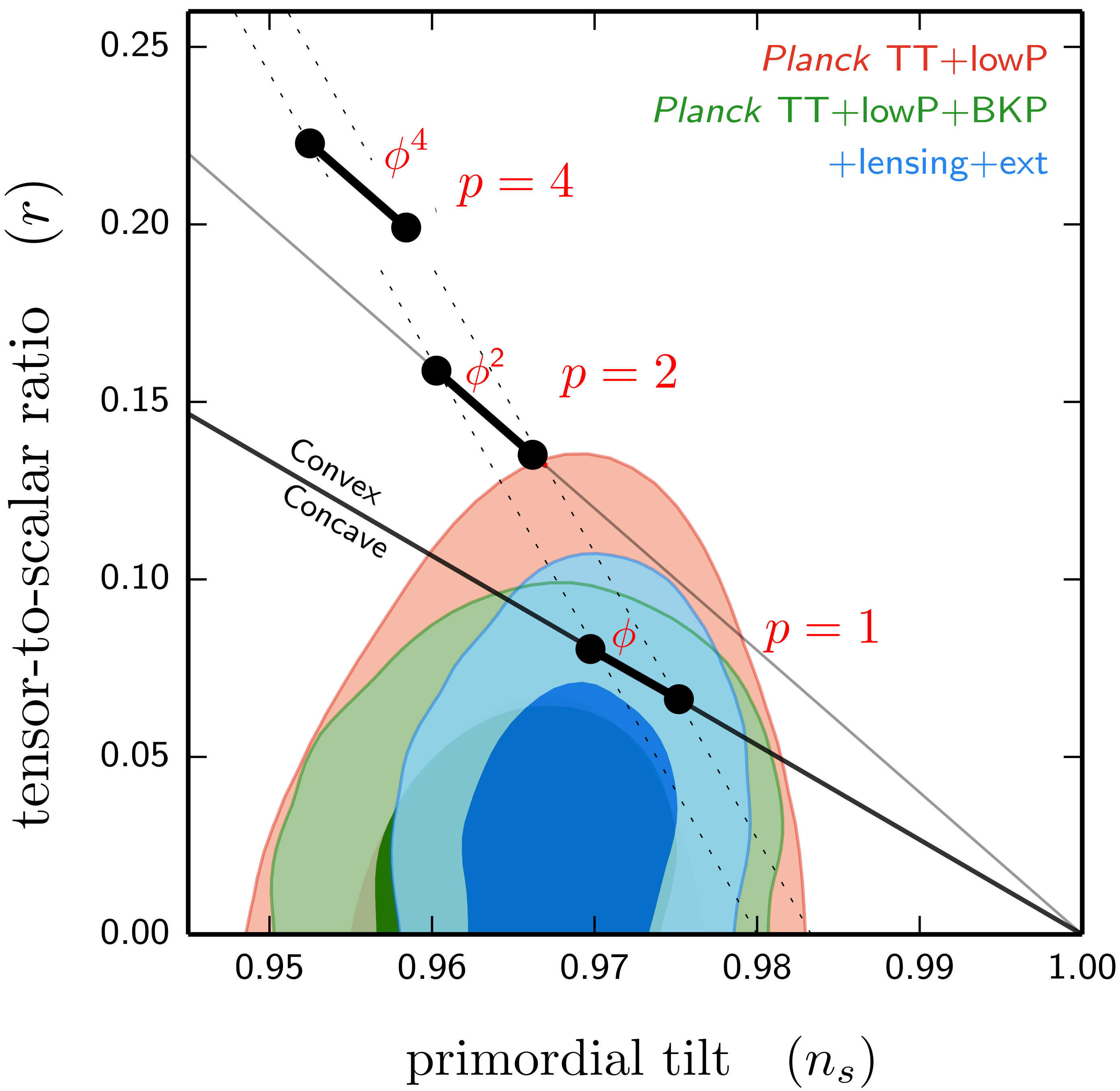
- **motivation & implications**
- **understanding what is happening in the movie**

1. relativistic but no gravitational clustering
   (w/ K. Lozanov, 1902.06736)
2. non-relativistic with gravitational clustering
   (w/ P. Mocz, 1902.07261)
3. + earlier papers with Shirokoff, Lozanov, Easther, Hertzberg, Finkel, Flauger …

inflation:
post-inflationary dynamics

$V(\phi) \propto \phi^p$

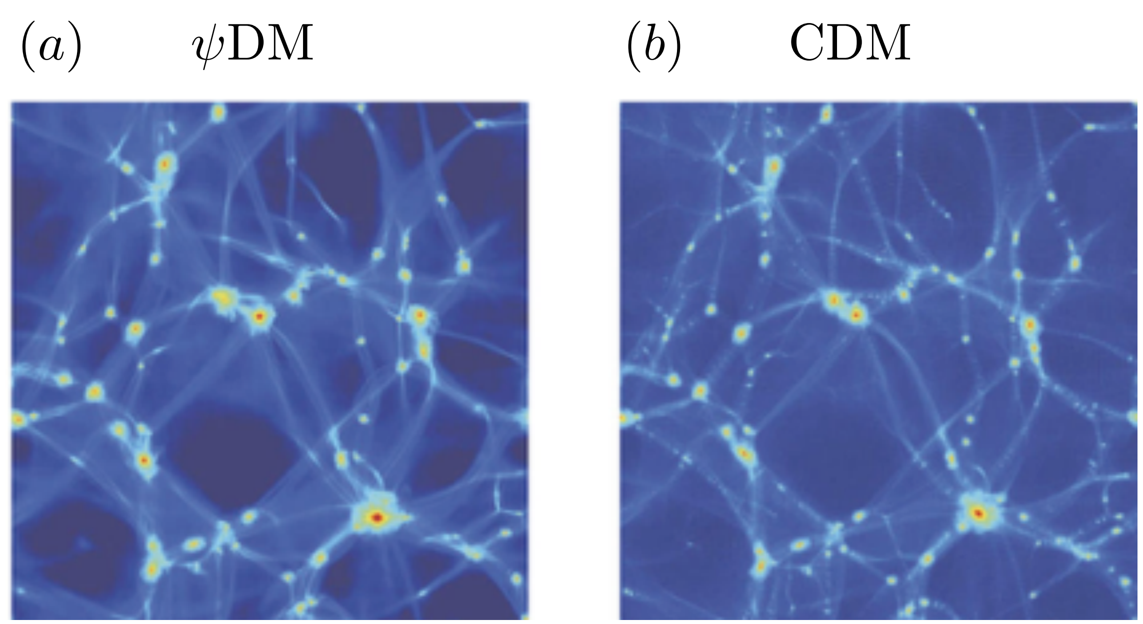


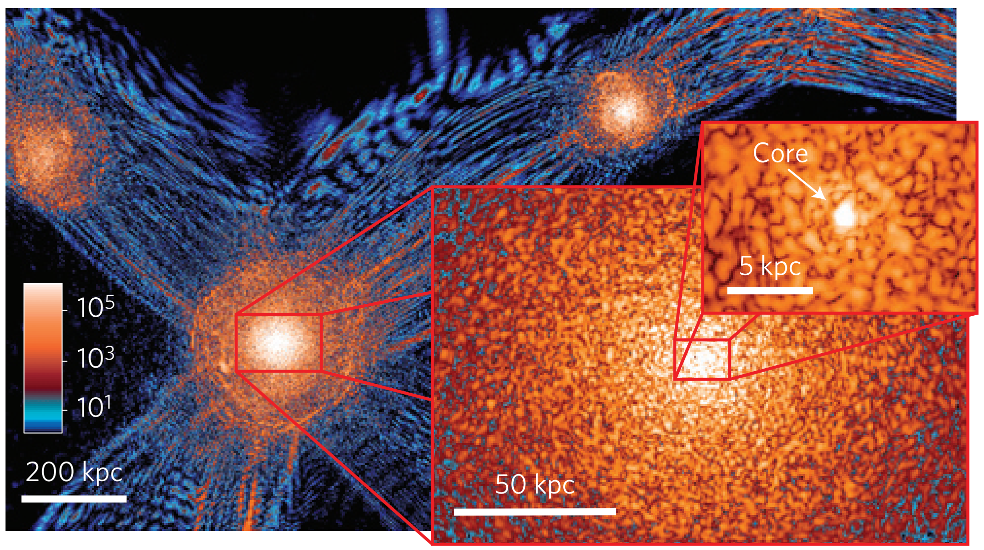


for example:

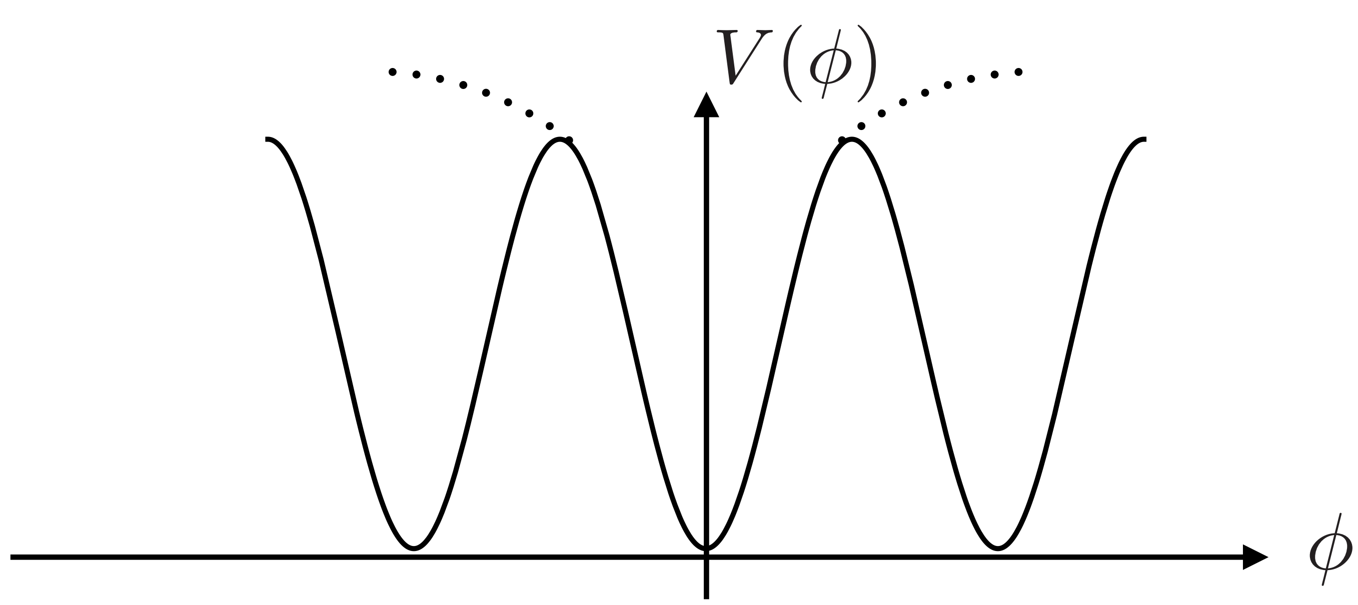
Starobinsky Inflation (1979)
Silverstein & Westhpal (2008)
Kallosh & Linde (2013)

axion-like fields





Schive et. al (2014)



for example:
Peccei & Quinn (1977)
Hu, Barkana & Gruzinov (2000)
Arvanitaki et. al (2009)

implications

- reheating after inflation ?
- stochastic gravitational wave-generation ?
  - constrained by $N_{\rm eff}$ or direct detection
- primordial black hole formation ?

....

- distinguishability from WIMPS ? (small scales)
- early structure formation
- compact objects
  - eg. sources of gravitational waves ?

eq. of motion for cosmological fields

$$\Box\phi + V'(\phi) = 0$$

nonlinear Klein-Gordon eq.

$$G_{\mu\nu} = \frac{1}{m_{\rm pl}^2} T_{\mu\nu}$$

Einstein Eq.

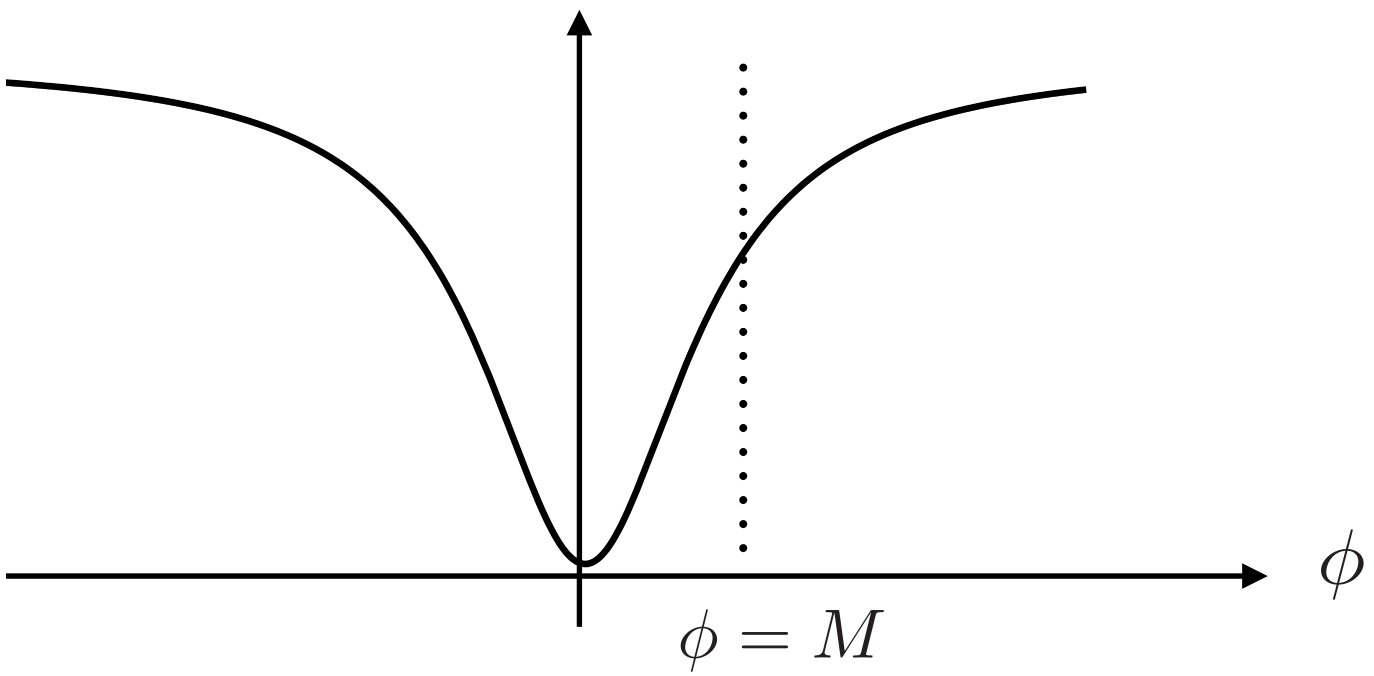
$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_{\rm nl}(\phi)$$

examples:

$$V(\phi) = \frac{m^2M^2}{2}\tanh^2\left(\frac{\phi}{M}\right)$$

$$V(\phi) = m^2M^2\left[\sqrt{1+\frac{\phi^2}{M^2}} - 1\right]$$

$$V(\phi) = m^2M^2\left[1 - \cos\frac{\phi}{M}\right]$$



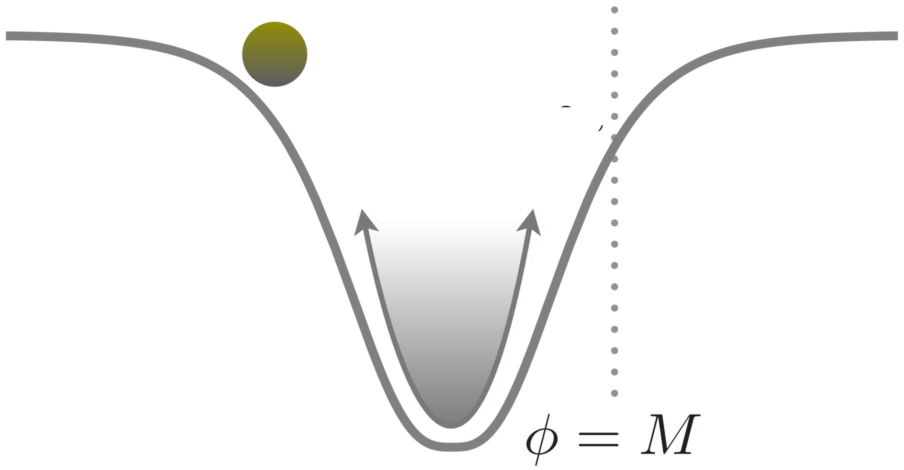
instabilities in oscillating fields

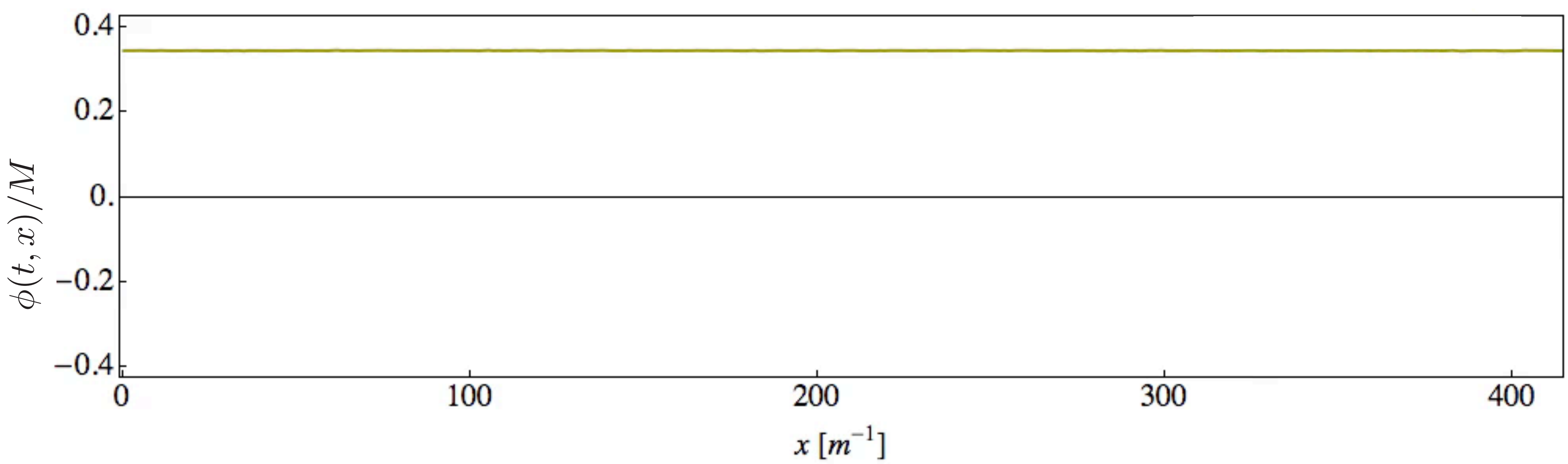
expansion ✓

self-interactions ✓

gravitational int. ✗

$$\Box\phi = V'(\phi)$$





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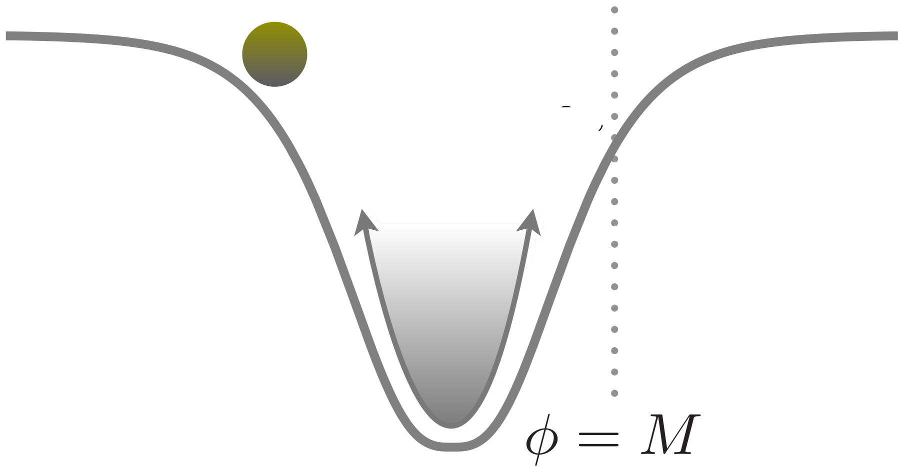
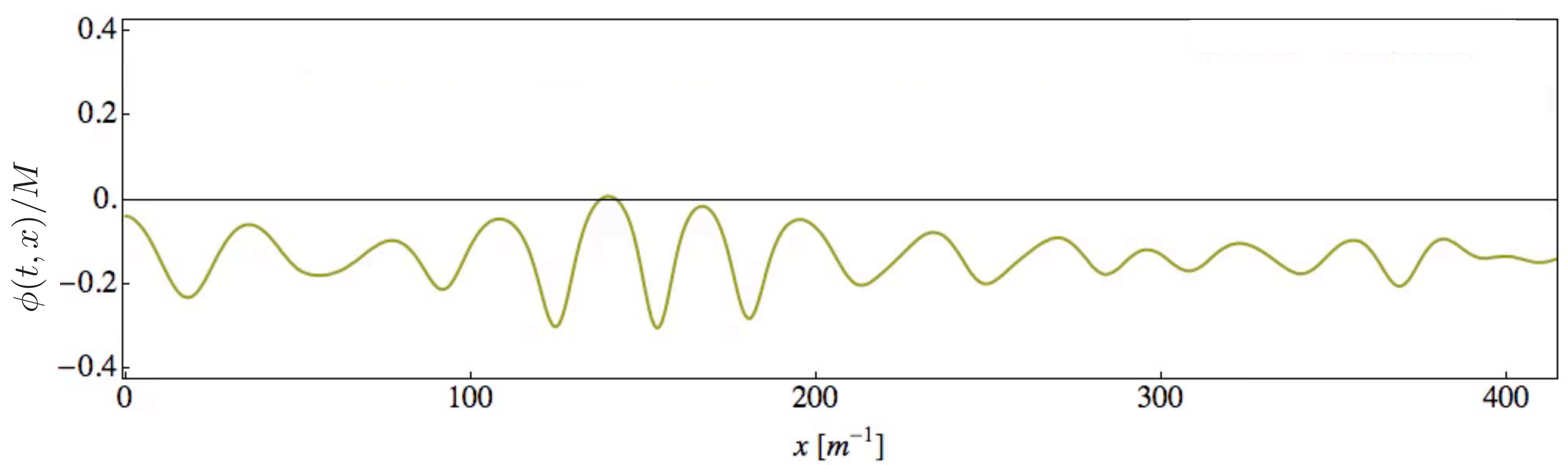


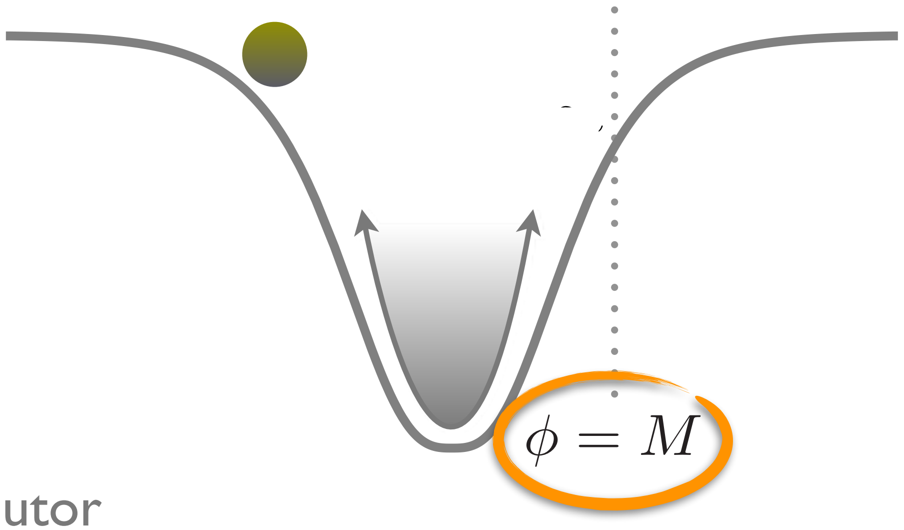


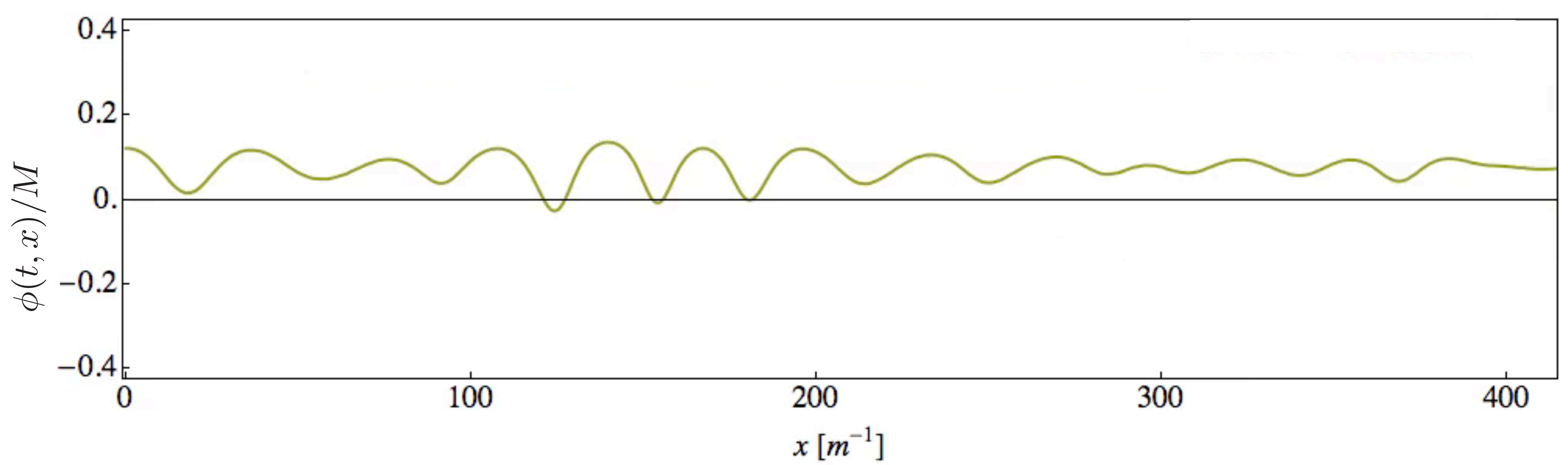
instabilities in an expanding universe

$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} = \frac{\Re[\mu_k]}{H} \sim \frac{m_{\rm pl}}{M} \gg 1$$

general

dominant contributor to energy density





dynamics of oscillating fields

expansion ✓

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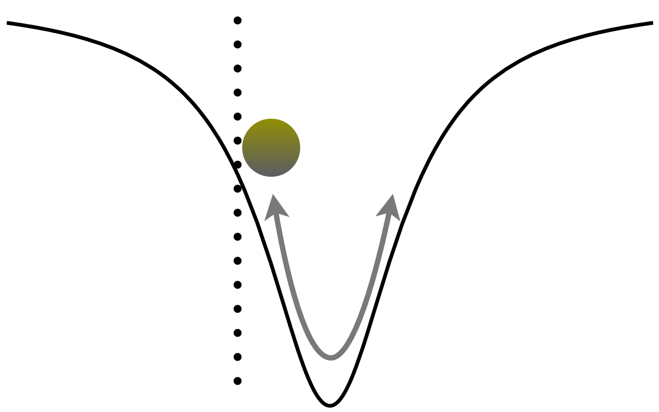
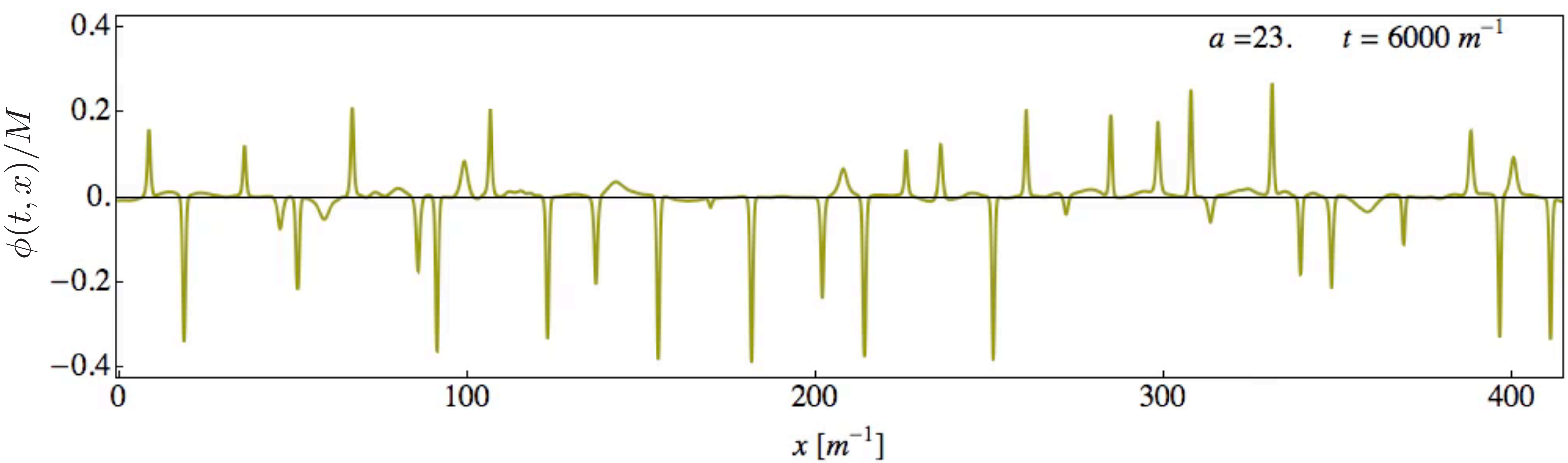
$$\Box\phi = V'(\phi)$$

