Fitting resonances in the early universe:
parametric resonance and oscillons

Francisco Torrentí
University of Basel (Switzerland)

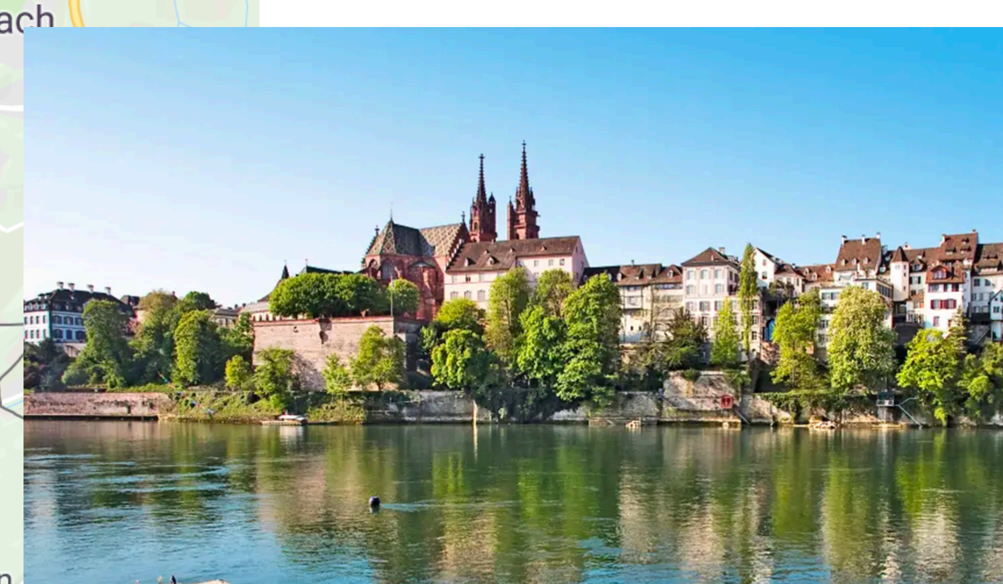
Resonant Instabilities in cosmology and their observational consequences

YITP, Kyoto, 17th May 2019

University of Basel

Where am I?

Three Country Corner
Lörrach
Vitra Design Museum
Village-Neuf
Blotzheim
Aeropuerto de
Basilea-Mulhouse-Friburgo
23 May 6:10
Huningue
Weil am Rhein
Saint-Louis
Riehen
Hésingue
Bettingen
Hégenheim
Buschwiller
Basilea
Rin
Allschwil
Zoo Basel
Birsfelden
Grenzach-Wyhlen
GUNDELDINGEN
Schönenbuch
Binningen
Deutschland
Schweiz
BRUDERHOLZ
Muttenz
Neuwiller
France
Bottmingen
Münchenstein
Pratteln
Oberwil
Schweiz
Biel-Benken
D66
D201
D105
D419
D473
D12B
D23
317
98
34



CONTENTS

1. **Fitting parametric resonance**

   JCAP 1702 (2017) 001 (with D. Figueroa)

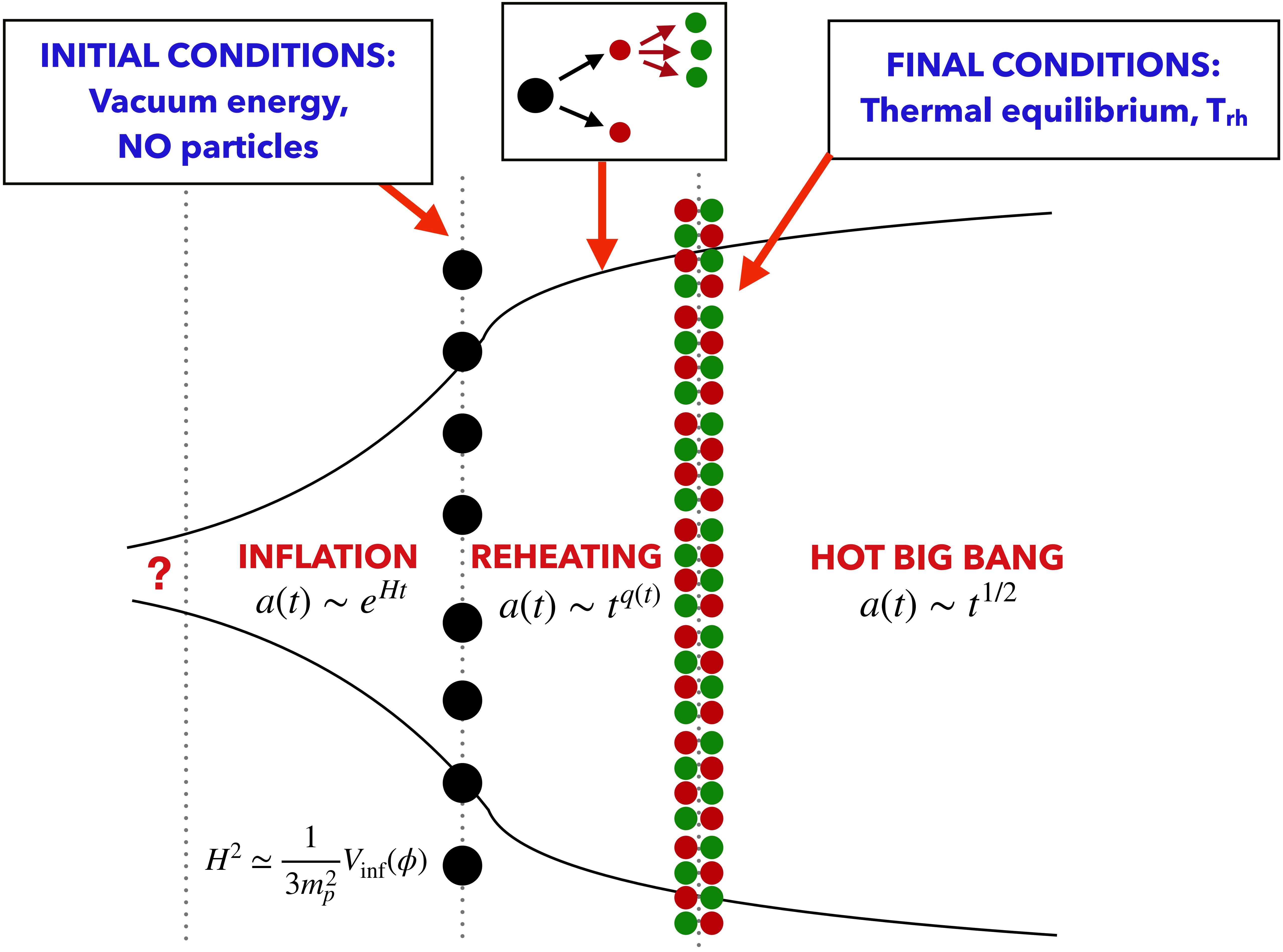
2. **Fitting GWs from parametric resonance**

   JCAP 1710 (2017) 057 (with D. Figueroa)

3. **Lifetime of oscillons in hilltop potentials**

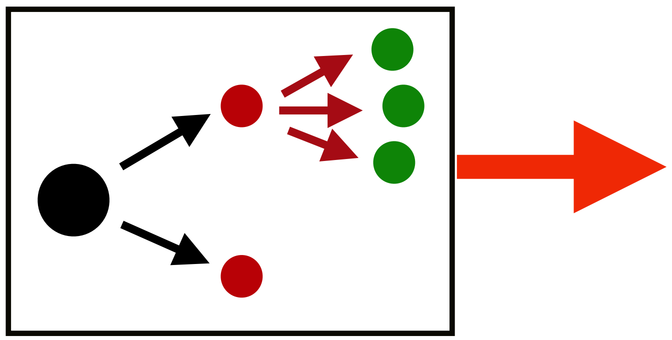
   In preparation (with S. Antusch, F. Cefala)

CONTENTS



INITIAL CONDITIONS:
Vacuum energy,
NO particles
FINAL CONDITIONS:
Thermal equilibrium, T_{rh}
?
INFLATION
$a(t) \sim e^{Ht}$
REHEATING
$a(t) \sim t^{q(t)}$
HOT BIG BANG
$a(t) \sim t^{1/2}$
$H^2 \simeq \frac{1}{3m_p^2} V_{\rm inf}(\phi)$

Introduction



$$\mathscr{L} = \mathscr{L}(\phi, \varphi_i, \psi_j, A_\mu, h_{\mu\nu}, \ldots)\textbf{??}$$

- **Poor understanding of reheating: details depend on high-energy physics model.**
- **Non-linear, non-perturbative, out-of-equilibrium physics.**
- **First stage is normally PREHEATING: an explosive production of particles due to non-perturbative effects.**
- **Resonant effects (e.g. parametric resonance, self-resonance, tachyonic resonance, flipping resonance…)**

1. Fitting parametric resonance

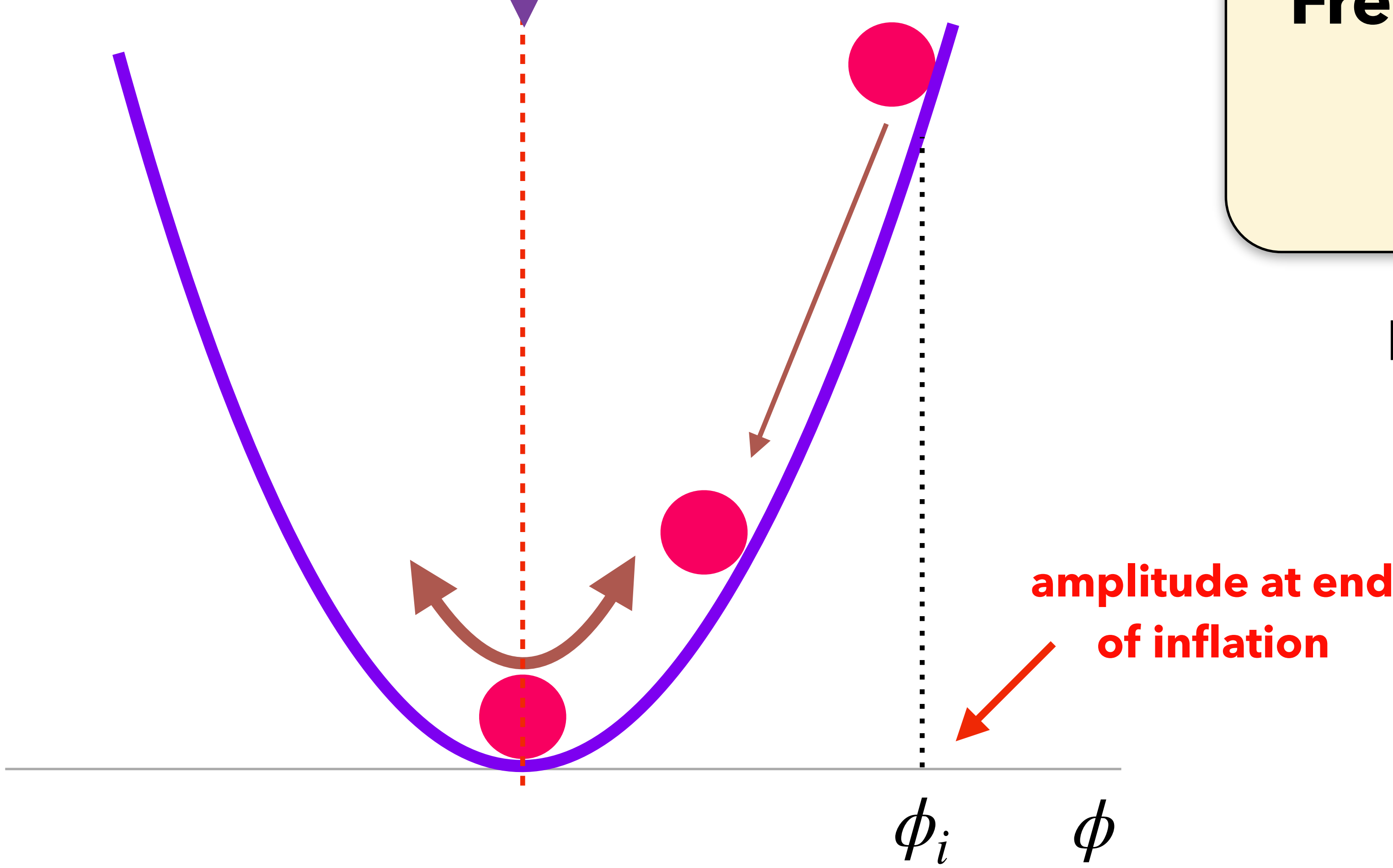
$$V_{\rm inf}(\phi) = \frac{1}{n}\lambda M^{4-n}\phi^n$$

$n = 2,4,6\ldots$

M : mass scale

λ : dimensionless parameter

$$\ddot\phi + 3H\dot\phi + \lambda M^{4-n}\phi^{n-1} \simeq 0$$



Frequency of oscillation:

$$\omega_*^2 \simeq \lambda M^{4-n}\phi_i^{n-2}$$

Natural time scale: ω_*^{-1}

1. Fitting parametric resonance

PARAMETRIC RESONANCE after inflation:
power-law potential + quadratic interaction term $g^2\varphi^2X^2$

- **Two scalar fields** $\begin{cases} \phi & \text{mother field (inflaton)} \\ X & \text{daughter field} \end{cases}$

- **Action:**

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu X\partial^\mu X + \frac{1}{2}g^2\phi^2X^2 + V_{\rm inf}(\phi)\right\}$$

- **Equations of motion:**

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} + g^2X^2\phi + \lambda M^{4-n}\phi^{n-1} = 0$$

$$\ddot{X} - \frac{1}{a^2}\nabla^2 X + 3H\dot{X} + g^2\phi^2 X = 0$$

1. Fitting parametric resonance

DAUGHTER EOM:

$$\ddot{X} - \frac{1}{a^2}\nabla^2 X + 3H\dot{X} + g^2\phi^2 X = 0$$

$\chi = a(t)X$

$\varphi = a(t)(\phi/\phi_i)$

$$\chi''_{\mathbf{k}} + \left(k^2 + q\varphi^2(t)\right)\chi_{\mathbf{k}} \simeq 0$$

$\omega^2_{\rm eff}(t)$

$$\frac{\omega'_{\rm eff}}{\omega^2_{\rm eff}} \gg 1$$

Floquet index

$$n_{\mathbf{k}} \sim |X_{\mathbf{k}}|^2 \sim e^{2\mu_\kappa(q,a)t}$$

Resonance parameter:

$$q \equiv \frac{g^2}{\lambda}\left(\frac{\phi_{\rm i}}{2M}\right)^{4-n}$$

depends on interaction, potential, and initial amplitude

For some values of (k,q,a), Re[μ_k]>0, and there is PARTICLE CREATION

Kofman et al (1994, 1997)

1. Fitting parametric resonance

- **Previously on (p)reheating…:**
  - **Analytical calculations:** for wide ranges of q, but valid only at initial times (linear regime)

    Kofman et al (1994, 1997), Greene et al (1997), …

  - **Lattice simulations:** valid at later times, but only for very specific q.

    Khlebnikov & Tkachev (1996), Prokopec & Ross (1996), …

- **Figueroa and F.T. (2017):**
  - With classical lattice simulations, we parametrize the dynamics of parametric resonance from the initial resonance until the later stationary regime.
  - **Power-law potentials:** $V(\phi) = \begin{cases} \frac{1}{4}\lambda\phi^4 \\ \frac{1}{2}m^2\phi^2 \end{cases}$

1. Fitting parametric resonance

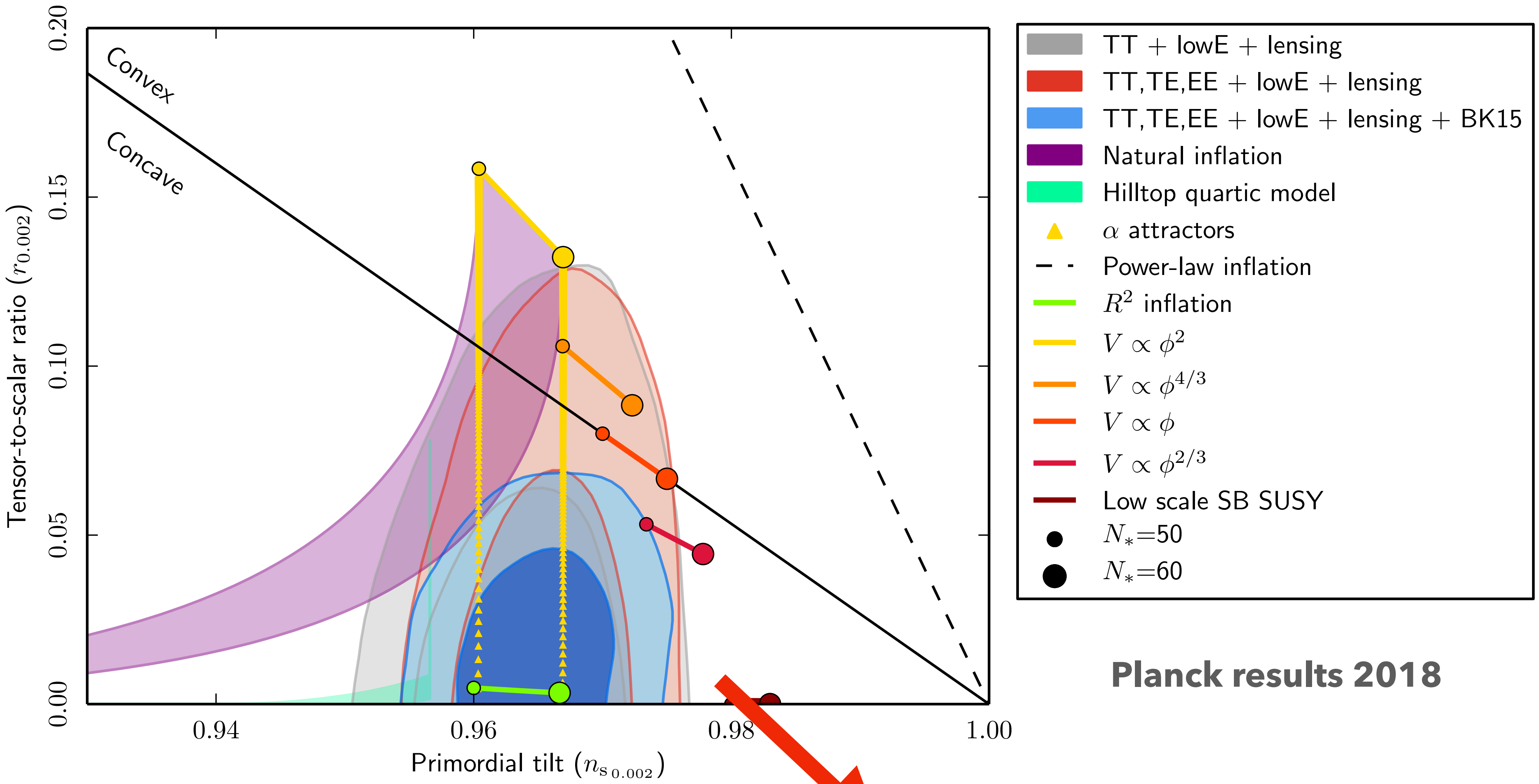
- **Related questions:**

  - **Is energy efficiently transferred from the inflationary sector to preheated species?**

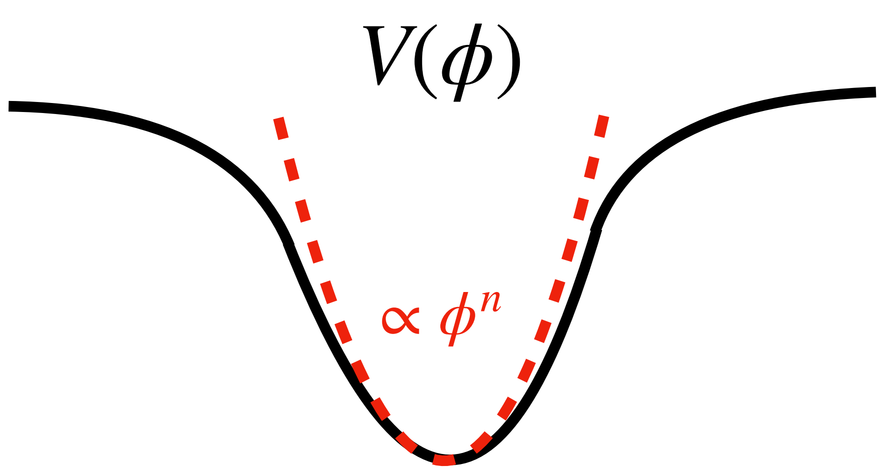
  - **Do we need perturbative decay channels?**

