Nonlinear Behaviour of Axions

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Two Popular Models of Dark Matter

- WIMPs and Axions

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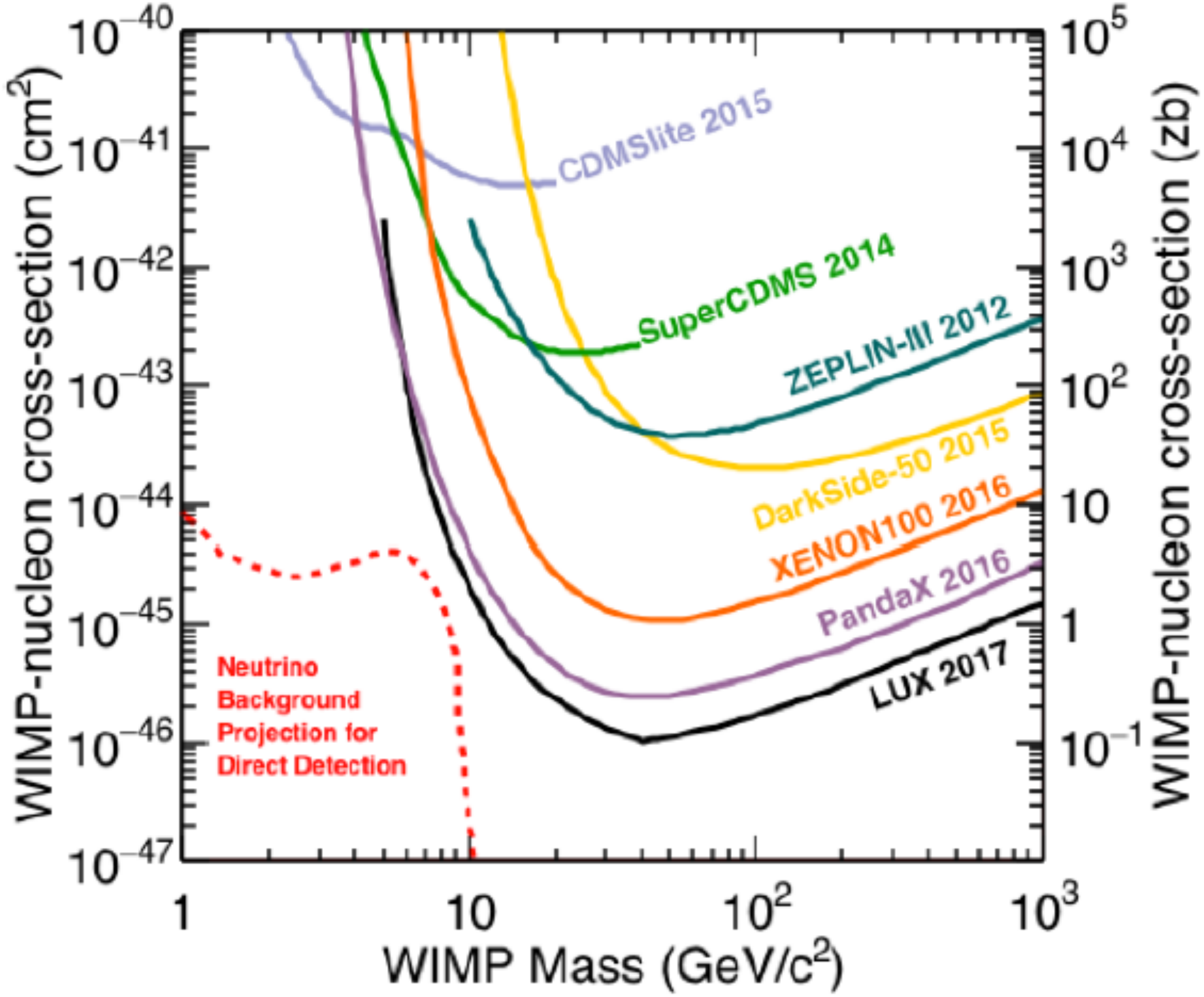
$$\Omega_{\rm WIMP} \sim \frac{10^{-37}\, cm^2}{\langle \sigma_{ann} v \rangle}$$



Mount et al 1703.09144v1

Two Popular Models of Dark Matter

- WIMPs and Axions

QCD-Axion

$$\Delta\mathcal{L}_{qcd} \sim \theta\, \mathbf{E}^a \cdot \mathbf{B}^a \qquad\qquad |\theta| \lesssim 10^{-10}$$

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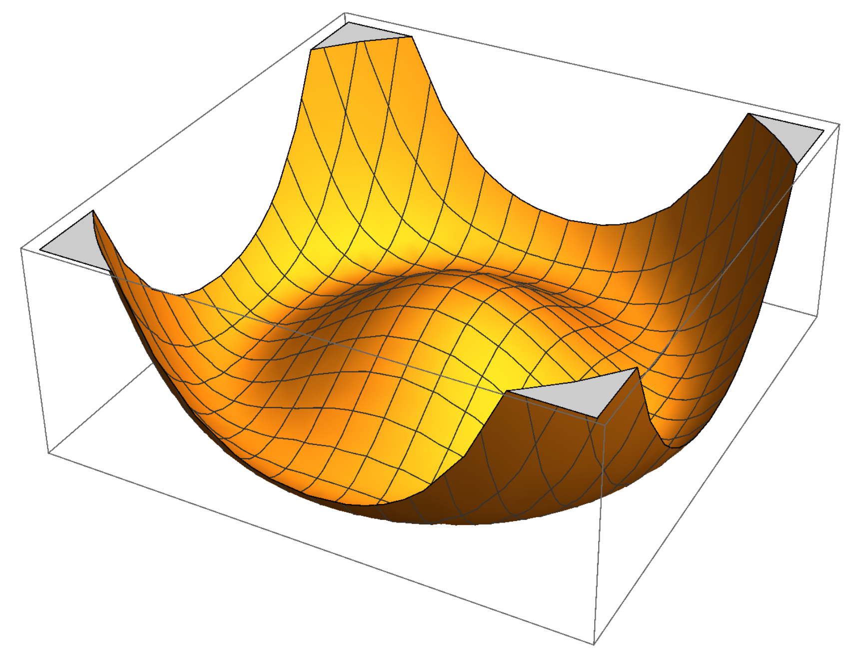
(Peccei-Quinn, Weinberg, Wilczek)

$$\theta \to \phi/f_a$$

$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2$$



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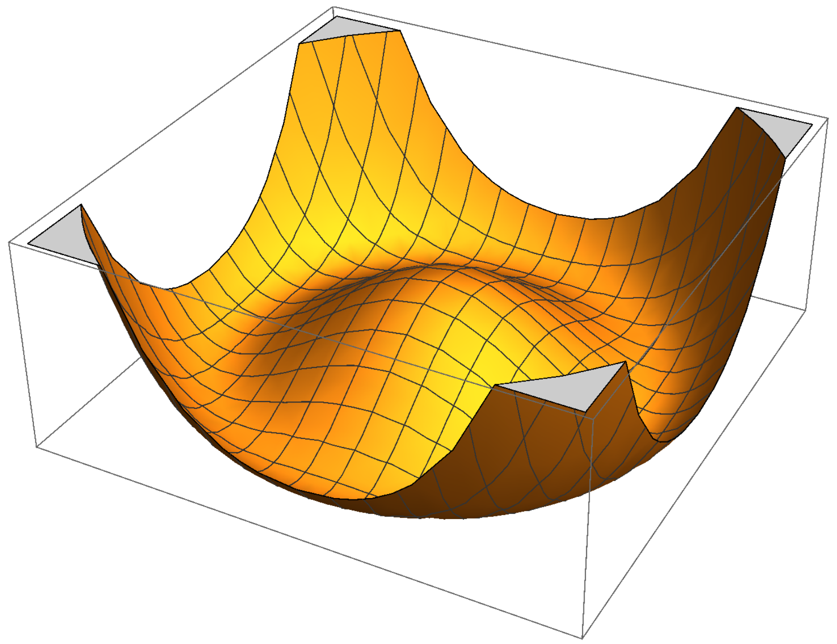
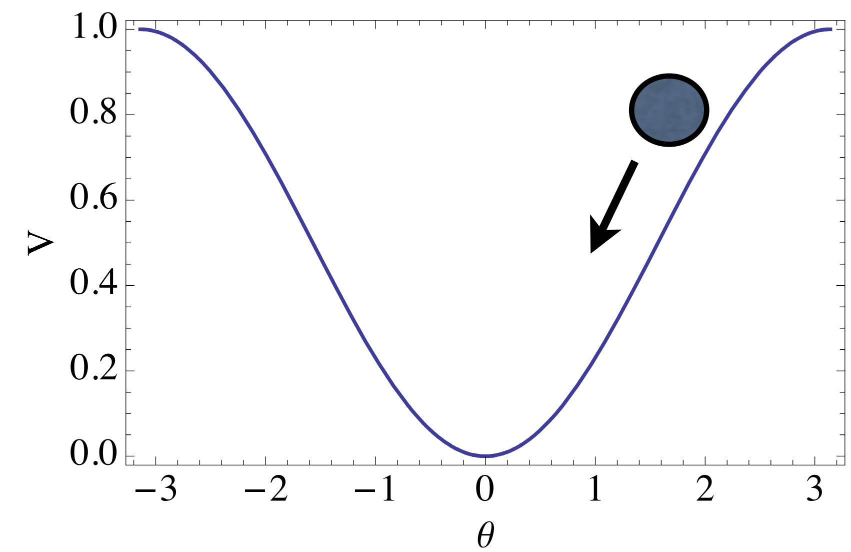


$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$



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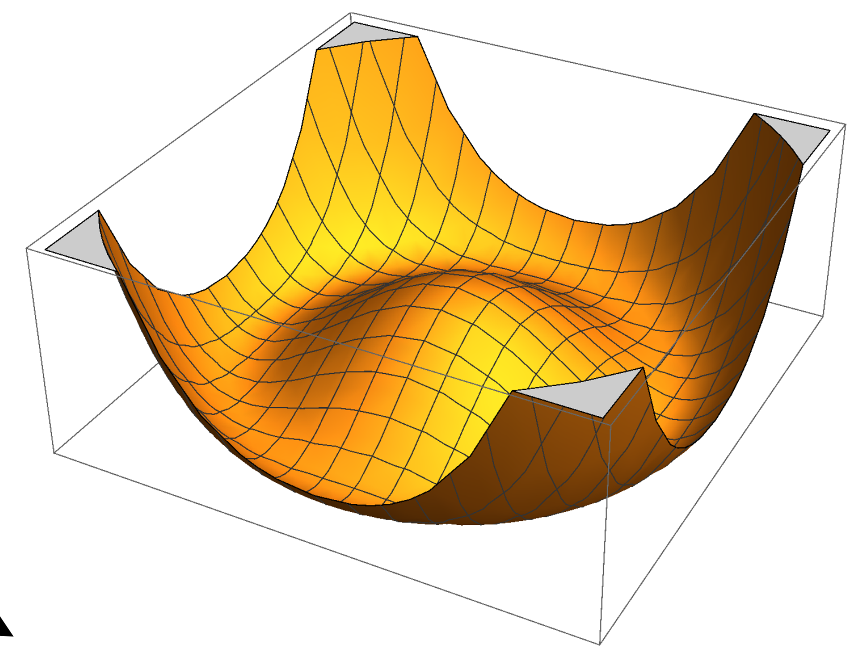
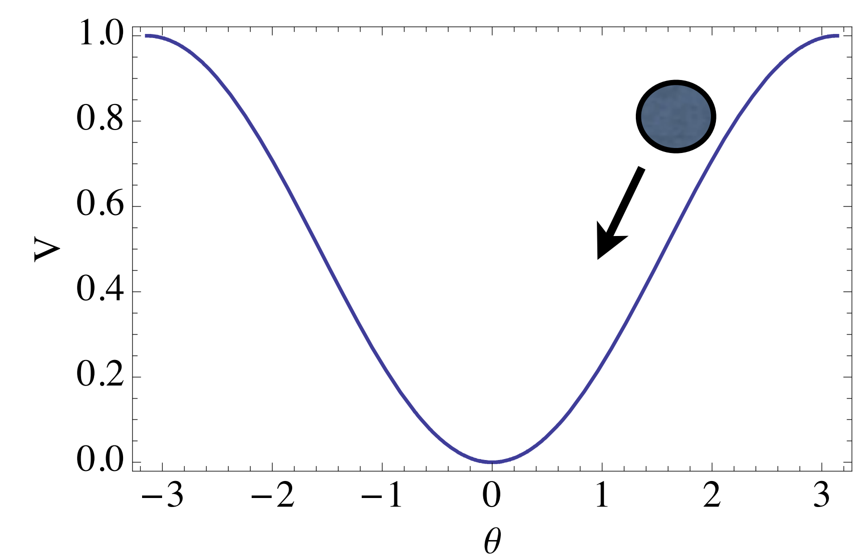


$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V(\phi) = \frac{1}{2}m_a^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$

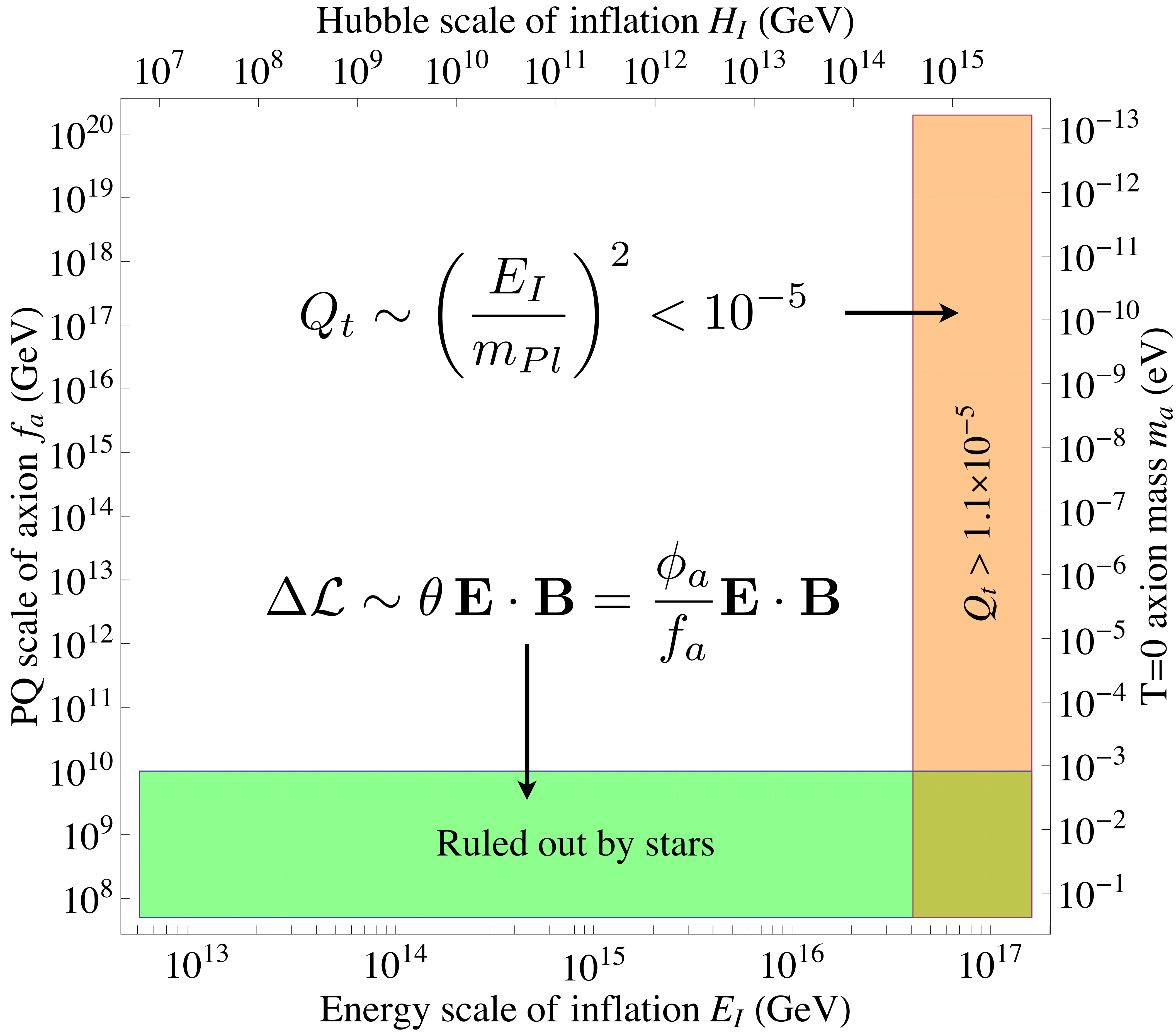


Axion mass:

$$m_a \sim \frac{\Lambda_{qcd}^2}{f_a}$$

(Attractive) Self-Coupling:

$$\lambda \sim -\frac{\Lambda_{qcd}^4}{f_a^4}$$



Hubble scale of inflation H_I (GeV)
10^7 10^8 10^9 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15}
PQ scale of axion f_a (GeV)
10^{20} 10^{19} 10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^9 10^8
T=0 axion mass m_a (eV)
10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1}
$Q_t \sim \left(\frac{E_I}{m_{Pl}}\right)^2 < 10^{-5}$
$Q_t > 1.1\times10^{-5}$
$\Delta\mathcal{L} \sim \theta\,\mathbf{E}\cdot\mathbf{B} = \frac{\phi_a}{f_a}\mathbf{E}\cdot\mathbf{B}$
Ruled out by stars
10^{13} 10^{14} 10^{15} 10^{16} 10^{17}
Energy scale of inflation E_I (GeV)

Axion Dark Matter Abundance

- Axion is neutral and stable

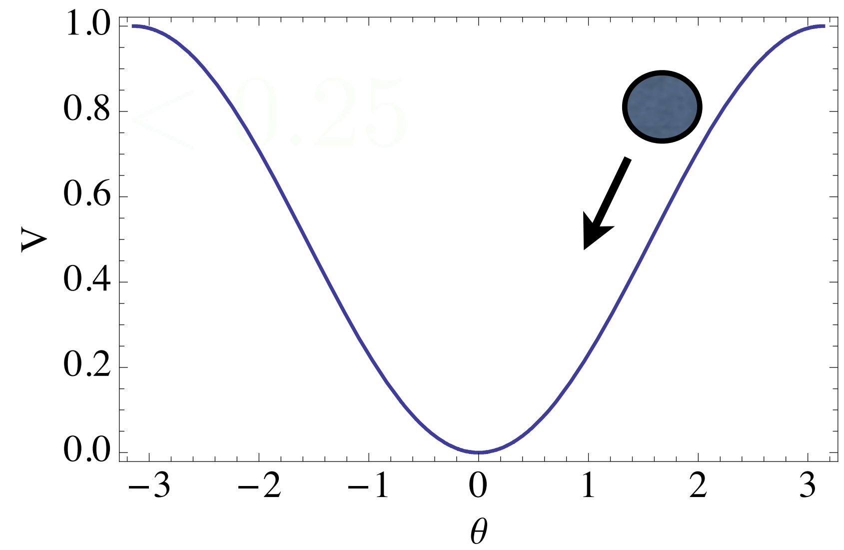
(For axion-string contribution, recall Masahide's talk on Thursday)

Axion Dark Matter Abundance

- Axion is neutral and stable

- Careful calculation from misalignment mechanism:

$$\Omega_a \approx \langle \theta_i^2 \rangle \left(\frac{f_a}{10^{12}\mathrm{GeV}} \right)^{7/6} < 0.25$$



- If f_a large we need θ_i small

- The allowed distribution for θ_i depends on inflation scale E_I relative to PQ SB scale f_a

(For axion-string contribution, recall Masahide's talk on Thursday)

If PQ SB after inflation

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−1.01	3.06	0.62	−1.98	−1.06	−1.67	2.23
0.63	3.12	1.56	1.6	−1.33	2.27	−2.95
1.1	1.38	−2.03	2.26	2.14	−1.79	−2.55
2.15	−1.38	1.97	0.01	2.17	−0.98	0.86
2.03	2.03	−2.78	−1.52	−2.18	−1.3	−2.44
1.14	2.48	−1.51	−3.1	−1.99	−2.38	−2.91
1.51	0.47	−0.94	−0.4	−0.54	0.41	0.19

$$\langle \theta_i^2 \rangle = \frac{\pi^2}{3}$$

High $f_a \gtrsim 10^{12}\,\mathrm{GeV}$ is disallowed

If PQ SB before inflation

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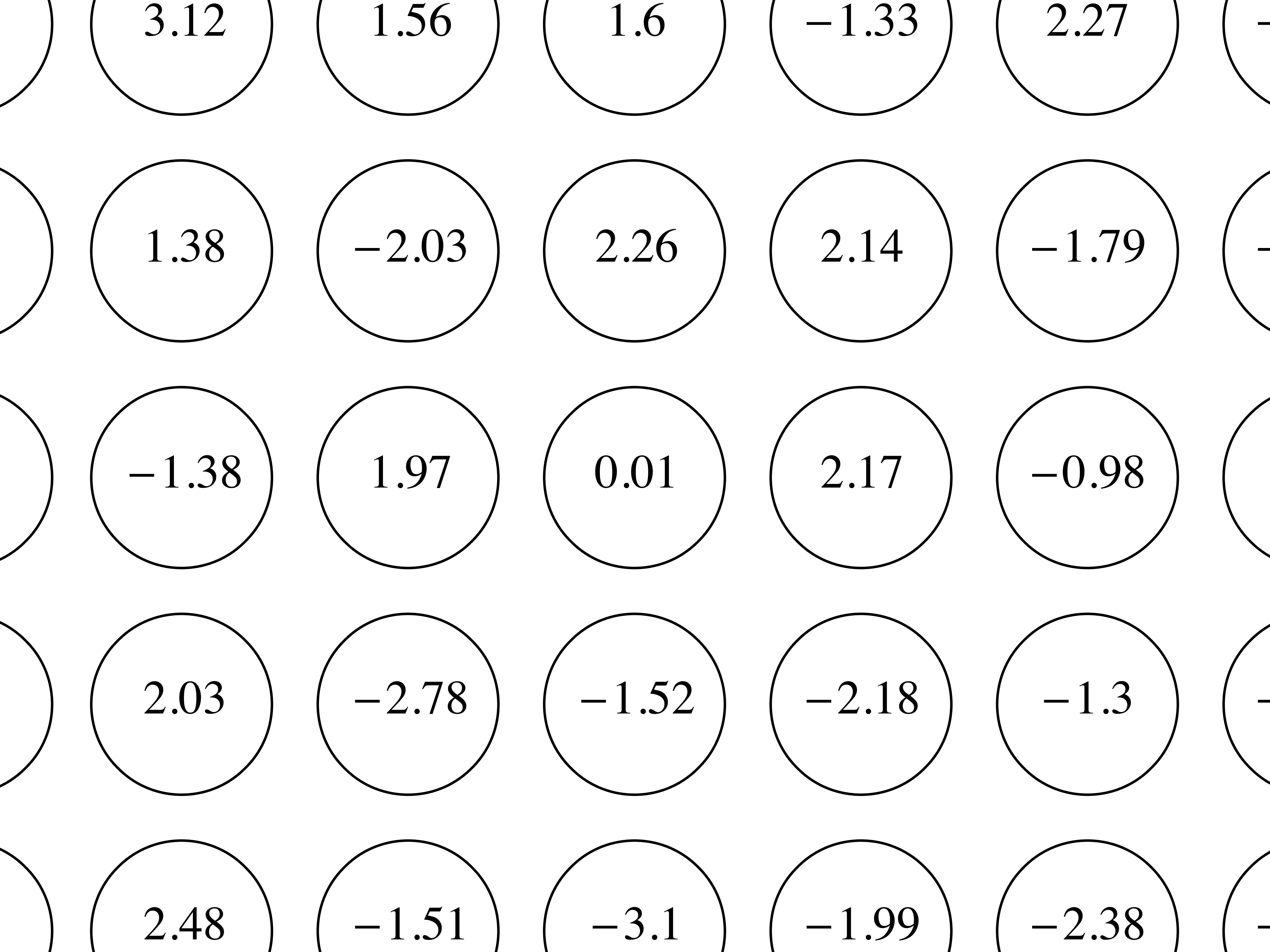
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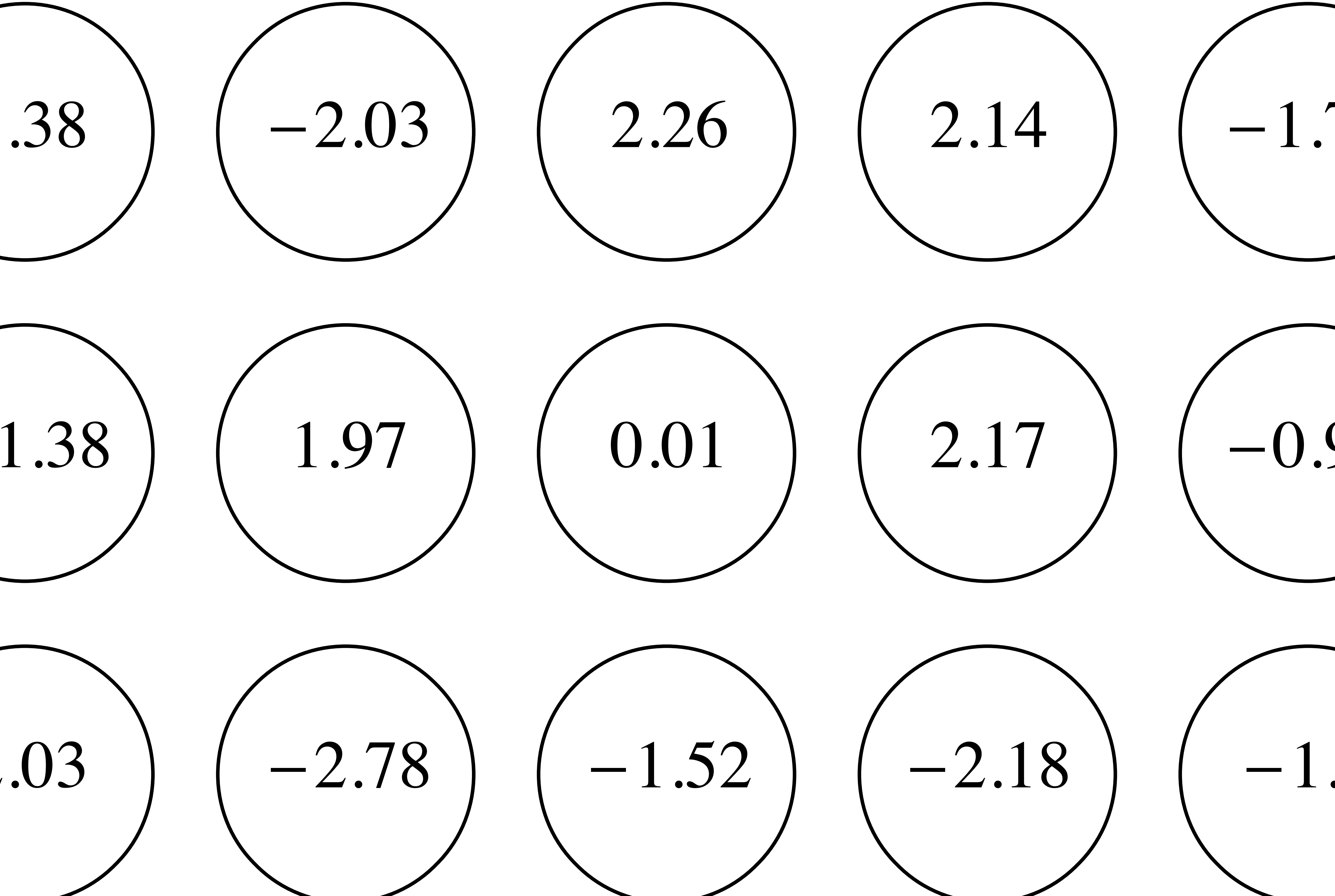