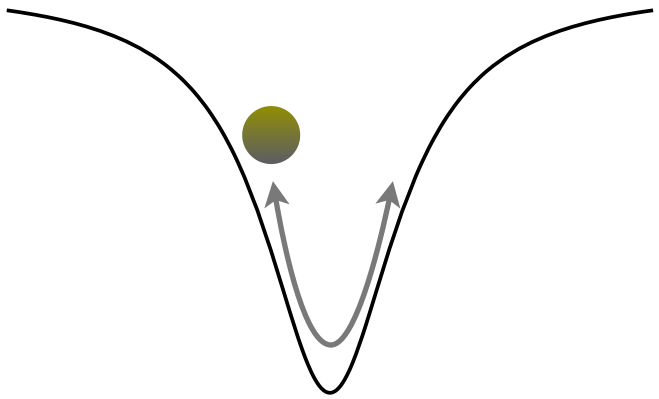


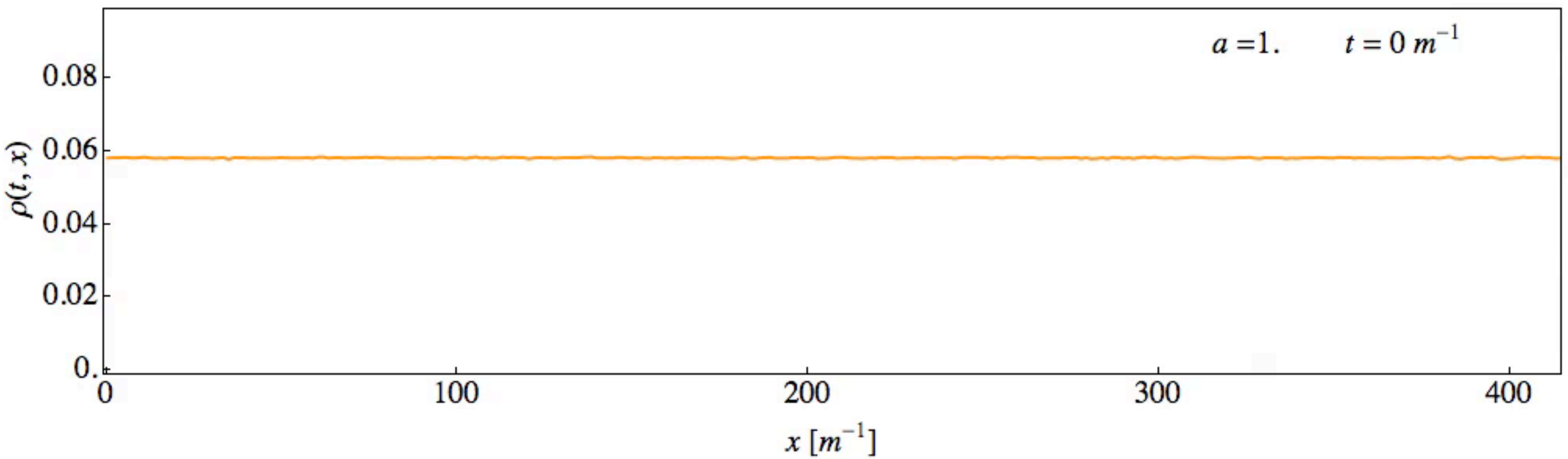


MA (2010)

dynamics of oscillating fields

$$\Box\phi = V'(\phi)$$





dynamics of oscillating fields

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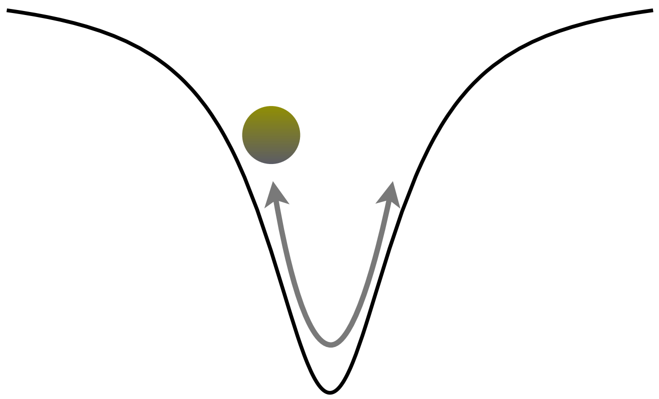
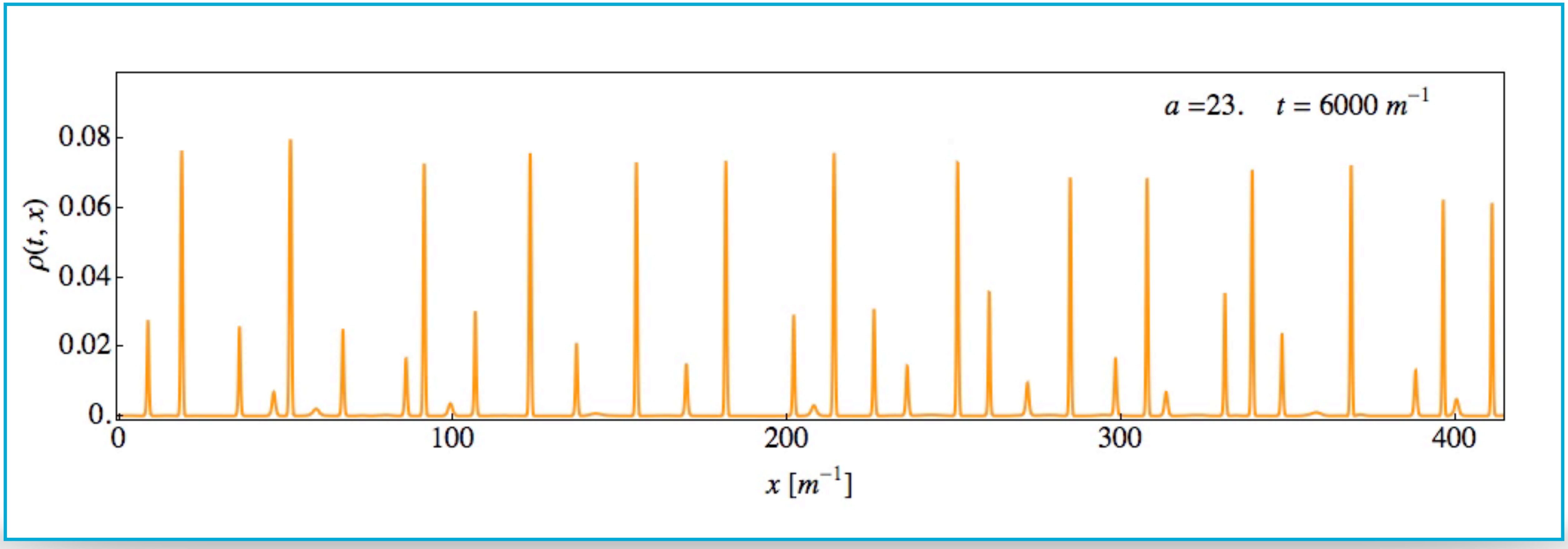




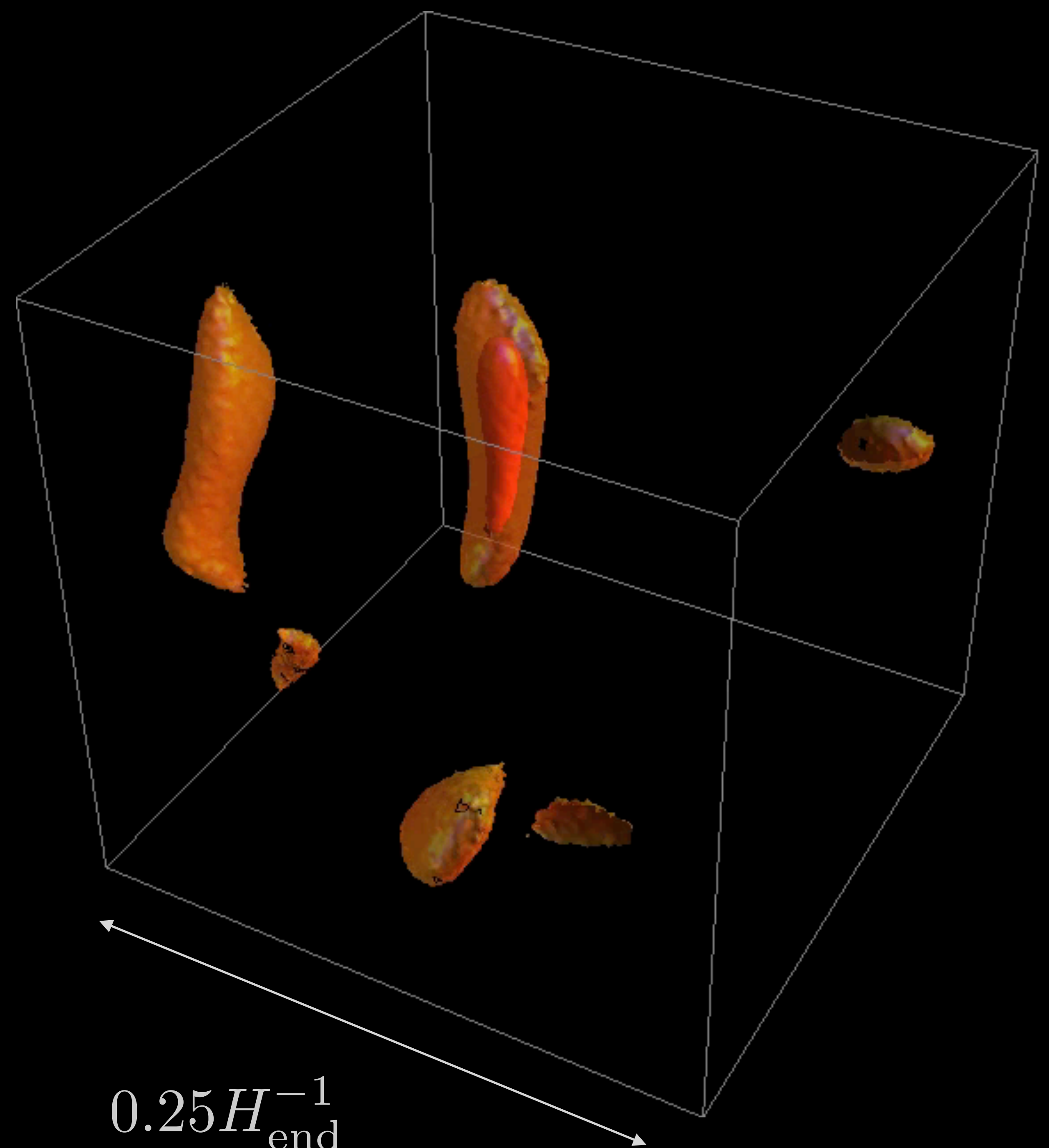
soliton formation in relativistic fields

expansion ✓

self-interactions ✓

gravitational int. ✗

solitons dominate the energy density of the field!



MA, Easther & Finkel (2010)

MA, Easther, Finkel, Flaugher & Hertzberg (2011)

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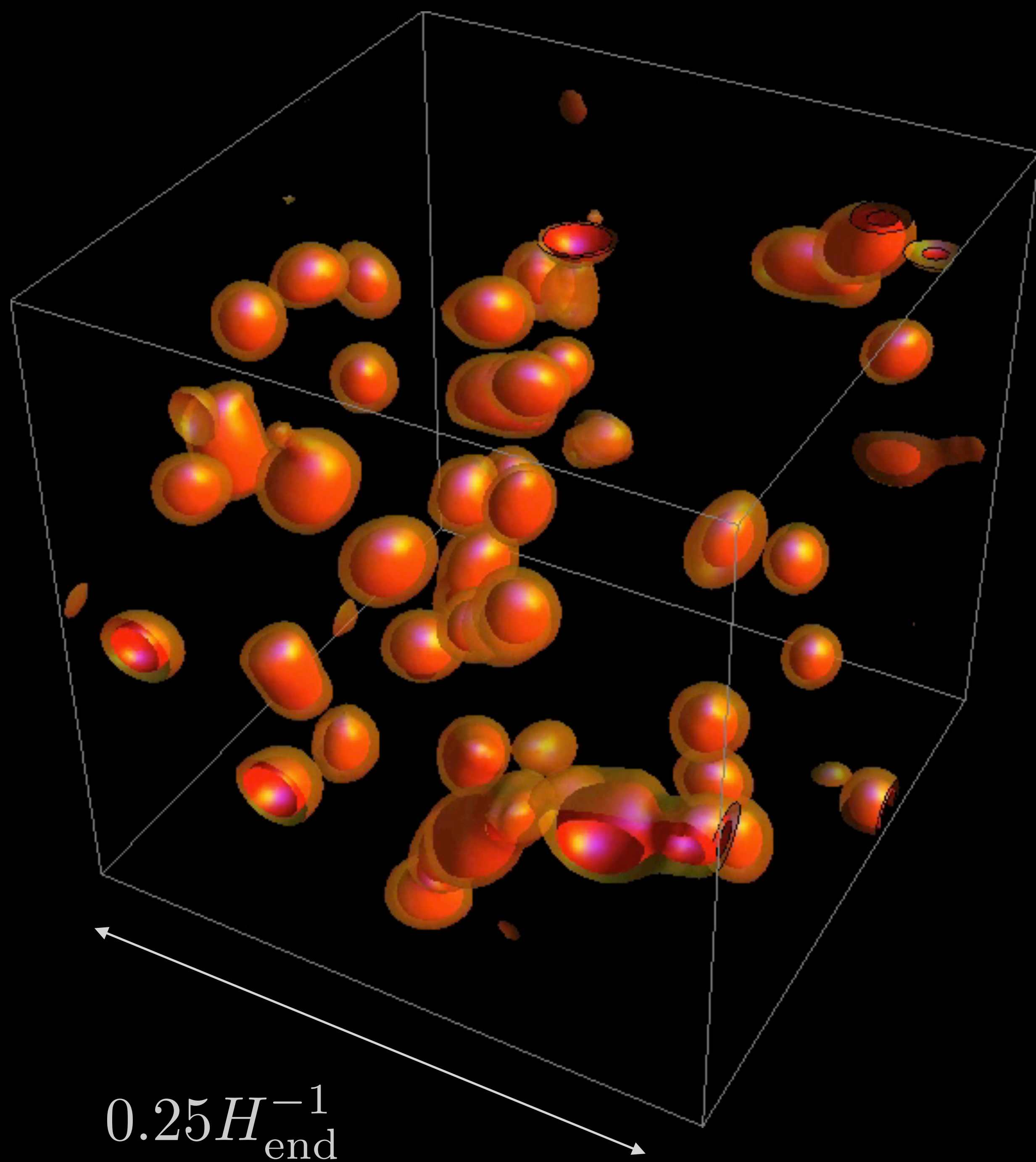
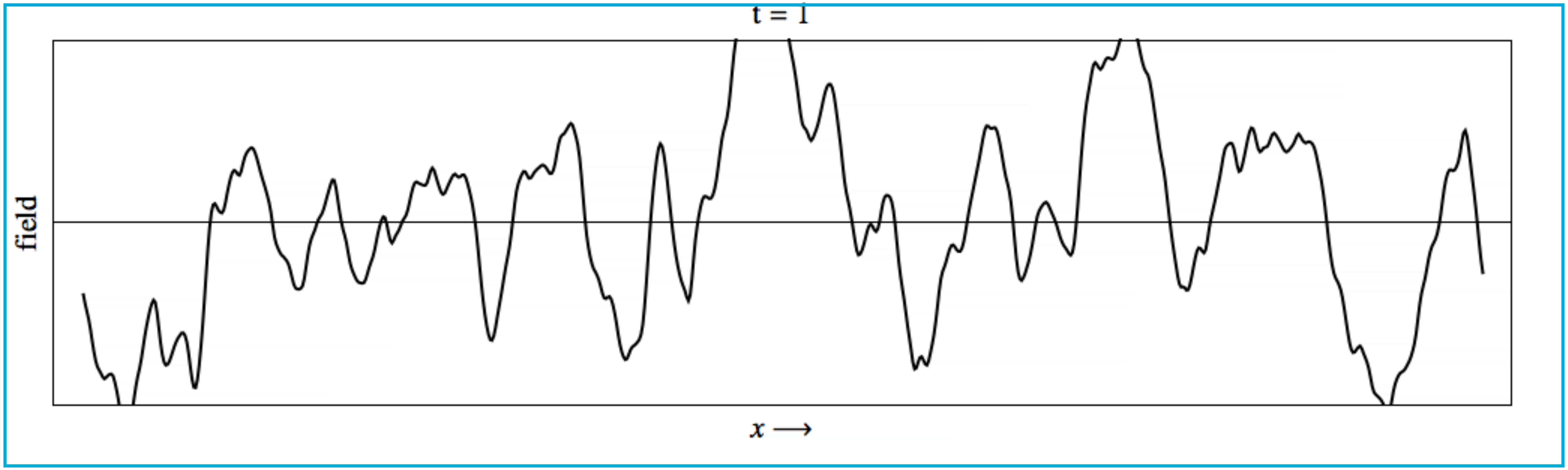


MA, Easther & Finkel (2010)

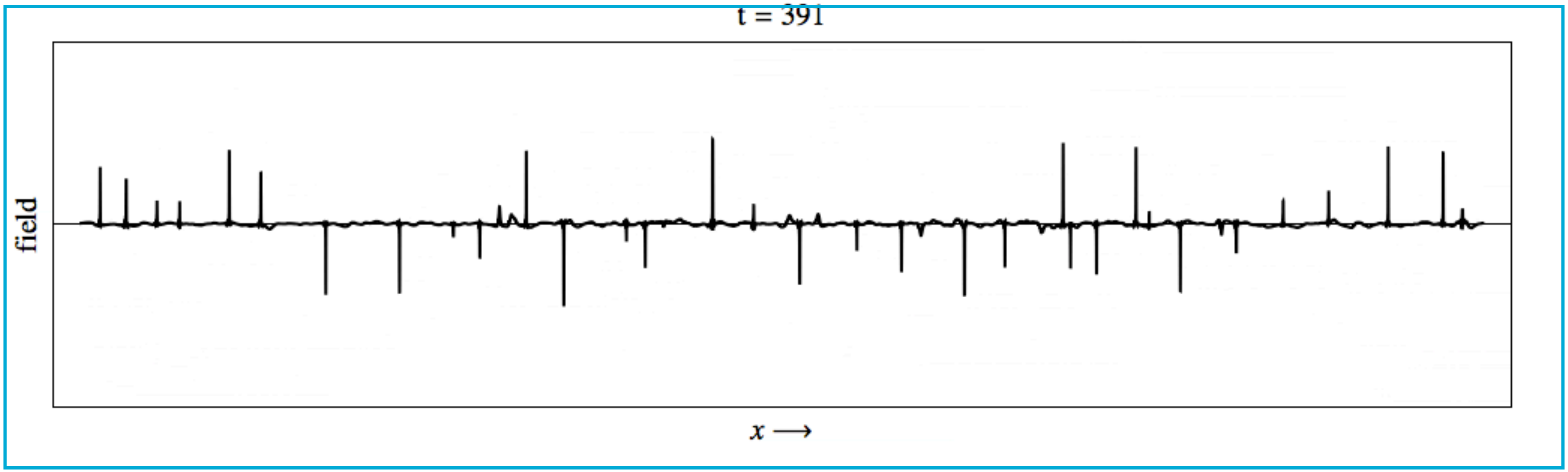
MA, Easther, Finkel, Flaugher & Hertzberg (2011)

insensitive to initial conditions



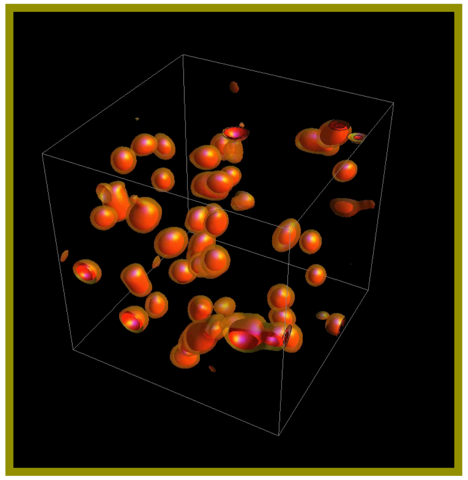
simulation of "quasi-thermal" example in Farhi et. al 2008

insensitive to initial conditions

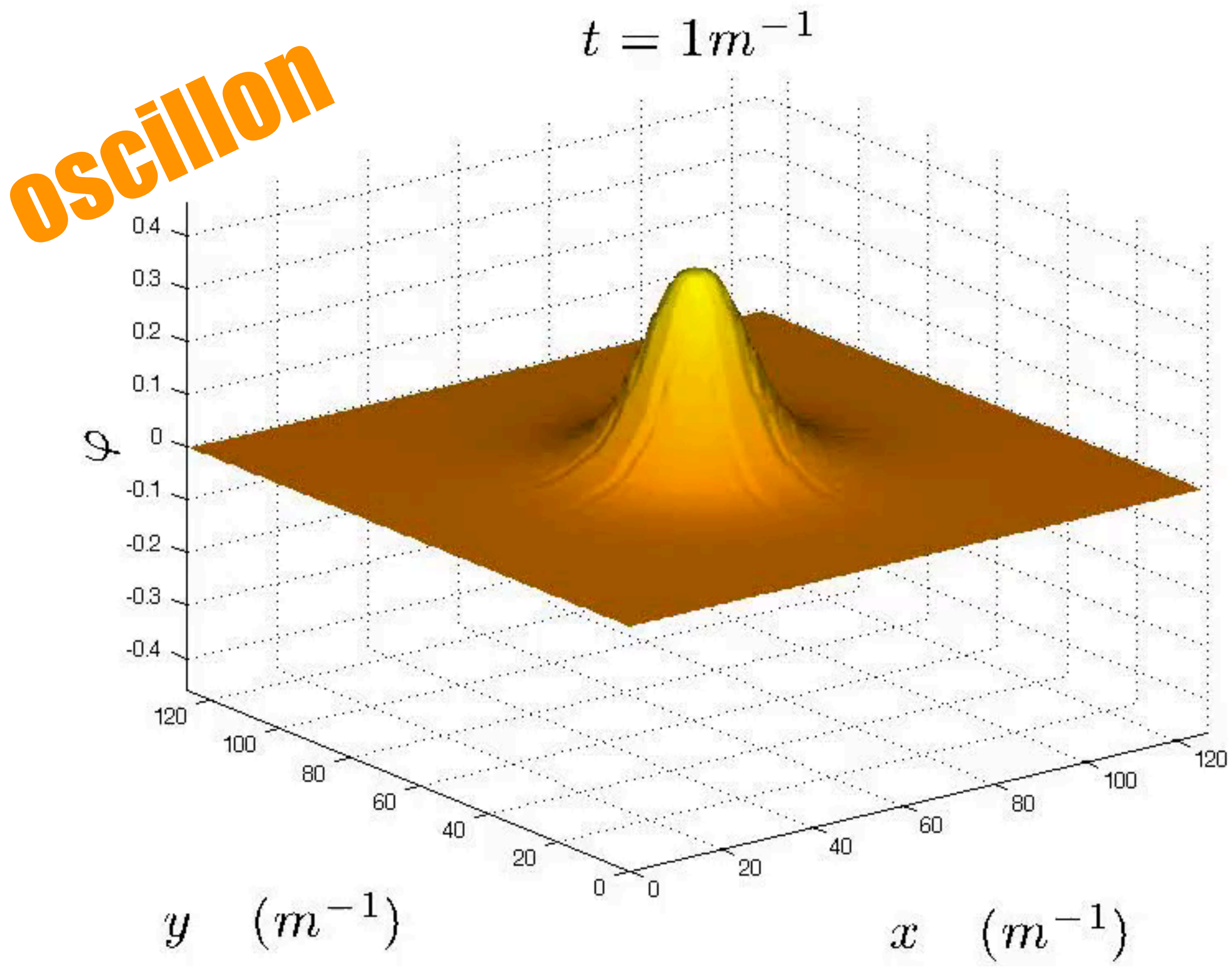


simulation of "quasi-thermal" example in Farhi et. al 2008

solitons?



(1) oscillatory (2) spatially localized (3) **very long lived**



existence and stability:

Segur & Kruskal (1987)
Kasuya, Kawasaki, Takashi (2003)
Hertzberg (2011)
Mukaido et. al (2016)
Salmi & Hindmarsh (2014)

MA (2013)
MA & Shirokoff (2010)

Bogolubsky & Makhankov (1976), Gleiser (1994), Copeland et. al (1995)

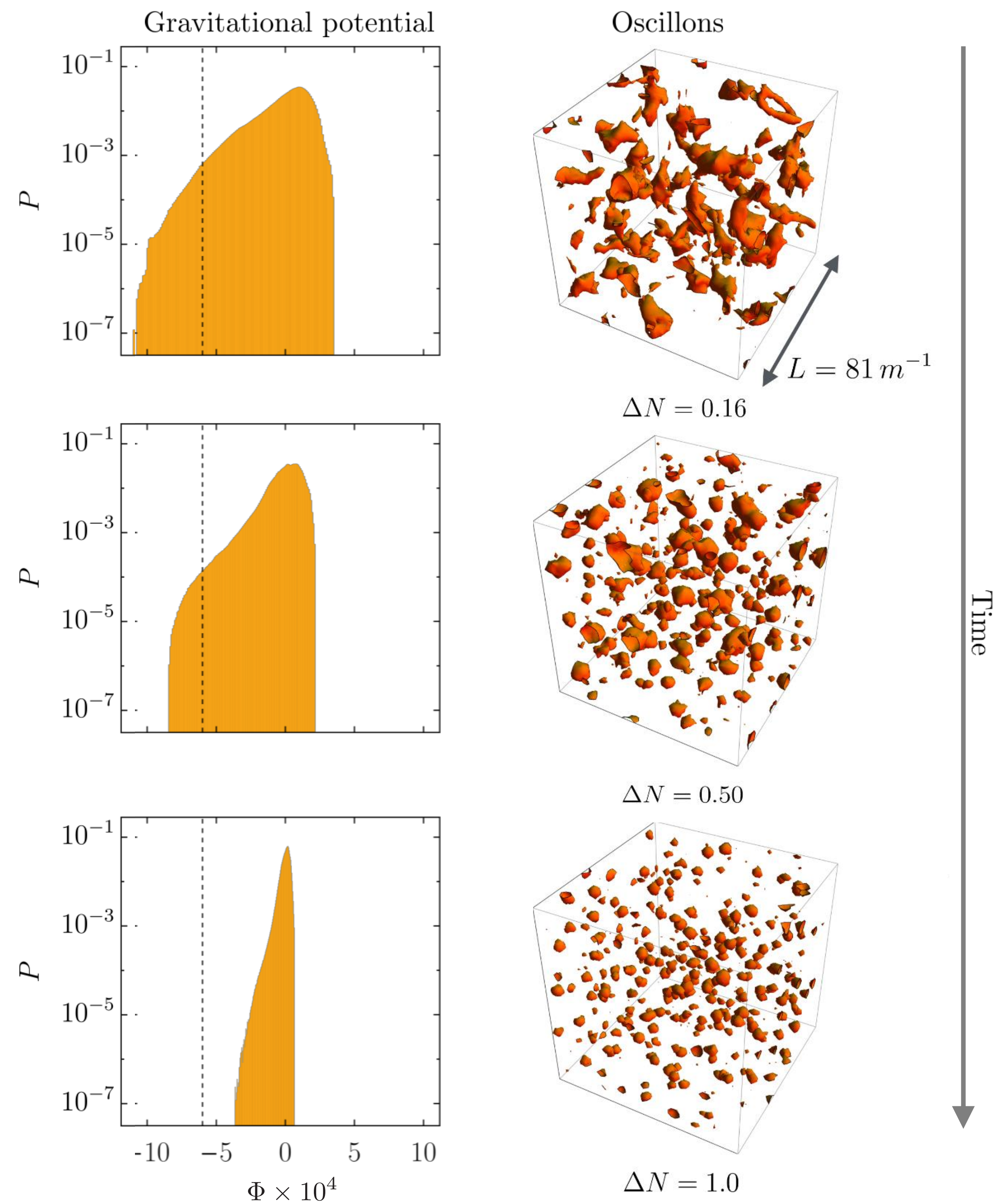
expansion ✓

self-interactions ✓

gravitational int. ✗

“passively” calculated gravitational potential

$$\Phi \lesssim \text{few} \times 10^{-3}$$



w/ K. Lozanov (2019)

expansion ✓
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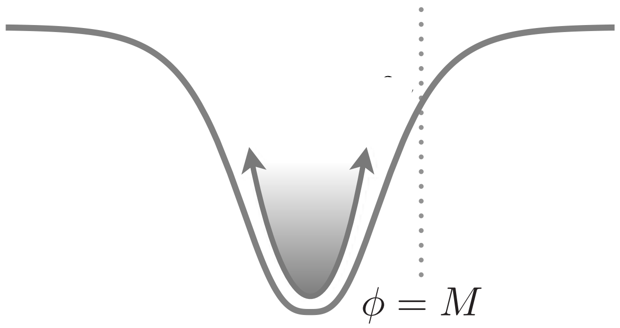
passively calculated gravitational potential

