Search for ultralight scalar dark matter with NANOGrav pulsar timing arrays

Ryo Kato and Jiro Soda,
Institute of Cosmophysics, Kobe University.

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Fast spinning neutron star

The gravitational effects can be observed as a delay of pulse arrival times.

Pulsars can be used as a galactic-scale detector, and sensitive to gravitational effects in a nanohertz frequency band.
Axion dark matter

Scalar field predicted by string theory

Feature: The axion behaves as a perfect fluid with an oscillating pressure.

- Oscillation of pressure
  - Einstein's equation
  - Oscillation of gravitational potential
    - Geodesic equation
    - Oscillation of pulse arrival times

If the axion mass is about $10^{-23} \text{ eV}$, pulse arrival times oscillates with a nanohertz frequency.
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To explain the energy density of the dark matter, a very large occupation number is required.

\[
\frac{\Delta N}{\Delta x^3 \Delta p^3} \sim 10^{96} \left( \frac{\rho}{0.4 \text{GeV/cm}^3} \right) \left( \frac{10^{-23} \text{eV}}{m} \right)^4
\]

\(\rho\) : the energy density of the dark matter

\(m\) : the axion mass

Axion can be treated as a classical scalar field.

The solution of the Klein-gordon equation:

\[
\phi(t, \vec{x}) = \phi_0(\vec{x}) \cos(mt + \theta(\vec{x}))
\]

because

\[
k \sim (10 \text{kpc})^{-1} \sim 10^5 H_0
\]

Galaxy Scale

\[
m \sim 10^{-23} \text{eV} \sim 10^{10} H_0
\]

dispersion relation:

\[
E_k^2 = k^2 + m^2 \approx m^2
\]

(monochromatic with \(m\))
Energy momentum tensor

\[ T_{\mu \nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}((\partial \phi)^2 + m^2 \phi) \]

Using the equation: \[ \phi(t, \vec{x}) = \phi_0(\vec{x}) \cos(mt + \theta(\vec{x})) \]

with \[ \partial_i \phi \ll \partial_0 \phi \quad \therefore k \ll m \]

\[ T_{00} = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 = \frac{1}{2} m^2 \phi_0^2 \]

\[ T_{ij} = \left( \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2 \right) \delta_{ij} = \frac{1}{2} m^2 \phi_0^2 \cos(2mt + \theta) \delta_{ij} \]

The axion behaves as a perfect fluid with an oscillating pressure.
Einstein's equation

\[ G_{\mu\nu} = 8\pi GT_{\mu\nu} \]

Newtonian gauge

\[ ds^2 = -(1 + 2\Psi)dt^2 + (1 + 2\Psi)\delta_{ij}dx^i dx^j \]

00 component: \[ \Delta \Psi = -4\pi G \rho \]

\[ \rho \equiv \frac{1}{2} m^2 \phi_0^2 \]

ij component: \[ \ddot{\Psi} = 4\pi G \rho \cos(2mt + \theta) \]

Separate \( \Psi \) into time-independent and time-dependent terms.

\[ \Psi = \Psi_0(\vec{x}) + \delta \Psi(t, \vec{x}) \]

Then

\[ \Delta \Psi_0 = -4\pi G \rho \]

Undetectable

\[ \delta \ddot{\Psi} = 4\pi G \rho \cos(2mt + \theta) \]

\[ \delta \Psi = \pi G \rho \cos(2mt + \theta) \]

Oscillation of the gravitational potential \( \delta \Psi \) is induced from the pressure oscillation.
To obtain the red shift of the light, the zero component of the geodetic equation is calculated.

\[
\frac{d^2 x^0}{d\lambda} + \Gamma^0_{\nu\rho} \frac{dx^\nu}{d\lambda} \frac{dx^\rho}{d\lambda} = 0 \quad ds^2 = -(1 + 2\delta \Psi) dt^2 + (1 + 2\delta \Psi) \delta_{ij} dx^i dx^j
\]

\[
\frac{d^2 x^0}{d\lambda} = -2\omega_0 \frac{d\delta \Psi}{d\lambda} \quad \omega_0 : \text{unperturbed frequency}
\]

Observed light frequency:

\[
\omega_{\text{obs}} = \omega_0 - \omega_0 [\delta \Psi(\lambda_{\text{Pulsar}}) - \delta \Psi(\lambda_{\text{Earth}})] + 2\omega_0 \int_{\lambda_{\text{Earth}}}^{\lambda_{\text{Pulsar}}} d\lambda' \frac{d^2 x^0}{d\lambda}
\]

\[
= \omega_0 + \omega_0 [\delta \Psi(\lambda_{\text{Pulsar}}) - \delta \Psi(\lambda_{\text{Earth}})]
\]
Red shift:

\[ z = \frac{\omega_0 - \omega_{\text{obs}}}{\omega_0} = \delta \Psi(\lambda_{\text{Earth}}) - \delta \Psi(\lambda_{\text{Pulsar}}) \]
\[ = \pi G [\rho(\vec{x}_E) \cos(2mt + \theta(\vec{x}_E)) - \rho(\vec{x}_P) \cos(2m(t - D) + \theta(\vec{x}_P))] \]