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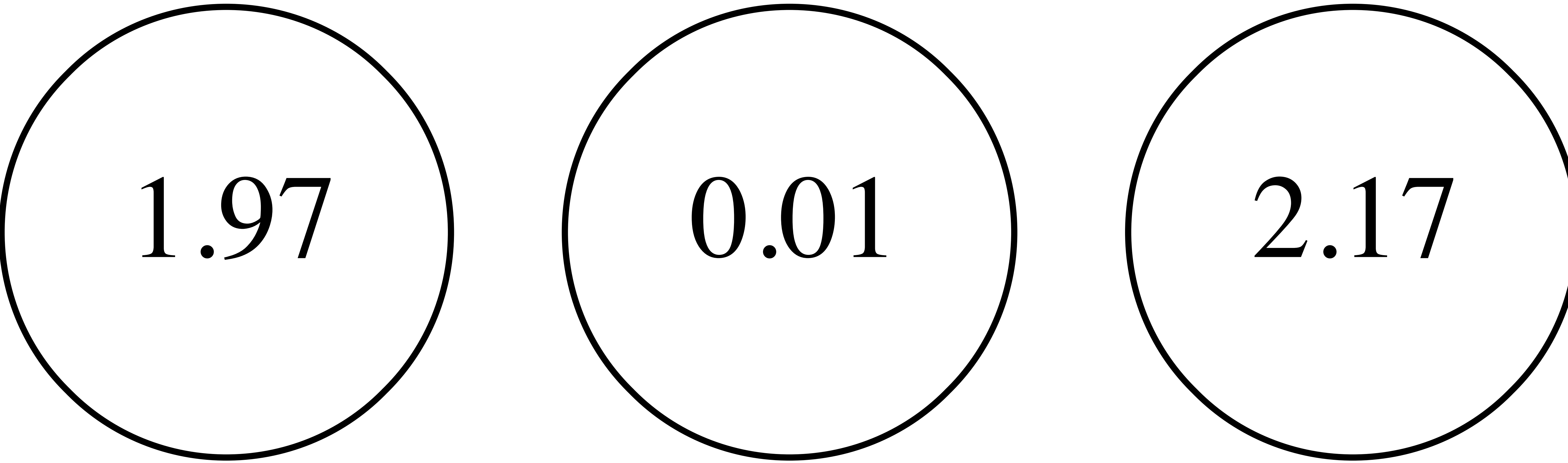
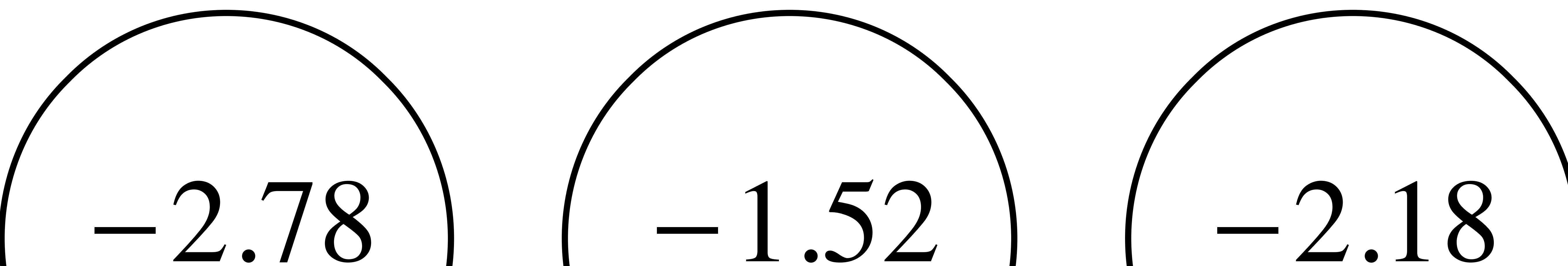
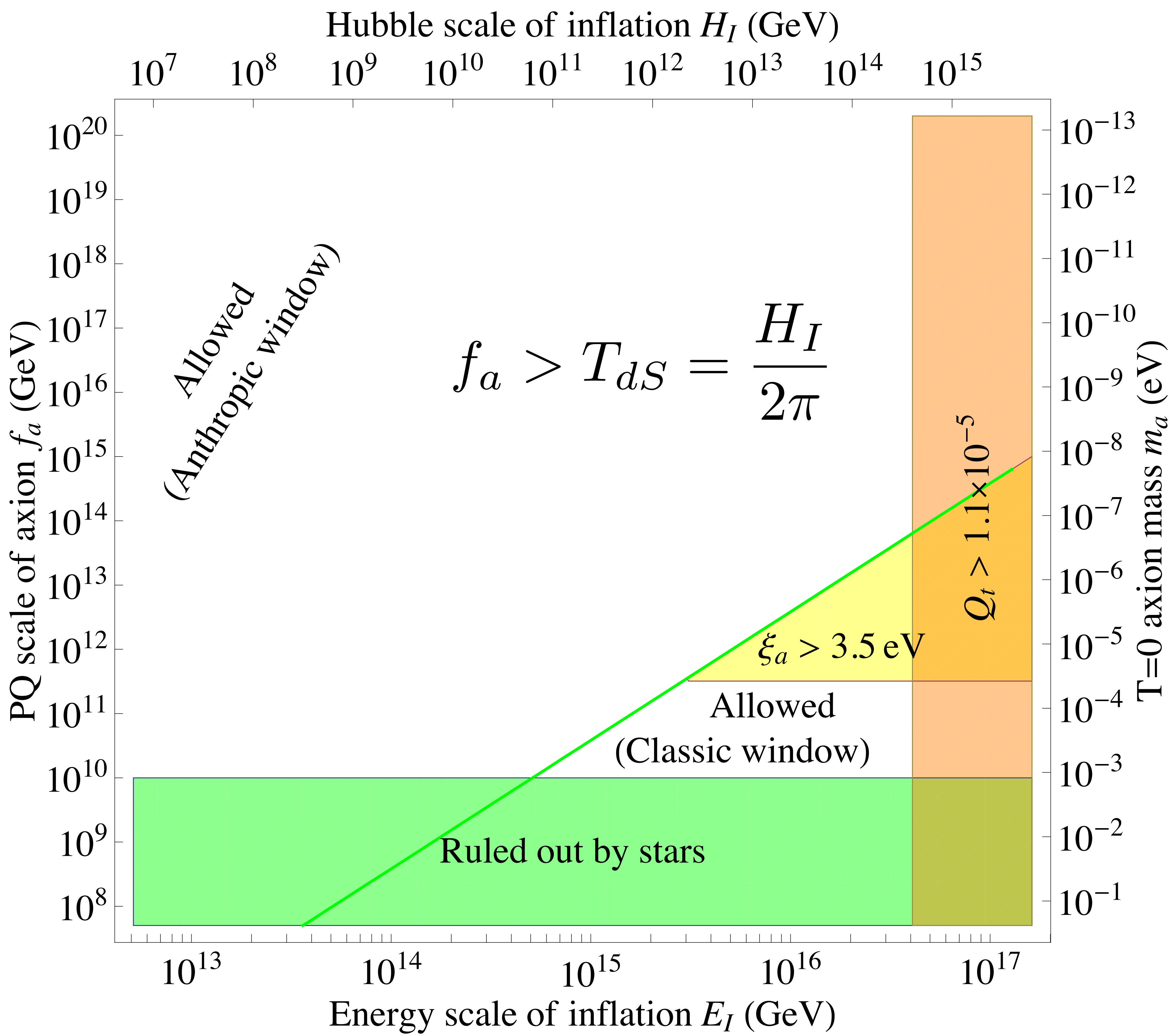
If PQ SB before inflation $0 < \langle \theta_i^2 \rangle < \pi^2$



High f_a is allowed (anthropics: Linde; Tegmark, Aguirre, Rees, Wilczek)





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Allowed
(Anthropic window)
$f_a > T_{dS} = \frac{H_I}{2\pi}$
$Q_t > 1.1\times10^{-5}$
$\xi_a > 3.5$ eV
Allowed
(Classic window)
Ruled out by stars
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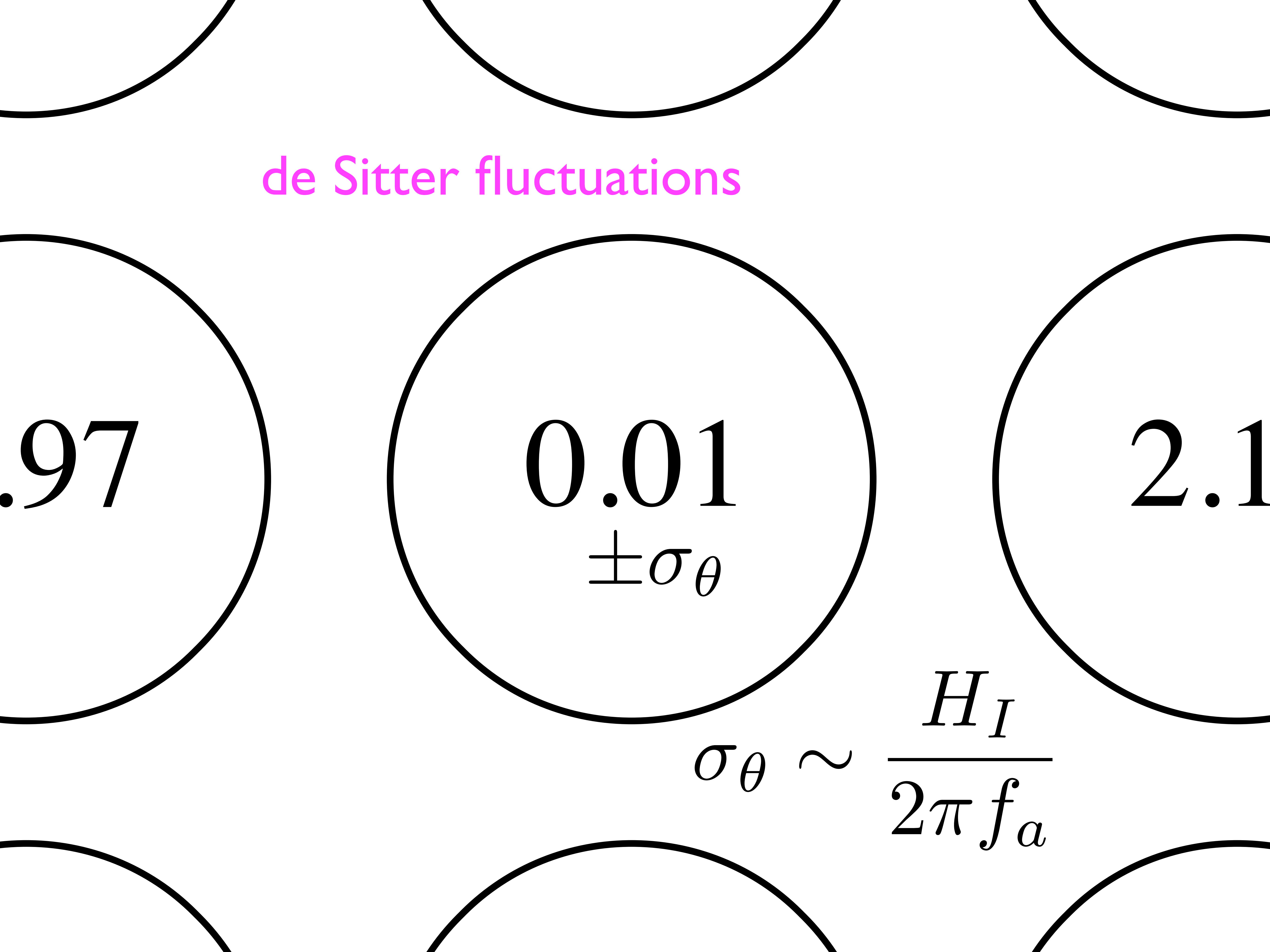
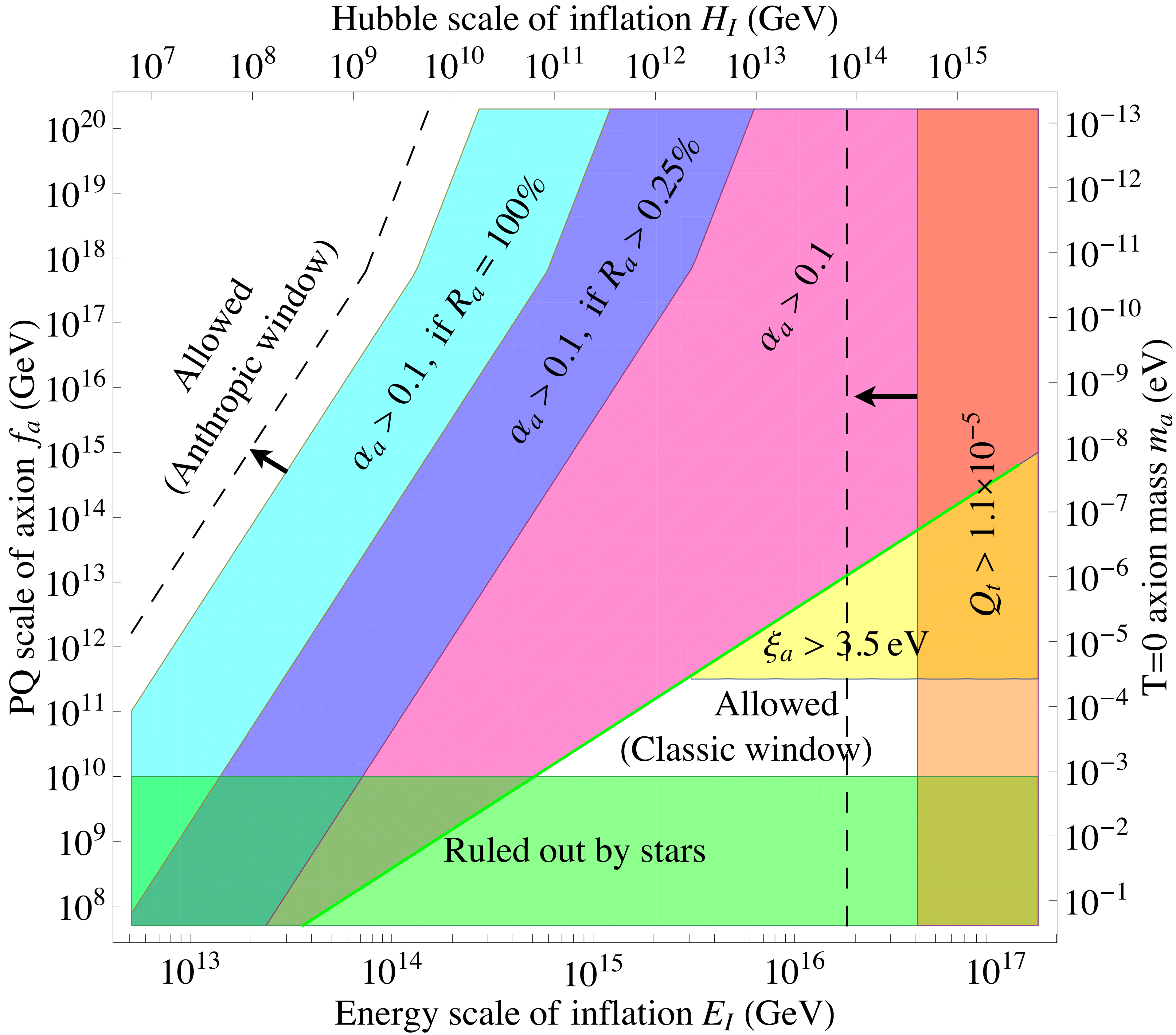
Energy scale of inflation E_I (GeV)

de Sitter fluctuations



$$\sigma_\theta \sim \frac{H_I}{2\pi f_a}$$

QCD-Axion Allowed Windows



Hertzberg, Tegmark, Wilczek 0807.1726

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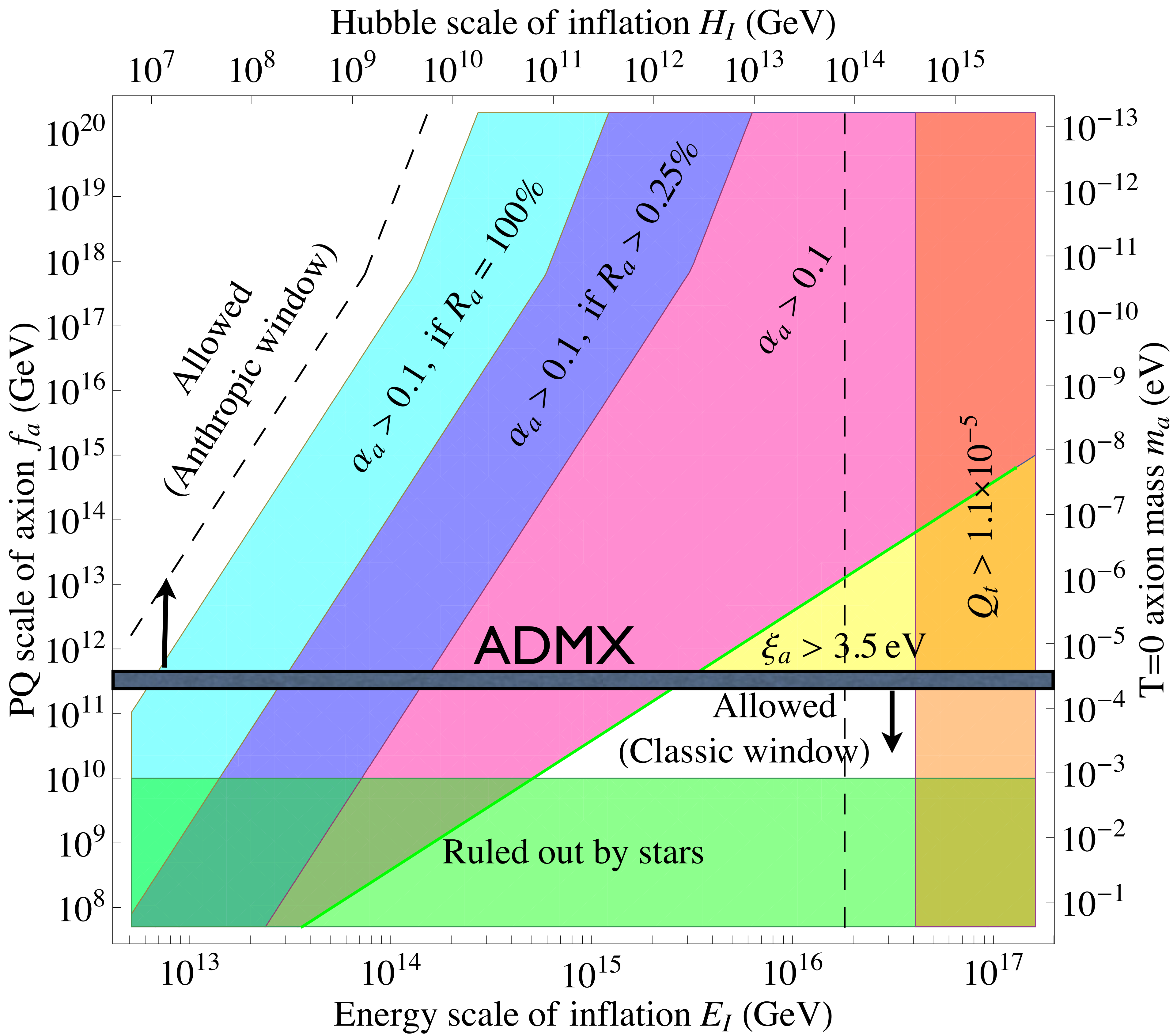
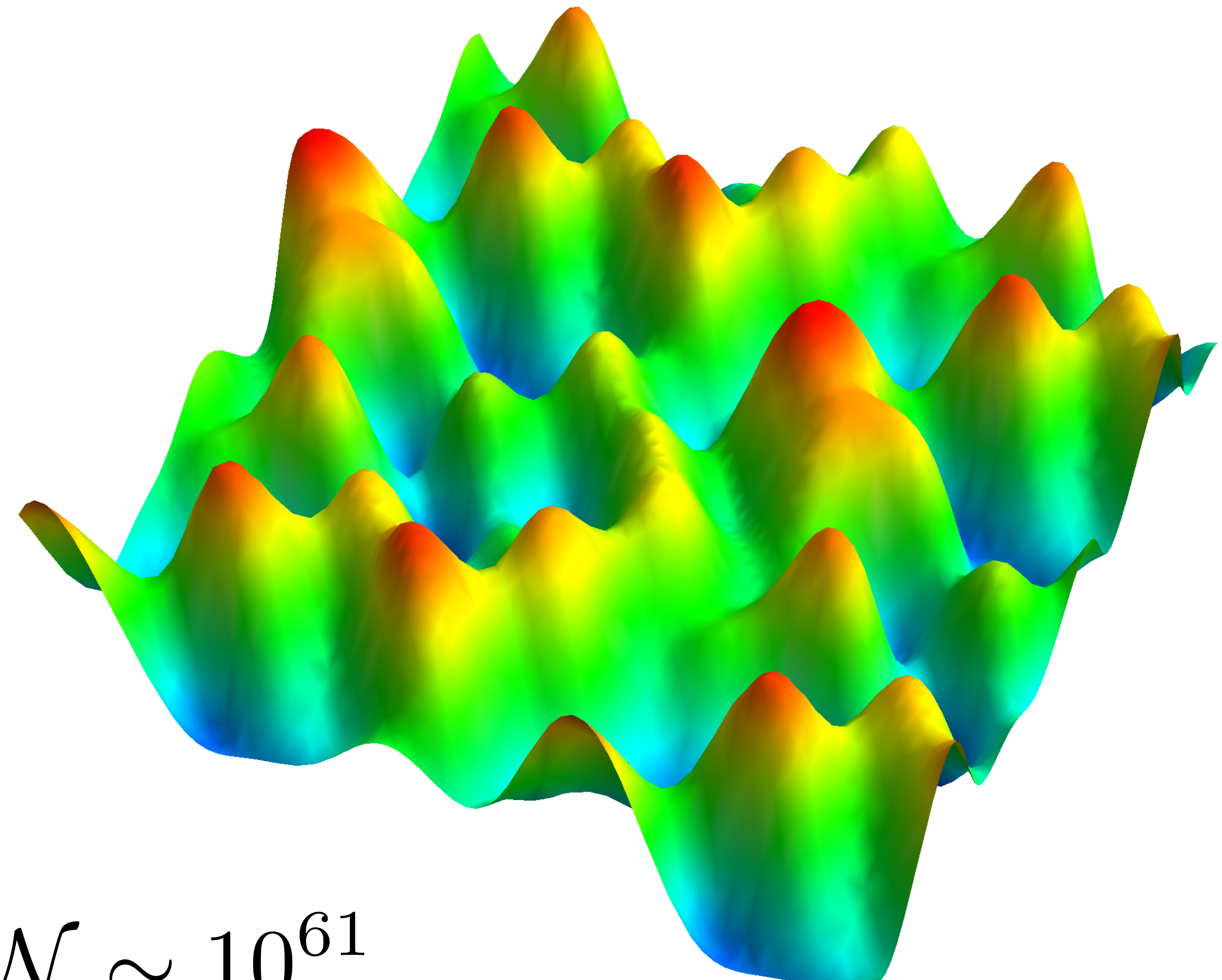


Hertzberg, Tegmark, Wilczek 0807.1726

Focus on Classic Window

In Classic Window; Axion Initial Distribution



$\mathcal{N} \sim 10^{61}$

Consider Non-Relativistic Behavior

$$\phi(\mathbf{x}, t) = \frac{1}{\sqrt{2\,m}} \left(e^{-imt}\psi(\mathbf{x}, t) + e^{imt}\psi^*(\mathbf{x}, t)\right)$$

(For rigorous treatment: Namjoo, Guth, Kaiser 2017)

Consider Non-Relativistic Behavior

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Hamiltonian

$$\hat{H} = \hat{H}_{\mathbf{kin}} + \hat{H}_{\mathbf{int}} + \hat{H}_{\mathbf{grav}}$$

$$\hat{H}_{\mathbf{kin}} = \int d^3x \frac{1}{2m} \nabla\hat{\psi}^\dagger \cdot \nabla\hat{\psi}$$

$$\hat{H}_{\mathbf{int}} = \int d^3x \frac{\lambda}{16m^2} \hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}$$

$$\hat{H}_{\mathbf{grav}} = -\frac{Gm^2}{2}\int d^3x \int d^3x' \frac{\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{x}')\hat{\psi}(\mathbf{x})\hat{\psi}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

Number Density

$$\hat{n}(\mathbf{x}) = \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x})$$

(For rigorous treatment: Namjoo, Guth, Kaiser 2017)

Approach to Equilibrium?

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

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Interaction Rate of Modes

$$\Gamma_k \equiv \frac{\dot{\mathcal{N}}_k}{\mathcal{N}_k}$$

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$\Gamma_k > H$ Early universe

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$$\Gamma_k \sim \frac{8\pi G\, m^2\, n_{\rm ave}}{k^2} \quad \propto \frac{1}{a}$$

$\Gamma_k > H$ Early universe

$\Gamma_k > H$ Late universe

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$\Gamma_k > H$ Early universe

$\Gamma_k > H$ Late universe

Equilibrium with high occupancy suggests BEC

Axion BEC Literature

- Sikivie, Yang (2009)
- Erken, Sikivie, Tam, Yang (2011)
- Chavanis (2012)
- Banik, Sikivie (2013)
- Davidson, Elmer (2013)
- Saikawa, Yamaguchi (2013)
- Noumi, Saikawa, Sato, Yamaguchi (2014)
- Vega, Sanchez (2014)
- Li, Rindler-Daller, Shapiro (2014)
- Berges, Haeckel (2014)
- Banik, Christopherson, Sikivie, Todarello (2015)
- Davidson (2015)
- .....

Classical Description of BEC Phase Transition

Free Theory

$$F[\psi] = \int \frac{d^3k}{(2\pi)^3} \left[\frac{k^2}{2m} - \mu(T) \right] |\psi_k|^2$$

Number

$$\langle N \rangle = \frac{\int \mathcal{D}\psi \, N[\psi] \, \exp\left(-F[\psi]/T\right)}{\int \mathcal{D}\psi \, \exp\left(-F[\psi]/T\right)}$$

Density

$$n_{\rm th} = \int \frac{d^3k}{(2\pi)^3} \frac{T}{\frac{k^2}{2m} - \mu(T)}$$

Critical Temperature

$$T_{\rm crit} = \frac{\pi^2 \, n_{\rm tot}}{m \, k_{\rm UV}}$$

Guth, Hertzberg, Prescod-Weinstein 1412.5930

Classical vs Quantum with Interactions

What About Interactions?

Fundamental claim of Sikivie, Todarello, 1607.00949

On time scales $t > \tau = 1/\Gamma$ the classical description breaks down, requiring the full quantum theory, which is the only way to see thermalization

Toy Model

Second Quantized Language

$$\hat{H} = \sum_i \omega_i \, \hat{a}_i^\dagger \hat{a}_i + \frac{1}{4} \sum_{ijkl} \Lambda_{ij}^{kl} \, \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l ,$$

Consider just 5 oscillators for simplicity

Initial quantum state $|\{N_i\}\rangle = |12, 25, 4, 12, 1\rangle$

Sikivie, Todarello, 1607.00949

Toy Model

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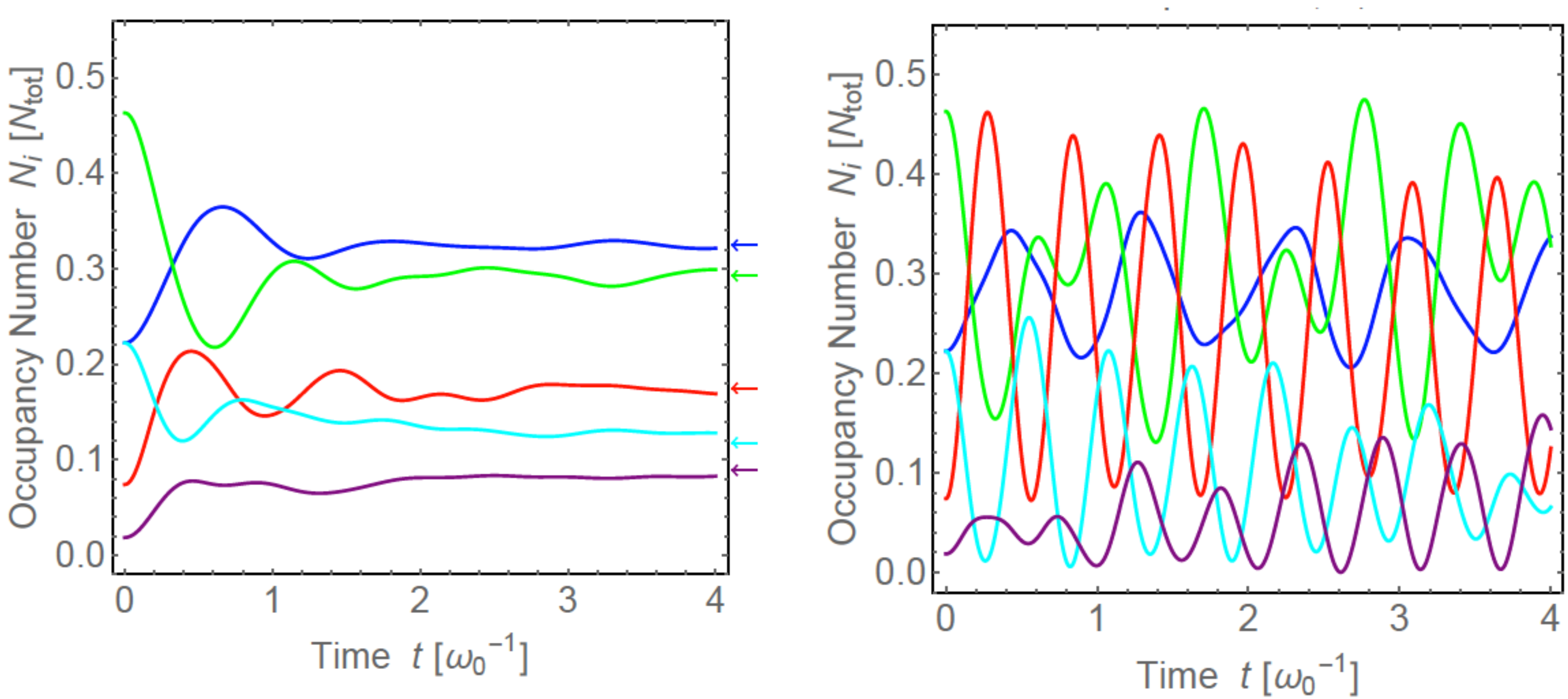
Consider just 5 oscillators for simplicity

Initial quantum state $|\{N_i\}\rangle = |12, 25, 4, 12, 1\rangle$

Initial classical state $a_i = \sqrt{N_i}$

Sikivie, Todarello, 1607.00949

Quantum vs Classical??