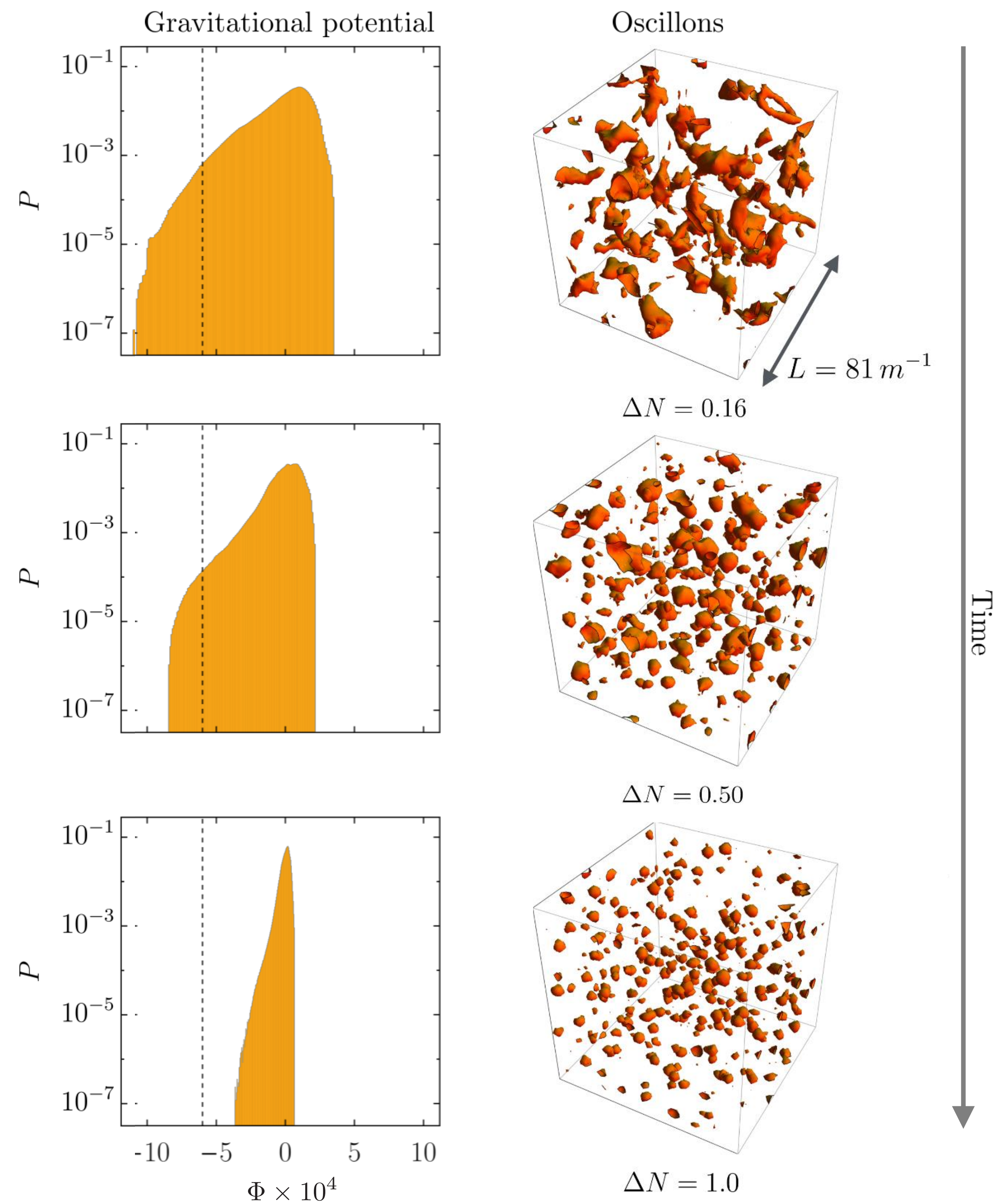


$$|\Phi|_{\rm sol} \lesssim 10 \times \left(\frac{M}{m_{\rm pl}}\right)^2$$

$$\Phi \lesssim \text{few} \times 10^{-3}$$

not easy to form black holes from individual solitons*

$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\rm pl}}{M} \gg 1$$

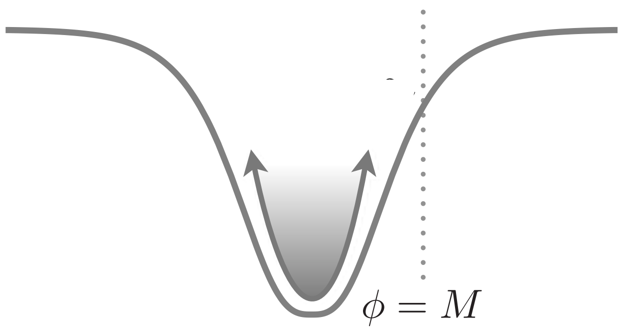


expansion ✓

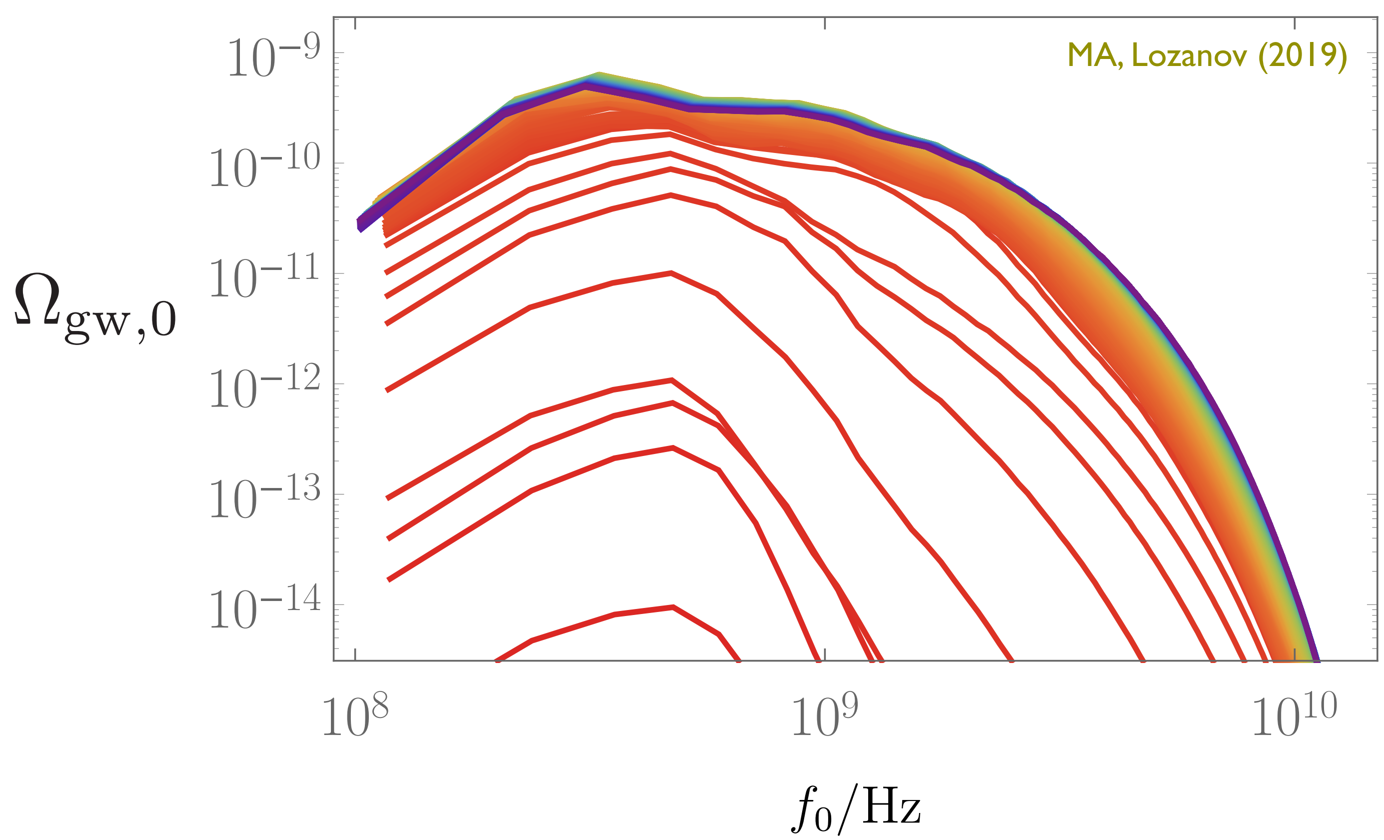
self-interactions ✓

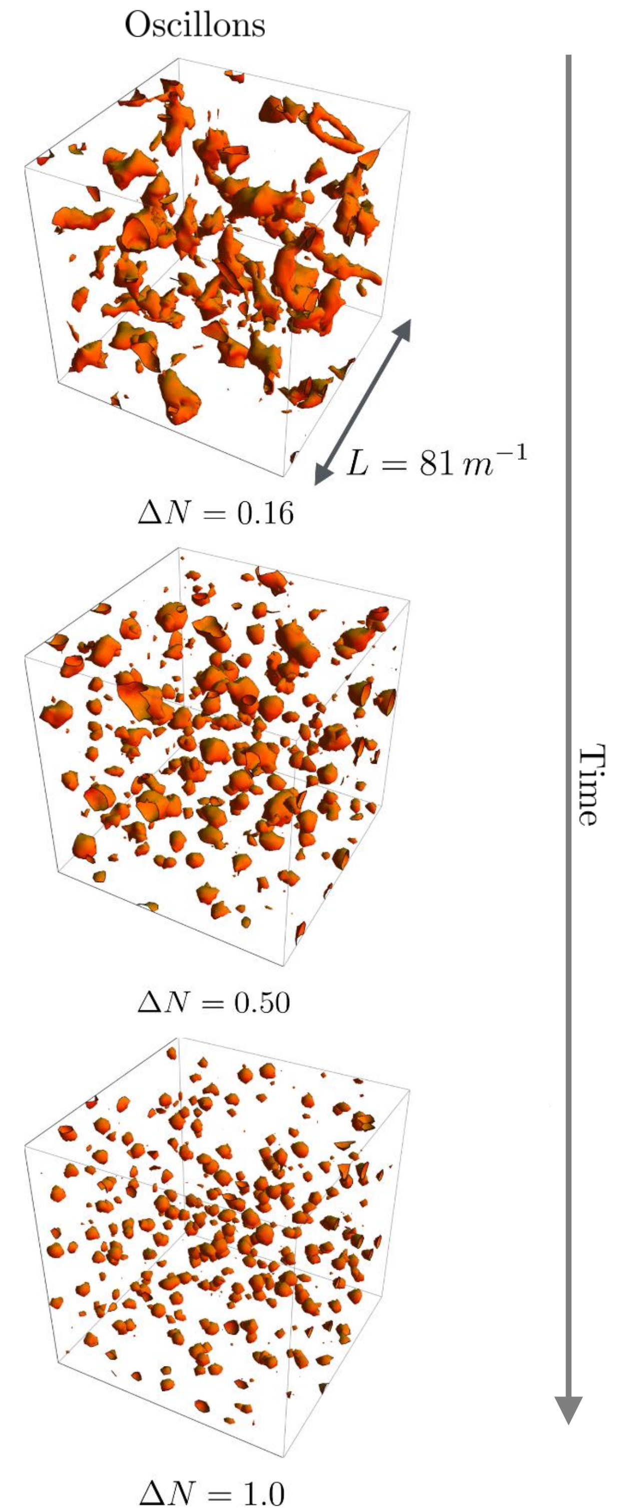
gravitational int. ✗

passively calculated gravitational waves



$$\Omega_{\mathrm{gw},0} \sim 10^{-6}\left(\frac{M}{m_{\mathrm{Pl}}}\right)^2 \lesssim \mathcal{O}[10^{-9}]$$



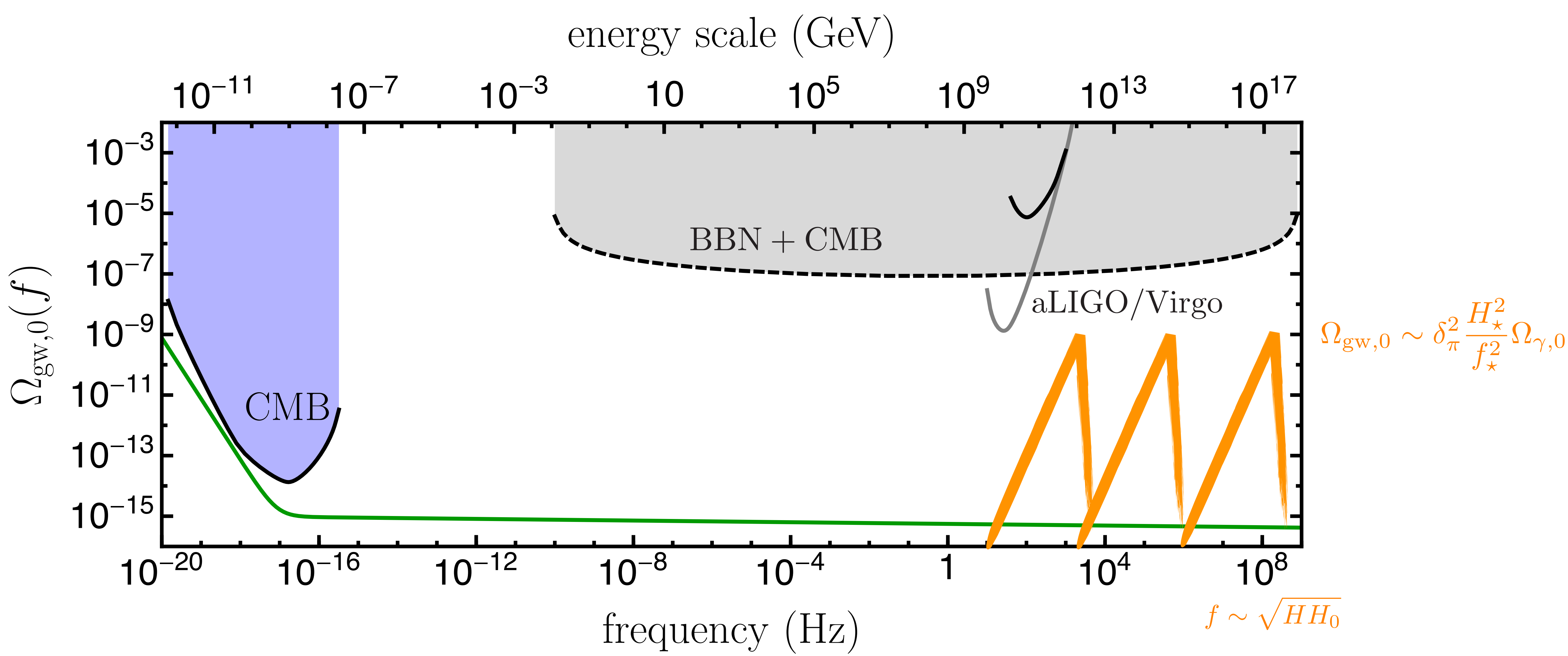


*assumes radiation domination afterwards

$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\mathrm{pl}}}{M} \gg 1$$

*also see Zhou et. al (2014)

can be constrained ?



$$\frac{\Omega_{\mathrm{gw},0} h^2}{\Omega_{\gamma,0} h^2} = \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Delta N_{\mathrm{eff}}$$

$$\Omega_{\mathrm{gw},0} h^2 \lesssim 1.12 - 1.68 \times 10^{-7} \quad (\mathrm{CMB} - \mathrm{S4})$$

* assumes radiation domination after production

Also see: Kitojima, Soda & Urakawa (2018)

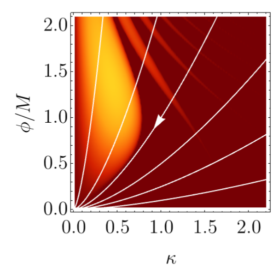
expansion ✓

self-interactions ✓

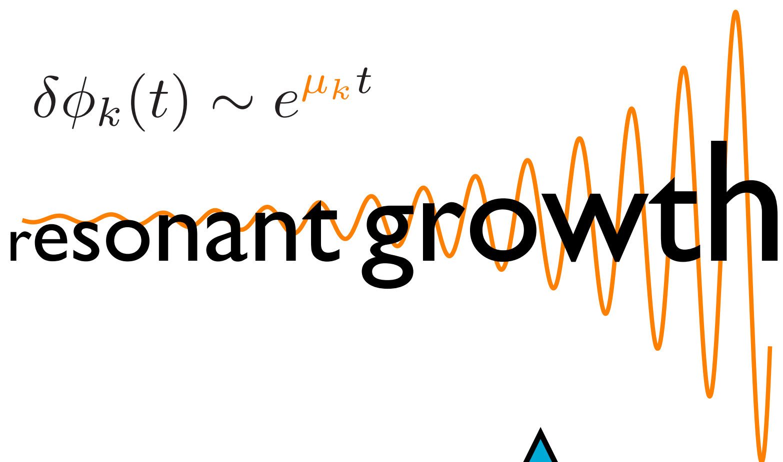
gravitational int. ✗

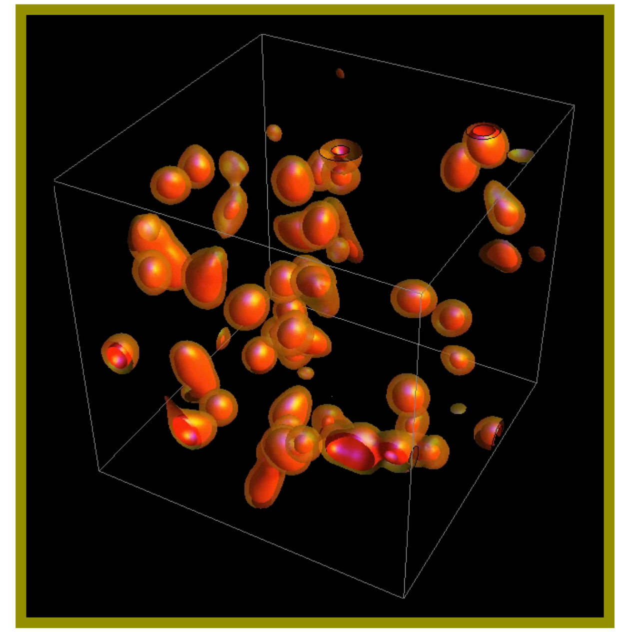
summary so far …

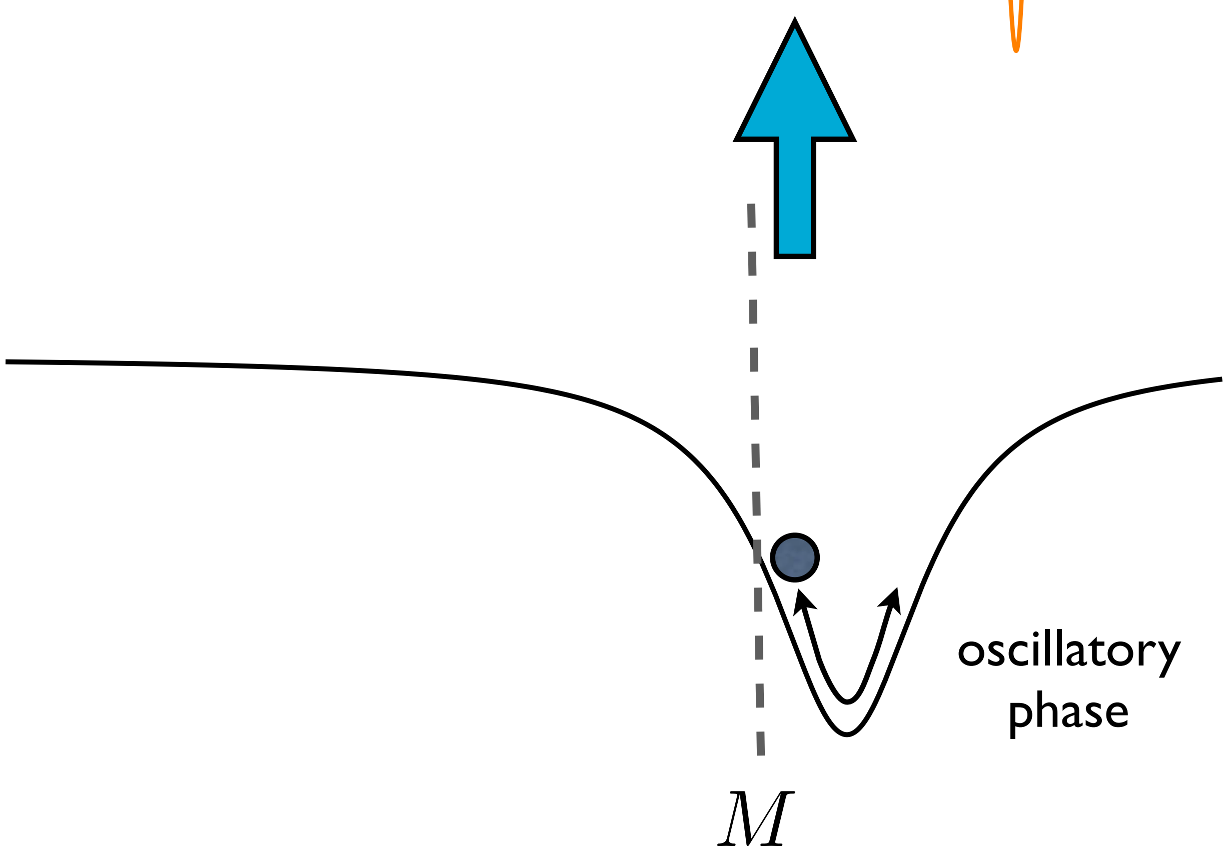
$$\frac{\mu_k}{H} \sim \frac{m_{\rm pl}}{M} \gg 1$$



$$\delta\phi_k(t) \sim e^{\mu_k t}$$







$$|\Phi|_{\rm sol} \lesssim 10 \times \left(\frac{M}{m_{\rm pl}}\right)^2$$

$$\Phi \lesssim \text{few} \times 10^{-3}$$

not easy to form black holes from individual solitons*

$$\Omega_{\rm gw,0} \sim 10^{-6} \left(\frac{M}{m_{\rm Pl}}\right)^2 \lesssim \mathcal{O}[10^{-9}]$$

expansion ✓

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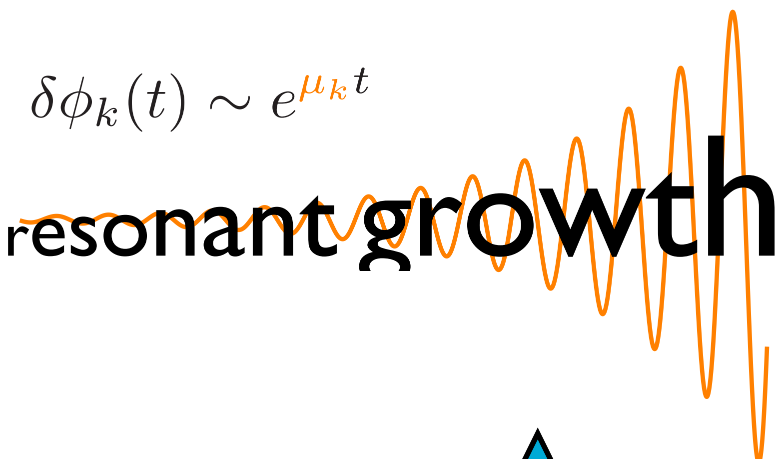

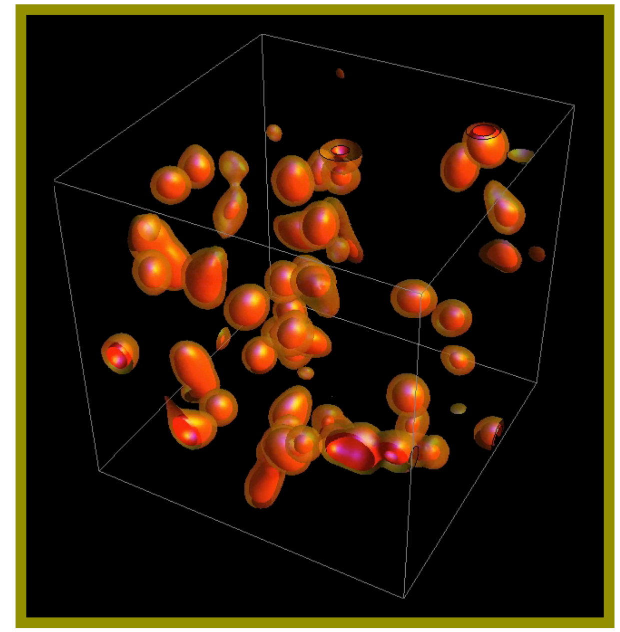

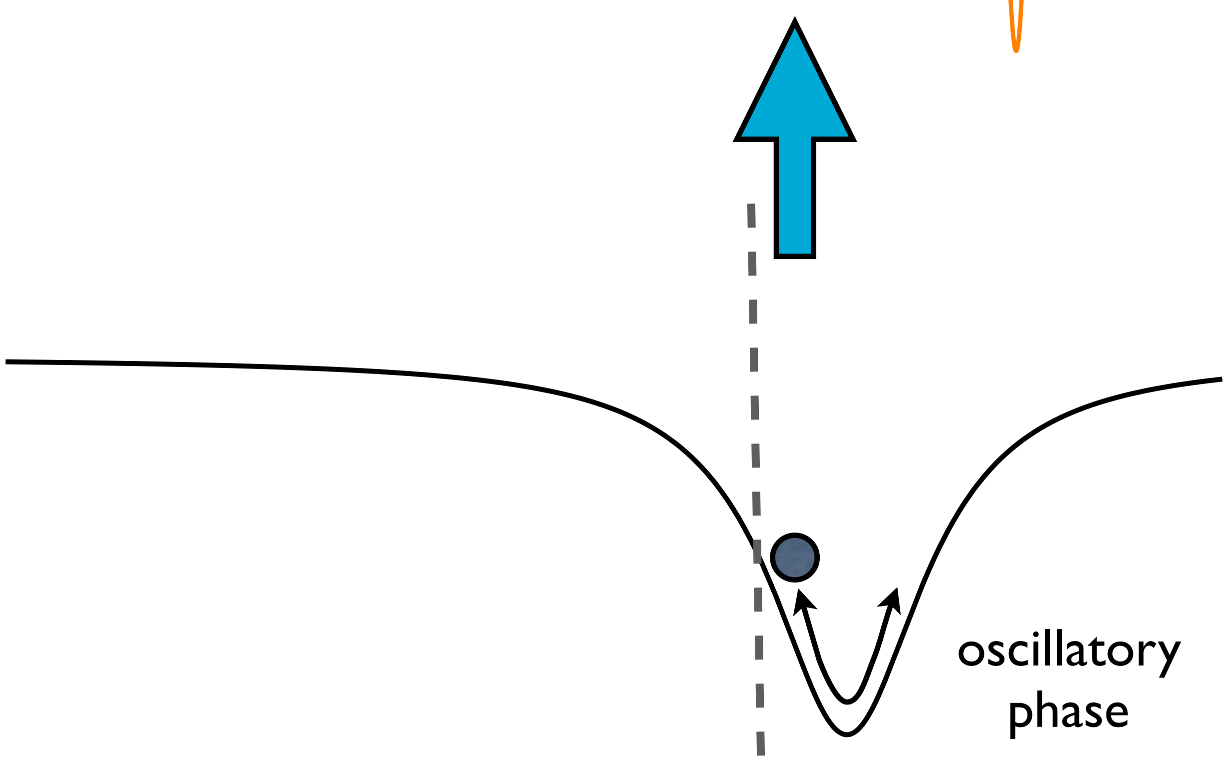

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$$\Omega_{\rm gw,0} \sim 10^{-6}\left(\frac{M}{m_{\rm Pl}}\right)^2 \lesssim \mathcal{O}[10^{-9}]$$

including gravitational interactions

- gravitational clustering takes time …
- long time makes it difficult to resolve very fast oscillatory time scale

a way forward …

- rapid oscillatory behavior of fields (integrate out)
- size of solitons and instability length scales* $\gg m^{-1}$
- gravity is weak
- non-relativistic simulations including local gravitational interactions

* by an order of magnitude or less, so care is needed

“non-relativistic” limit

$$\Box\phi + V'(\phi) = 0$$

$$G_{\mu\nu} = \frac{1}{m_{\rm pl}^2} T_{\mu\nu}$$

$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re[e^{-imt}\psi(t,\mathbf{x})]$$

$$\frac{|\nabla|}{m}, \frac{\partial_t}{m} \ll 1$$

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Phi)d\mathbf{x}^2$$

$$|\Phi| \ll 1$$

$$\downarrow$$

non-linear Schrodinger eq.

Poisson eq. + Friedmann eq.

$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_{\rm nl}(\phi)$$

* some dragons hide here …

non-relativistic case

$$\left[i\left(\partial_t + \frac{3}{2}H\right) + \frac{1}{2a^2}\nabla^2 - U'_{\rm nl}(|\psi|^2) - \Phi\right]\psi = 0\,,$$

nonlinear Schrodinger eq.

$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re\left[e^{-imt}\psi(t,\mathbf{x})\right]$$

non-relativistic case

$$\left[i\left(\partial_t + \frac{3}{2}H\right) + \frac{1}{2a^2}\nabla^2 - U'_{\rm nl}(|\psi|^2) - \Phi\right]\psi = 0\,,$$

nonlinear Schrodinger eq.

$$\frac{\nabla^2}{a^2}\Phi = \frac{\beta^2}{2}\left[|\psi|^2 + \frac{1}{2a^2}|\nabla\psi|^2 + U_{\rm nl}(|\psi|^2)\right] - \frac{3}{2}H^2\,,$$

Poisson eq.

$$H^2 = \frac{\beta^2}{3}\overline{\left[|\psi|^2 + \frac{1}{2a^2}|\nabla\psi|^2 + U_{\rm nl}(|\psi|^2)\right]}\,,$$

Friedmann eq.

$$mx^\mu \to x^\mu$$

length/time units

$$\frac{\psi}{mM} \to \psi$$

non-linearity

$$\beta \equiv \frac{M}{m_{\rm pl}}$$

$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re\left[e^{-imt}\psi(t,\mathbf{x})\right]$$

relativistic* ✗

self-interactions ✓

gravitational int. ✓

two linear instabilities

$$\psi(t,\mathbf{x}) = \bar{\psi}(t)\left[1+\varepsilon\frac{\delta\psi_{\mathbf{k}}(t)}{\bar{\psi}(t)}e^{i\mathbf{k}\cdot\mathbf{x}}\right]$$

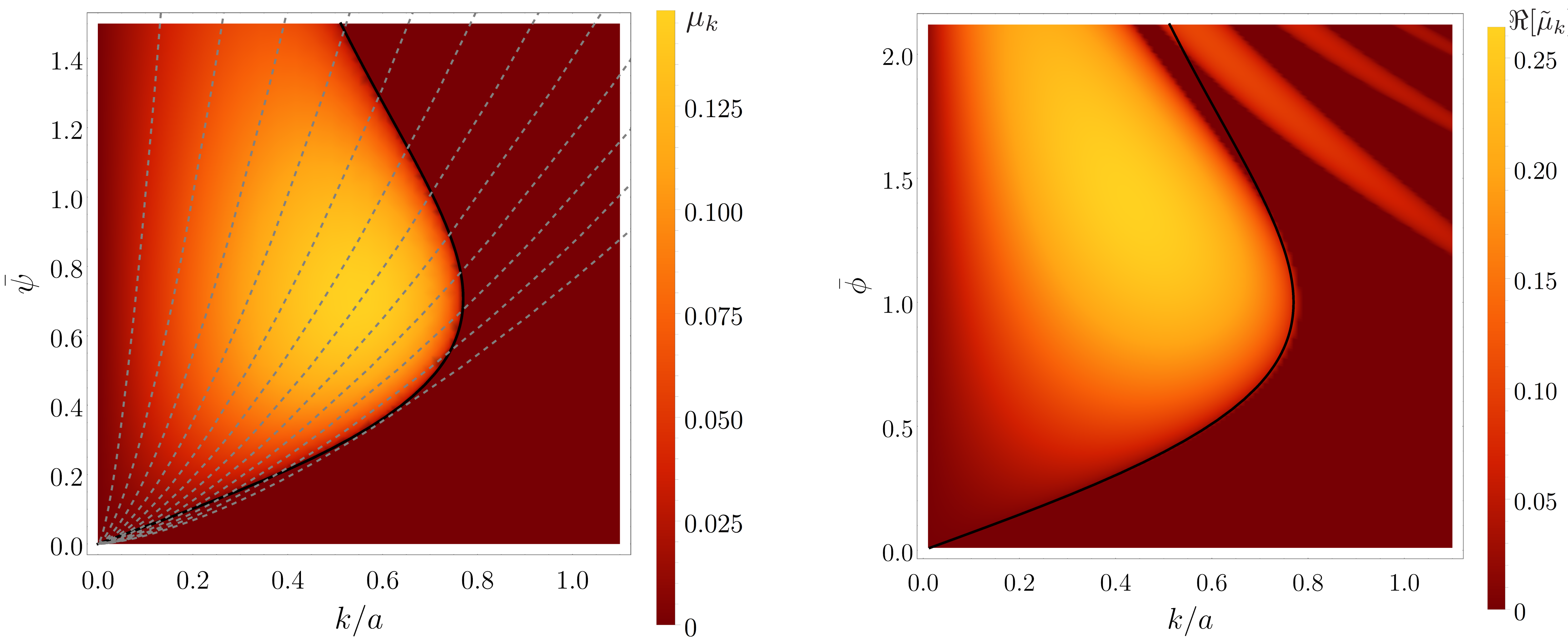
$$|\delta\psi_{\mathbf{k}}/\bar{\psi}| \sim e^{\mu_k t}$$

self-interactions	gravitational interactions
$k^2 < -4\lvert\bar{\psi}\rvert^2 U''_{\rm nl}(\lvert\bar{\psi}\rvert^2)\,,$	$k < k_J \approx \sqrt{\sqrt{2}\beta\lvert\bar{\psi}\rvert}$
$\mu_k = \left\lvert i\frac{k}{2}\sqrt{k^2+4\lvert\bar{\psi}\rvert^2 U''_{\rm nl}(\lvert\bar{\psi}\rvert^2)}\right\rvert$	$\mu_k = \sqrt{\frac{1}{2}\beta^2\lvert\bar{\psi}\rvert^2 - \frac{k^4}{4}}$
$\frac{\mu_k}{H} \sim \frac{1}{\beta}$	$\frac{\mu_k}{H} \sim 1$

For $\beta \equiv \frac{M}{m_{\rm pl}} \ll 1$ self-interaction instability dominates