  - **Equation-of-state evolution? (MD → RD → MD). Effect on inflationary constraints?** (see Kaloian talk)

1. Fitting parametric resonance



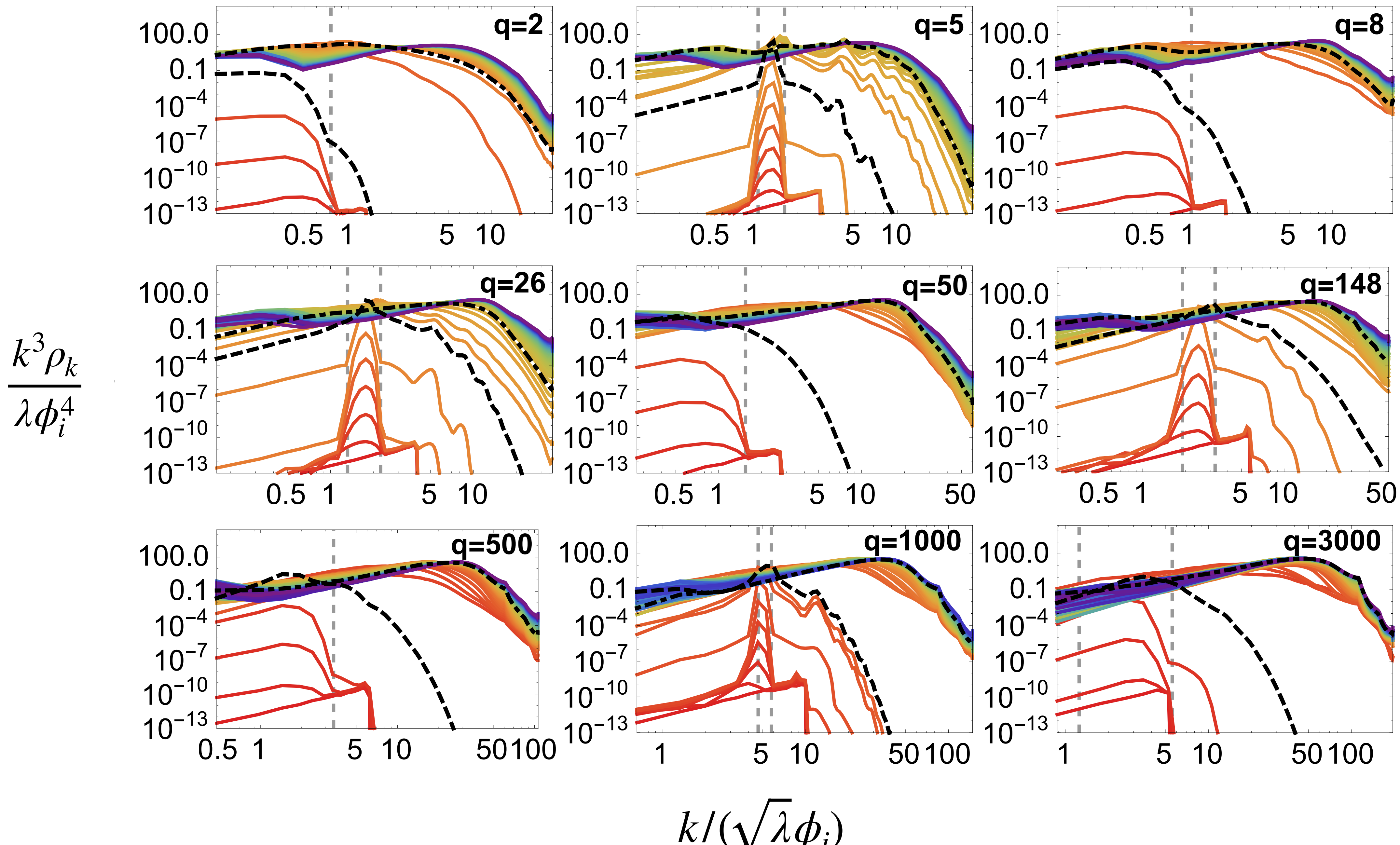
Planck results 2018



1.1. Parametric resonance in λφ⁴

$$q = \frac{g^2}{\lambda}$$

DAUGHTER FIELD SPECTRA



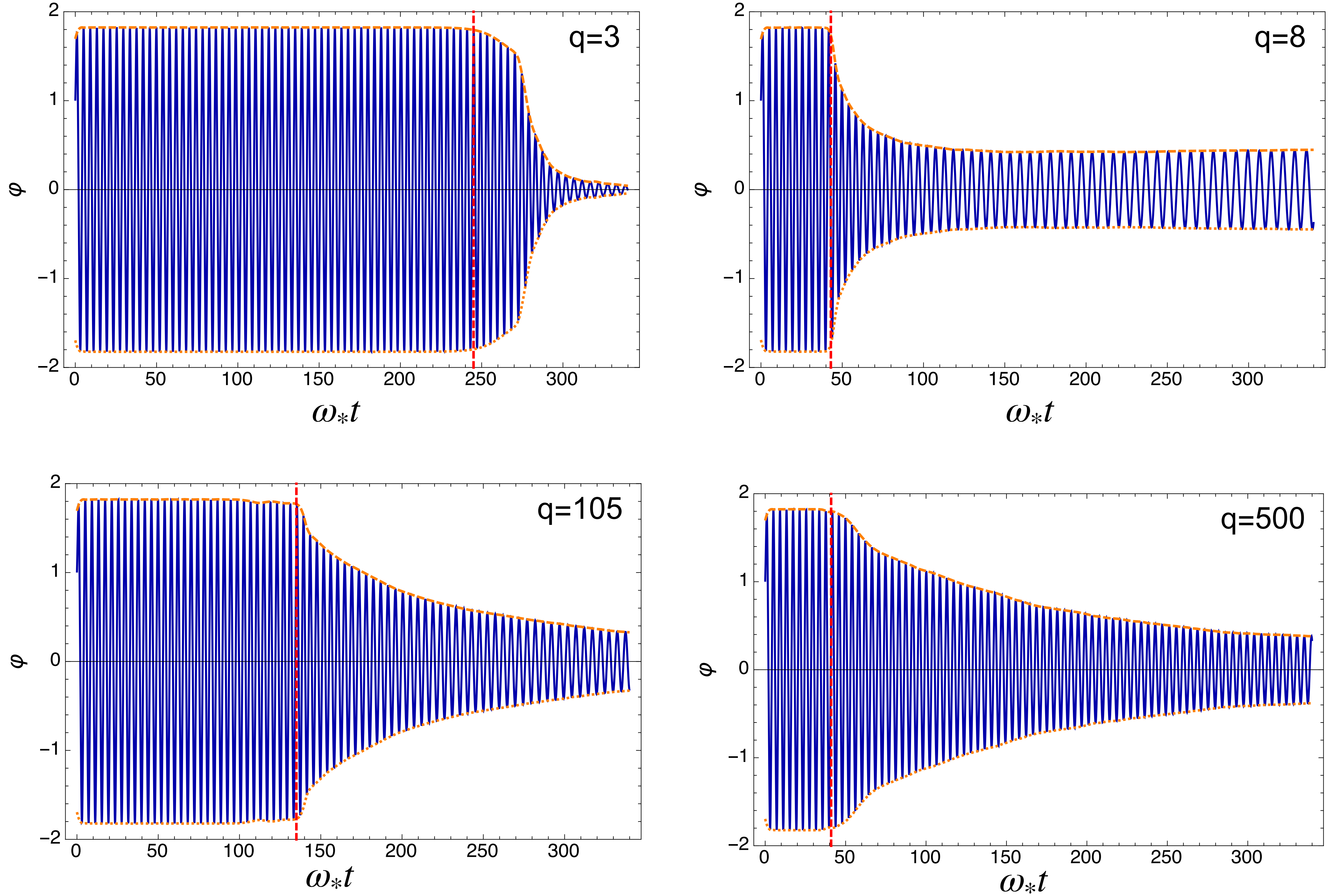
$\omega_* t = 0, 10, 20, \ldots 670, 680, 690$

1.1. Parametric resonance in λφ⁴

INFLATON (CONFORMAL) AMPLITUDE

$$q = \frac{g^2}{\lambda}$$

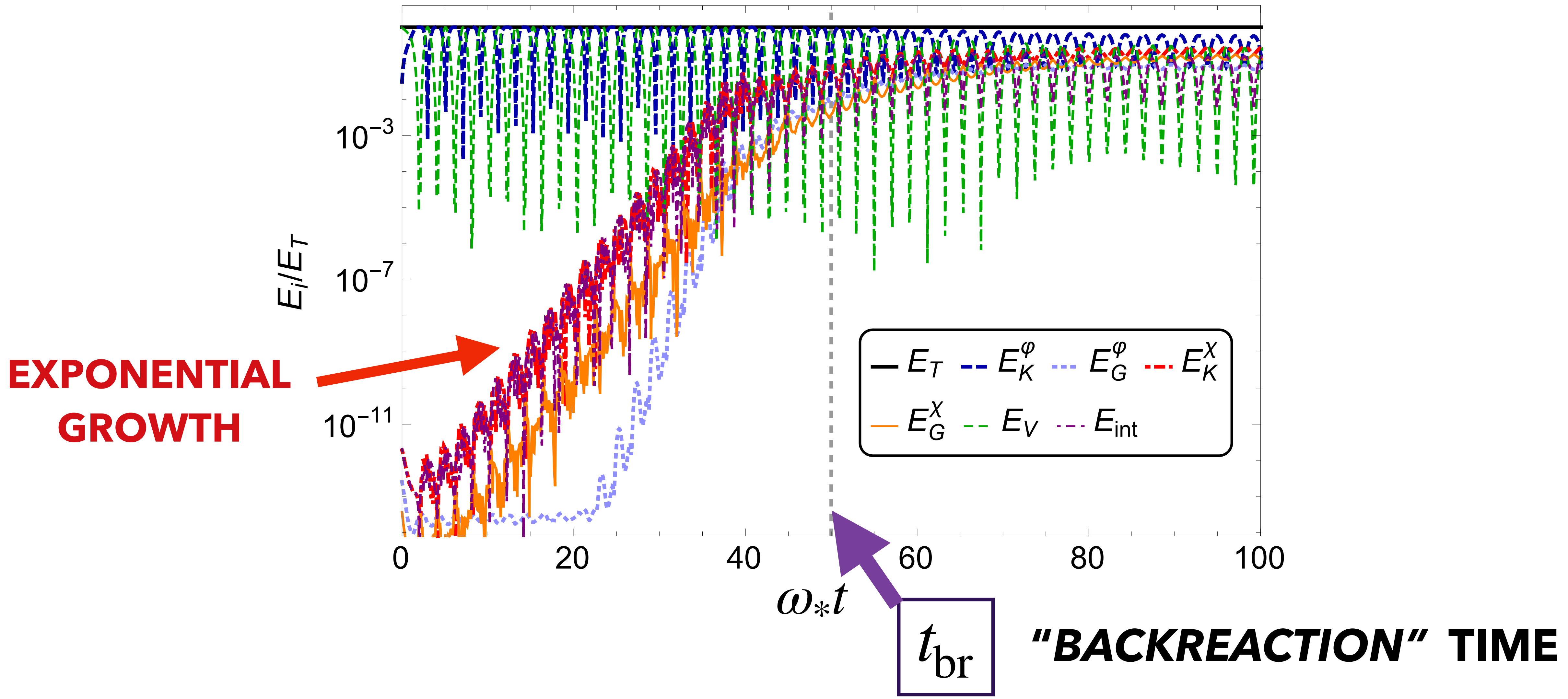
$$\omega_* = \sqrt{\lambda}\phi_i$$



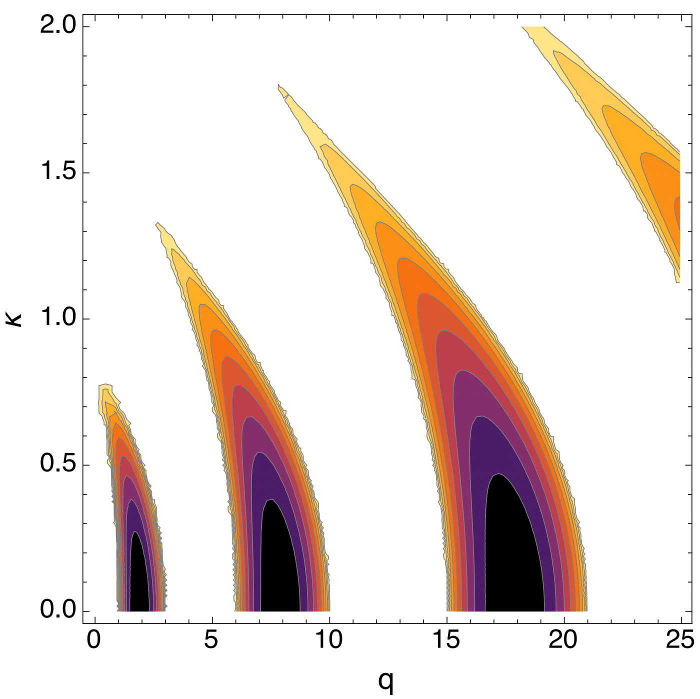
1.1. Parametric resonance in λφ⁴

$$\rho_t(z) \equiv \frac{\lambda\phi_{\rm i}^4}{a^4}E_t \equiv \frac{\lambda\phi_{\rm i}^4}{a^4}\left(E_{K,\varphi} + E_{\rm V} + E_{G,\varphi} + E_{K,\chi} + E_{G,\chi} + E_{\rm int}\right)$$

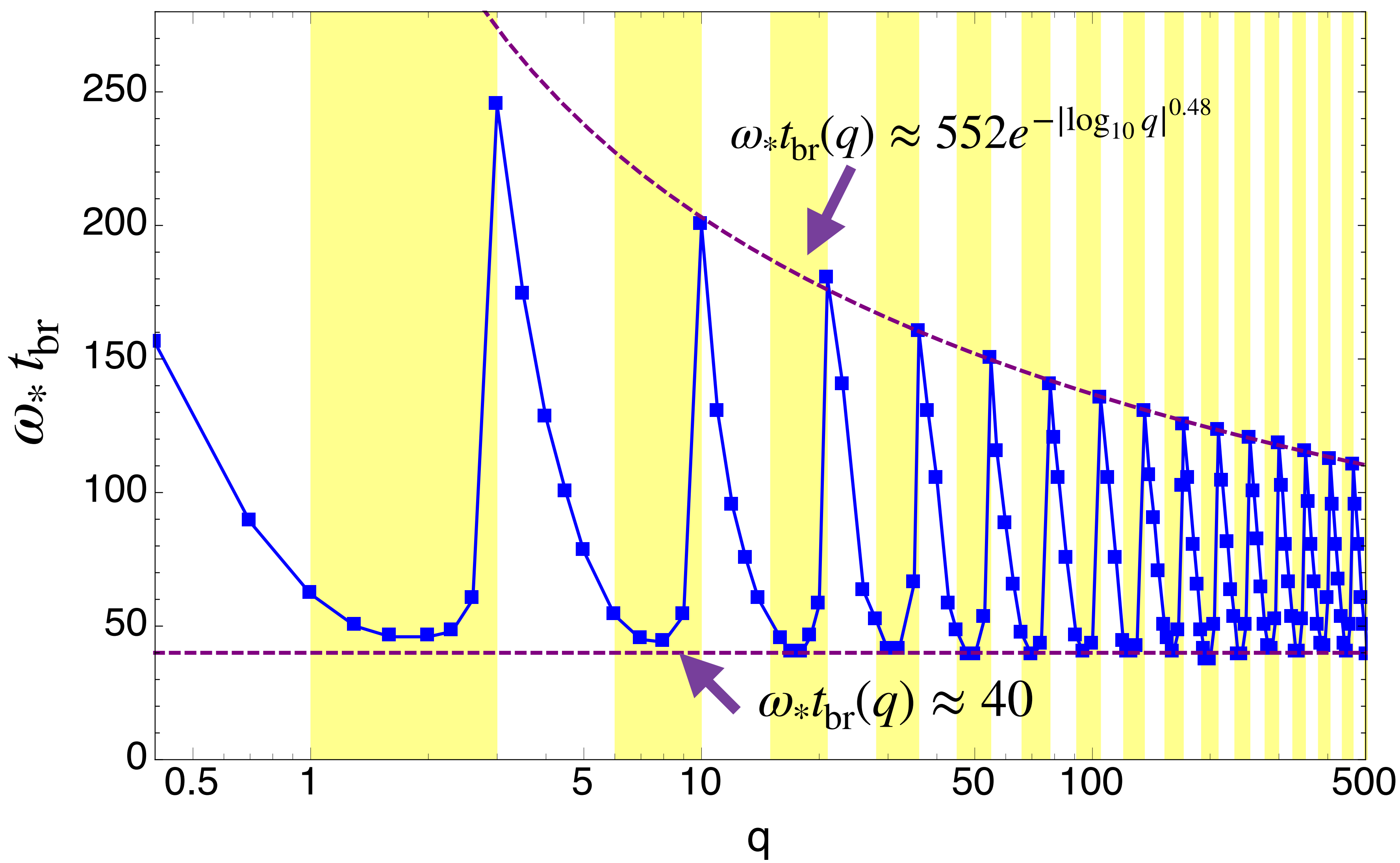
Kinetic inflaton | Potential inflaton | Gradient inflaton | Kinetic daughter | Gradient daughter | Interaction energy



1.1. Parametric resonance in $\lambda\phi^4$



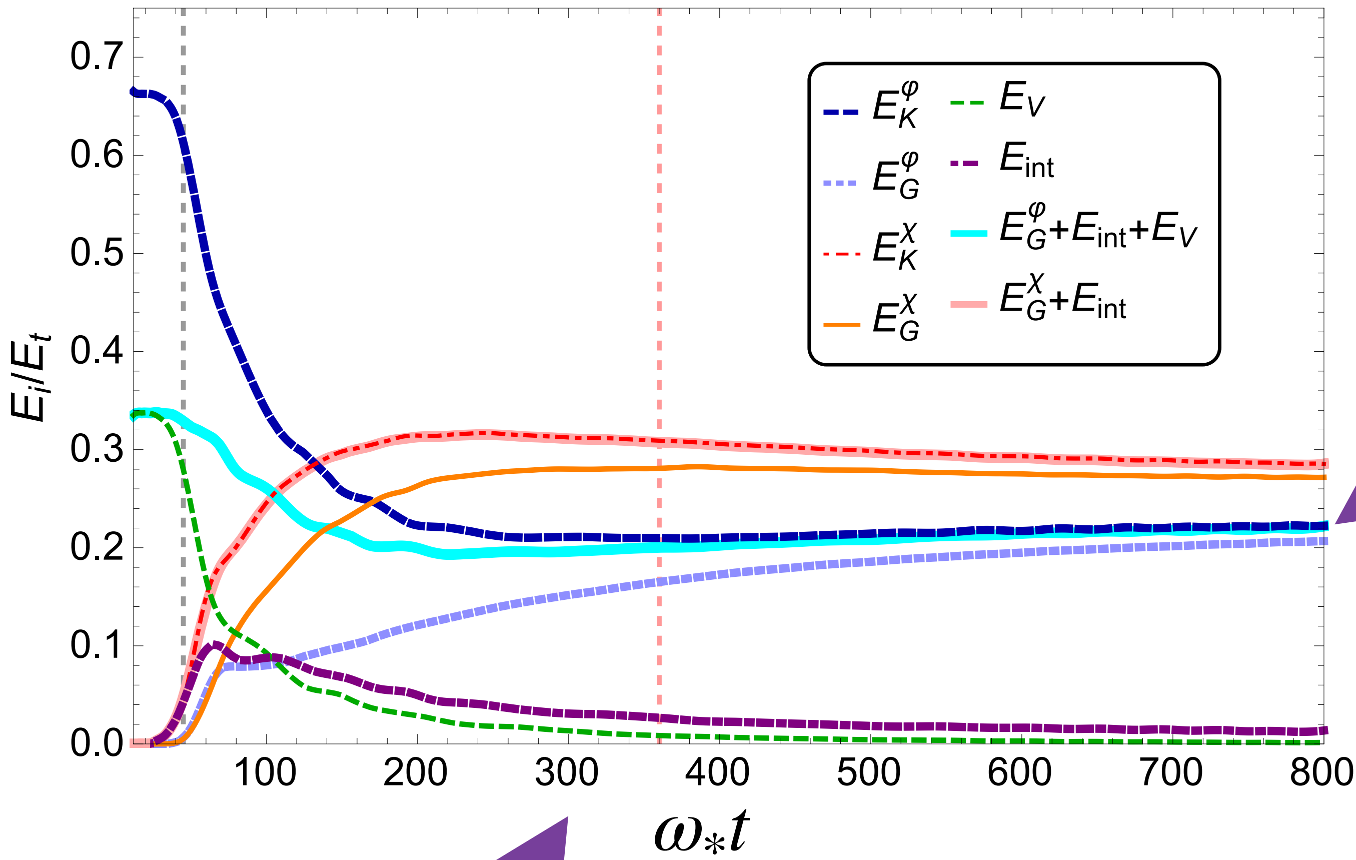
BACKREACTION TIME t_{br}



1.1. Parametric resonance in λφ⁴

$$\rho_t(z) \equiv \frac{\lambda \phi_i^4}{a^4} E_t \equiv \frac{\lambda \phi_i^4}{a^4}\left(E_{K,\varphi} + E_V + E_{G,\varphi} + E_{K,\chi} + E_{G,\chi} + E_{\rm int} \right)$$