Sikivie, Todarello, 1607.00949

Correct Classical Treatment

Initial classical state $a_i = \sqrt{N_i}\, e^{I\theta_i}, \quad \theta_i \in [0, 2\pi)$

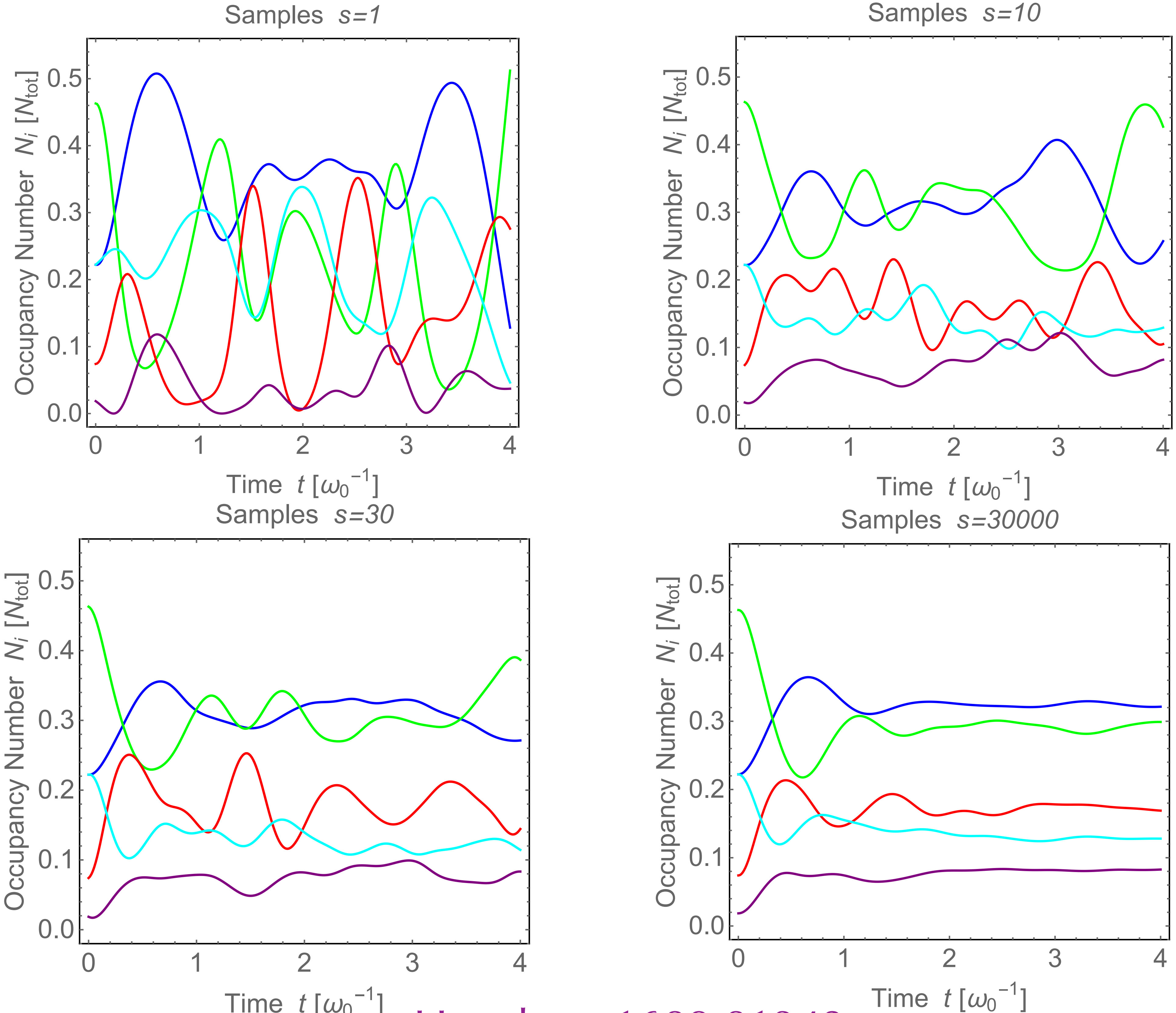
Ensemble average over random initial phases

Meaningful comparison

Connects to uncertainty in branch of wavefunction

Hertzberg 1609.01342

Correct Classical Treatment



Hertzberg 1609.01342

Implication for Correlation Functions

Implication for Correlation Functions

At high occupancy

$$\langle \{N_i\} | \hat{\psi}^\dagger(\mathbf{x},t)\, \hat{\psi}(\mathbf{y},t) | \{N_i\} \rangle \approx \langle \psi^*(\mathbf{x},t)\, \psi(\mathbf{y},t) \rangle_{ens}$$

Ergodic theorem

$$\langle \psi^*(\mathbf{x},t)\, \psi(\mathbf{y},t) \rangle_{ens} = \frac{1}{V} \int_V d^3z\, \psi_\mu^*(\mathbf{x}+\mathbf{z},t)\, \psi_\mu(\mathbf{y}+\mathbf{z},t)$$

Hertzberg 1609.01342

Implication for Correlation Functions

Correlation functions of quantum and classical micro-states agree at high occupancy, despite the spread of the quantum wave-function in these chaotic systems

(Note: this is not some trivial consequence of Ehrenfest theorem; more akin to billiard balls which exhibit chaos)

Hertzberg 1609.01342

Related Issues for Classical Pre-heating Simulations

$$\psi(\mathbf{x}, t) = \psi_c(t) + \delta\psi(\mathbf{x}, t)$$

(Recall Mustafa's talk. Kofman, Linde, Starobinsky 1994, 1997; Felder, Tkachev 2001; Amin, Easther, Finkel, Flauger, Hertzberg 2011; Lozanov, Amin 2016, 2017,2019; Kitajima, Soda, Urakawa 2018)

Related Issues for Classical Pre-heating Simulations

$$\psi(\mathbf{x},t) = \psi_c(t) + \delta\psi(\mathbf{x},t)$$

- True classical behavior is $\delta\psi(\mathbf{x},t) = 0$, but that only means classical field theory fails at zero occupancy (no surprise)

- In LINEAR regime, ensemble average is correct (despite small occupancy)

- In NONLINEAR regime, ensemble average is correct (since occupancy is large)

(Recall Mustafa's talk. Kofman, Linde, Starobinsky 1994, 1997; Felder, Tkachev 2001; Amin, Easther, Finkel, Flauger, Hertzberg 2011; Lozanov, Amin 2016, 2017,2019; Kitajima, Soda, Urakawa 2018)

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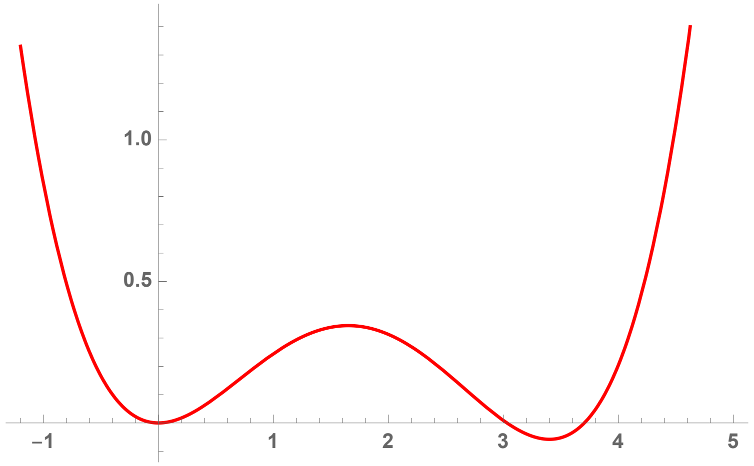

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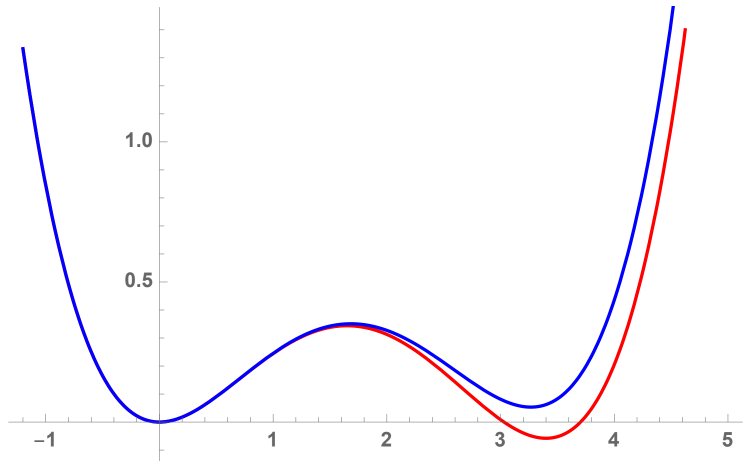

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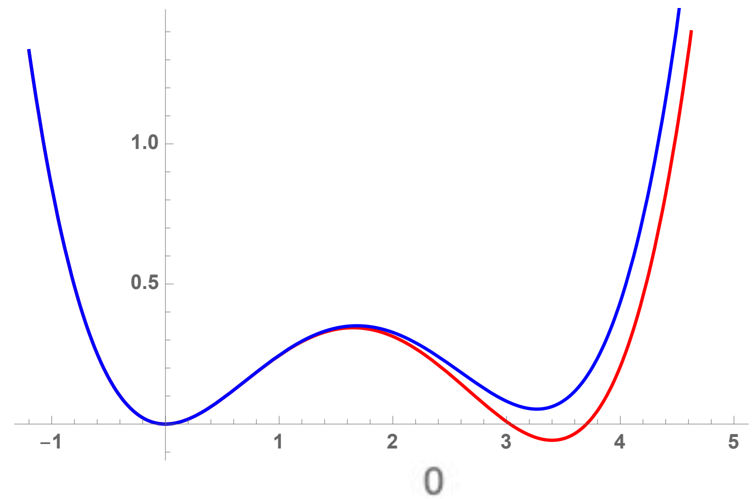

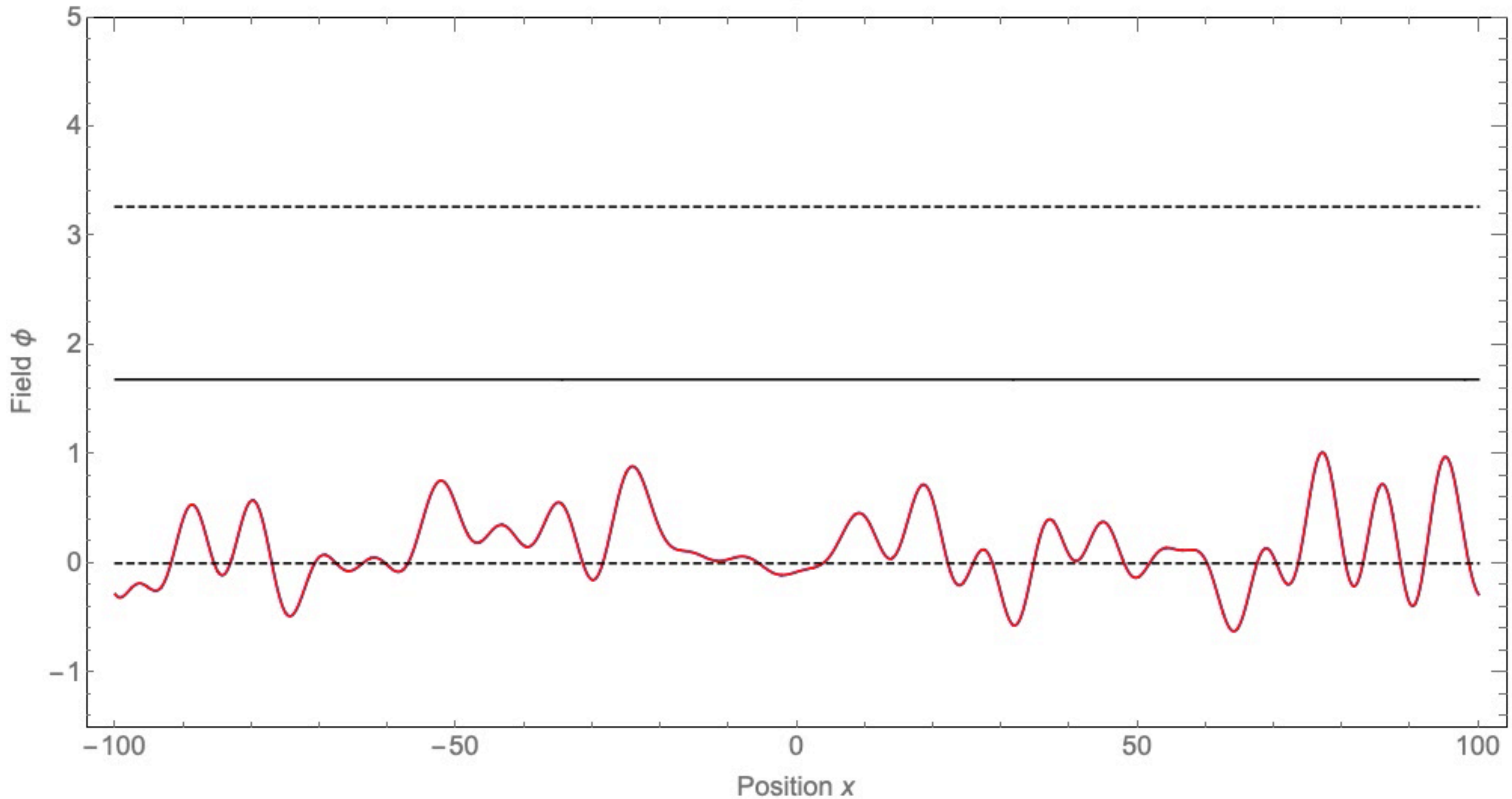

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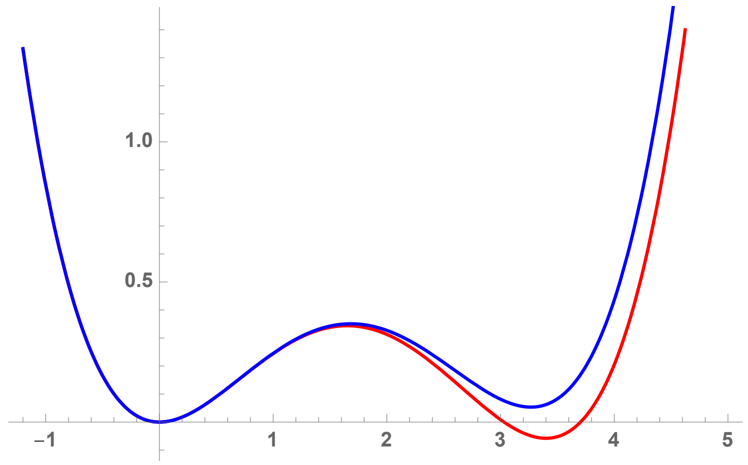
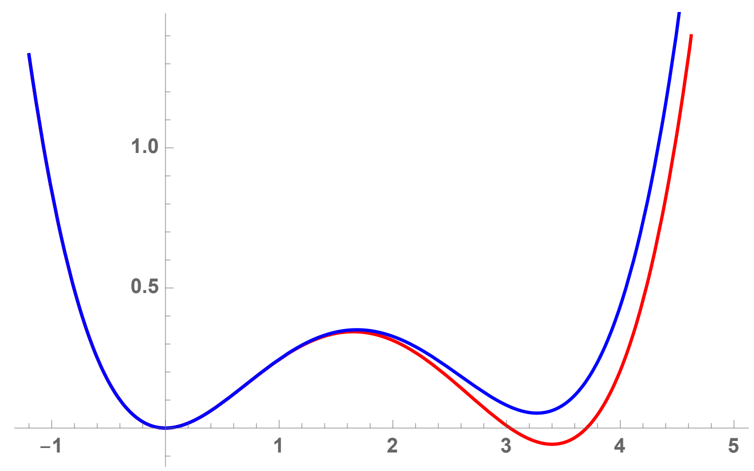


- Numerical agreement found in: Braden, Johnson, Peiris, Pontzen, Weinfurtner 2018

Related Issues for Classical Tunneling Simulations



- Numerical agreement found in: Braden, Johnson, Peiris, Pontzen, Weinfurtner 2018

- In LINEAR regime, ensemble average is correct (despite small occupancy)

- In NONLINEAR regime; ensemble average can (sometimes) describe tunneling (since bubble's have high occupancy)

Hertzberg, Yamada 1904.08565

Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

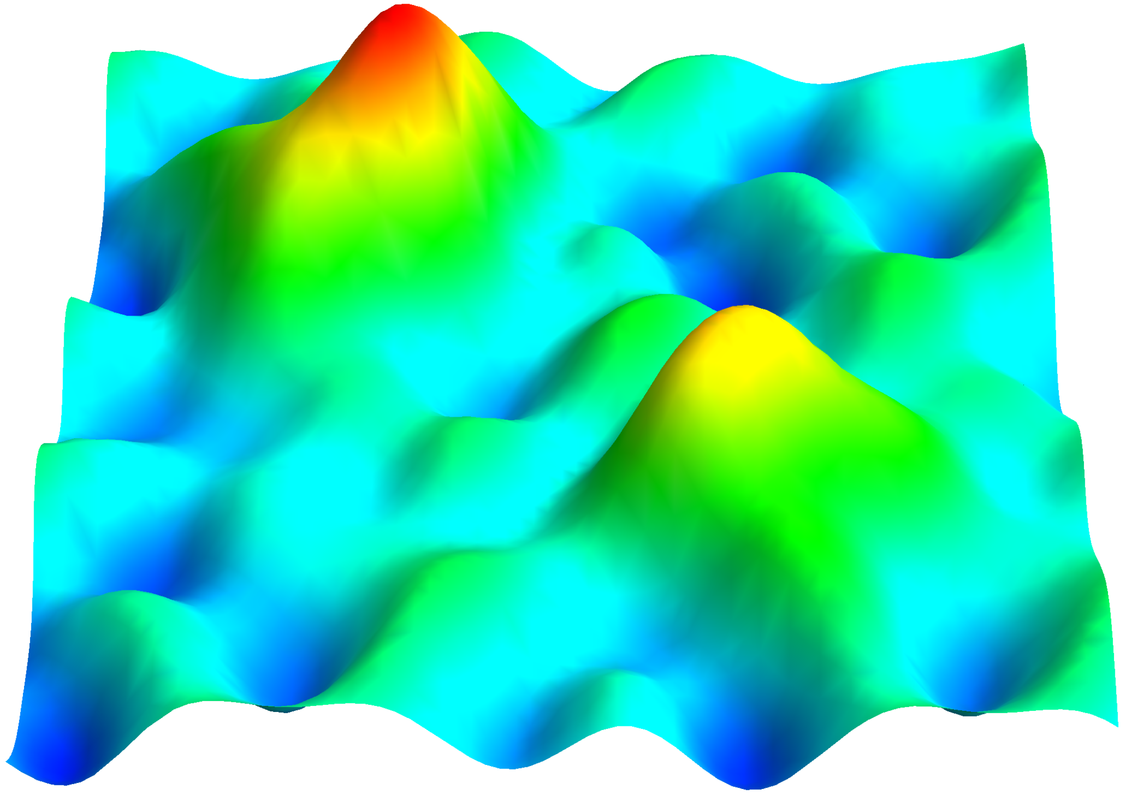
Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

Small clumps (miniclusters)

that may populate the galaxy



Hogan, Rees 1988; Kolb, Tkachev 1993, 1994, 1995; Barranco, Bernal 2001; Guth, Hertzberg, Prescod-Weinstein 2014; Fairbairn, Marsh, Quevillon, Rozier 2017; Kitajima, Soda, Urakawa 2018; Amin, Mocz 2019

Axion Clumps in Detail

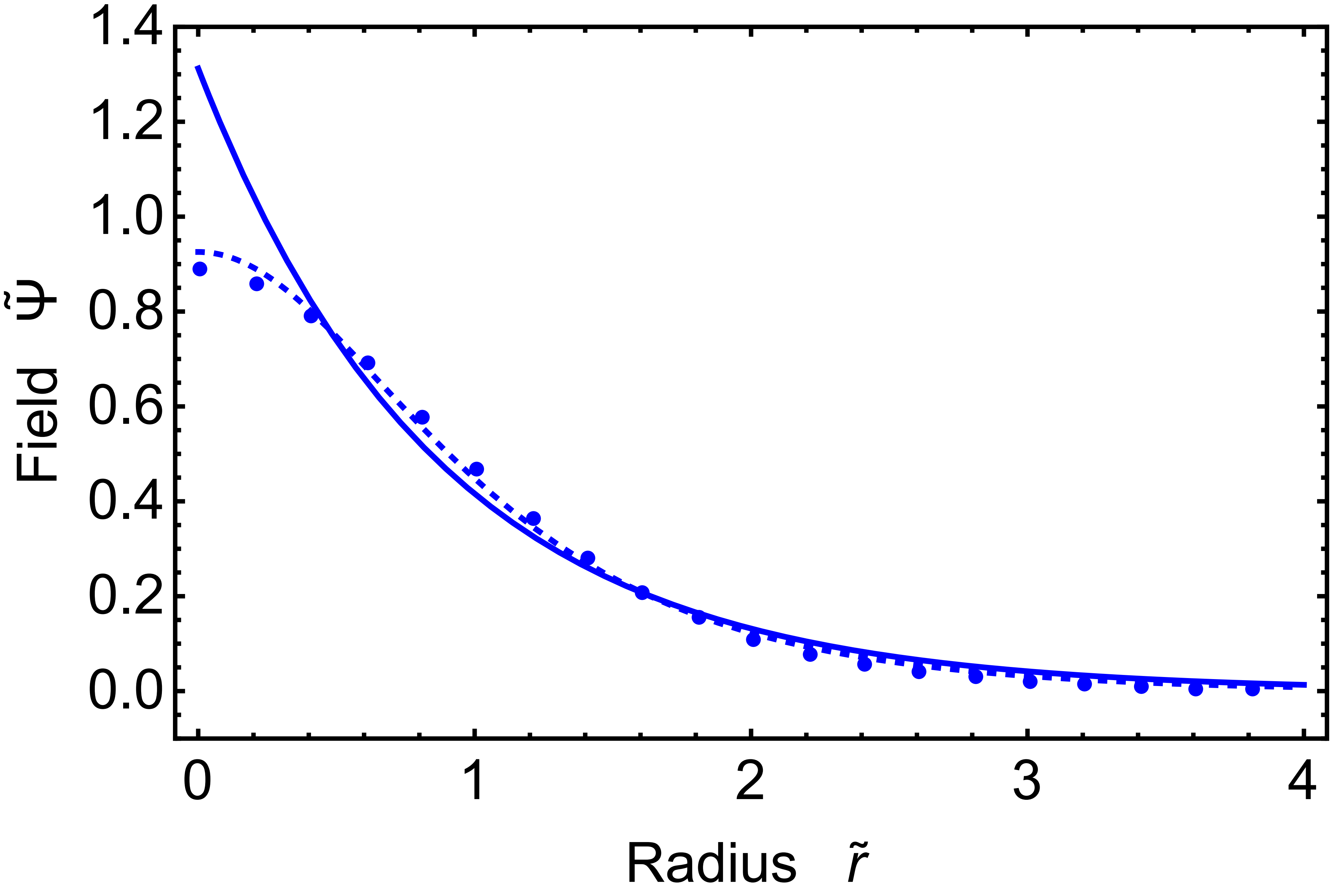
Return to Non-Relativistic Classical Field Theory

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

$$(\lambda < 0)$$

Clump Solutions (BEC) at fixed N



Schiappacasse, Hertzberg 1710.04729

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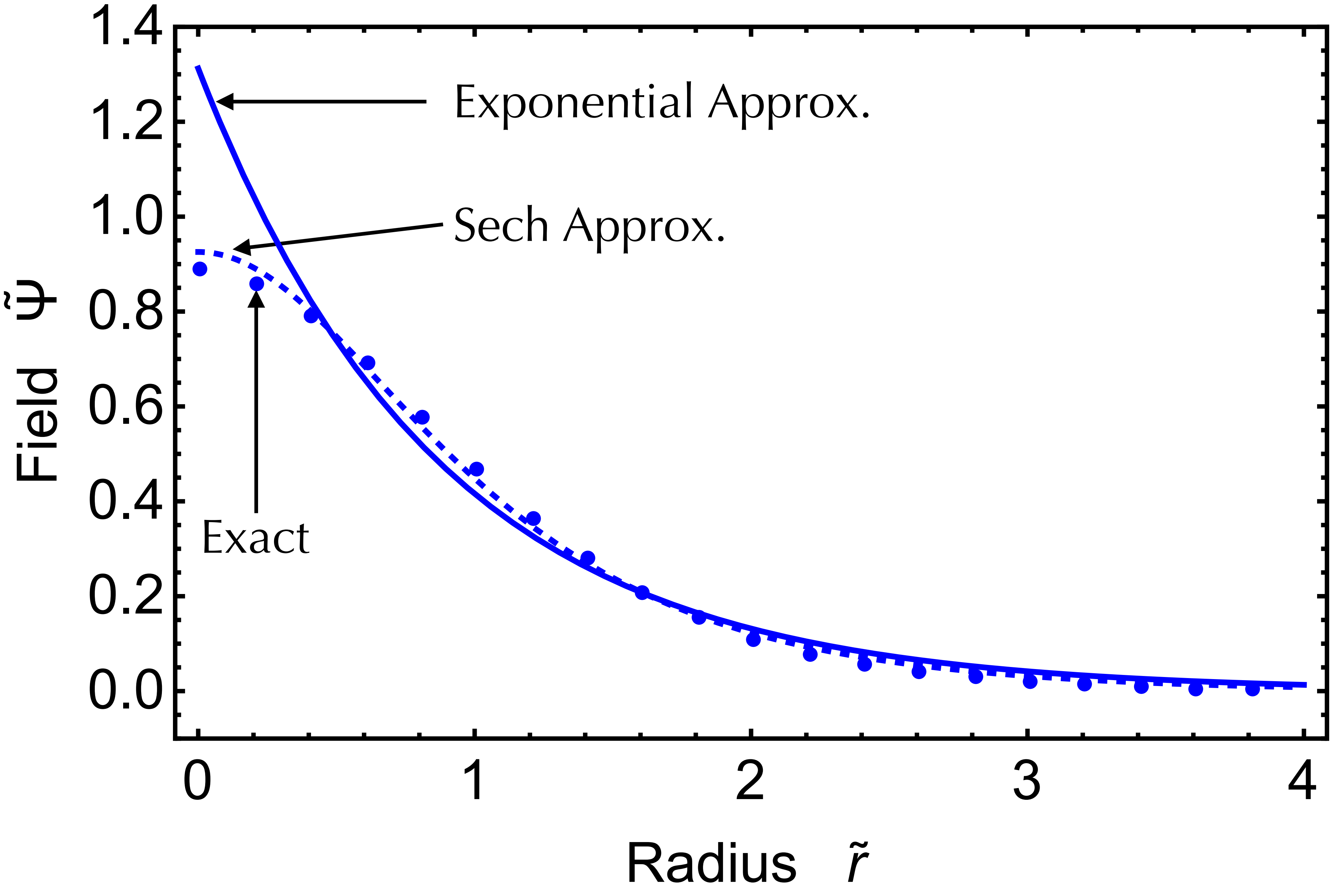
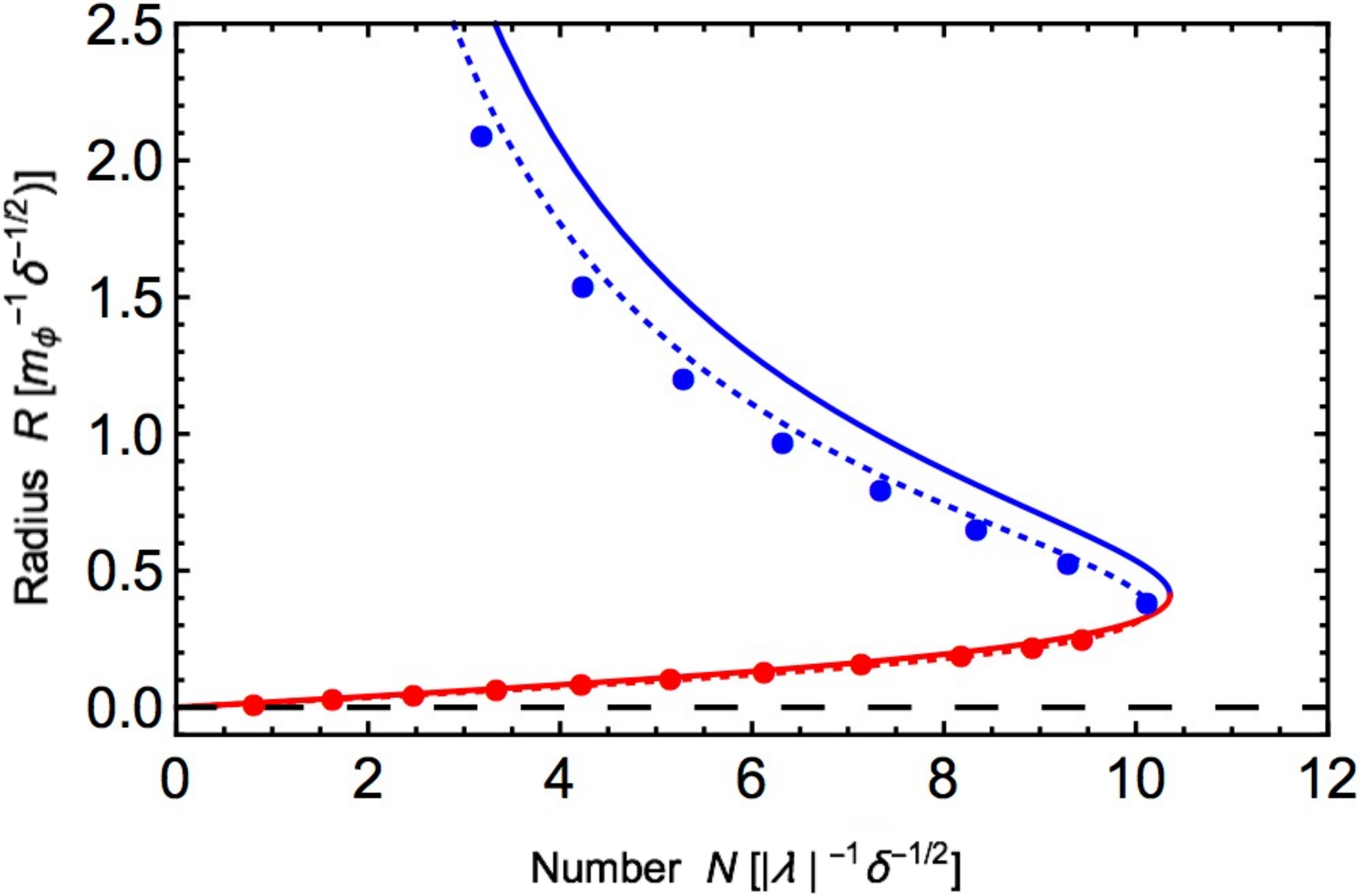


