2 day mini-workshop:

Axion Cosmology

Yukawa Institute for Theoretical Physics, Kyoto University
May 14 - 15, 2019

Exploring the string axiverse and parity violation in gravity with gravitational waves

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Kobe University

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1. Traces of the string axiverse
2. Axion dark matter
3. Dynamical Chern-Simons gravity
4. Parametric Resonance
5. Estimations
6. Conclusion

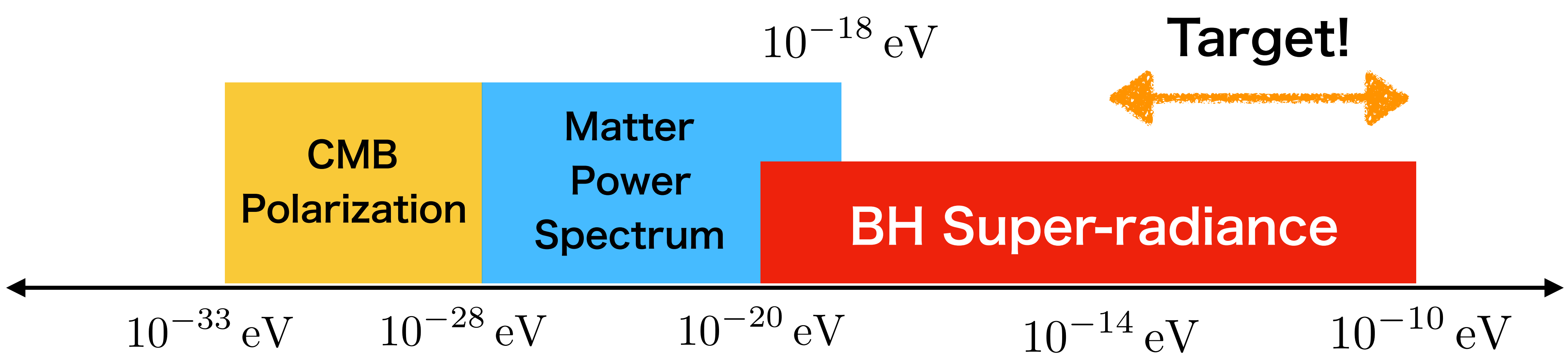
Menu

Traces of String Axiverse

- **String theory gives a lot of massive pseudo-scalar fields (Axion).**

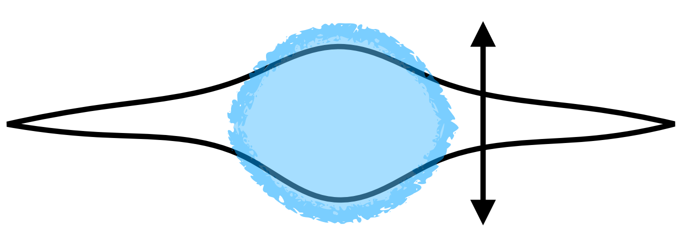
A. Arvanitaki, et al (2010),
P. Svrcek and E. Witten (2006)

- **Their mass is $10^{-33} \sim 10^{-10}\,\mathrm{eV}$. Its range is the very wide.**



- **It is indicated that the axion can behave as the cold dark matter.**

W. Hu, R. Barkana, and A. Gruzinov (2000)



- **The string axiverse gives the parity-violated interactions as follows, $\Phi\tilde{F}F$, $\Phi\tilde{R}R$. In this talk, we concentrate on the modified gravity.**

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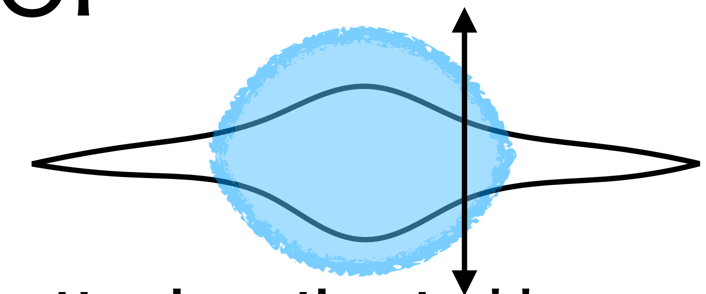
Axion Dark Matter



- The cold dark matter is the dust which is pressureless.

- The occupation number of the state of the axion dark matter is estimated by

$$\frac{N}{\Delta x^3\,\Delta p^3} \sim \frac{n}{p^3} = \frac{\rho_{\rm DM}}{m\,p^3} \sim 10^{43}\left(\frac{\rho_{\rm DM}}{0.3\,{\rm GeV/cm^3}}\right)\left(\frac{10^{-10}{\rm eV}}{m}\right)^4 .$$

A. Khmelnitsky & V. Rubakov, 2014

- The oscillation of the axion as a classical field is given by $\Phi(t) \simeq \Phi_0 \cos(mt)$.

$$v \sim 10^{-3},\, E \simeq m + \frac{1}{2}mv^2 \sim m$$

→ **Energy density :** $T_{00} = \rho_{\rm DM} = \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}m^2\Phi^2 \sim \frac{1}{2}m^2\Phi_0^2$

$$\Phi_0 \simeq 2.1\times 10^7\,{\rm eV}\left(\frac{10^{-10}\,{\rm eV}}{m}\right)\sqrt{\frac{\rho}{0.3\,{\rm GeV/cm^3}}}$$

Pressure : $p_{\rm DM} = \frac{1}{2}\dot{\Phi}^2 - \frac{1}{2}m^2\Phi^2 \sim -\frac{1}{2}m^2\Phi_0^2\cos 2mt$

- The period of the oscillation of the axion can be estimated as $1/m$. Then, by the average under the long time scale $T \gg 1/m$, the pressure of the oscillating axion can be regarded as zero. Then, the oscillating axion behaves as the CDM.