Kinetic inflaton — Potential inflaton — Gradient inflaton — Kinetic daughter — Gradient daughter — Interaction energy

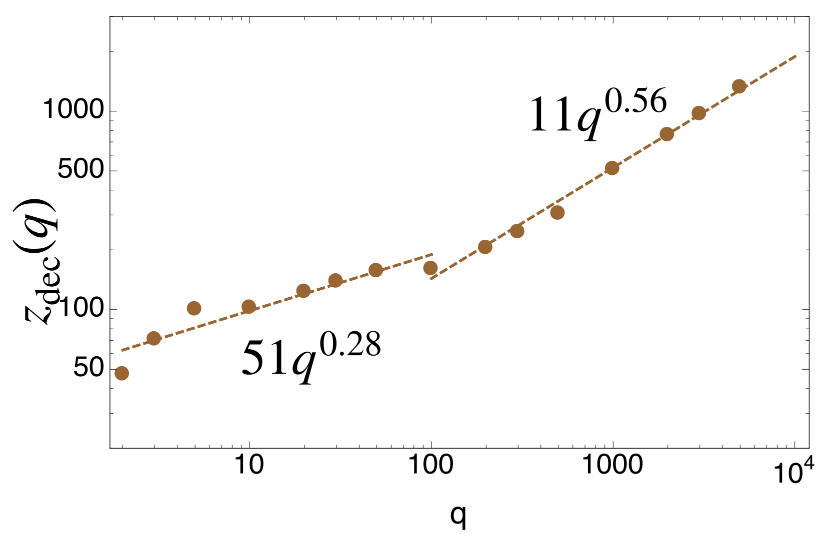


EQUIPARTITION at late times:

$$\langle E_{K,\chi}\rangle_{\rm osc} \simeq \langle E_{G,\chi} + E_{\rm int}\rangle_{\rm osc}$$

$$\langle E_{K,\varphi}\rangle_{\rm osc} \simeq \langle E_{G,\varphi} + E_{\rm int} + E_V\rangle_{\rm osc}$$

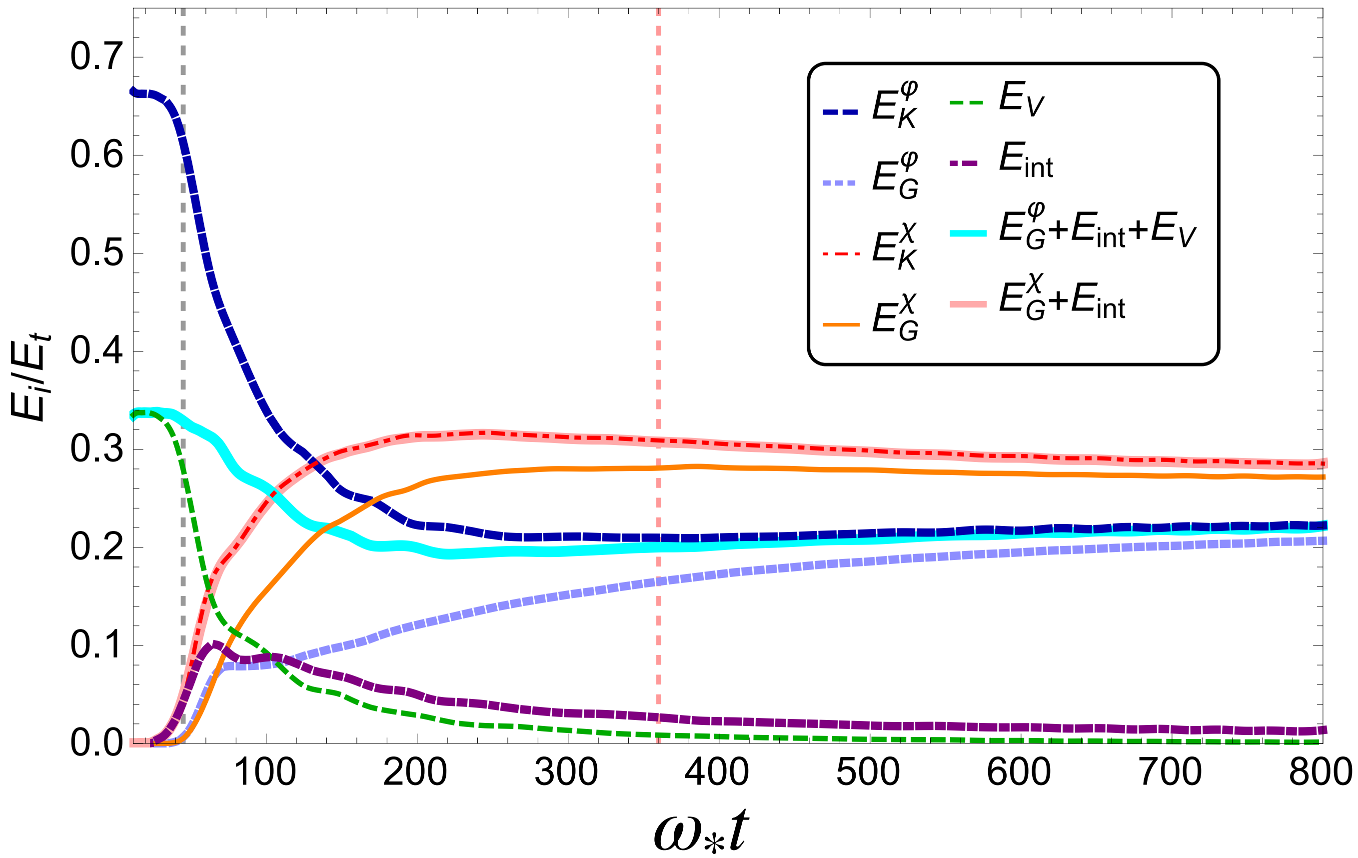
$t_{\rm dec}$ **"DECAY" TIME**



1.1. Parametric resonance in λφ⁴

$$\rho_t(z) \equiv \frac{\lambda \phi_i^4}{a^4} E_t \equiv \frac{\lambda \phi_i^4}{a^4} \left(E_{K,\varphi} + E_V + E_{G,\varphi} + E_{K,\chi} + E_{G,\chi} + E_{int} \right)$$

Kinetic inflaton, Potential inflaton, Gradient inflaton, Kinetic daughter, Gradient daughter, Interaction energy



$$\frac{E_{K,\chi}}{E_t} \simeq (29.5 \pm 3.3)\,\%$$

$$\frac{E_{G,\chi}}{E_t} \simeq (26.2 \pm 3.4)\,\%$$

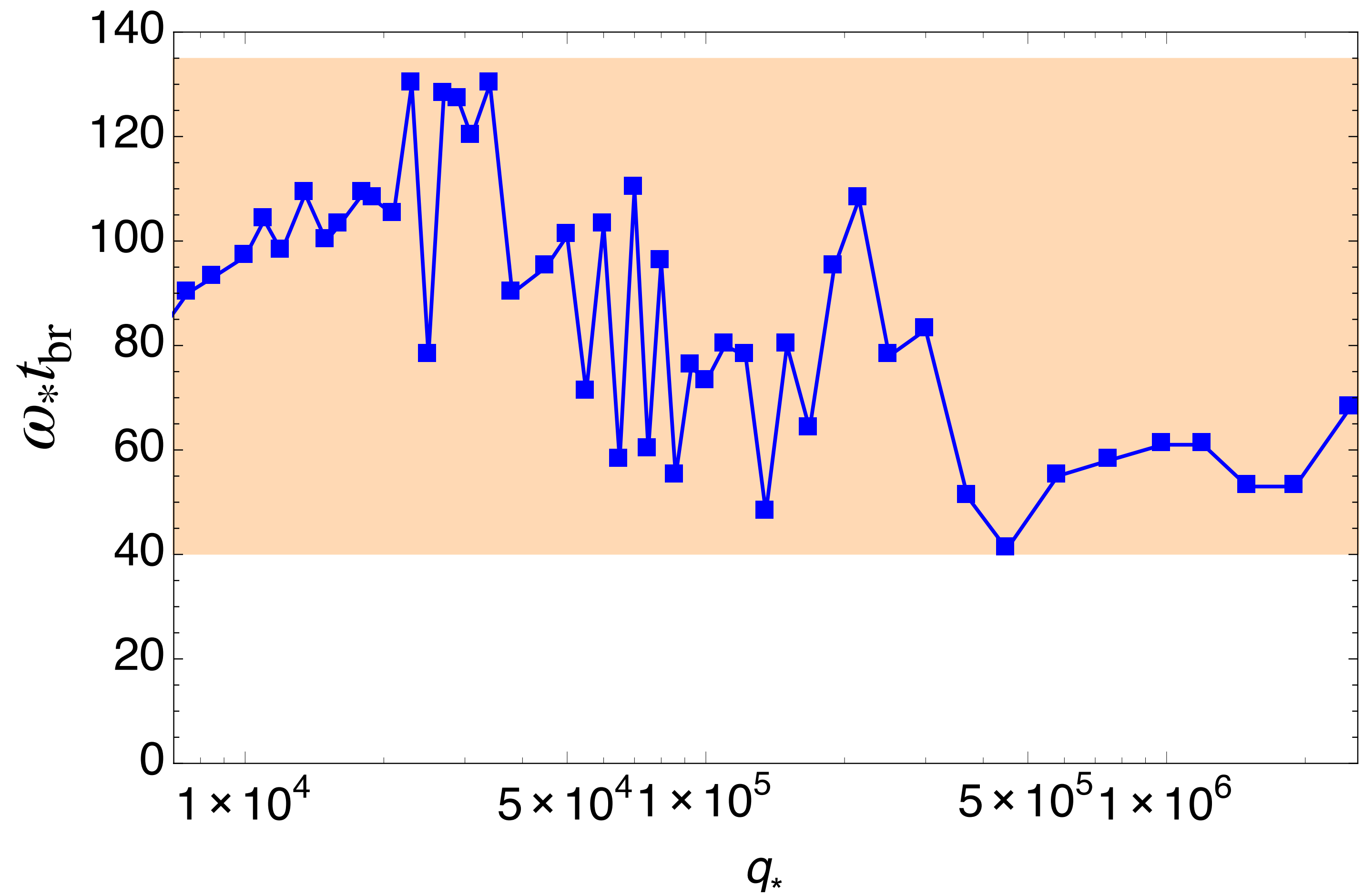
$$\frac{E_{K,\varphi}}{E_t} \simeq (22.6 \pm 3.4)\,\%$$

$$\frac{E_{G,\varphi}}{E_t} \simeq (17.7 \pm 3.0)\,\%$$

Approximately 40% of the energy remains on the inflaton. This result is independent on q

1.2. Parametric resonance in m²φ²

BACKREACTION TIME t_{br}



$$q = \frac{g^2\phi_i^2}{4m^2}$$

$$\chi_k'' + [A_k(z) - 2q_{\rm eff}(z)\ \cos 2z]\chi_k = 0$$

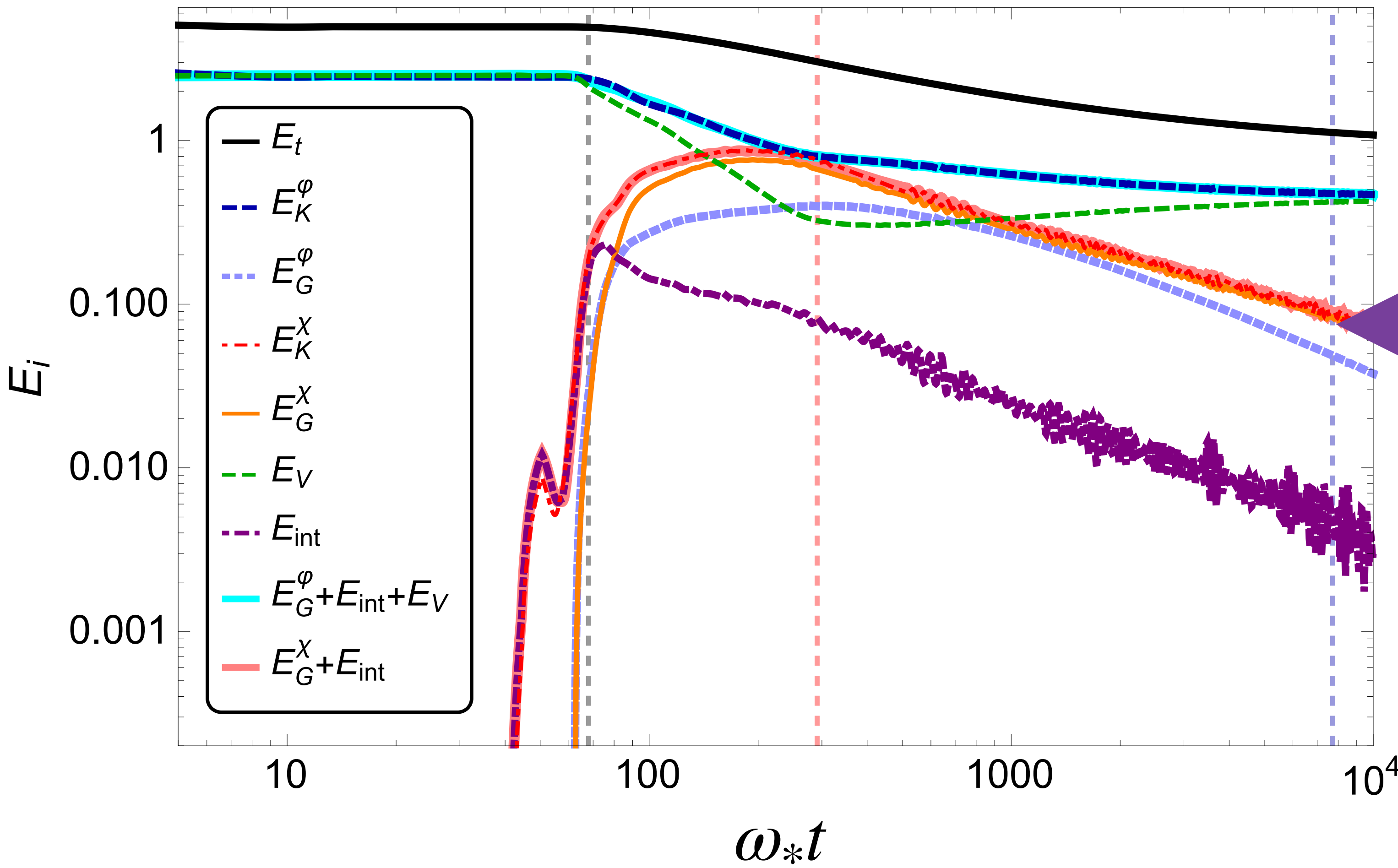
$$q_{\rm eff}(z) = \frac{q}{a^3}$$

Due to the expansion of the Universe, a given mode redshifts through many resonance bands → **stochastic dependence**

1.2. Parametric resonance in $m^2\varphi^2$

$$\rho_t(z) \equiv \frac{m^2\phi_i^2}{a^3} E_t \equiv \frac{m^2\phi_i^2}{a^3}\left(E_{K,\varphi} + E_V + E_{G,\varphi} + E_{K,\chi} + E_{G,\chi} + E_{int}\right)$$

Kinetic inflaton ($E_{K,\varphi}$), **Potential inflaton** (E_V), **Gradient inflaton** ($E_{G,\varphi}$), **Kinetic daughter** ($E_{K,\chi}$), **Gradient daughter** ($E_{G,\chi}$), **Interaction energy** (E_{int})



EQUIPARTITION at all times:

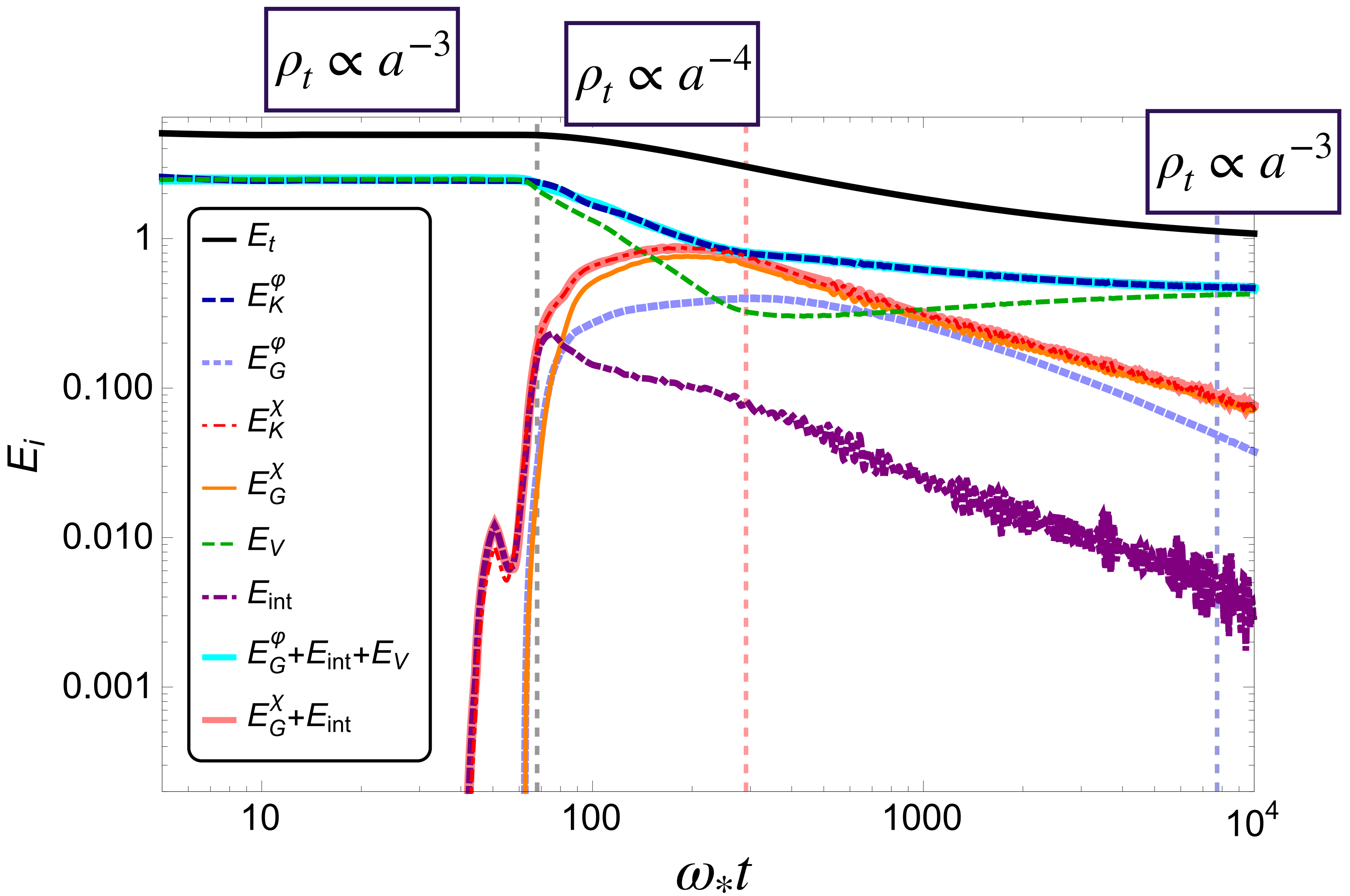
$$\langle E_{K,\chi}\rangle_{osc} \simeq \langle E_{G,\chi} + E_{int}\rangle_{osc}$$

$$\langle E_{K,\varphi}\rangle_{osc} \simeq \langle E_{G,\varphi} + E_{int} + E_V\rangle_{osc}$$