Schiappacasse, Hertzberg 1710.04729

Two Branches of Solutions



Schiappacasse, Hertzberg 1710.04729

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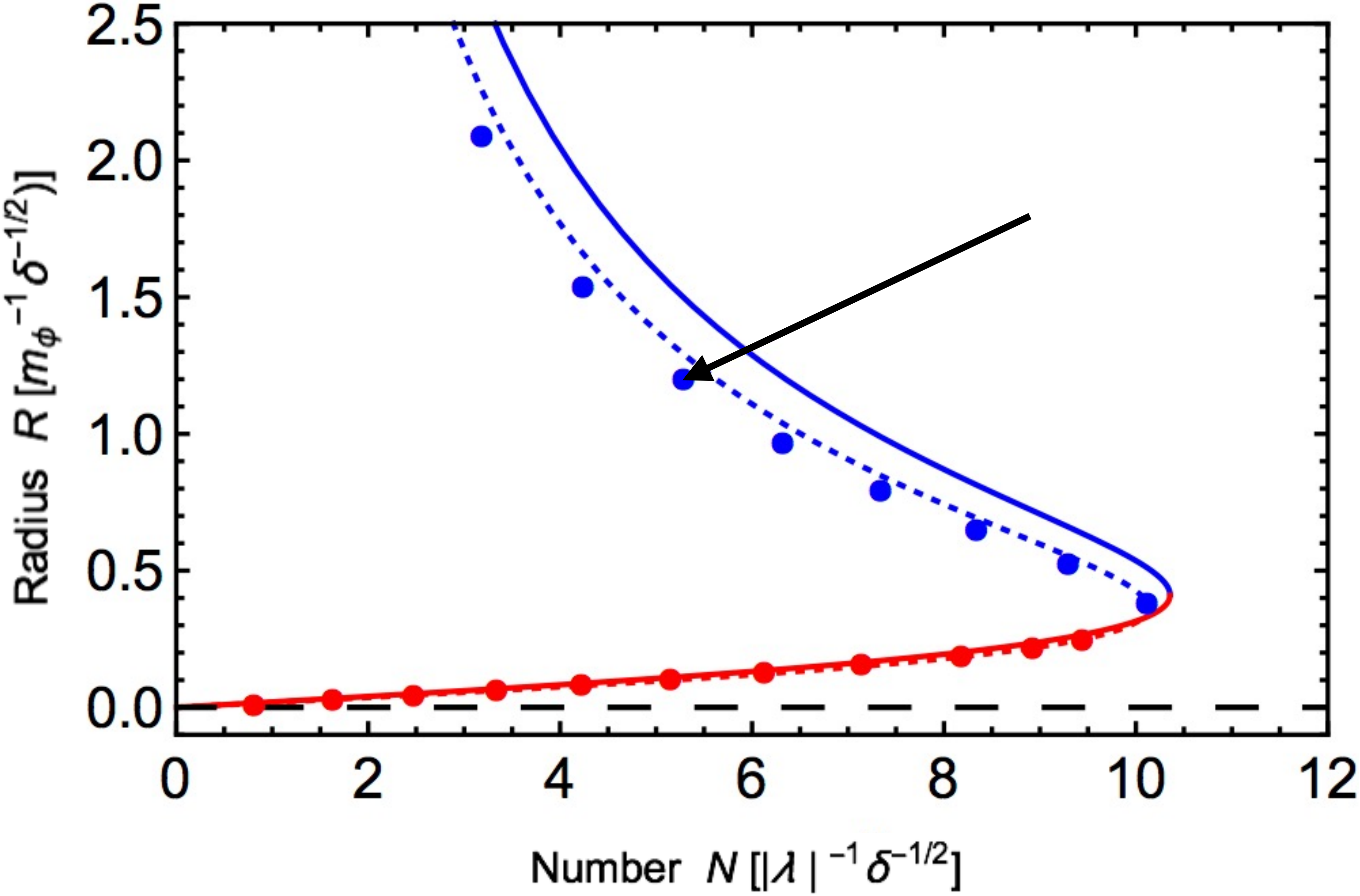
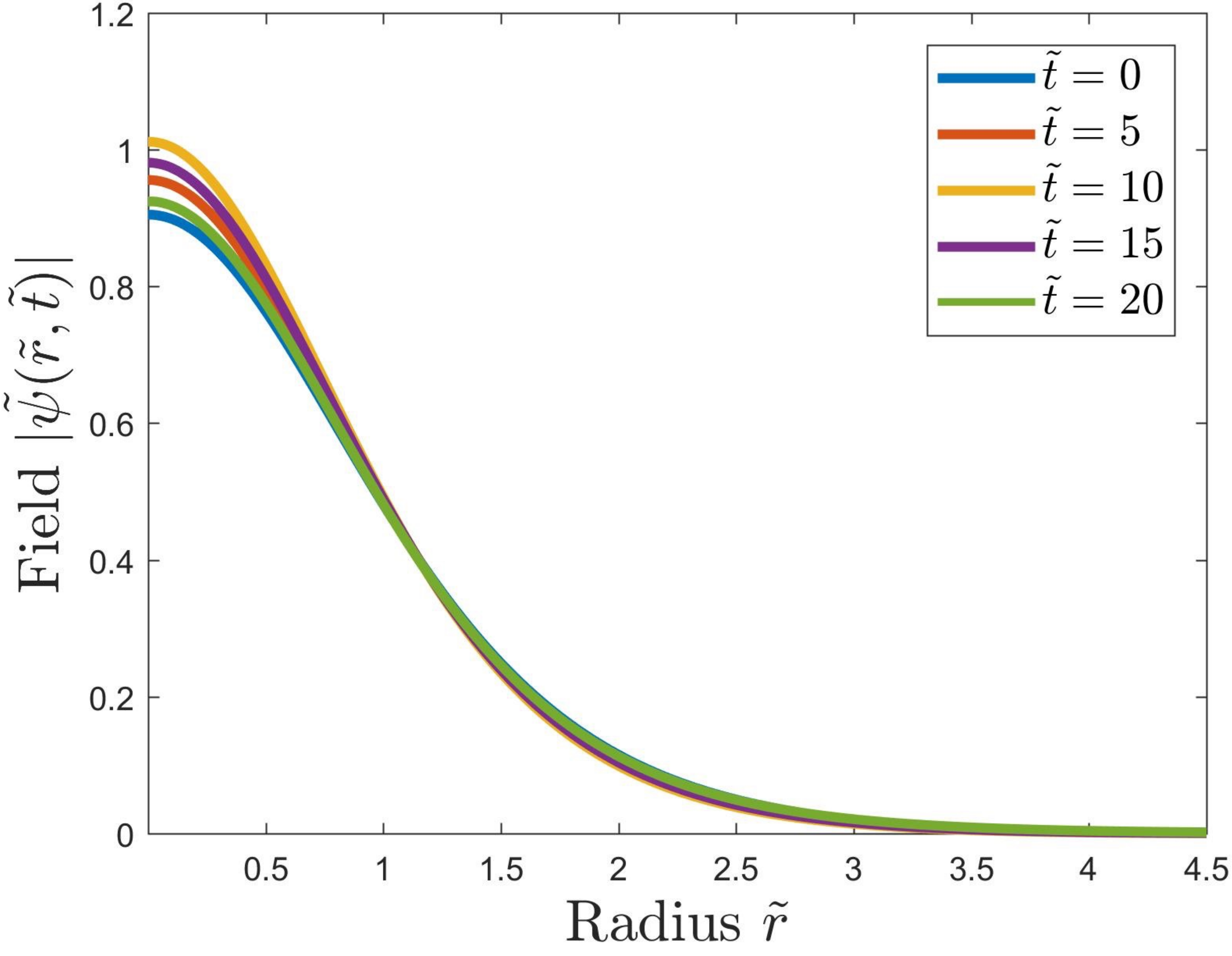


Schiappacasse, Hertzberg 1710.04729

Perturbing Upper Branch



Schiappacasse, Hertzberg 1710.04729

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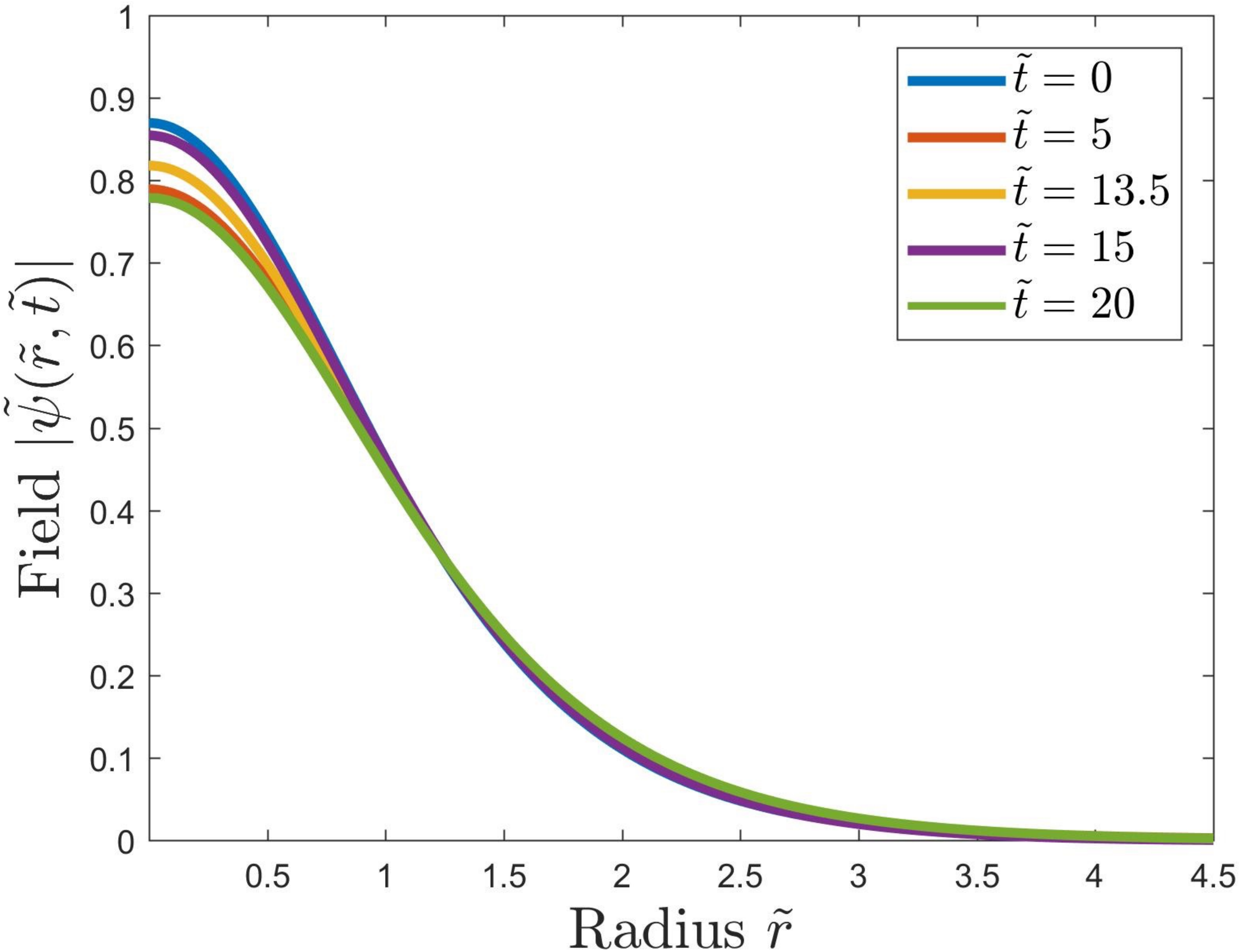

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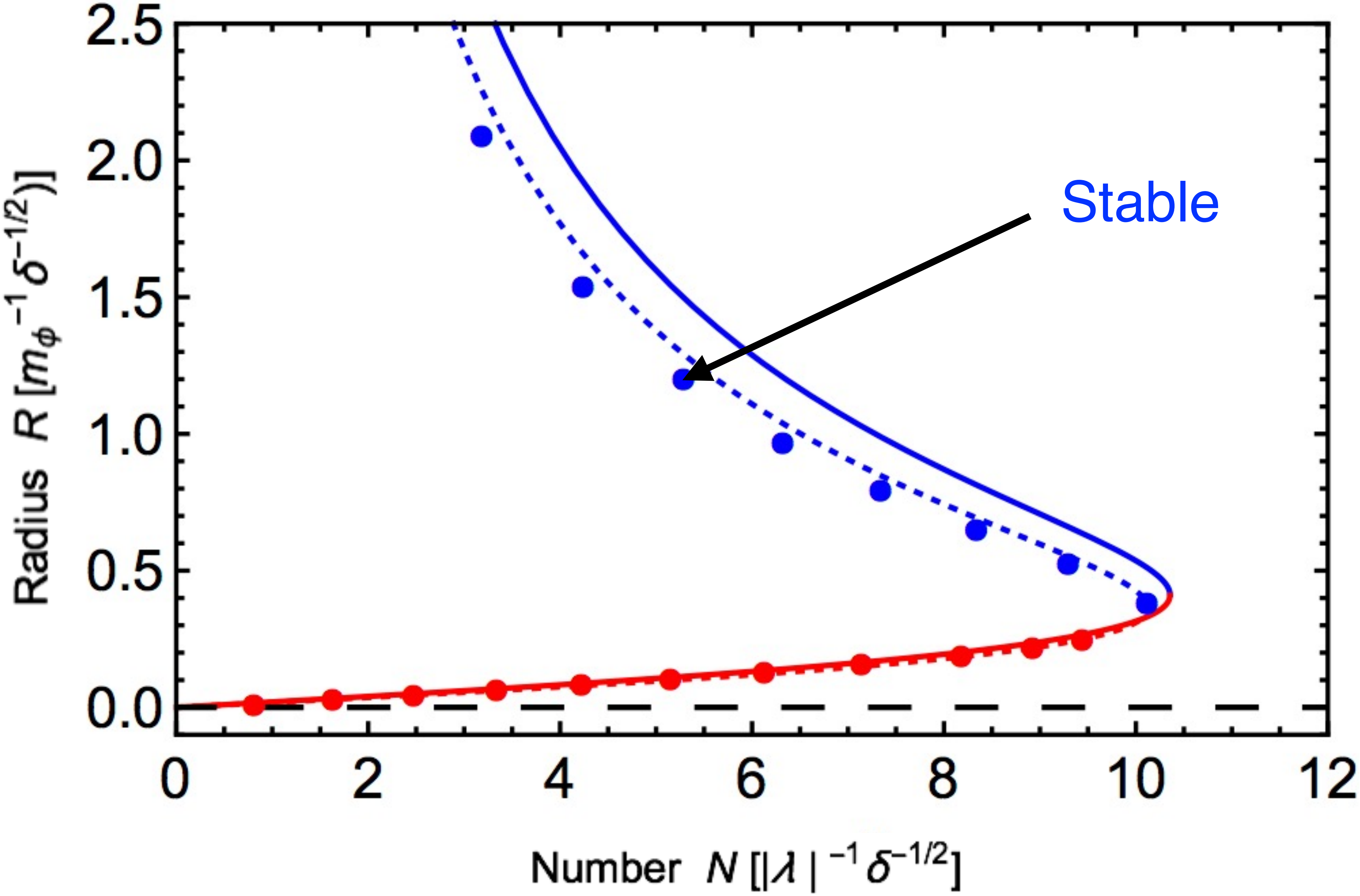

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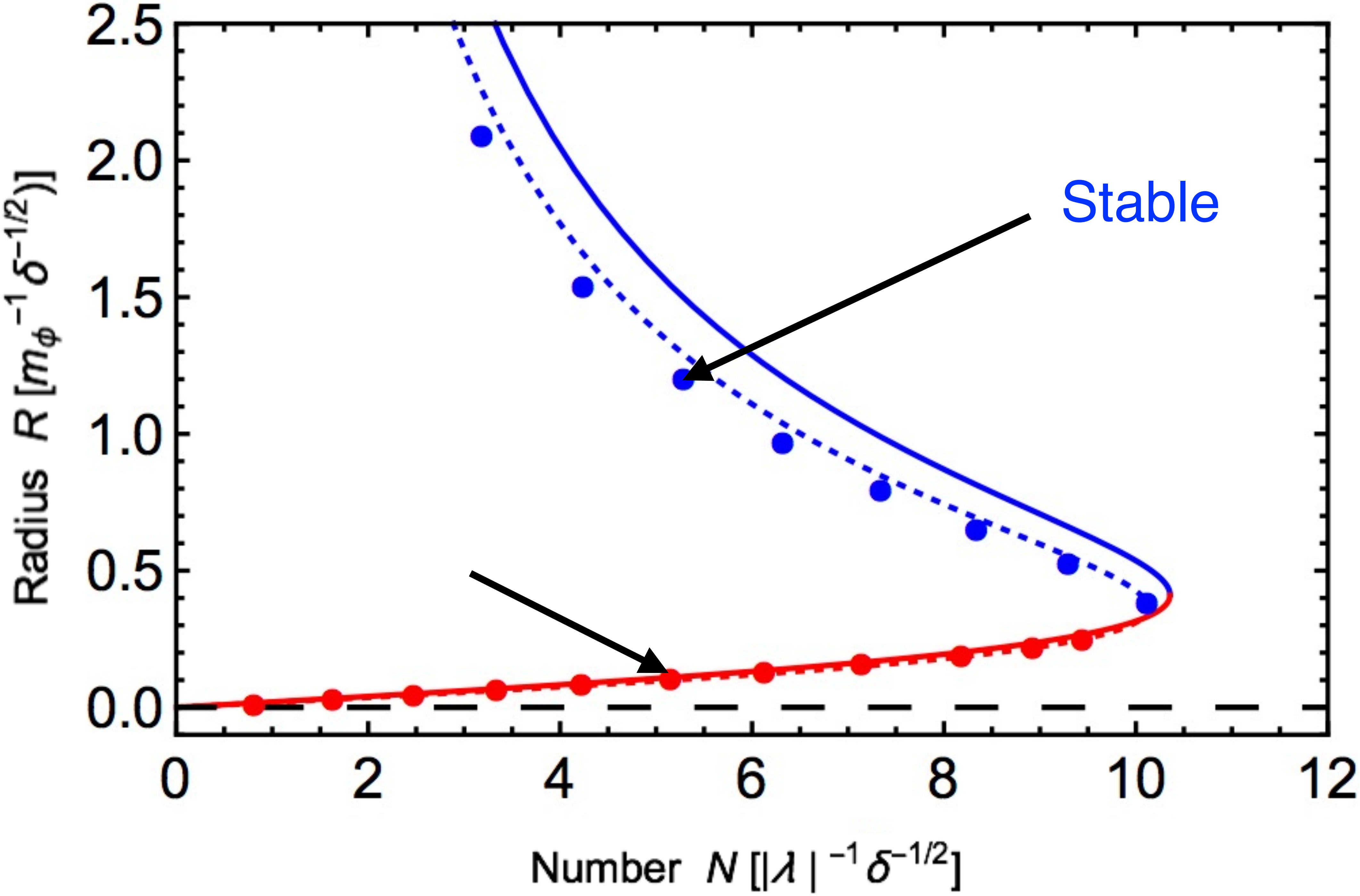
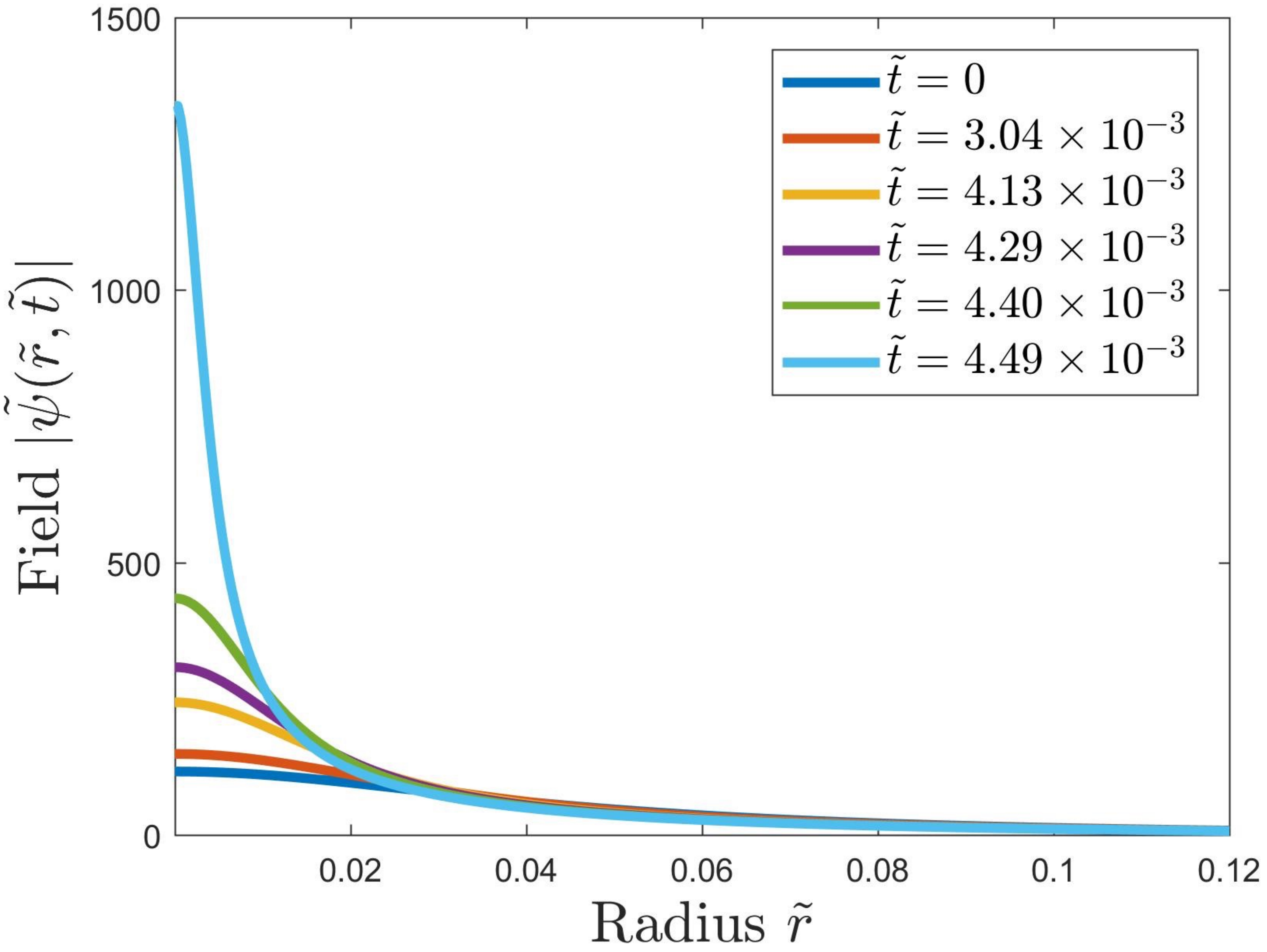


Schiappacasse, Hertzberg 1710.04729

Perturbing Lower Branch



Schiappacasse, Hertzberg 1710.04729

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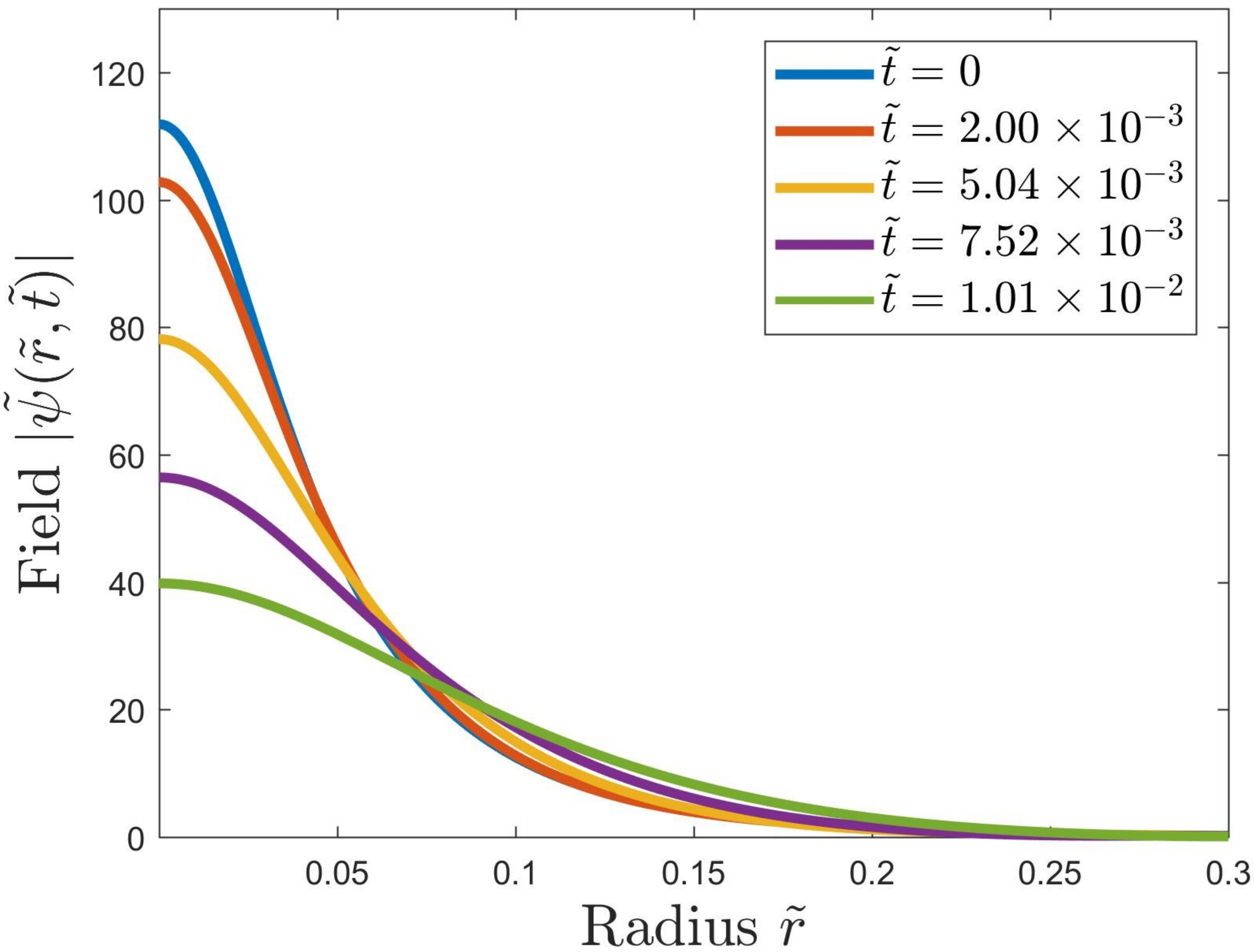

