Parametric resonance of axions

Yuko Urakawa (Nagoya Univ., Bielefeld Univ.)

J.Soda & Y.U. Euro. Phys. J. C 78, 9, 779 (2018)

Kitajima, Soda & Y.U. JCAP 10, 008 (2018)

Fukunaga, Kitajima & Y.U. arXiv: 1903. 02119

Kitajima, Soda & Y.U. in progress

Patel, Kobayashi, Tashiro, & Y.U. in progress

w/ Hayato Fukunaga, Naoya Kitajima (Nagoya U.), Jiro Soda (Kobe U.), Teerthal Patel (Nagoya U.)

String axiverse

Arvanitaki et al. (10), ⋯

Superstring theory in compact 6D

review of QCD axion
recall Mark's talk

↓

4D low energy EFT + Axions + Moduli ….

Wide mass ranges ➞ Probe of exDim

$m_a^2 \sim \frac{\mu^4}{f_a^2} e^{-\#\sigma_i}$

↓

Inflaton, DM candidate (Fuzzy DM)

Hu et al.(00), ⋯

Demirtas, Long, McAllister, Stillmann(18)

Type IIB compactified on orientifold

$h_{1,1} > 22$ → lightest axion mass $m_a < 10^{-33}$ ($\sim H_0$)

Axion as dark matter

$m_a << H$

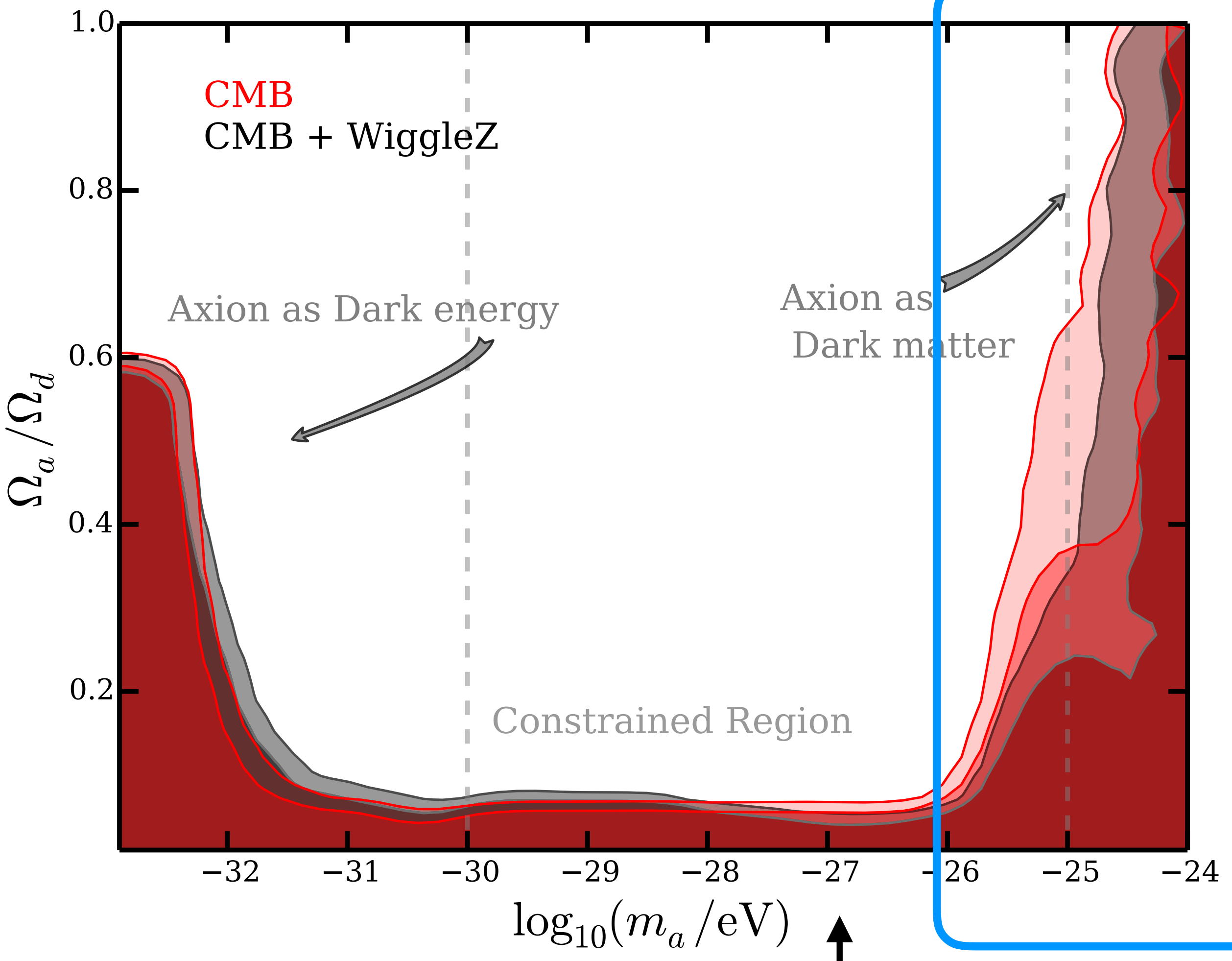
axion → Λ

$m_a >> H$

axion → DM

$$\Omega_d = \Omega_a + \Omega_c$$

Marsh (15)

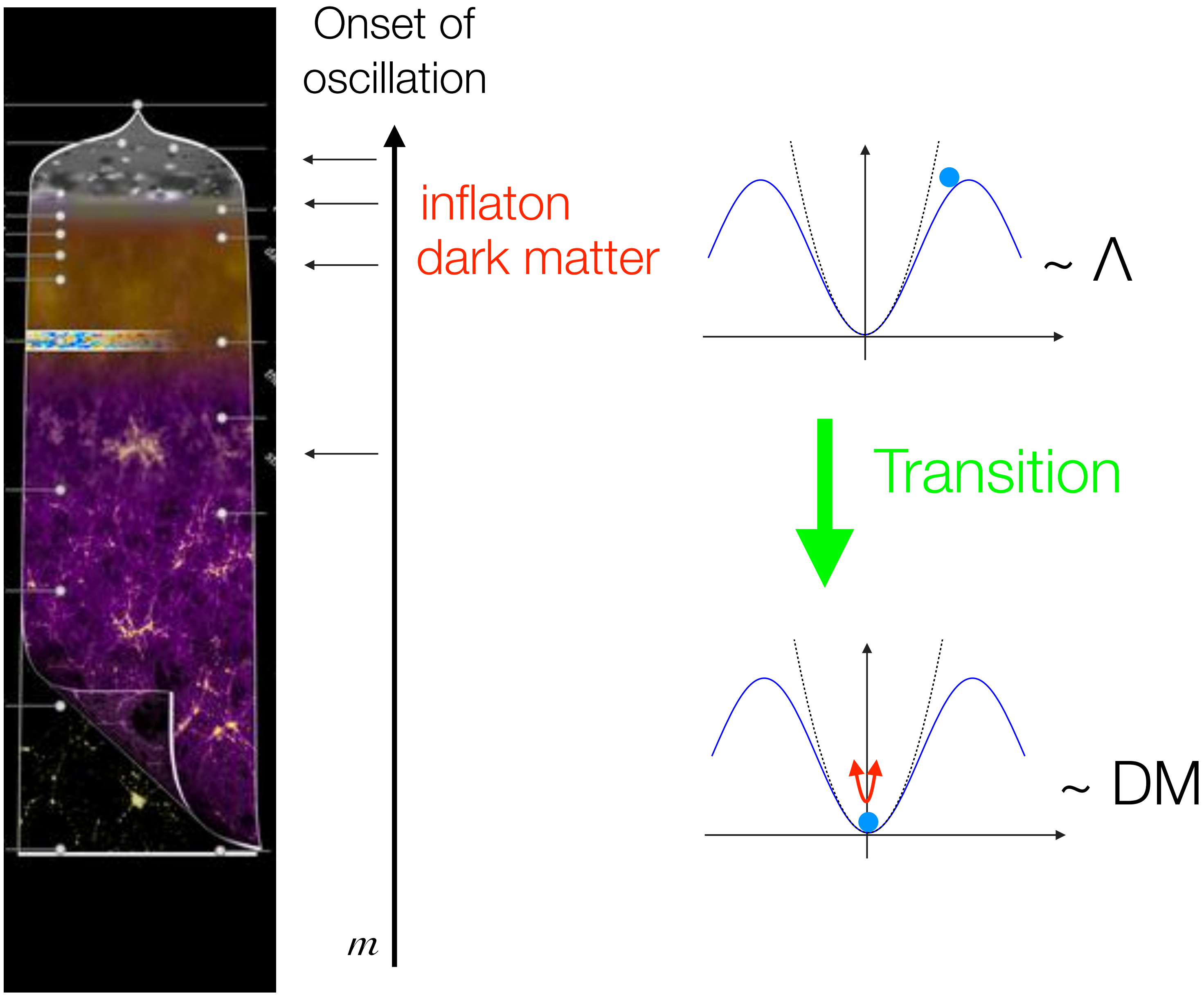


H_{eq}

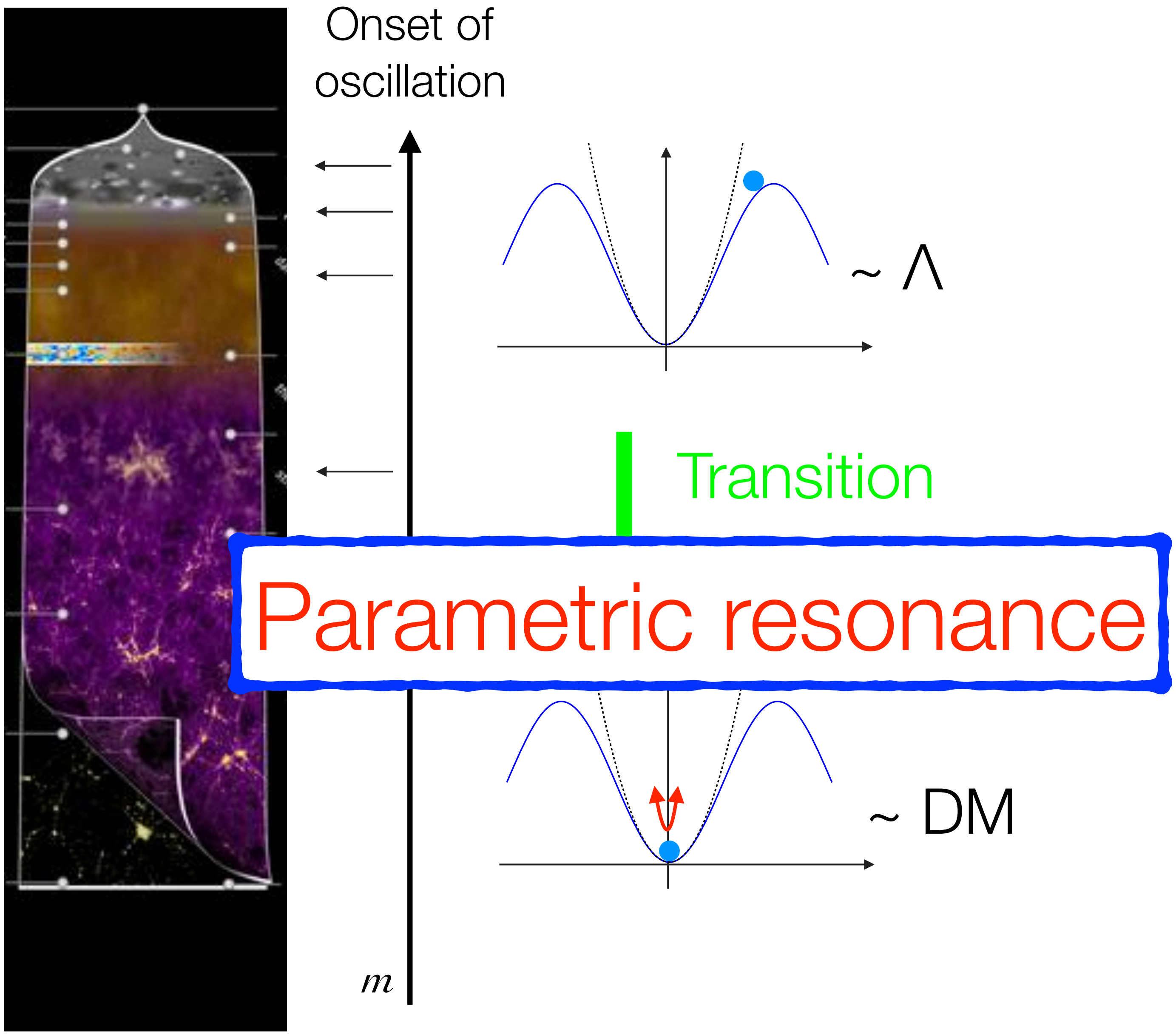
axion ~ CDM?

