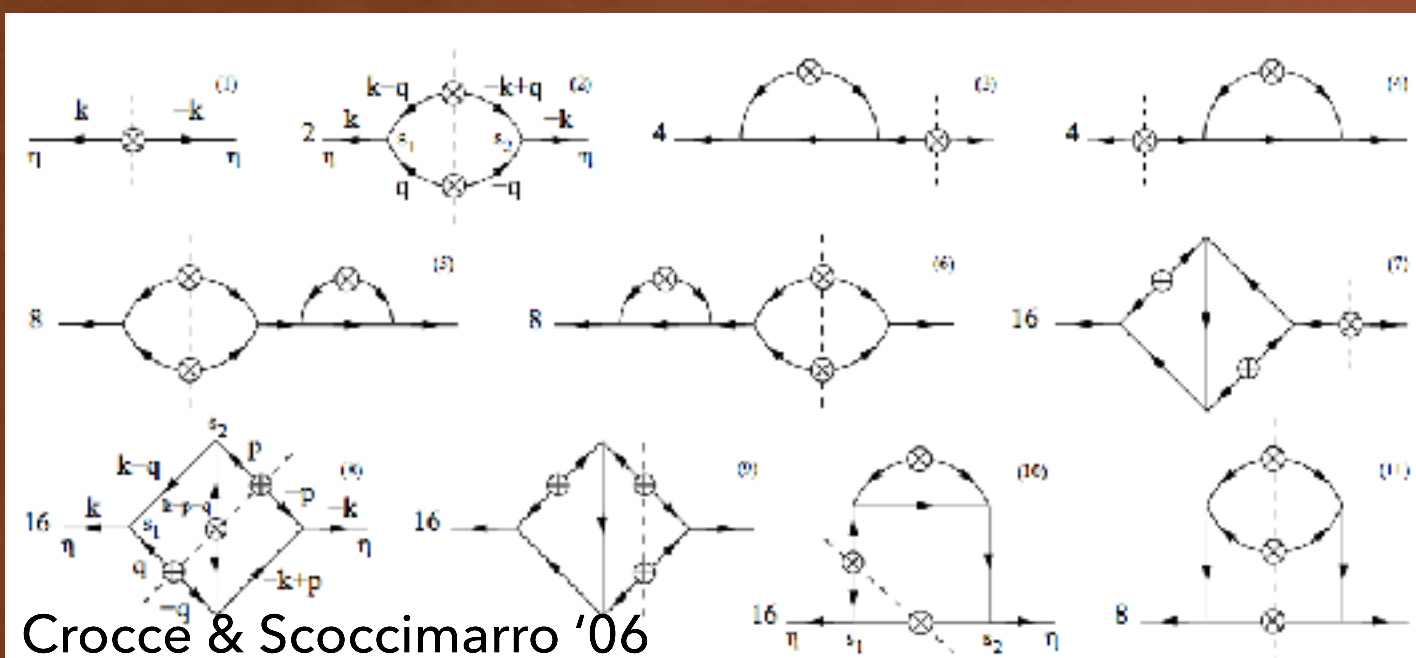
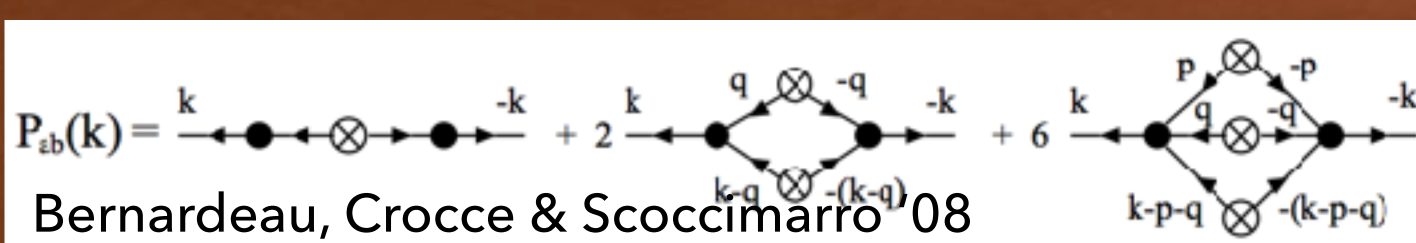
宇宙大規模構造理論予言の現状と展望

西道 啓博 (KAVLI IPMU → 1月より基研)

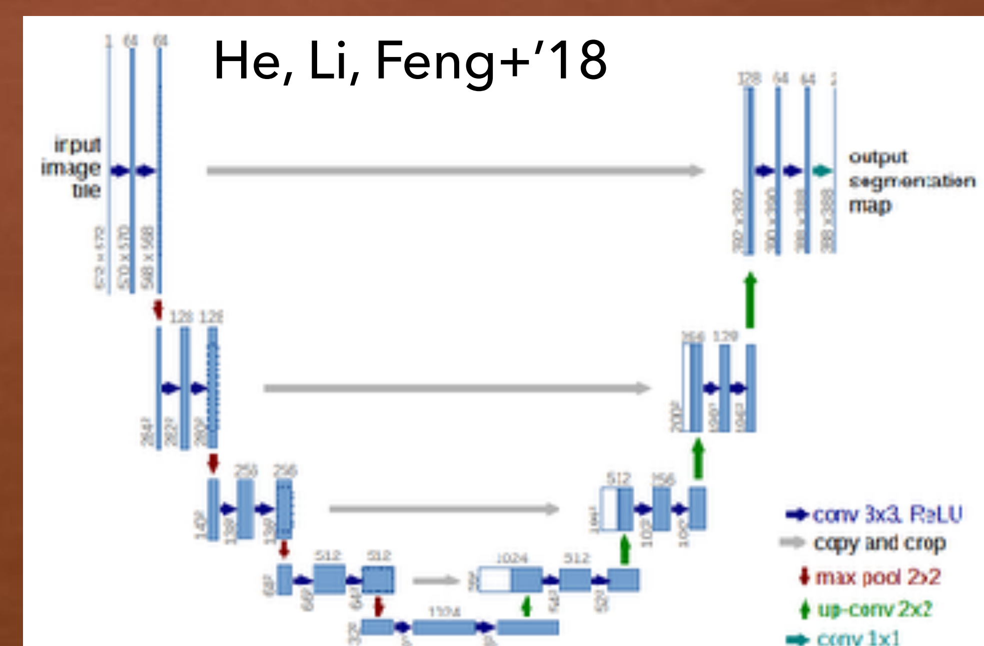


Crocce & Scoccimarro '06



Bernardeau, Crocce & Scoccimarro '08

Mathuriya, Bard, Mendygral+'18



He, Li, Feng+'18

理論懇シンポジウム, 京大基研, 20 Dec 2018

宇宙の揺らぎと情報

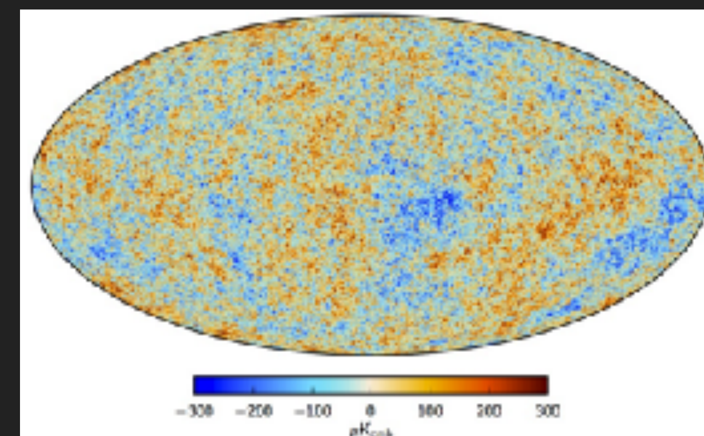
- 宇宙の揺らぎは確率場（と普通考える）
- ナイーブにはオーダー $10^6 \sim 10^7$ 要素のデータベクトルの同時確率分布を相手にすることになる
- 求めたいのは少数の宇宙論パラメタ
- 主に2つのチャンネル
  - Early time: 宇宙マイクロ波背景放射
  - Late time: 大規模構造
    - 重力レンズ, 銀河分布, Lyα forest, intensity mapping, …
- 現在の標準的な理解によれば、
  - 初期条件: インフラトンの量子揺らぎ、ほぼガウシアン
  - 時間発展: 重力相互作用 + correction

宇宙論的統計解析: CMBの場合

$$\Theta(\hat{p}) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\hat{p})$$

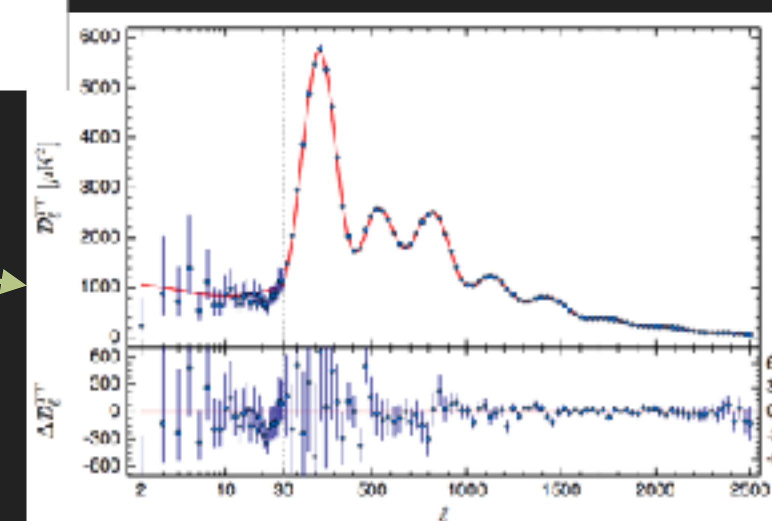
$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$