1.2. Parametric resonance in $m^2\varphi^2$

$$\rho_t(z) \equiv \frac{m^2\phi_i^2}{a^3}E_t \equiv \frac{m^2\phi_i^2}{a^3}\left(E_{K,\varphi} + E_V + E_{G,\varphi} + E_{K,\chi} + E_{G,\chi} + E_{int}\right)$$

Kinetic inflaton — Potential inflaton — Gradient inflaton — Kinetic daughter — Gradient daughter — Interaction energy

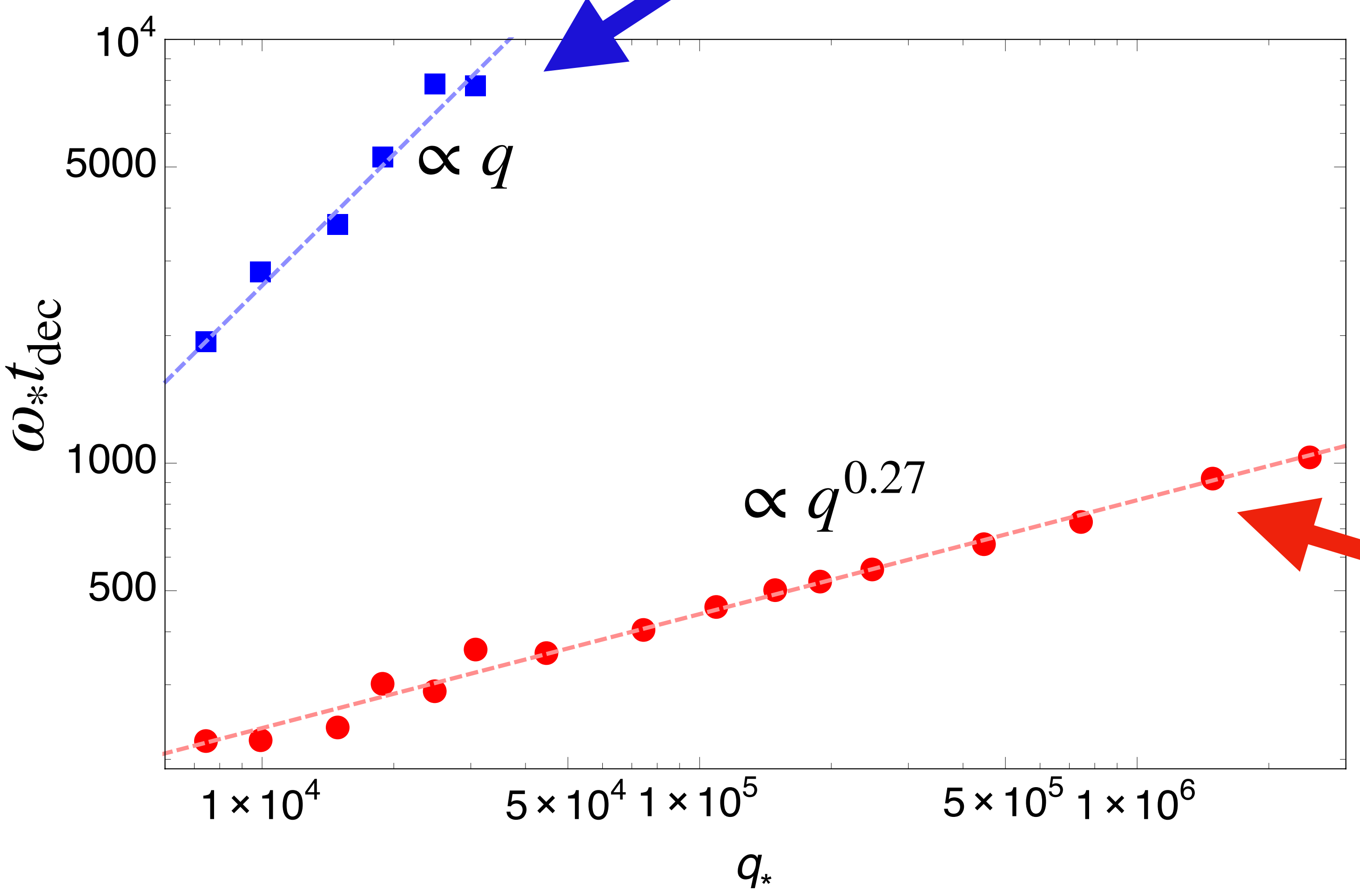


Equation of state:

MD → **RD** → **MD**

1.2. Parametric resonance in $m^2\varphi^2$

80% of energy recovered by the inflaton



~50% of energy remains in the inflaton

The inflaton slowly recovers the energy transferred to the daughter field (the stronger the interaction, the slower the recovery)

CONTENTS

1. Fitting parametric resonance

   JCAP 1702 (2017) 001 (with D. Figueroa)

2. Fitting GWs from parametric resonance

   JCAP 1710 (2017) 057 (with D. Figueroa)

3. Lifetime of oscillons in hilltop potentials

   In preparation (with S. Antusch, F. Cefala)

2. Fitting GWs from (p)reheating

➤ Gravitational waves are spatial perturbations of the FLRW metric:

$$ds^2 = a^2(\tau)\left(-d\tau^2 + \delta_{ij} + h_{ij}\right) dx^i dx^j$$

↓

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} - \nabla^2 h_{ij} = \frac{2}{m_p^2}\Pi_{ij}^{\rm TT}$$

➤ Gradients of all field species contribute to GWs:

$$\Pi_{ij} = T_{ij} - p g_{ij}$$

$$\Pi_{ij}^{\rm TT} \equiv \left\{ \partial_i\phi\, \partial_j\phi + \Re\mathrm{e}[(D_i\varphi)^*(D_j\varphi)] + \frac{4}{g^2 a^2(t)} F_i^{\alpha} F_{j\alpha} + \dots \right\}^{\rm TT}$$

Real scalars — Complex scalars — Gauge fields

➤ GW spectra:

$$h^2\Omega_{\rm GW} \equiv \frac{h^2}{\rho_c}\frac{d\rho_{\rm GW}}{d\log k} = \frac{h^2}{\rho_c}\frac{k^3 m_p^2}{8\pi^2 a^2}\mathcal{P}_{h'}(k,\tau)$$

$$\langle h'(\mathbf{k},\tau) h^{*'}(\mathbf{k}',\tau)\rangle = (2\pi)^3 \mathcal{P}_{h'}(\kappa,\tau)\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

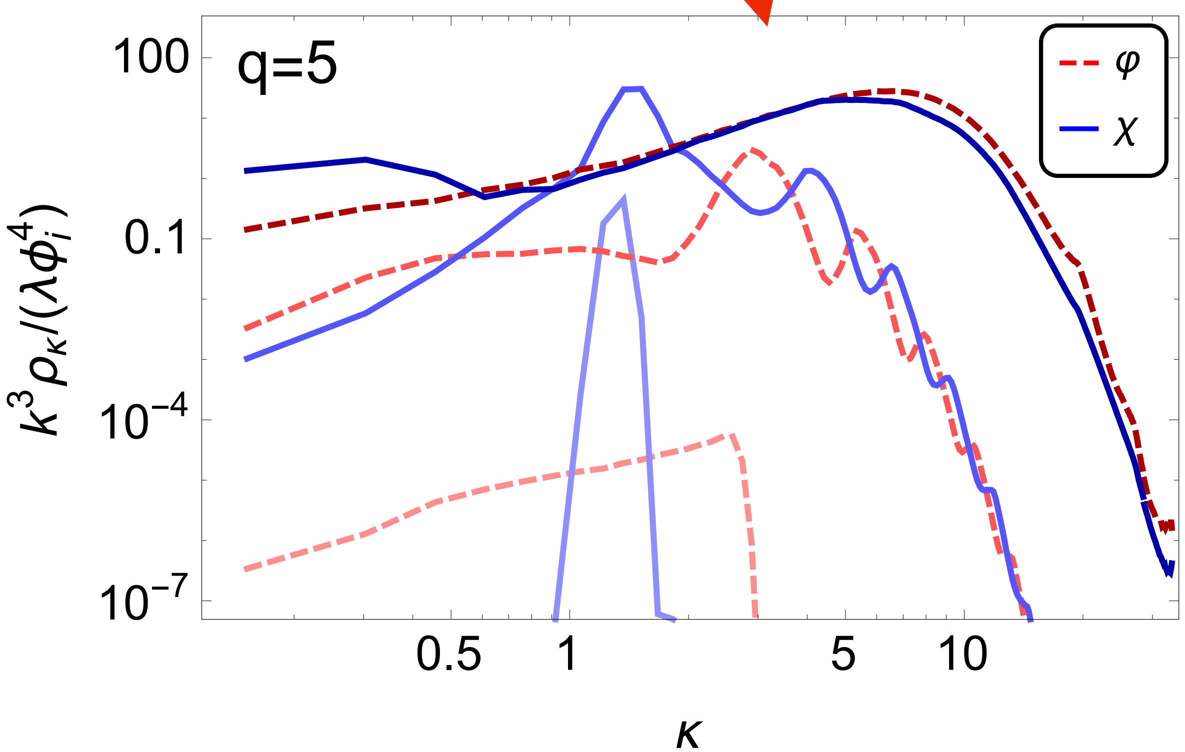
2. Fitting GWs from (p)reheating

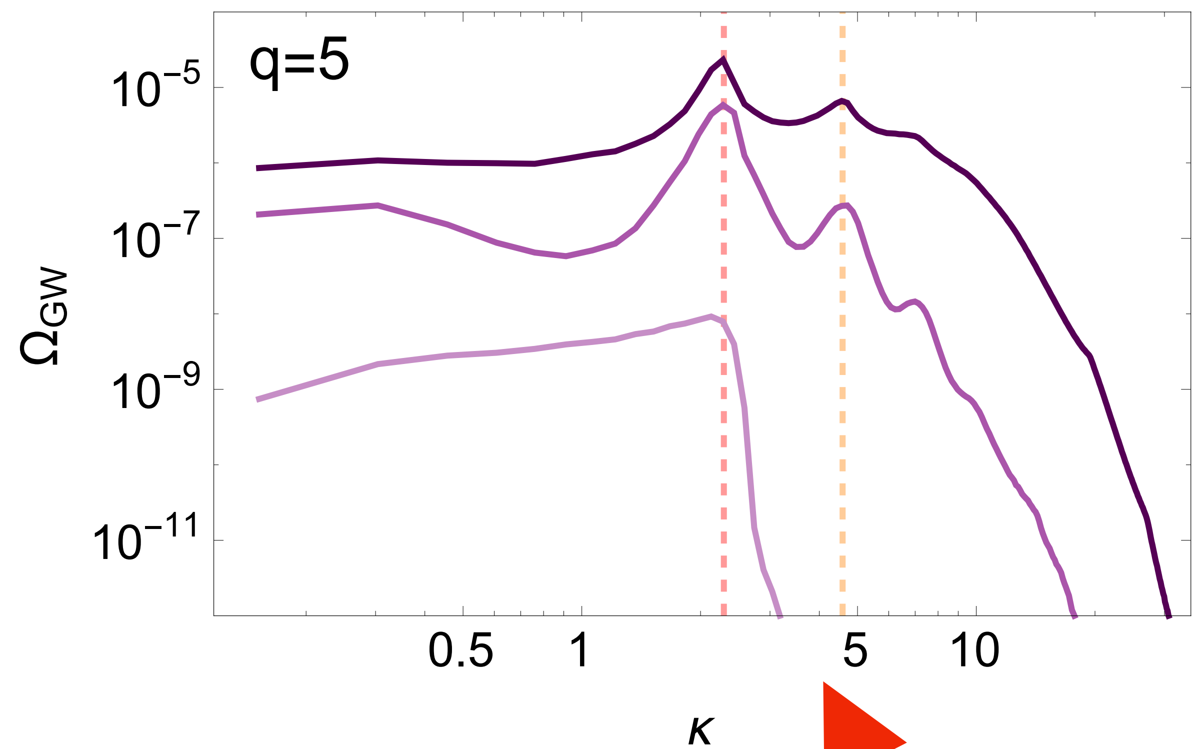
➤ **GWs from preheating (parametric resonance):**

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} - \nabla^2 h_{ij} = \frac{2}{m_p^2}\left\{\partial_i X\,\partial_j X + \partial_i\phi\partial_j\phi\right\}^{\rm TT}$$

$$V(\phi) = \frac{1}{4}\lambda\phi^4$$

FIELD GRADIENTS…

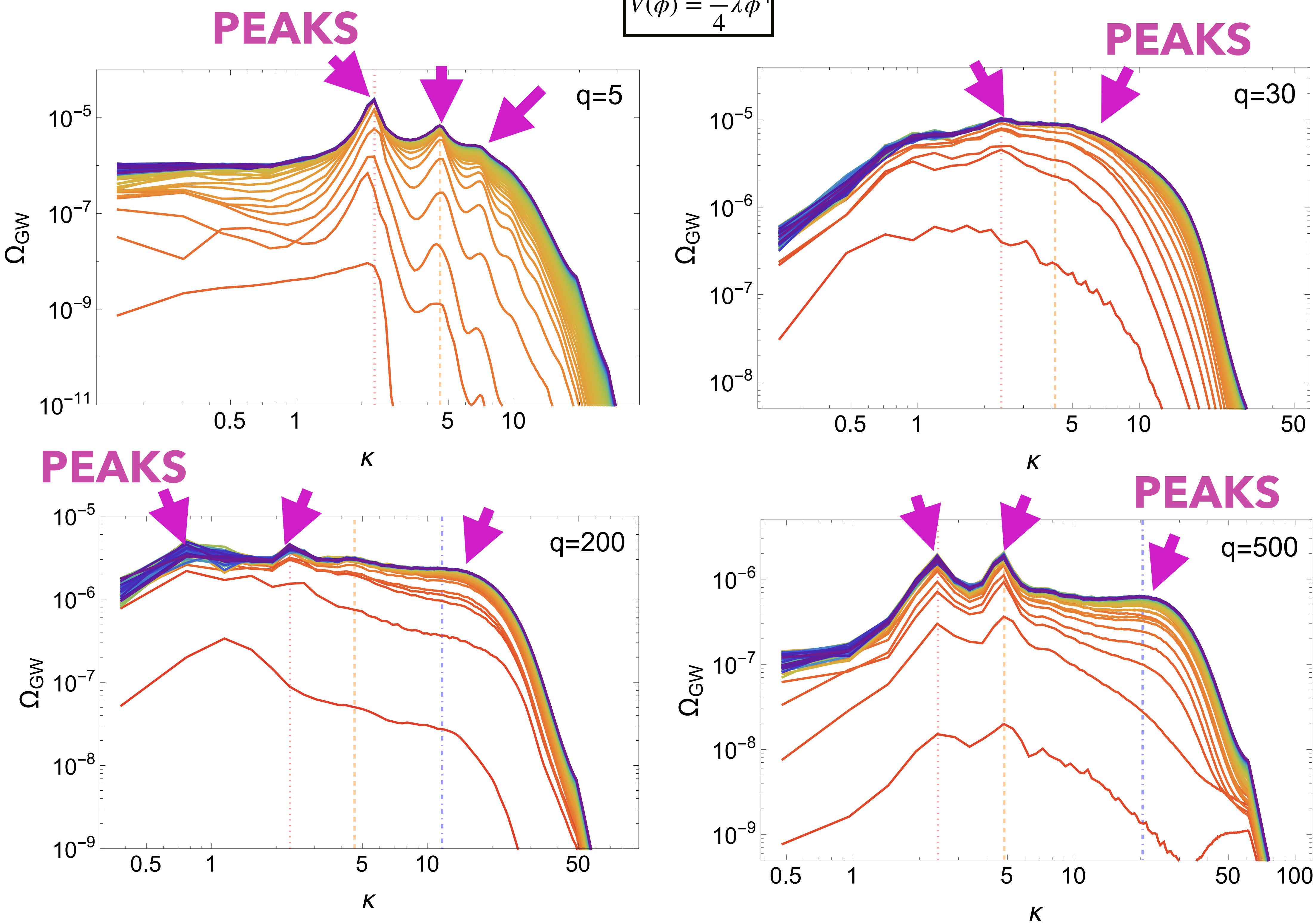




…SOURCE GWs!

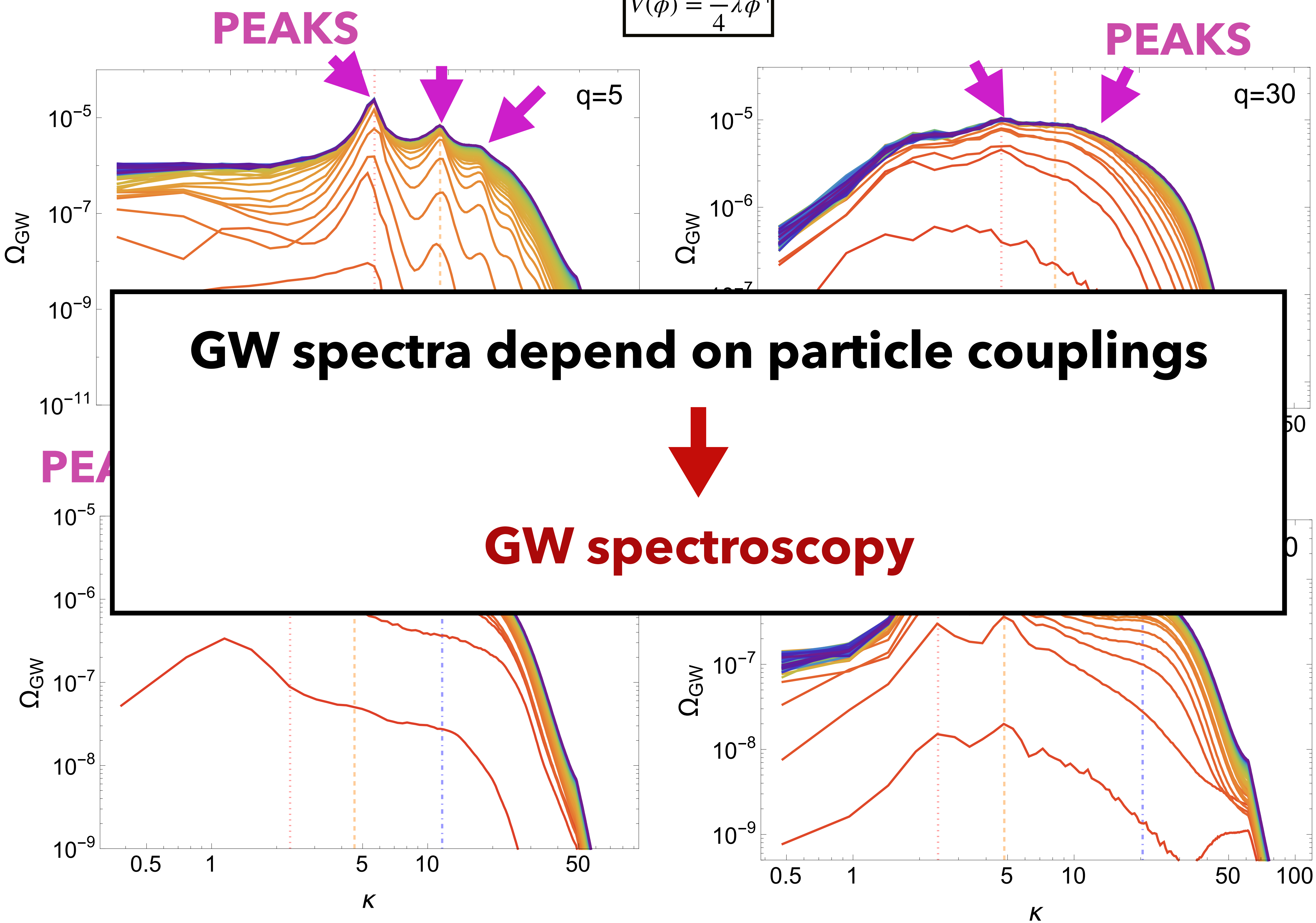
2. Fitting GWs from (p)reheating

$$V(\phi) = \frac{1}{4}\lambda\phi^4$$



2. Fitting GWs from (p)reheating

$$V(\phi) = \frac{1}{4}\lambda\phi^4$$



GW spectra depend on particle couplings

↓

GW spectroscopy

2. Fitting GWs from (p)reheating

Figueroa and F.T. (2017)