…yet, see also Hertzberg, Hayashi, Chiueh’s talks

New window of Axion (like particle) search



New window of Axion (like particle) search



- GW emissions

  Soda and Y.U. (17)

  Soda, Kitajima, Y.U. (18)

  Soda, Kitajima, Y.U. (in progress)

- Magnetogenesis

  Patel, Tashiro, Y.U., .. (in progress)

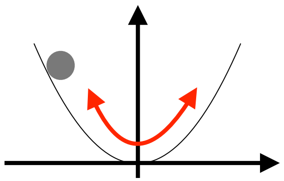
Contents

Parametric Resonance in cosmology

Ex1: Reheating

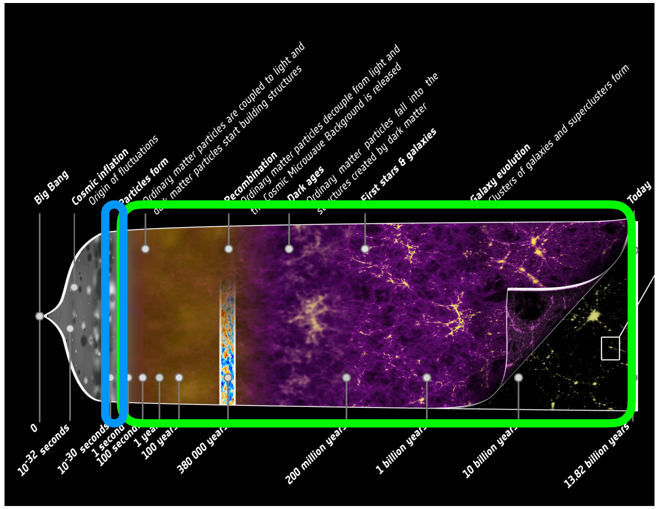
inflation

creation of
SM particles

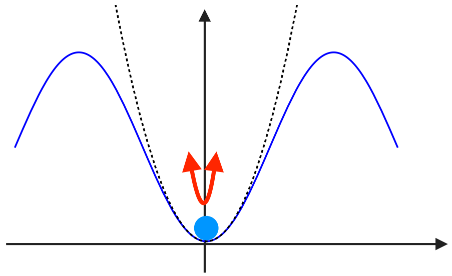


radiation dom.

coherent
oscillation



Ex2: Axion cosmology



~ dark matter

coherent
oscillation

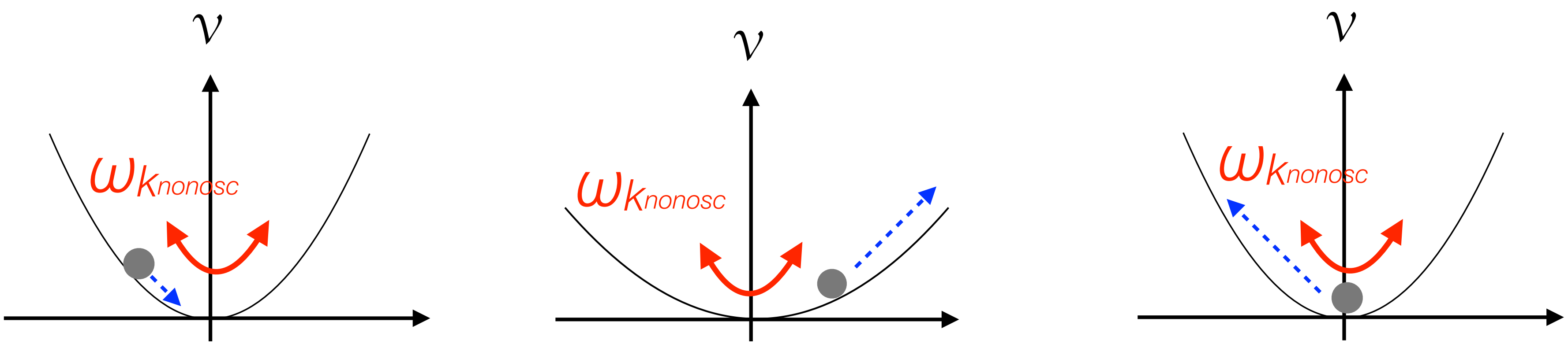
Parametric resonance

free field in flat space w/ periodic mass $M(t+T) = M(t)$

$$\omega_k^2(t) = k^2 + M^2(t) = \omega_{k\ \mathrm{nonosc}}^2 + \delta M^2(t)$$

parametric resonance

e.g., $\omega_{k\ \mathrm{nonosc}} >> |\delta M|$



potential opens

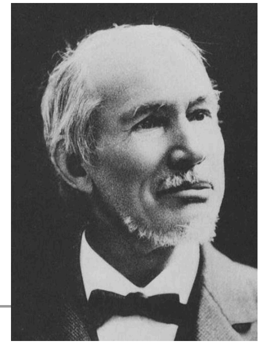
only $\omega_{k_{\mathrm{nonosc}}}$ whose phases match to $\delta M(t)$ can grow

even if the amp. of oscillation is tiny → Resonant growth

Physics of swing

Hill's equation

George William Hill



linearized equation for $\delta\phi(\tilde{t},\, \tilde{x}^i) \equiv \phi(\tilde{t},\, \tilde{x}^i) - \phi_{\mathrm{bg}}(\tilde{t})$

gauge field $\phi F \tilde{F}$

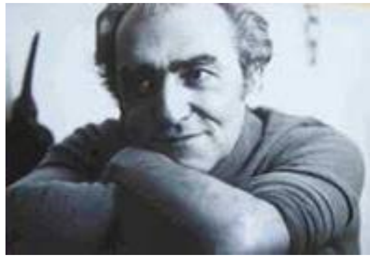
$$\frac{d^2 y_k(\tilde{t}\,)}{d\tilde{t}^2} + \omega_k^2(\tilde{t}\,) y_k(\tilde{t}\,) = 0 \qquad \tilde{t} \equiv mt$$

$$\omega_k^2 = A_k - 2q\psi(\tilde{t}\,) \qquad \psi(\tilde{t}\,) = \psi(\tilde{t} + T)$$

$A_k,\, q \in$ **R** e.g. Mathieu eq. $\psi(\tilde{t}\,) = \sin(2\tilde{t}\,)$

Floquet theorem

Gaston Floquet (1883)



Floquet theorem states

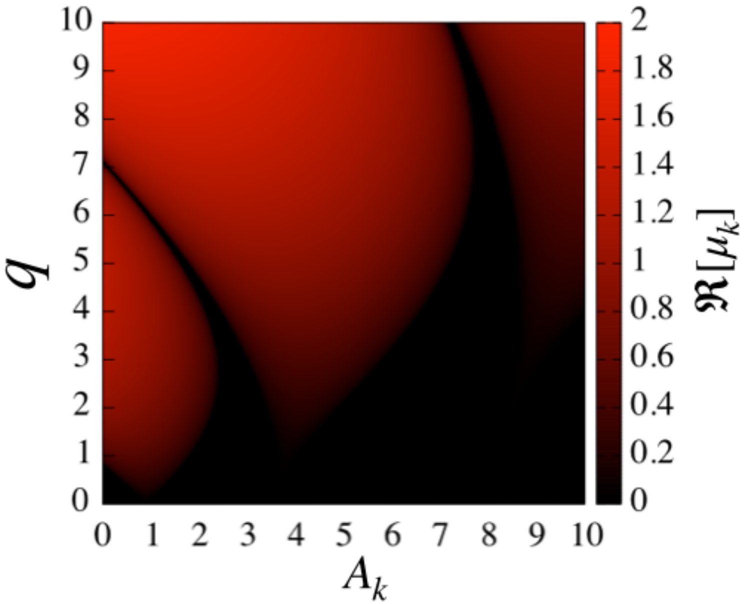
A solution of Hill's equation $y_k(\tilde{t}\,)$ is given by

$$y_k(\tilde{t}\,) = P_1(\tilde{t}\,)\, e^{\mu_k \tilde{t}} + P_2(\tilde{t}\,)\, e^{-\mu_k \tilde{t}} \qquad P_i(\tilde{t}\,) = P_i(\tilde{t} + T)$$

μ_k : Floquet exponent

$$\psi(\tilde{t}\,) = \sin(2\tilde{t}\,)$$

Mathieu eq.



red: unstable

black: stable

A_k → which k?

q → which model?

Parametric Resonance in cosmology

Cosmic expansion makes Bose enhancement inefficient.

$q(t) \propto \phi_{\rm bg}^{\#}(t) \propto \rho_{\rm bg}^{\#\#}(t) \quad (\#, \#\# > 0)$

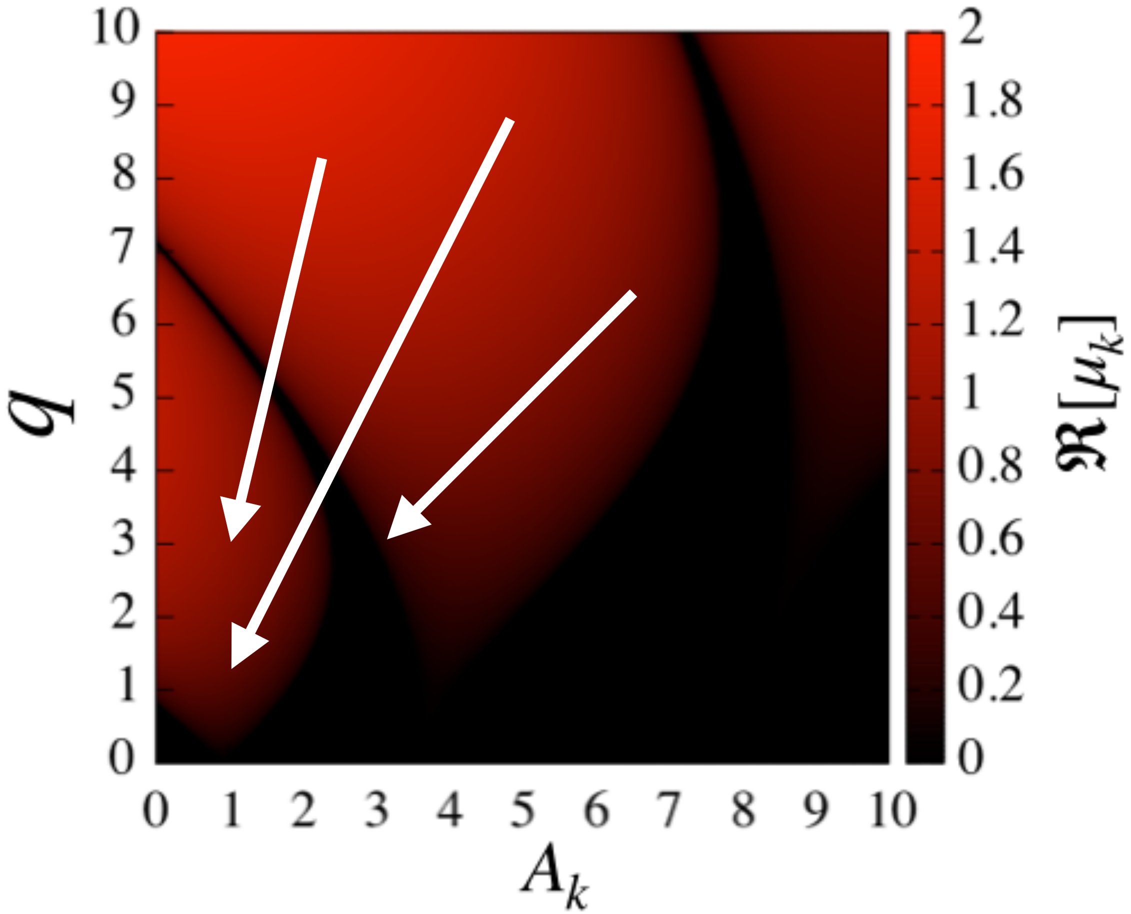
$$A_k(t) \sim \left(\frac{k}{a(t)m}\right)^2$$

How fast?

oscillation *m*

vs

expansion *H*



Parametric Resonance in cosmology

Cosmic expansion makes Bose enhancement inefficient.

$$q(t) \propto \phi_{\rm bg}^{\#}(t) \propto \rho_{\rm bg}^{\#\#}(t) \quad (\#,\ \#\# > 0) \qquad A_k(t) \sim \left(\frac{k}{a(t)m}\right)^2$$