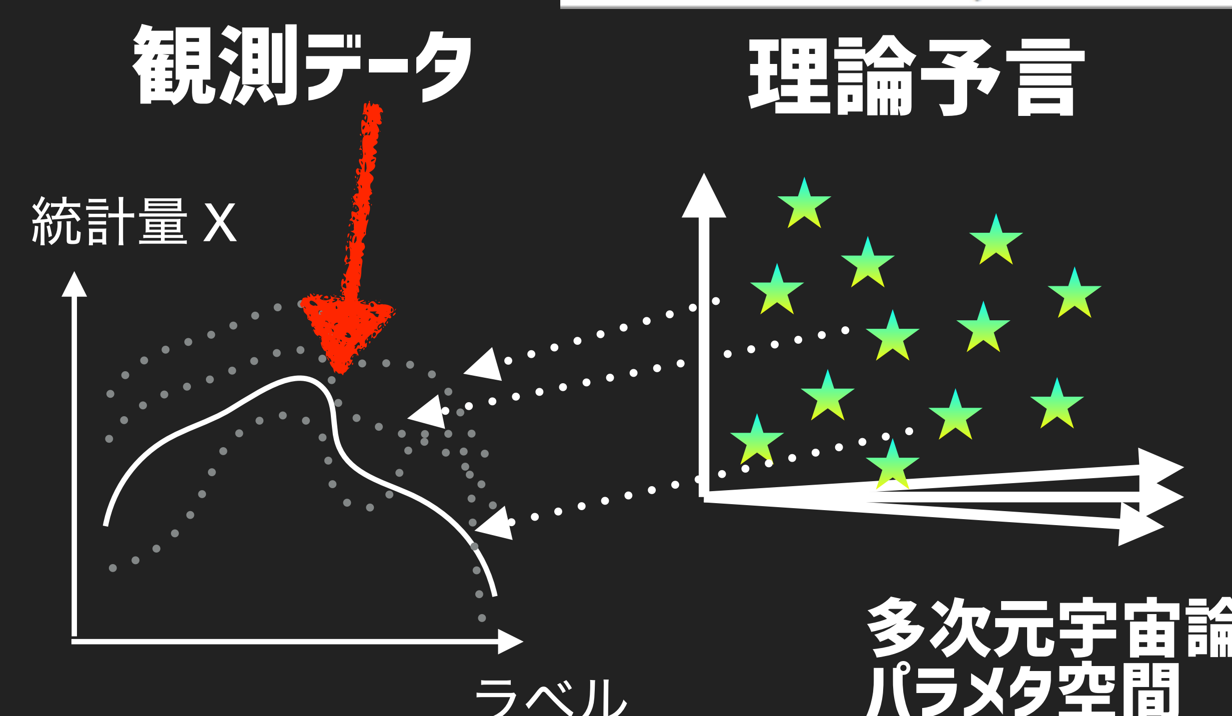
- 宇宙背景放射の温度揺らぎマップ
  - 非常に良い近似でガウスランダム場
  - 理論、観測ともにこれを示唆
  - a_ℓ,mが独立なランダム変数
  - パワースペクトルC_ℓで尽くされる
- 揺らぎは小さい (~10^-4)
  - 線形理論: 与えられた宇宙論パラメタに対してシグナル、ノイズともすぐに計算可能
  - ***MCMC法によるパラメタサーチ(ベイズ推定)***



次元削減

Planck 2015 results. XIII.



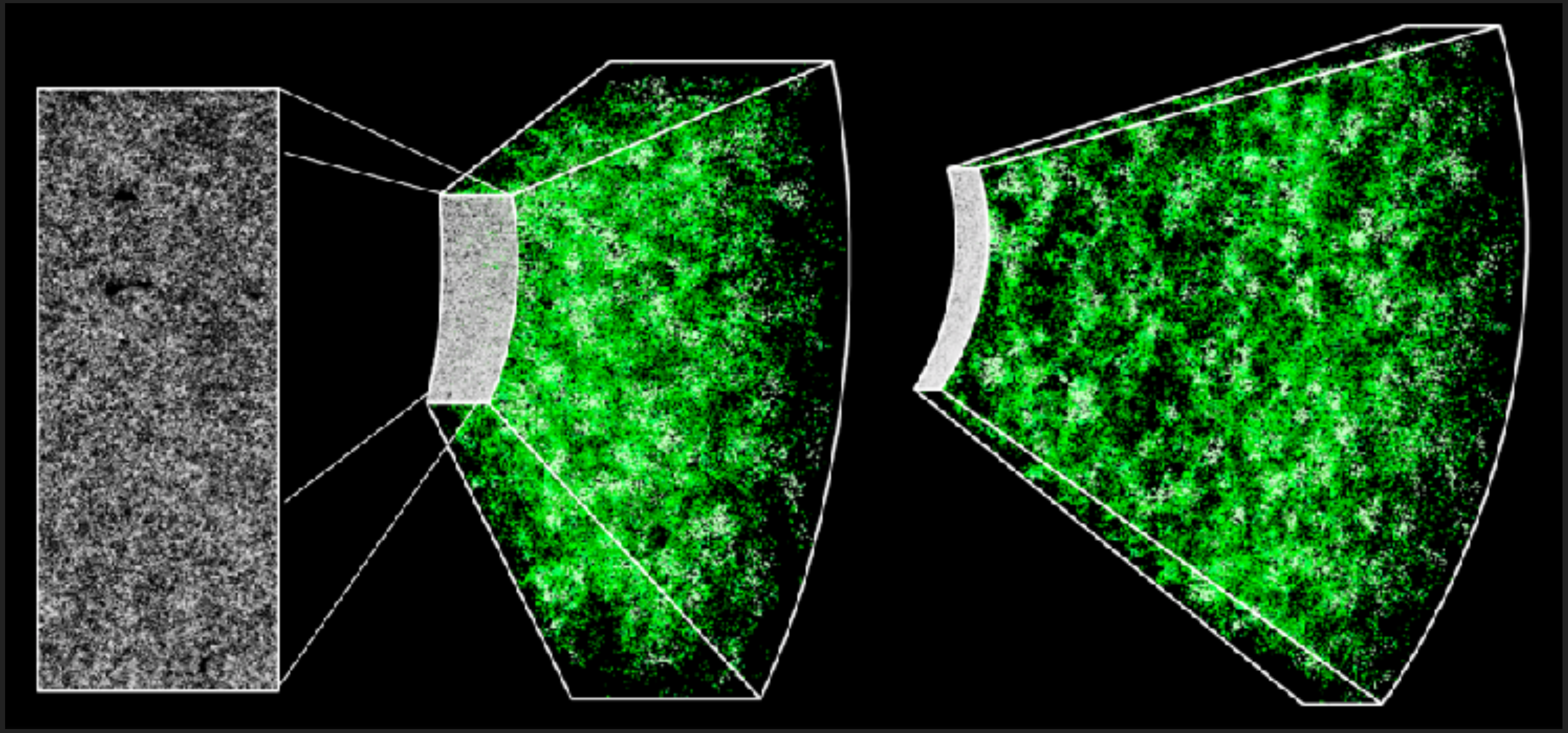


宇宙論的統計解析: LSSの場合

- 銀河の大域的空間分布
  - 元をたどるとCMBと同起源
  - 球面調和関数展開 → フーリエ展開
  - 密度揺らぎδ_kがランダム変数
  - パワースペクトルが基本的な統計量
- ただし、δ_k1, δ_k2, ..., δ_knは独立でない

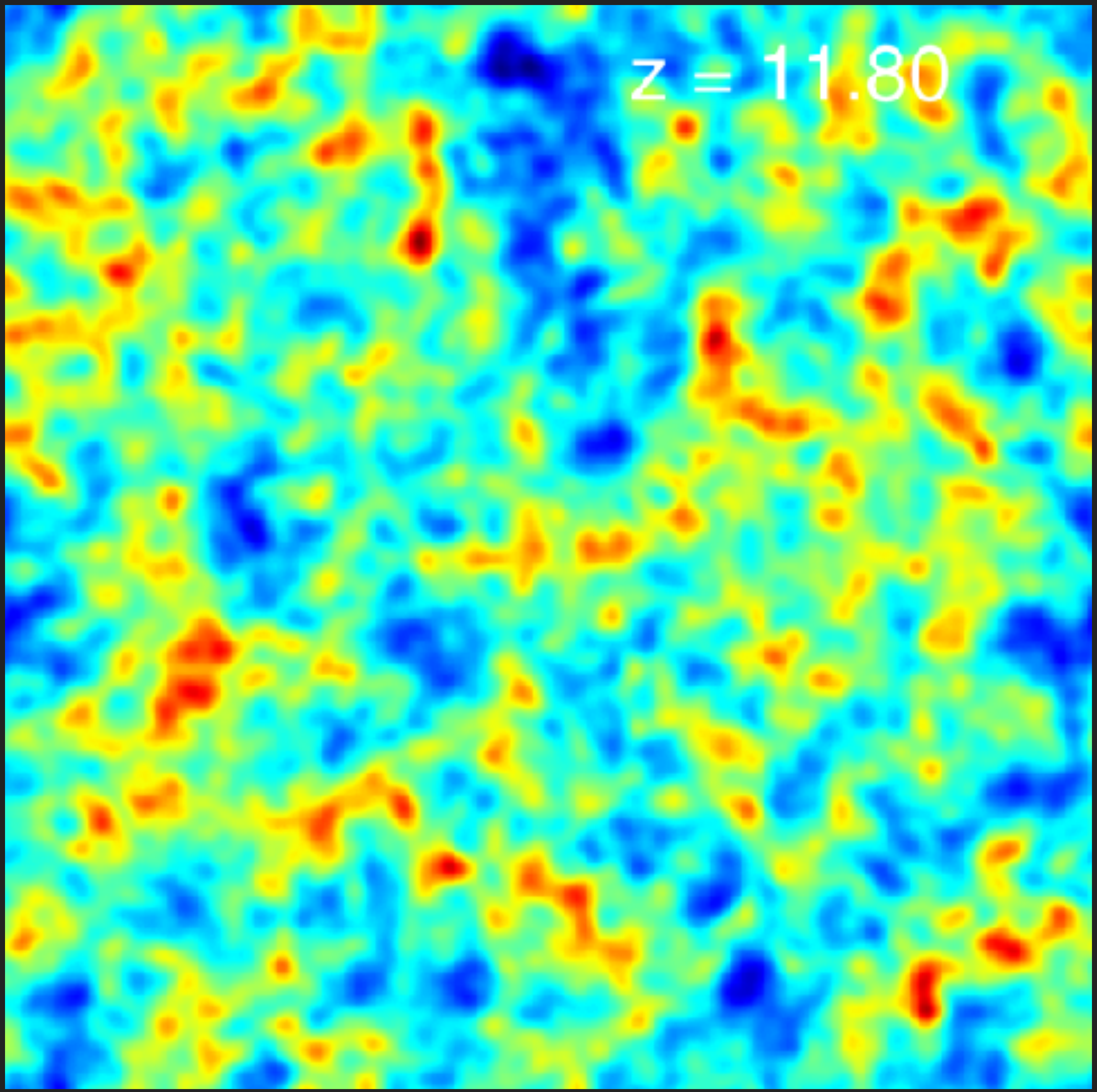
$$\delta(\vec{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\delta_{\vec{k}}e^{i\vec{k}\cdot\vec{x}}$$