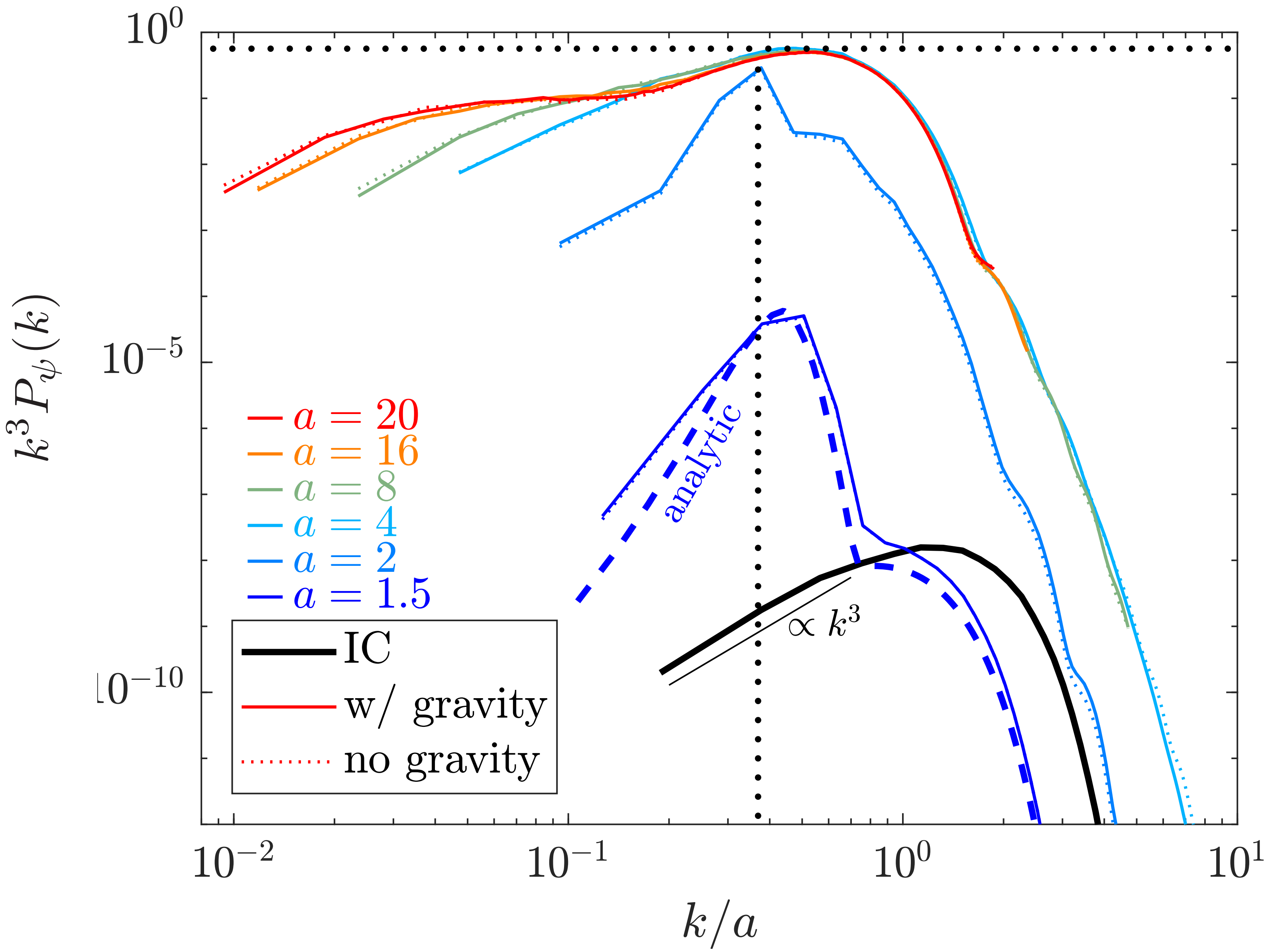
MA & Mocz (2019), Johnson & Kamionkowski (2008)

digression —
Floquet vs. non-relativistic instability



MA & Mocz (2019)

power spectrum



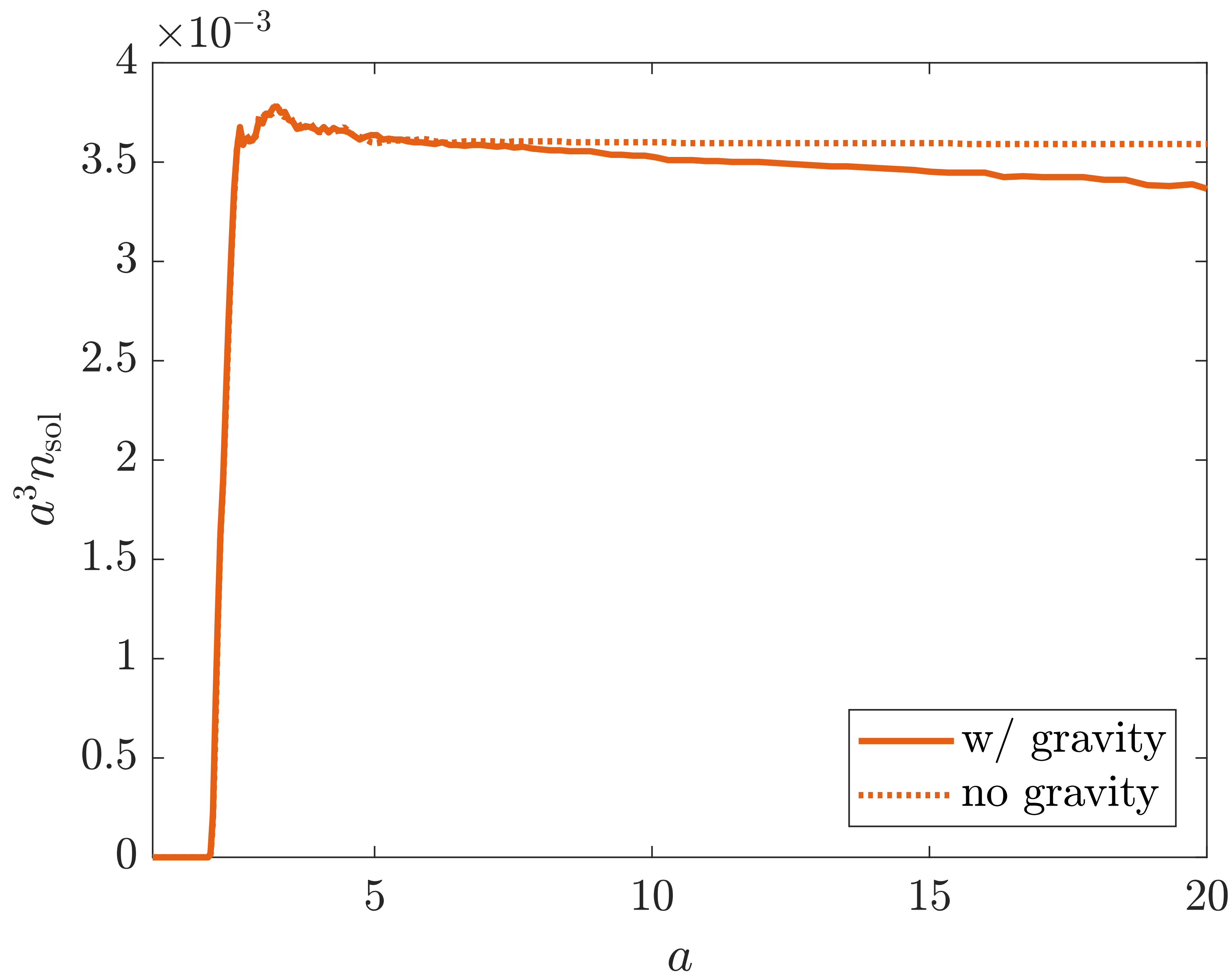
perturbations become non-linear when:

$$\sqrt{k^3 P_\psi(k,a)} \sim \bar{\psi}$$

$$a_{\rm nl} \sim 2 \qquad k_{\rm nl} \sim 0.7$$

$$a^3 n_{\rm sol} \sim (k_{\rm nl}/2\pi)^3$$

co-moving number density of solitons



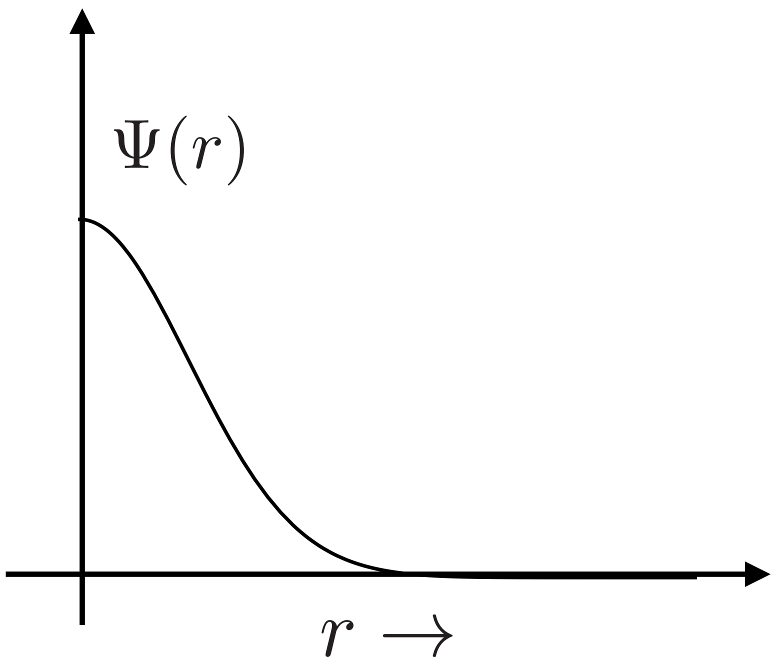
$$\frac{d\ln(a^3 n_{\mathrm{sol}})}{d\ln a} \simeq 0.1$$

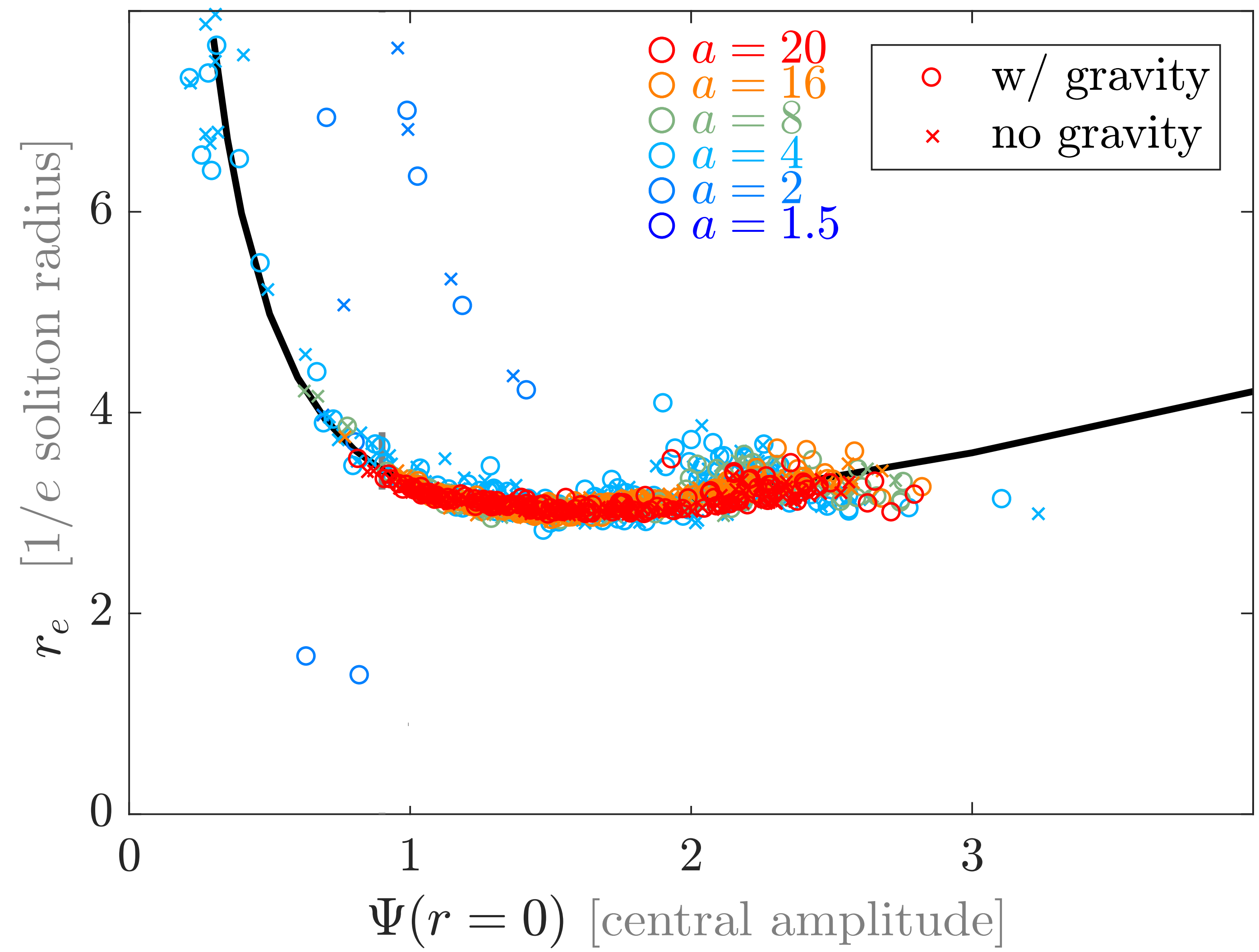
few x 10^3 /Hubble volume at time of formation

individual solitons

$$\phi(t,r) = \sqrt{2}\Re[e^{-it}\psi(t,r)] = \sqrt{2}\Psi(r)\cos[(1+\nu)t]$$

$$\psi(t,r) = e^{-i\nu t}\Psi(r)$$





$$\nu < 0, \qquad |\nu| \ll 1$$

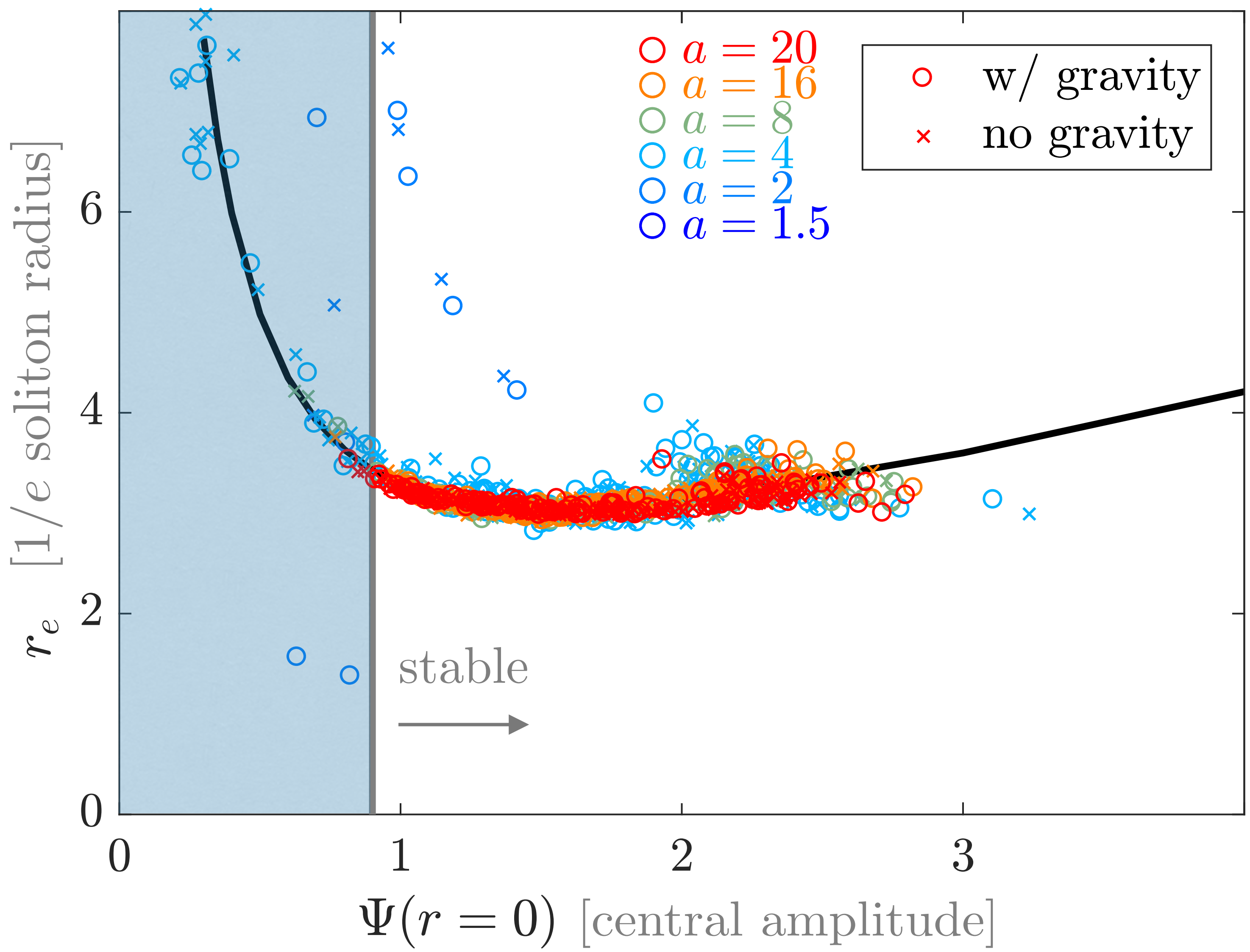
individual solitons stability

$$\psi(t,r) = e^{-i\nu t}\Psi(r)$$

$$\mathcal{N} \equiv \int d^3r\Psi^2(r)$$

stable iff:
Vakhitov Kolokolov (1973)

$$\frac{d\mathcal{N}}{d(-\nu)} > 0$$

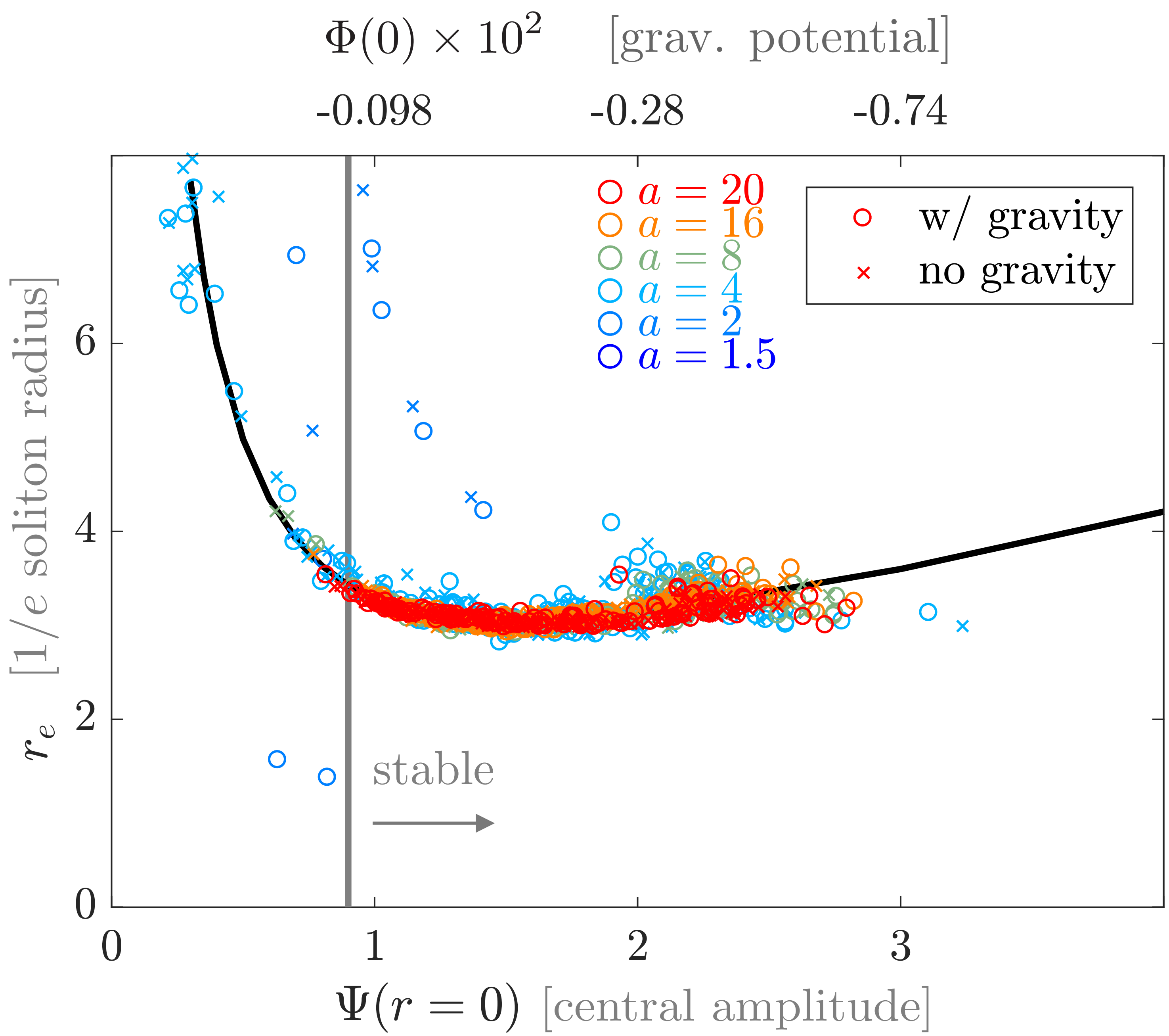


— caution, relativistic + quantum effects missing

gravity remains weak

$$|\Phi| \sim \mathcal{O}[10^{-3}]$$

not easy to form black holes from individual solitons still

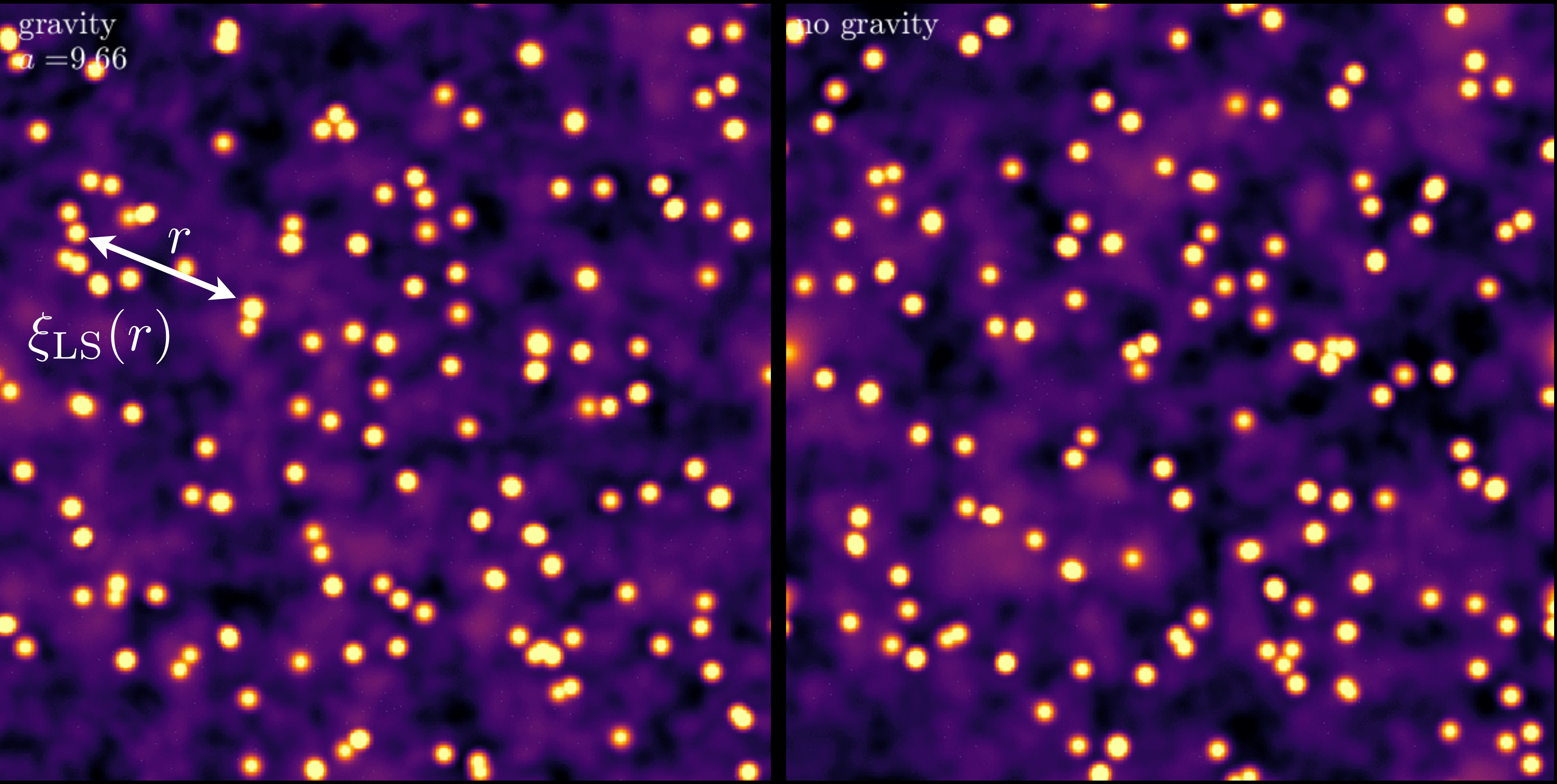


— caution, relativistic + quantum effects missing

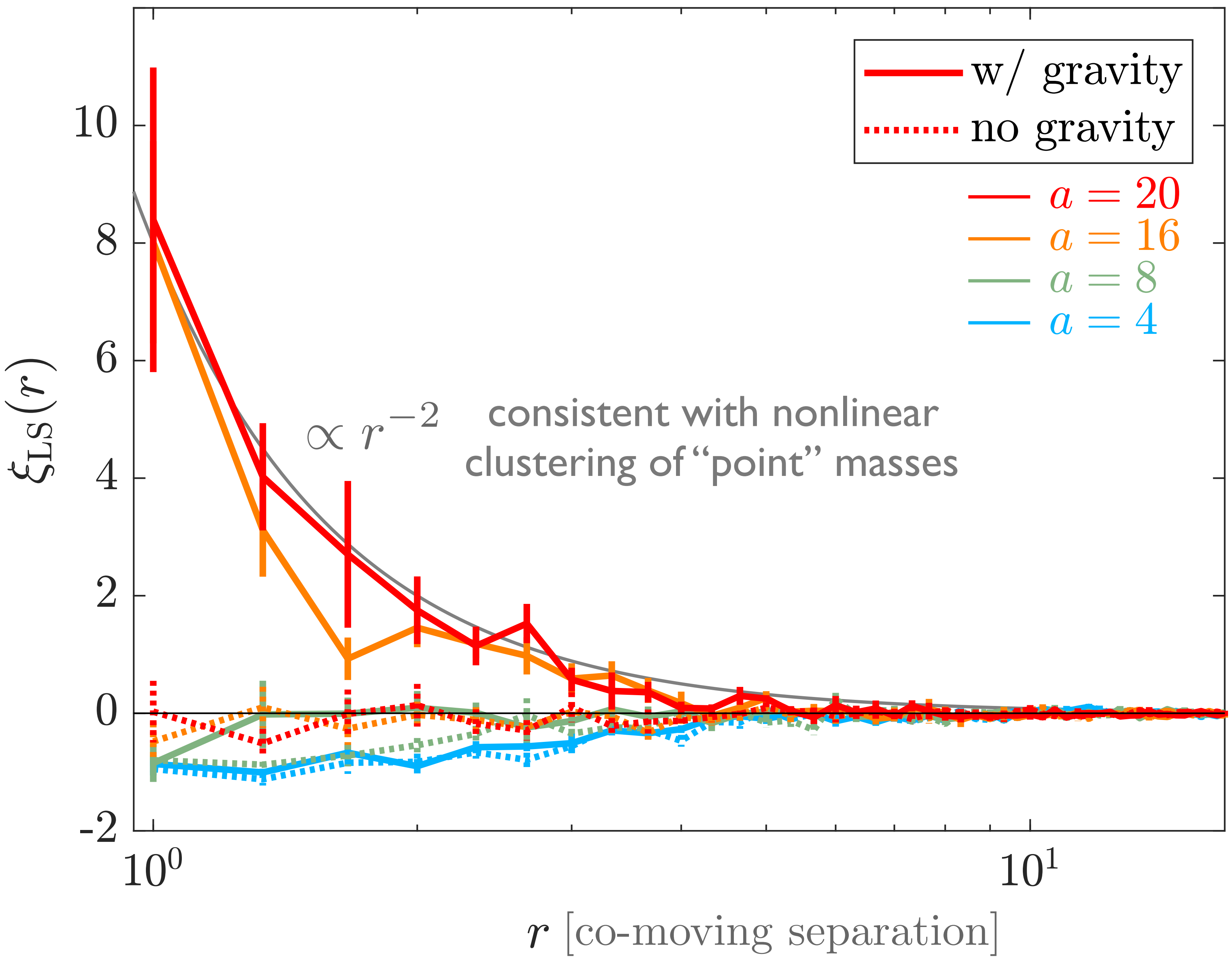
gravitational clustering of solitons

gravity

no-gravity

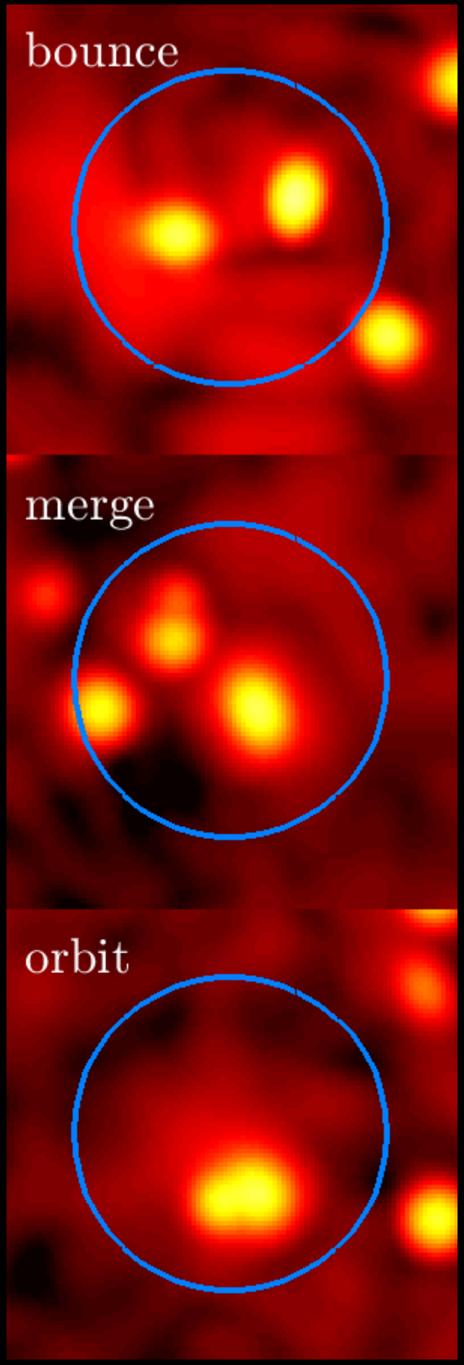


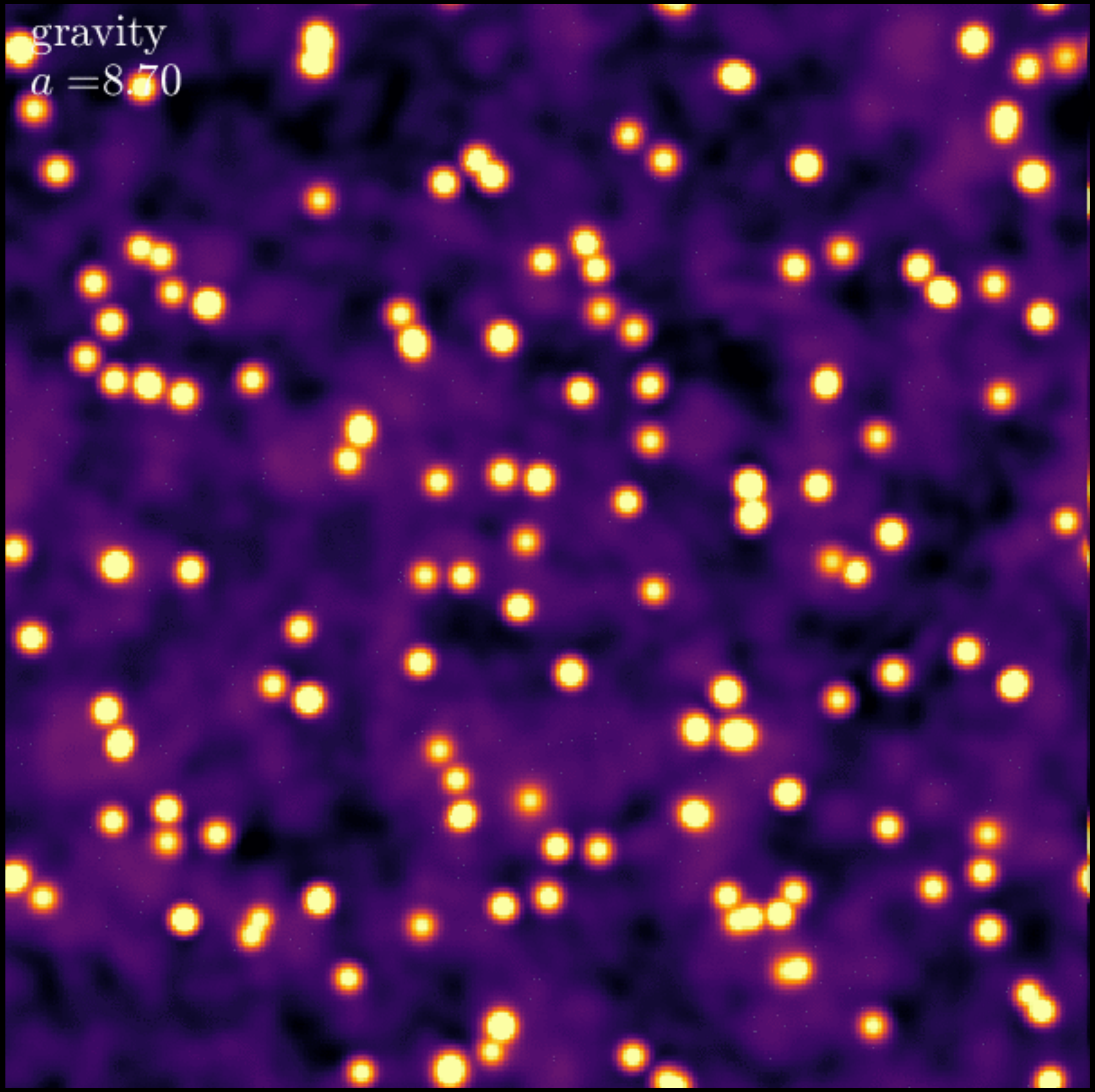
gravitational clustering of solitons



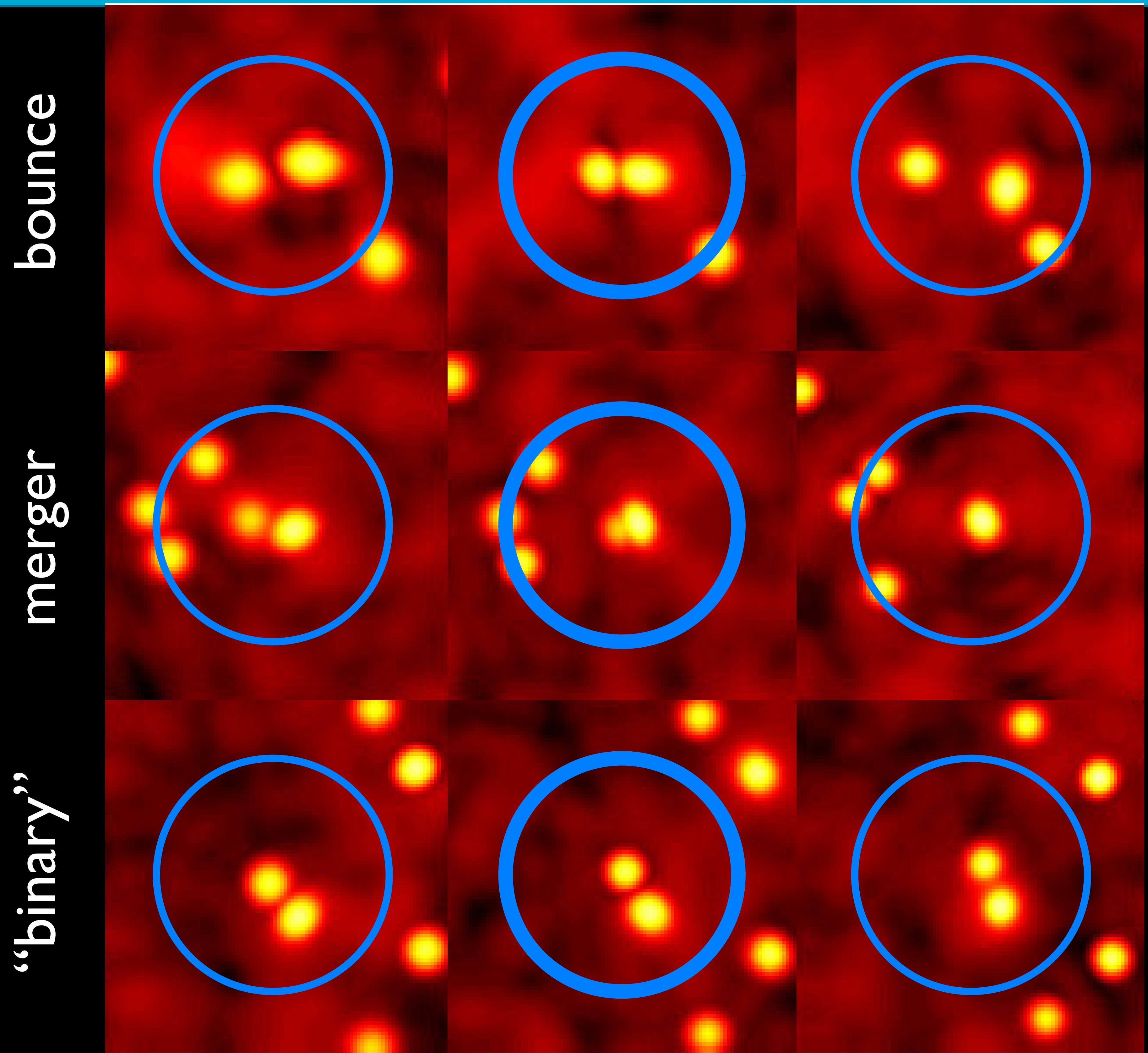
MA & Mocz (2019)