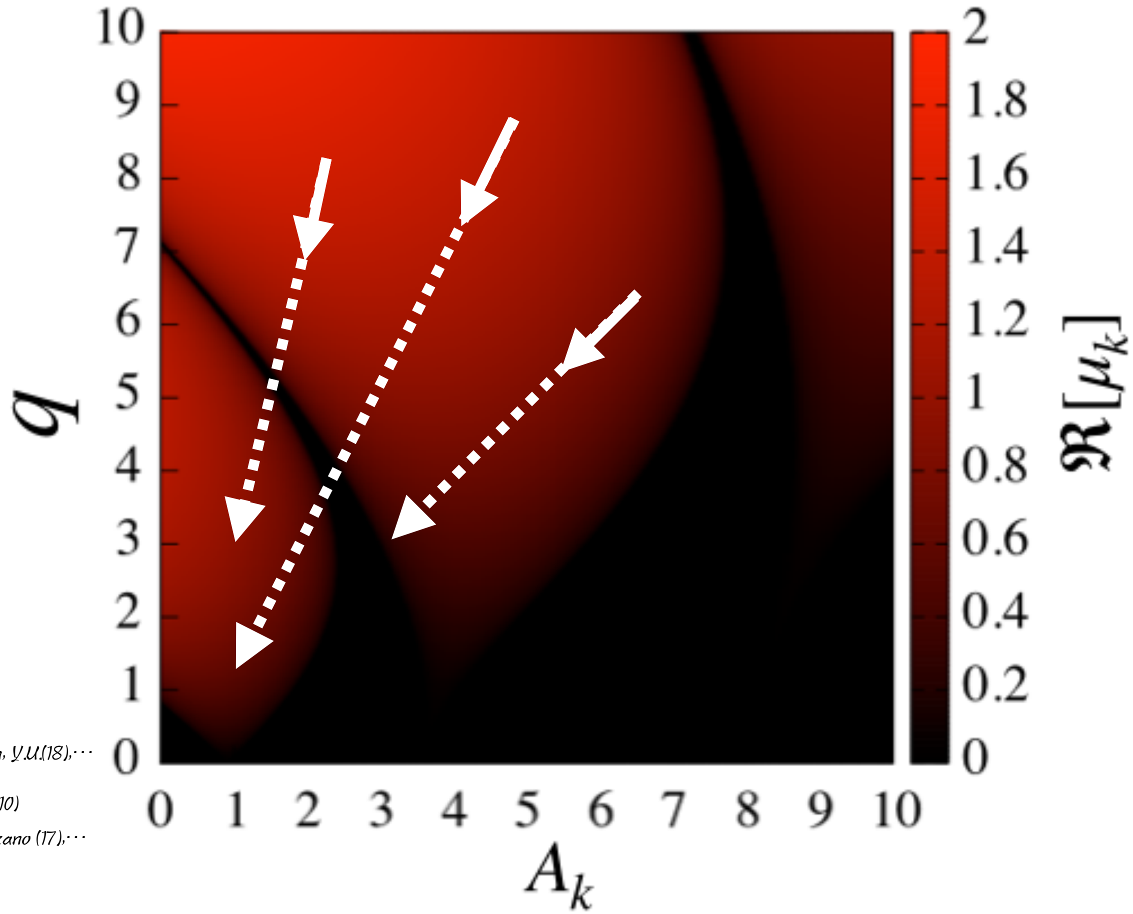
How fast?

oscillation m

$\vee$
$\vee$

expansion H

~ PR in flat sp.



GW emission Antusch, Cefala, Orani,, (14, 17), Soda, Kitajiam, Y.U.(18),⋯

Oscillon formatic Kasuya et al.(03) Amin & Shirokoff(10) Amin et al.(2010) Amin et al.(2014) Amin & Lozano (17),⋯

Efficient particle production,…

Setup of the problem (Parametrization)

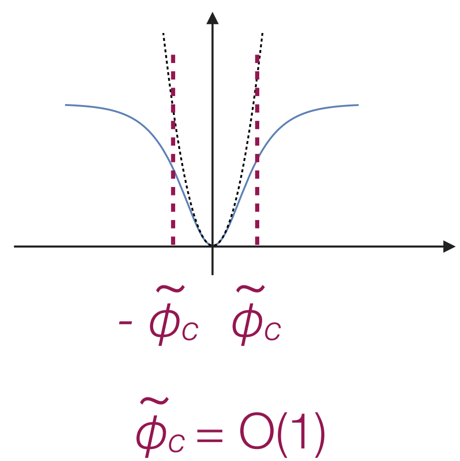
$$V(\phi) = (mf)^2\,\tilde{V}(\tilde{\phi})\,, \qquad \tilde{\phi} \equiv \frac{\phi}{f}$$

where $\tilde{V}(\tilde{\phi})$ satisfies (in canonically normalized fame)

(1) Z_2 symmetry

(2) $\tilde{V}(\tilde{\phi}) \to \tilde{\phi}^2/2 \text{ in the limit } \tilde{\phi} \to 0$

$$m^2 \equiv \left.\frac{d^2V}{d\phi^2}\right|_{|\phi|\ll f}$$



Scalar potential of axion

continuous shift sym.

$\phi \to \phi + c$

→ NP effects, e.g. instanton effects →

$\phi \to \phi + 2\pi n/f$

$n \in \mathbf{Z}$

$V(\phi) \sim \Lambda^4 \cos\phi/f$

Potential can be more flatten than $\cos\phi/f$

i) ~~Dilute instanton gas approximation~~

see. implications for axion=inflaton, *Nomura + (17, 18)* $V(\phi) = M^4 \left[1 - \frac{1}{(1+(\phi/F)^2)^p}\right]$

ii) Non-min. coupling w/gravity, Non-canonical kinetic term

Recall α attractor model for Re[T] *Kallosh, Linde, Roest, $\cdots$. (13, 14, $\cdots$)*

etc…

When m >> H ? : ϕ = dominant
e.g. ϕ: inflaton

i.e., Oscillation becomes much faster than cosmic exp.

Friedmann equation

$$\left(\frac{H}{m}\right)^2 = \frac{f^2}{6M_{\rm pl}^2}\left[\underbrace{\left(\frac{d\tilde{\phi}}{dmt}\right)^2}_{O(1)} + \underbrace{2\tilde{V}(\tilde{\phi})}_{O(1)}\right] \longrightarrow \frac{H_{\rm osc}}{m} \simeq \frac{f}{M_{\rm pl}}$$

if $\frac{f}{M_{\rm pl}} \ll 1$ recall WGC $\longrightarrow \frac{H_{\rm osc}}{m} \ll 1$

Recall natural inflation w/ $\tilde{V}(\tilde{\phi}) = 1 - \cos\tilde{\phi}$ f > $M_{\rm pl}$

Analysis w/o mass term, see *Amin & Lozanov (17)*

When m >> H ? : $\phi \neq$ dominant   e.g. ϕ: dark matter

i.e., Oscillation becomes much faster than cosmic exp.

(Time scale of cosmic exp.)		(Time scale of V driven motion)
$1/H$		$\sqrt{\lvert V_\phi/\phi \rvert} = \sqrt{\lvert \tilde{V}_{\tilde{\phi}}/\tilde{\phi} \rvert}/m$
Slow-roll	<<	
Onset	~	
Oscillation	>>	

recall $\partial_t^2 \phi + V_{,\phi} = 0$

if initially $\left| \frac{\tilde{V}_{,\tilde{\phi}}}{\tilde{\phi}} \right|$ <<1, $\frac{H_{\rm osc}}{m} \sim \sqrt{\left|\frac{\tilde{V}_{\tilde{\phi}}}{\tilde{\phi}}\right|} \ll 1$

e.g. $\tilde{V}(\tilde{\phi}) = 1 - \cos\tilde{\phi}$

with $\tilde{\phi}_i \sim \pi$

N.B. The dynamics is independent of *f*.

Hill’s equation in cosmology

a field coupled w/ coherently oscillating scalar field

$$\frac{d^2 y_k(\tilde{t}\,)}{d\tilde{t}^2} + \omega_k^2(\tilde{t}\,) y_k(\tilde{t}\,) = 0 \qquad \tilde{t} \equiv mt$$

$$\omega_k^2 = A_k - 2q\psi(\tilde{t}\,) \qquad A_k,\, q \in \mathbf{R}$$

for $H_{osc}/m << 1$ $\qquad \psi(\tilde{t}) \simeq \psi(\tilde{t} + T)$

What characterizes the resonance for general Hill’s eq.?

efficiency, structure of the spectrum, etc…

What characterizes general Hill's equation?

Violation of the adiabatic condition? *Fukunaga, Kitajima, Y.U. (19)*

No, even for Mathieu eq.

e.g. $$\left|\frac{d\omega_k/d\tilde{t}}{\omega_k^2}\right| = \frac{q|d\psi(\tilde{t}\,)/d\tilde{t}|}{|A_k - 2q\psi(\tilde{t}\,)|^{3/2}} \gg 1$$ $A_k \sim q$

Coming back to the basics,…

$$\omega_k^2 = \boxed{A_k} - 2\boxed{q}\psi(\tilde{t}\,)$$

amplitude of non-oscillatory part

amplitude of oscillatory part

Fukunaga, Kitajima, Y.U. (19)

see Hayato's talk tomorrow

Brute force analysis of Hill's equation

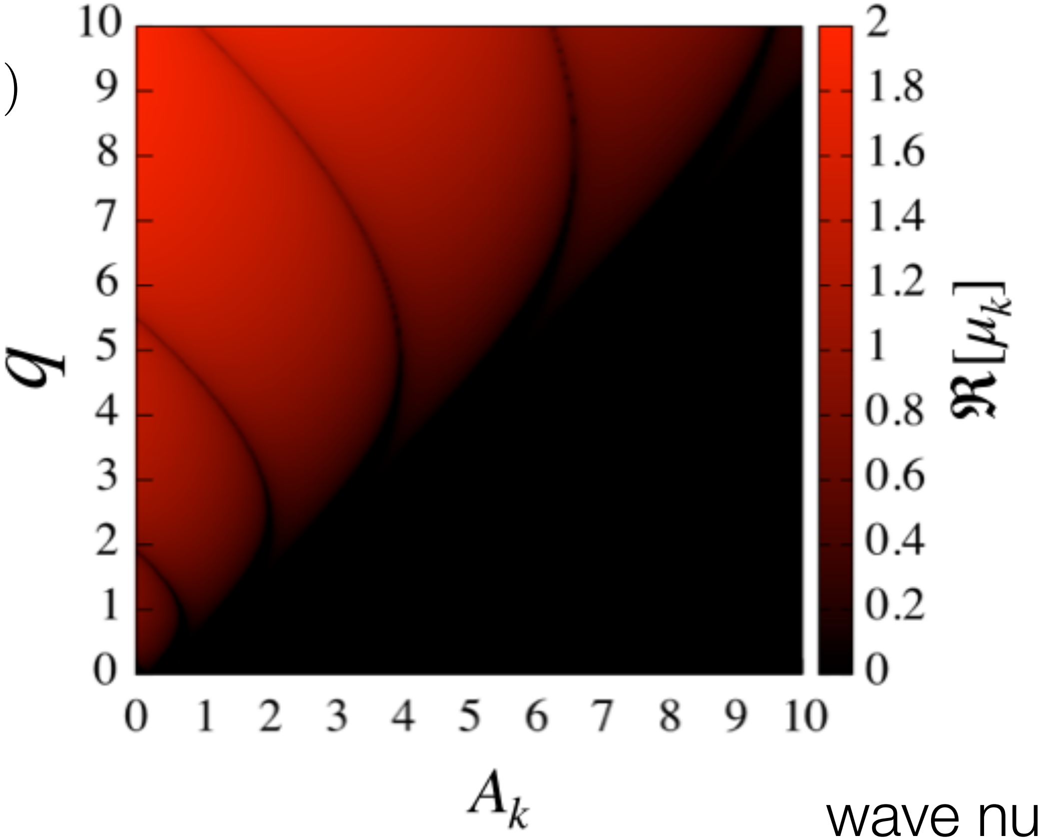
stability/instability chart is "generically" characterized by (A_k, q)

$$\omega_k^2 = A_k - 2q\psi(\tilde{t}\,) \qquad q = \sqrt{\frac{\langle(\omega_k^2 - \langle\omega_k^2\rangle)^2\rangle}{2}} \qquad A_k = \langle\omega_k^2\rangle$$

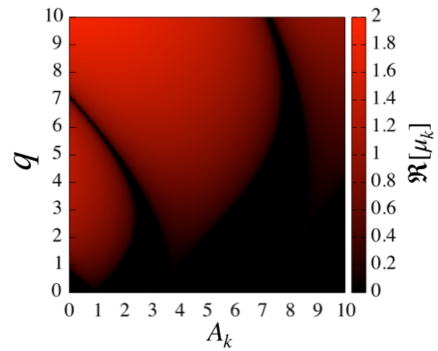
$$\langle F(\tilde{t})\rangle \equiv \frac{1}{T}\int_{\tilde{t}-\frac{T}{2}}^{\tilde{t}+\frac{T}{2}} d\tilde{t}'\,F(\tilde{t}')$$

$\psi(\tilde{t}\,) = \sin(2\tilde{t}\,)$

model



$\psi(\tilde{t}\,) = \sin(2\tilde{t}\,)$



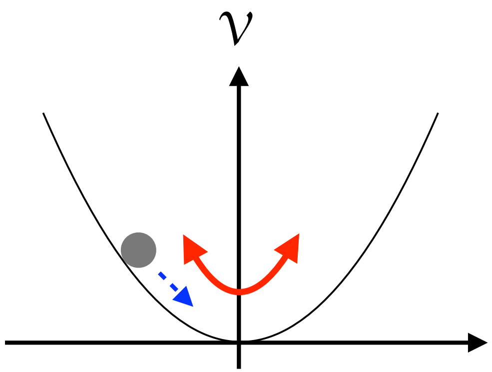
wave number

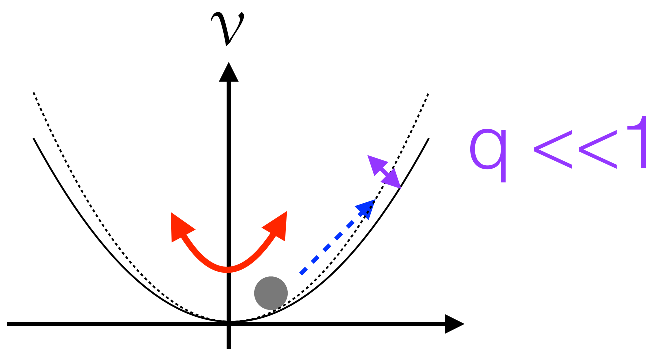
Fukunaga, Kitajima, Y.U. (19)