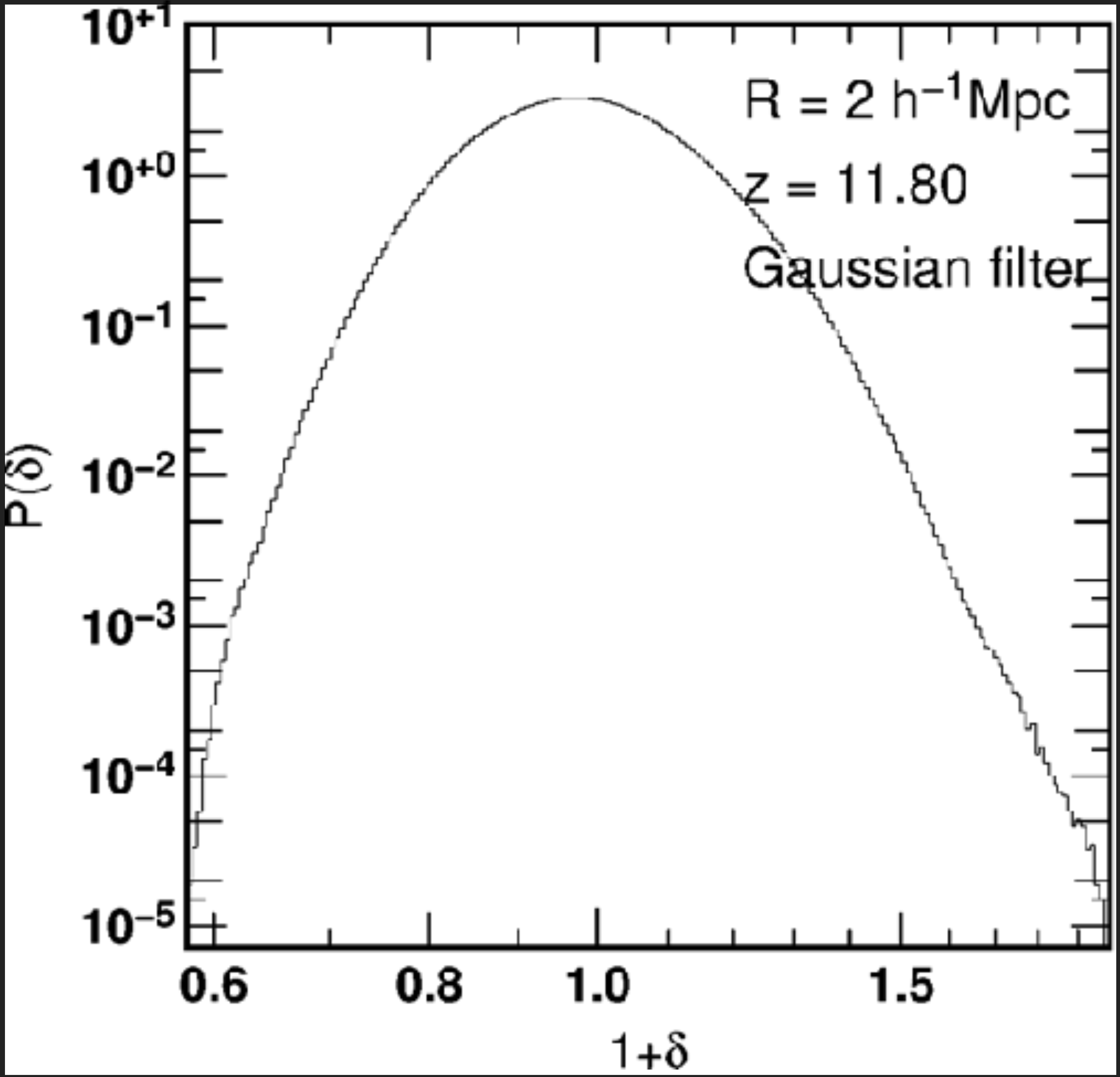
$$\langle\delta_{\vec{k}}\delta_{\vec{k}'}\rangle = (2\pi^3)\delta^3_{\mathrm{D}}(\vec{k}+\vec{k}')P(k)$$



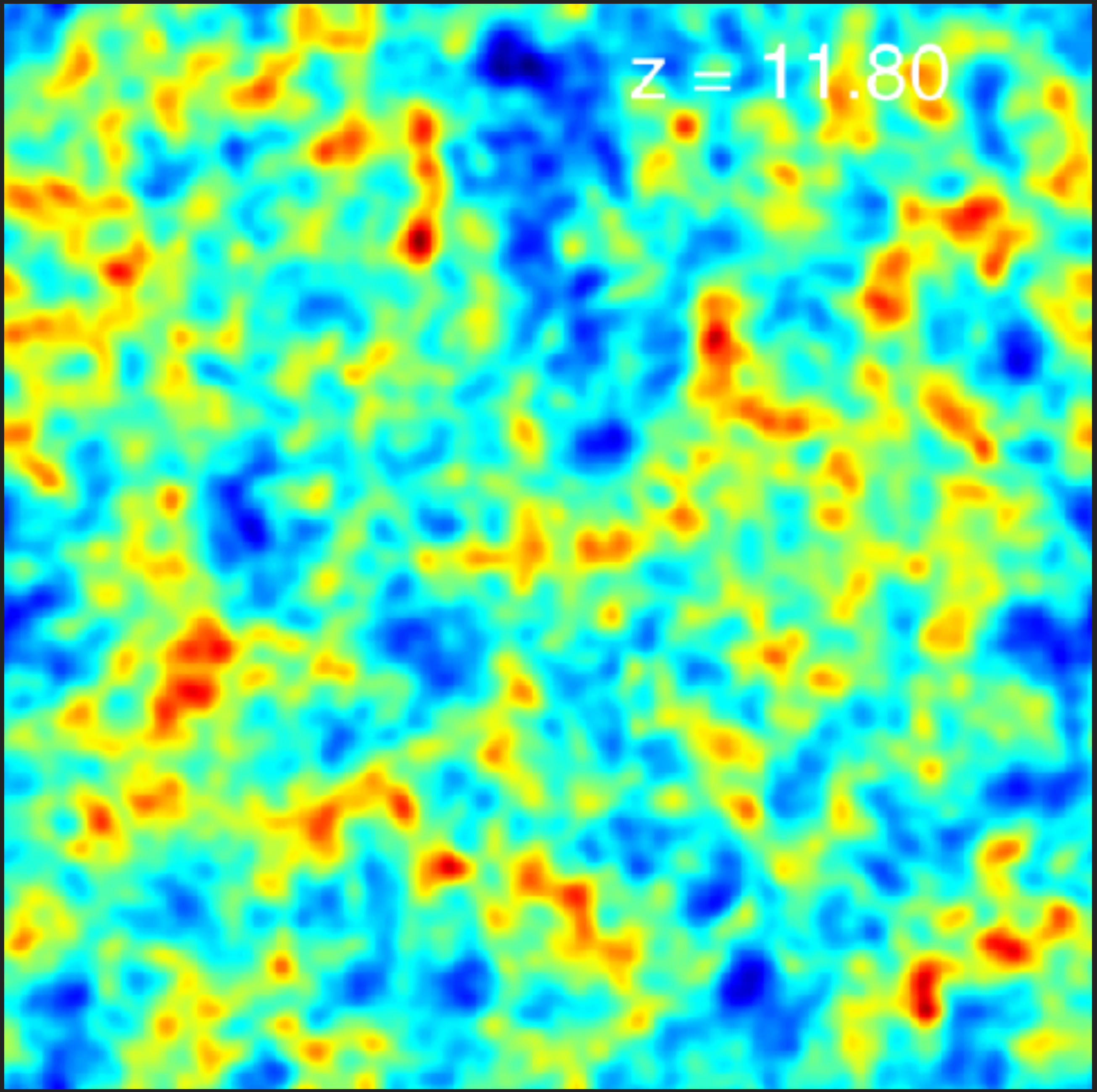
SDSS BOSS final data release ©Jeremy Tinker and the SDSS-III collaboration