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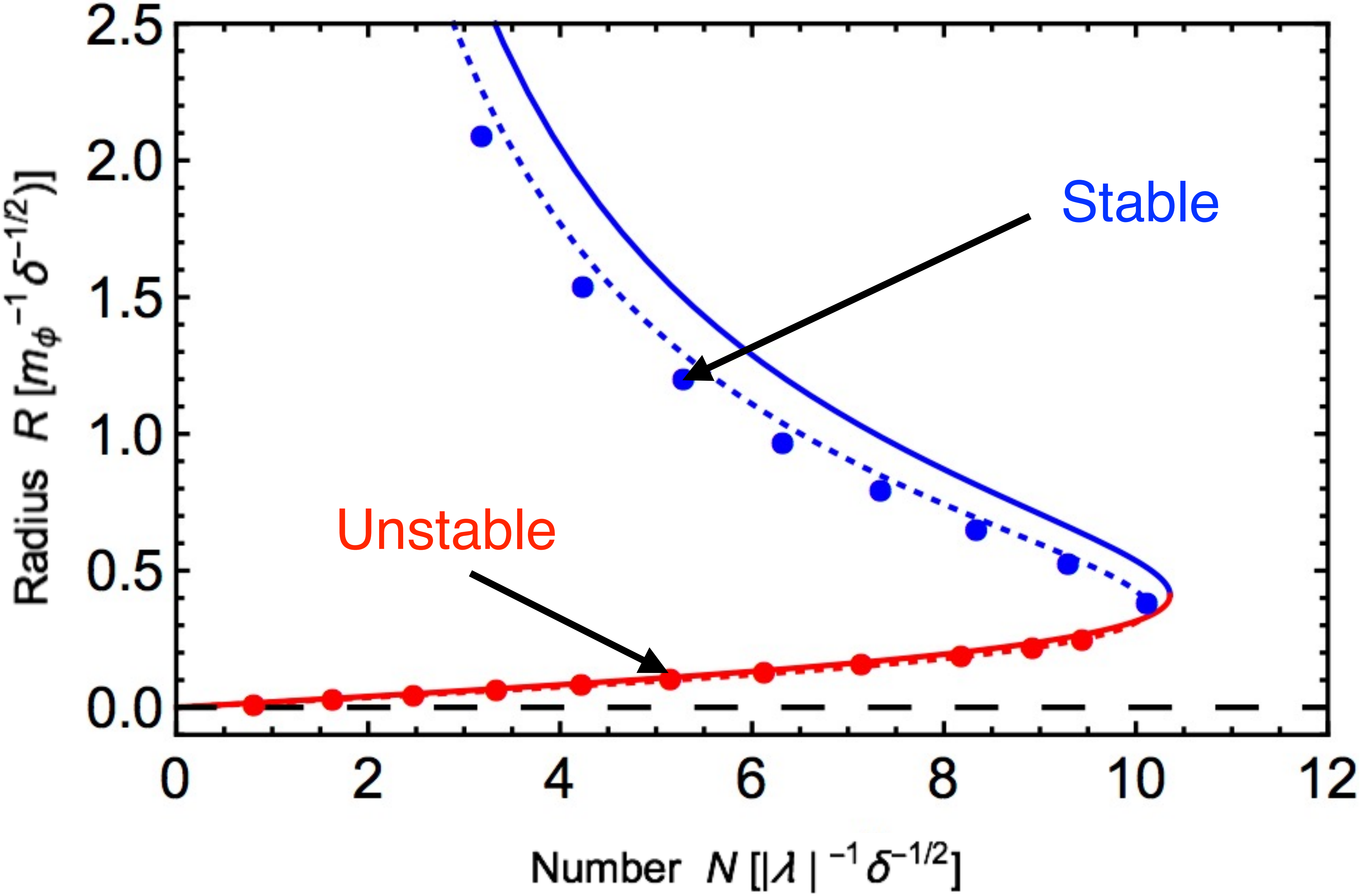
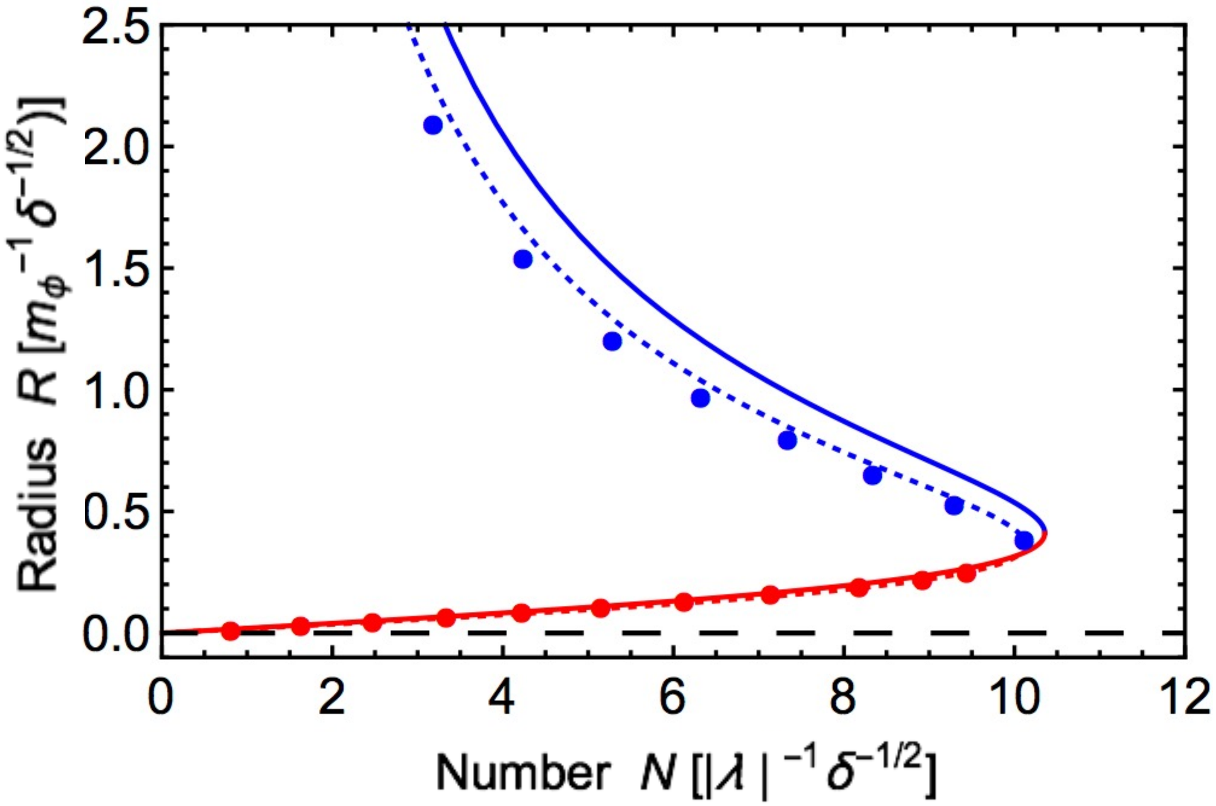


Schiappacasse, Hertzberg 1710.04729

Two Branches of Solutions



$$N_{max} = \frac{f_a}{m^2\sqrt{G}}\tilde{N}_{max} \sim 8\times 10^{59}\,(\tilde{m}^{-2}\tilde{f}_a)\,,$$

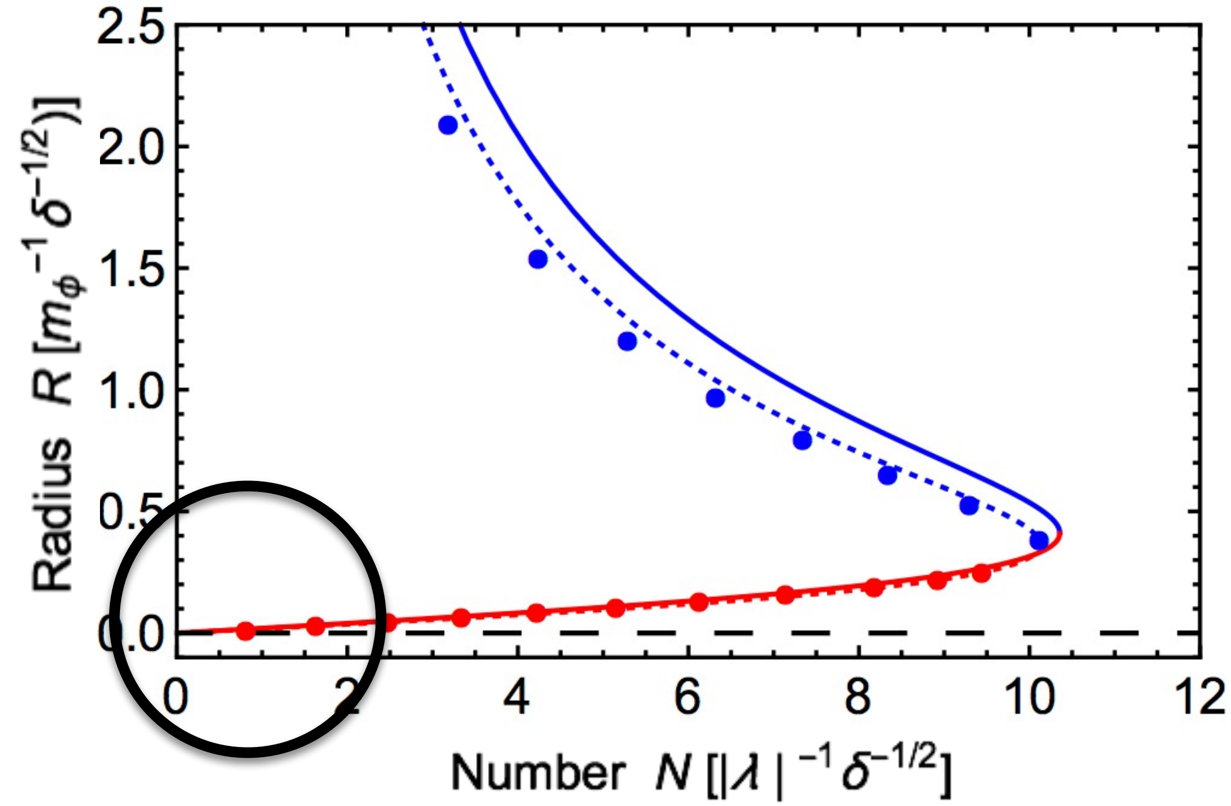
$$M_{max} = N_{max}\,m \sim 1.4\times 10^{19}\,\mathrm{kg}\,(\tilde{m}^{-1}\tilde{f}_a)\,,$$

$$R_{90,min} = \frac{a\,(\tilde{R}_{90}/\tilde{R})}{bN_{max}Gm^3} \sim 130\,\mathrm{km}\,(\tilde{m}^{-1}\tilde{f}_a^{-1})\,,$$

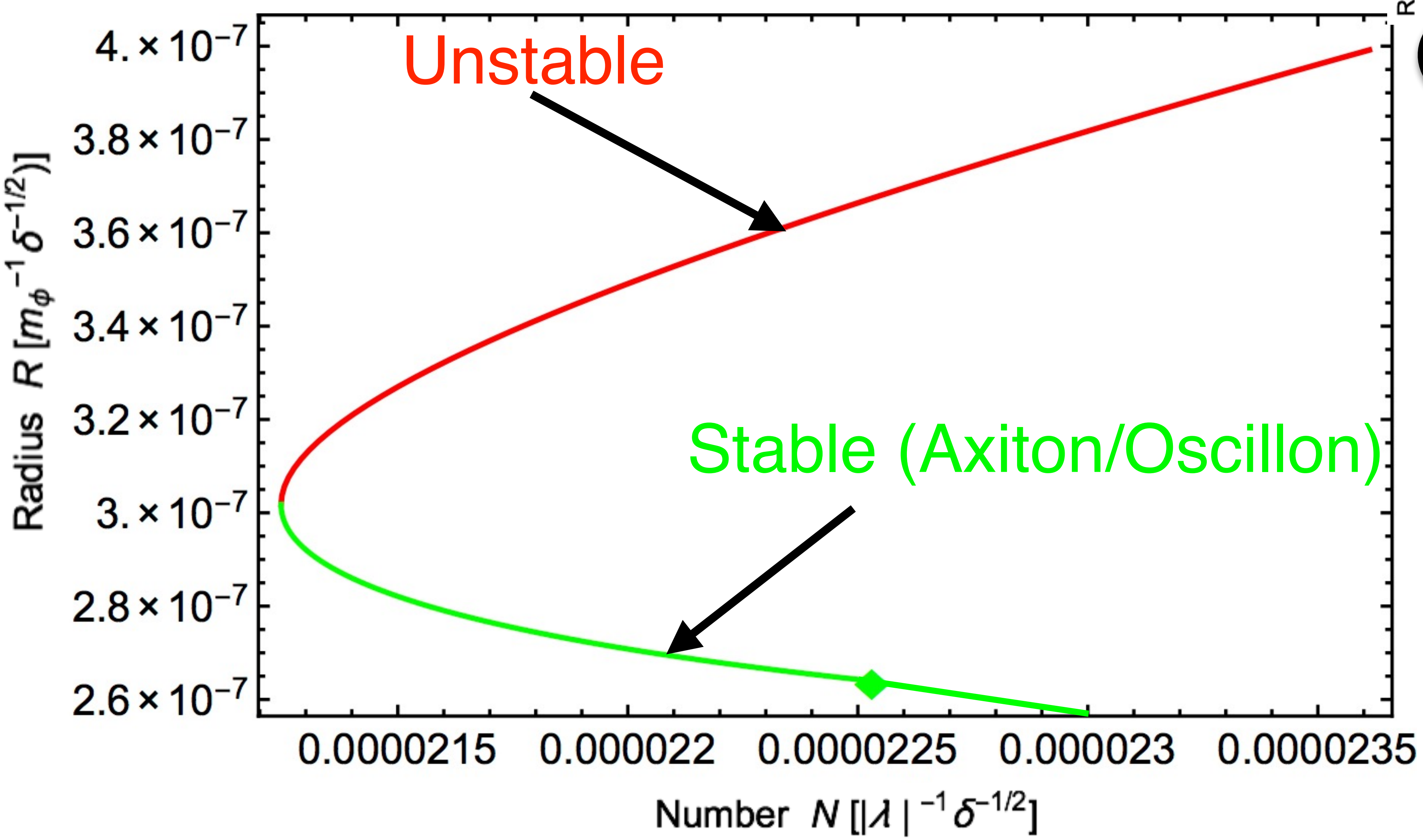
where $\tilde{f}_a \equiv f_a/(6\times 10^{11}\,\mathrm{GeV})$ and $\tilde{m} \equiv m/(10^{-5}\,\mathrm{eV})$.

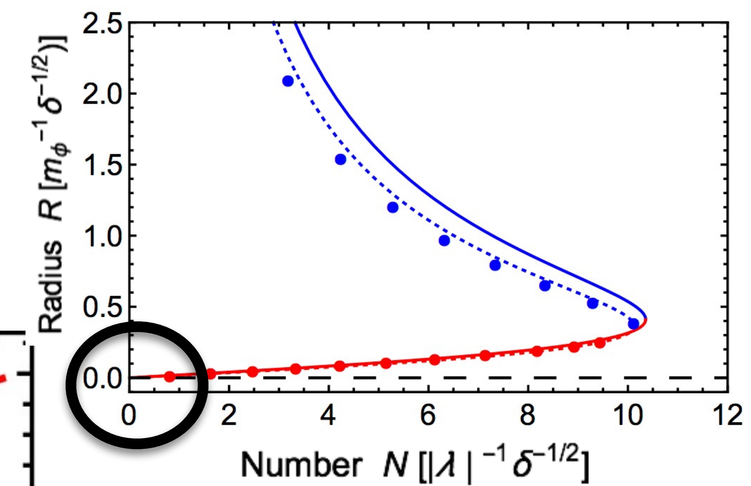
Schiappacasse, Hertzberg 1710.04729

Relativistic Branch (Axiton)



Relativistic Branch (Axiton/Oscillon)





Kolb, Tkachev astro-ph/9311037; Schiappacasse, Hertzberg 1710.04729;
Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910

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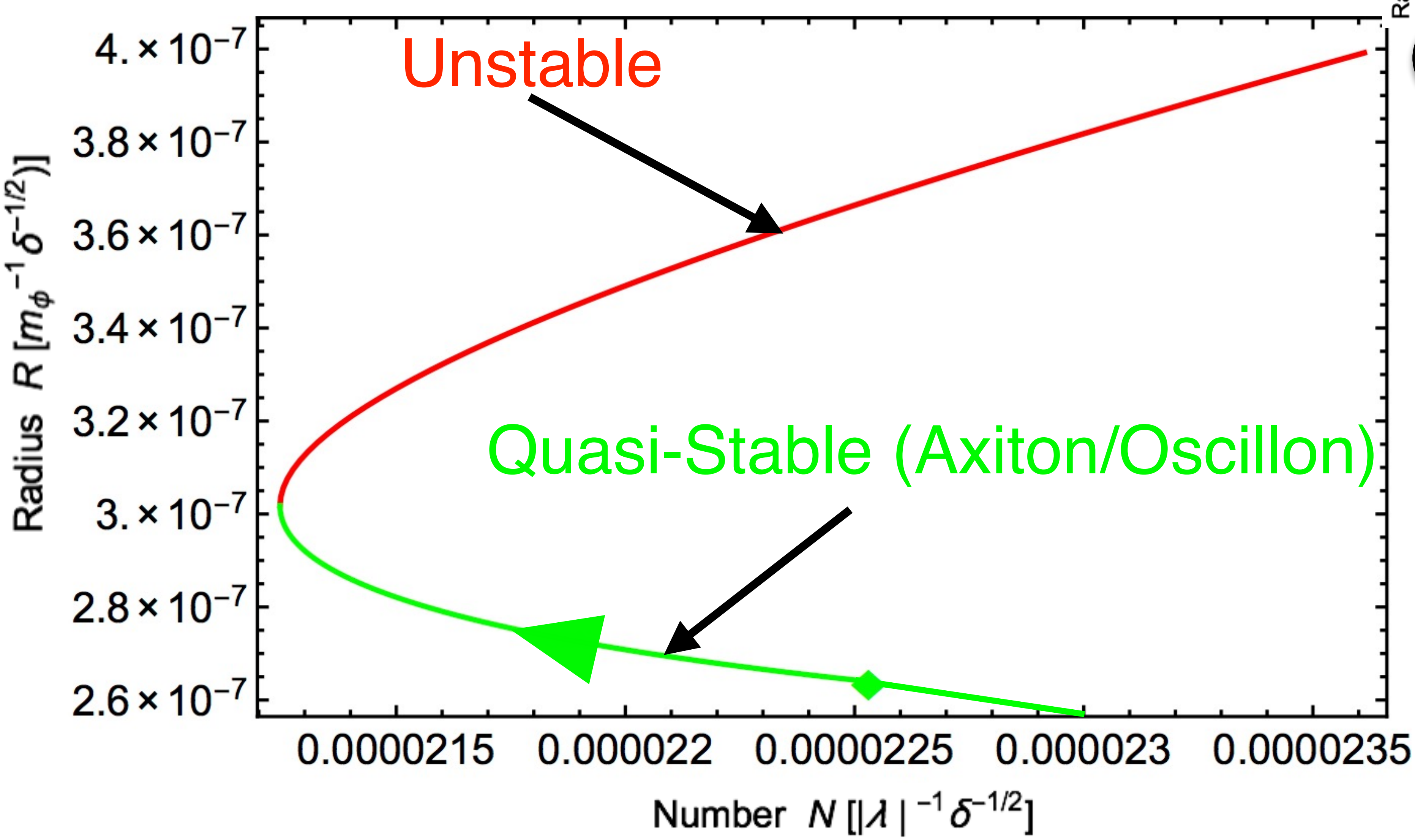
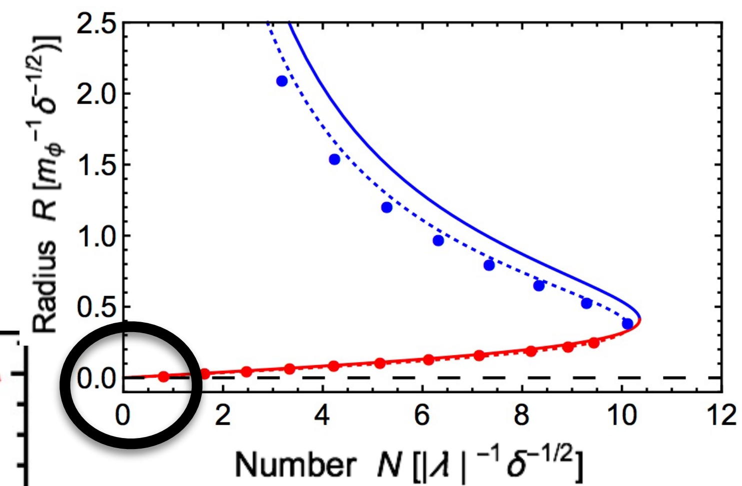
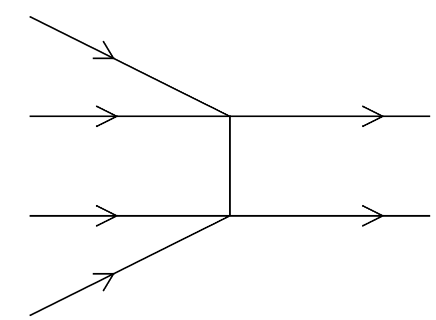
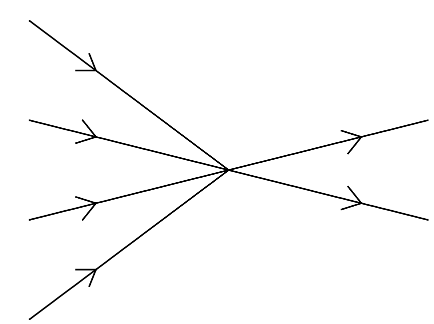






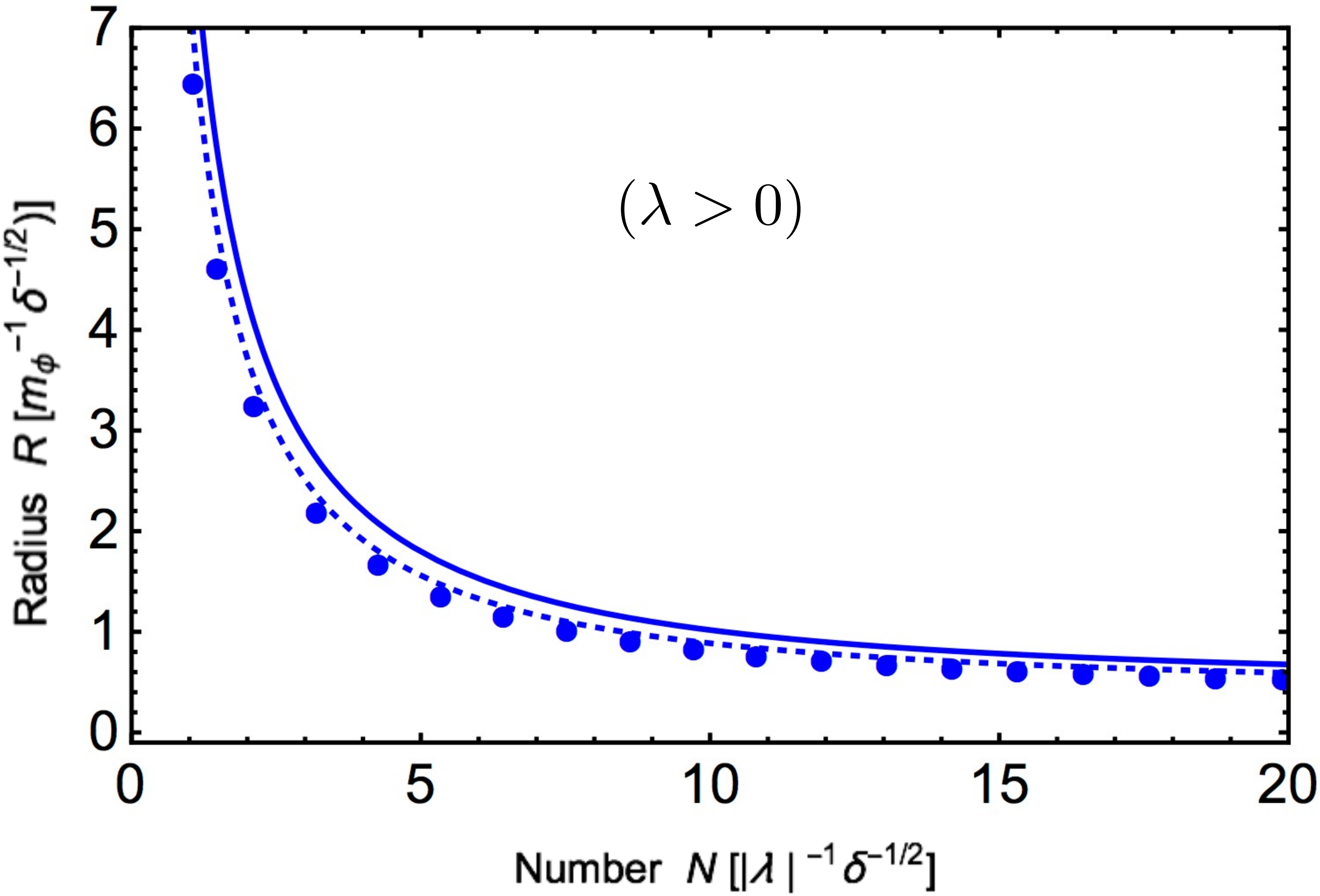


Kolb, Tkachev astro-ph/9311037, Schiappacasse, Hertzberg 1710.04729, Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910

Repulsive Self Interactions

(see; Fan 2016)