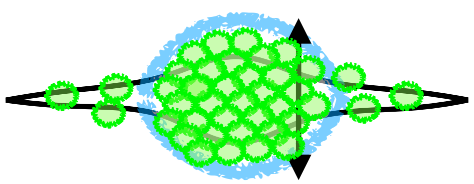
Size of Axion Dark Matter

- In space, the matters are collected by the gravitational forces.
  → It appears the equilibrium between the characteristic pressure of matters and the gravitational force induced by matters themselves.

  → This equilibrium is expressed by the size of the halo of matters.

  → Jeans length of the axion dark matter is given by

$$r_{\rm J} = 4.3 \times 10^{-3}{\rm pc} \times \left(\frac{0.3\,{\rm GeV/cm^3}}{\rho}\right)^{1/4} \left(\frac{10^{-10}\,{\rm eV}}{m}\right)^{1/2}$$



- Jeans length is the maximum size which the matters can exist stably in space.
  → In this talk, we assume the axion dark matters have the enough energy density and behave as the classical oscillating field.

  → In low energy scale, the potential of the axion can be approximated by

$$V \sim \frac{1}{2}m^2\Phi^2.$$

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3. **Dynamical Chern-Simons gravity**
4. Parametric Resonance
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Dynamical Chern-Simon Gravity

- The action is given by

$$S = \int_{\mathcal{V}} dx^4 \sqrt{-g} \left[\kappa R + \frac{1}{4}\alpha \Phi \tilde{R}R - \frac{1}{2}\left(\nabla_\mu \Phi\right)\left(\nabla^\mu \Phi\right) - V(\Phi) \right]$$

$$\alpha = \sqrt{\frac{\kappa}{2}}\ell^2 \qquad \tilde{R}R \equiv \underbrace{\frac{1}{2}\epsilon^{\gamma\delta\rho\sigma} R^\alpha{}_{\beta\rho\sigma}}_{\equiv \tilde{R}^\alpha{}_\beta{}^{\gamma\delta}} R^\beta{}_{\alpha\gamma\delta}$$

- Dynamical or non-dynamical
  → This is switched by the existence of the dynamical term of the axion field.

$$i\epsilon_{ilm} n_l e^{\mathrm{R/L}}_{mj}(\boldsymbol{n}) = \pm e^{\mathrm{R/L}}_{ij}(\boldsymbol{n})$$

- Because the axion field is the pseudo scalar, this interaction term induces the circular polarized gravitational waves.

- The upper bound of the coupling constant ℓ is given as follows,

  $\ell \sim 10^8 \mathrm{km}$ in non dynamical theory by Y. Ali-Haimoud & Y. Chen (2011).
  in dynamical theory by Y. Nakamura et. al. (2018).

- The ghost modes exist in this theory. We assume this theory as the effective theory of gravity.

Equations of Motion of dCS Gravity

- The action of the dynamical Chern-Simons gravity is

$$S = \kappa \int_{\mathcal{V}} dx^4 \sqrt{-g}\, R + \frac{1}{4}\alpha \int_{\mathcal{V}} dx^4 \sqrt{-g}\, \Phi \tilde{R}R$$
$$-\frac{1}{2}\int_{\mathcal{V}} dx^4 \sqrt{-g}\, \left[g^{\mu\nu}(\nabla_\mu \Phi)(\nabla_\nu \Phi) + 2V(\Phi)\right]$$

- The equations of motion are derived by

$$G_{\mu\nu} + \frac{\alpha}{\kappa} C_{\mu\nu} = \frac{1}{2\kappa} T_{\mu\nu}$$ for the gravitational field and

$$C^{\mu\nu} \equiv (\nabla_\alpha \Phi)\epsilon^{\alpha\beta\gamma(\mu}\nabla_\gamma R^{\nu)}{}_\beta + (\nabla_\alpha \nabla_\beta \Phi)\tilde{R}^{\beta(\mu\nu)\alpha}$$

$$\nabla_\mu \nabla^\mu \Phi - \frac{dV(\Phi)}{d\Phi} = -\frac{\alpha}{4}\tilde{R}R$$ for the axion field.

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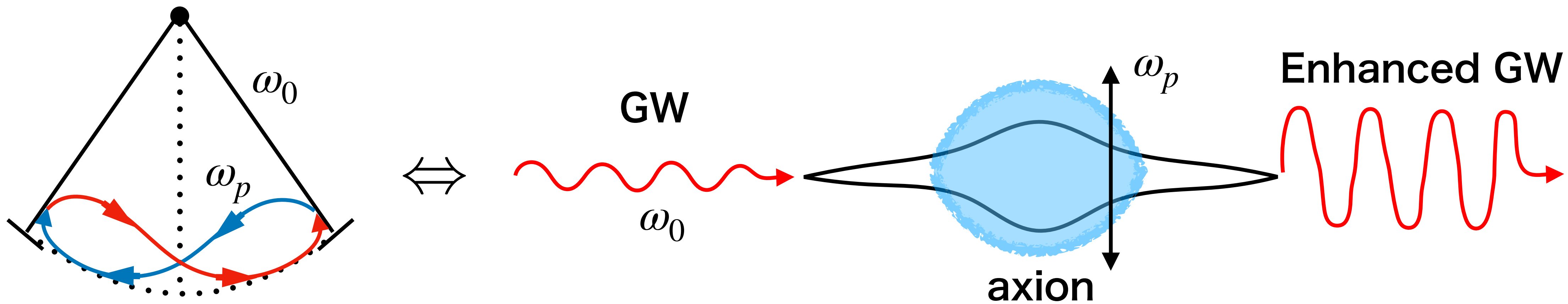
Parametric Resonance

- The Swings
  → The parameters vary at roughly twice the natural frequency of the arms of the swing, the amplitude of it will grow.