The delay of the light:

\[ \Delta t = \int_0^t dt \ z \]
\[ = \frac{\pi G}{2m} [\rho(\vec{x}_E) \sin(2mt + \theta(\vec{x}_E)) - \rho(\vec{x}_P) \sin(2m(t - D) + \theta(\vec{x}_P))] \]

The delay of the light \( \Delta t \) can be detected by the pulser timing array as a change in the pulse arrival times.
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\[
\frac{1}{2\pi f} \left[ \Psi \sin(2\pi ft + \alpha) - \Psi \sin(2\pi ft + \alpha_p) \right]
\]

\[\Psi = \frac{\pi G \rho}{m^2}, \quad f = \frac{m}{\pi}\]

- \(m\) : Axion mass
- \(\rho\) : Axion energy density
- \(\alpha, \alpha_p\) : Phases at earth and pulsar

In this study:

- We search for the axion, and if it cannot be detected, we give an upper limit for \(\Psi\).
- We investigate whether the axion is necessary for a model which describes the observation data.
Bayesian statistics

- **Parameter estimation**

  Bayes' theorem:
  \[
  p(\theta|d) \propto p(d|\theta)p(\theta)
  \]

  \(d\) : observation data
  \(\theta\) : parameter
  \(p(\theta)\) : prior knowledge
  \(p(\theta|d)\) : posterior knowledge

  The goal of the Bayesian estimation is to update the prior knowledge of the parameter \(p(\theta)\) to the posterior knowledge \(p(\theta|d)\) using the data \(d\).

- **Bayesian hypothesis testing**

  Bayes factor:
  \[
  B_{10} = \frac{p(d|M_1)}{p(d|M_0)}
  \]

  \(M_0, M_1\) : hypothesis

  The goal of the Bayesian hypothesis testing is to compute the Bayes factor \(B_{10}\).

**Interpretation of the Bayes factor (Jeffreys’ scale)**

<table>
<thead>
<tr>
<th>(B_{10})</th>
<th>Evidence in favor of (M_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>Week</td>
</tr>
<tr>
<td>3-20</td>
<td>Positive</td>
</tr>
<tr>
<td>20-150</td>
<td>Strong</td>
</tr>
</tbody>
</table>
Pulsar observation data: The NANOGrav Eleven-Year Data Set

https://data.nanograv.org

Program: Based on PAL2 (Bayesian inference package for pulsar timing data)

https://github.com/jellis18/PAL2

First Analysis

Model: \( s + n_{\text{red}} \)

\( s \): Axion signal

\( n_{\text{red}} \): Red noise

Red noise: The power spectral density of the noise has most of their power at low frequencies in a given data set.

Source:

- Fluctuations of the rotation speed of the pulsar
- Diffractive and refractive interstellar effects

\[
\langle n_{\text{red}} n_{\text{red}}^T \rangle = \int_{-\infty}^{\infty} df \ e^{-2\pi ift} P(f) \quad P(f) \equiv Af^{-\gamma}
\]
First analysis

Upper limit on the axion amplitude $\Psi$ as a function of the frequency

The upper limit on the axion amplitude is still one order of magnitude larger than the expected amplitude.

The Bayes factor exceeds 20, and so the existence of the axion can not be denied.
Uncertainty of earth orbit

In the case of gravitational waves

Uncertainties of the earth orbit affects upper limits and Bayes factors.


Second Analysis

**Model:** \( s + n_{\text{red}} + n_{\text{orbit}} \)

- \( s \): Axion signal
- \( n_{\text{red}} \): Red noise
- \( n_{\text{orbit}} \): Uncertainties of earth orbit

Sources of \( n_{\text{orbit}} \):

- Error of ecliptic longitude
- Mass errors of Jupiter, Saturn, Uranus, and Neptune
- Orbit error of Jupiter

The Jupiter has a orbital period of 11.86 years and a large mass.

The Jupiter produces a large noise in the low frequency region of pulser timing arrays.
Almost the same upper limit as in the first analysis is obtained, but most of the Bayes factors are less than 3, and so no axion is detected.
Comparison with previous study

Upper limit on the axion amplitude $\Psi$ as a function of the frequency

Previous study: Parkes Pulsar Timing Array Twelve-Year Data Set (PPTA2018)

Up to three times stronger upper limit is obtained in the blue circle.
Bayes factor is less than 3 $\iff$ Probability that axion is not required is more than 75%

We would like to know whether the noise is sufficient to absorb the signal of the axion.

Last Analysis

**Model:** $n_{\text{red}} + n_{\text{orbit}}$ then **Model:** $s$

$s$: Axion signal $n_{\text{red}}$: Red noise $n_{\text{orbit}}$: Uncertainty of earth’s orbit

Note that,

It is difficult to distinguish the signal and the noise. Therefore, this analysis cannot be regarded as giving upper limits to the axion.
The model of only $n_{\text{red}}$ and $n_{\text{orbit}}$ absorbs the axion signal sufficiently. It suggests that the axion is unnecessary for the model of the observation data.
Summary

We set the upper limit of the axion amplitude using the recent pulsar observation data.

- By considering the uncertainty of the earth’s orbit, we revealed that the axion is not detected.

- In comparison with the previous study (Porayko et.al.2018), we obtain up to three times stronger upper limit.

- We clarified that the only the red noise and the uncertainty of the earth’s orbit are sufficient for the model of observation data.