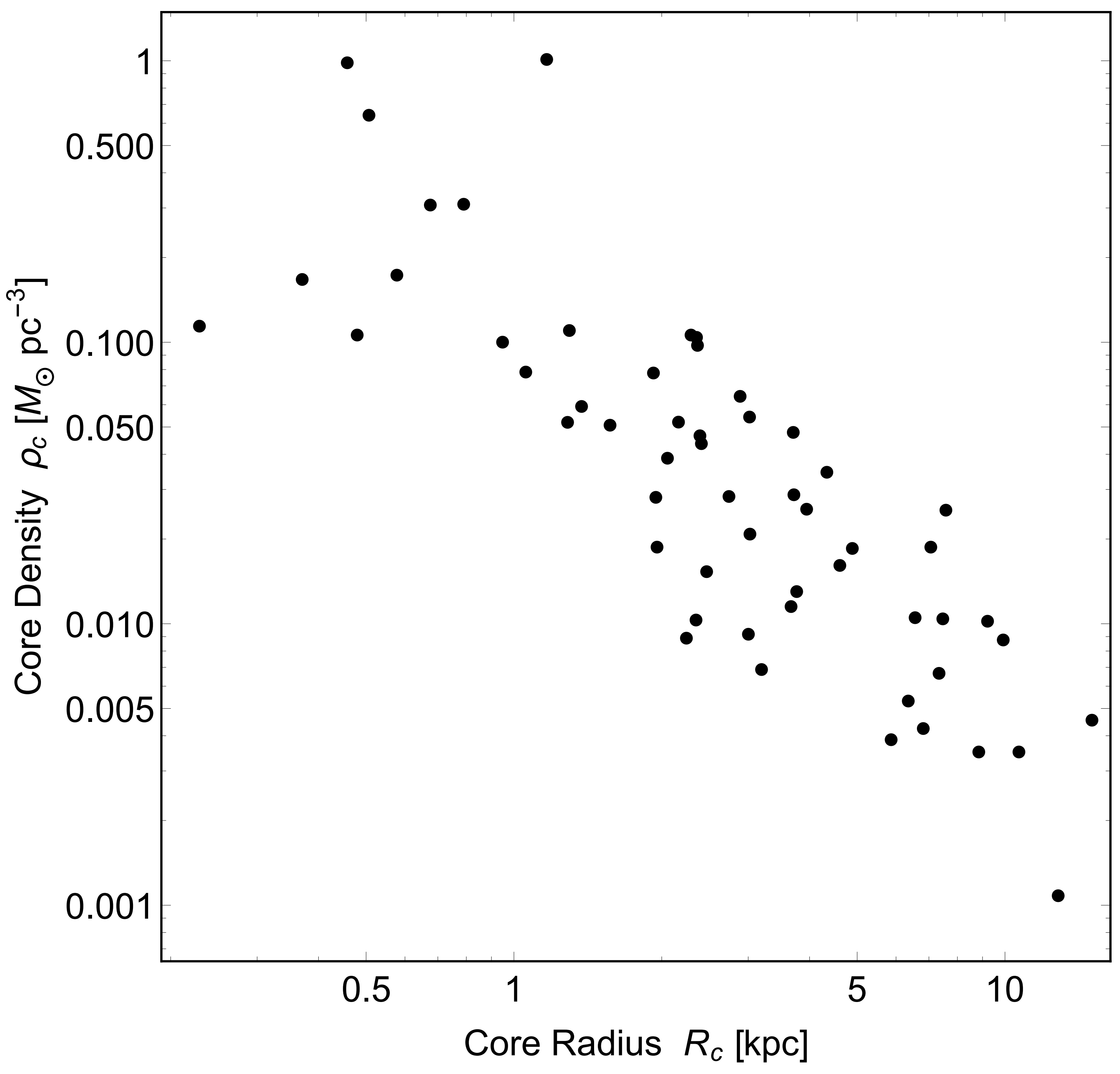
Repulsive Self Interaction (Axion-Like Particle)



Schiappacasse, Hertzberg 1710.04729

Implications for Fuzzy Dark Matter

Core Density Vs Core Radius (Data)



Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

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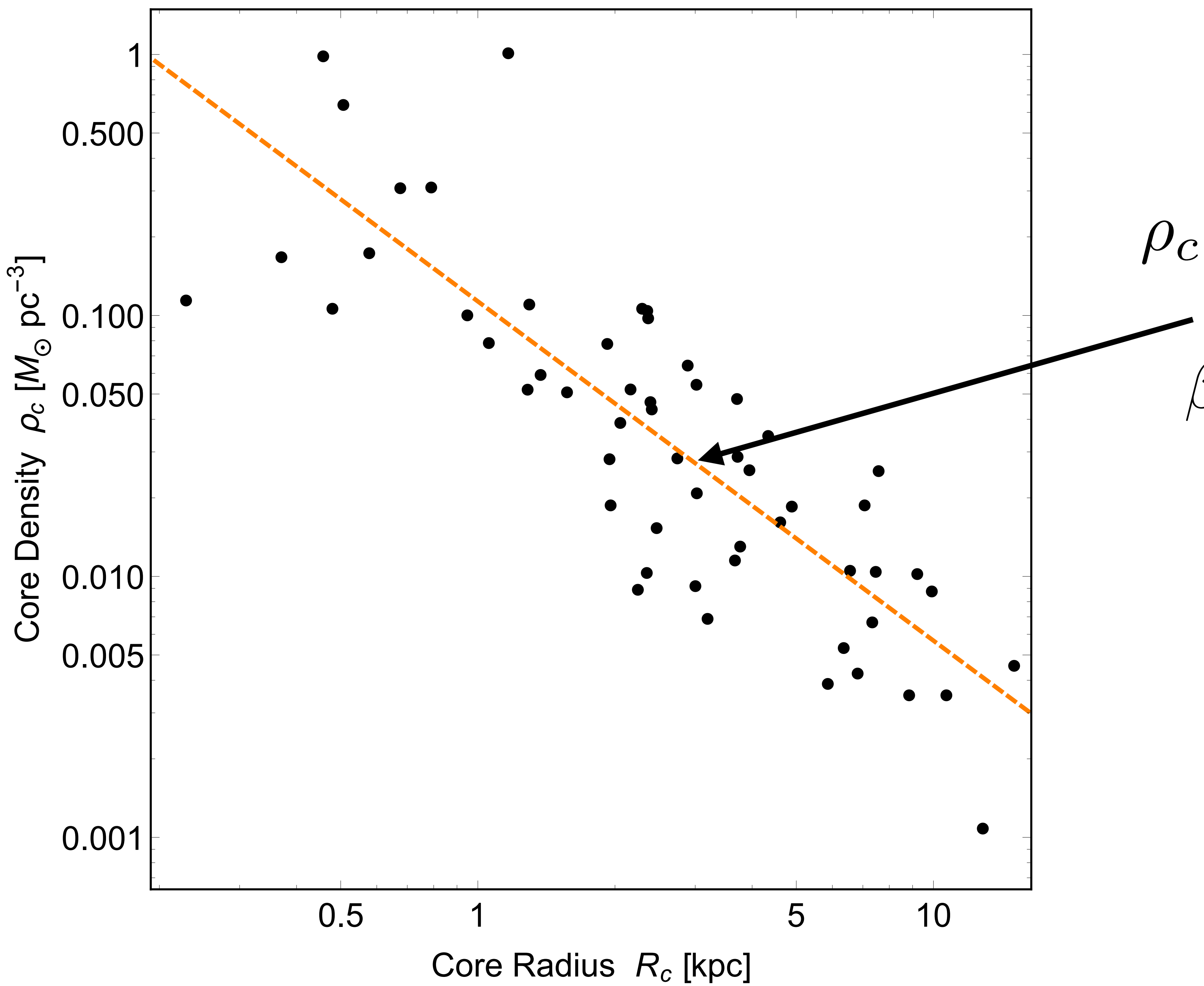
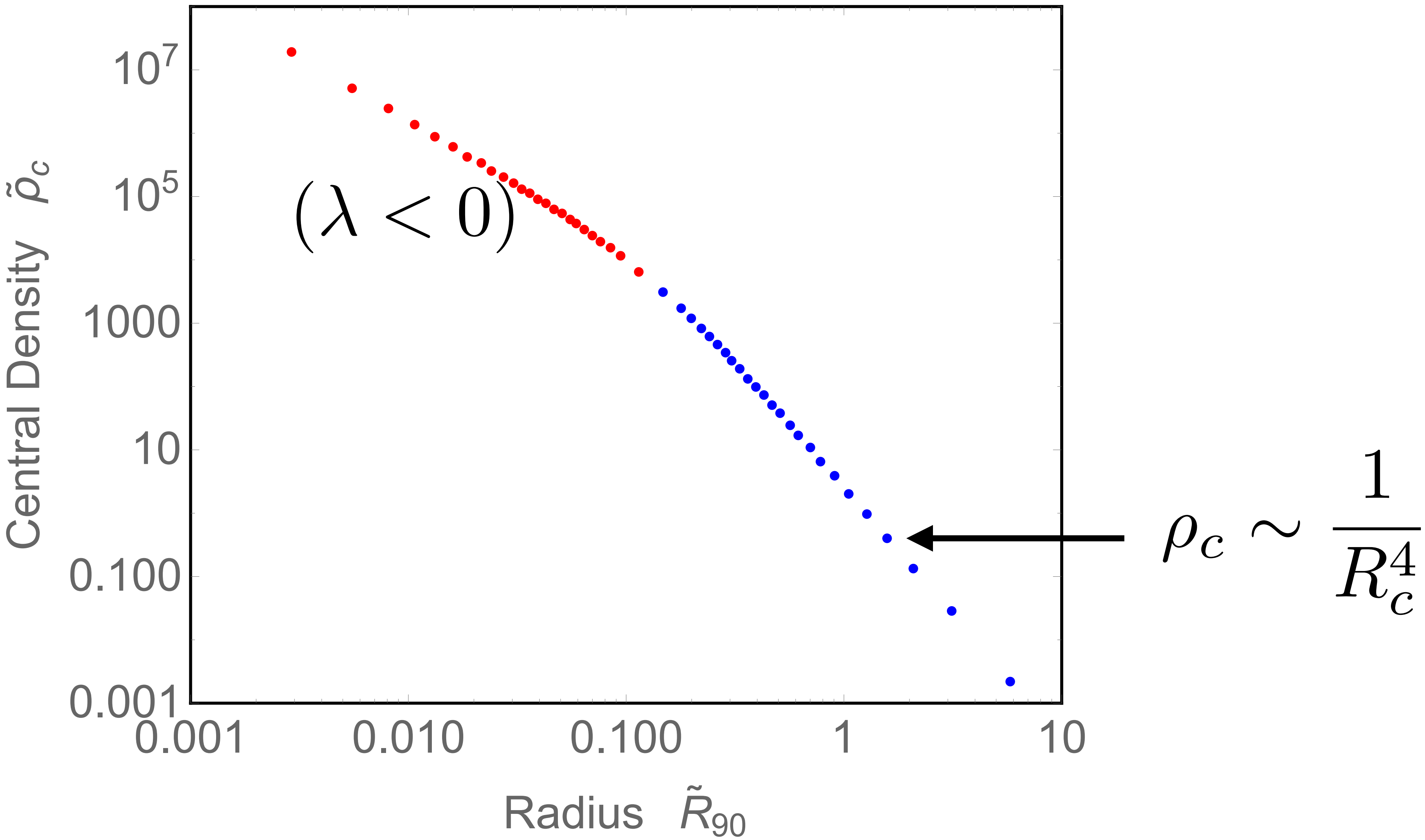


$$\rho_c \sim \frac{1}{R_c^\beta}$$

$$\beta \approx 1$$

Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

Core Density Vs Core Radius (Light Scalar in BEC)



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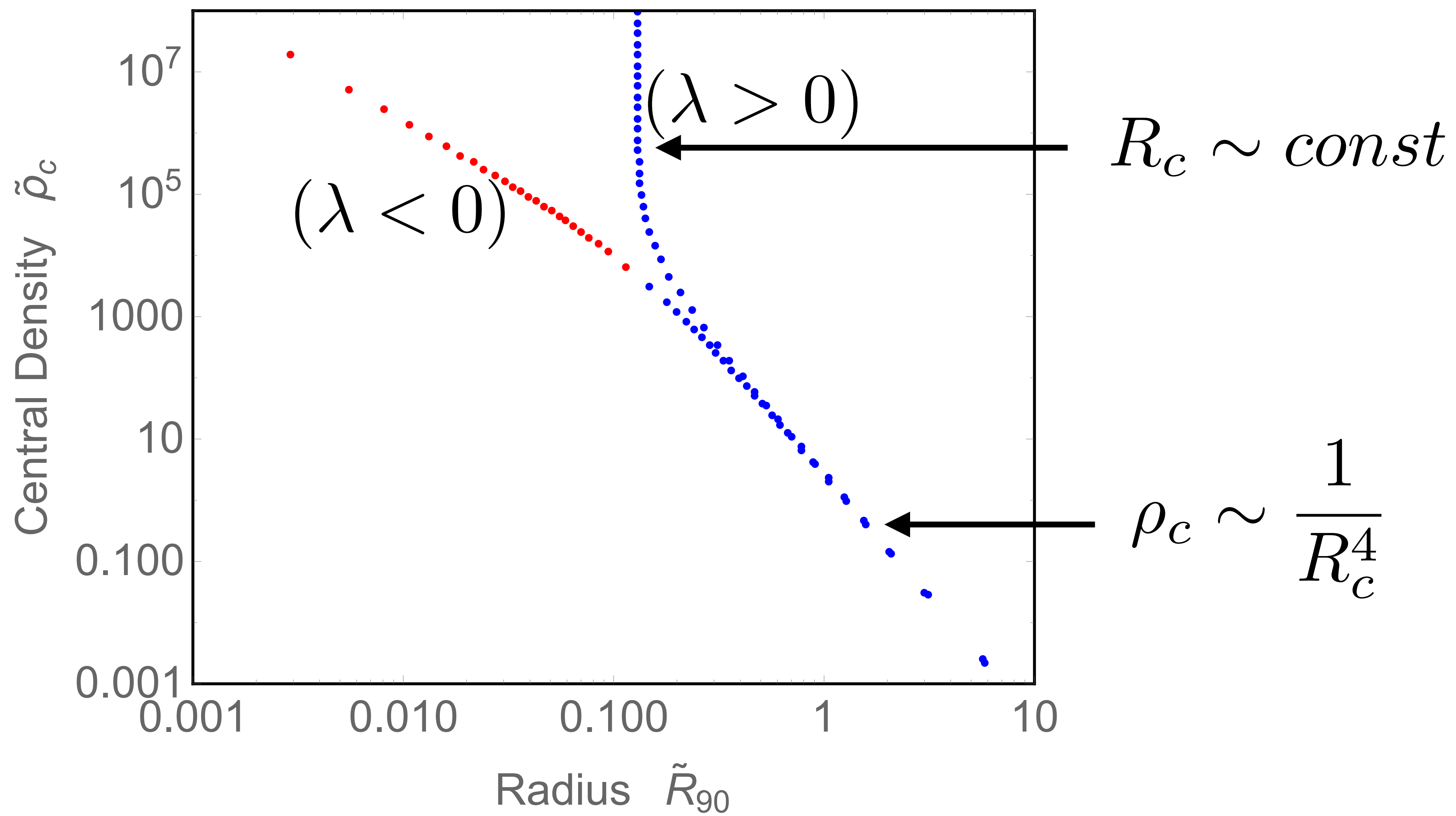
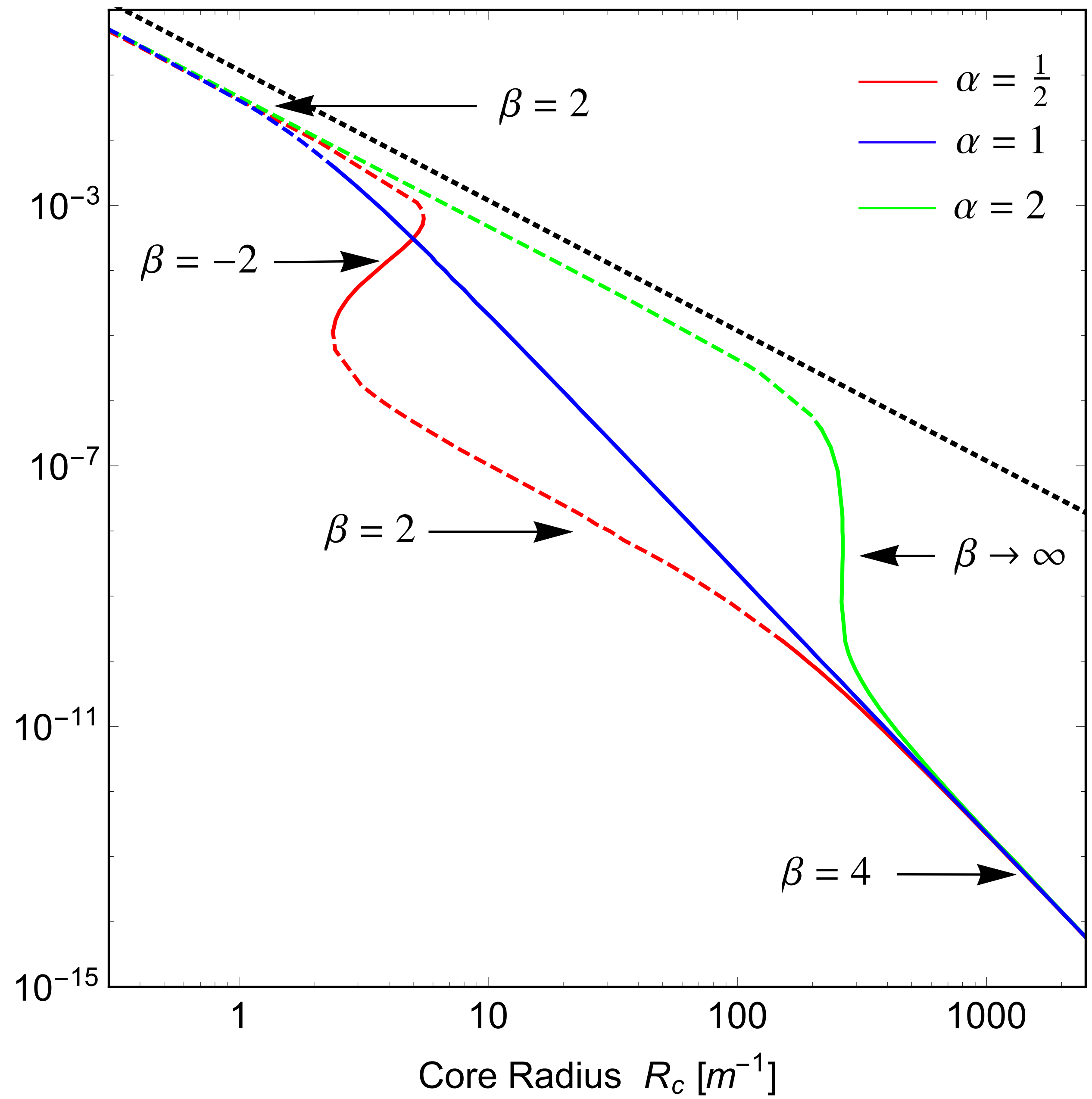


Core Density Vs Core Radius (Light Scalar in BEC)

Extension to general potentials, polytropes, full relativistic

$$V(\phi) \propto ((1+\phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable



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Deng, Hertzberg, Namjoo, Masoumi 1804.05921

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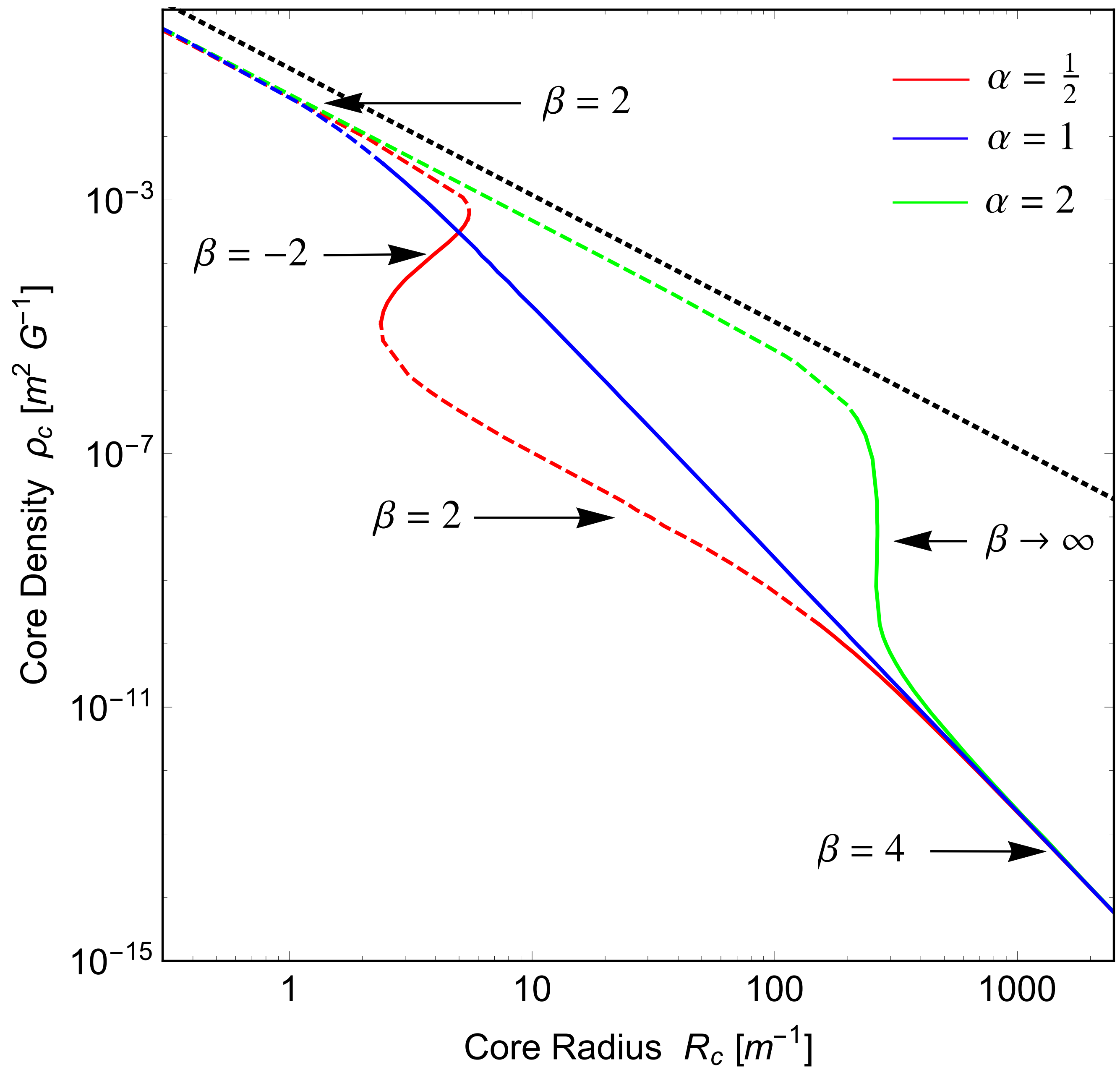
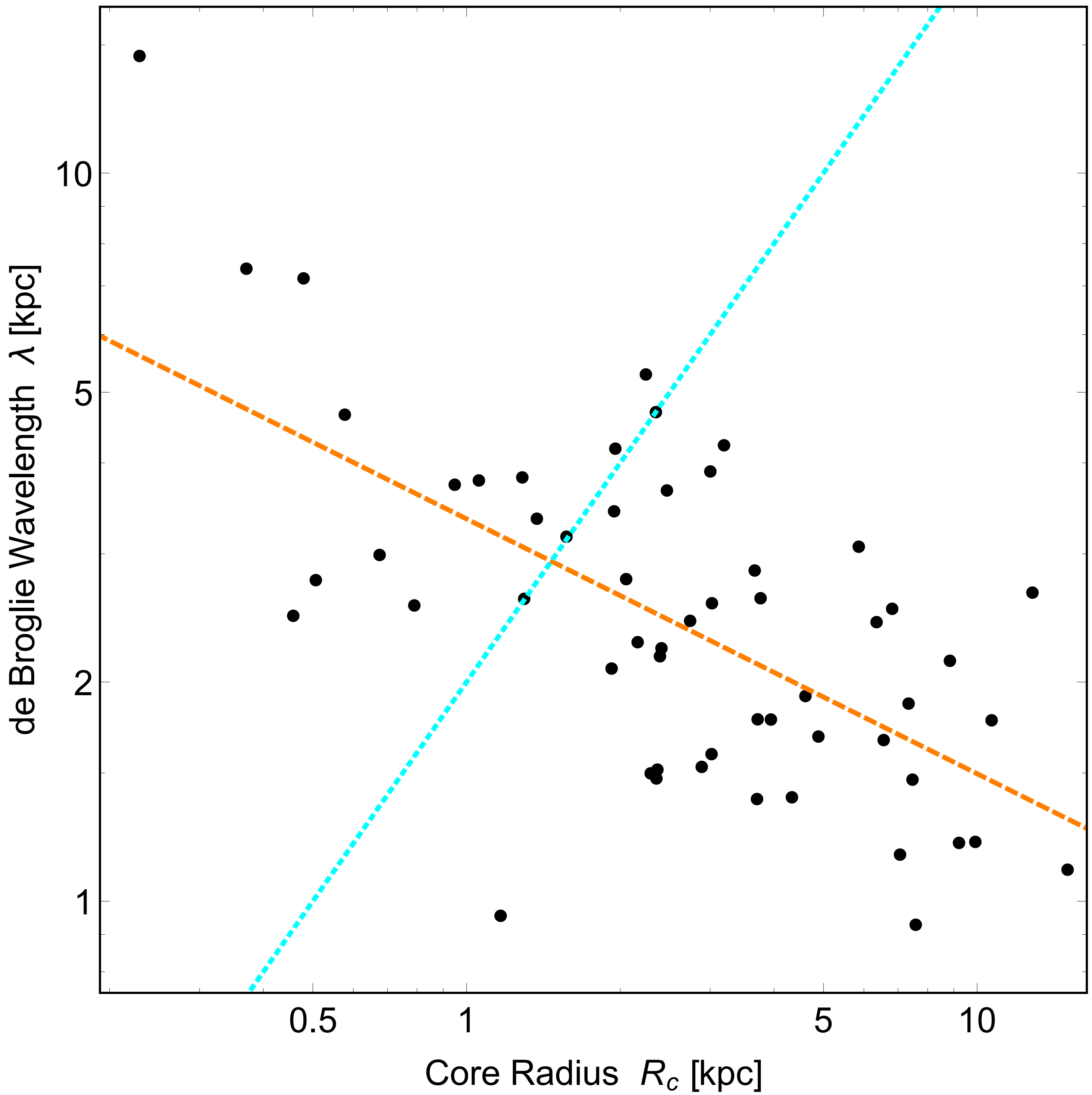


$$\rho_c \sim \frac{1}{R_c^\beta}$$

Never obtain $\beta \sim 1$ and stable

Deng, Hertzberg, Namjoo, Masoumi 1804.05921

Core Density Vs Core Radius (Light Scalar in BEC)



Deng, Hertzberg, Namjoo, Masoumi 1804.05921

Axion Clump Resonance into Photons

Consider Axion to Photon Coupling

Photon Lagrangian

$$\mathcal{L}_\gamma = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + g_{a\gamma}\,\phi\,\mathbf{E}\cdot\mathbf{B}$$

(Sikivie 1983; Adshead, Giblin, Scully, Sfakianakis 2015, 2016; Masaki, Aoki, Soda 2017)

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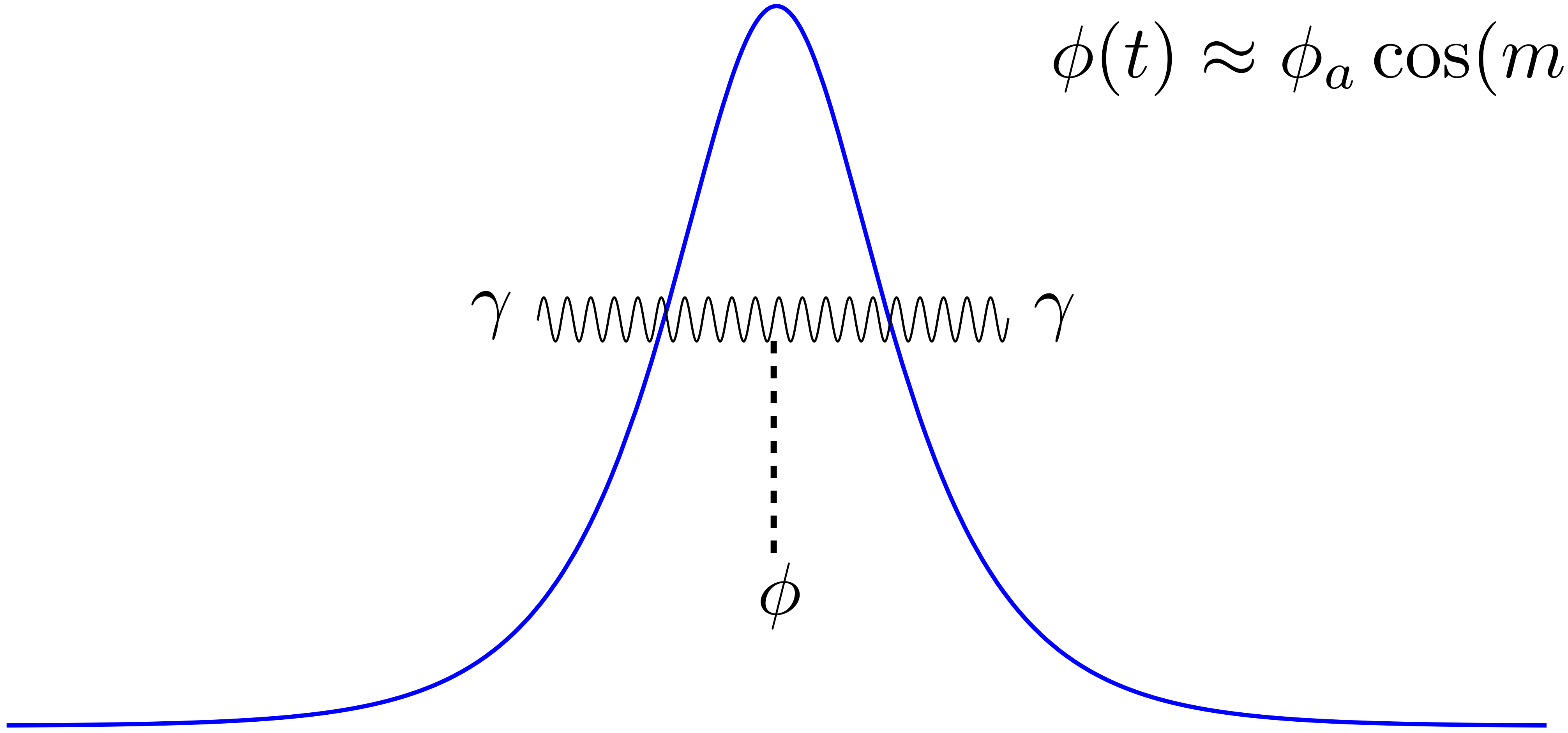
(Sikivie 1983; Adshead, Giblin, Scully, Sfakianakis 2015, 2016; Masaki, Aoki, Soda 2017)