“General” property of Hill’s equation

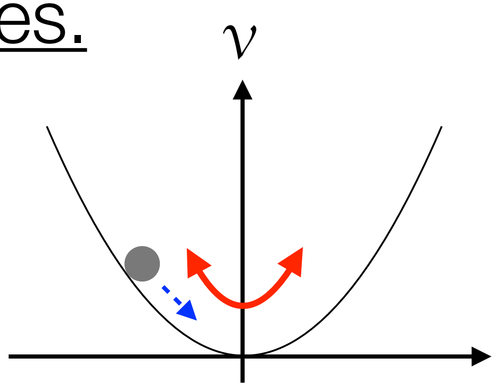
large q (large opening angle)→ rapid growth, wide band

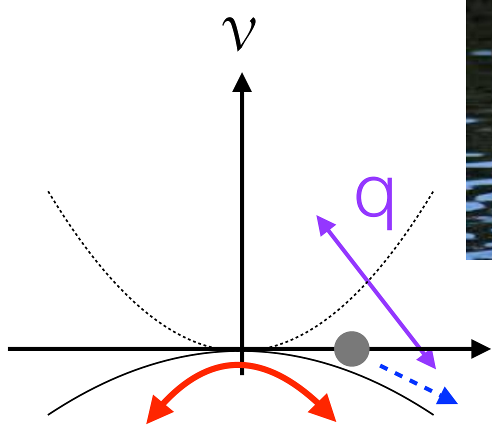
Narrow res.





Flapping res.





Kitajima, Soda, Y.U. (18)

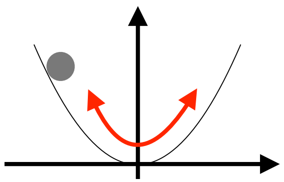
Parametric Resonance in cosmology

Parametric Resonance can be efficient and sustainable also in expanding universe.

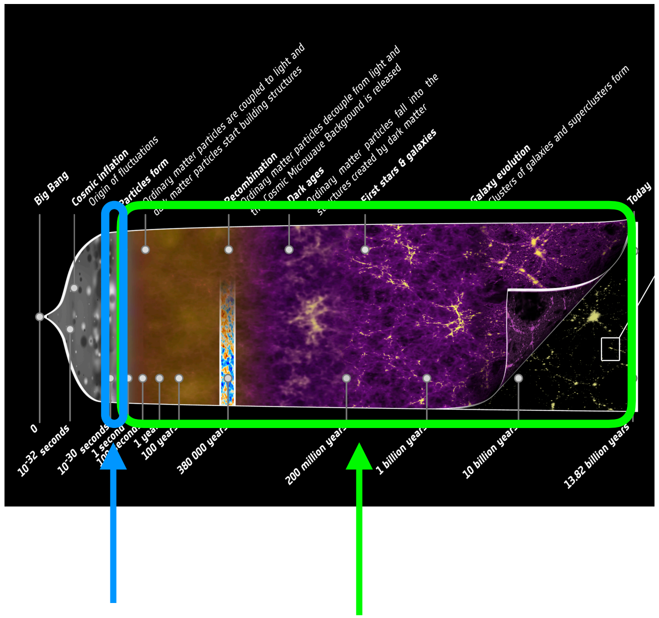
Ex1: Reheating

inflation

creation of SM particles



radiation dom.



Antusch, Cefala, Orani,, (14, 17), ⋯.

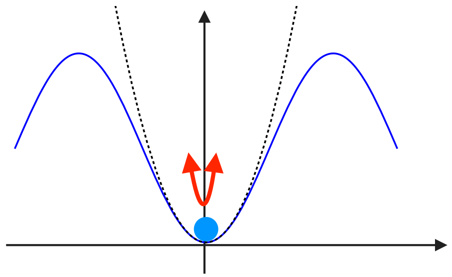
Kasuya et al.(03) Amin & Shirokoff(10)

Amin et al.(2010) Amin et al.(2014) Amin & Lozano (17),⋯

see Kaloian & Paco's talks

HERE

Ex2: Axion cosmology



~ dark matter

Contents

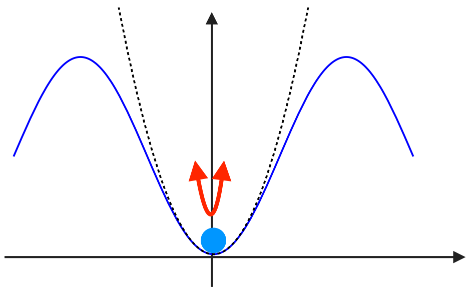
Bottom-line story

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{\rm TT} \qquad \partial_i h_{ij} = 0 \text{ and } h_{ii} = 0$$

non-linear

h: +, -

homogeneous axion



resonance instability

$\phi F\tilde{F}$

$\delta\phi \longrightarrow h_{ij,\,h}$

$$\Pi_{ij}^{\rm TT} = \frac{1}{a^2}P_{ij}^{lm}\partial_l\phi\partial_m\phi$$

$A_+ \neq A_- \longrightarrow h_{ij,\,+} \neq h_{ij,\,-}$

$$\Pi_{ij}^{\rm TT} = \frac{1}{a^2}P_{ij}^{lm}F_{l\mu}F_m^{\ \mu}$$

$$\epsilon_{ij}^{(\pm)} \sim \epsilon_i^{(\pm)}\epsilon_j^{(\pm)}$$

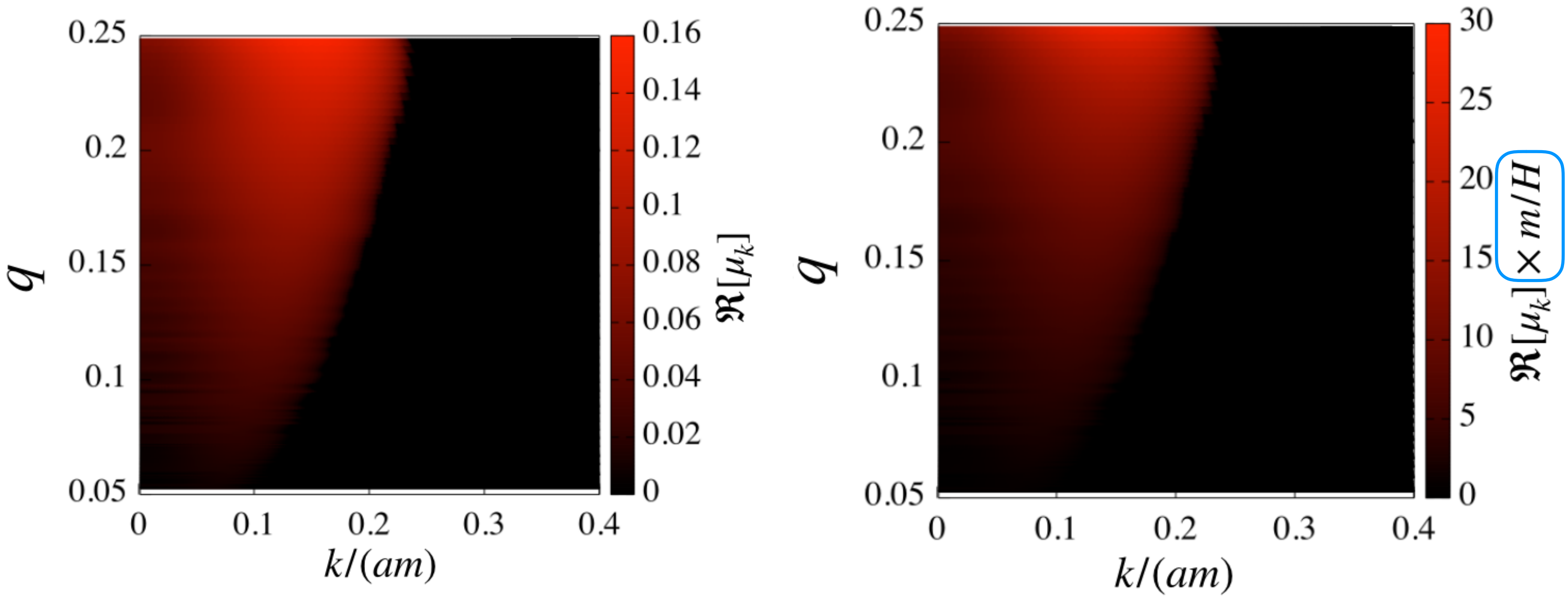
Fukunaga, Kitajima, Y.U. (19)

Parametric res. during RD

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}\left[1 - \frac{1}{(1+\tilde{\phi}^2/c)^c}\right] \qquad (c>0)$$

Nomura + (17, 18)

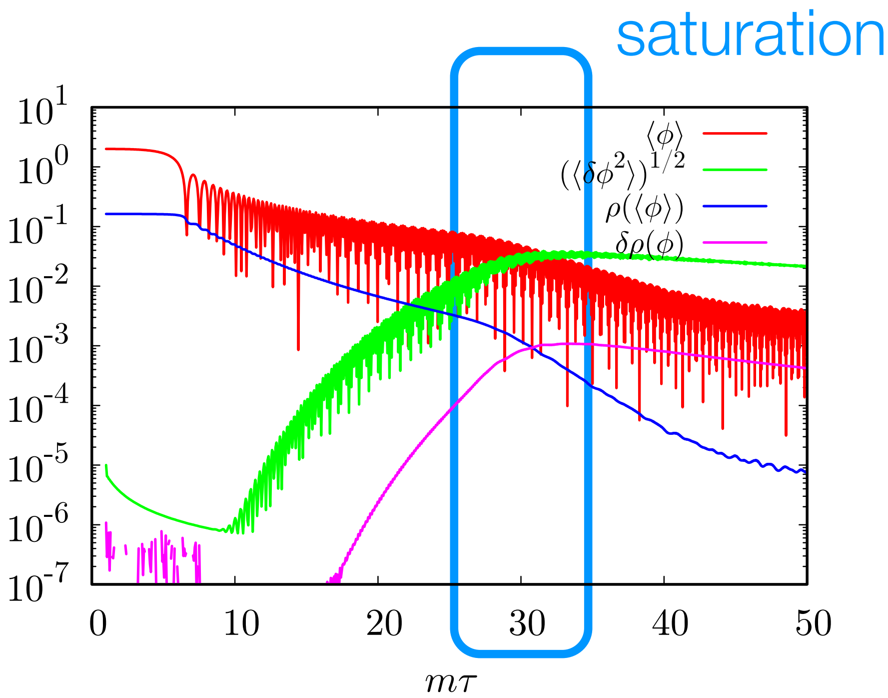
Growth rate Re[μ_k] for linear perturbation (in RD) eg. axion DM



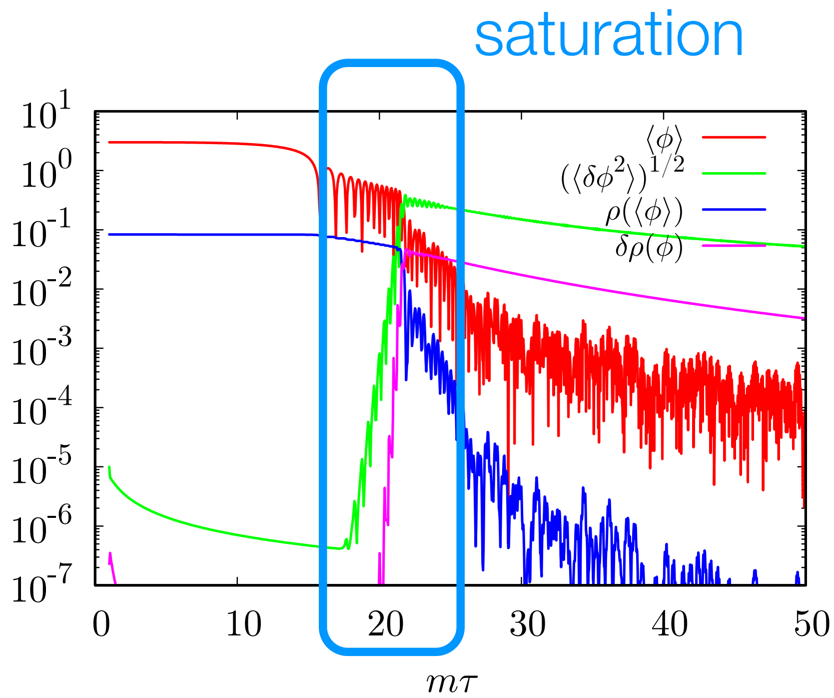
Expo. growth much faster than cosmic exp.

see Naoya's talk tomorrow

Lattice simulation



(b) $c = 2, \phi_i = 2f$



(a) $c = 5, \phi_i = 3f$

Narrow res. dominant

q << 1

Flapping res. dominant

q = O(1)

Cosmic exp. does not stop growth, but backreaction does.