- The parametric resonance is induced by Mathieu equation.

$$\ddot{x}(t) + \omega_0^2 \left(1 - f_0 \sin\left(\omega_p t\right)\right) x(t) = 0$$

- The properties of the Mathieu equation are noted.
  → The (first) resonance frequency is given by $\omega_p \simeq 2\omega_0$ $\quad h \propto e^{\Gamma\eta}$

  → The growth rate of the amplitude is given by $\Gamma_{\max} = \frac{1}{4} |f_0| \, \omega_0$

Gravitational Wave in dCS Gravity

- We choose the ansatz of the spacetime below,

$$ds^2 \simeq a(\eta)^2(-d\eta^2 + \delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j)$$

- For the interest in the circular polarization, we take the circular polarization basis which are defined by

$$h_{ij}^{\mathrm{TT}} = \sum_A h_A e_{ij}^A$$

$$e_{ij}^{\mathrm{R}}(\boldsymbol{n}) = \frac{1}{\sqrt{2}}\left(e_{ij}^{+}(\boldsymbol{n}) + ie_{ij}^{\times}(\boldsymbol{n})\right) \qquad h_{\mathrm{R}} = \frac{1}{\sqrt{2}}(h_+ - ih_\times)$$

$$e_{ij}^{\mathrm{L}}(\boldsymbol{n}) = \frac{1}{\sqrt{2}}\left(e_{ij}^{+}(\boldsymbol{n}) - ie_{ij}^{\times}(\boldsymbol{n})\right) \qquad h_{\mathrm{L}} = \frac{1}{\sqrt{2}}(h_+ + ih_\times)$$

Then, the equations of the gravitational wave is derived by

$$\left(a^2 - \epsilon_A \Phi' \frac{\alpha}{\kappa}k\right)h_A'' + \left(2aa' - \epsilon_A \Phi'' \frac{\alpha}{\kappa}k\right)h_A' + \left(a^2 - \epsilon_A \Phi' \frac{\alpha}{\kappa}k\right)k^2 h_A = 0$$

- We can neglect the effect of the expansion of space in the time scale when the gravitational waves pass Galaxy. Then, the scale factor can be estimated as

$a(\eta) \simeq 1$. Then, we can get the solution of the axion field, $\Phi \simeq \Phi_0 \cos(m\eta)$.

- Finally, the equations of the gravitational wave is derived as

$$h_A'' + \frac{\epsilon_A \delta \cos(m\eta)}{1 + \epsilon_A \frac{k}{m}\delta \sin(m\eta)} k\, h_A' + k^2 h_A = 0$$

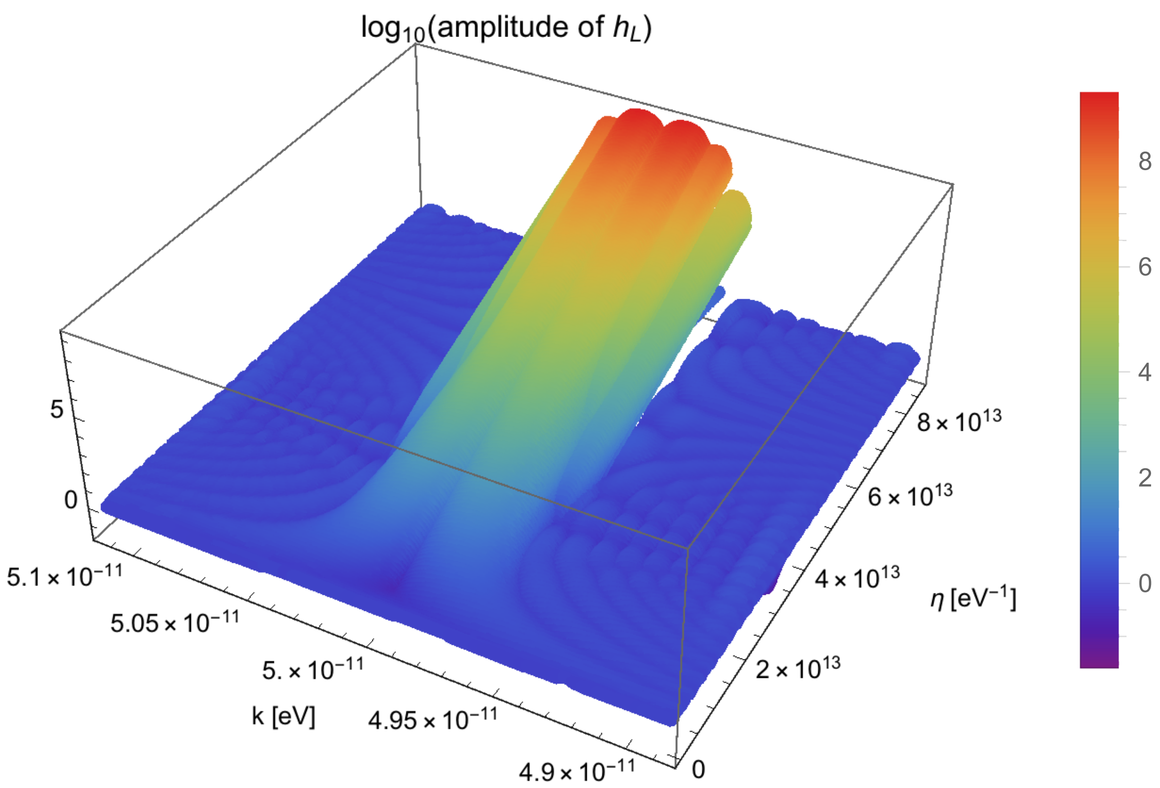
$$\epsilon_A \equiv \begin{cases} 1 & : \ A = \mathrm{R} \\ -1 & : \ A = \mathrm{L} \end{cases} \qquad \delta \equiv \frac{\alpha}{\kappa} m^2 \Phi_0$$