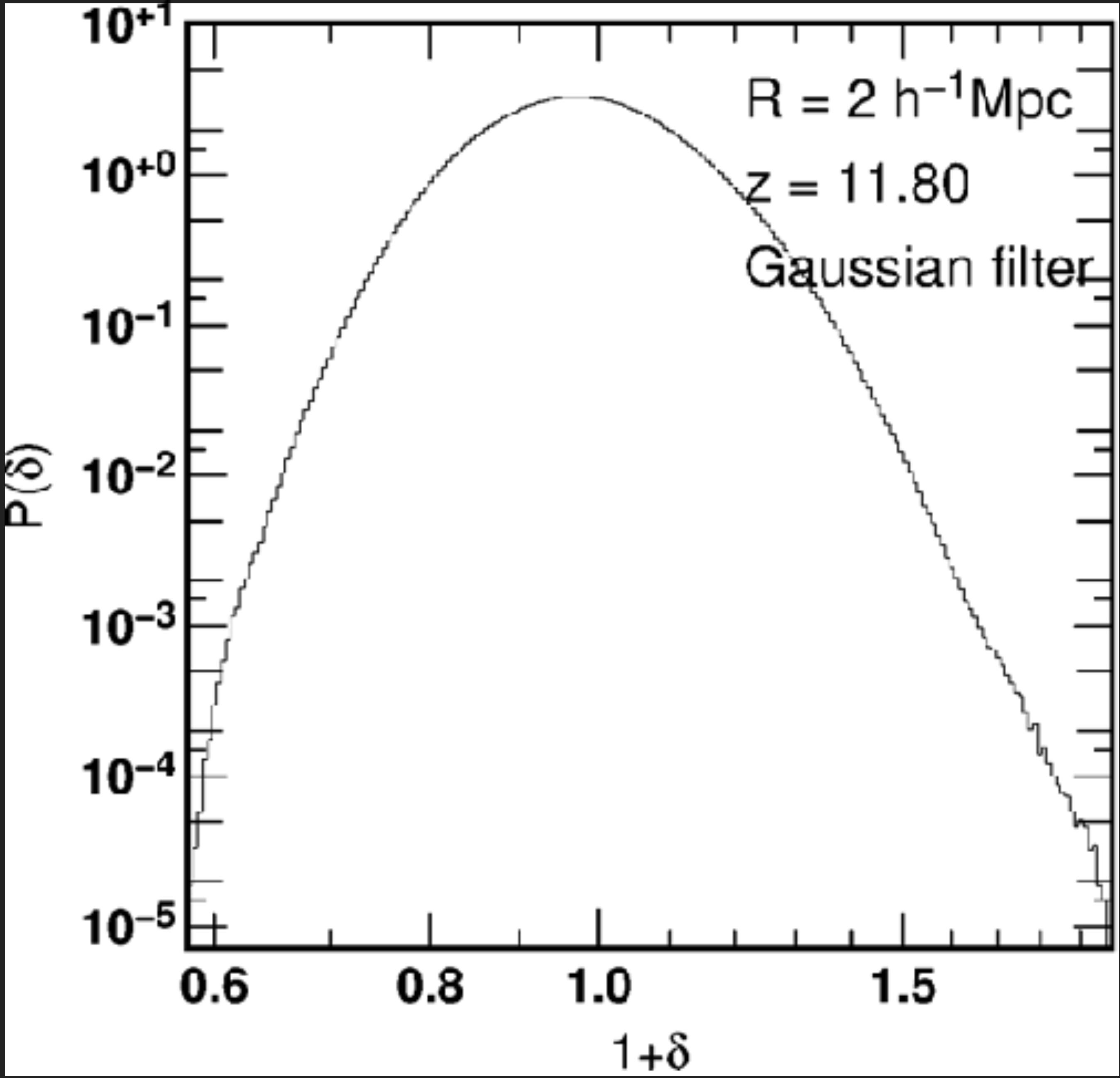
重力進化と非ガウス性/非線形性



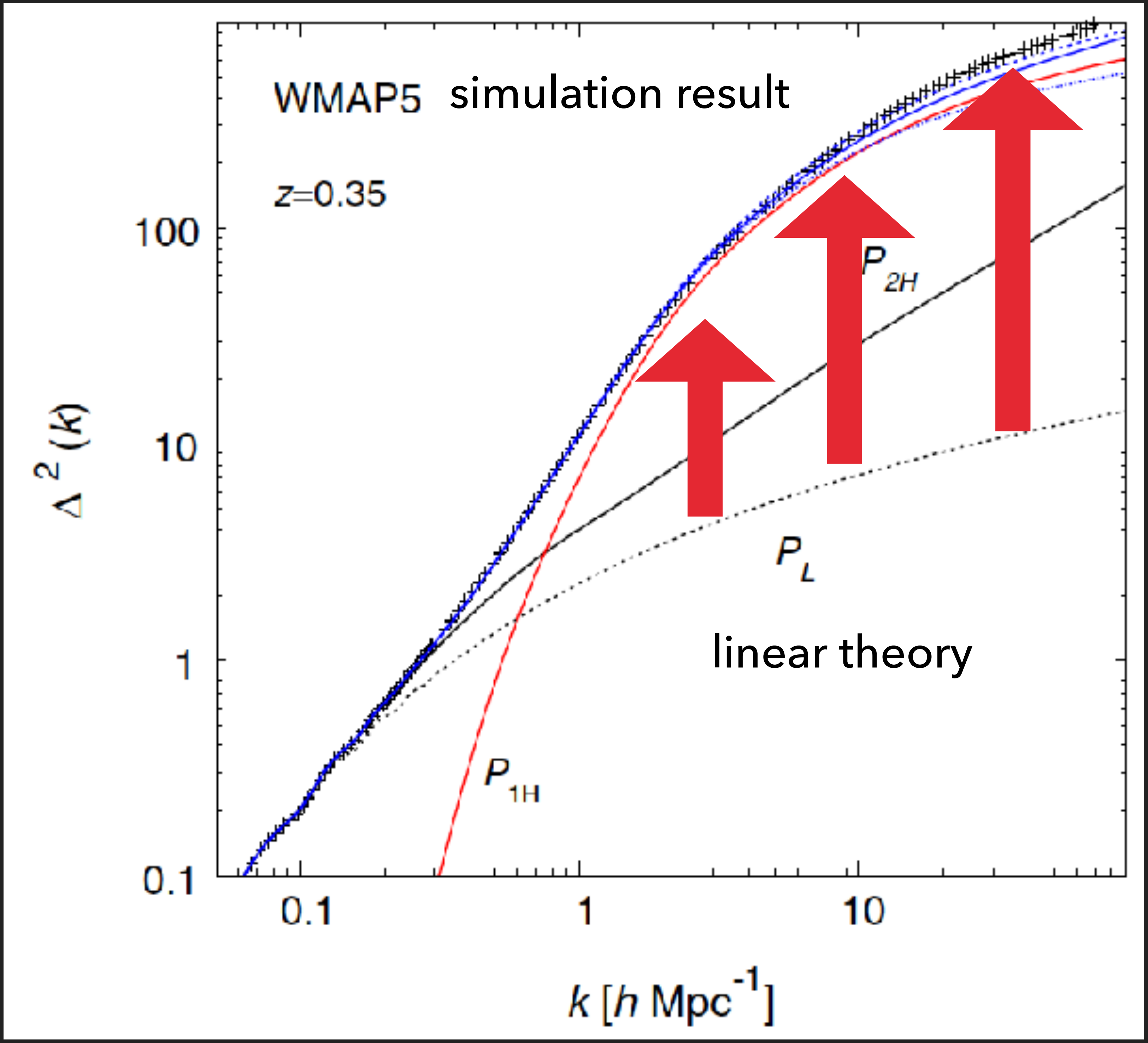


重力進化と非ガウス性/非線形性



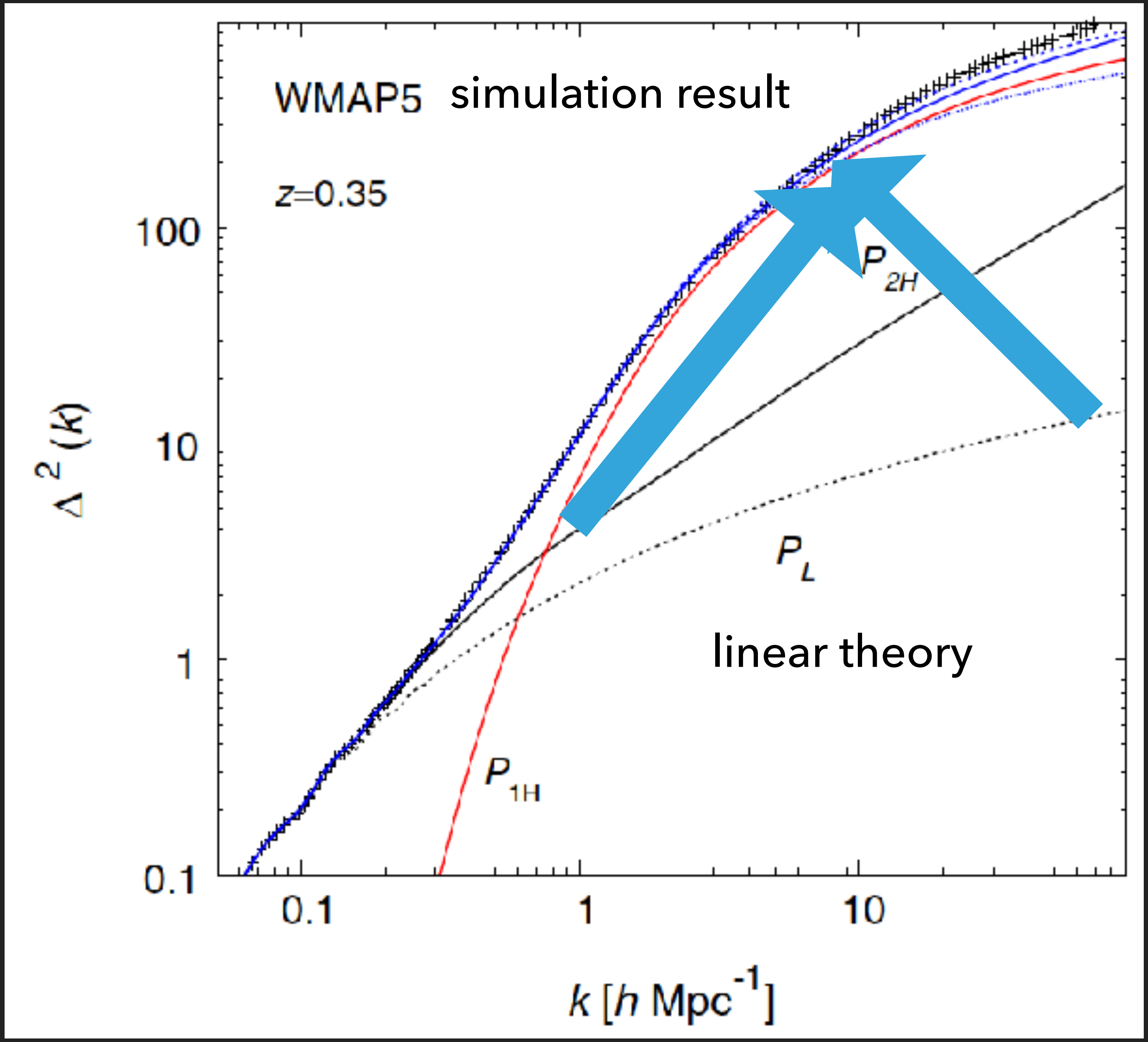


重力進化と非ガウス性/非線形性



Valageas,TN, Taruya '13

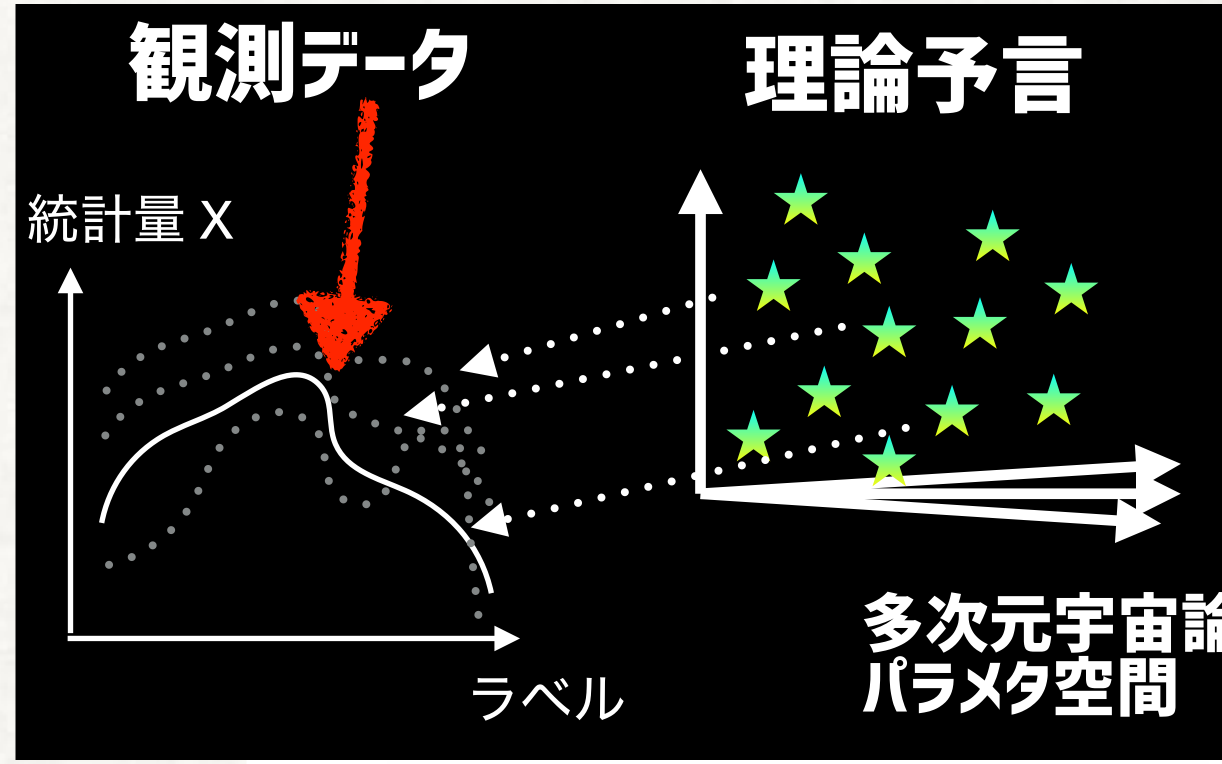
重力進化と非ガウス性/非線形性

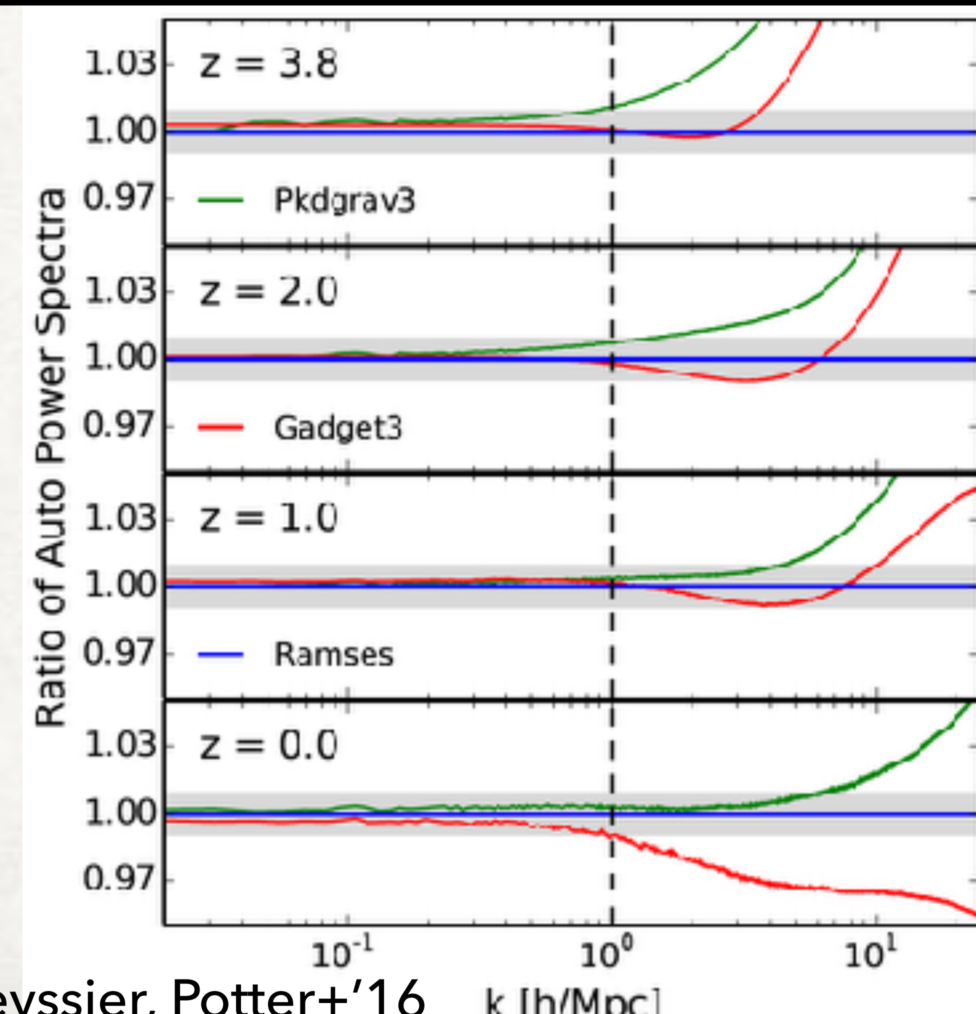


Valageas,TN, Taruya '13

宇宙大規模構造の理論予言

- 将来観測からの要請
  - ひとこえ、波数10h/Mpcまで誤差1%以内
- N体シミュレーション
  - 計算コスト
    - MCMCに乗るか？
  - 精度を担保できる？
  - 機械学習の導入？
- できるだけ手で解きたい
  - 揺らぎが小さいうちは摂動展開で良さそう
  - より定量的な理解が必要





Schneider, Teyssier, Potter+'16

BASIC EQUATIONS FOR LARGE-SCALE STRUCTURE

Juszkiewicz '81, Vishnac '83, Goroff '86,
Suto, Sasaki '91, Makino, Sasaki, Suto '92