strong interactions





strong interactions



$$\psi_a(t, \mathbf{x}) = \Psi_a(\mathbf{x}) e^{-i\nu_a t + \theta_a}$$

$$|\theta_1 - \theta_2| \simeq \pi$$

$$|\theta_1 - \theta_2| \simeq 0$$

- need better understanding
- & simulations

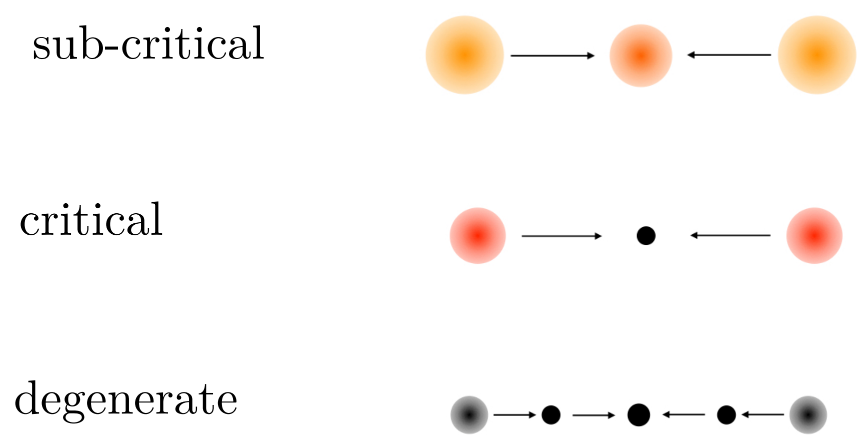
Numerical GR ✓

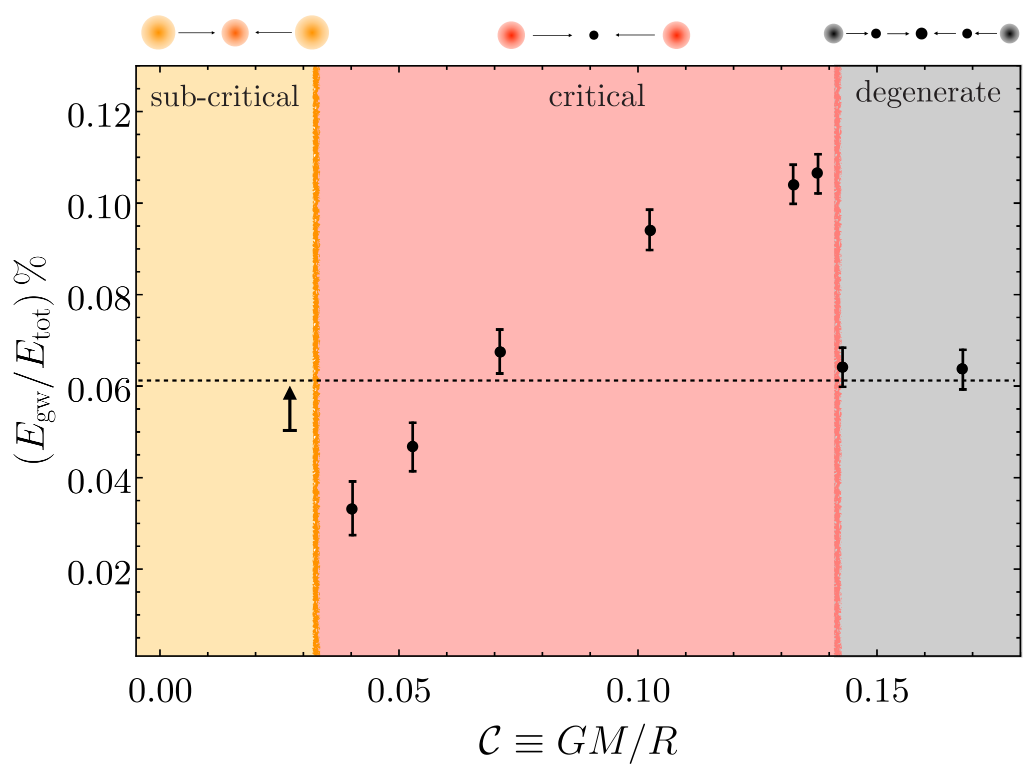
self-interactions ✗

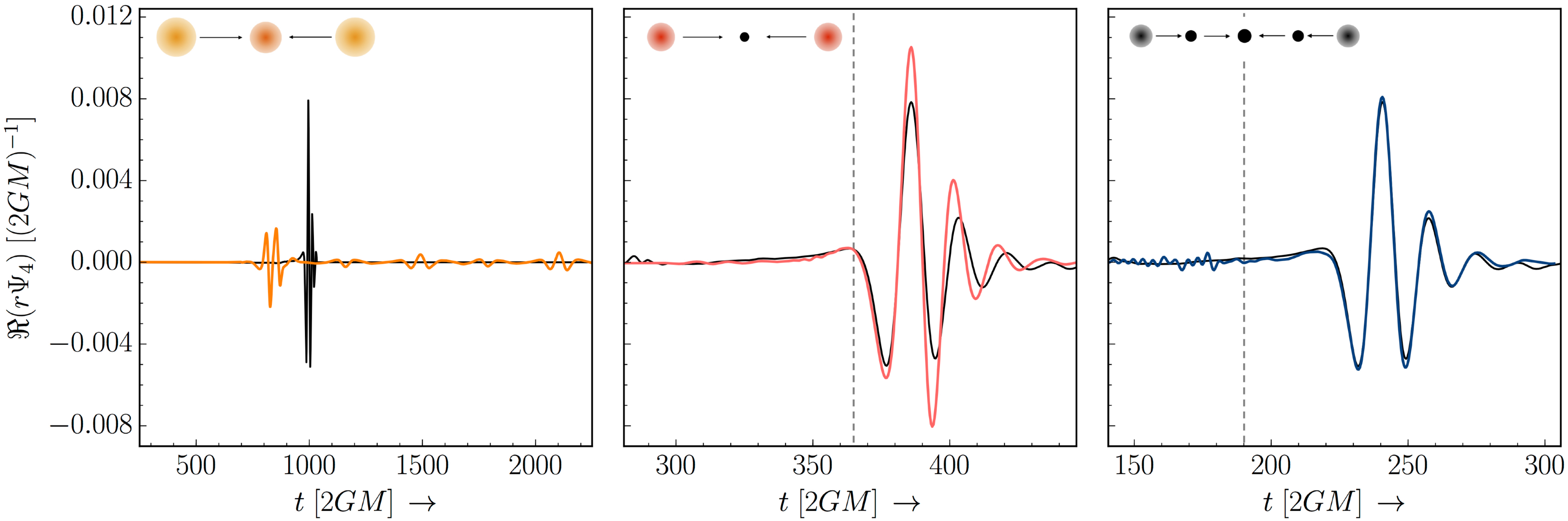
formation ✗

“ultra-compact” soliton collision

Helfer, Lim, Garcia & MA (2018)







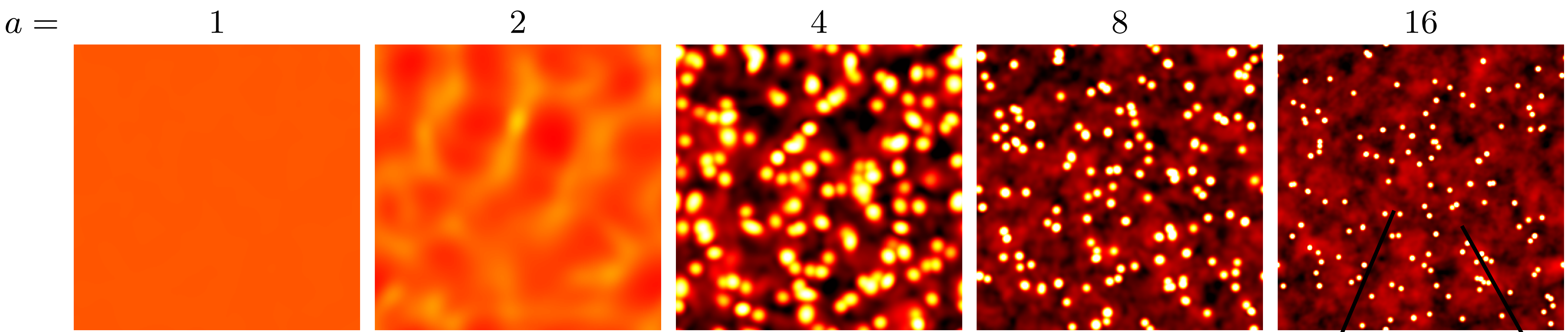
summary I

self-interaction instability

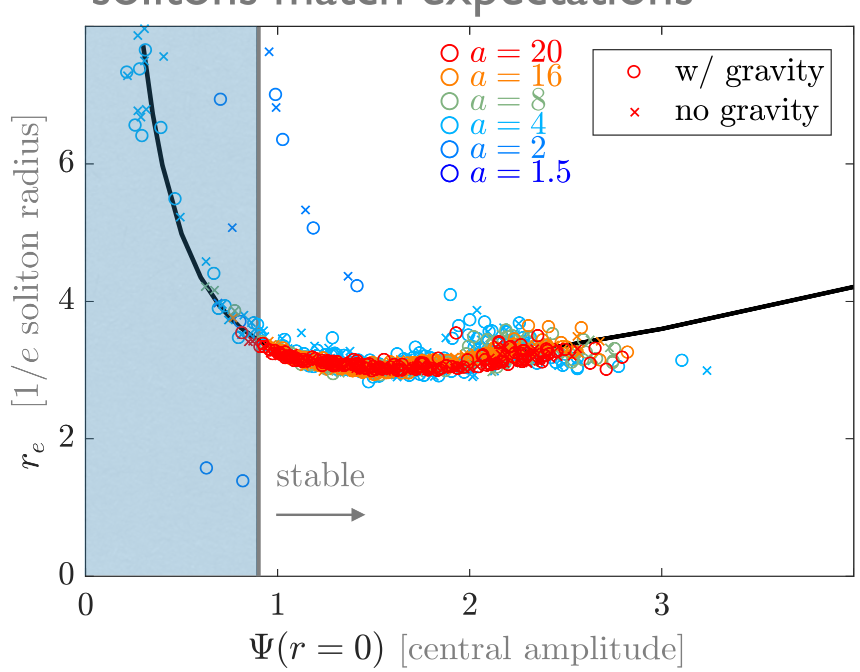
soliton formation

gravitational clustering

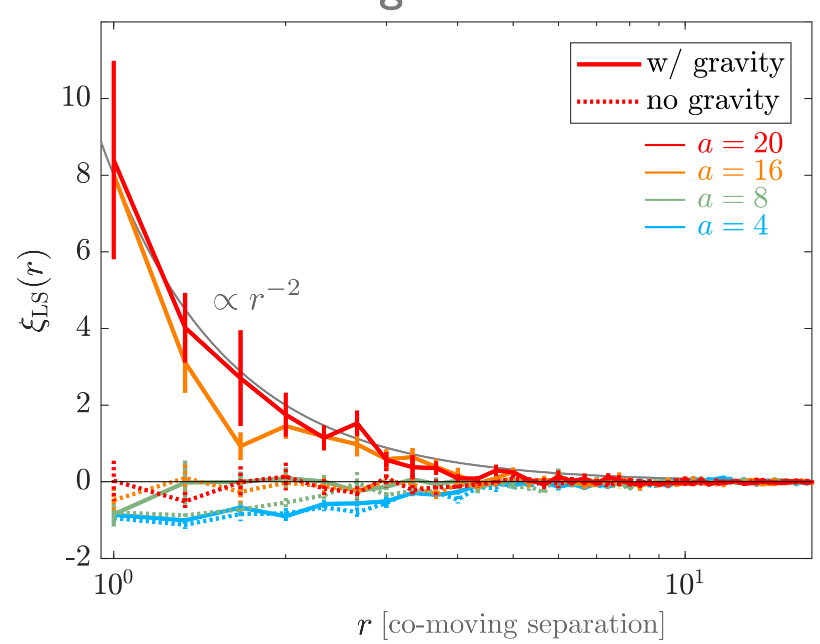
some late-time strong interactions

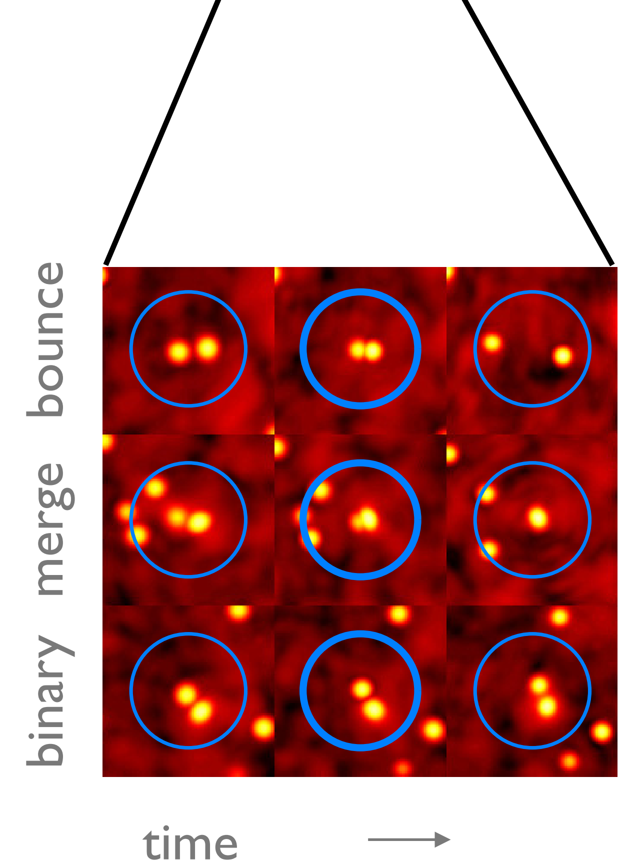


solitons match expectations



evidence for gravitational clustering





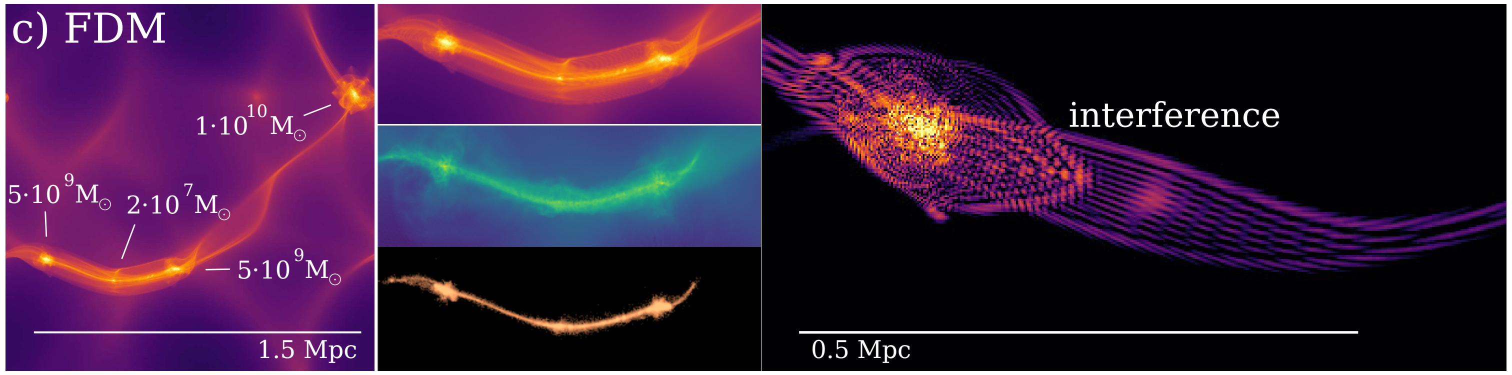
MA & Mocz (2019)

things that need more work

- long term state of the strongly interacting soliton gas — does probability of PBH formation increase ?
- additional source of g-waves ?
- *velocity distribution …*
- *relativistic vs. relativistic fields + classical fields vs. quantum aspects*

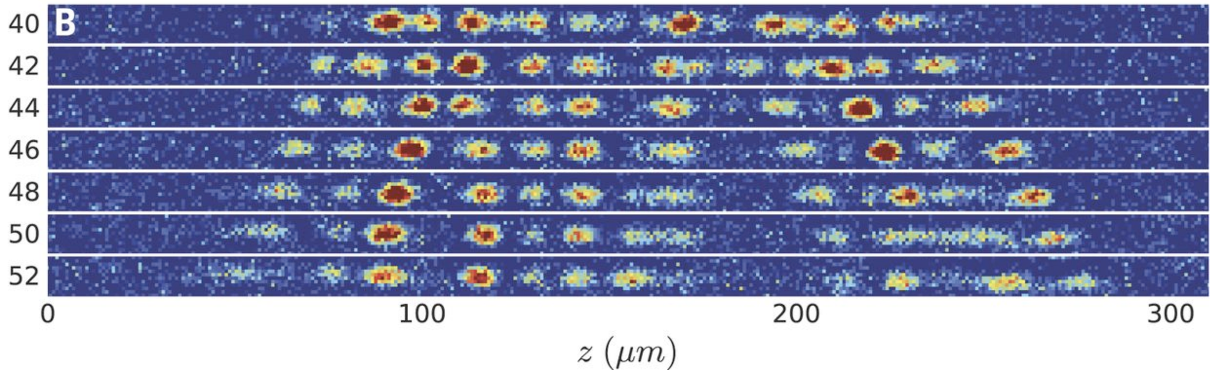
things in progress that I did not discuss

- dynamics in late-time ultra-light axions
  - solitons at centers of halos/galaxies
  - small-scale structure of CDM (including baryons — in progress)



Mocz et. al (in prep)

- solitons in Bose-Einstein condensates
  - 1D already demonstrated experimentally
  - first examples of 3D solitons in the BECs in the lab



Nguyen, Luo & Hulet (2017)

Non-perturbative Dynamics of Cosmological Fields

1. Gravitational Aspects of Oscillons/Solitons

Lozanov, Mocz + earlier collaborators

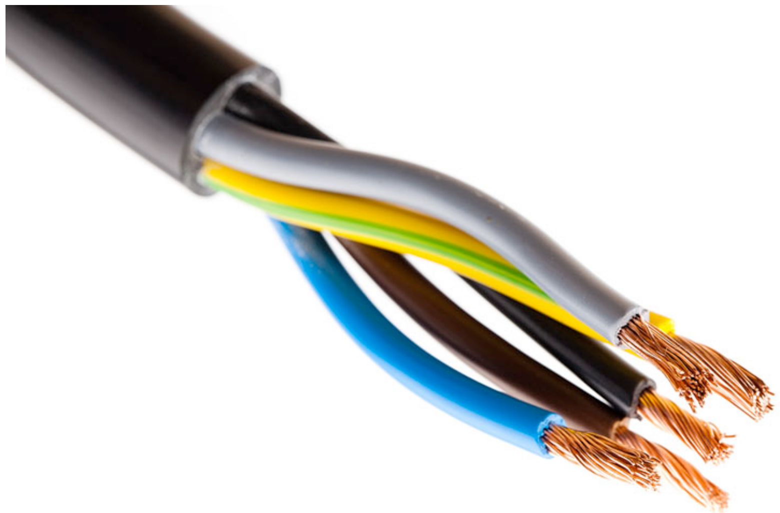
2. Stochastic Particle Production In Cosmology

Garcia, Wen, Carlsten, Baumann, Xie, Green …

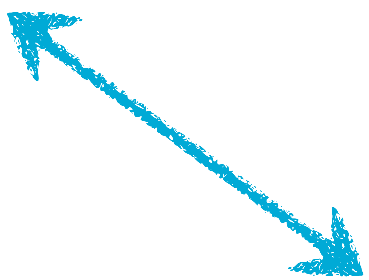
Mustafa A. Amin

RICE

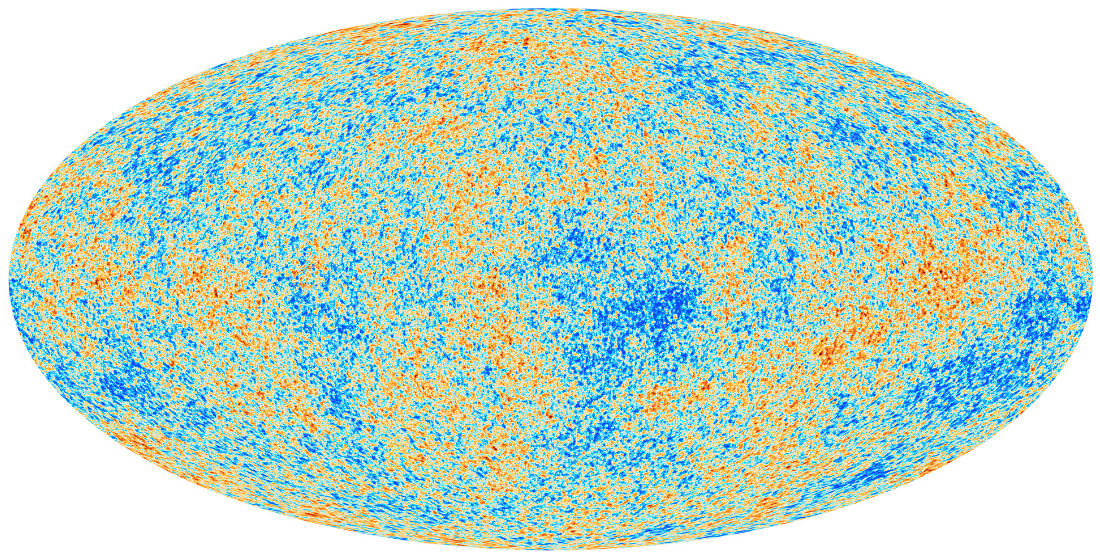
plan for the talk (part II)

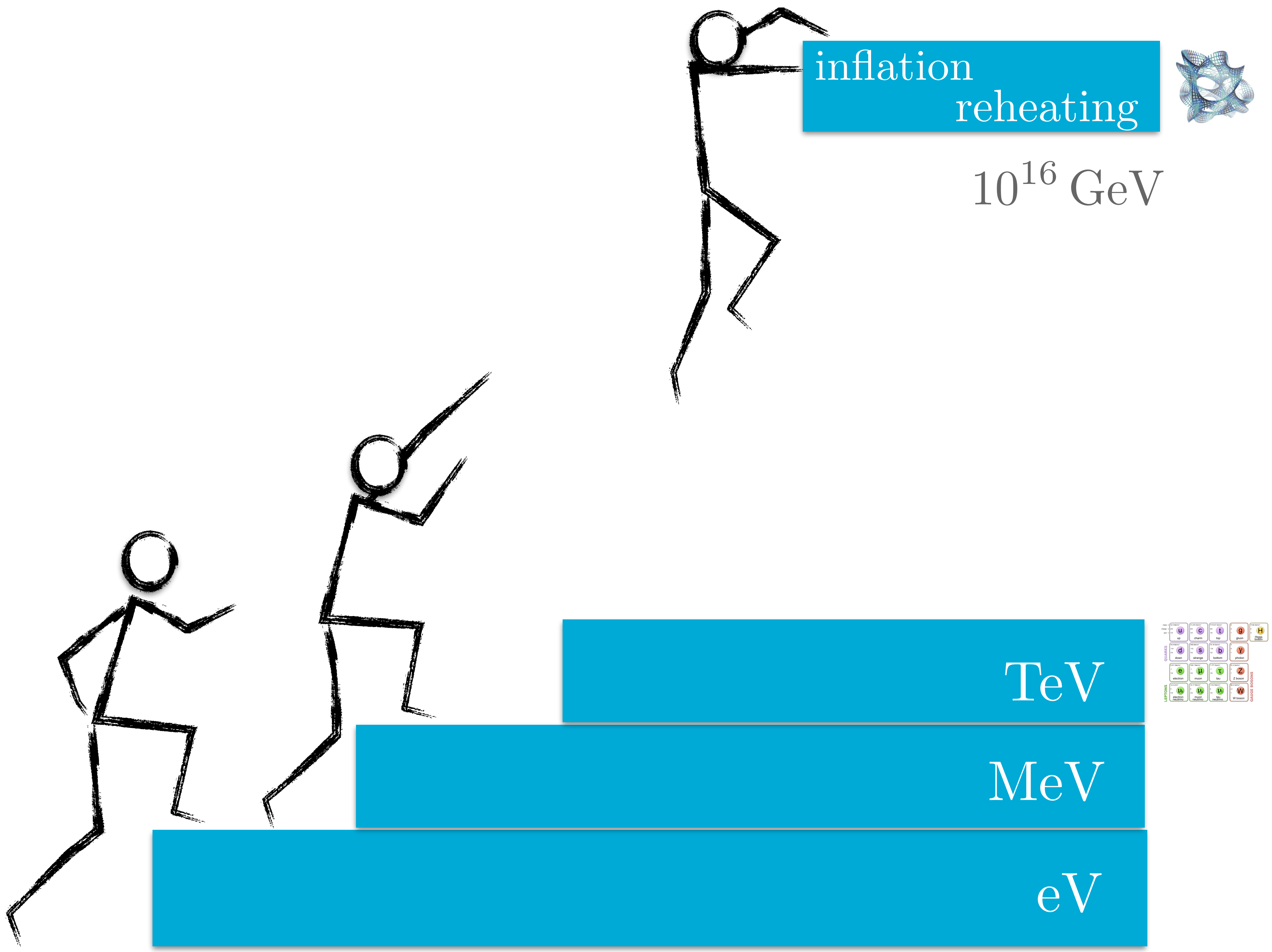


- motivation
- framework
- implications



- Wires to Cosmology
  (w/ Baumann 1512.02637)
- Multifield Stochastic Particle Production
  (w/ Garcia, Wen & Xie 1706.02319)
- Stochastic Particle Production in deSitter Space
  (w/ Garcia, Carlsten & Green 1902.06736)
- Curvature Perturbations from Stochastic Particle Production
  (in progress)