Growth of Amplitudes of GWs

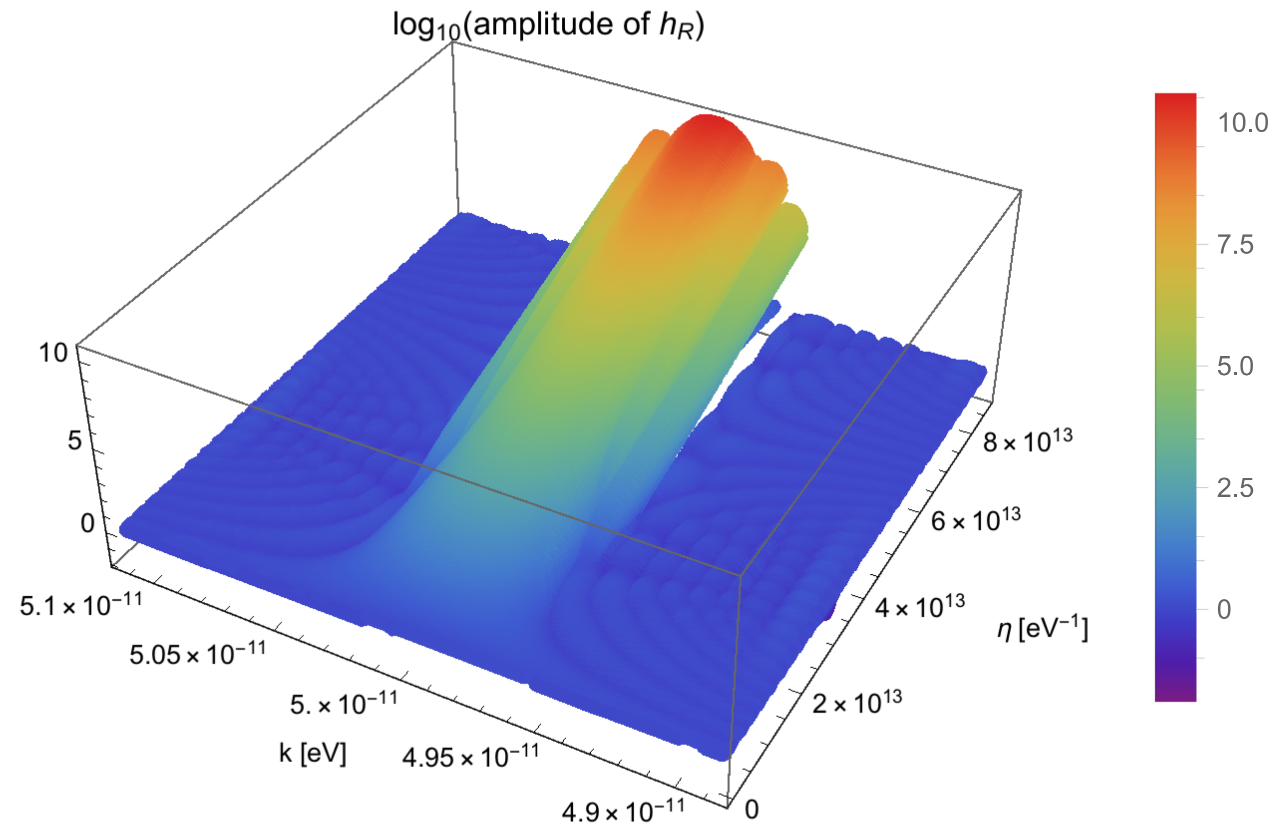
- **These Plots shows the growth of the amplitude in the circular polarization basis. We used the values below**

$$\ell = 10^8\,\mathrm{km},\;\; m = 10^{-10}\,\mathrm{eV},\;\; \rho = 0.3\times10^6\,\mathrm{GeV/cm^3}$$

$$\delta \simeq 0.02$$



h_{L}



h_{R}

→ **There is the difference between the growth of the amplitude in R- and L- circular polarization basis. In these conditions, the amplitude of h_{R} becomes 10^4 times bigger than the amplitude of h_{L}. So, the fully circular polarized wave appears.**

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Strength of Circular Polarization

- This plot shows the time evolution of the strength of the circular polarization.
  We used the same values before slide.