- Density and velocity kernel $\{F_n(\mathbf{q}_1, \ldots, \mathbf{q}_n), G_n(\mathbf{q}_1, \ldots, \mathbf{q}_n)\}$

continuity + Euler + Poisson eqs.

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \cdot [(1+\delta)\boldsymbol{v}] = 0,$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + H\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{1}{a}\nabla\phi,$$

$$\nabla^2\phi = 4\pi\mathcal{G}\bar{\rho}a^2\delta.$$

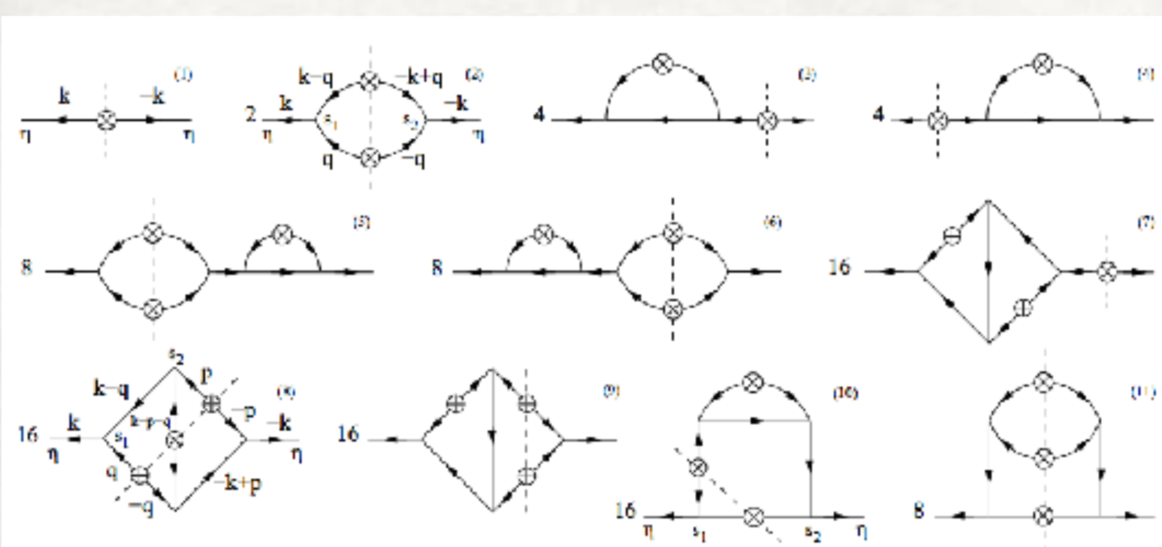
$$\delta(\mathbf{x}) = \rho(\mathbf{x})/\bar{\rho} - 1$$

$$\theta(\mathbf{x}) = \nabla \cdot \mathbf{v}(\mathbf{x})$$

$$\tilde{\delta}(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\mathbf{k}), \qquad \tilde{\theta}(\mathbf{k},\tau) = -\mathcal{H}(\tau)\sum_{n=1}^{\infty} a^n(\tau)\theta_n(\mathbf{k})$$