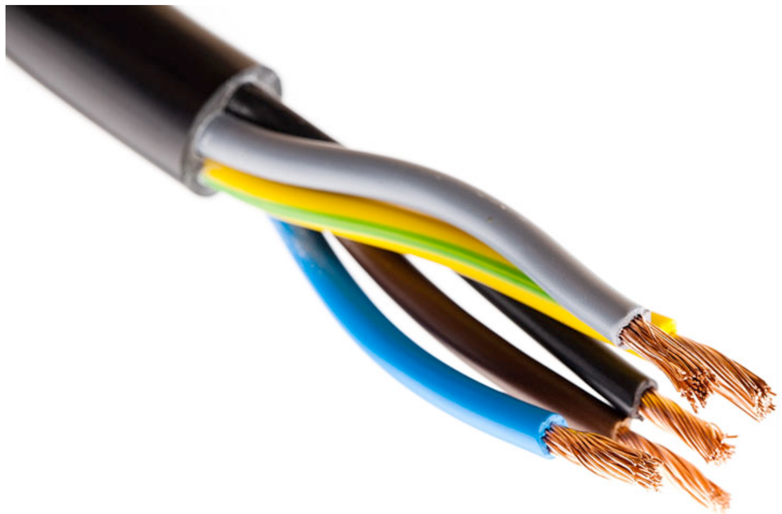

inflation
reheating
10^16 GeV
TeV
MeV
eV

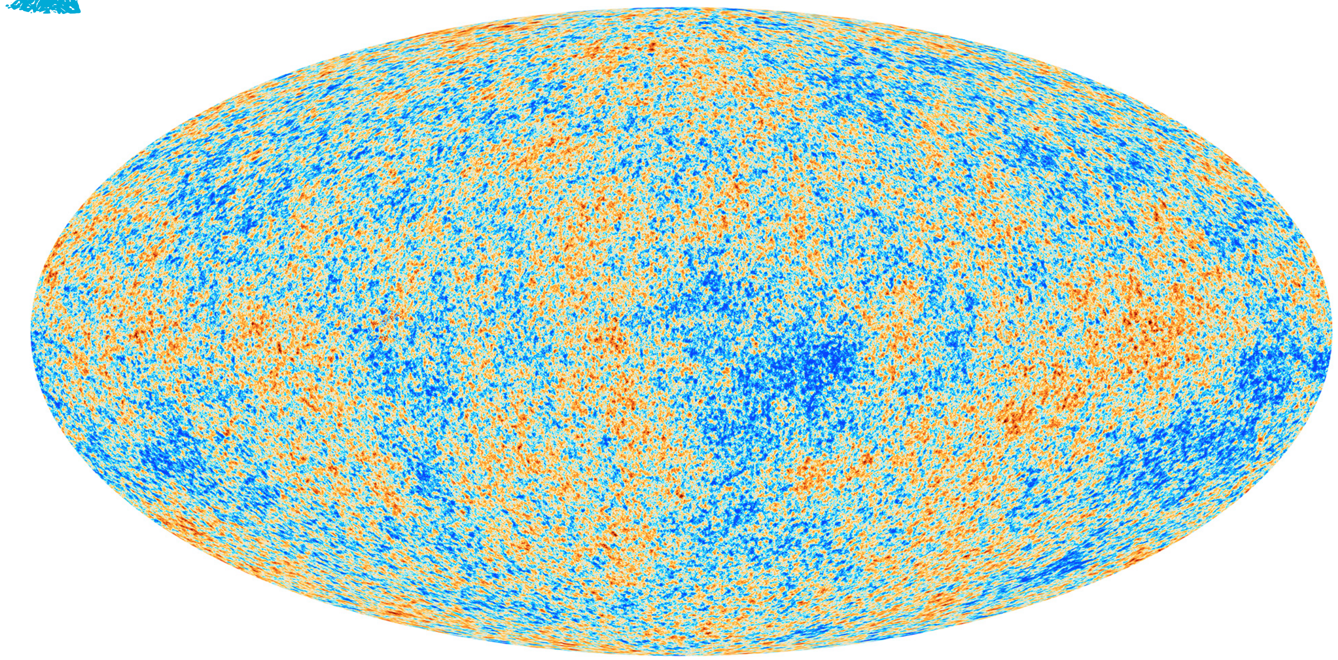
two approaches

SIMPLE enough

COMPLEX enough

inspiration from disordered wires

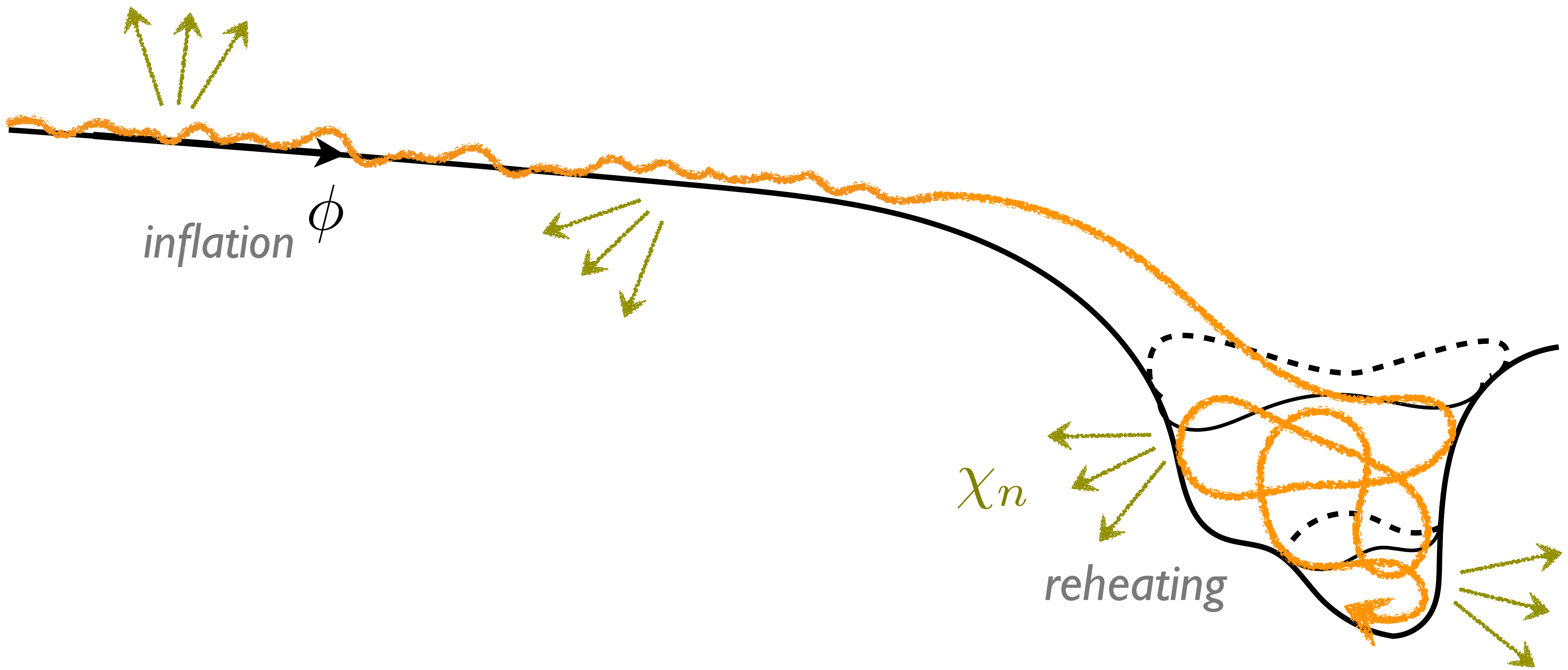




MA & Baumann 2015

multifield inflation/reheating

- inflation/reheating: many coupled fields (spectators)
- fluctuations: coupled, non-perturbative

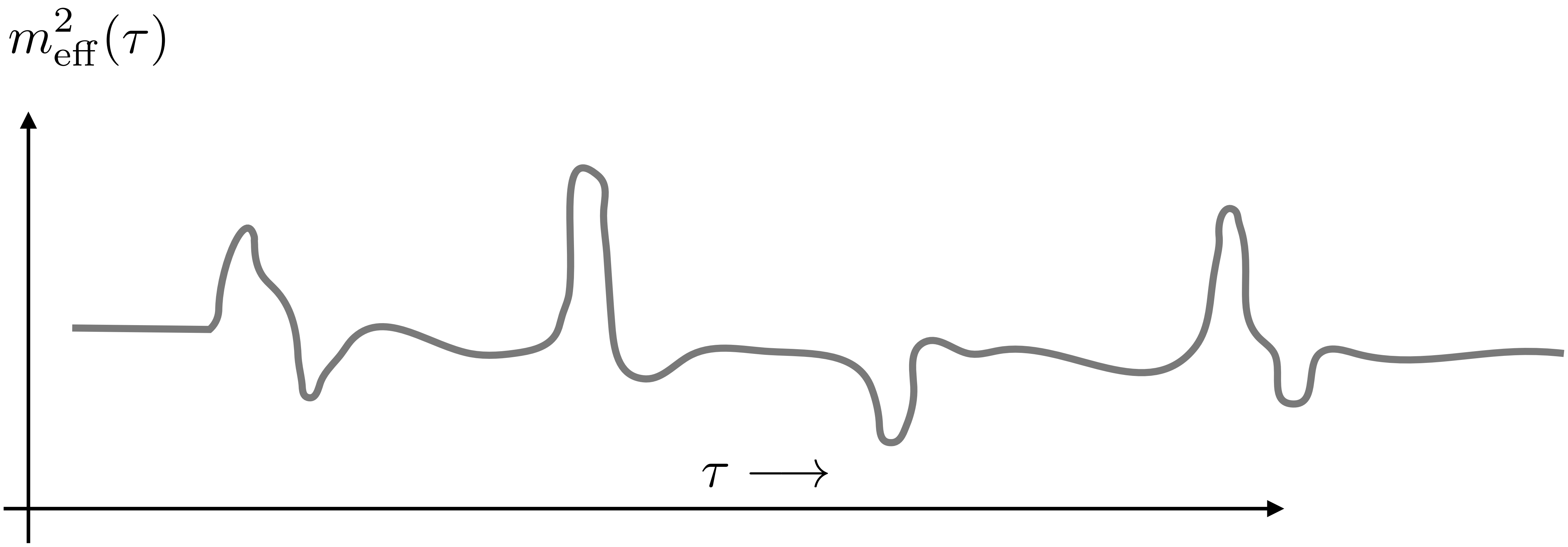


complexity in the "effective mass"/ interactions

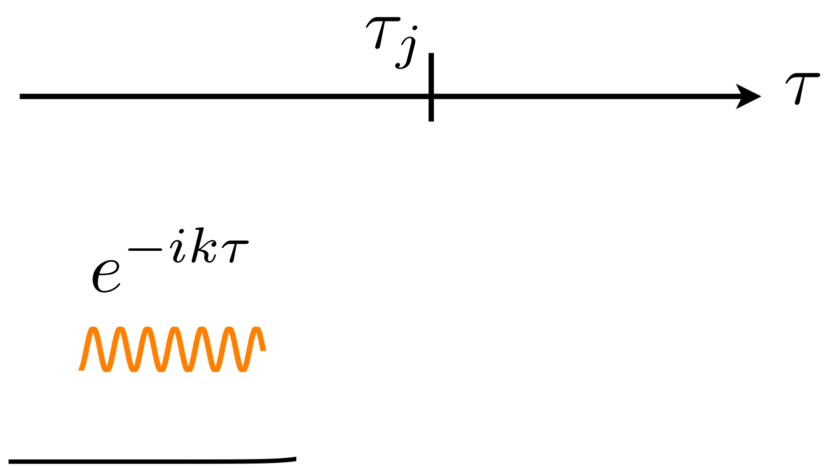
simplified version!

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$

$$m_{\rm eff}^2(\tau) = -\frac{\ddot{a}(\tau)}{a(\tau)} + a^2(\tau)\left[M^2 + \sum_j g_j^2(\phi(\tau) - \phi_j^*)^2 + \ldots\right]$$



particle production as "scattering"



$$|\chi_k(\text{after})\rangle = \underbrace{\begin{pmatrix} 1/t_j^* & -r_j^*/t_j^* \\ -r_j/t_j & 1/t_j \end{pmatrix}}_{\mathsf{M}_j} |\chi_k(\text{before})\rangle$$

Kofman, Linde & Starobinsky 1997

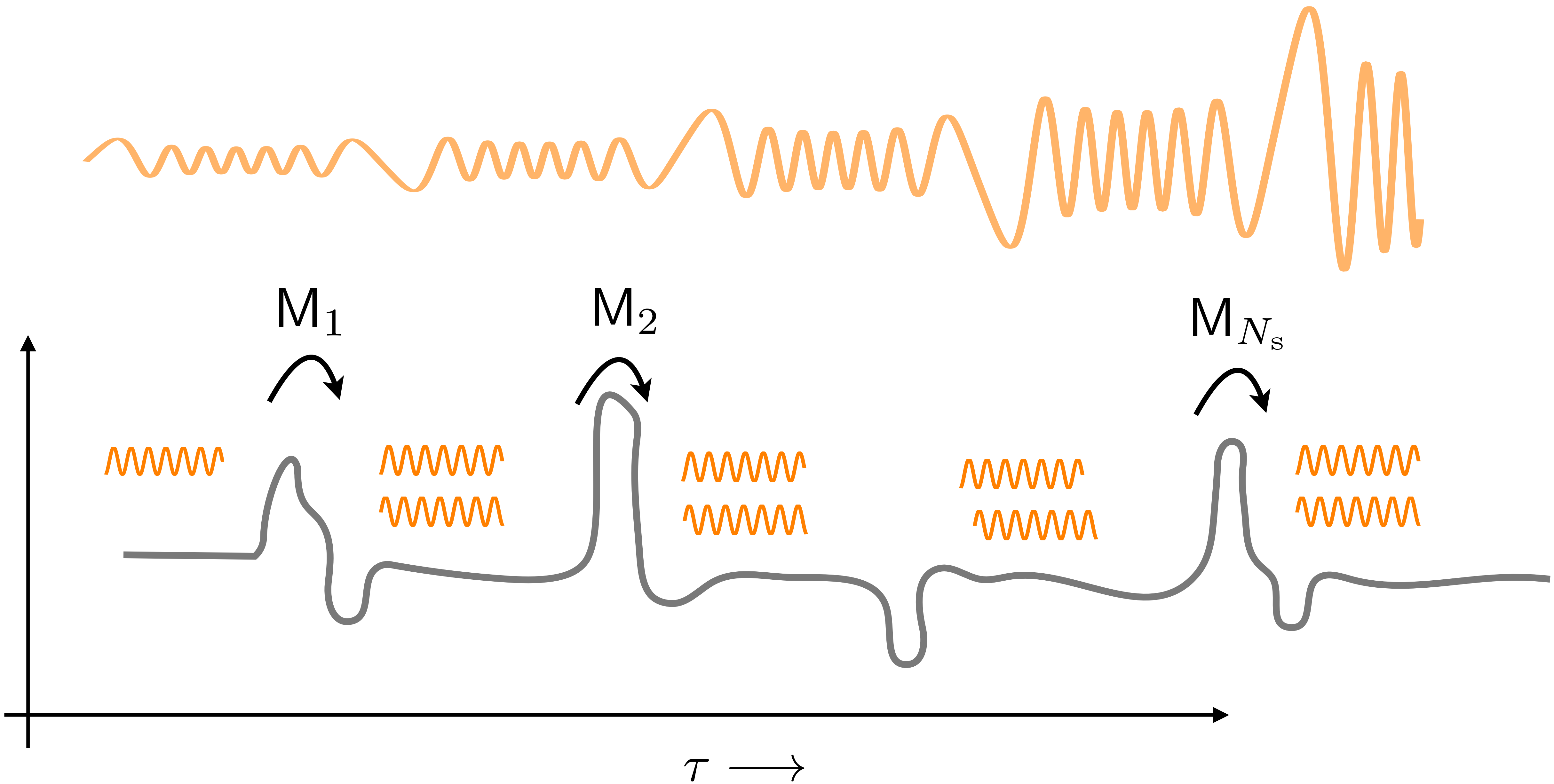
occupation number per mode

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$

$$T_j = |t_j|^2 \qquad n_j \equiv \frac{|r_j|^2}{|t_j|^2} = T_j^{-1} - 1$$

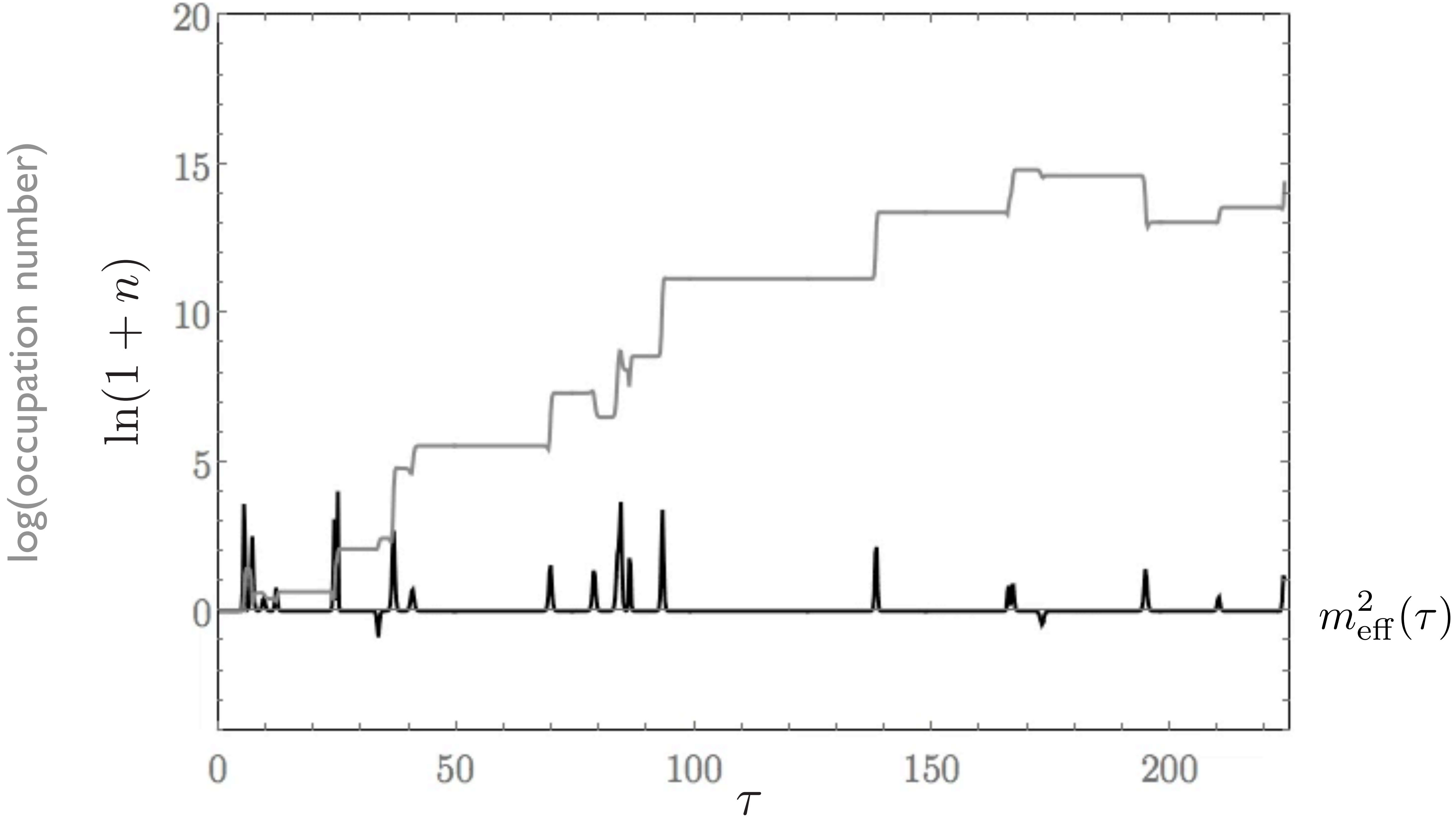
chaining transfer matrices

$$|\chi_k(N_\mathrm{s})\rangle = \mathsf{M}|\chi_k(0)\rangle \qquad \text{where} \qquad \mathsf{M} \equiv \mathsf{M}_{N_\mathrm{s}} \cdots \mathsf{M}_2\mathsf{M}_1$$



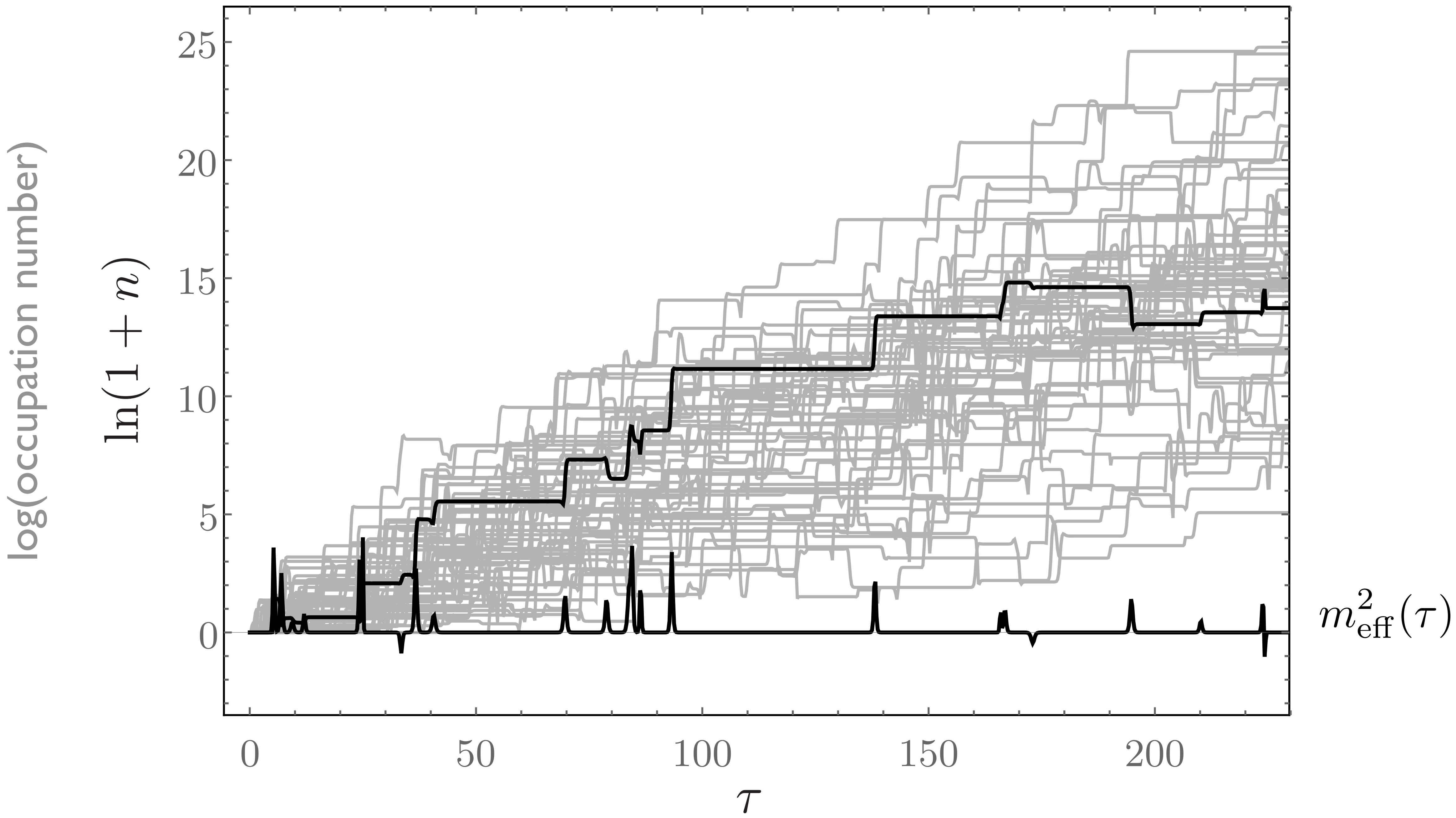
occupation number performs a drifted random walk

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$

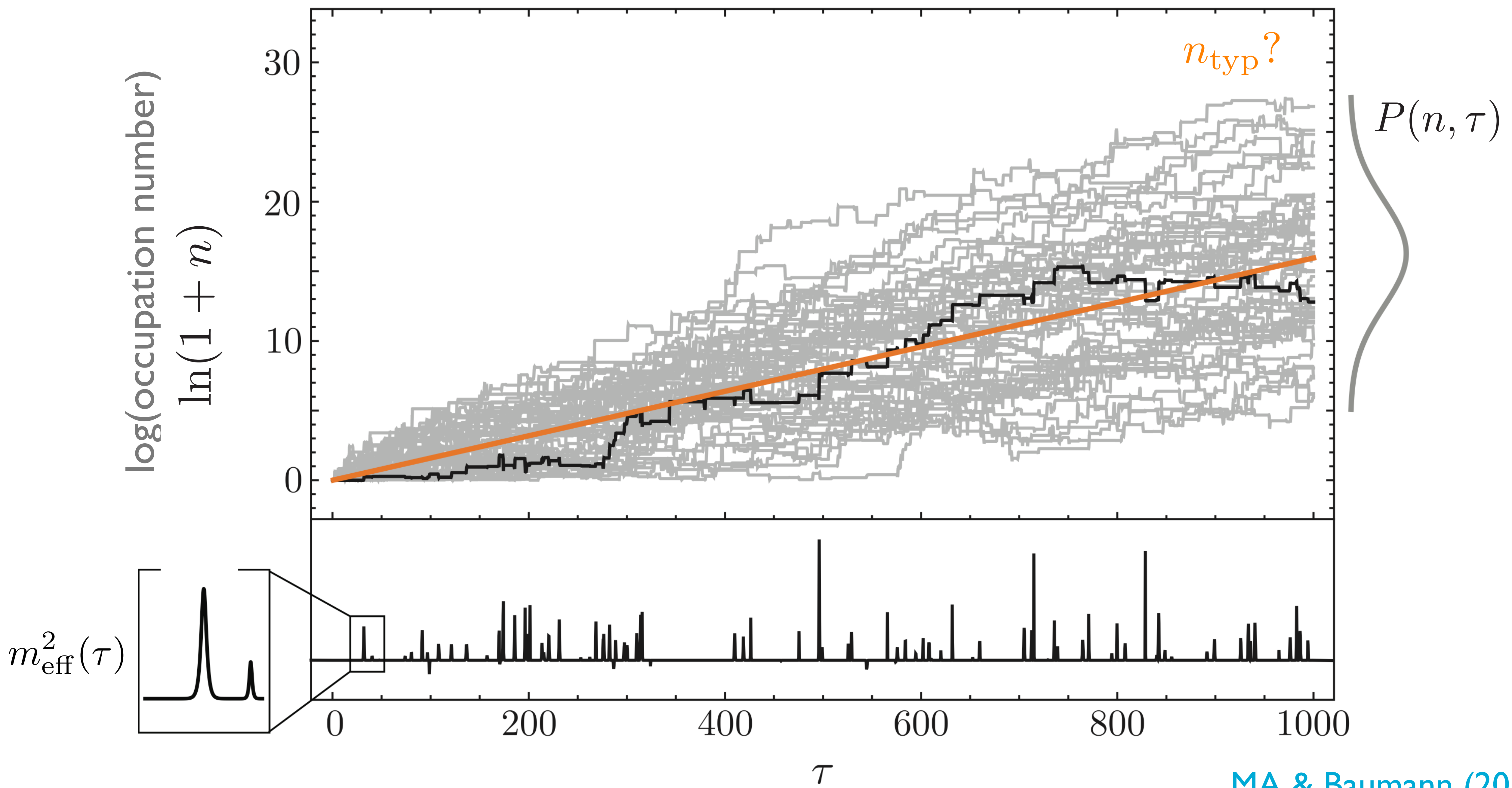


a drifted random walk
different realizations

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$



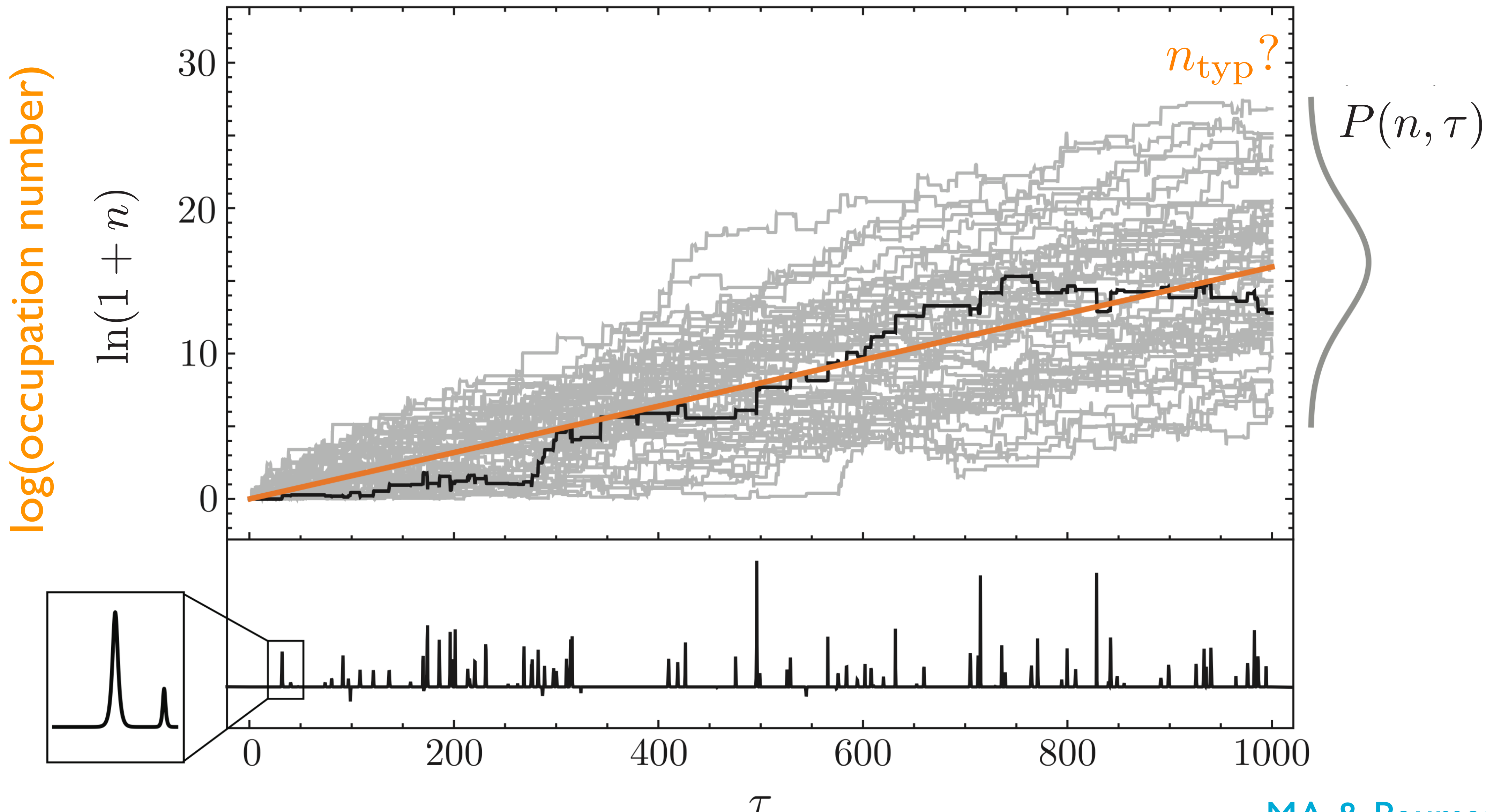
probability distribution ?
typical occupation number ?