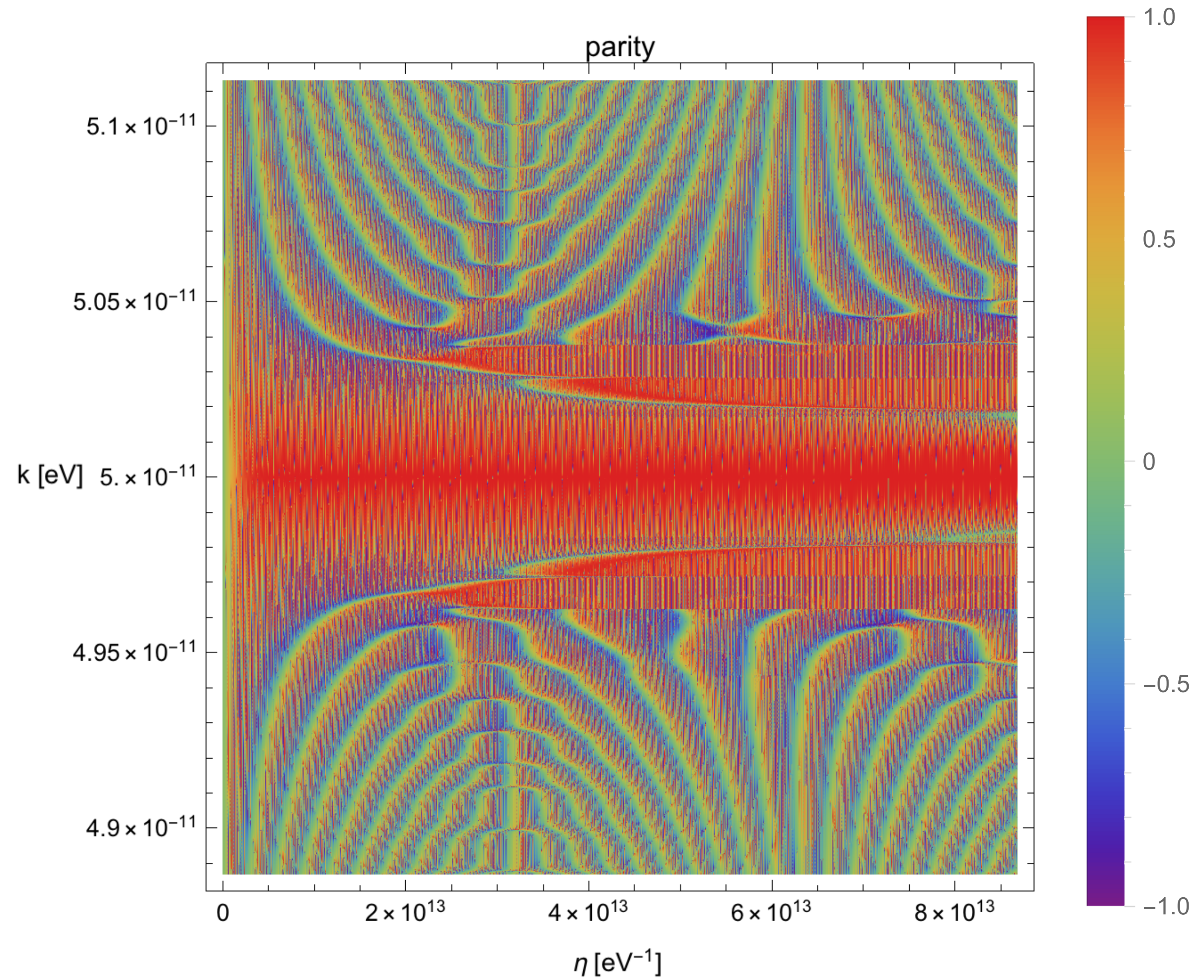
  → The horizontal axe shows the time η .

  → The vertical axe shows the frequency k of the GWs.

  → The plotted function is defined by

$$\mathrm{parity}(\eta) \equiv \frac{|h_{\mathrm{R}}|^2 - |h_{\mathrm{L}}|^2}{|h_{\mathrm{R}}|^2 + |h_{\mathrm{L}}|^2}$$

$\mathrm{parity} \sim \pm 1$: R or L



- The color shows the strength of the circular polarization.
  → At the resonance frequency, the GW is completely circular polarized.

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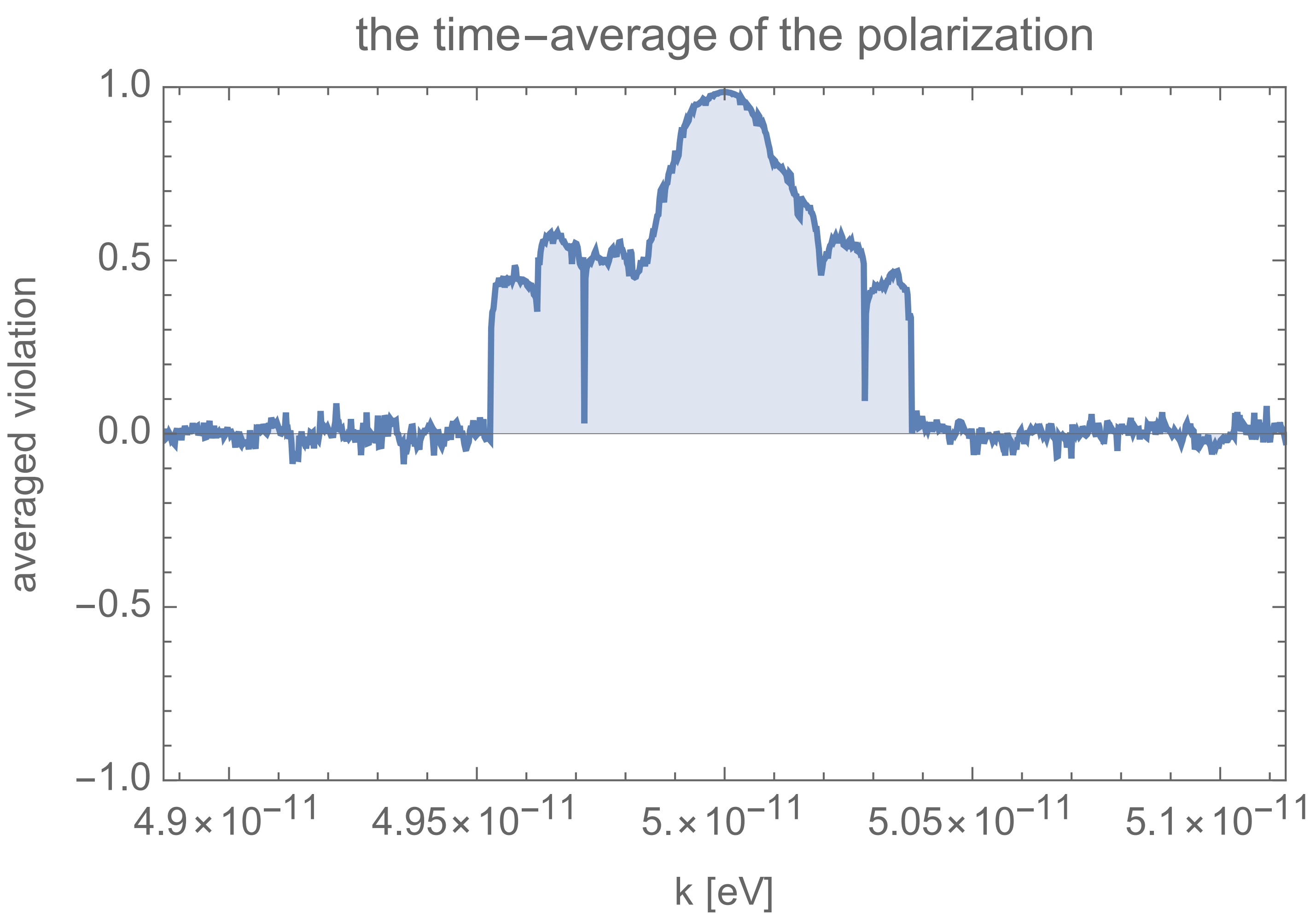