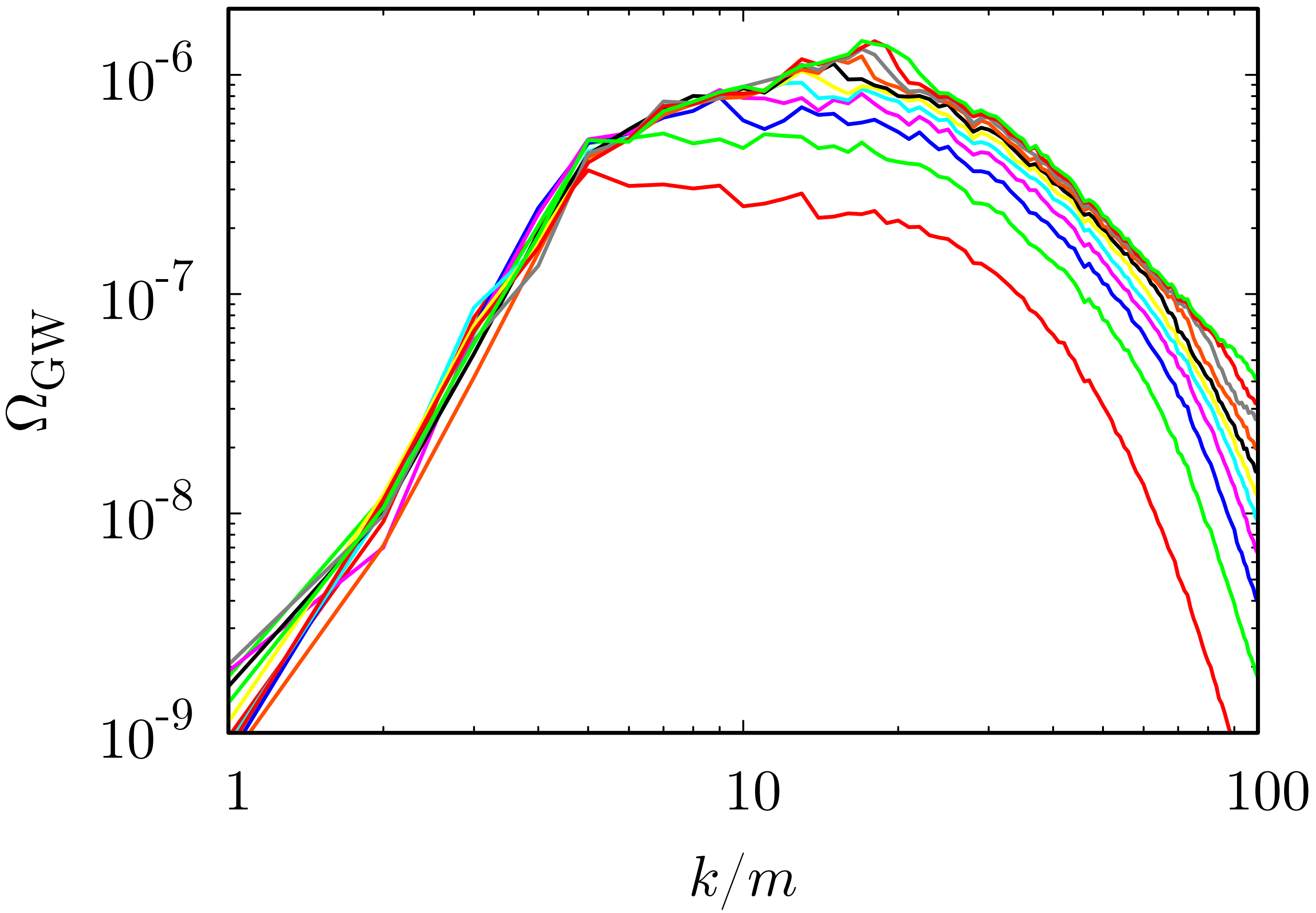
Lattice simulation $N_{grid}=(256)^3$

Kitajima, Soda & Y.U. (18)

see Naoya's talk tomorrow

GW spectrum

α=0

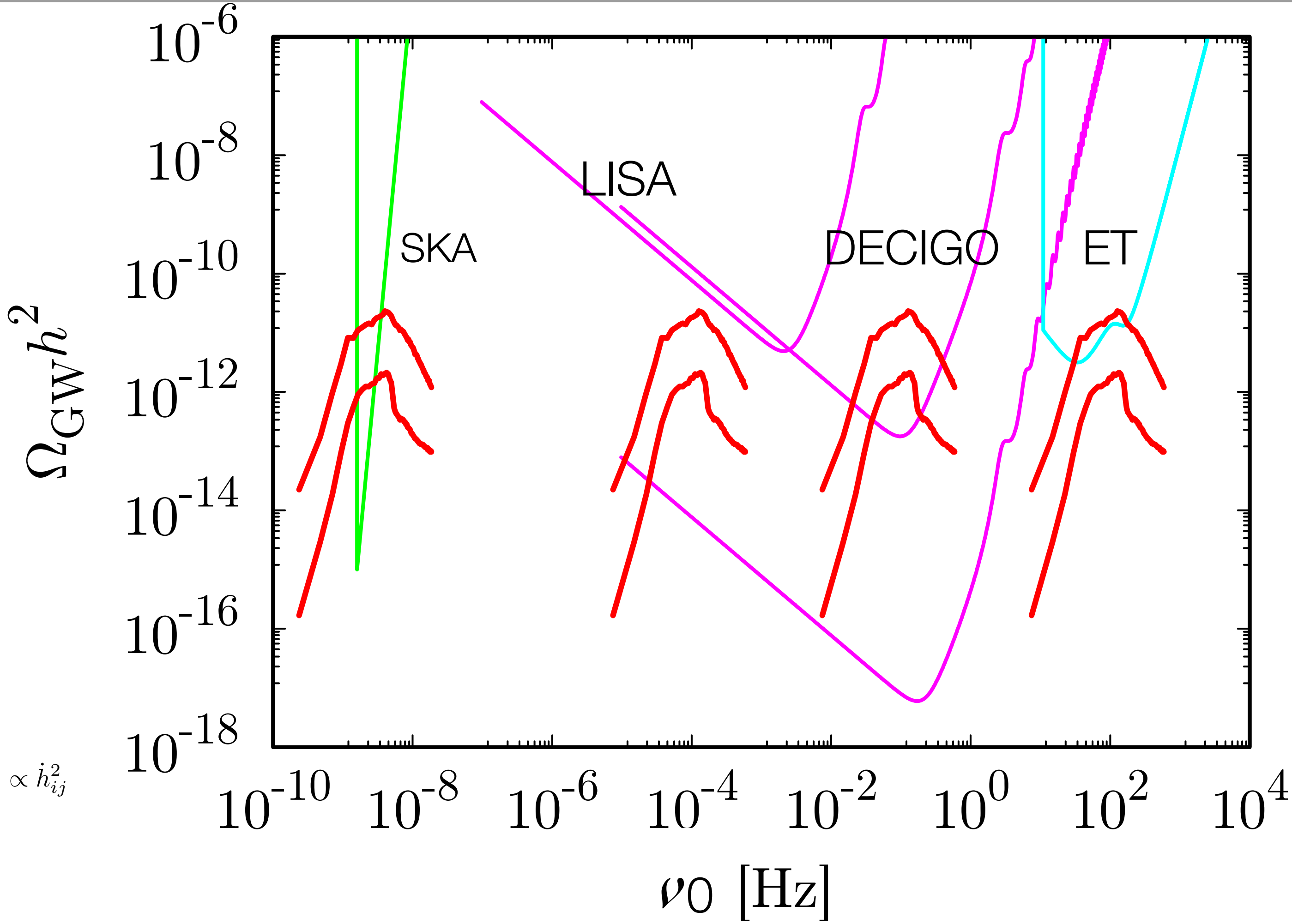


$f = 10^{16}$ GeV, $c = 5$ and $\phi_i = 3f$

to evaluate the present value, x Ω_r

Detectability

Kitajima, Soda & Y.U. (18)

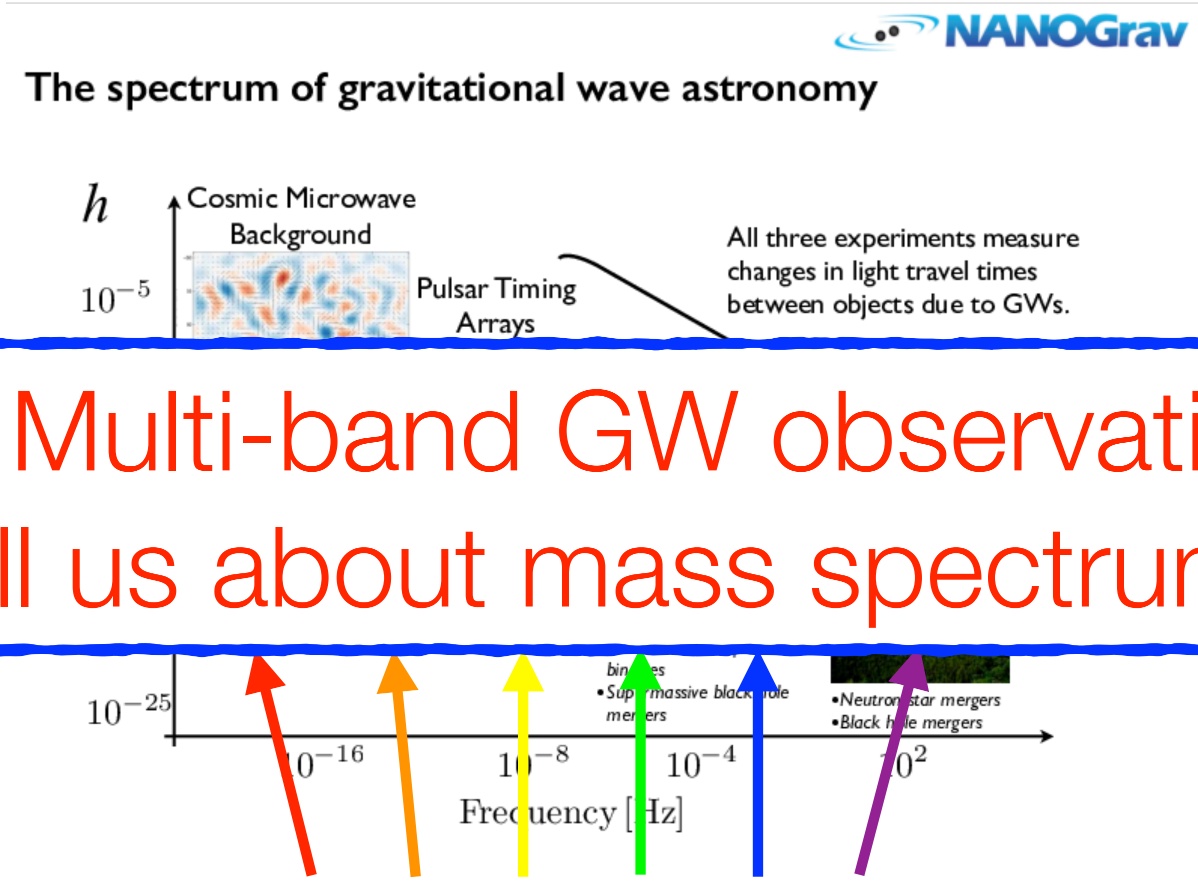


$$\Omega_{\rm GW} = \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln k} \propto \dot{h}_{ij}^2$$

Redshifted frequency $\quad \nu_0 = \frac{\kappa m_{\rm a}}{2\pi}\left(\frac{a_{\rm em}}{a_0}\right) \propto m_{\rm a}^{1/2}$ *Soda & Y.U. (17)*

Amplitude $\quad \Omega_{\rm GW} h^2 \propto f_a^4 \quad$ f_a: decay const.