MA & Baumann (2015)

a Fokker Planck equation

$$\boxed{\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right]}$$



MA & Baumann (2015)

generalization to <u>many fields</u>

MA, Garcia, Xie & Wen (2017)

- simple *Trace formula* for estimating particle production rate when the number of fields is large (*without being statistically similar*)

$$\boxed{n_{\rm typ} \sim \exp\left[\frac{2}{1+N_{\rm f}}\langle {\rm Tr}\,\Lambda^2\rangle\ \tau\right]}$$

- Λ contains all the information about the strengths of interactions

stochastic particle production in an expanding universe

MA, Garcia, Carlsten, Green (2019)

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2\chi}{a^2} + m_{\rm eff}^2(t)\chi = 0$$

spectator field

$$m_{\rm eff}^2(t) = M^2 + m^2(t) \text{ with } m^2(t) = \sum_j m_j\delta(t - t_j)$$

$$\langle m_j \rangle = 0\,, \qquad \langle m_j m_i \rangle = \sigma^2 \delta_{ij}$$

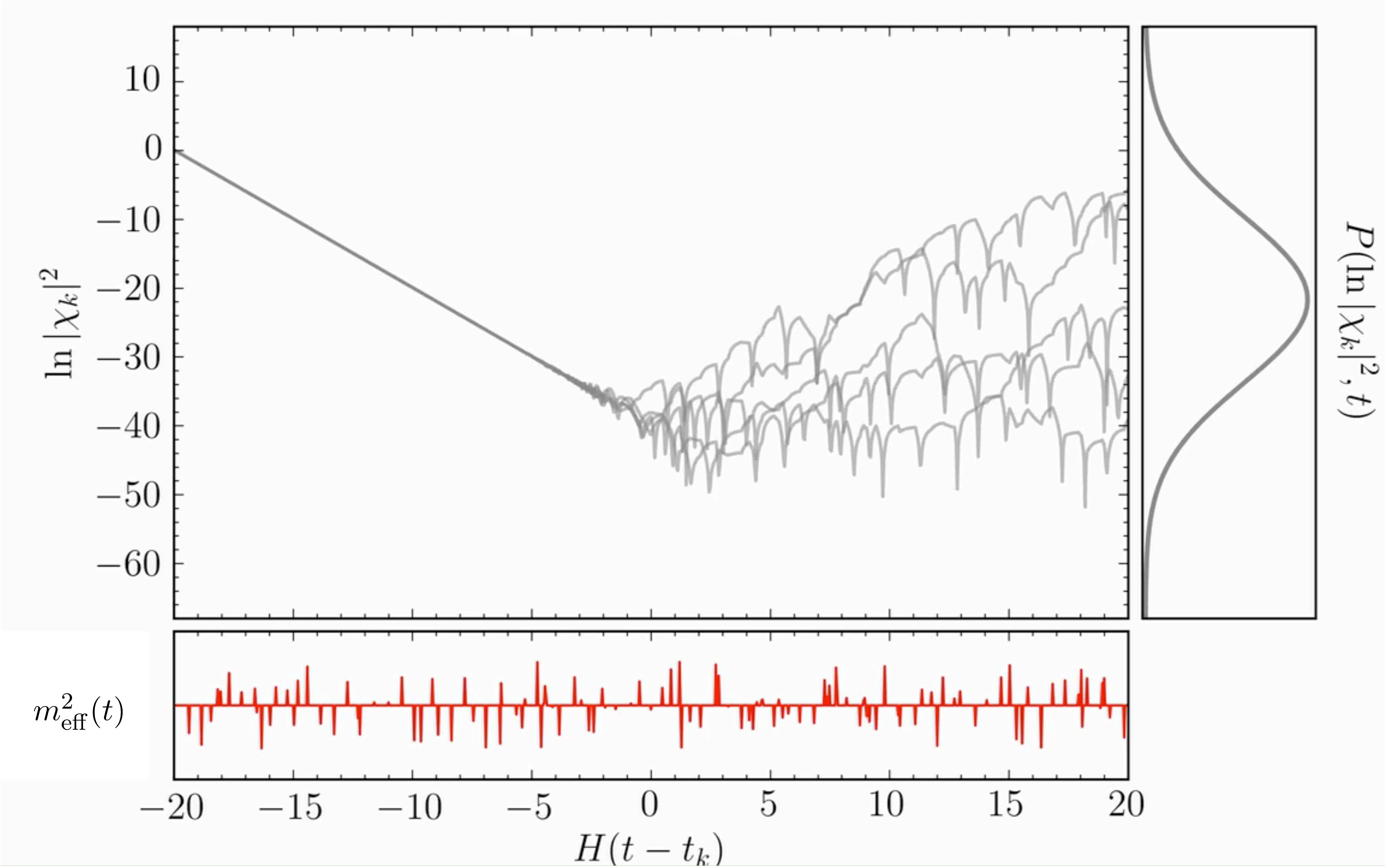
$$\mathcal{N}_s \left(\frac{\sigma}{H}\right)^2$$

useful dimensionless parameter for superhorizon scale evolution

# of scatters per Hubble time

* delta functions not necessary, good approx. when momentum less than inverse width

stochastic particle production in de Sitter universe



universal features

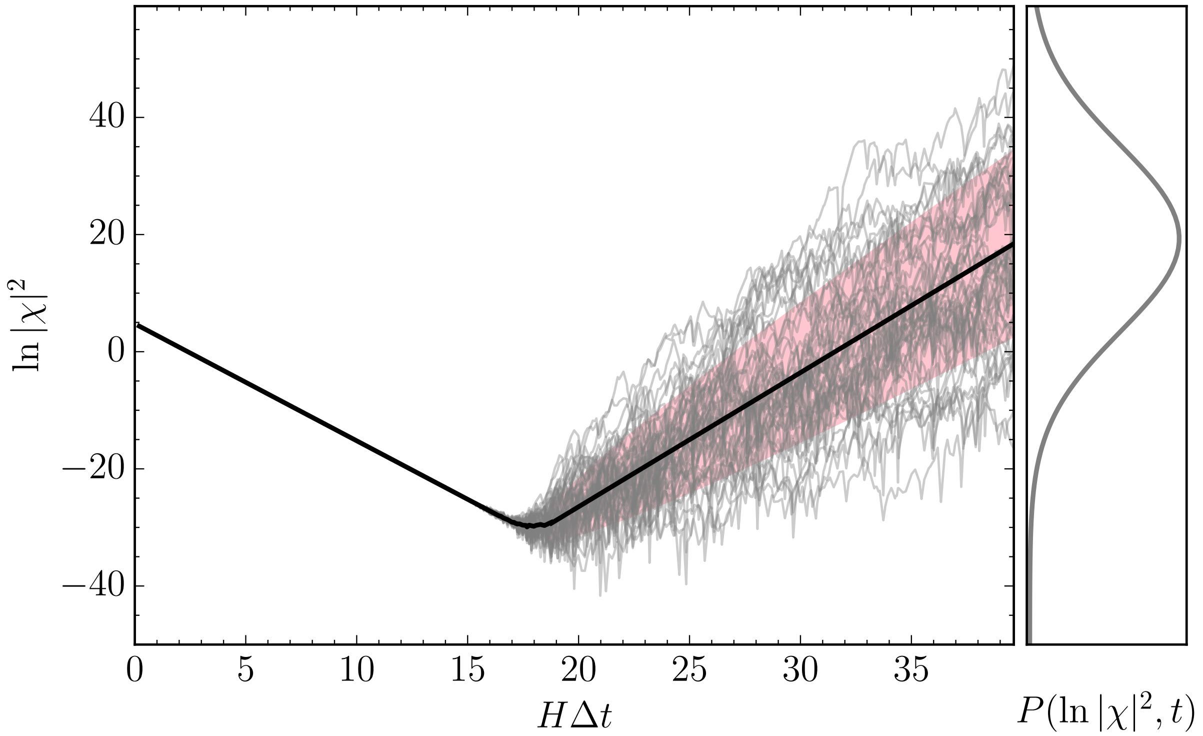
MA, Garcia, Carlsten, Green (2019)

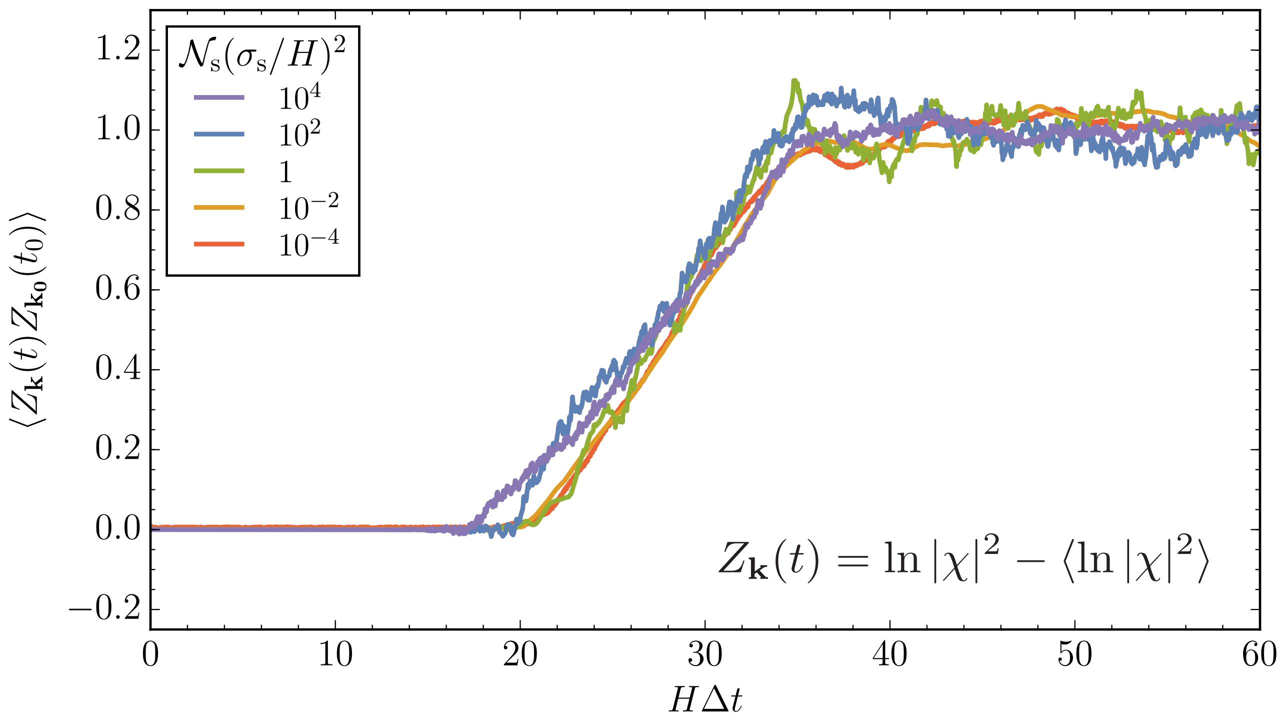
1. *typical* mode amplitudes grow/decay exponentially with cosmic time outside the horizon

$$\mathcal{N}_s \frac{\sigma_s^2}{H^2}$$

2. the distribution of field amplitudes is log-normal

3. correlation functions have the characteristics of a geometric random walk.





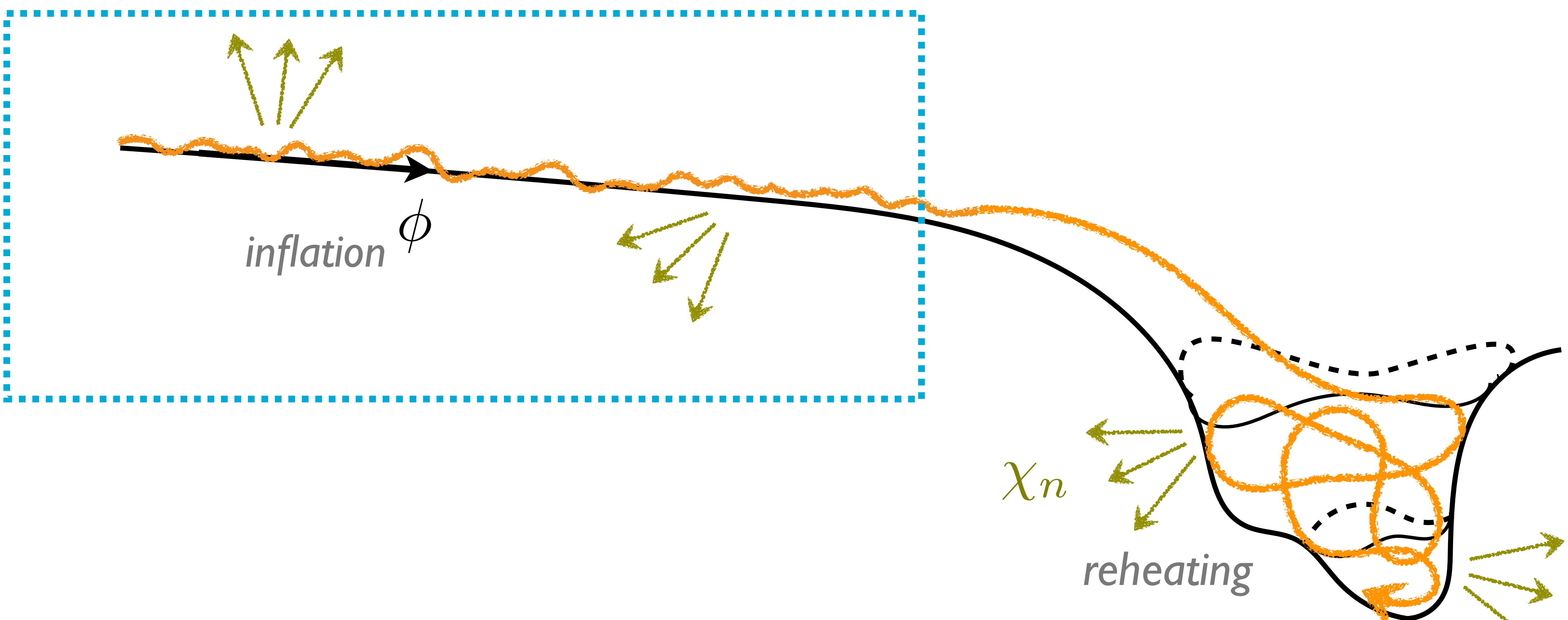
applications: inflation

WORK IN PROGRESS

MA, Garcia, Carlsten, Green, Baumann & Chia (in progress)

background dynamics → particle production ↔ curvature fluctuations

$\langle \chi_{k_1} \chi_{k_2} \dots \rangle$ $\langle \zeta_{k_1} \zeta_{k_2} \dots \rangle$

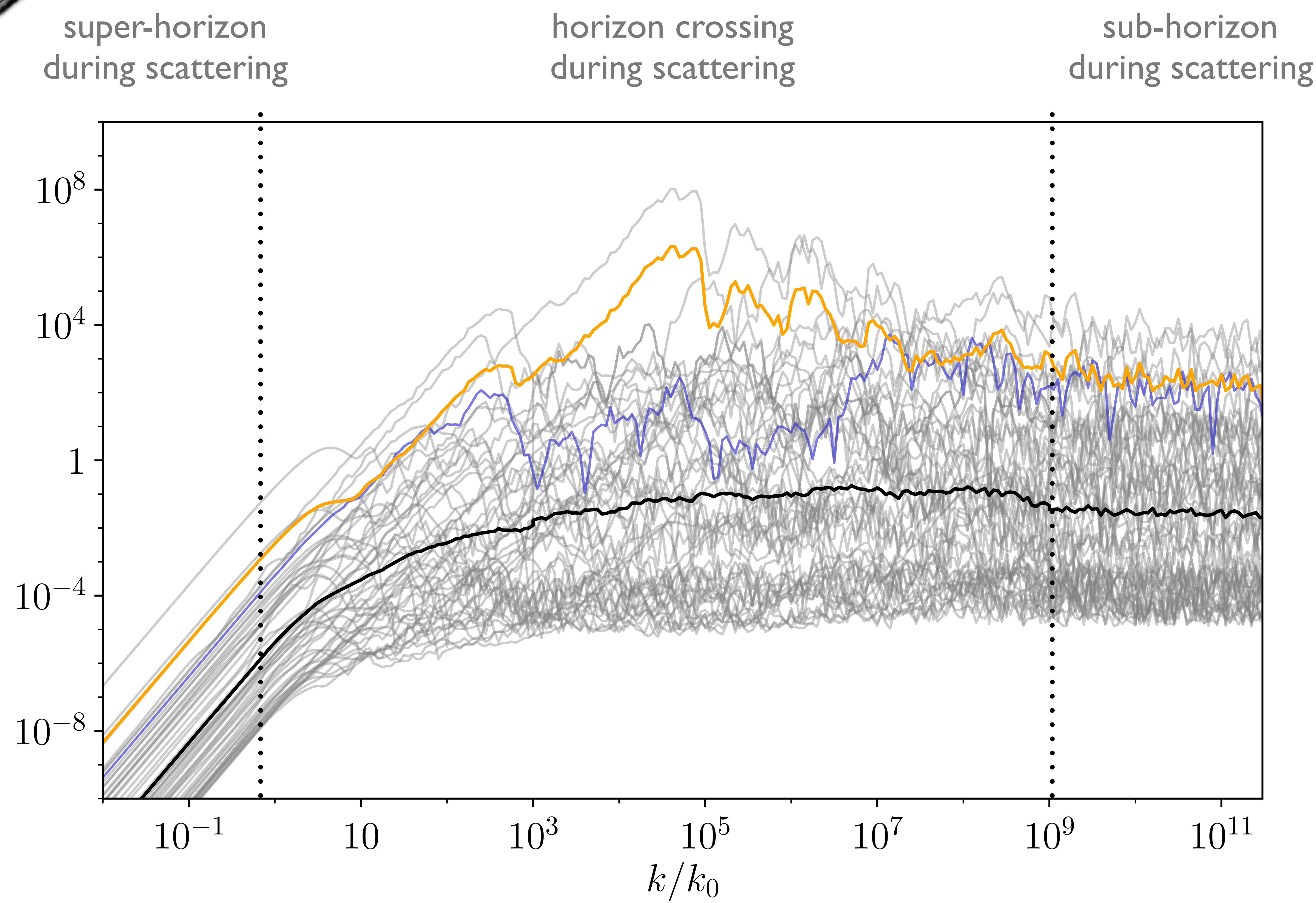


also see: Dias, Fraser & Marsh (2015)

curvature perturbations

MA, Garcia, Carlsten, Green, Baumann &Chia (in progress)

WORK IN PROGRESS

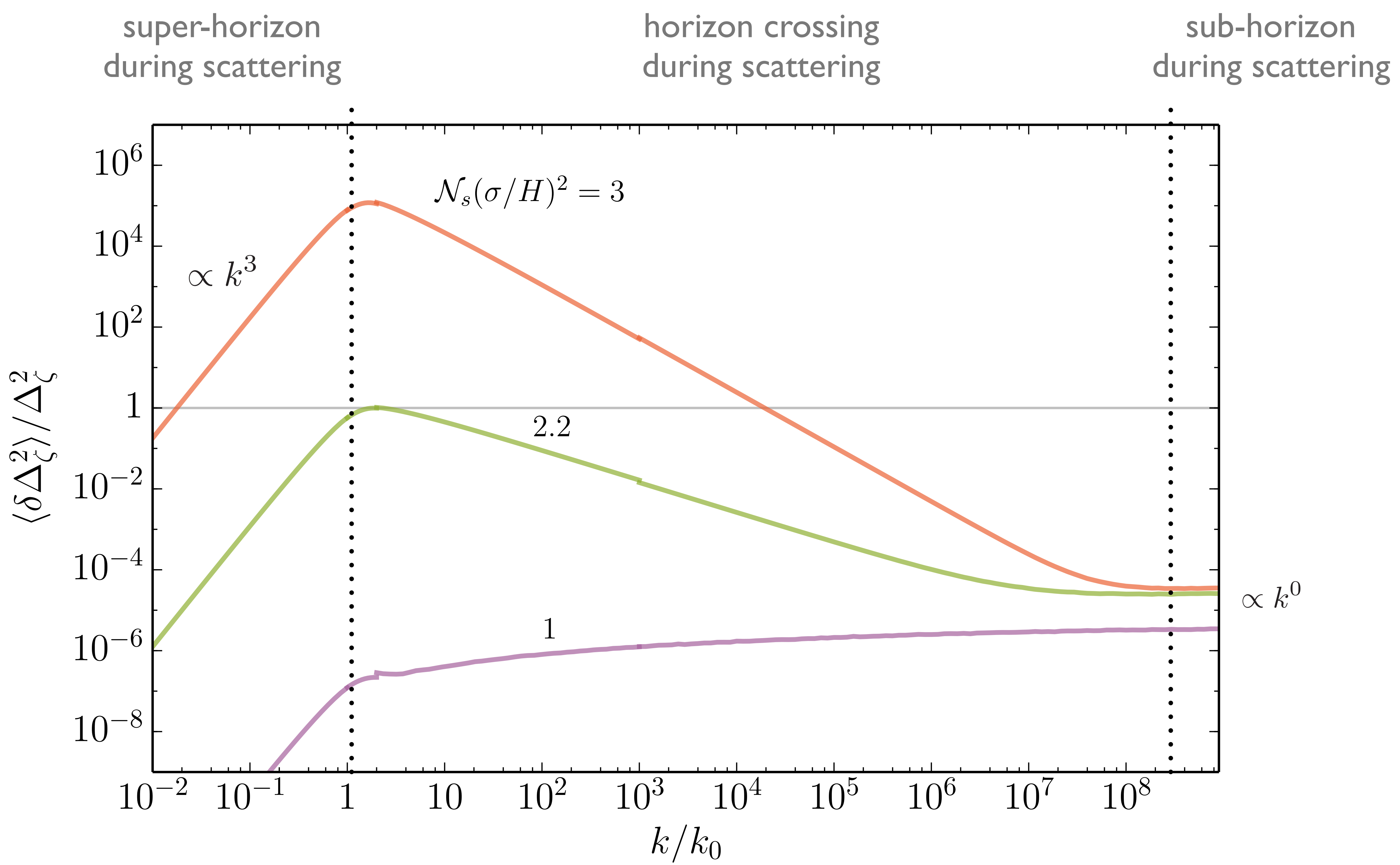


$N_{\rm tot} = 20 \qquad \mathcal{N}_{\rm s}(\sigma^2/H^2) = 25$

“mean” curvature perturbations

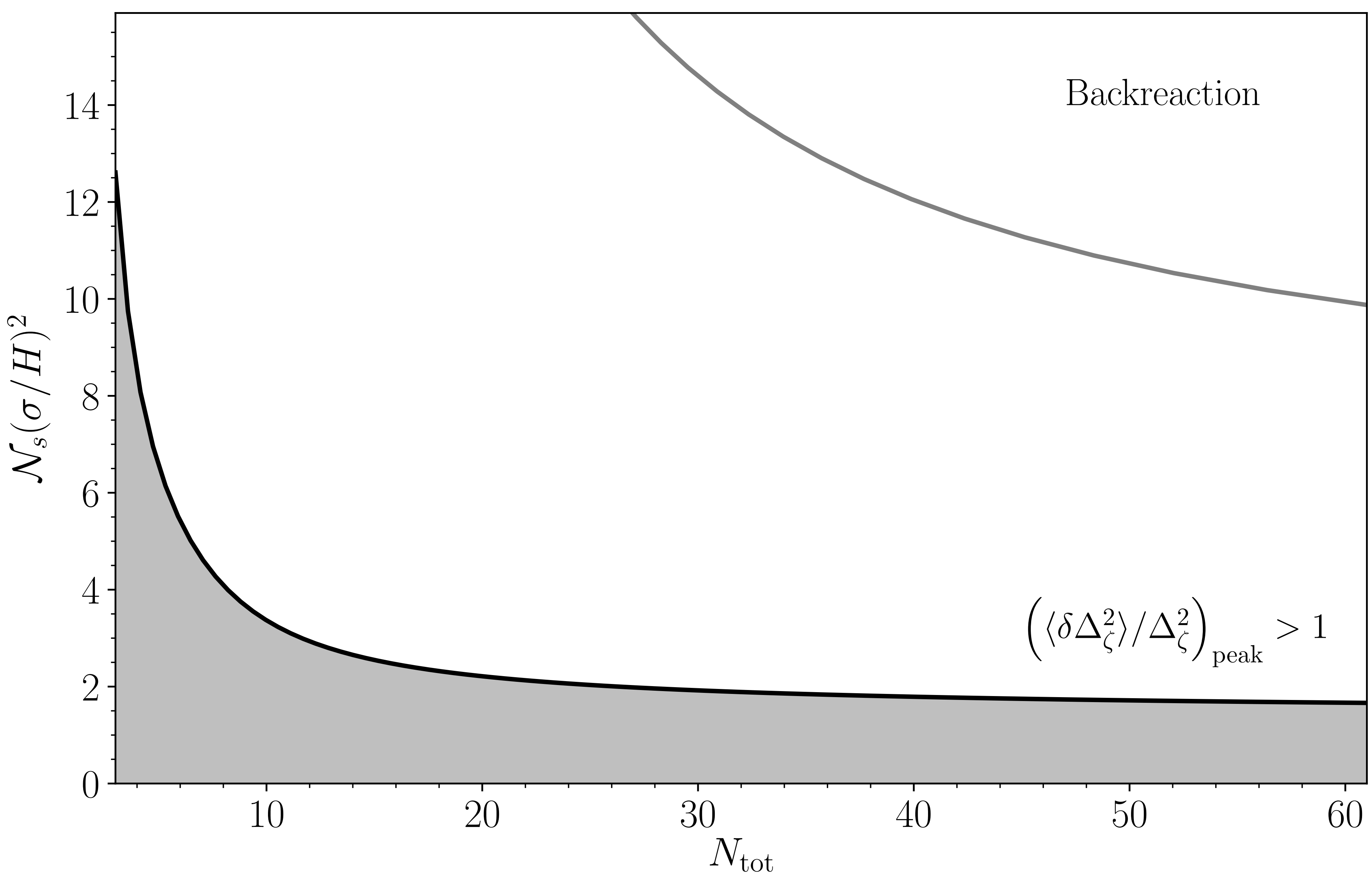
MA, Garcia, Carlsten, Green, Baumann &Chia (in progress)

WORK IN PROGRESS



WORK IN PROGRESS

backreaction? detectable?

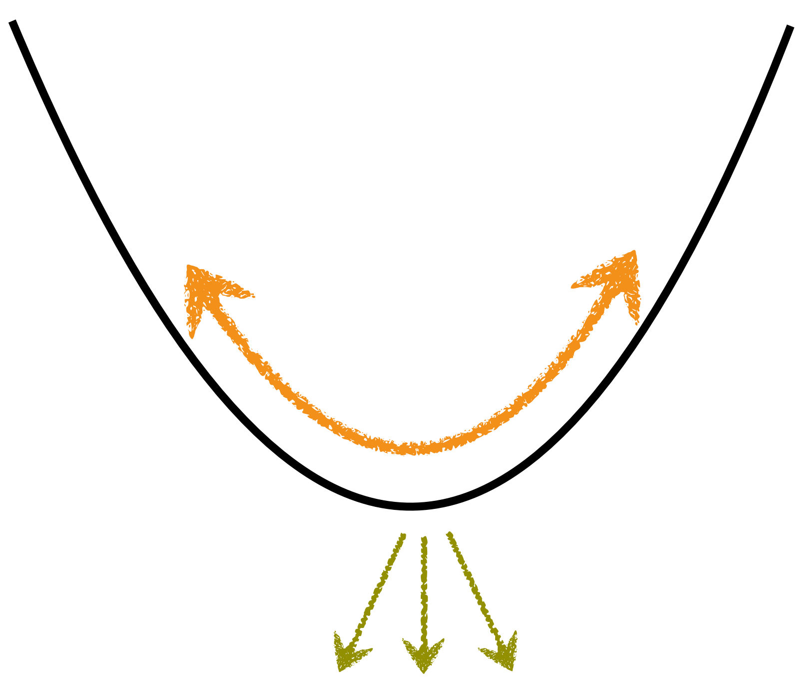


implications ?

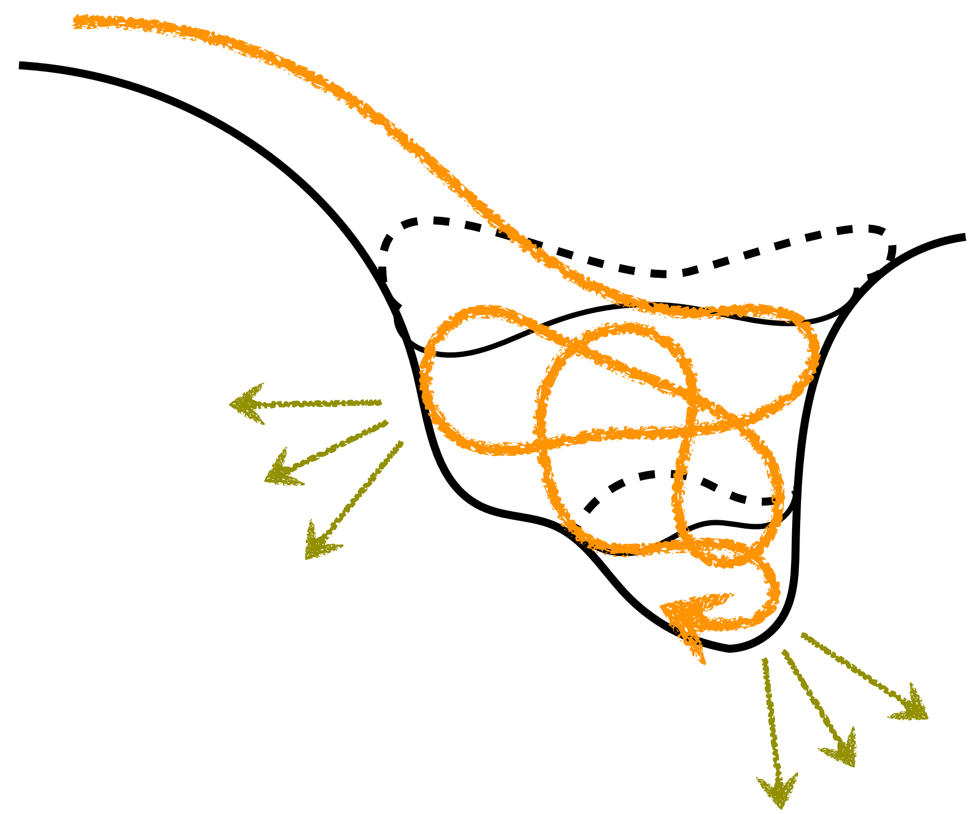
1. implications - PBH ?
2. primordial gravitational waves
3. higher point correlation functions
4. application to reheating …

WORK IN PROGRESS

applications : reheating



multichannel — multifield — statistical

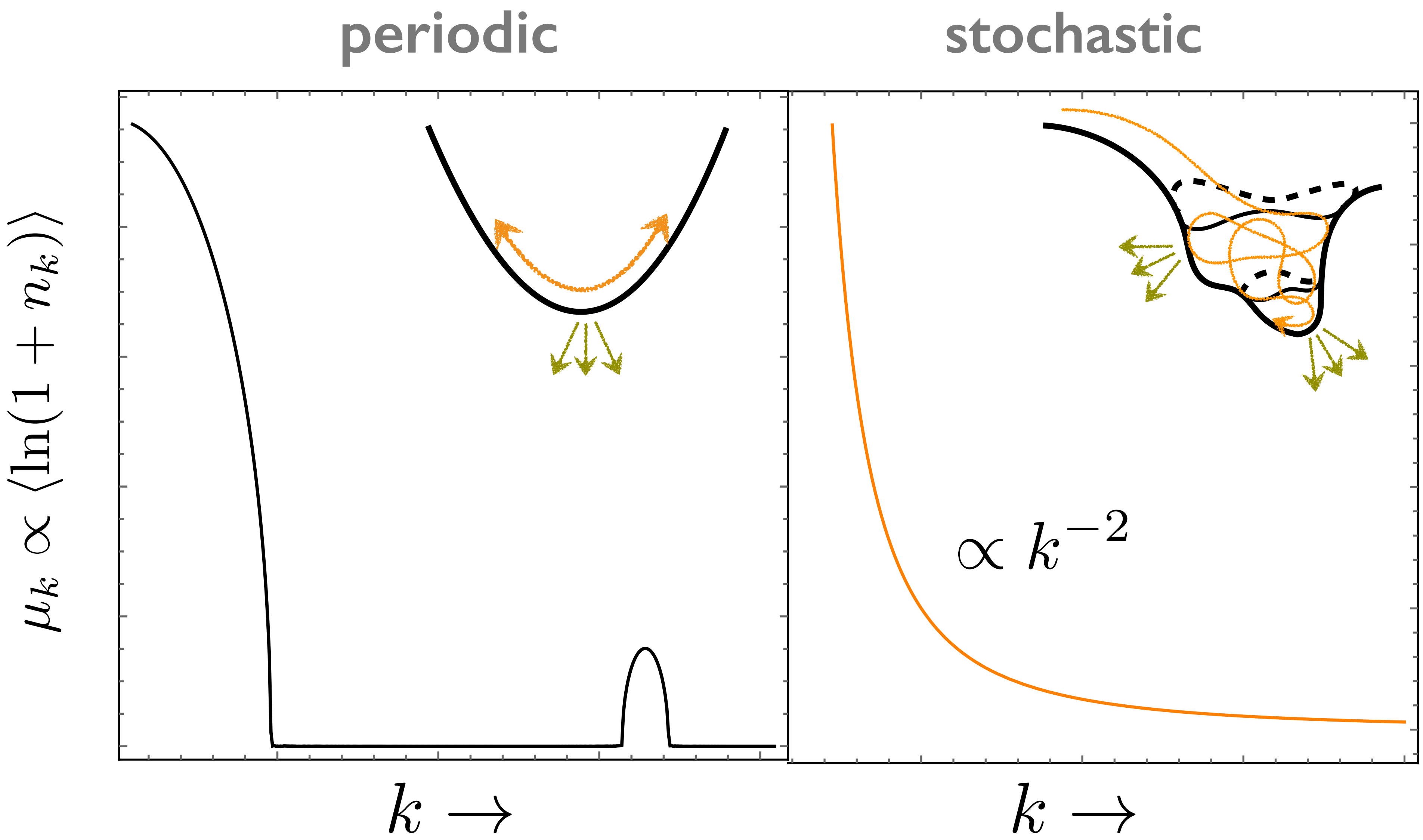


Kofman, Linde & Starobinsky (1997)
Traschen & Brandenberger (1997)
Zanchin et. al (1998) & Bassett (1998) [with noise]

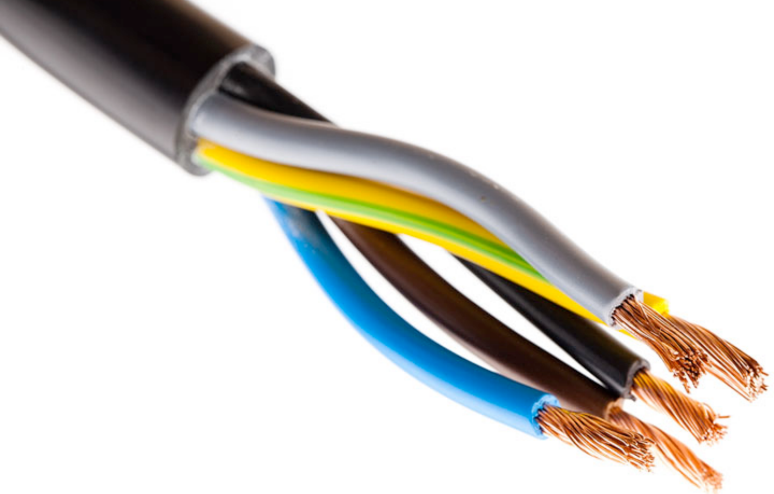
model-insensitive description of a complicated reheating process.