Proceedings, 4th Workshop on Cosmology and the Quantum Vacuum: Segovia, Spain, September 4-8, 2017

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Tools for Estimations

- From the general theory of the parametric resonance, we can give the formulae for the estimations of the resonance frequency and the rate of the amplification in the length.

- Resonance

→ Resonance frequency : $k_{\mathrm{r}} = \frac{m}{2} \Rightarrow f_{\mathrm{r}} = \frac{k_{\mathrm{r}}}{2\pi} \simeq 1.2 \times 10^4\,\mathrm{Hz} \left(\frac{m}{10^{-10}\,\mathrm{eV}} \right)$

→ Width of the resonance : $\frac{m}{2} - \frac{m}{8}\delta \lesssim k_{\mathrm{r}} \lesssim \frac{m}{2} + \frac{m}{8}\delta \quad \delta \equiv \frac{\alpha}{\kappa} m^2 \Phi_0 \quad \alpha = \sqrt{\frac{\kappa}{2}} \ell^2$

- Amplification of the amplitude of GW

We can convert Γ into the length $R_{\times 10}$ in which the amplitude of the GWs becomes 10 times bigger.

→ Typical length :

$$R_{\times 10} \simeq 5.2 \times 10^{-8}\,\mathrm{pc} \times \left(\frac{10^{-10}\,\mathrm{eV}}{m} \right)^2 \left(\frac{10^8\,\mathrm{km}}{\ell} \right)^2 \sqrt{\frac{0.3\,\mathrm{GeV/cm^3}}{\rho}}$$

Simple Estimation

- The length in which the GWs becomes 10 times bigger is estimated as

$$R_{\times 10} \simeq 5.0 \times 10^{-8}\,\mathrm{pc} \times \left(\frac{10^{-10}\,\mathrm{eV}}{m}\right)^2 \left(\frac{10^8\,\mathrm{km}}{\ell}\right)^2 \sqrt{\frac{0.3\,\mathrm{GeV/cm^3}}{\rho}}$$

- If the GWs which have the resonance frequency through in Galaxy for $10\,\mathrm{kpc}$, the amplitude of the GWs become $10^{10^{12}}$ times bigger.

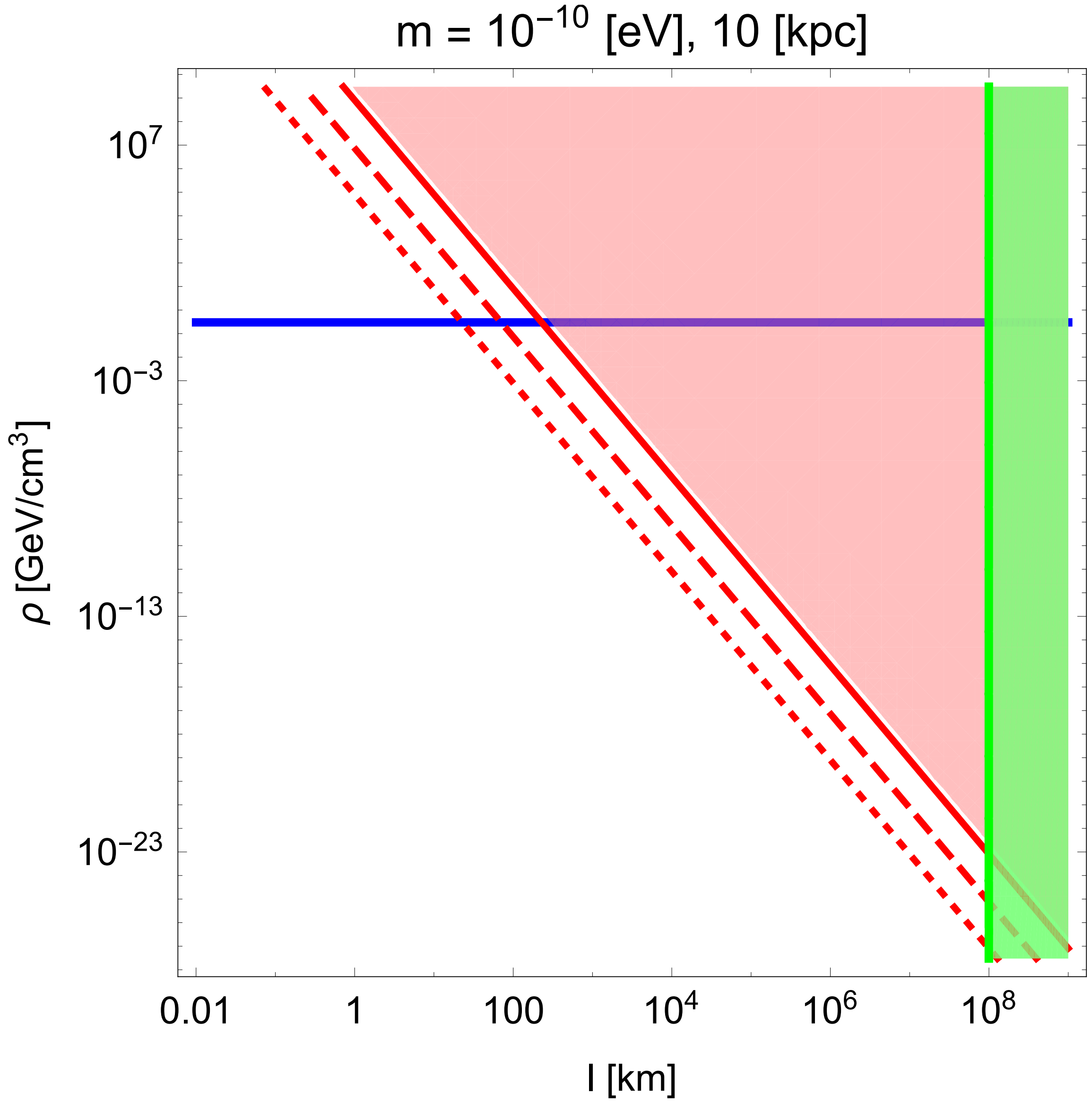
- This effect will be detected in the difference between the waveform from the interferometers and from the numerical template of the GWs in pure GR.