Multi-wavelength GW era

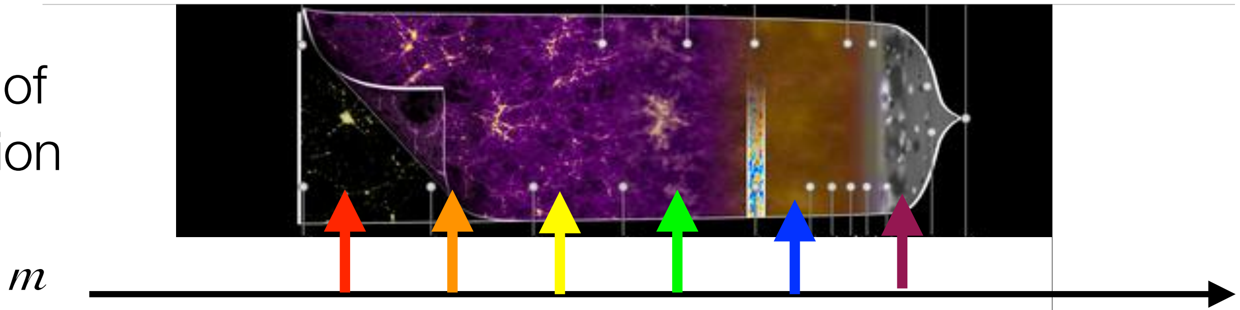


Multi-band GW observations may tell us about mass spectrum of axions

higher freq.

earlier onset of oscillation

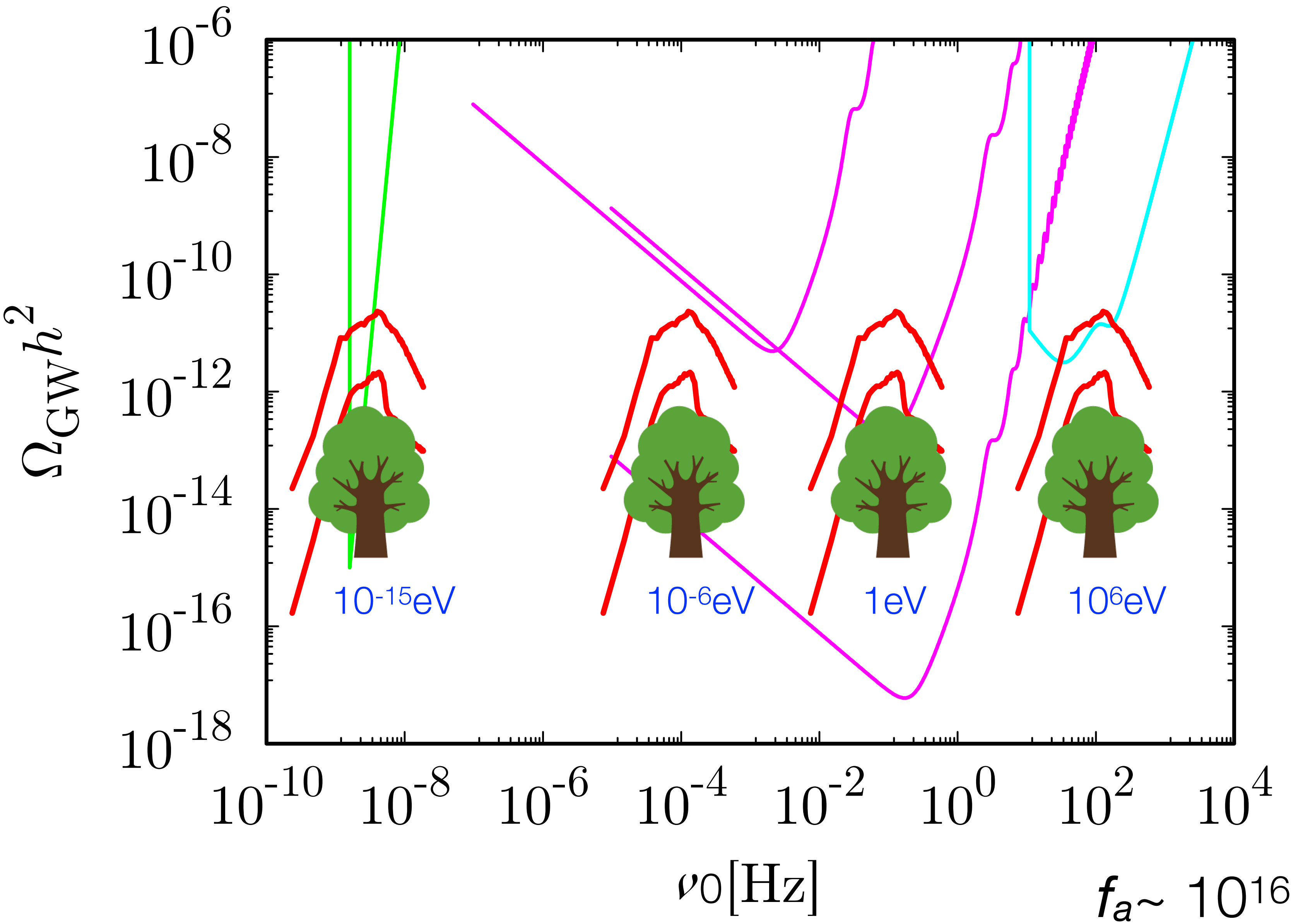
Onset of oscillation



heavier axion

GW forest

Kitajima, Soda & Y.U. (18)



$f_a \sim 10^{16}$ GeV

e.g.,Svrcek & Witten (06)

$\alpha = 0$

GWs from axion DM

Kitajima, Soda, Y.U. (18)

freq. of GW ν_0 ⟶ mass m

\+

abundance of axion ⟶ decay const. f x mass m

Crude Order estimation

using $\varphi(t, x) \sim f\,(a_{\rm osc}/a)^{3/2}$

Δ : Sym. suppression (< 1)

$$\Omega_{\rm GW}h^2 \simeq 0.8\times 10^{-18}\kappa^4\Delta^2\left(\frac{\rm nHz}{\nu_0}\right)^2(\Omega_\phi h^2)^2$$

for κ=10 $\Omega_{\rm GW}h^2 \simeq 10^{-16}$ at $\nu_0 = $nHz

or lower frequency btwn CMB & PTAs?

Prospects on polarized GW forest

$\alpha \neq 0$

Kitajima, Soda & Y.U. (in prep.)

What about $\alpha \neq 0$?

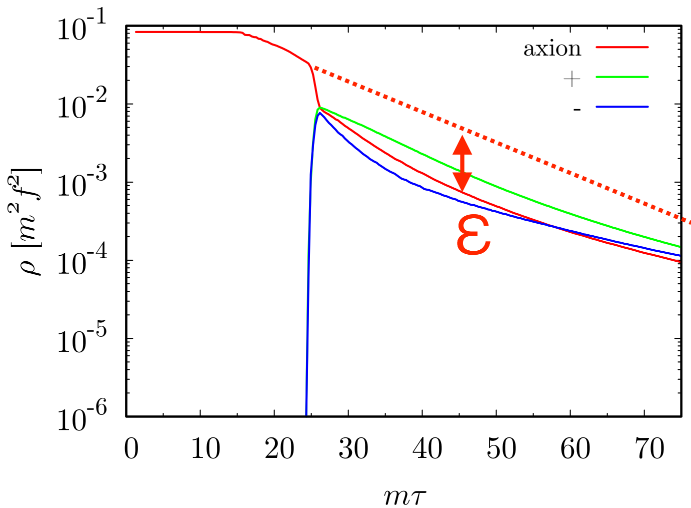
- GW circularly polarized

  see also *Adshead* +(18)

- More prominent GW emission

  - Larger Δ (Less symmetric)

  - Weaker abundance restriction



GW from axion DM

$$\Omega_{\rm GW}h^2 \simeq 0.8\times 10^{-18}\kappa^4\Delta^2\left(\frac{\rm nHz}{\nu_0}\right)^2(\Omega_\phi h^2)^2 \quad \times\frac{1}{\varepsilon^2}$$

Contents

Chern-Simon coupling $\mathcal{L}_{int} = \frac{\alpha}{4}\frac{\phi}{f}F\tilde{F}$

Coulomb gauge ($A_0=0$, $\partial_i A_i=0$)

$$\frac{d^2\mathcal{A}_h}{d\tilde{t}^2} + \frac{H}{m}\frac{d\mathcal{A}_h}{d\tilde{t}} + \omega_h^2\mathcal{A}_h = 0 \qquad \omega_h^2 \equiv \left(\frac{k}{am}\right)^2 - h\alpha\frac{d\tilde{\phi}}{d\tilde{t}}\frac{k}{am}$$

Circular polarization bases h = +, -

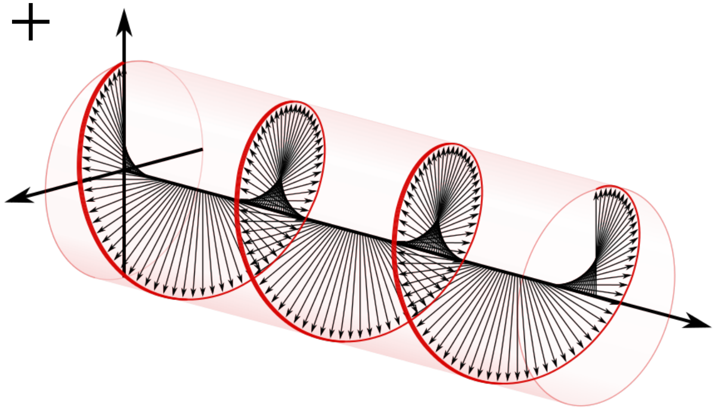
2 polarizations

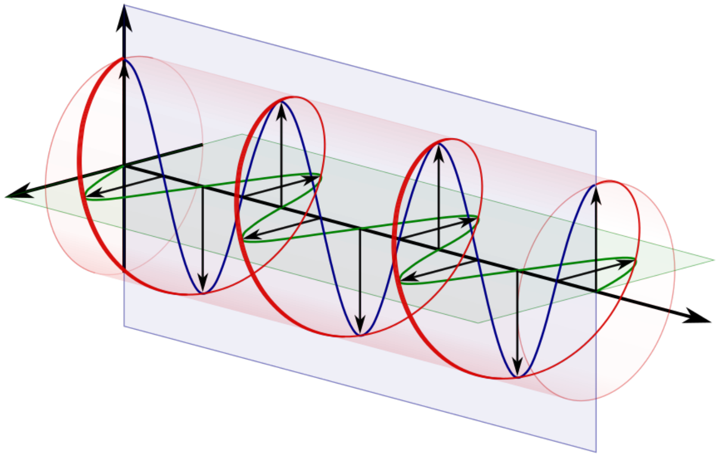
Recall electromagnetic wave

from receiver

Right-handed (Clockwise)

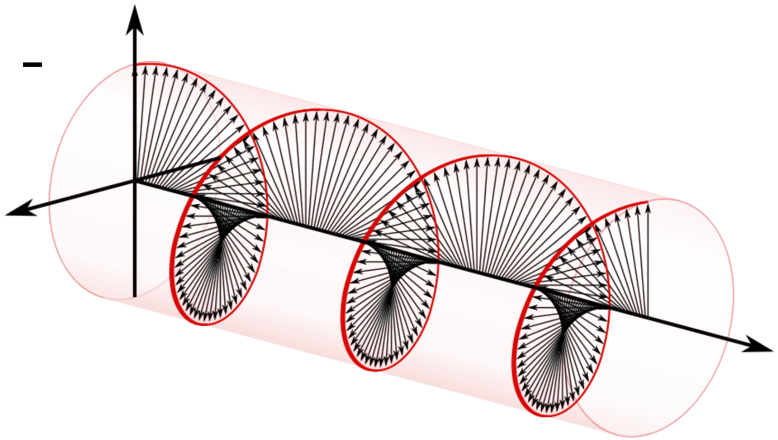
h = +

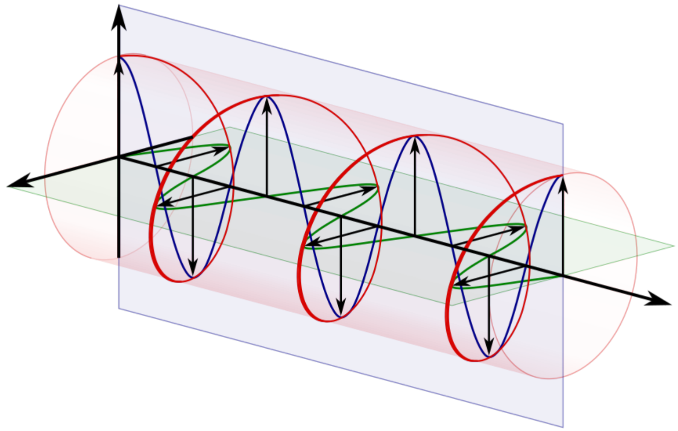




Left-handed (Anti-Clockwise)

h = -



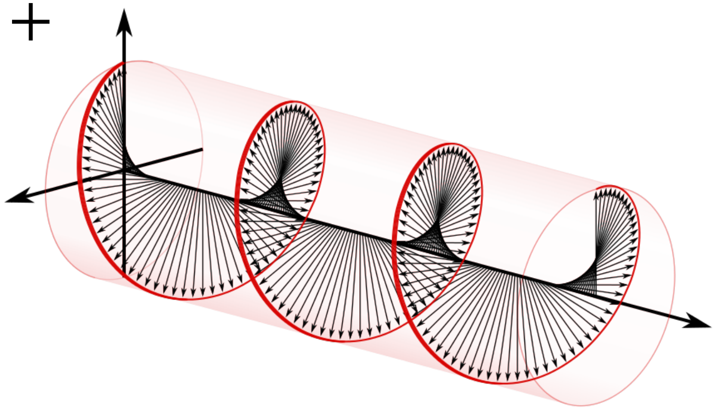


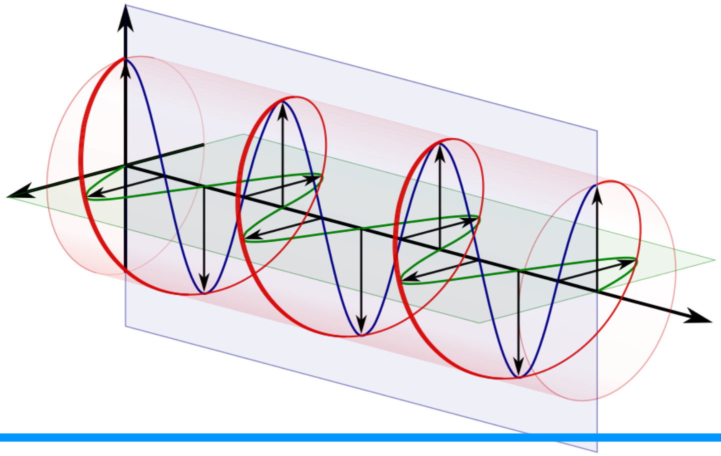
Circular polarization

with $\phi F\tilde{F}$ parity

Right-handed (Clockwise)