simplicity from stochasticity

WORK IN PROGRESS

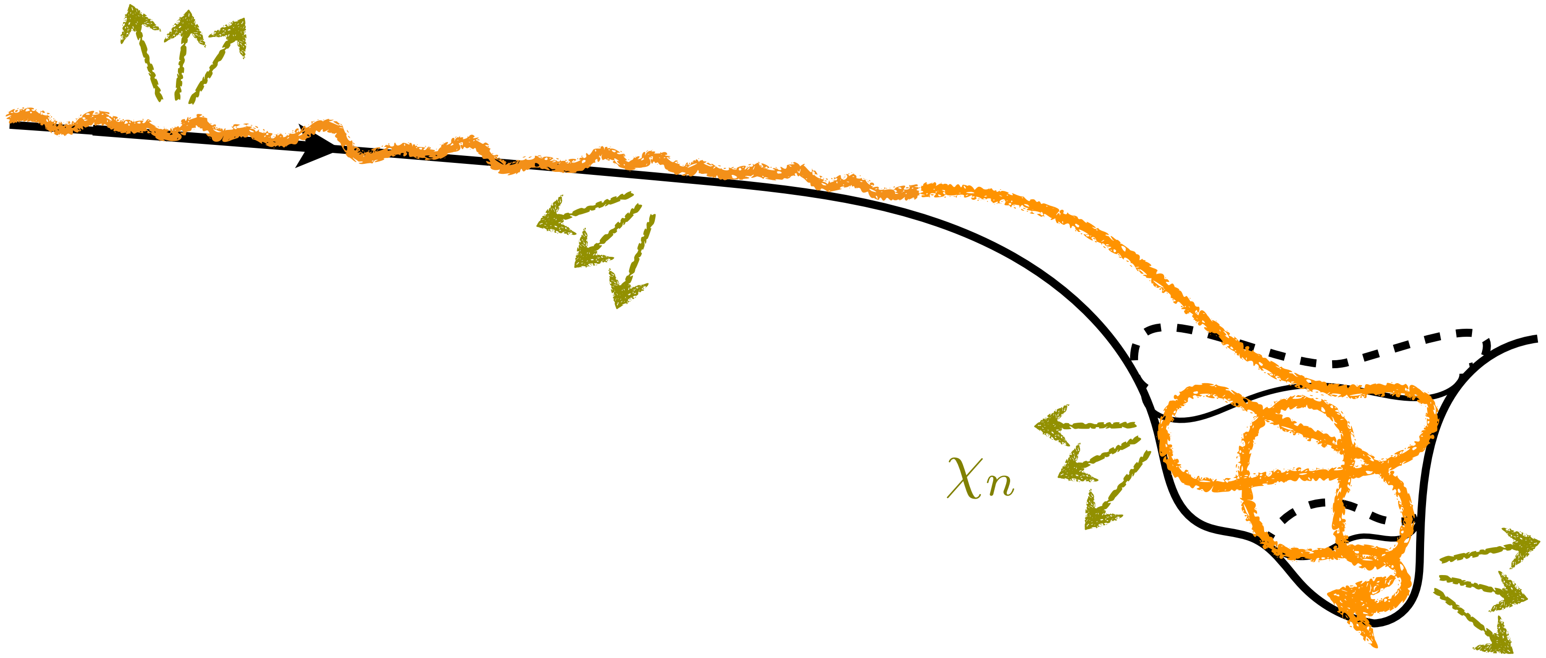


see hints in: Bassett (1998), Barnaby, Kofman & Braden et. al 2010

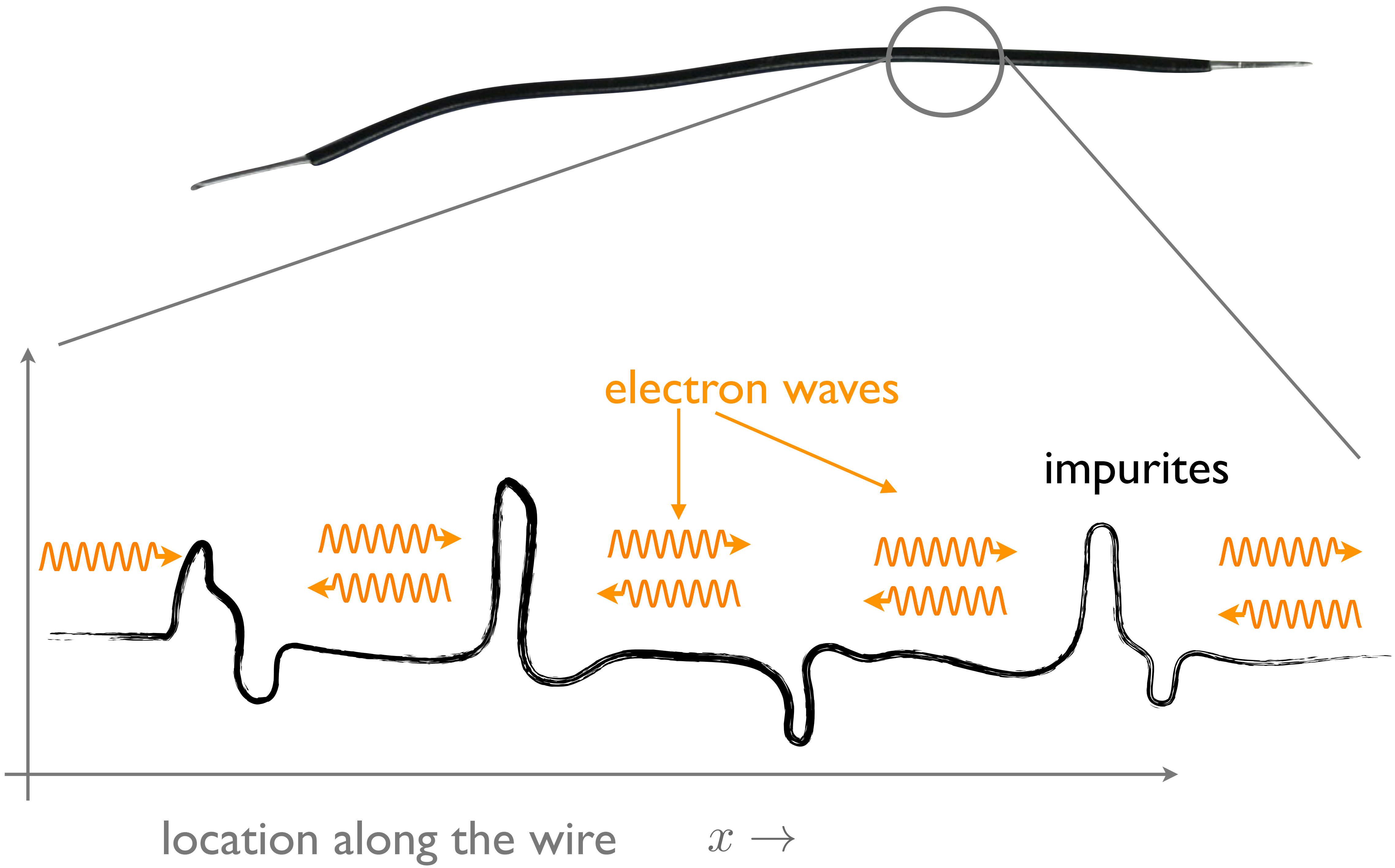


what is the connection to wires?

From Wires to Cosmology [MA & Baumann 2016]

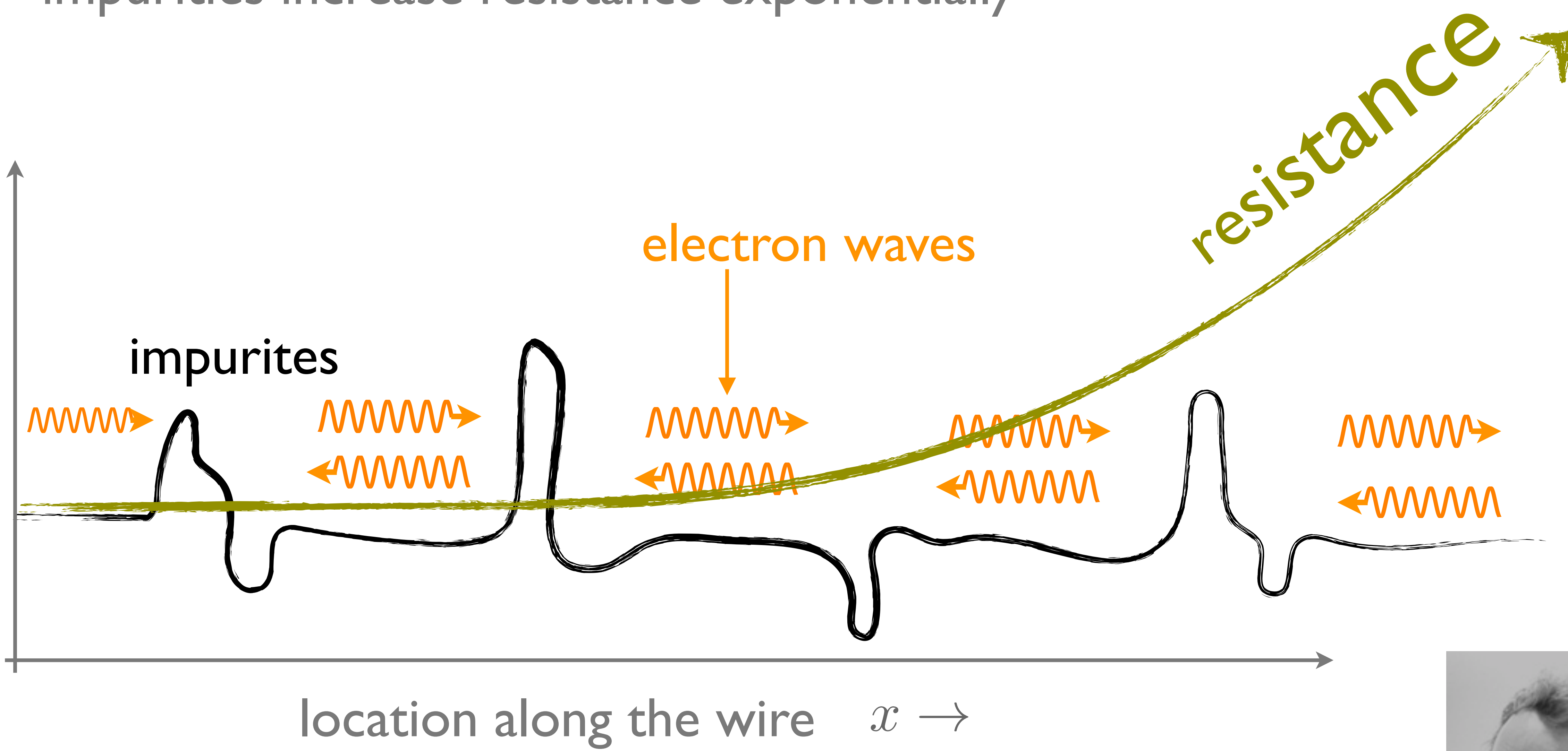


scattering inside disordered wires



universal behavior: Anderson Localization

- impurities increase resistance exponentially



$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

at low temperatures, one dimensional wires are insulators

complexity in time ⟷ complexity in space
cosmology ⟷ wires

exponential growth in occupation number

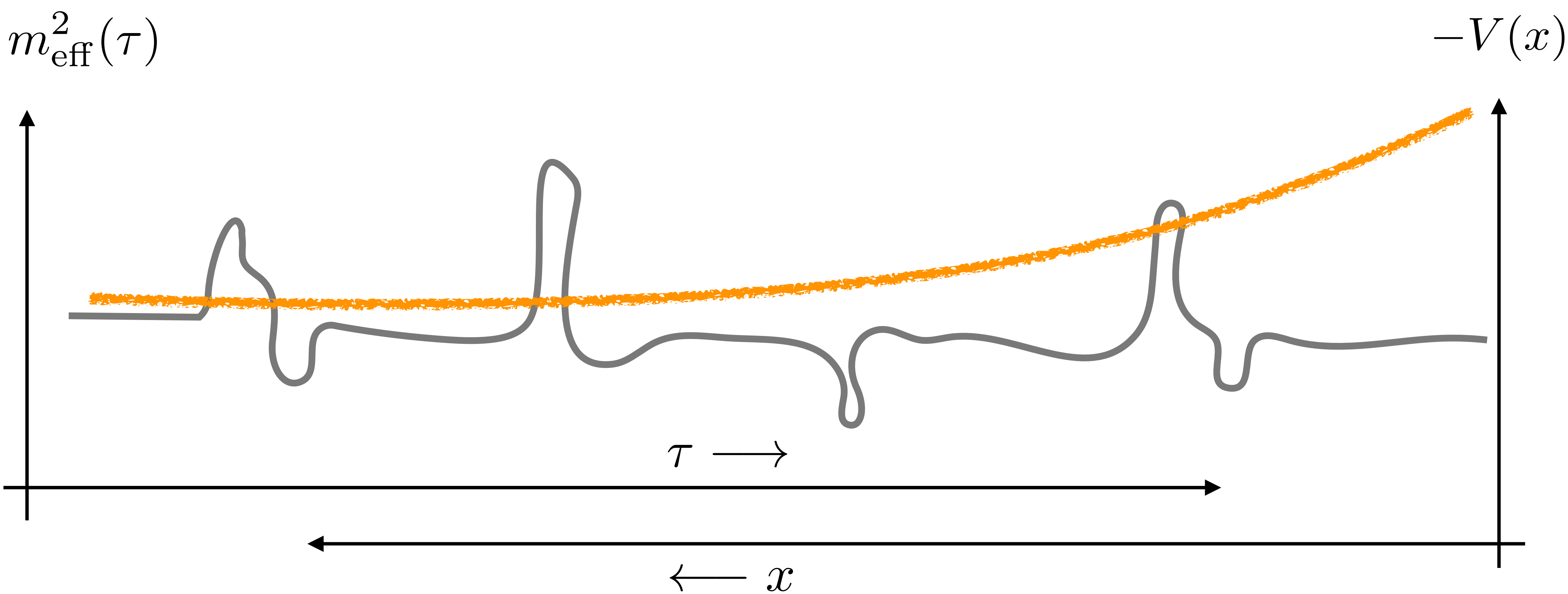
$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right] \chi_k(\tau) = 0$$

$\longleftrightarrow$

Anderson localization

$$\psi''(x) + \left[k^2 - V(x)\right] \psi(x) = 0$$

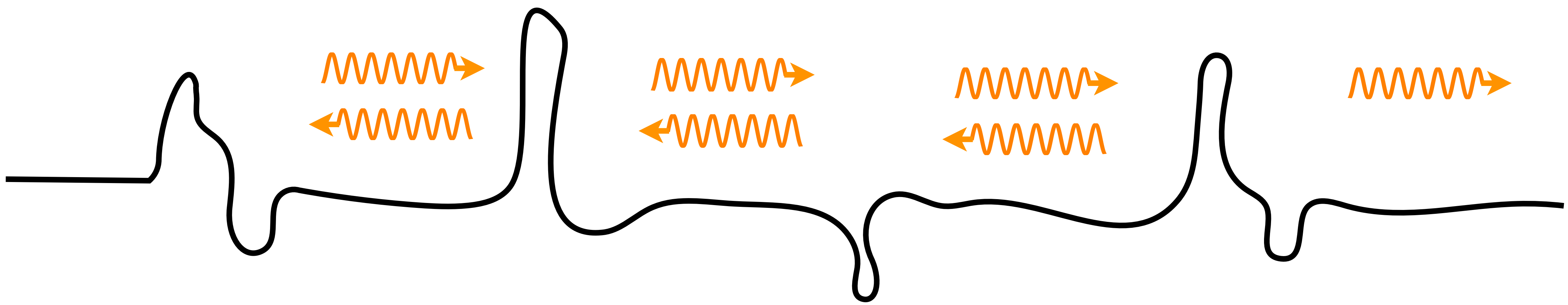
simplified version!



for periodic case with noise see Zanchin et. al 1998, Brandenburger & Craig 2008

dictionary

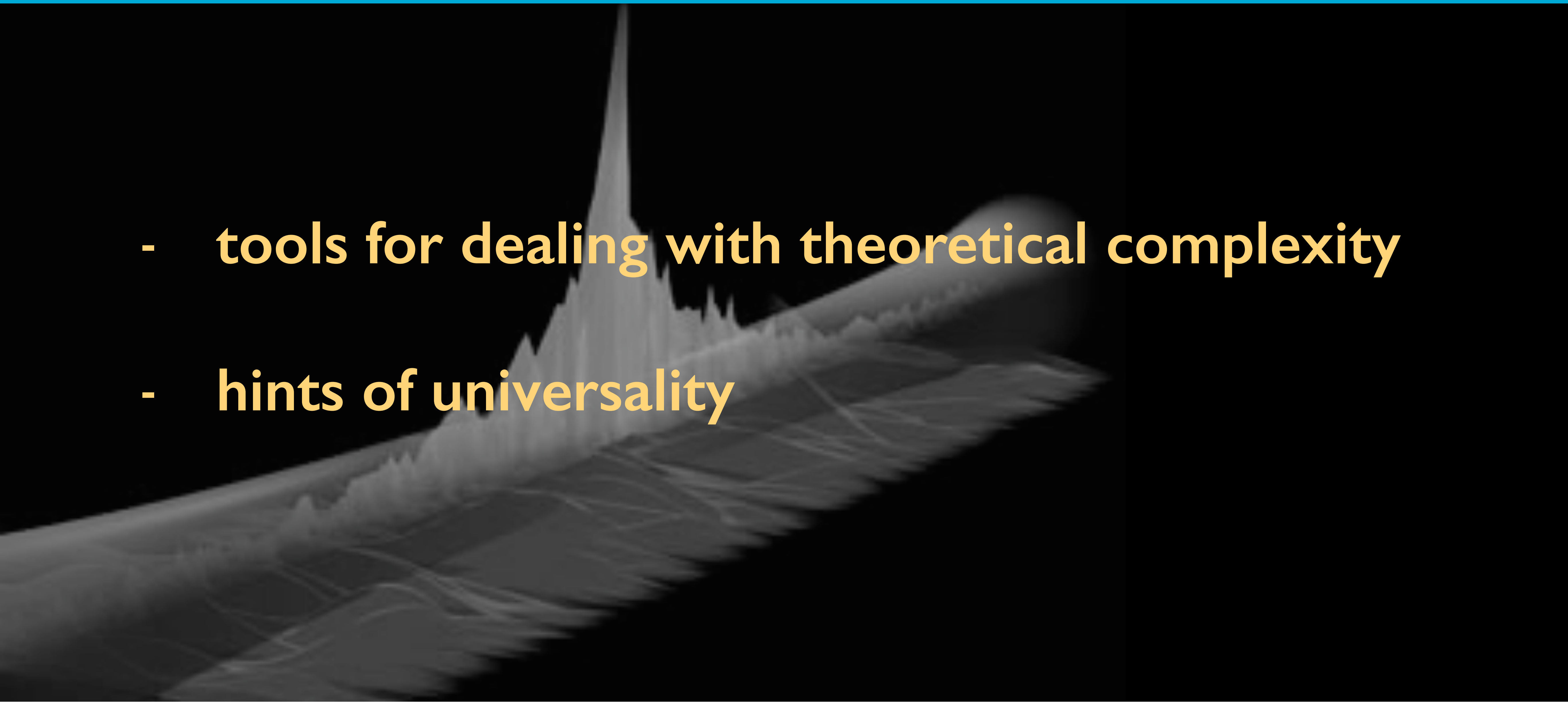
Time-dependent "Klein-Gordon"	Time-independent Schrödinger
$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$	$\frac{d^2\psi}{dx^2} + \big(E - V(x)\big)\psi = 0$

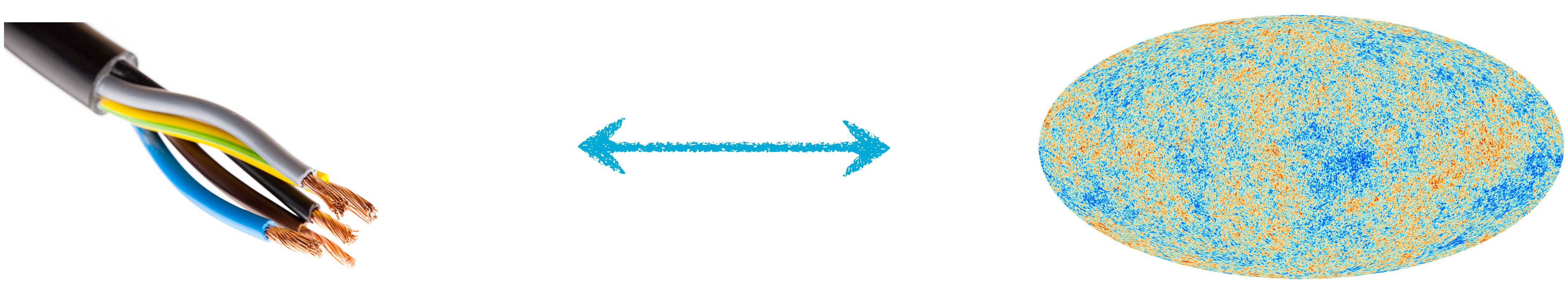


occupation number	resistance
(local) particle production rate	mean free path
multiple fields	**multiple channels**

summary

- tools for dealing with theoretical complexity
- hints of universality

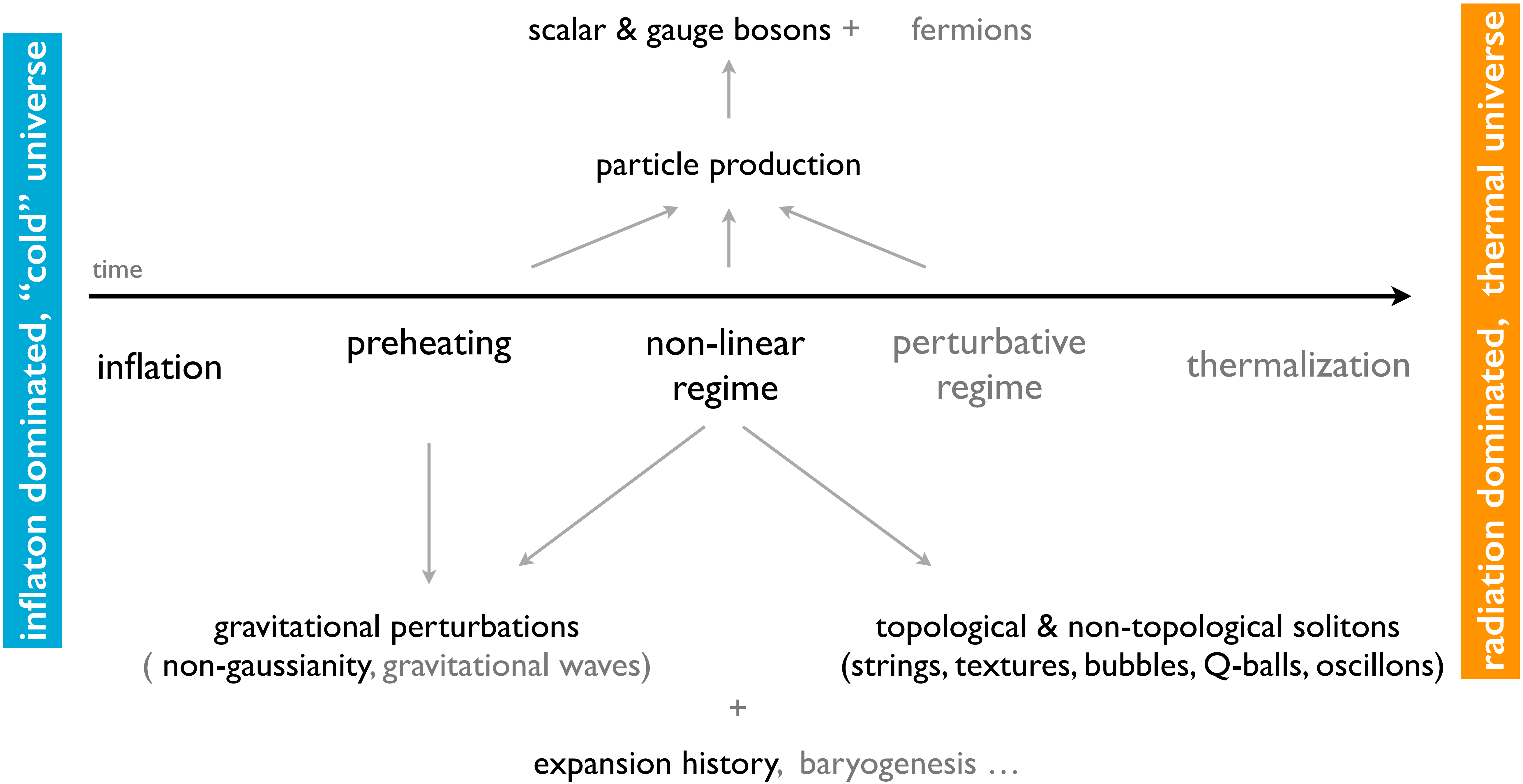




thanks

general non-perturbative dynamics from early universe

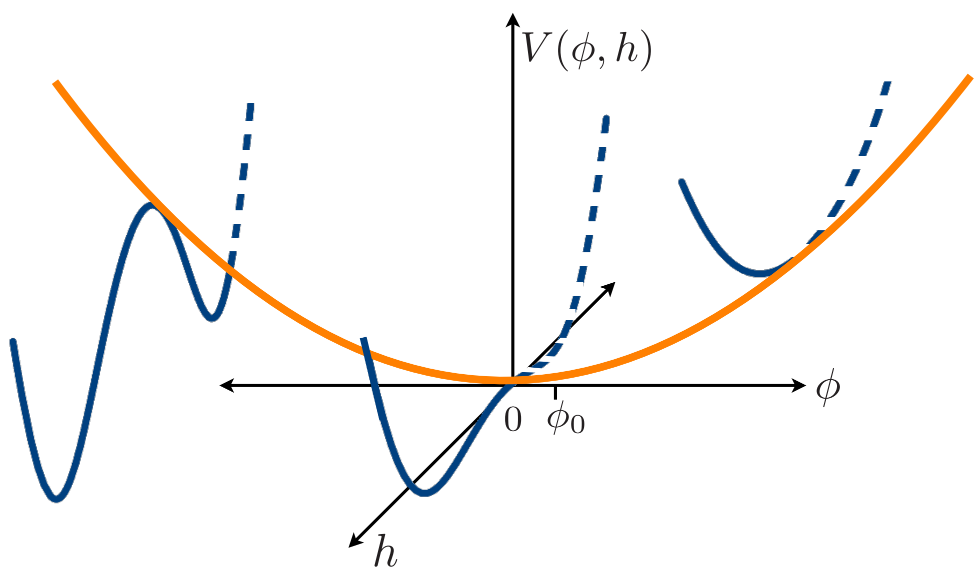
from review: MA, Kaiser, Karouby & Hertzberg (2013)



Cosmological Dynamics & Higgs Fine Tuning

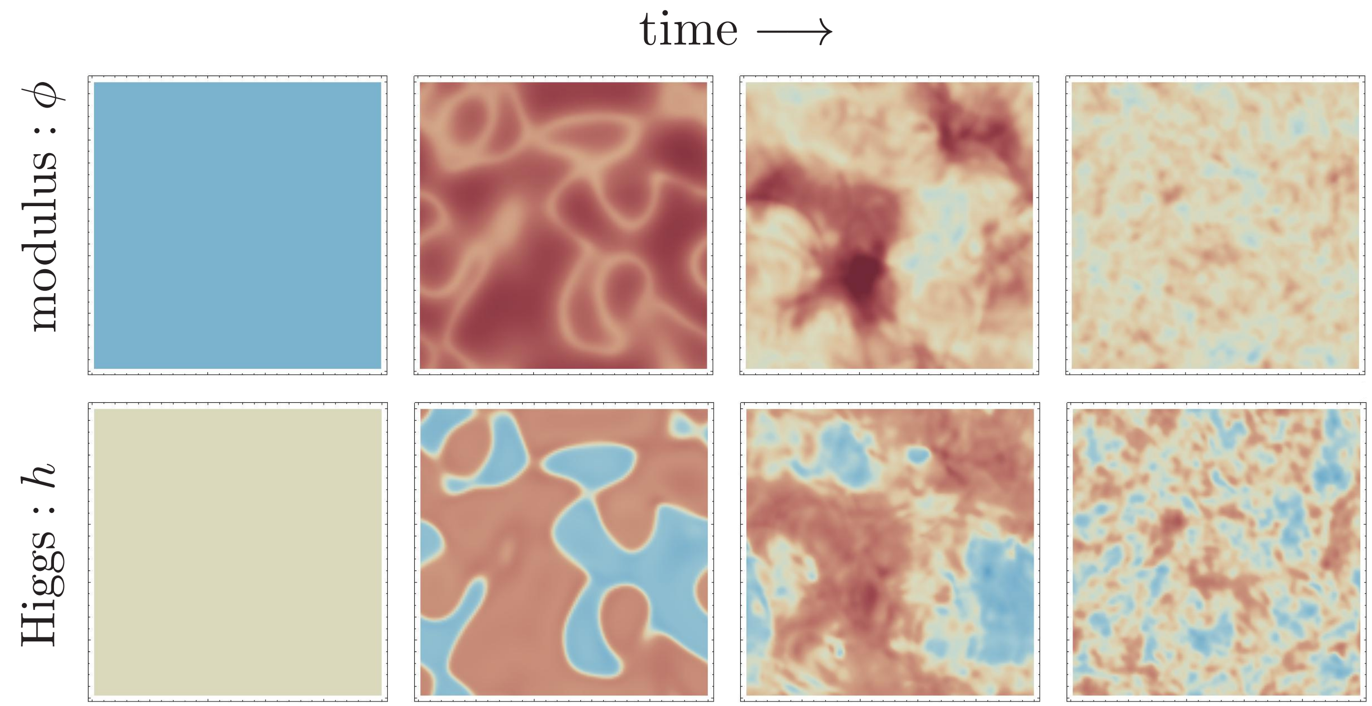
arXiv: 1802.00444

MA, Fan, Lozanov & Reece



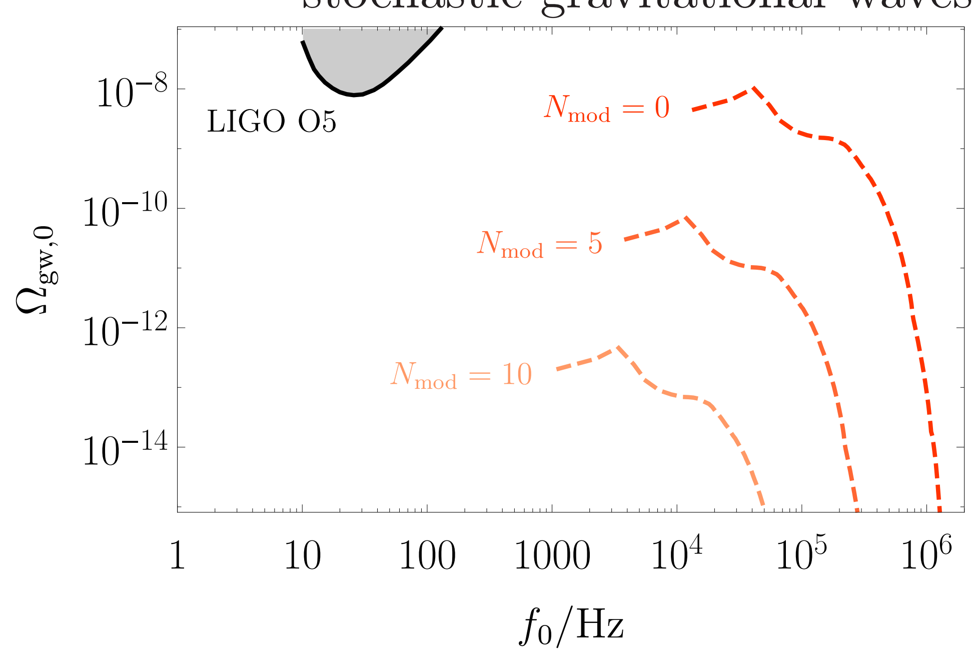
$$\frac{1}{2}m_\phi^2\phi^2 + \frac{M^2}{f}(\phi - \phi_0)\left(h^\dagger h - \frac{v^2}{2}\right) + \lambda(h^\dagger h)^2$$

$$\text{fine tuning} \Leftrightarrow \frac{\phi_0}{f} \ll 1$$



$$\frac{M^4}{2\lambda f^2 m_\phi^2} \to 1 \Leftrightarrow \text{rapid fragmentation}$$

stochastic gravitational waves



non-trivial eq. of state