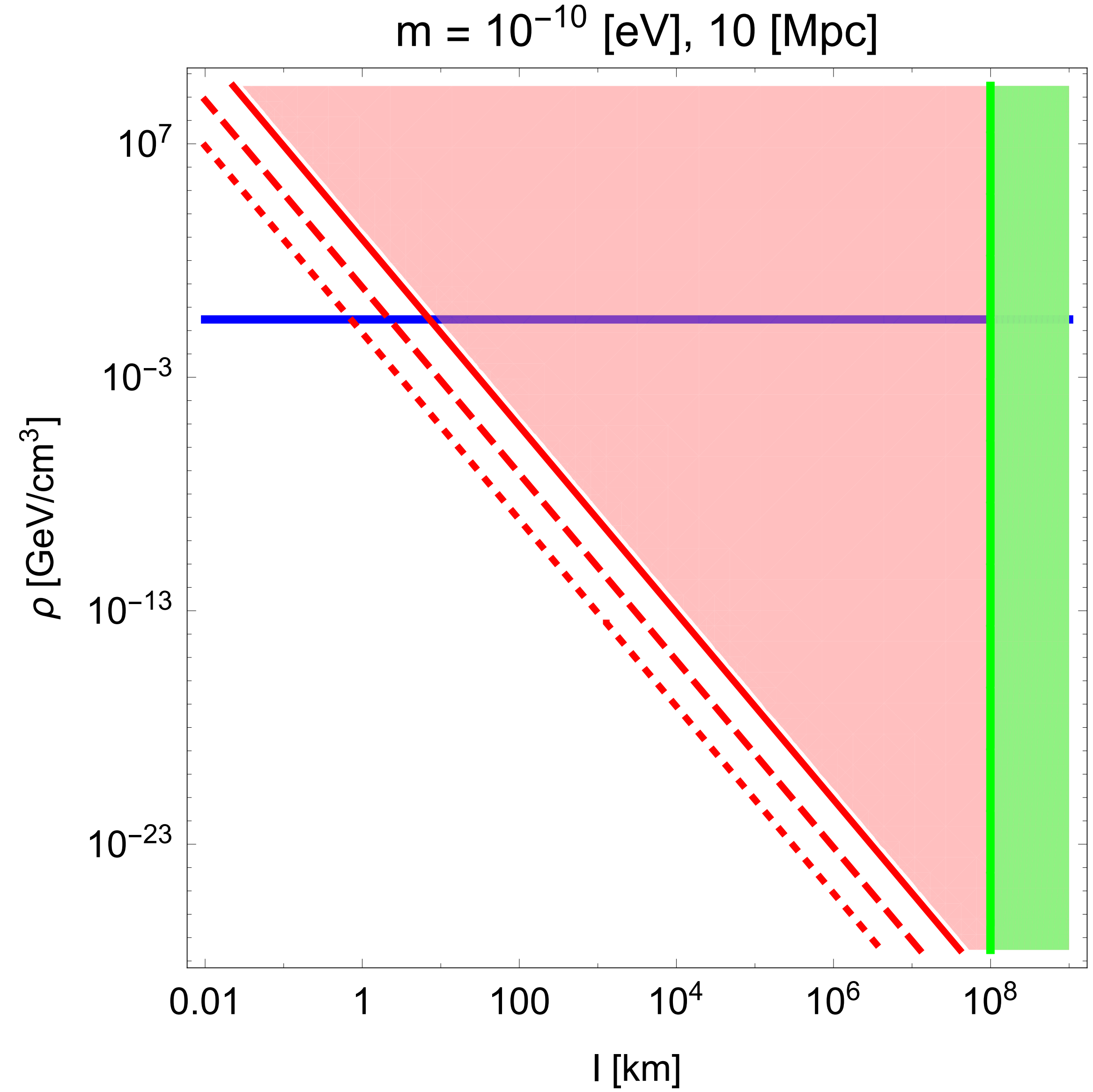
- If we found no difference in the waveform, we can give the constraint to the coupling constants or the energy density of the axion dark matter.

→ If you believe $\ell \sim 10^8\mathrm{km}$, then ρ will be constraint by $\rho \leq 10^{-26}\,\mathrm{GeV/cm^3}$.