Equation of motion

$$\ddot{\mathbf{A}} - \nabla^2\mathbf{A} + g_{a\gamma}\,\partial_t\phi\,\nabla\times\mathbf{A} = 0$$

$$\phi(t) \approx \phi_a \cos(m_\phi t)$$



Homogeneous Axion Field

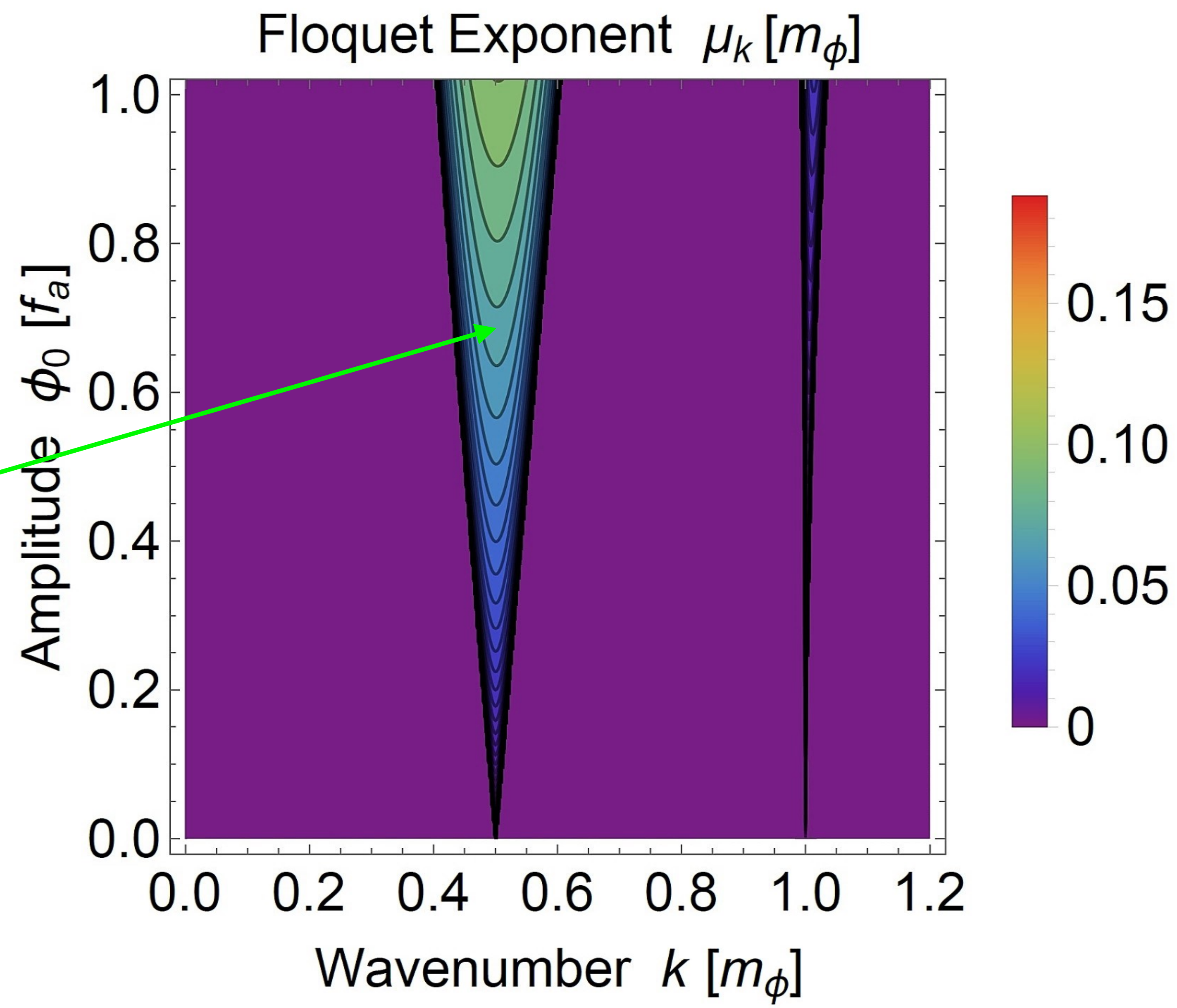
Mathieu Equation

$$\ddot{\mathbf{A}}^T_{\mathbf{k}} + k^2 \mathbf{A}^T_{\mathbf{k}} + g_{a\gamma}\, k\, \partial_t \phi(t) \mathbf{A}^T_{\mathbf{k}} = 0$$

Parametric resonance always present

$$k \approx \frac{m_a}{2}$$

$$\mu^*_H \approx \frac{1}{4} g_{a\gamma}\, m_\phi\, \phi_a$$



e.g., Yoshimura 1996

Comment on Photon Plasma Mass

In plasma, the photon acquires an effective mass $$\omega_p^2 = \frac{4\pi\alpha\, n_e}{m_e}$$

In early universe, this is huge; preventing resonance

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In early universe, this is huge; preventing resonance

Clumps in halo: $$\omega_p^2 \approx \frac{n_e}{0.03\,\mathrm{cm}^{-3}} \left(6 \times 10^{-12}\,\mathrm{eV}\right)^2$$

Negligibly small; allowing for resonance

Inhomogeneous (Spherical) Axion Clump

Decomposition into vector spherical harmonics

$$\mathbf{A}(\mathbf{r},t) = \sum_{lm} \int \frac{d^3k}{(2\pi)^3} \left[a_{lm}(k,t)\mathbf{N}_{lm}(k,\mathbf{r}) + b_{lm}(k,t)\mathbf{M}_{lm}(k,\mathbf{r})\right]$$

$$\mathbf{M}_{lm}(k,\mathbf{r}) = i\frac{j_l(kr)}{\sqrt{l(l+1)}}\nabla \times [Y_{lm}(\theta,\varphi)\mathbf{r}]$$

where

$$\mathbf{N}_{lm}(k,\mathbf{r}) = \frac{i}{k}\nabla \times \mathbf{M}_{lm}$$

Hertzberg, Schiappacasse 1805.00430

Inhomogeneous (Spherical) Axion Clump

Instability channel $\quad l = 1, m = 0, \quad b_{10} = -i\, a_{10}$

$$\ddot{a}_{10}(k,t) + k^2 a_{10}(k,t) + g_{a\gamma} k \int \frac{dk'}{(2\pi)} \partial_t \tilde{\phi}(k-k') a_{10}(k',t) = 0$$

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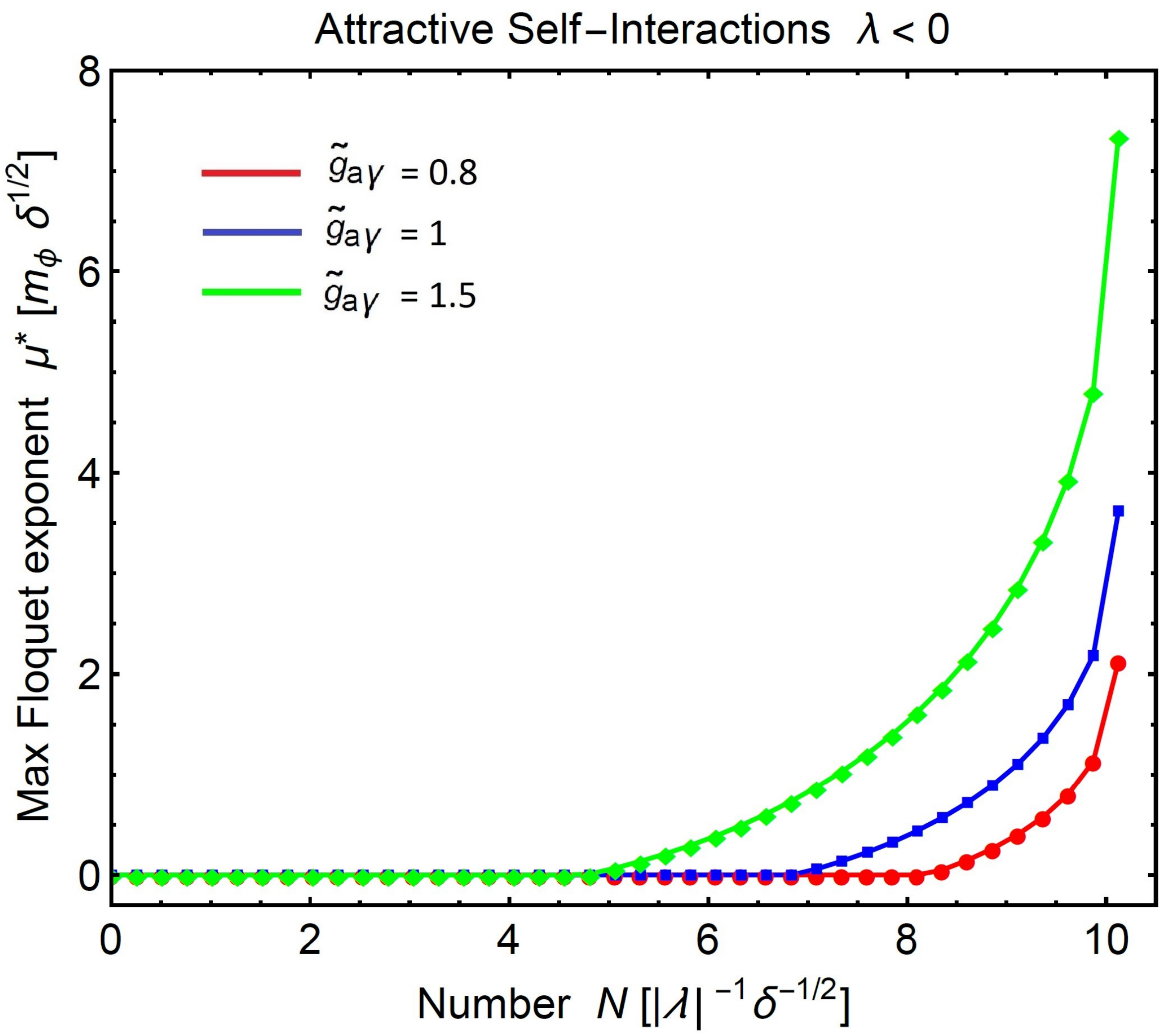

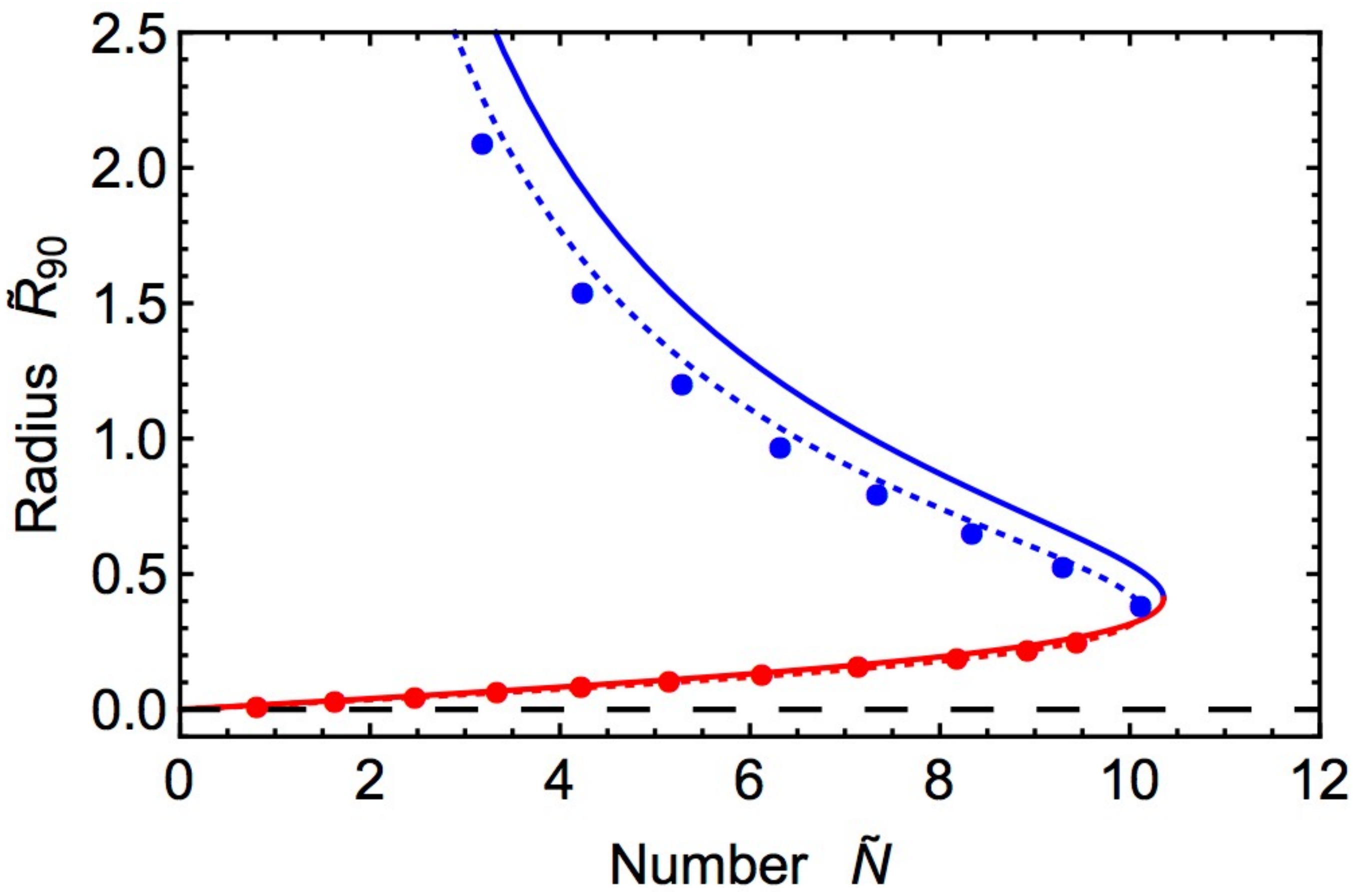

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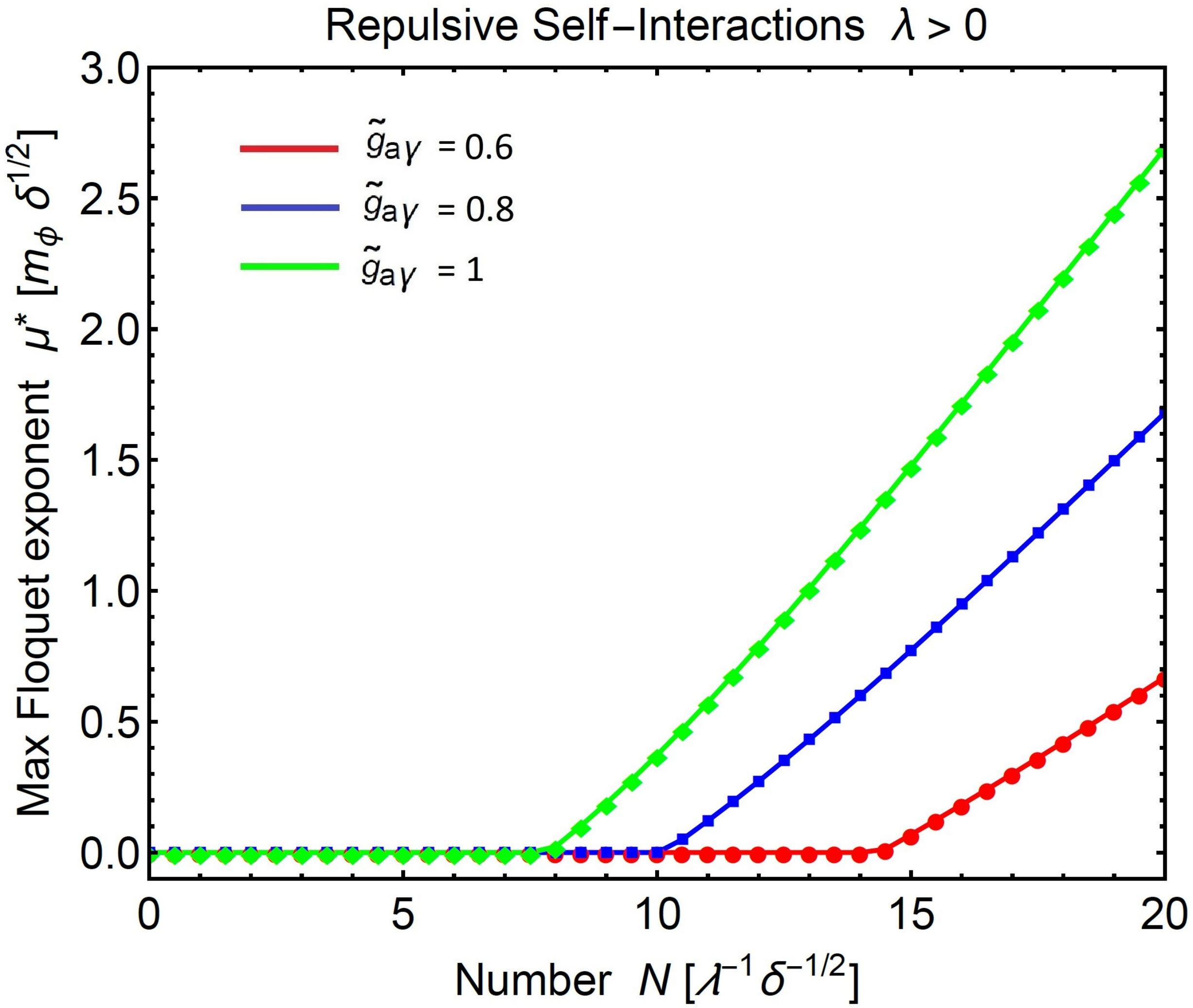

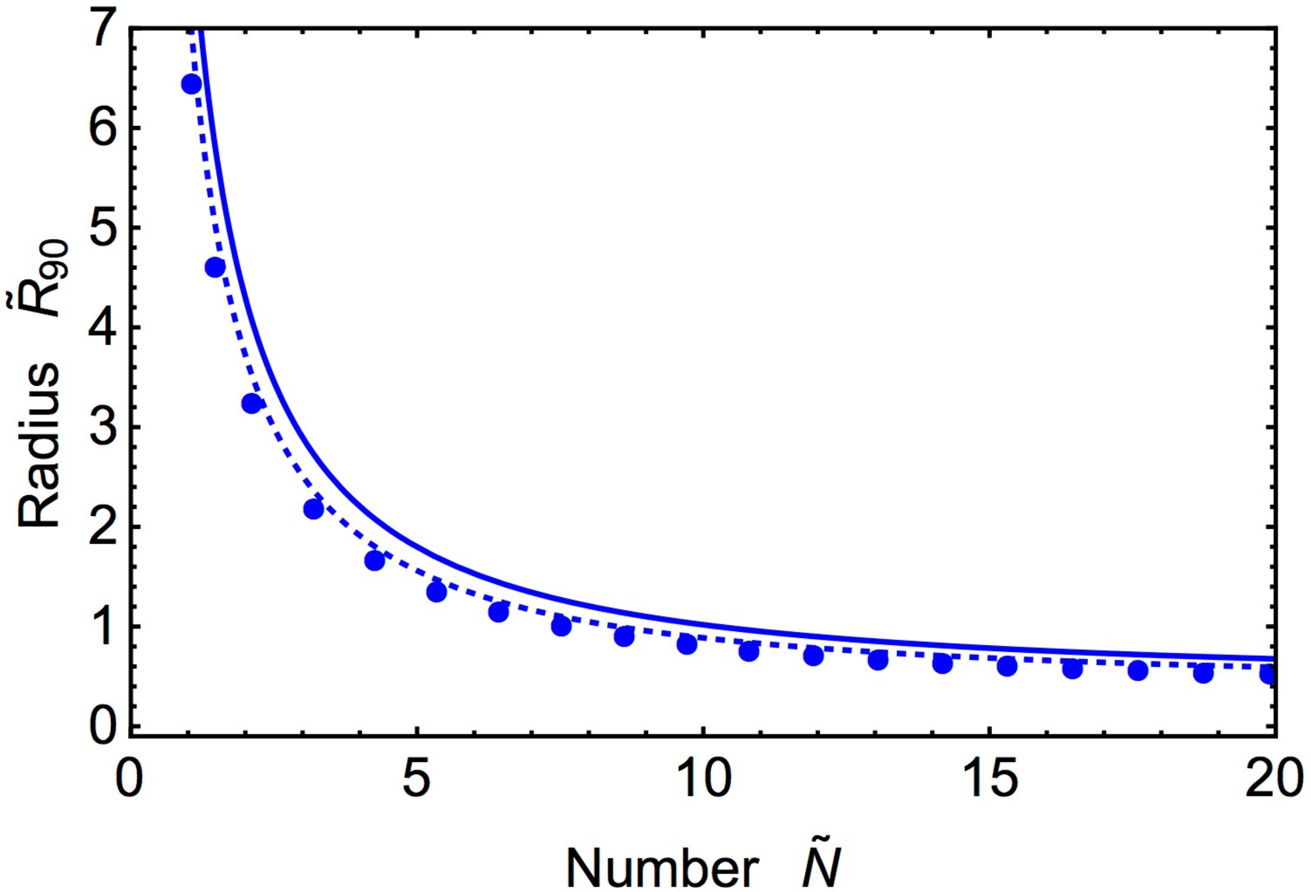

Hertzberg, Schiappacasse 1805.00430

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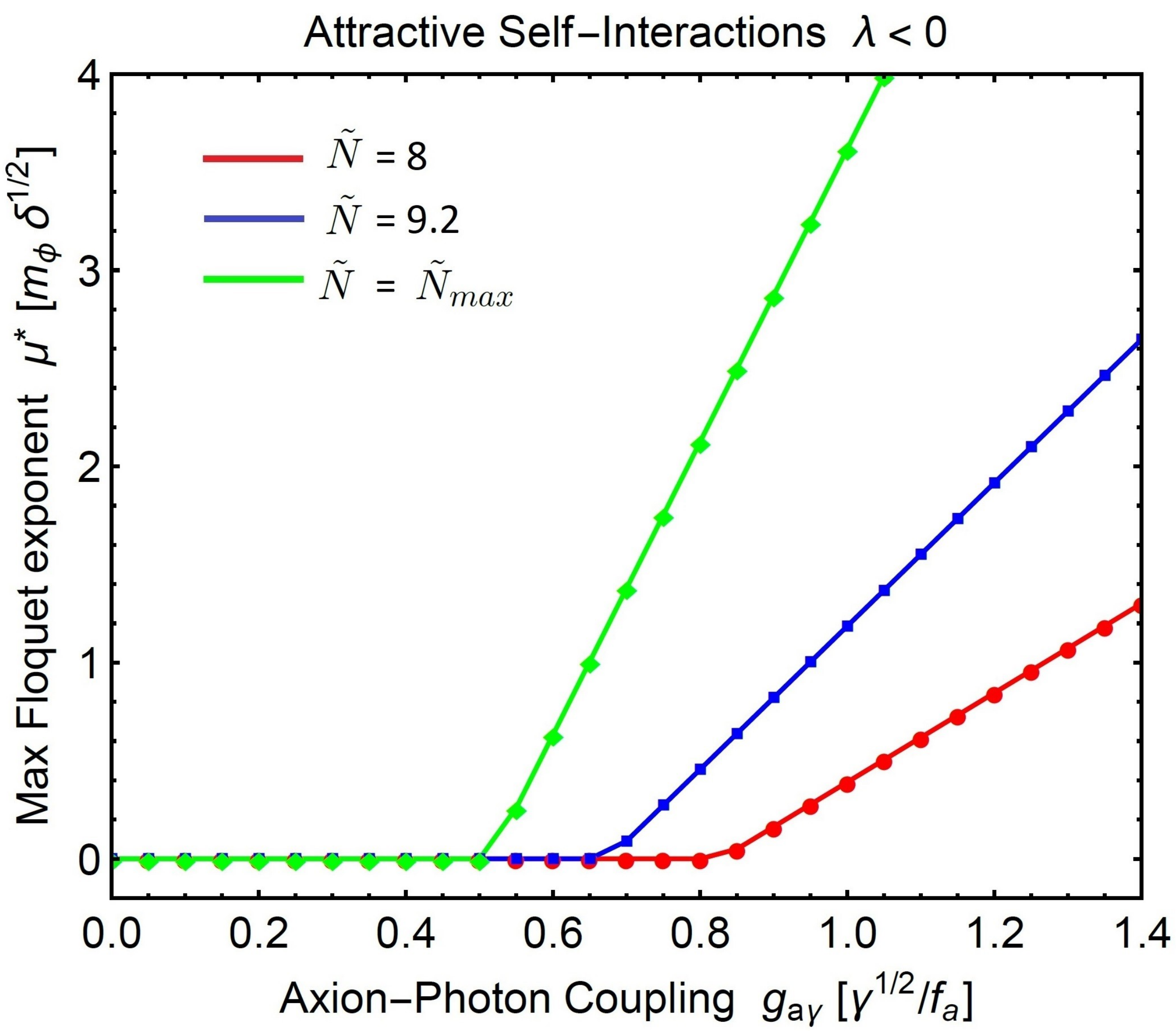