- **Analytical prediction for peaks in GW spectra from preheating:**

$$\Omega^{(\mathrm{f})}_{\mathrm{GW}}(\kappa_p) = \frac{C}{8\pi^4}\frac{\omega_*^6}{\rho_i m_p^2}\, q^{-\frac{1}{2}+\delta} \qquad (\eta, \delta \ll 1\,?)$$

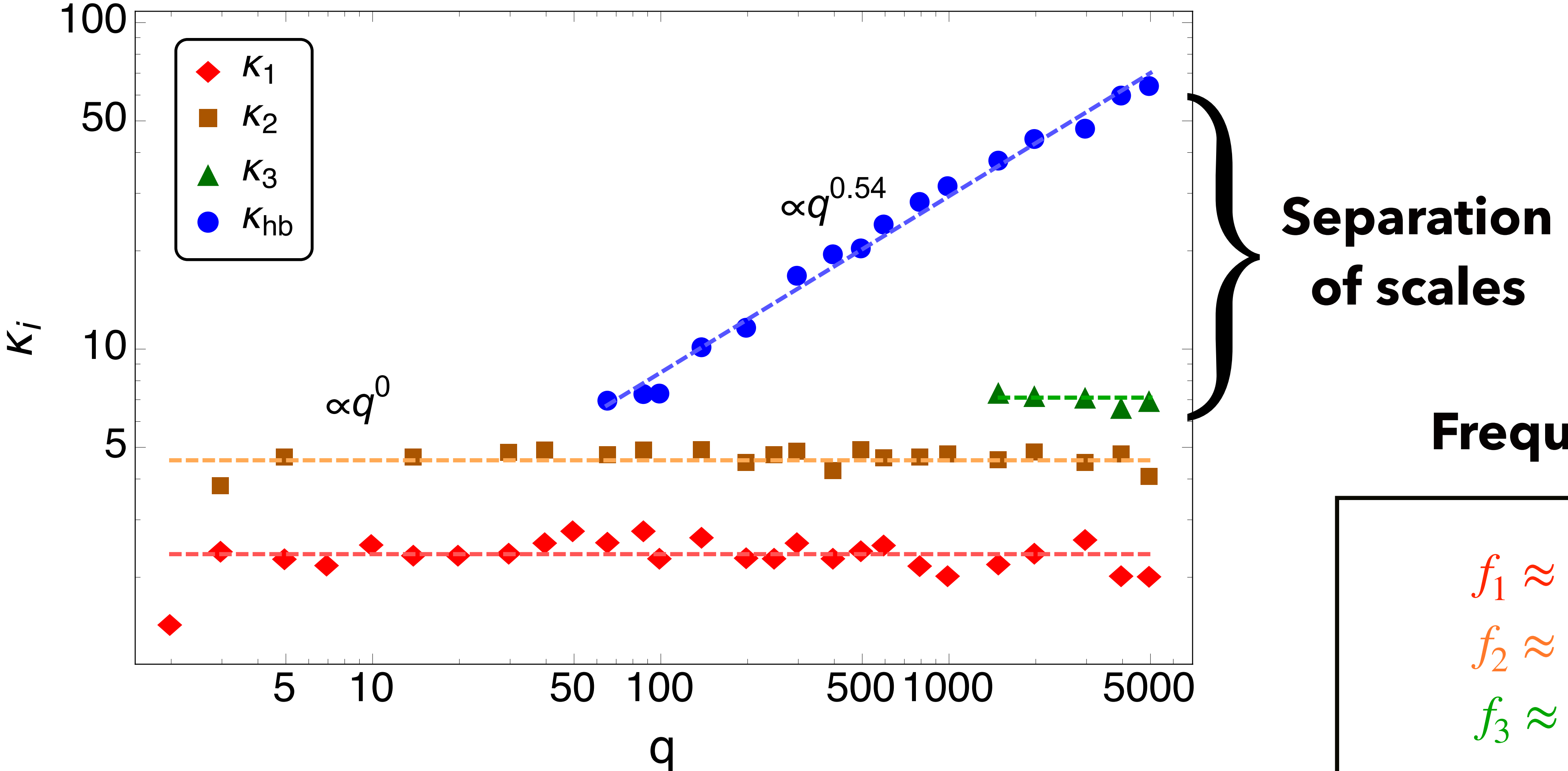
$$f_p \simeq 8\cdot 10^9 \left(\frac{\omega_*}{\rho_{\mathrm{i}}^{1/4}}\right) \epsilon_i^{\frac{1}{4}}\, q^{\frac{1}{4}+\eta}\ \mathrm{Hz} \times \begin{cases} 1 & , V(\phi) \propto \phi^4 \\ \left(\frac{a_{\mathrm{f}}}{a_{\mathrm{i}}}\right)^{\frac{1}{4}} & , V(\phi) \propto \phi^2 \end{cases} \qquad \epsilon_i \equiv \left(\frac{a_i}{a_{\mathrm{RD}}}\right)^{1-3w}$$

Frequency increases with q. Amplitude decreases with q

- **Parameters C, δ, η: fixed with lattice simulations**

$$q = \frac{g^2}{\lambda}$$

Peaks frequency:



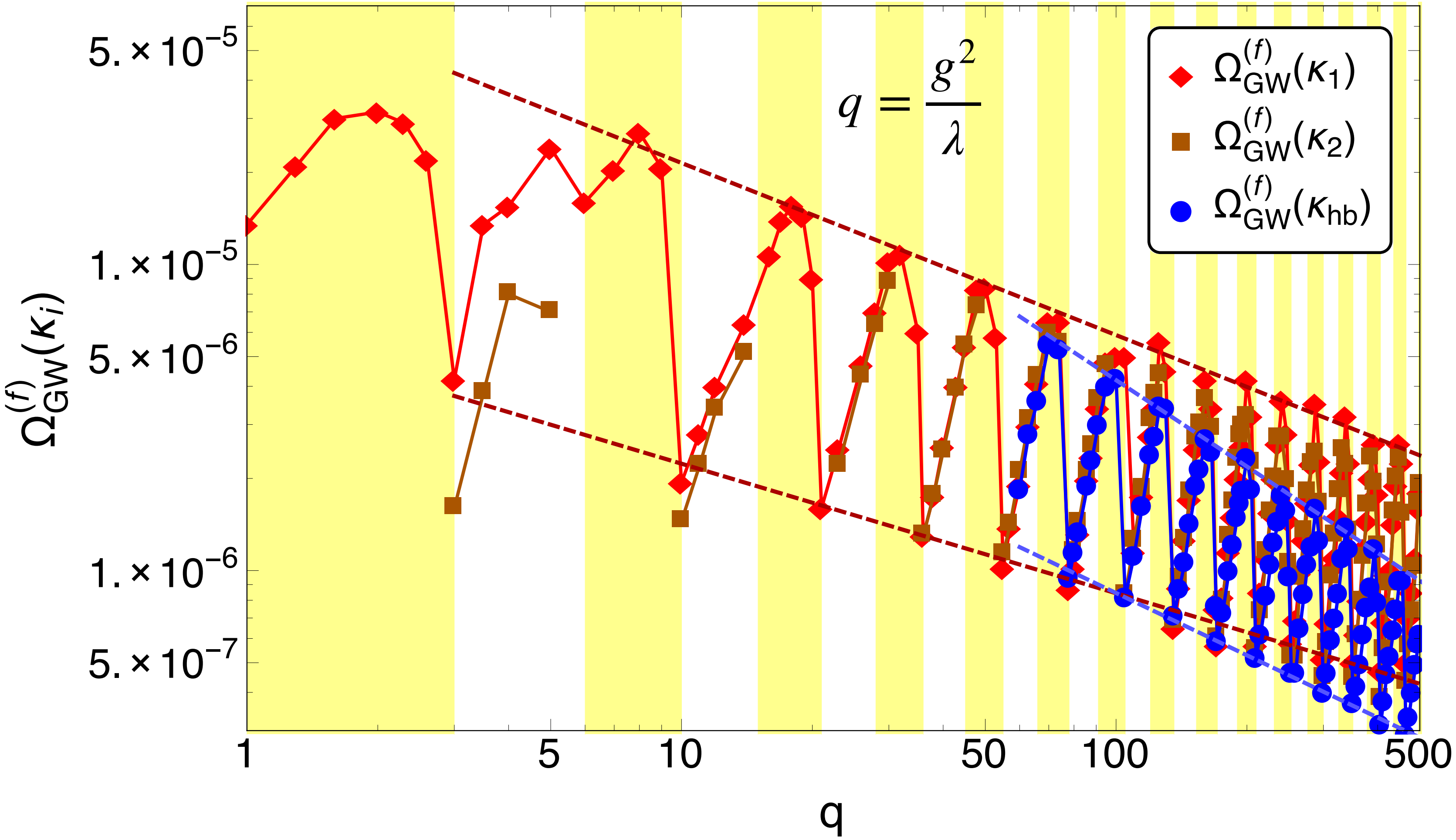
Frequency today:

$$f_1 \approx 1.5 \cdot 10^7 \text{ Hz}$$
$$f_2 \approx 2.8 \cdot 10^7 \text{ Hz}$$
$$f_3 \approx 4.5 \cdot 10^7 \text{ Hz}$$
$$f_{hb} \approx \left(\frac{q}{100}\right)^{0.54} \times 5.3 \cdot 10^7 \text{ Hz}$$

2.1. GWs from (p)reheating in λφ⁴

Peaks amplitude in λφ⁴:

$$q = \frac{g^2}{\lambda}$$

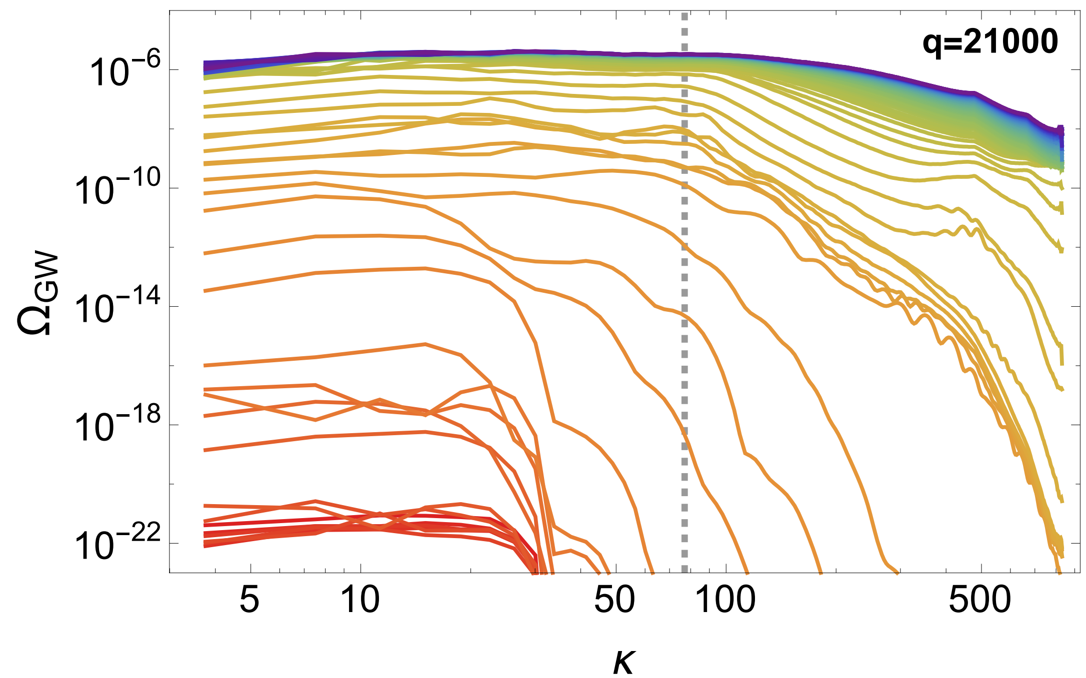


GW amplitude today:

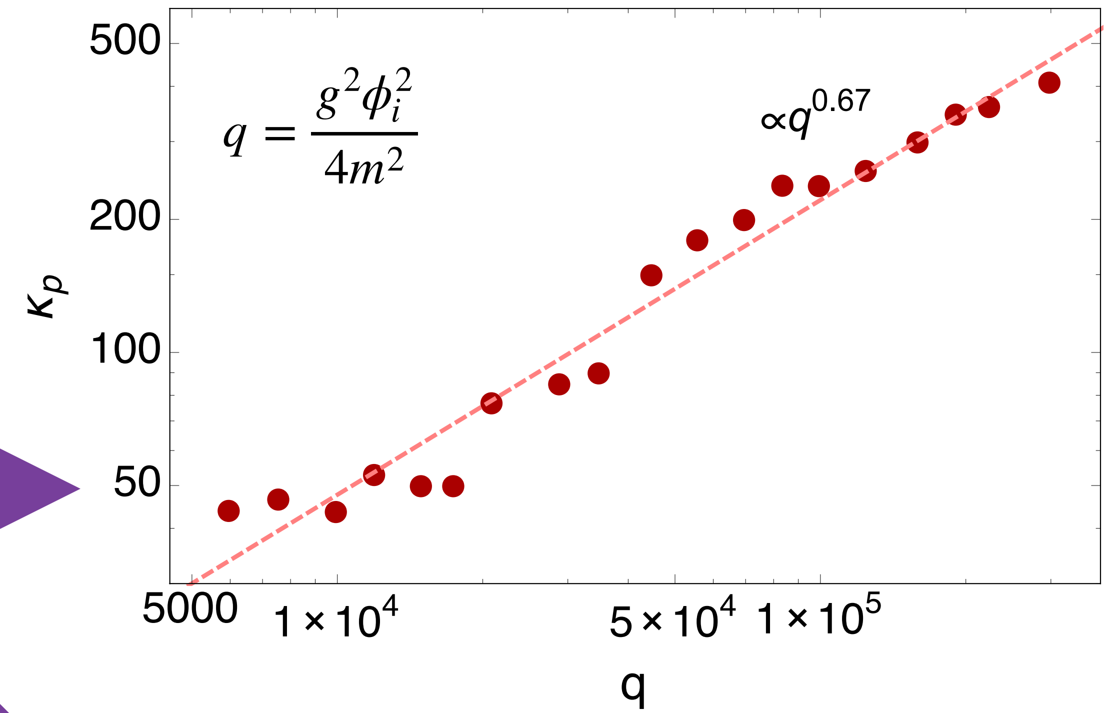
$$3.4 \cdot 10^{-12} \left(\frac{q}{100}\right)^{-0.68} \lesssim h^2\Omega_{GW}(f_{hb}) \lesssim 1.6 \cdot 10^{-11} \left(\frac{q}{100}\right)^{-0.94}$$

$$3.4 \cdot 10^{-12} \left(\frac{q}{100}\right)^{-0.42} \lesssim h^2\Omega_{GW}(f_{1,2}) \lesssim 2.4 \cdot 10^{-11} \left(\frac{q}{100}\right)^{-0.56}$$

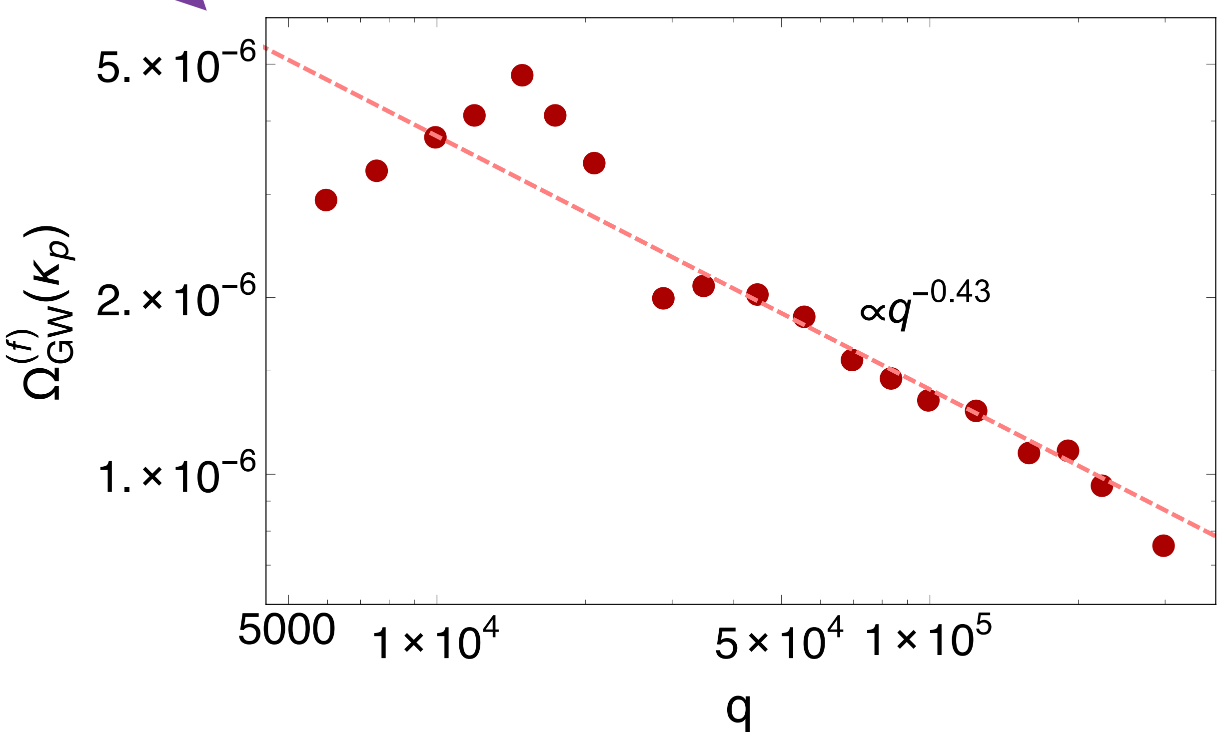
2.2. GWs from (p)reheating in m²φ²



PEAKS FREQUENCY



PEAKS AMPLITUDE



GW signal today:

$$f_p = \epsilon_f^{1/4}\left(\frac{q}{10^4}\right)^{0.67} \times 2.0\cdot 10^8\ \text{Hz}$$

$$h^2\Omega_{GW}(f_p) = \epsilon_f\left(\frac{q}{10^4}\right)^{-0.43} \times 1.5\cdot 10^{-11}$$

$$\epsilon_f \equiv \left(\frac{a_f}{a_{RD}}\right)$$ ***Redshift factor***

2.3. GWs from par. res.: other cases

- GW from parametric resonance of **spectator fields:**

Spectator field → $\Omega_{\rm GW}^{(s)}$

Inflationary field → $\Omega_{\rm GW}^{(i)}$

$$\frac{\Omega_{\rm GW}^{(s)}}{\Omega_{\rm GW}^{(i)}} \sim \left(\frac{H}{m_p}\right)^4 \ll 1$$

$$H \lesssim H^{(\max)} \approx 10^{13}{\rm GeV} \;\longrightarrow\; \frac{\Omega_{\rm GW}^{(s)}}{\Omega_{\rm GW}^{(i)}} \sim 10^{-20}$$

SUPPRESSED!