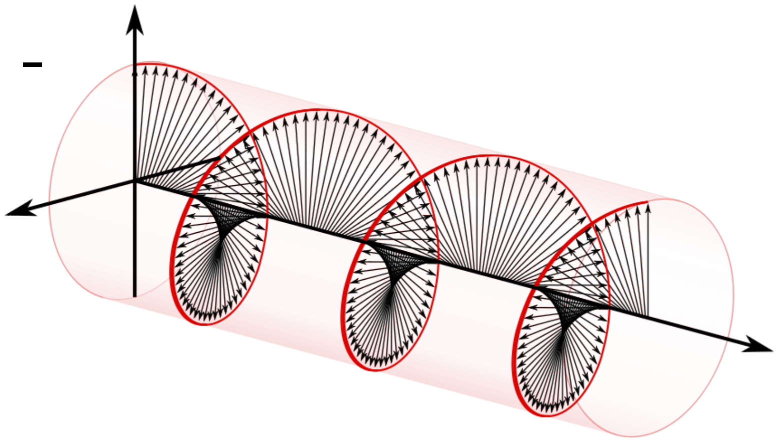
h = +

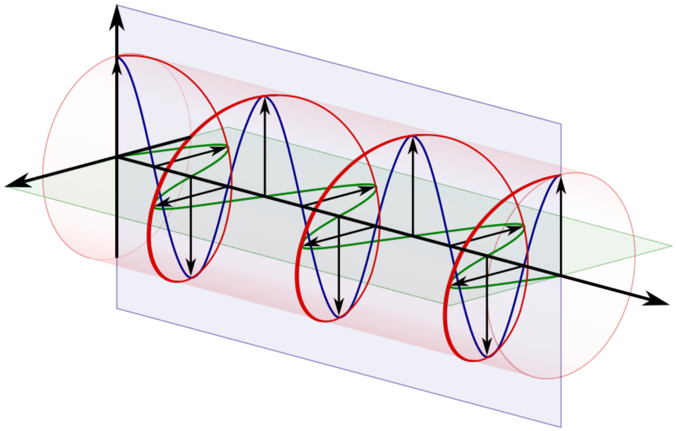




Left-handed (Anti-Clockwise)

h = -



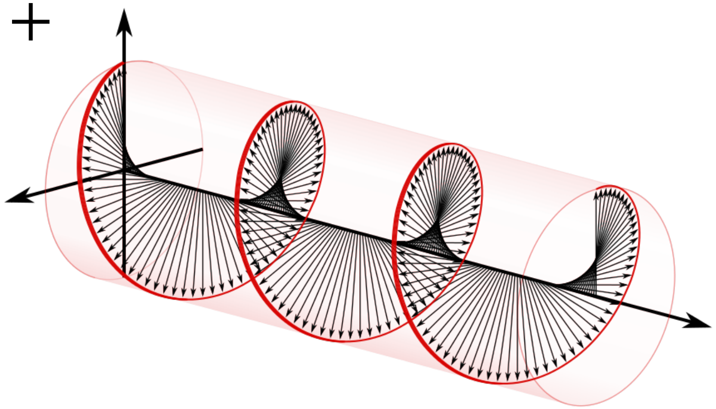


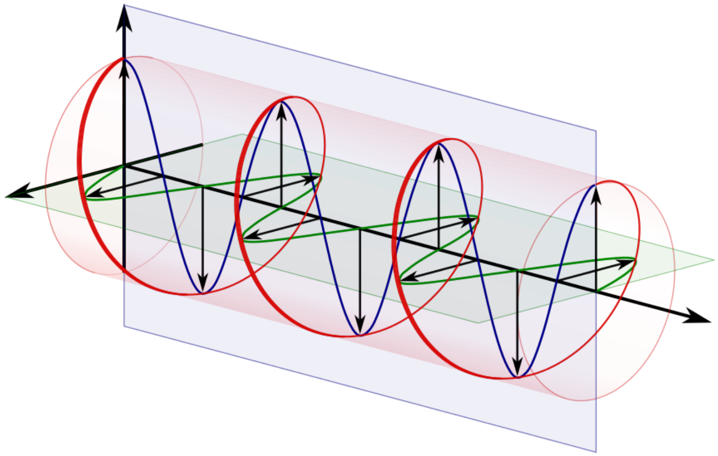
Circular polarization

with $\phi F\tilde{F}$ parity

Right-handed (Clockwise)

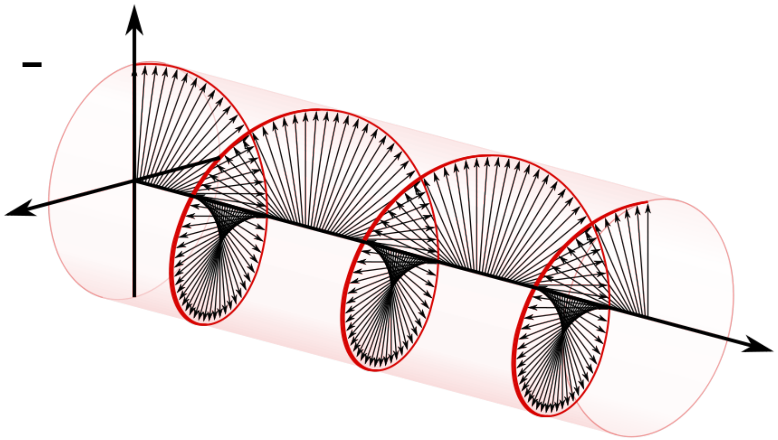
h = +

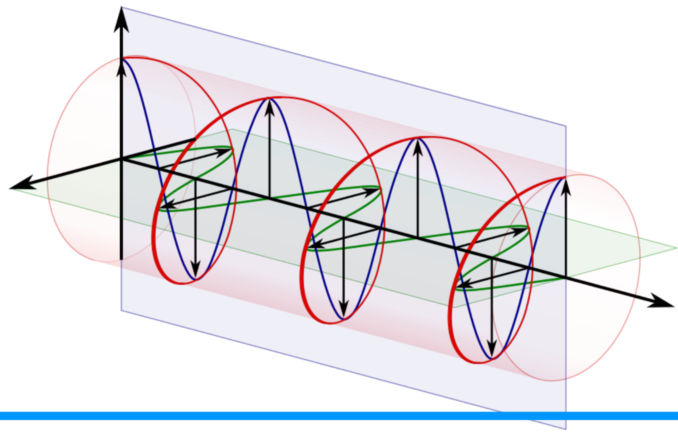




Left-handed (Anti-Clockwise)

h = -



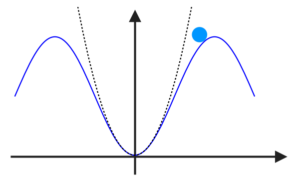


Chern-Simons coupling $\mathcal{L}_{int} = \frac{\alpha}{4}\frac{\phi}{f}F\tilde{F}$

$$\frac{d^2\mathcal{A}_h}{d\tilde{t}^2} + \frac{H}{m}\frac{d\mathcal{A}_h}{d\tilde{t}} + \omega_h^2\mathcal{A}_h = 0 \qquad \omega_h^2 \equiv \left(\frac{k}{am}\right)^2 - h\alpha\frac{d\tilde{\phi}}{d\tilde{t}}\frac{k}{am}$$

1) Monotonic evolution of $\frac{d\tilde{\phi}}{d\tilde{t}}$

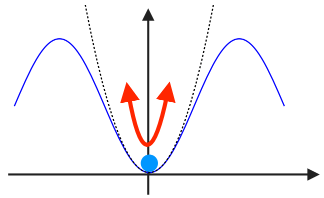
→ Generate circular polarization



2) Oscillation of $\frac{d\tilde{\phi}}{d\tilde{t}}$

Parametric resonance

q ∝ α



α >> 1 Broad resonance

α ~O(1) Flapping resonance

α << 1 Narrow resonance

} persistent for $m/H_{osc} >> 1$

Patel, Kobayashi, Tashiro, Y.U. (in progress)

see Teerthal's talk tomorrow

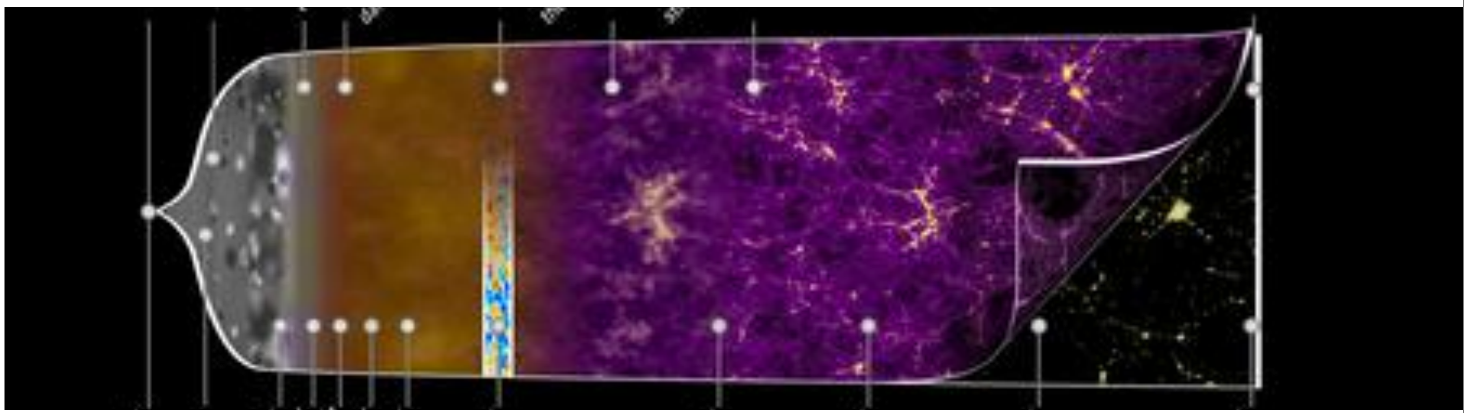
Magnetogenesis from axions

During inflation

1) axion: dominant

2) axion: sub-dominant

During reheating



After reheating till recombination

2) axion: sub-dominant

After recombination

Patel, Kobayashi, Tashiro, Y.U. (in progress)