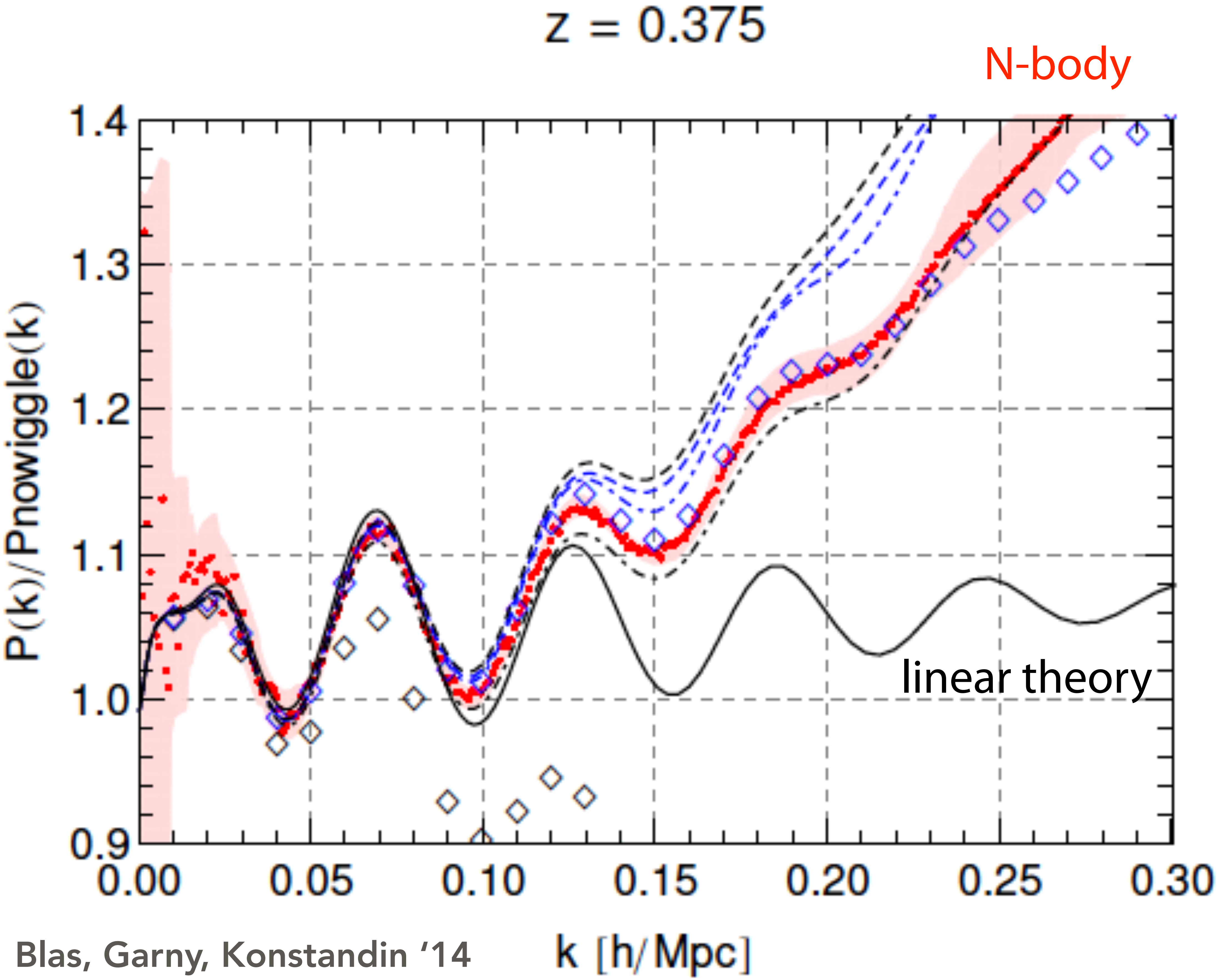
$$\delta_n(\mathbf{k}) = \int d^3\mathbf{q}_1 \ldots \int d^3\mathbf{q}_n\, \delta_D(\mathbf{k} - \mathbf{q}_{1\ldots n}) F_n(\mathbf{q}_1, \ldots, \mathbf{q}_n)\delta_1(\mathbf{q}_1)\ldots\delta_1(\mathbf{q}_n),$$

$$\theta_n(\mathbf{k}) = \int d^3\mathbf{q}_1 \ldots \int d^3\mathbf{q}_n\, \delta_D(\mathbf{k} - \mathbf{q}_{1\ldots n}) G_n(\mathbf{q}_1, \ldots, \mathbf{q}_n)\delta_1(\mathbf{q}_1)\ldots\delta_1(\mathbf{q}_n)$$