→ If you believe $\rho \sim 0.3\,\mathrm{GeV/cm^3}$, then ℓ will be constraint by $\ell \leq 10\,\mathrm{km}$.

Maps of Constraints





The green region is excluded by the observation.

The blue line shows the local dark matter density $0.3\,\mathrm{GeV/cm^3}$**.**

The red line represents the 10 times enhancement of GWs.

The red dashed line represents the 0.1 times enhancement.

The red dotted line represents the 0.01 times enhancement.

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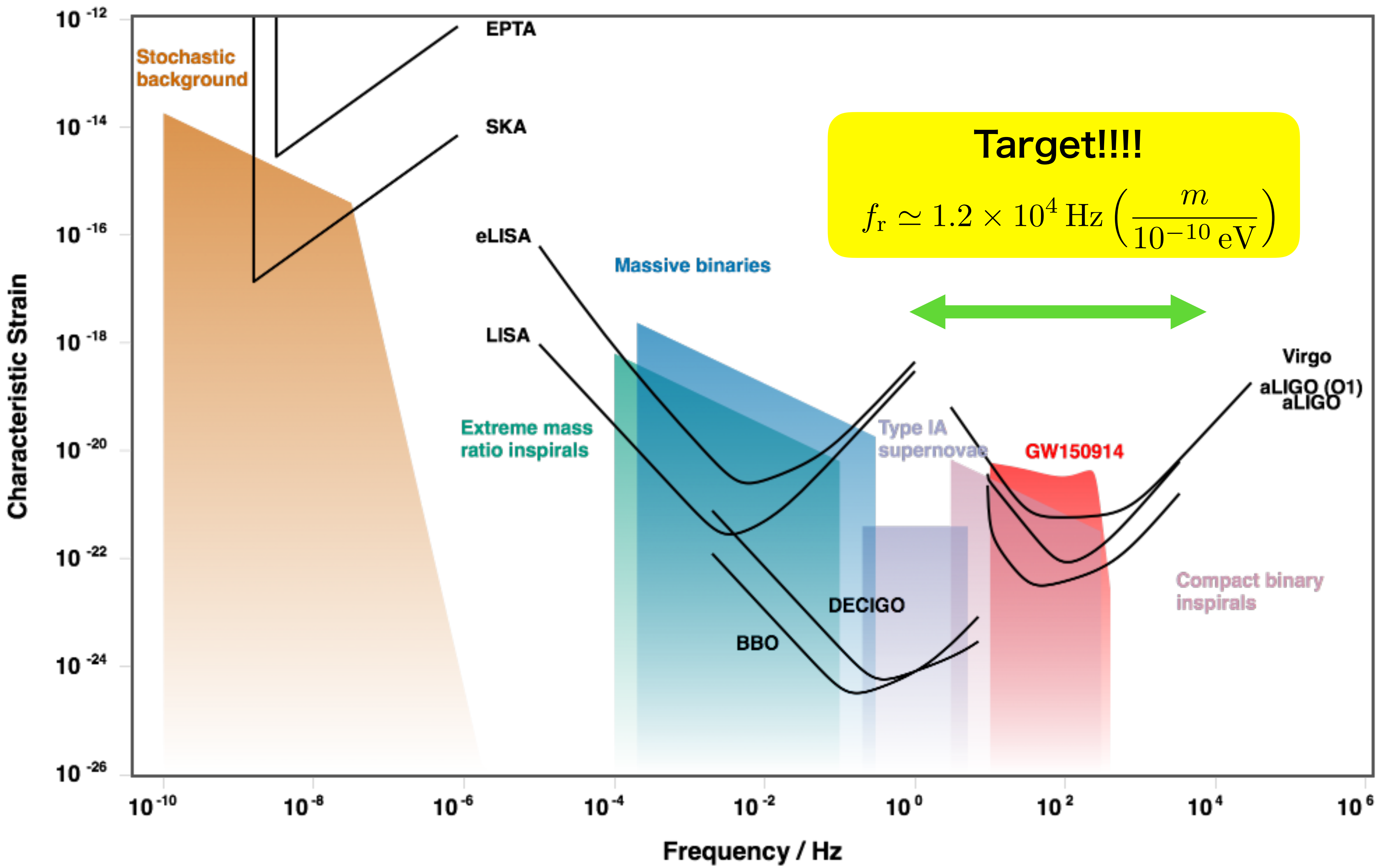
Sensitivity Curves and Axion Dark Matter



These Sources are from "http://rhcole.com/apps/GWplotter/".

Menu

Conclusion

- String axiverse generates the axions which have the light mass and the dCS coupling between the axion field and the gravitational field.

- The coherently oscillating axion can behave as the cold dark matter well, so, through the above coupling, the monochromatic and parity violated gravitational waves are generally induced.

- This effect may use to detect the counterpart of the GR.
  → They might give the new constraint to the abundance of the axion dark matter or the CS-coupling.

- This analysis can connect the problems about the CDM and the modified gravity.

Related Researches

- **Articles**

  “Electromagnetic memory effect induced by axion dark matter”
  DY and Jiro Soda, Phys. Rev. D **96**, 064005 (2017)

  “Exploring the string axiverse and parity violation in gravity with gravitational waves”,
  DY and Jiro Soda, International Journal of Modern Physics D Vol. 27, No. 9 (2018) 1850096

  “Electromagnetic waves propagating in the string axiverse”,
  DY and Jiro Soda, Progress of Theoretical and Experimental Physics, Volume 2018, Issue 4, 1 April 2018, 041E01

- **Proceedings**

  “Exploring String Axions with Gravitational Waves”,
  Jiro Soda and DY, Galaxies 5 (2017) no.4, 96