- GW from parametric resonance of **other species:**
  - **BOSONS** $\mathscr{L} \in g^2\chi^2\phi^2$ $\Omega_{\rm GW} \propto q^{-1/2}$
  - **FERMIONS** $\mathscr{L} \in g\bar{\psi}\psi\phi$ $\Omega_{\rm GW} \propto q^{3/2}$ Figueroa (2014)
  - **GAUGE BOSONS** $\mathscr{L} \in (D_\mu\phi)^\dagger(D_\mu\phi)$ $\Omega_{\rm GW} \propto q^{3/2}$ Figueroa, Garcia-Bellido, F.T. (2015 + ongoing work)

    $D_\mu\varphi = \partial_\mu\phi - ieA_\mu$

CONTENTS

1. Fitting parametric resonance

   JCAP 1702 (2017) 001 (with D. Figueroa)

2. Fitting GWs from parametric resonance

   JCAP 1710 (2017) 057 (with D. Figueroa)

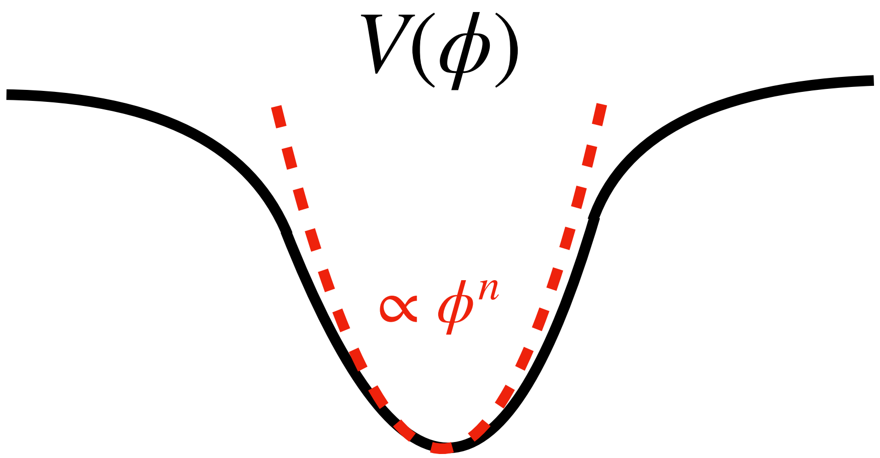
3. **Lifetime of oscillons in hilltop potentials**

   In preparation (with S. Antusch, F. Cefala)

3. Lifetime of oscillons in hilltop potentials

Potentials with flat regions give rise to oscillons: localised strong fluctuations of a scalar field.

(see Mustafa talk)



They continuously lose energy through the emission of scalar waves…

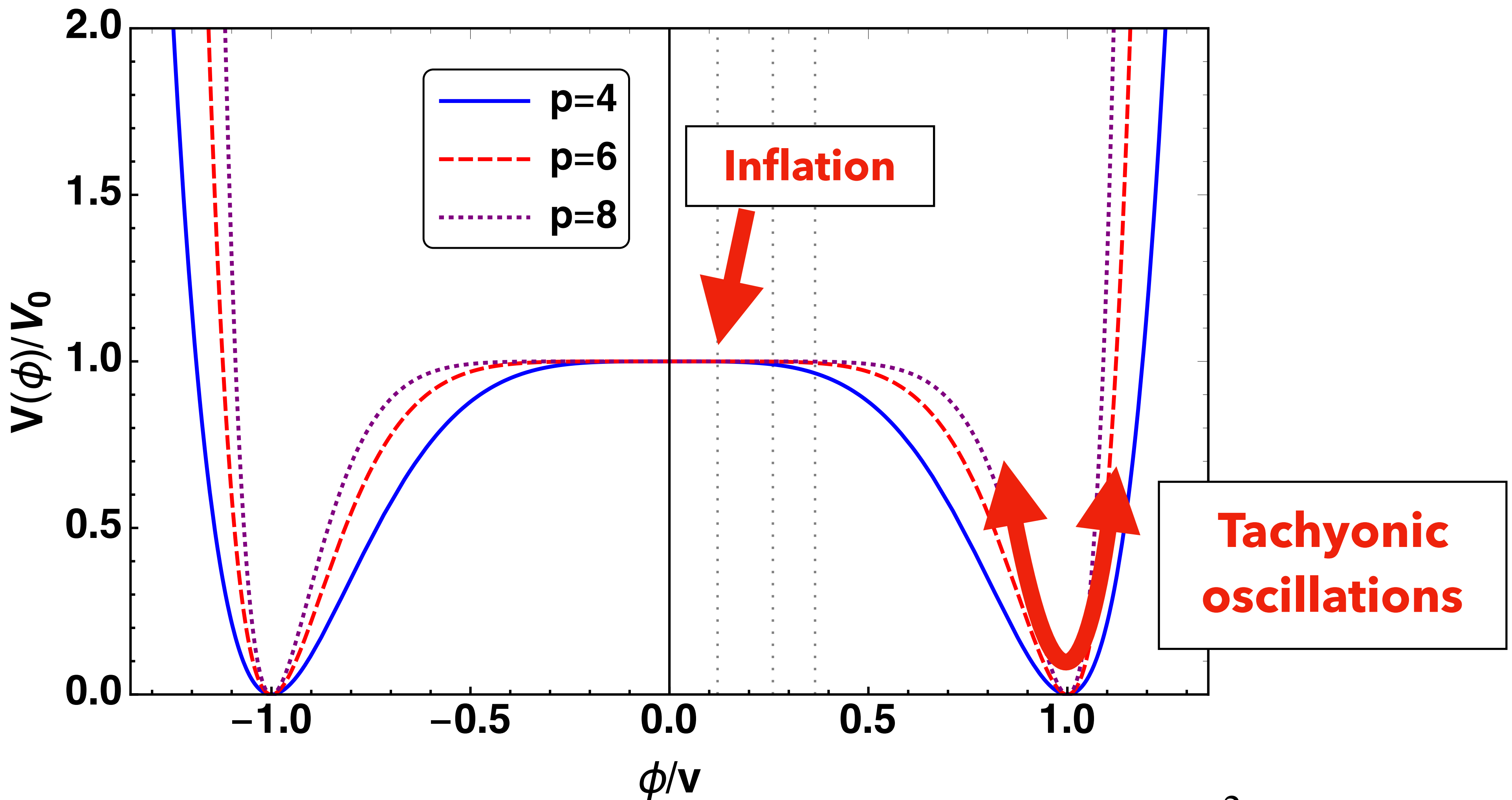
…but they are extremely long-lived: impossible to capture with full 3D lattice simulations!

Some references:

Amin, Easther, Finkel, Flauger, Hertzberg (2011)
Zhou, Copeland, Easther, Finkel, Mou, Saffin (2013)
Achilleos et al (2013)
Amin (2013)
Gleiser, Graham (2014)
Bond, Braden, Mersini-Houghton (2015)
Antusch, Cefala, Orani (2015,2016, 2017)
Antusch et al (2017)
Hong, Kawasaki, Yamazaki (2017)
Liu, Guo, Cai, Shiu (2017, 2018)
Gleiser, Stephens, Sowinski (2018)
Lozanov, Amin (2019)…

3. Lifetime of oscillons in hilltop potentials

Hilltop potentials: $V(\phi) = V_0 \left(1 - \frac{\phi^p}{v^p}\right)^2$



$$m_\phi^2 \equiv \frac{\partial^2 V}{\partial \phi^2}\bigg|_{\phi=v} = \frac{2p^2 V_0}{v^2}$$

3. Lifetime of oscillons in hilltop potentials

- **Oscillons properties in hilltop potentials studied in:**

  Antusch, Nolde, Orani (2015)
  Antusch, Orani (2015)
  Antusch, Cefala, Orani (2016)

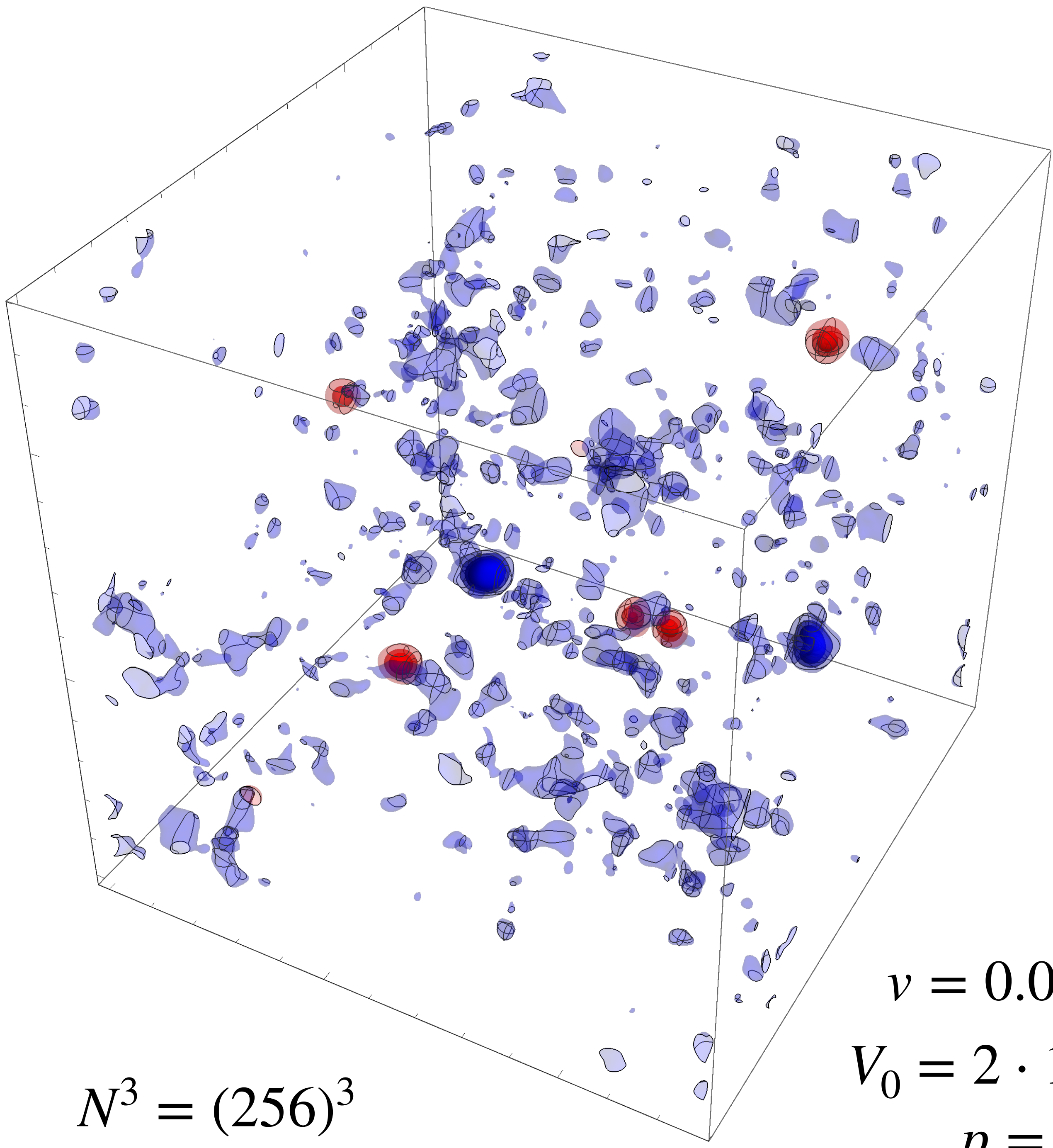
Antusch, Cefala, F.T. (in preparation):

We study the **lifetime of oscillons** in *hilltop* potentials:

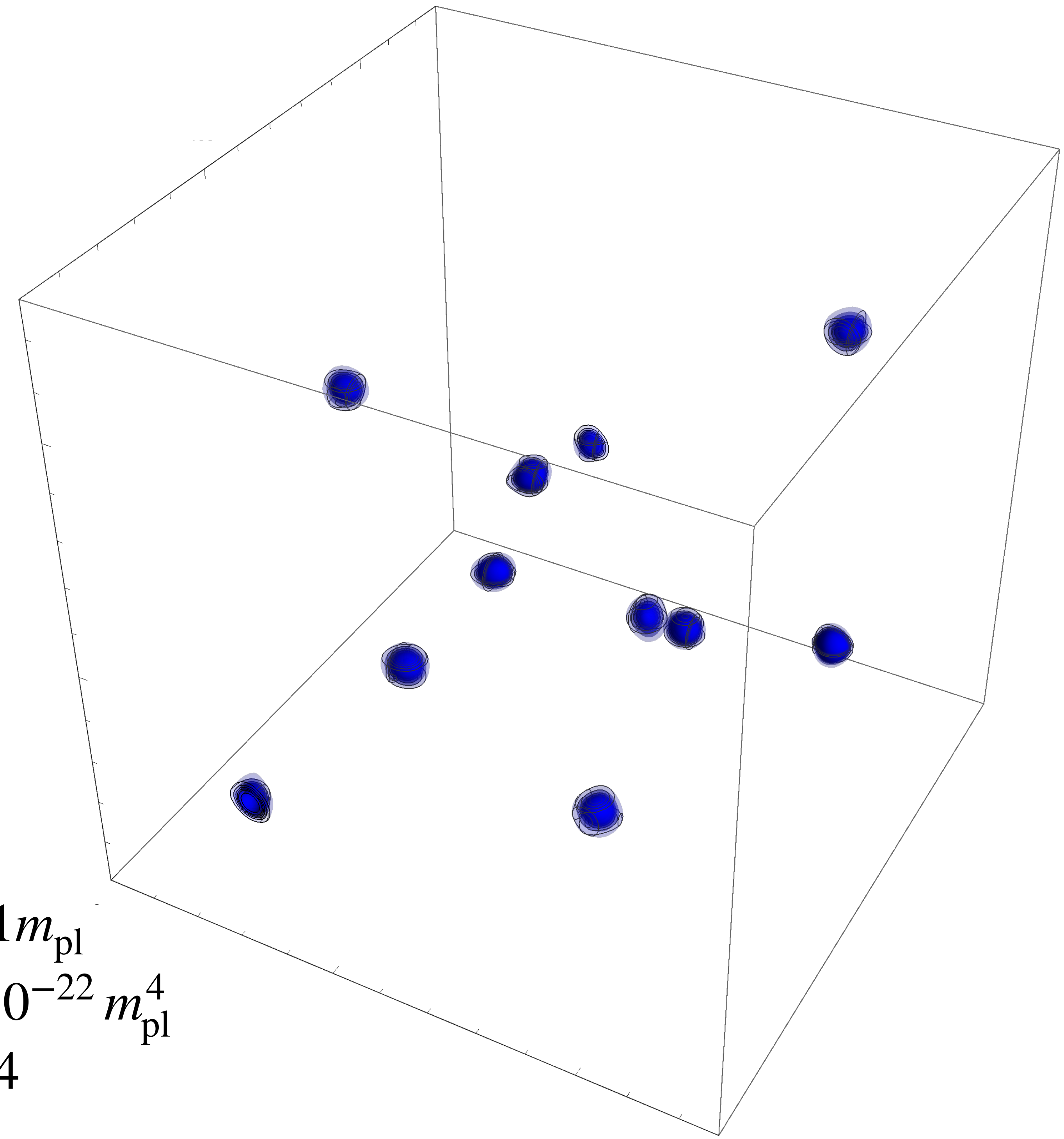
- **Part 1:** Full (3+1) classical lattice simulations: fitting oscillon shapes
- **Part 2:** Radially symmetric simulations: we observe the oscillon decay
  - *Single oscillon*
  - *Truncation technique*
  - *4th order spatial derivatives*

3. Lifetime of oscillons in hilltop potentials

FIELD AMPLITUDE



OVERDENSITY REGIONS



$N^3 = (256)^3$

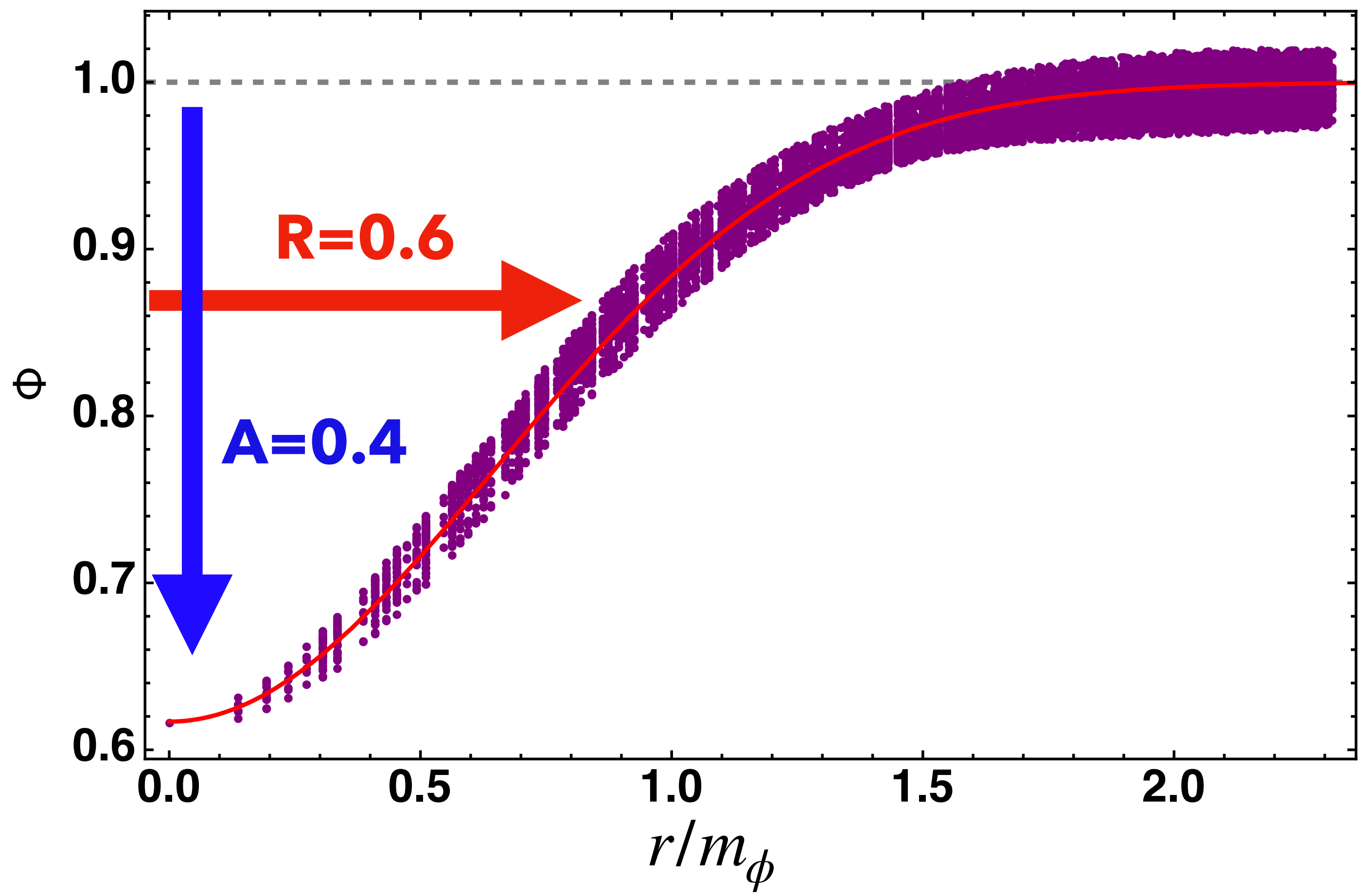
$v = 0.01 m_{\rm pl}$

$V_0 = 2 \cdot 10^{-22}\, m_{\rm pl}^4$

$p = 4$

3. Lifetime of oscillons in hilltop potentials

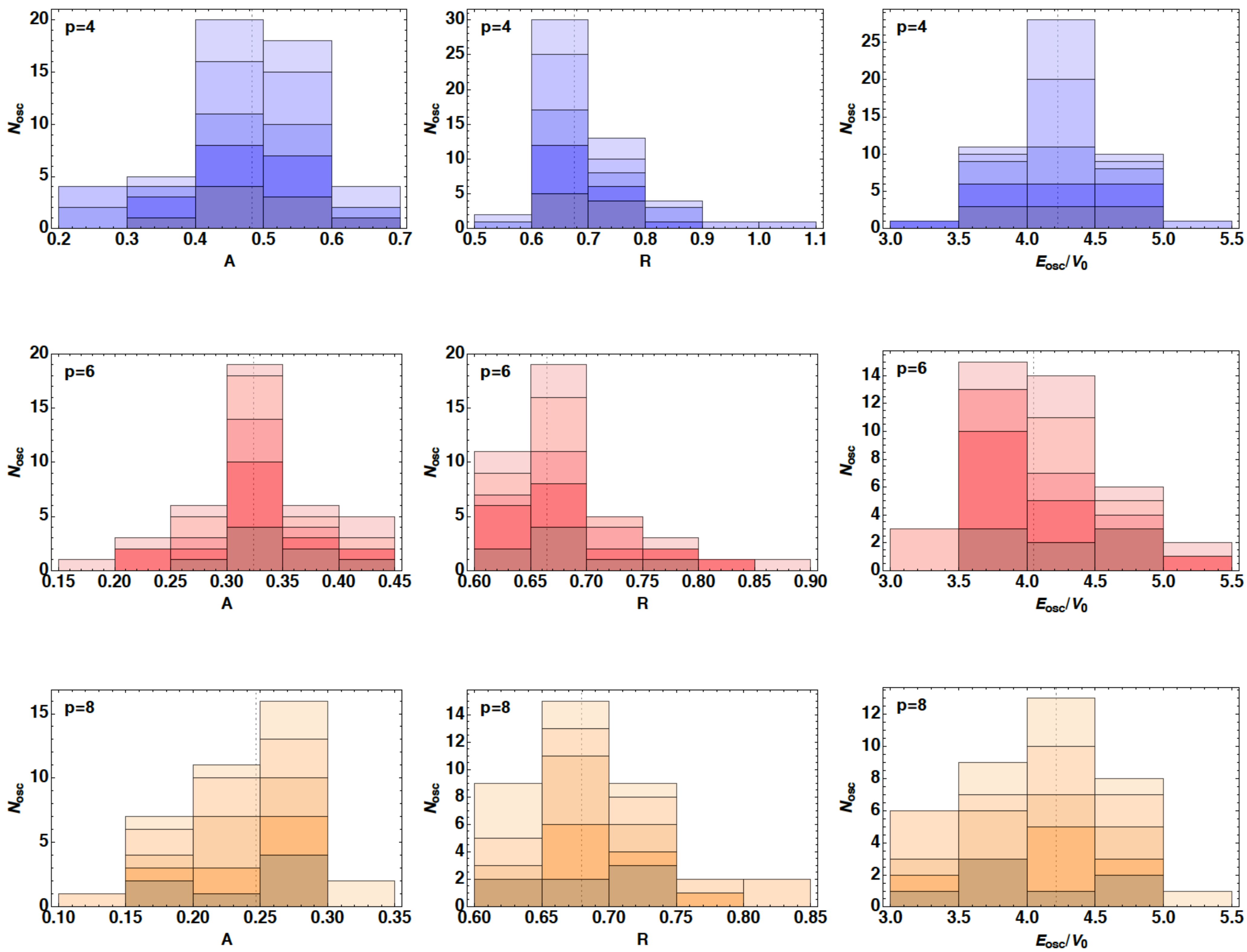
Oscillons in hilltop models are approximately spherically symmetric with Gaussian shape