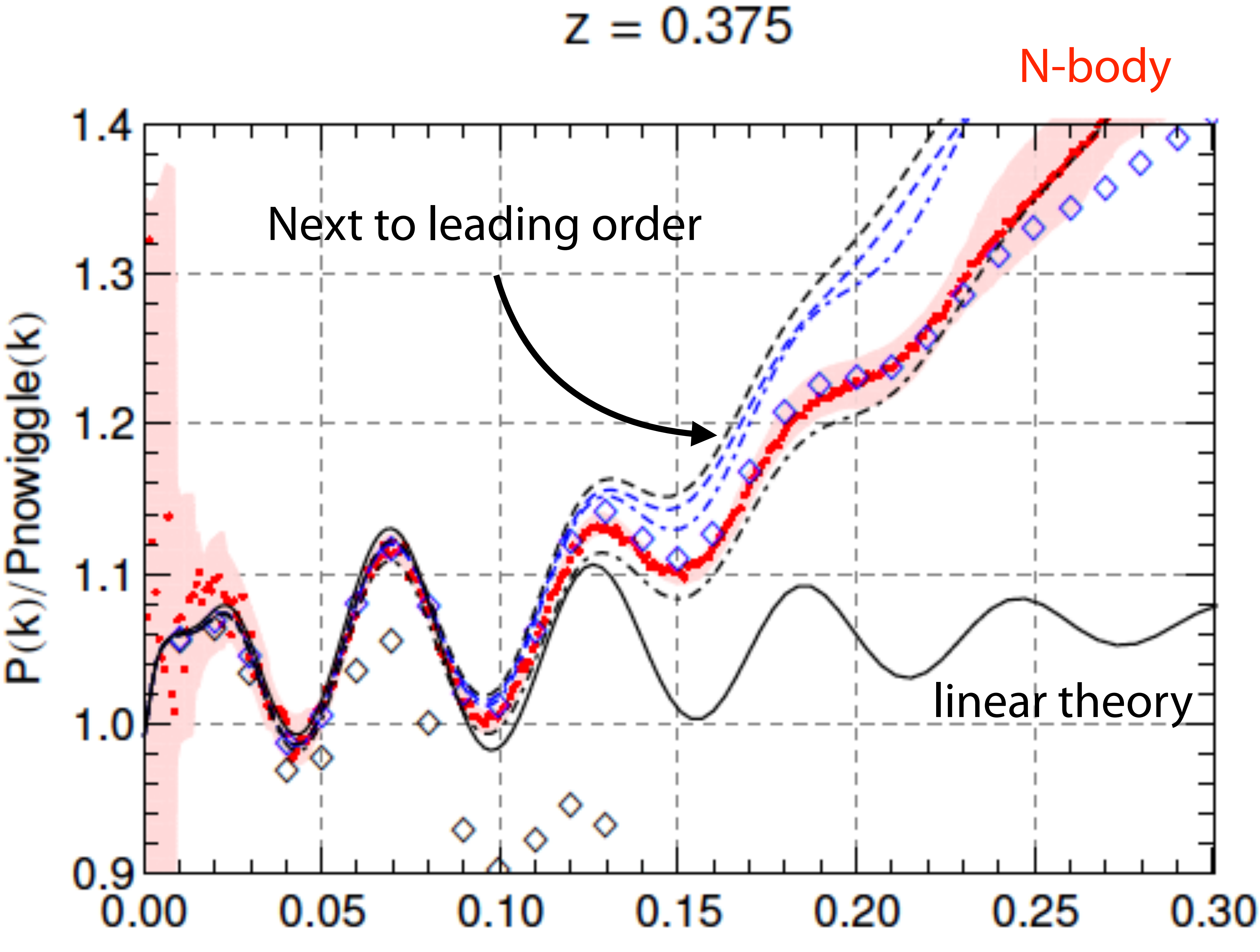
Crocce & Scoccimarro 06

PERTURBATION THEORY IS IN CRISIS



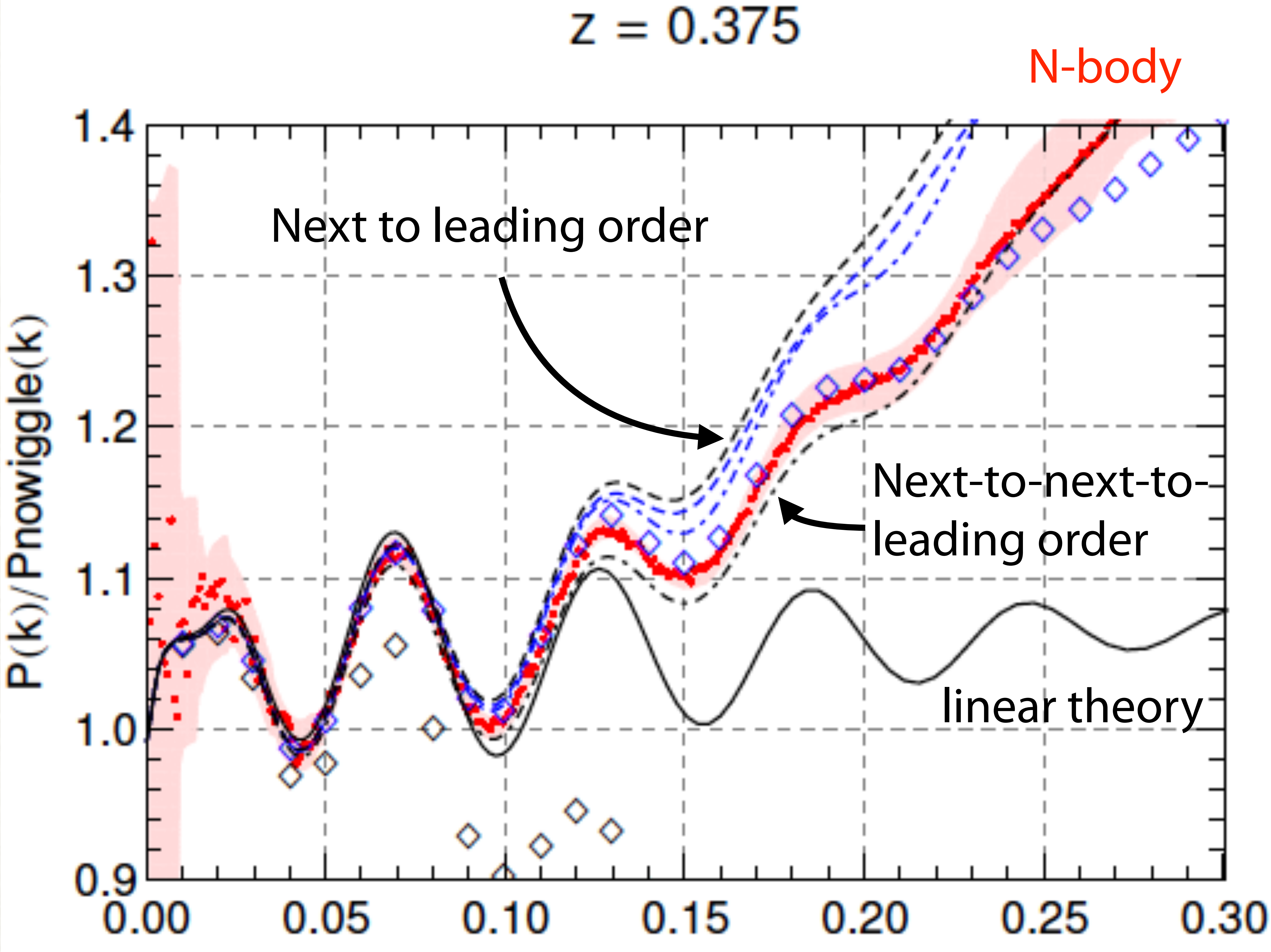
Blas, Garny, Konstandin '14

PERTURBATION THEORY IS IN CRISIS



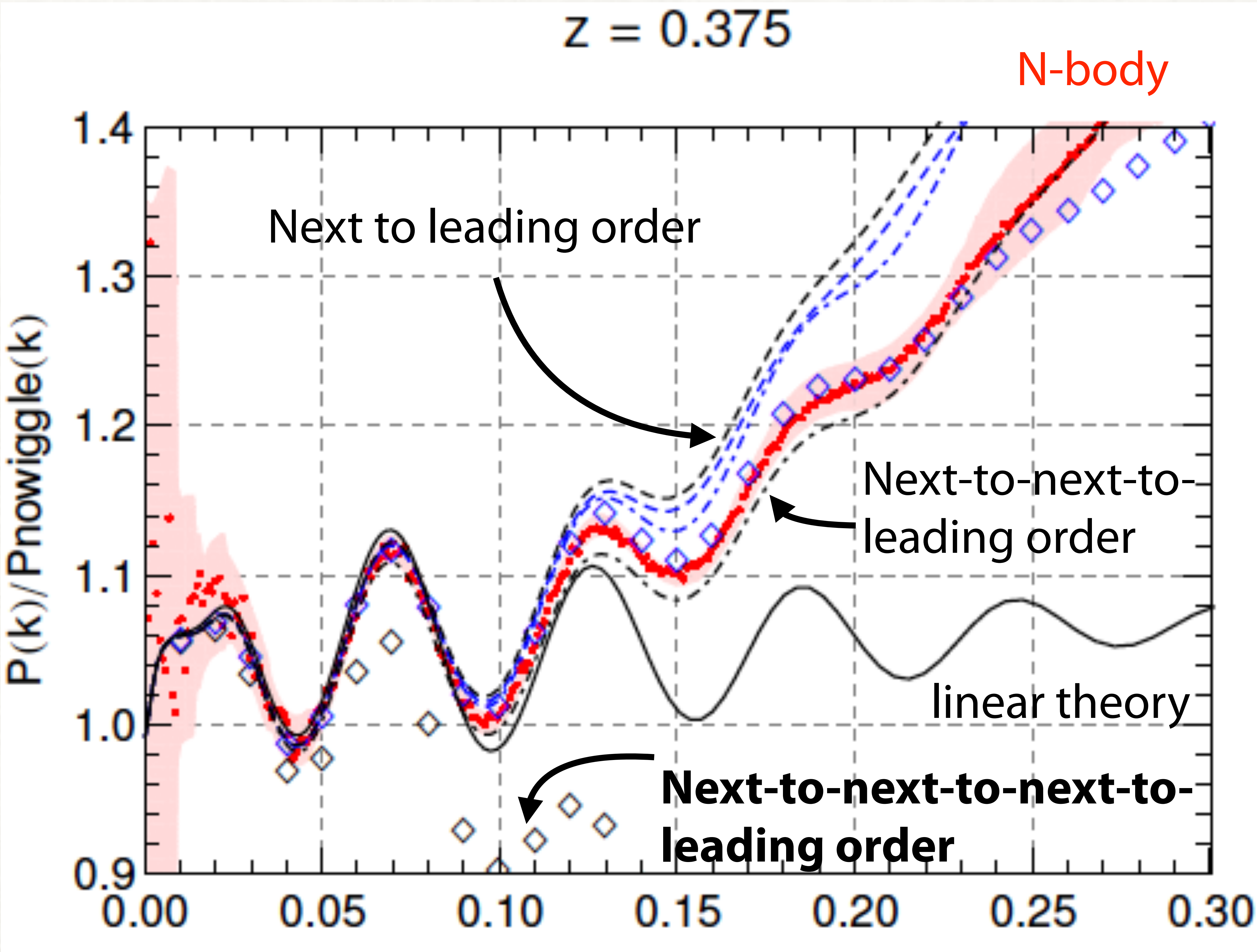
Blas, Garny, Konstandin '14

PERTURBATION THEORY IS IN CRISIS



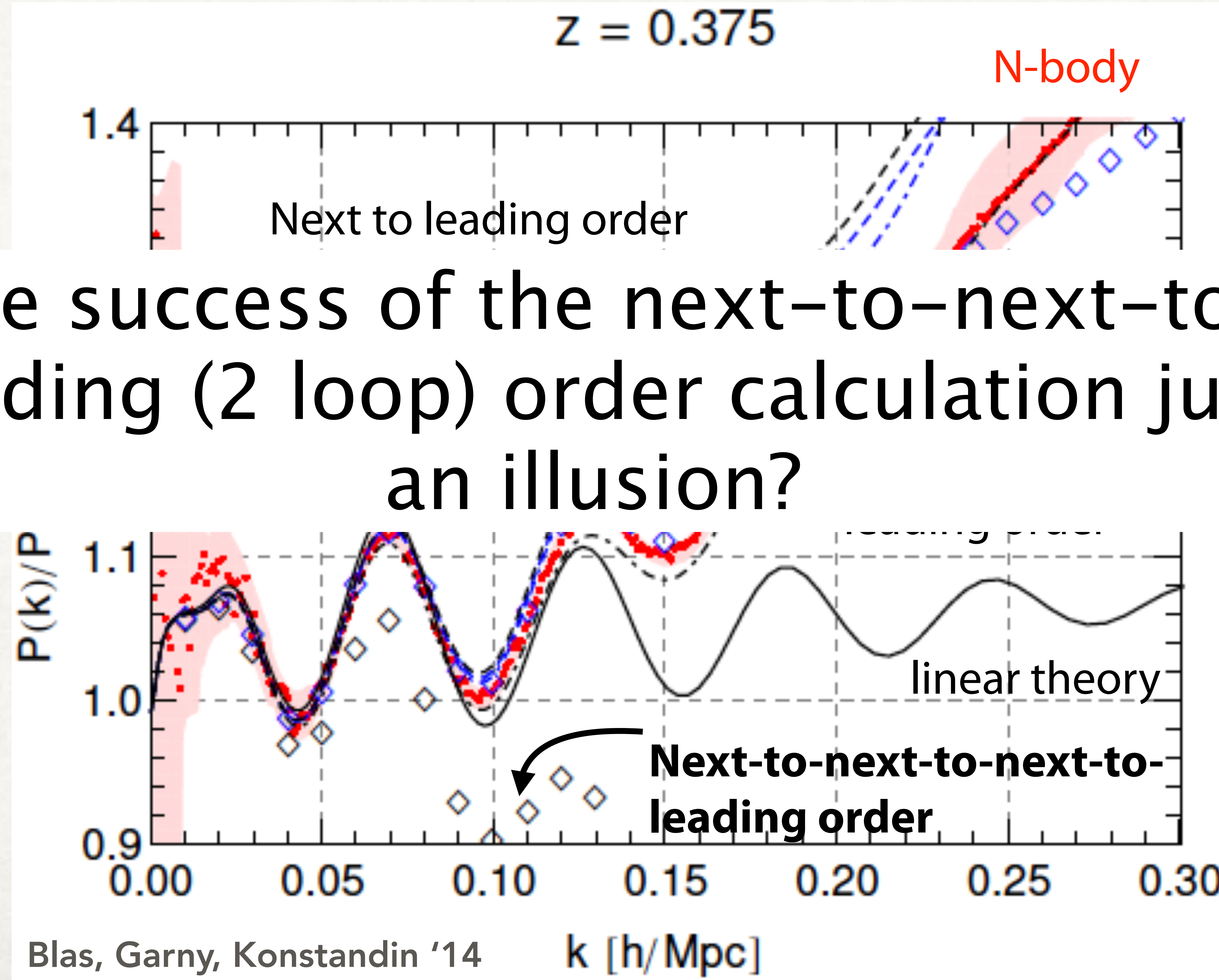
Blas, Garny, Konstandin '14

PERTURBATION THEORY IS IN CRISIS



Blas, Garny, Konstandin '14

PERTURBATION THEORY IS IN CRISIS



EFFECTIVE FIELD THEORY APPROACHES

continuity + Euler + Poisson eqs.

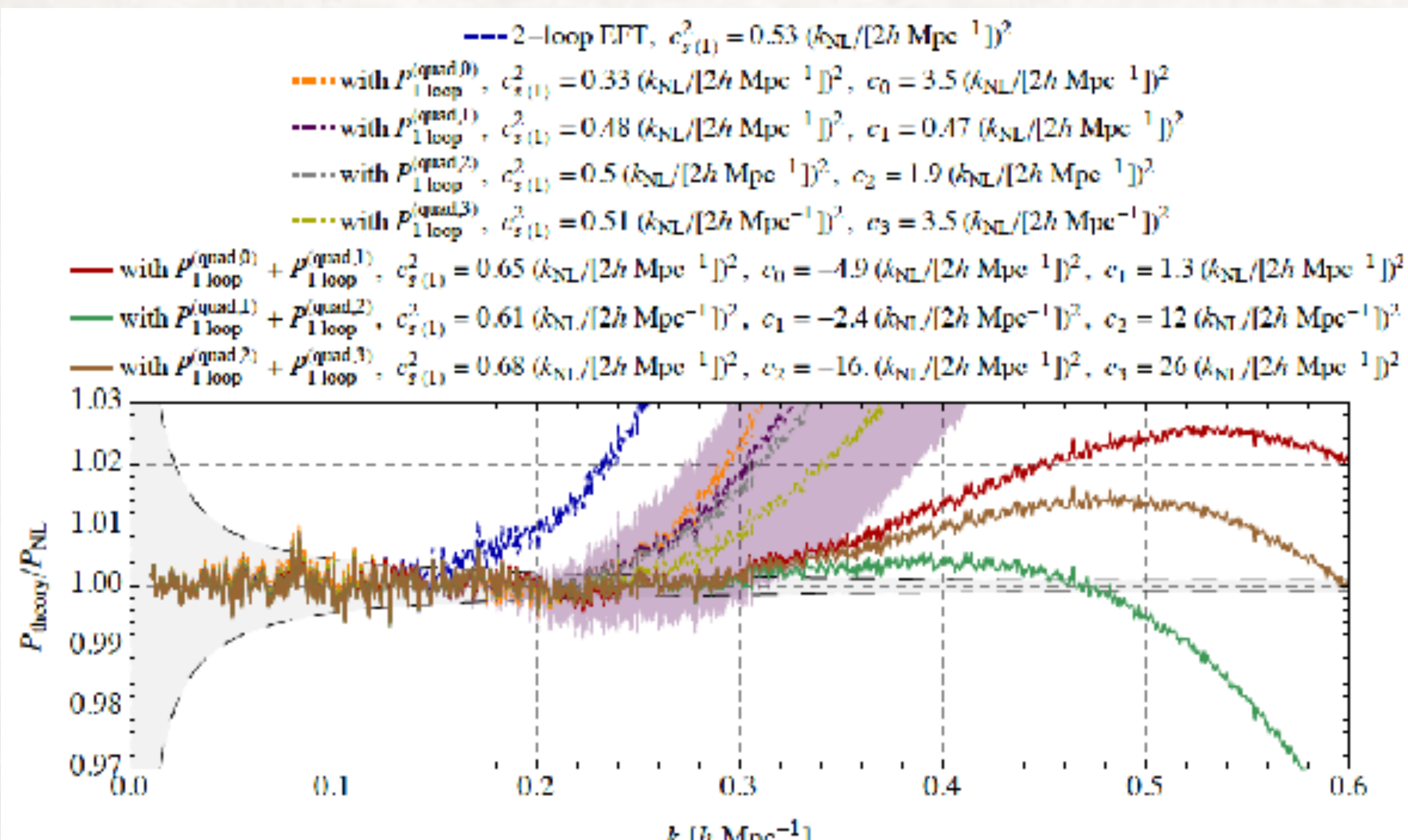
Baumann+'12, Carrasco, Herzberg, Senatore'12 …