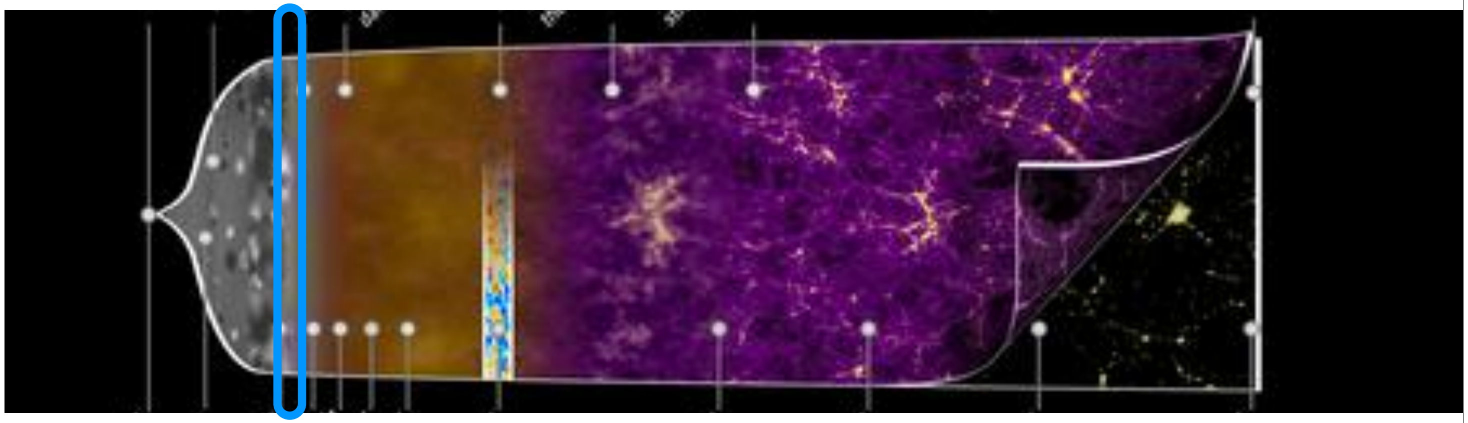
Magnetogenesis from axions

During inflation

👍 Resonant production is possible

👎 coherent length at generation is small

During reheating



After reheating till recombination

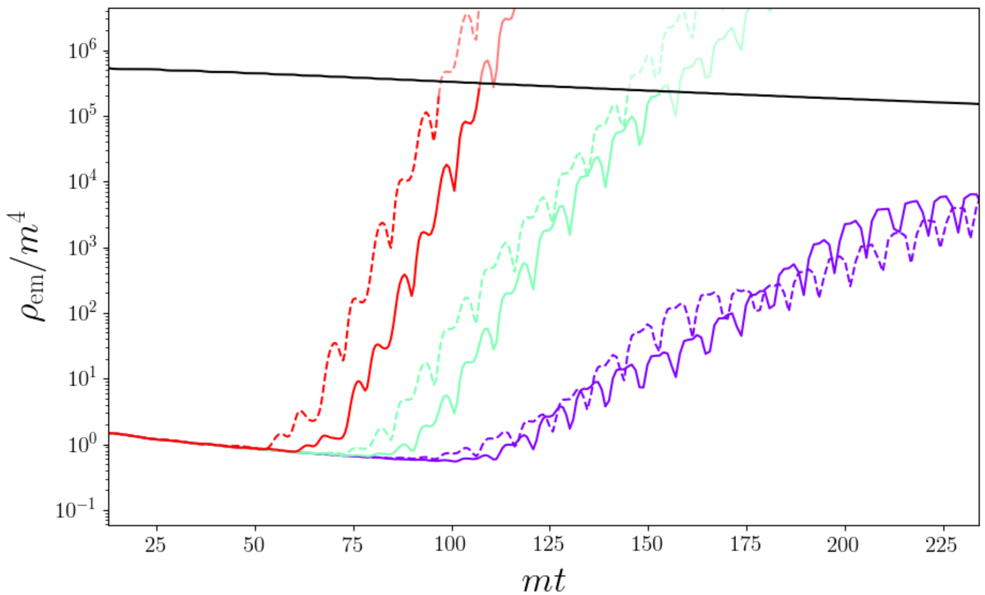
After recombination

Patel, Kobayashi, Tashiro, Y.U. (in progress)

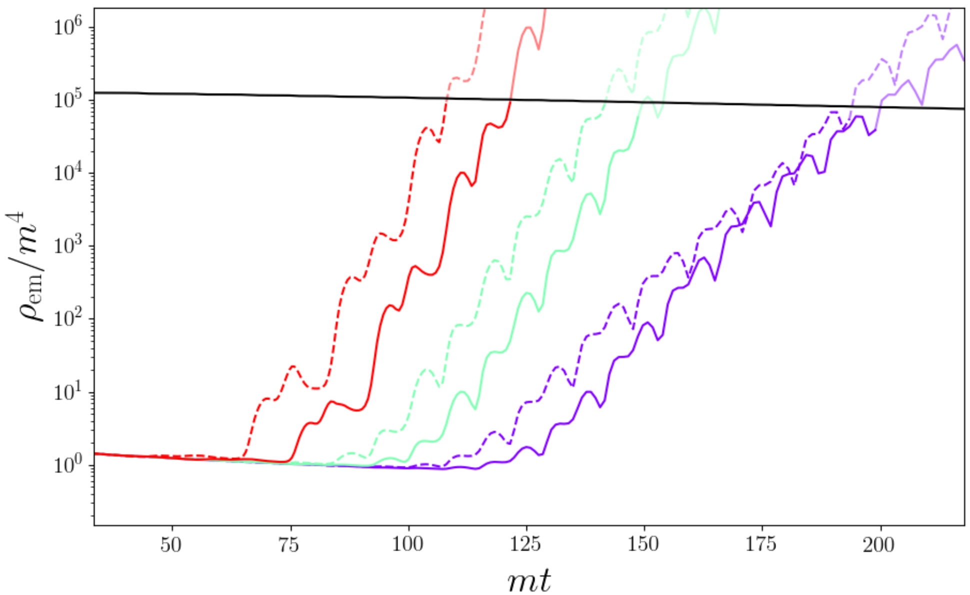
Preliminary

Efficient energy transfer

ϕ=inflaton w/ $\tilde{V}(\tilde{\phi}) = \frac{1}{2}\left[1 - \frac{1}{(1+\tilde{\phi}^2/c)^c}\right]$ $(c > 0)$



$f/M_{\rm pl} = 0.01$

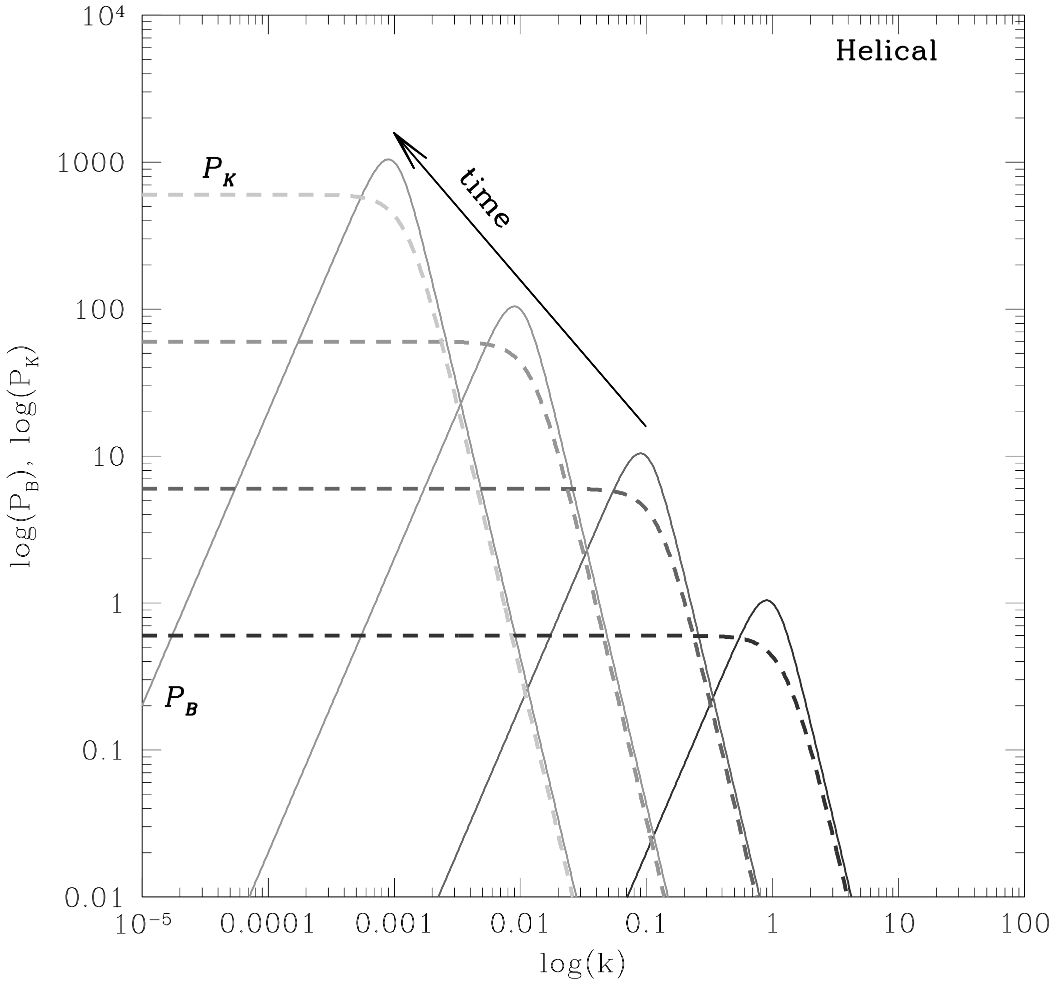


$f/M_{\rm pl} = 0.005$

$\alpha = 0.75$ (purple), $\alpha = 1$ (cyan), and $\alpha = 1.5$ (red)

Inverse cascade

Circularly polarized electro-magnetic field

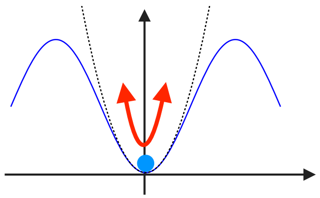


Durrer & Neronov (13)

Patel, Kobayashi, Tashiro, Y.U. (in progress)

Magnetogenesis from axions during inflation

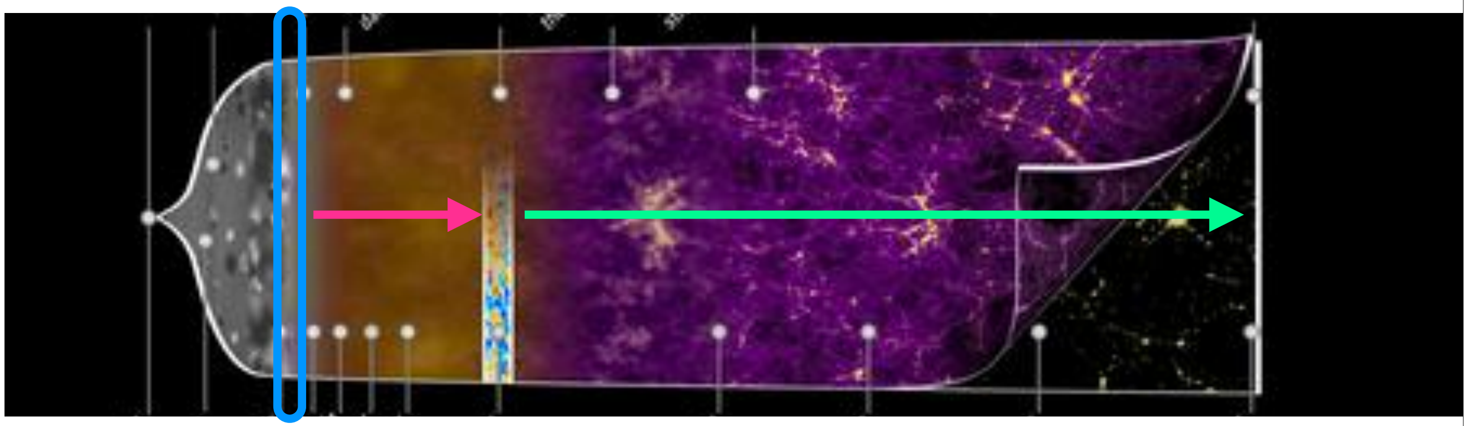
axion=inflaton



$\phi \to A_\mu$

for $\alpha \gg 1$ or $H_{osc}/m \ll 1$

During reheating



Inverse cascade

Adiabatic decay $B \propto 1/a^2(t)$

Order estimation

Inverse cascade until LSS

- helicity density is conserved after the transition

$$\langle h \rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} k \left(|\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 \right) \simeq \Delta(\ln k)\, a^3 \frac{a}{k_m} \mathcal{B}_{k_m}^2$$

(k_m: peak)

- dissipation below the turbulence scale

$$\frac{a_{rec}}{k_{m,rec}} \simeq v_{A,rec} H_{rec}^{-1} \simeq \frac{\mathcal{B}_{k_{m,rec}}(\eta_{rec})}{\sqrt{\rho_{rec}}} H_{rec}^{-1} \longrightarrow B_{k_m,0} = 10\,\mathrm{nG} \times \frac{\lambda_{m,0}}{\mathrm{Mpc}}$$

Adiabatic evolution after recombination

$$\lambda_{m,0} = \frac{a_0}{a_{\mathrm{rec}}} \frac{a_{\mathrm{rec}}}{k_{m,\mathrm{rec}}} \qquad B_{k_m,0} = \left(\frac{a_0}{a_{\mathrm{rec}}} \right)^2 B_{k_m,\mathrm{rec}}$$

Patel, Kobayashi, Tashiro, Y.U. (in progress)

Preliminary

Crude Order estimation

Assumptions

- helicity density is conserved after generation (→ reheating)
- (Hyper) B field generates completes before plasma creation
- B dissipates below the turbulence scale (after plasm creation)

$$B_{k_m,0} = 10\,\mathrm{nG} \times \frac{\lambda_{m,0}}{\mathrm{Mpc}}$$

$$\lambda_{m,0} \simeq 3.3 \times 10^5 \mathrm{Mpc} \left(\frac{a_\star}{a_0}\right) \left(\frac{\lambda_{m,\star}}{\mathrm{GeV}^{-1}}\right)^{\frac{1}{3}} \left[\frac{\Delta(\ln k)_\star}{\Delta(\ln k)_{rec}}\right]^{\frac{1}{3}} \left(\frac{B_{k_{m,\star}}}{\mathrm{GeV}^2}\right)^{\frac{2}{3}}$$

during reheating $\lambda_{m,0} \lesssim 10^3 \mathrm{Mpc} \left(\frac{10^5 \mathrm{GeV}}{T_{\gamma,\mathrm{R}}}\right) \left(\frac{f/M_{\mathrm{pl}}}{10^{-3}}\right)^{1/3} \left(\frac{r}{0.1}\right)^{\frac{1}{6}} \left(\frac{\mathcal{P}_\zeta}{10^{-9}}\right)^{\frac{1}{6}}$

Summary / Discussion

Parametric Resonance can be efficient and sustainable also in expanding universe.

- GW emission

- Magnetogenesis

Next issue

- Chiral fermion production?