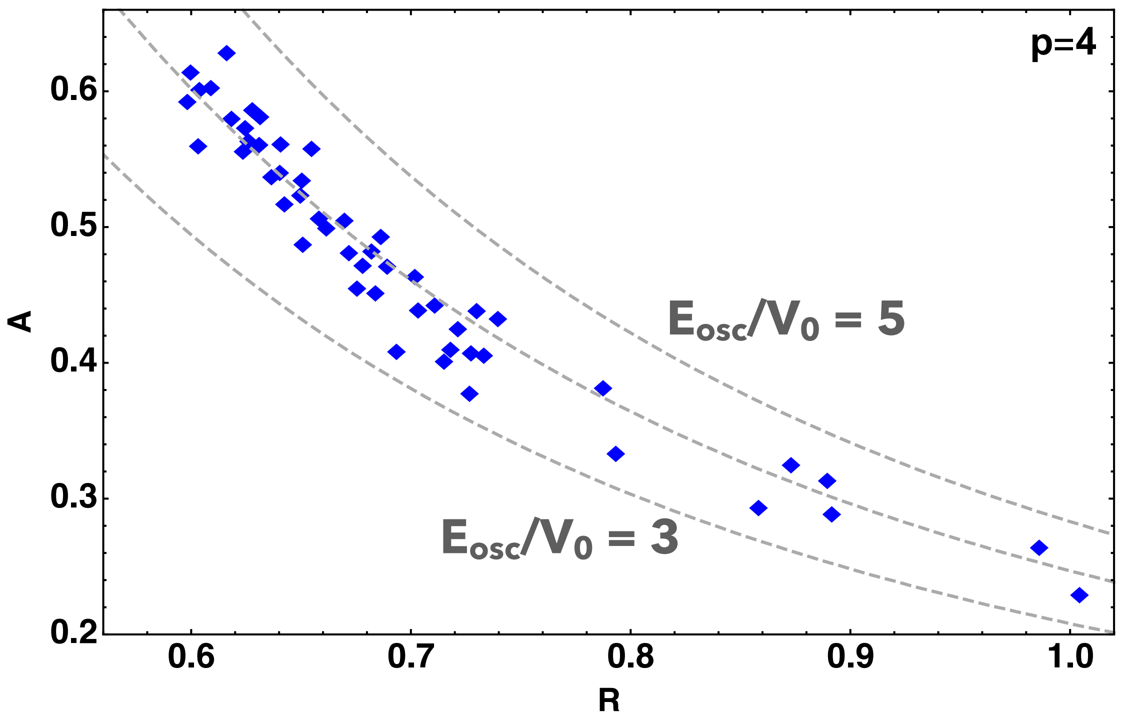
We fit the amplitude an radius of the oscillons with the full (3+1)-dim lattice simulations

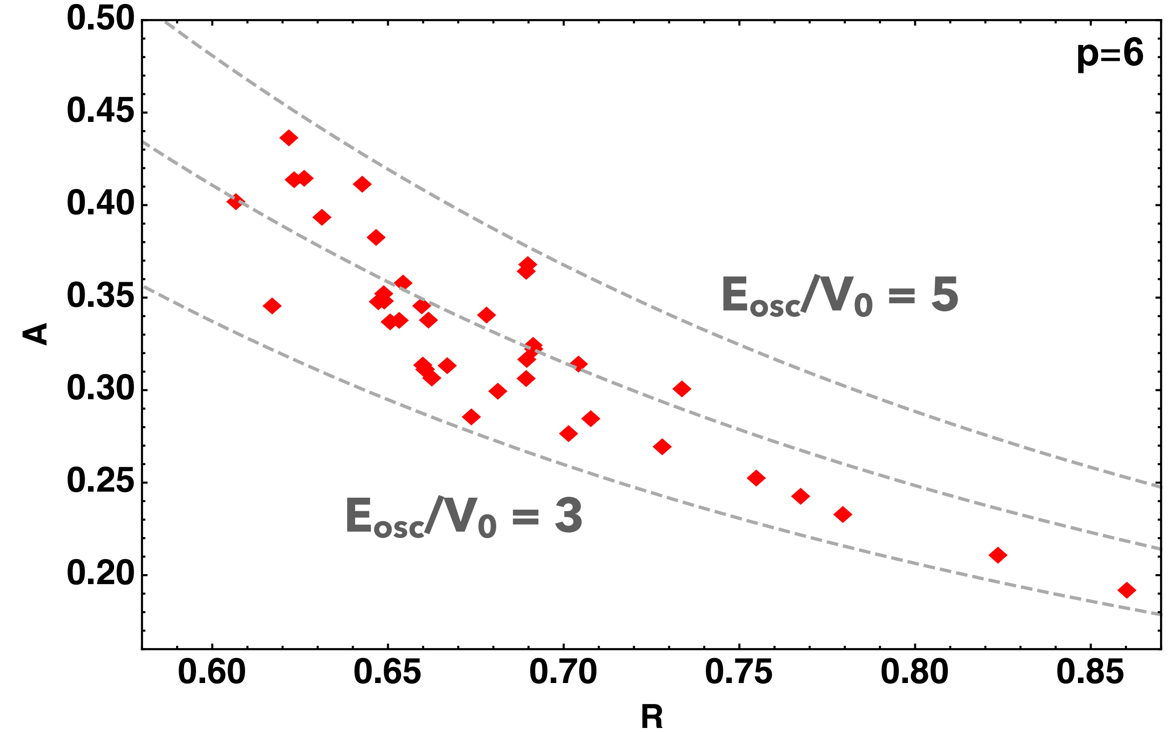
$$\Phi \equiv \frac{\phi}{v} = 1 - A e^{-\frac{1}{2}\left(\frac{r}{R}\right)^2}$$

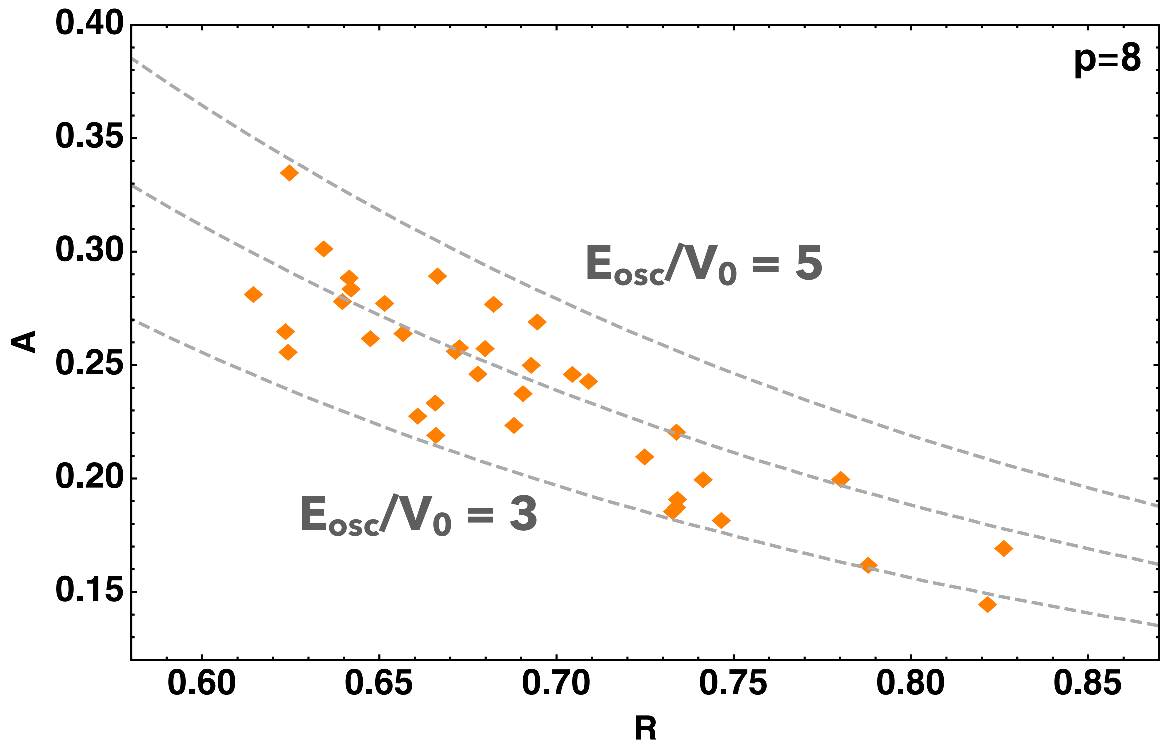
3. Lifetime of oscillons in hilltop potentials



3. Lifetime of oscillons in hilltop potentials

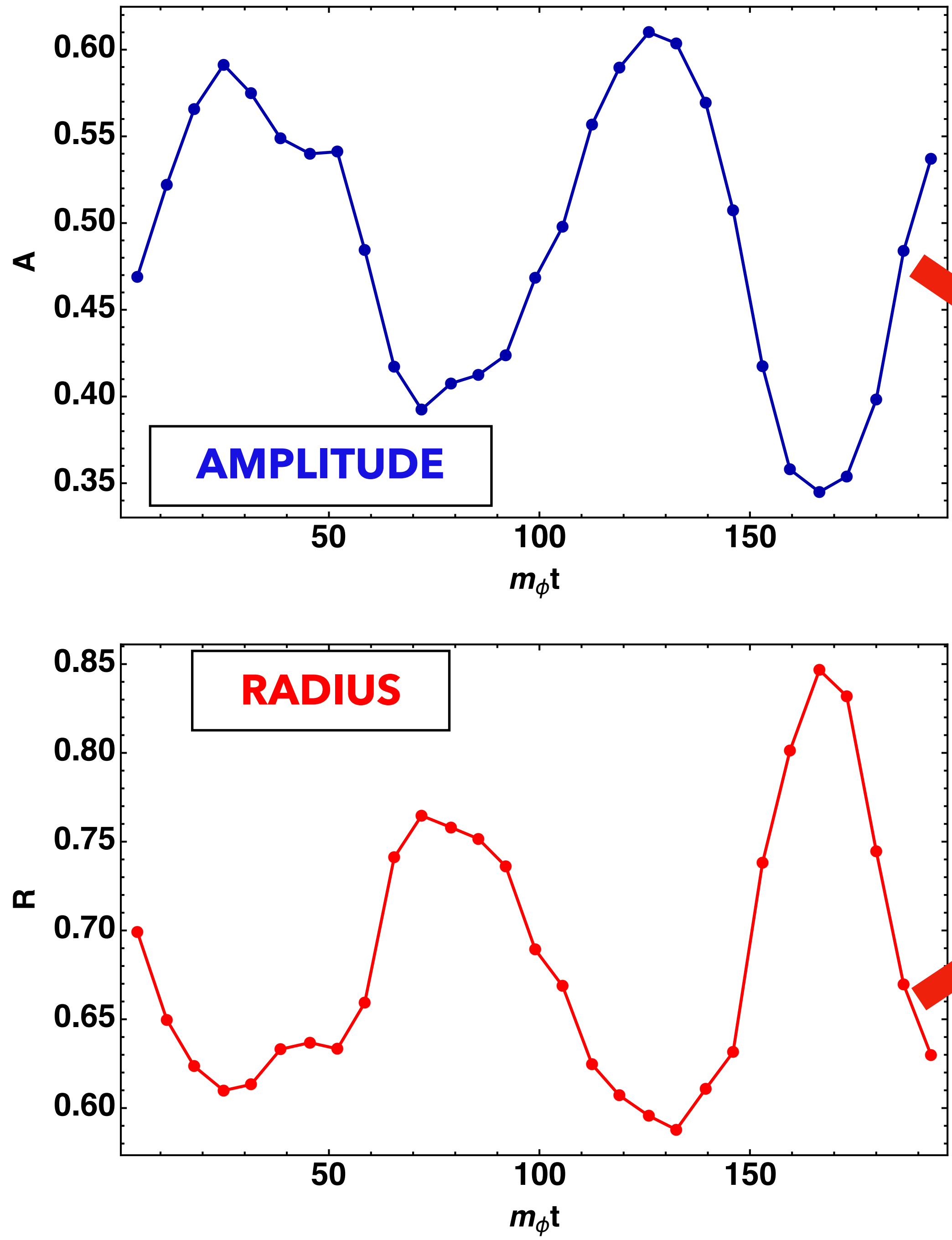




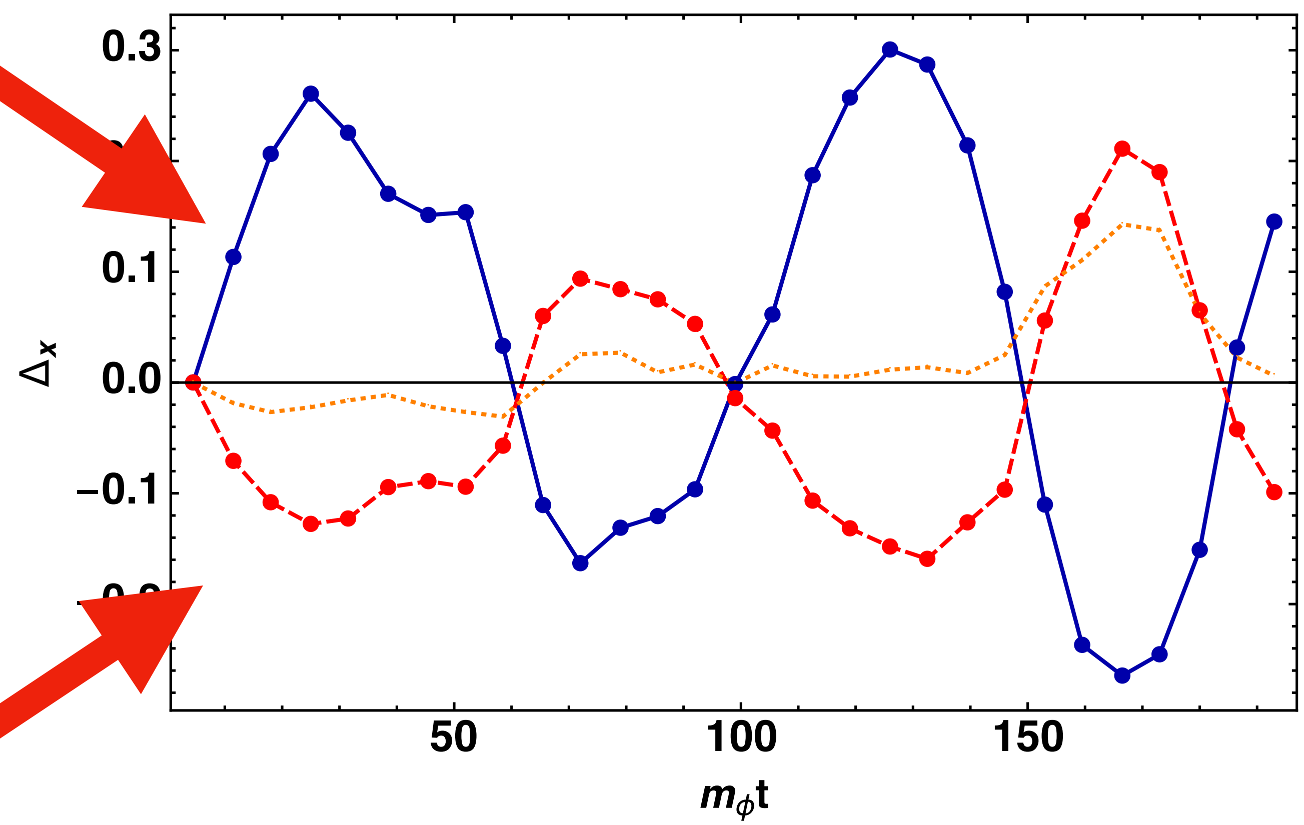


Oscillons form along lines of equal energy

3. Lifetime of oscillons in hilltop potentials

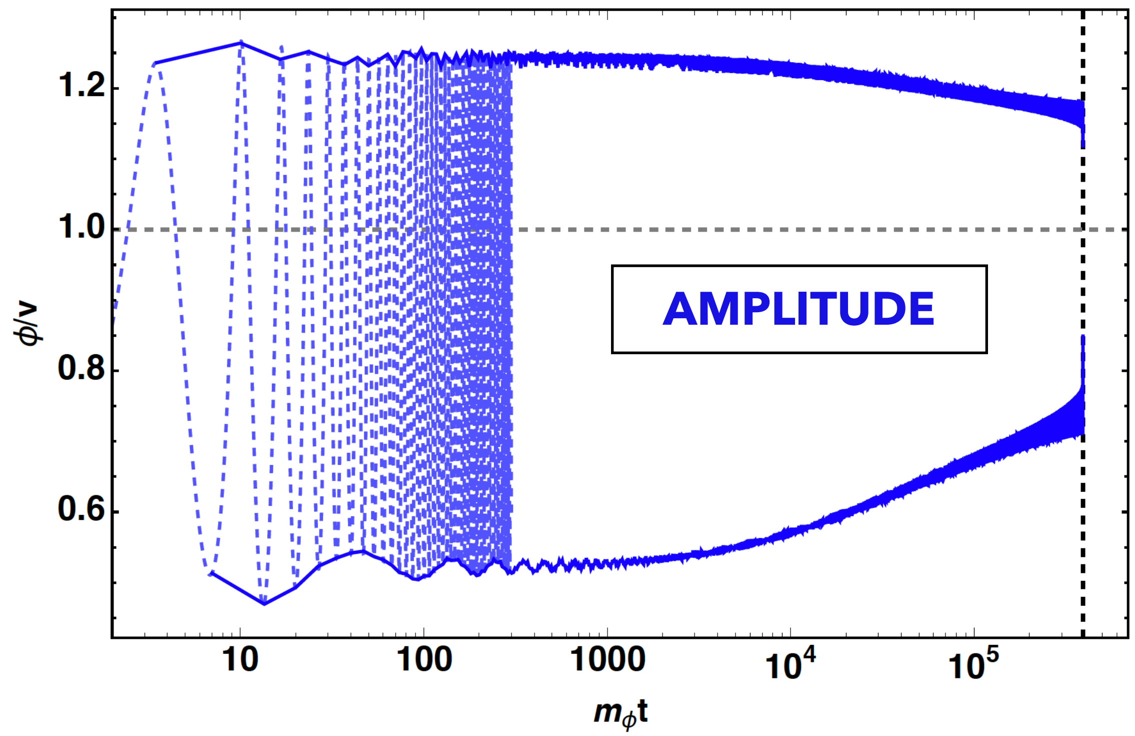


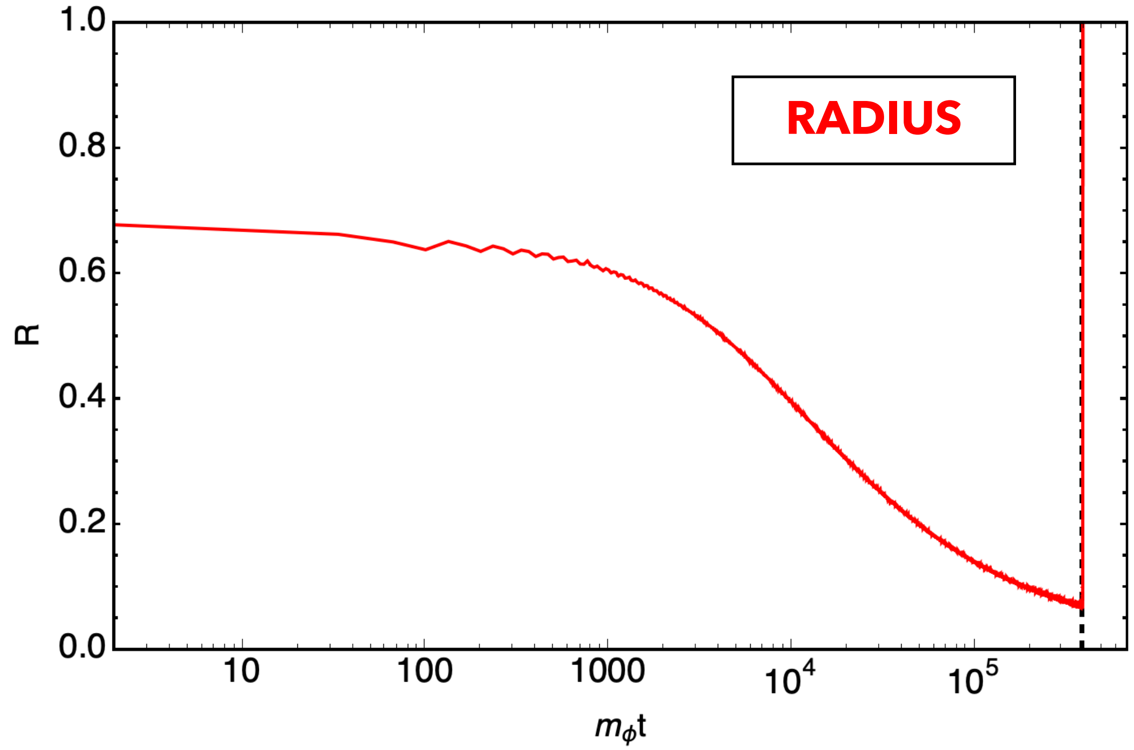
Oscillons ***breathe***
(contract and expand)