Hertzberg, Schiappacasse 1805.00430

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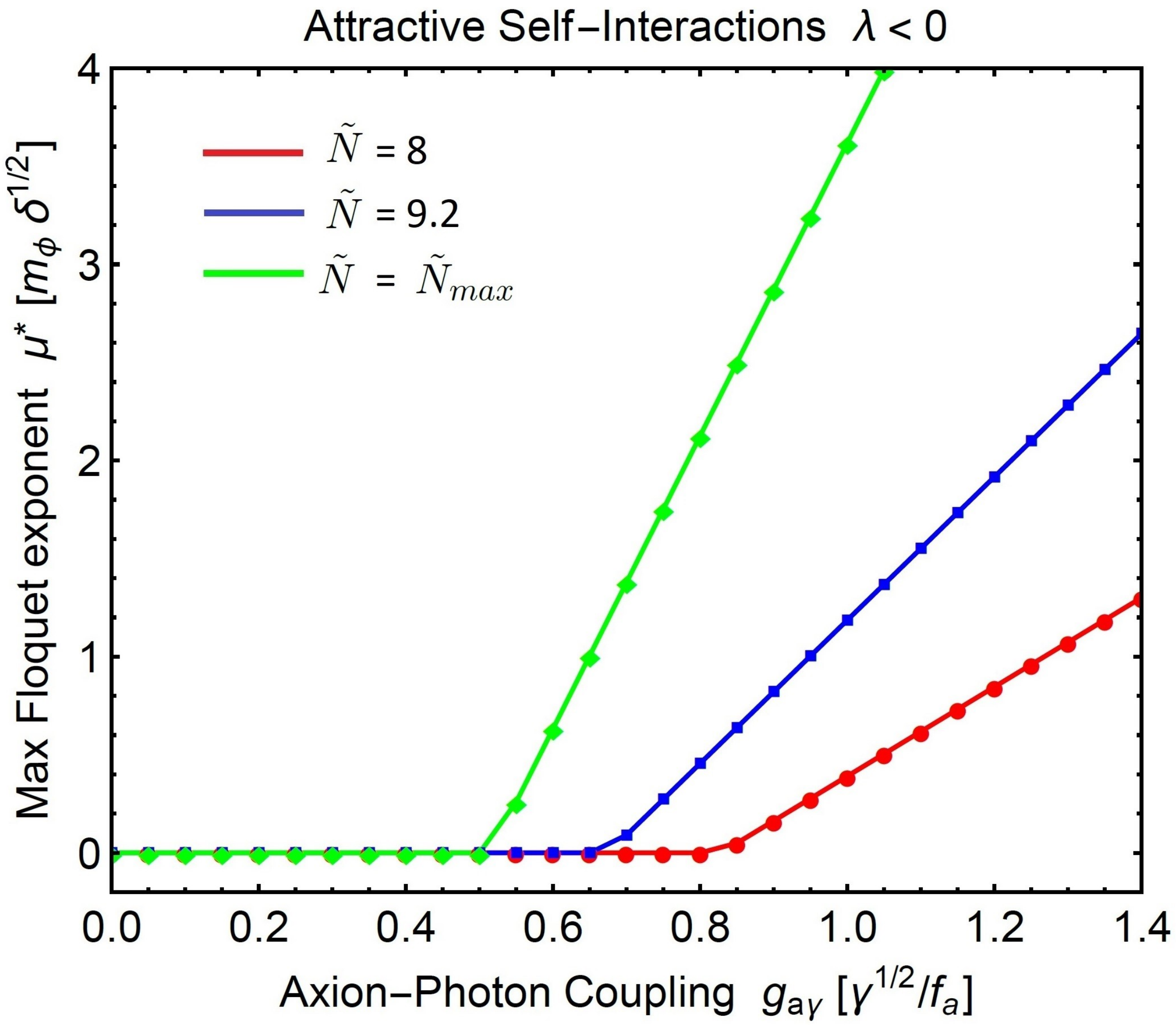
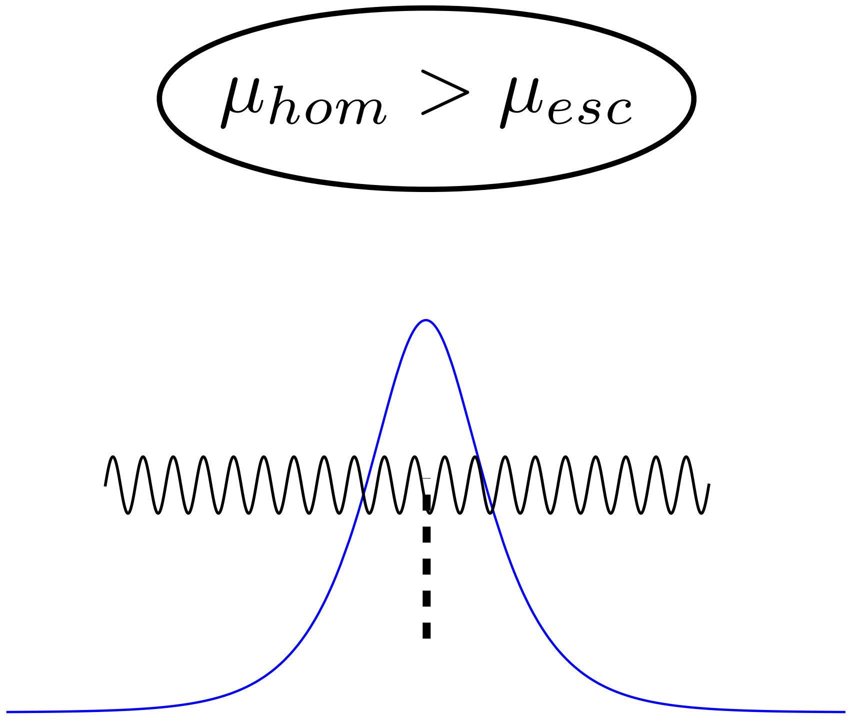




Hertzberg, Schiappacasse 1805.00430 Hertzberg 2010, Kawasaki, Yamada 2014

Resonance Condition (Spherical) Axion Clump

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a}}$$

$$(\lambda < 0)$$

No resonance for standard QCD axion-photon coupling $\quad g_{a\gamma} \sim \dfrac{\alpha}{f_a}$

Allowed for models with enhanced couplings, decay to hidden sectors, or repulsive interactions

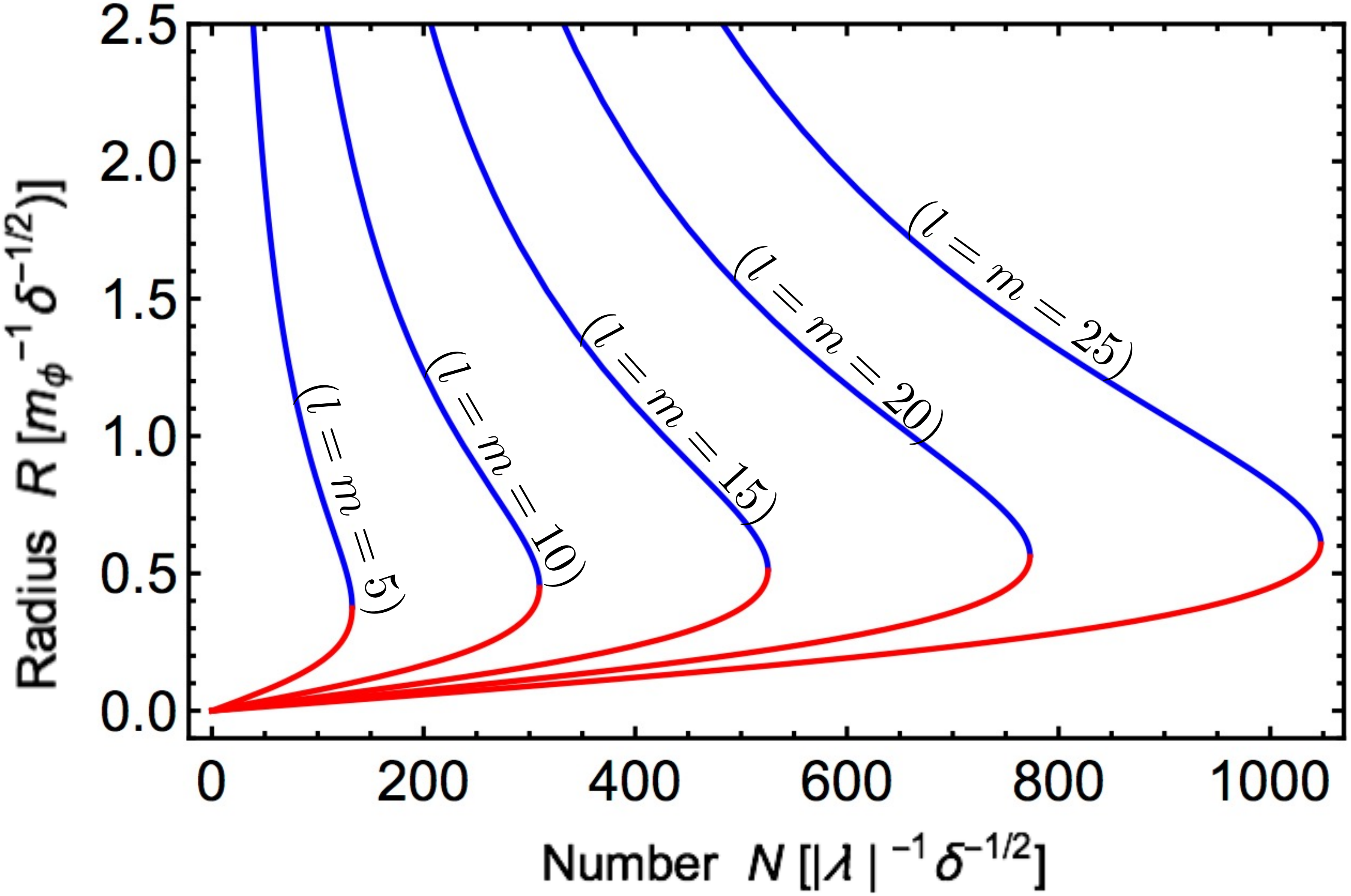
↓ (see; Fan 2016)

↓ (see; Daido, Takahashi, Yokozaki 2018)

Hertzberg, Schiappacasse 1805.00430

Including Angular Momentum

Two Branches of Solutions (with Angular Momentum)

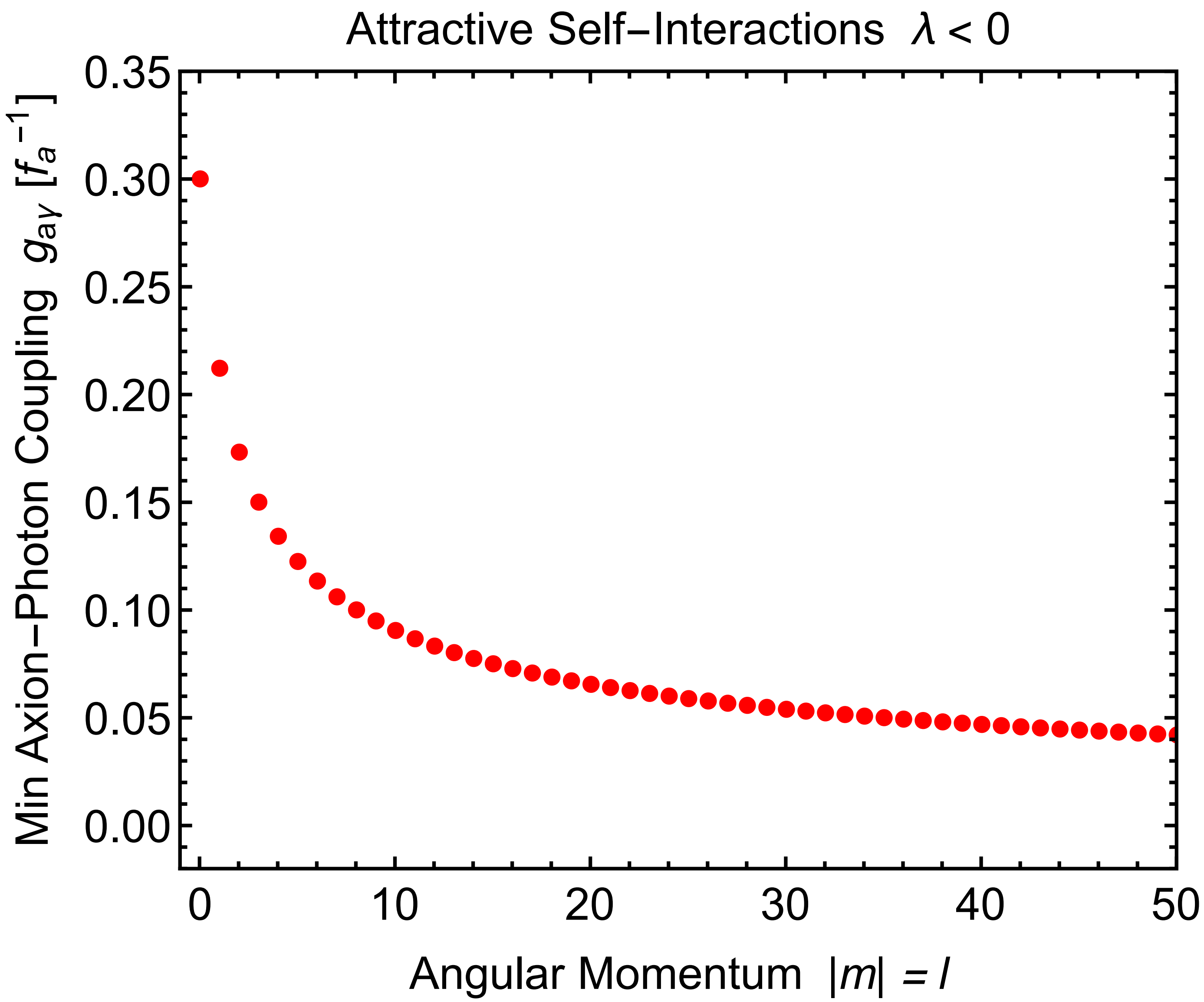


High angular momentum allows higher amplitude at core, which helps for resonance into photons

Hertzberg, Schiappacasse 1804.07255

Resonance Condition (Non-Spherical) Axion Clump

$$g_{a\gamma} > \frac{0.3}{f_a\sqrt{l+1}} \qquad (\lambda < 0)$$

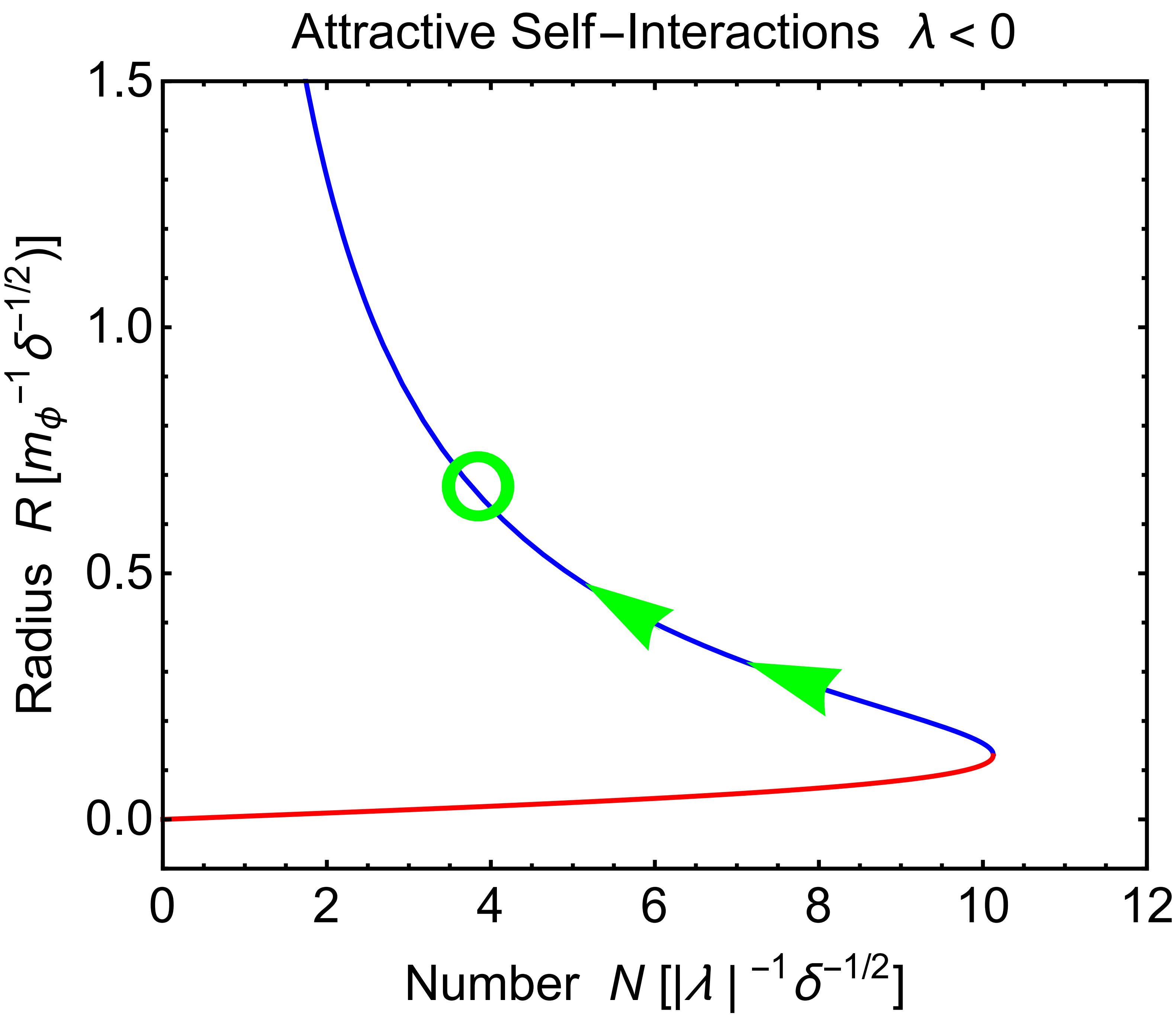


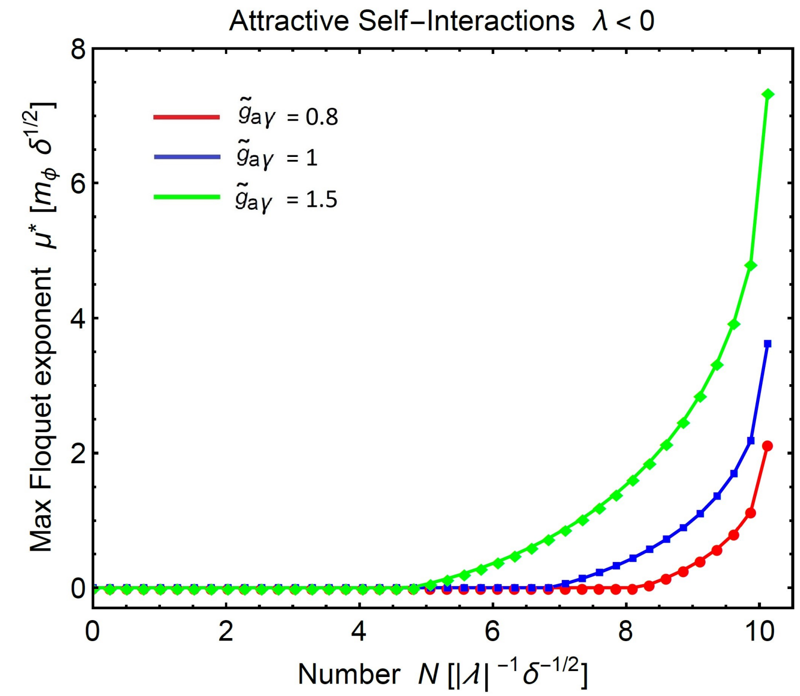
Resonance allowed for standard QCD axion-photon couplings, with high angular momentum

Hertzberg, Schiappacasse 1805.00430

Astrophysical Consequences

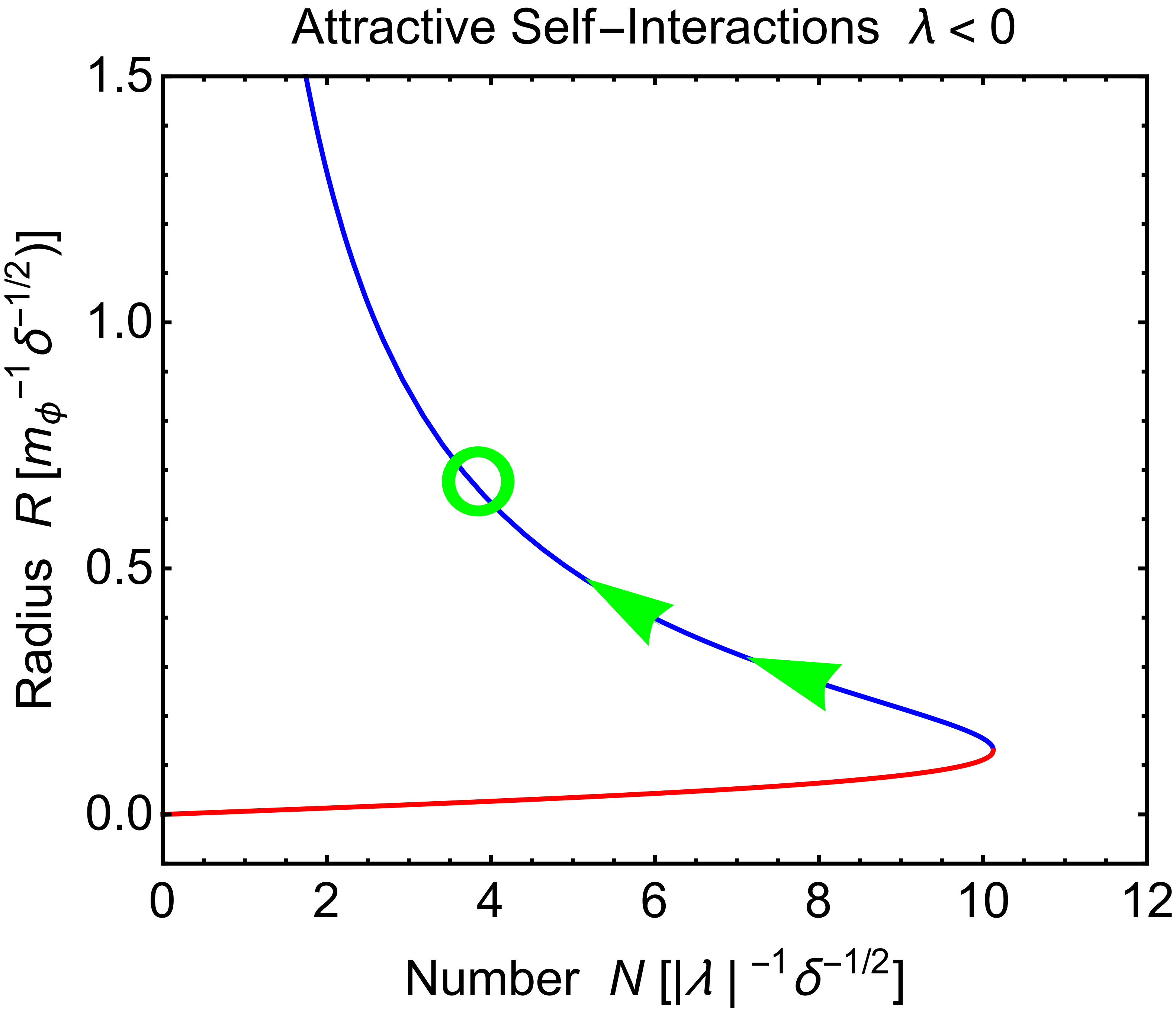
Energy Loss

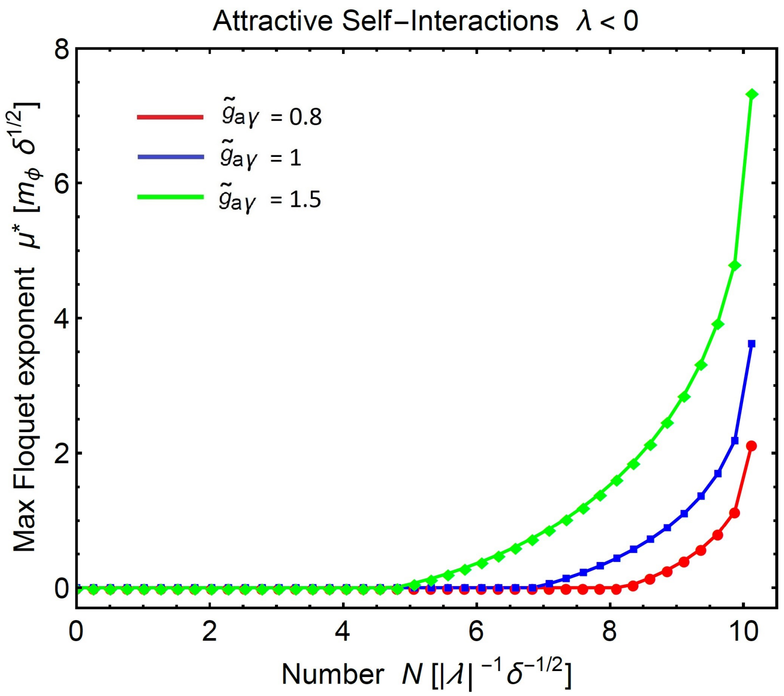




Hertzberg, Schiappacasse 1805.00430

Energy Loss

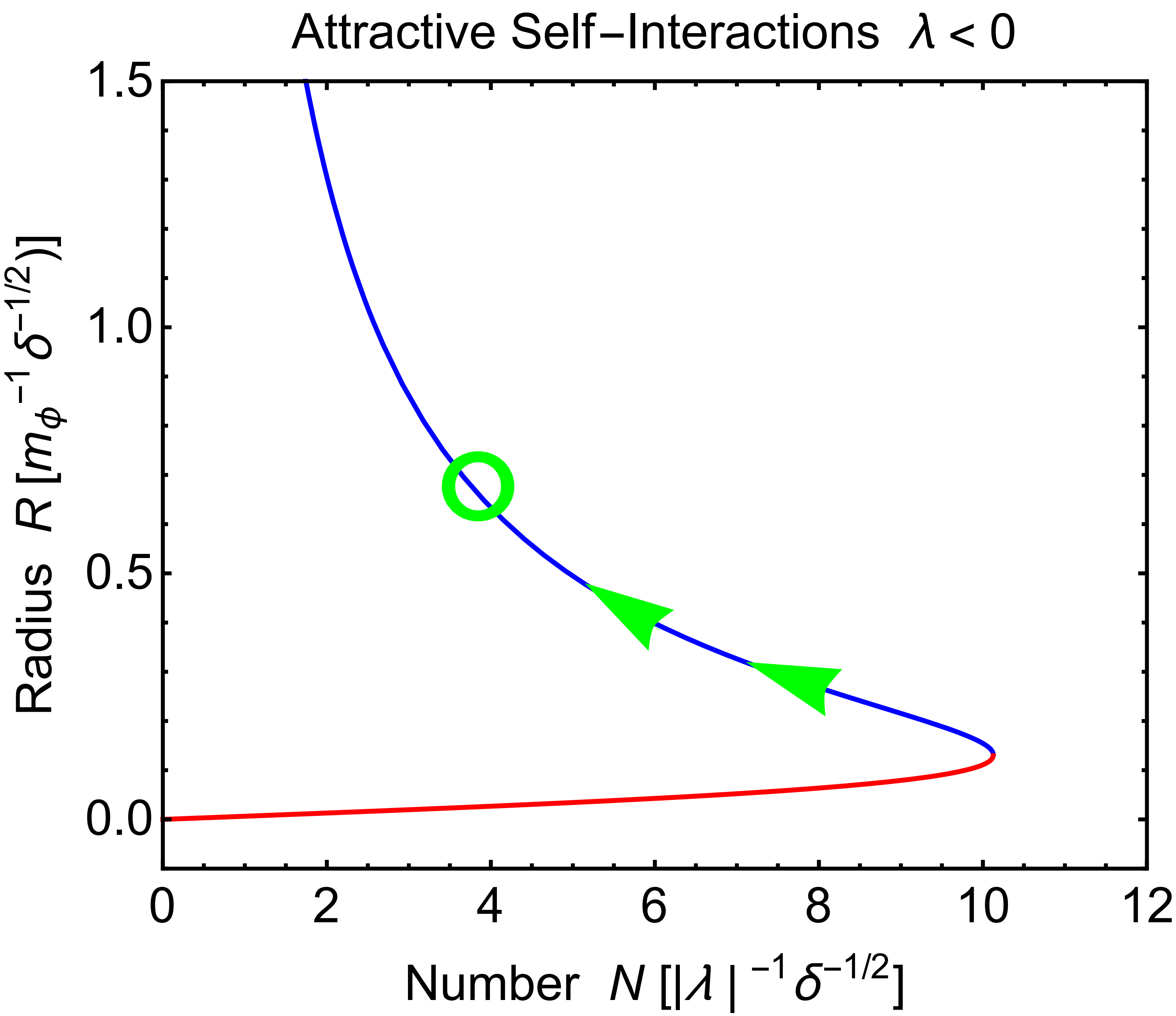


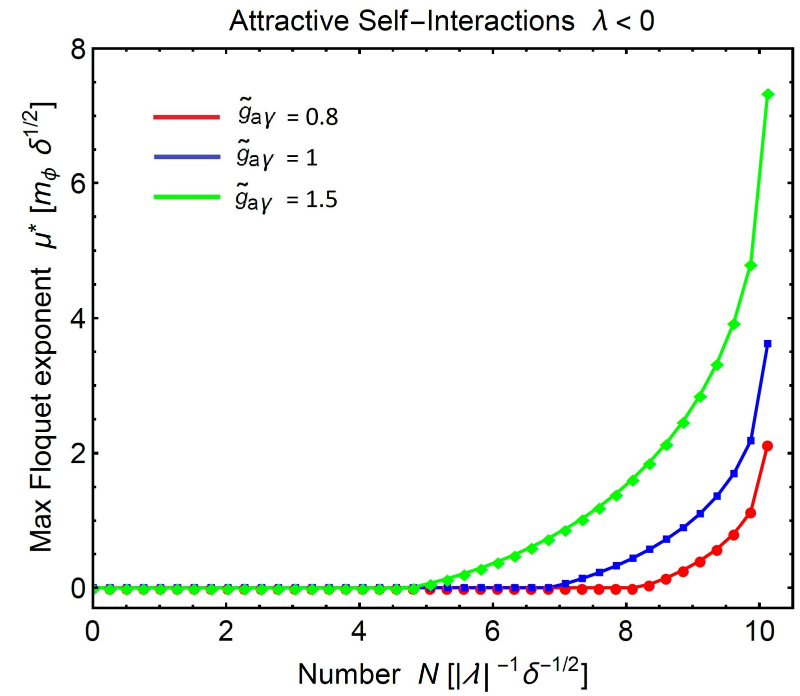


(i) Mass Pile Up

Hertzberg, Schiappacasse 1805.00430

Energy Loss





(i) Mass Pile Up

(Amin, Mocz 2019)

(ii) Late Time Mergers;
Radio-wave Bursts

$$\lambda_{EM} = \frac{2\pi}{k} \approx \frac{4\pi}{m_a}$$
$$= \mathcal{O}(1)\,meters$$

Hertzberg, Schiappacasse 1805.00430

(also Tkachev 2015)

Thank you