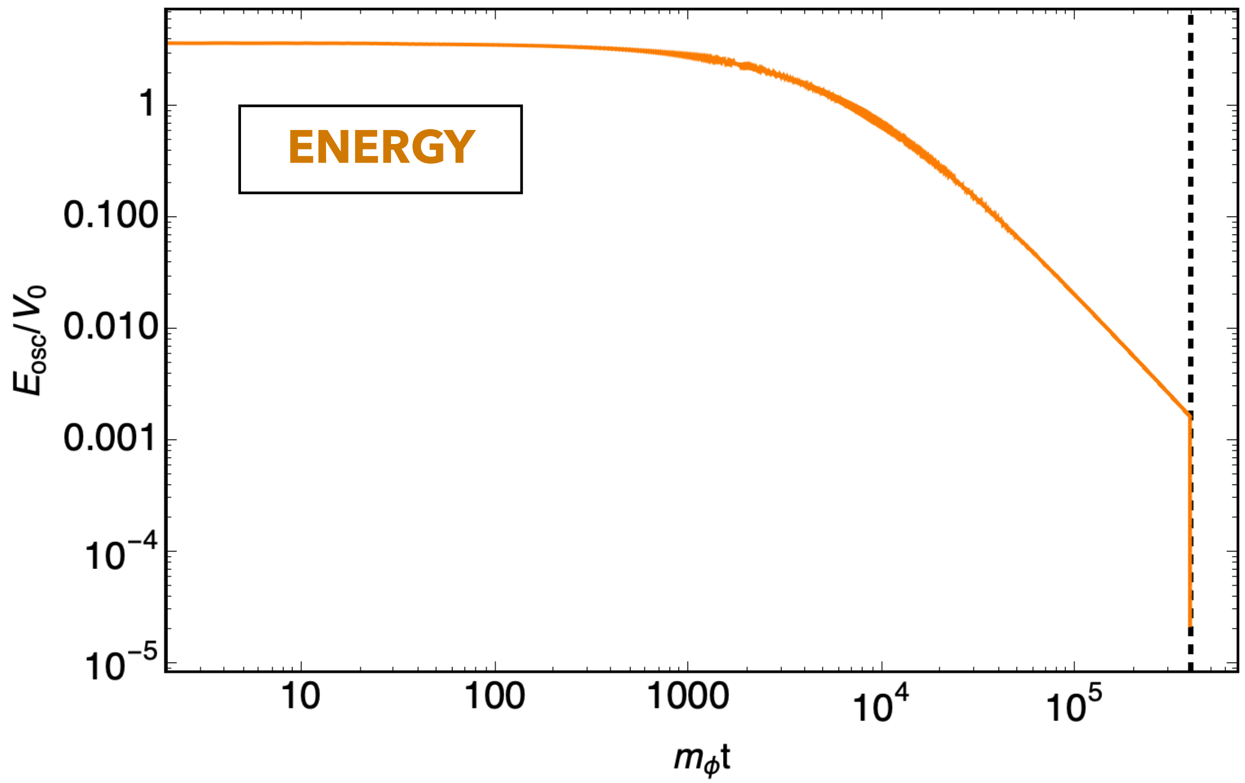
From full (3+1) lattice simulations

3. Lifetime of oscillons in hilltop potentials



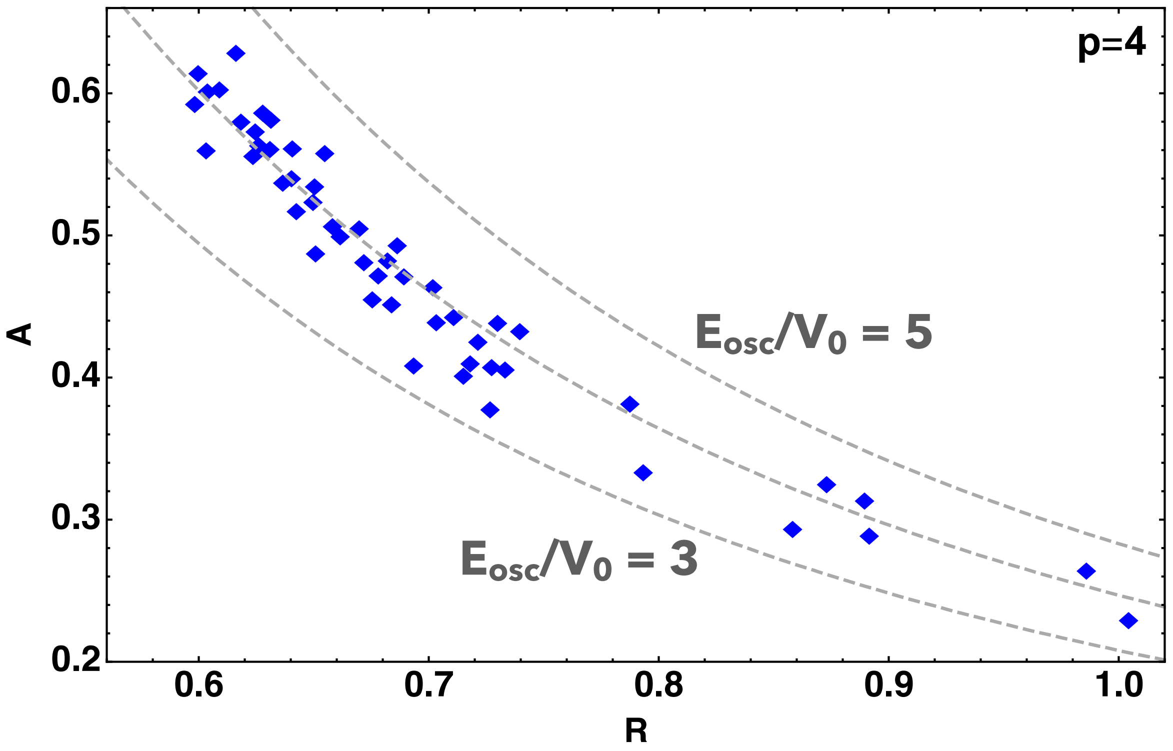


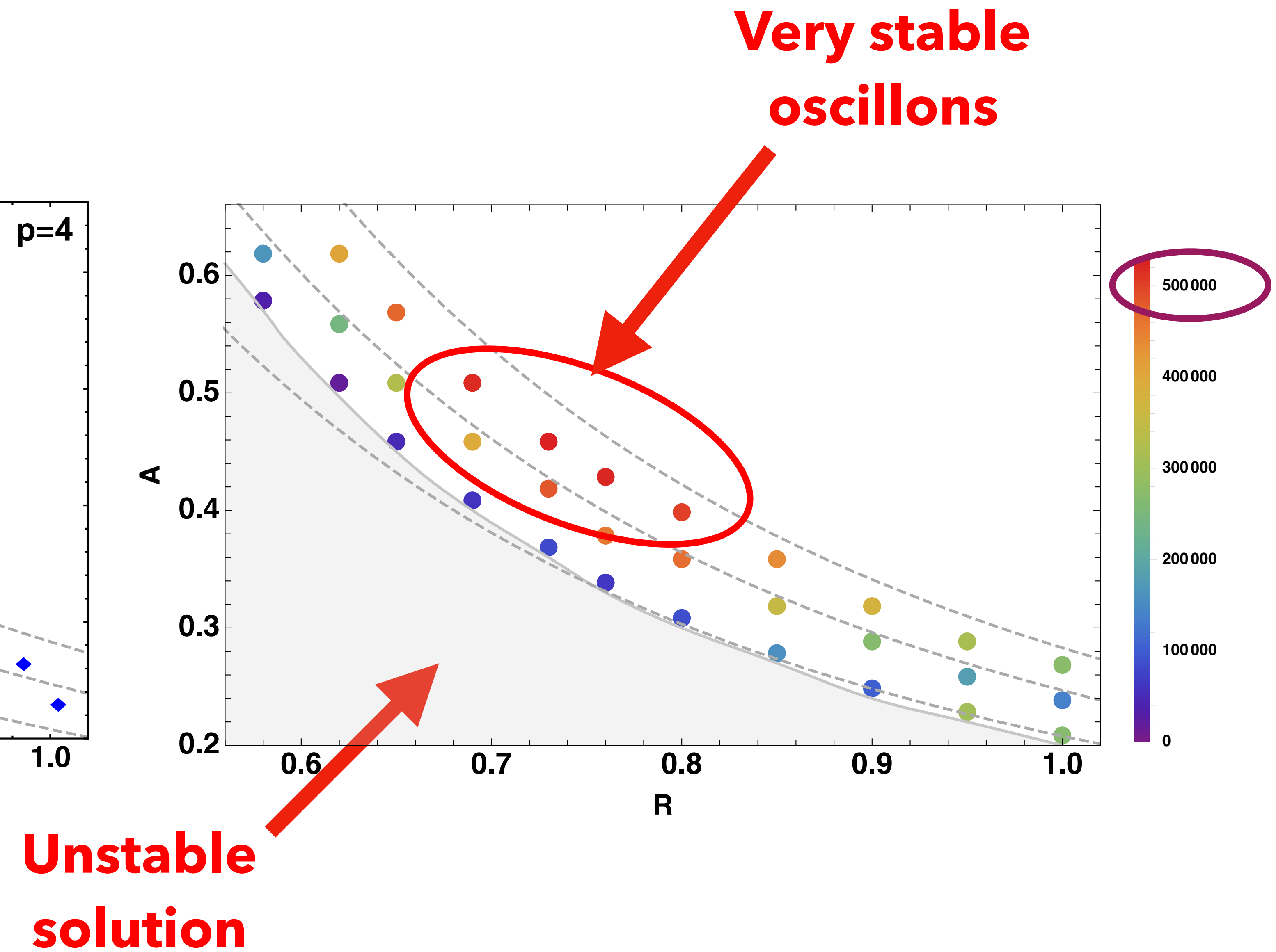
Oscillons lose energy due (mainly) to a decreasing volume



From radially symmetric simulations

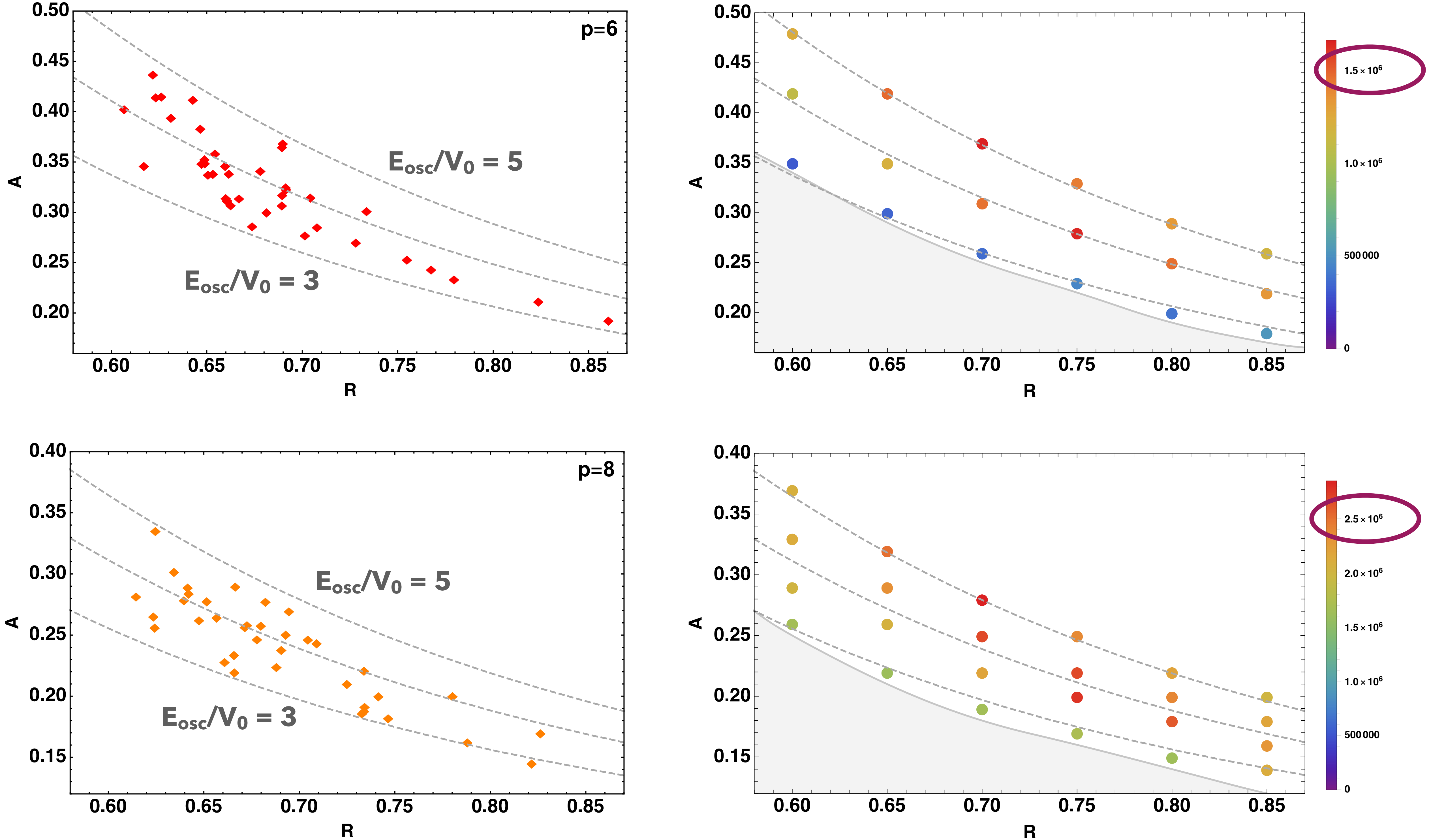
3. Lifetime of oscillons in hilltop potentials





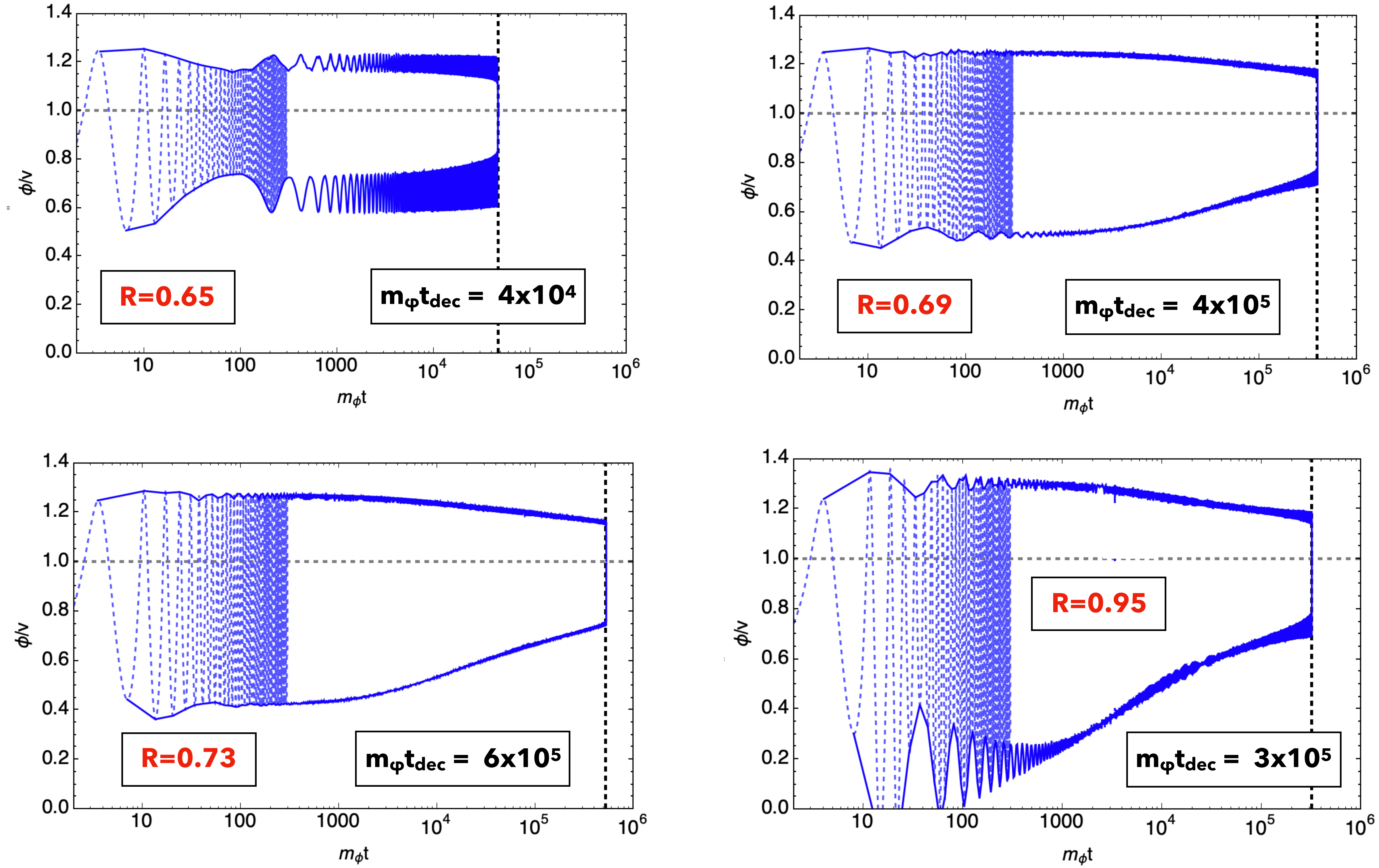
Oscillons live approximately 5 e-folds in hilltop models

3. Lifetime of oscillons in hilltop potentials



3. Lifetime of oscillons in hilltop potentials

Oscillons with same initial amplitude (A=0.46):



THANK YOU