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \cdot [(1+\delta)\boldsymbol{v}] = 0,$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + H\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{1}{a}\nabla\phi - \frac{1}{\rho_{\mathrm{m}}}\frac{1}{a}\nabla\tau_{ij}$$

$$\nabla^2\phi = 4\pi\mathcal{G}\bar{\rho}a^2\delta.$$

- Neglecting the stress tensor would be a reasonable approximation for a CDM-dominated universe at least at early times
- EFT estimates the functional form for the corrections from viscosity and anisotropic stress in an empirical manner
- Introduce free parameters and determine them by simulations

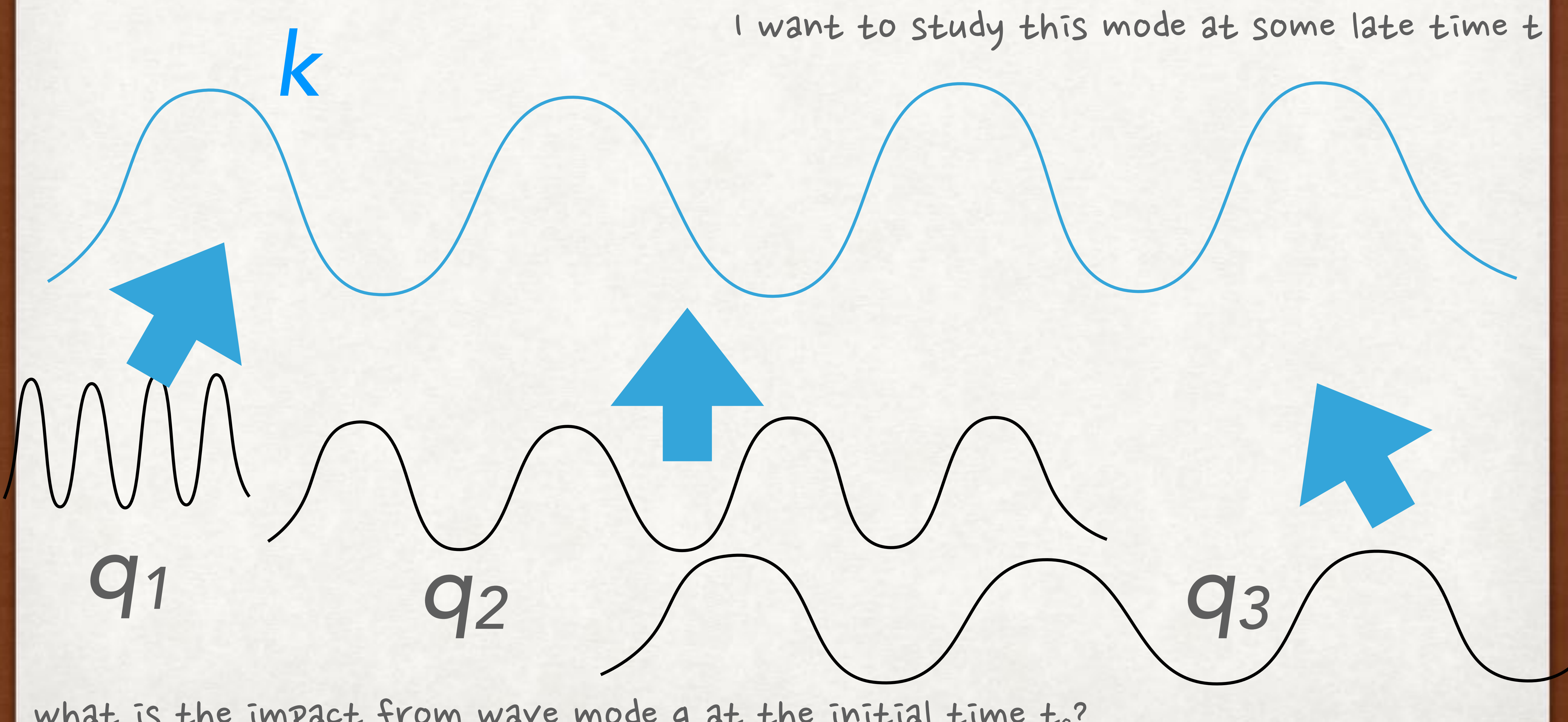


Foreman, Perrier, Senatore'16

SYSTEM-LEVEL RESPONSE FUNCTION

$$K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}$$

TN, Bernardeau, Taruya '16



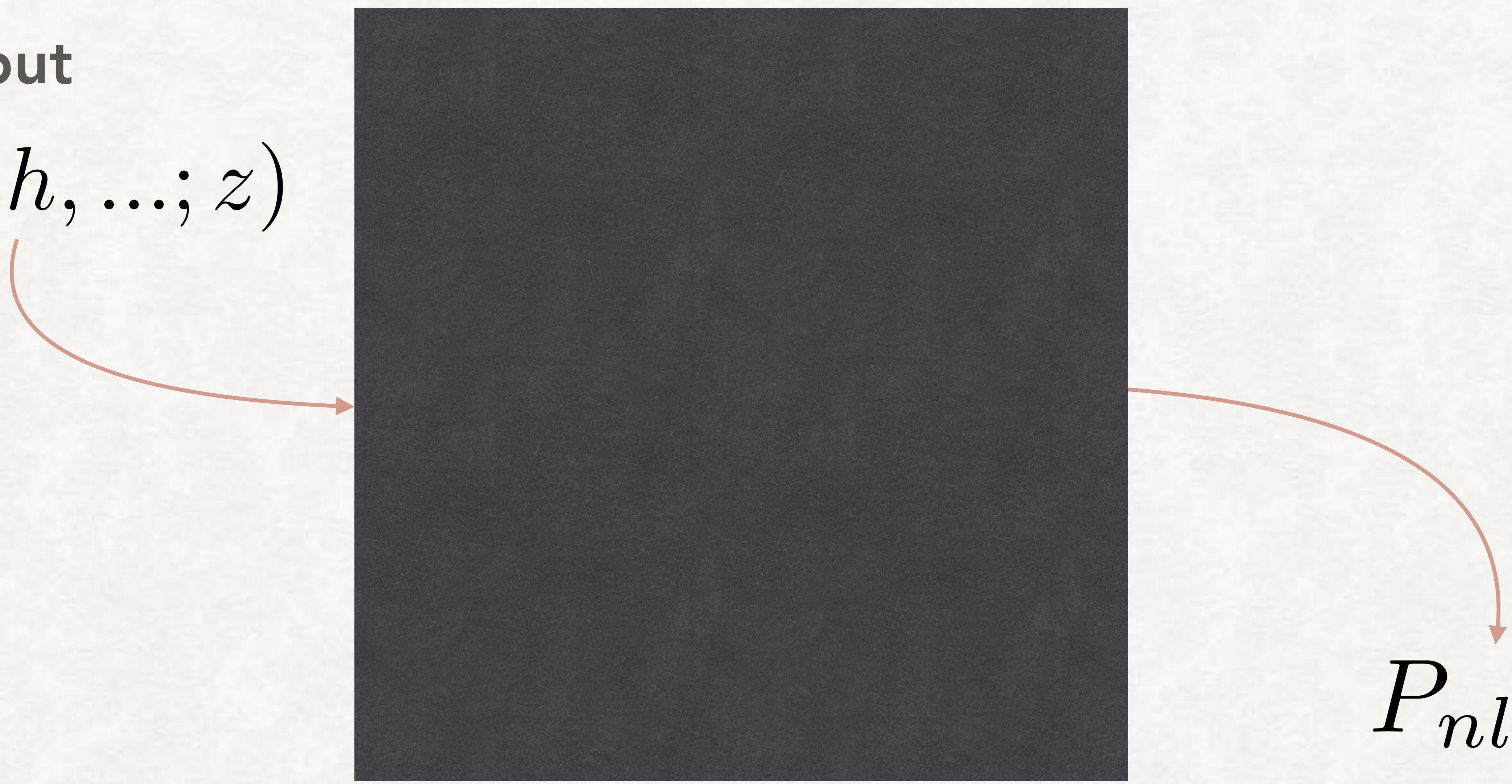
SYSTEM-LEVEL RESPONSE FUNCTION

TN, Bernardeau, Taruya '16

large scale structure gravitational evolution

Input

$(\Omega_m, h, ...; z)$



$P_{nl}(k)$

Output

SYSTEM-LEVEL RESPONSE FUNCTION

TN, Bernardeau, Taruya '16

large scale structure gravitational evolution

Input

$$(\Omega_m, h, ...; z)$$

$$P_{lin}(k)$$

$$P_{nl}(k)$$

To a very good approximation

Output

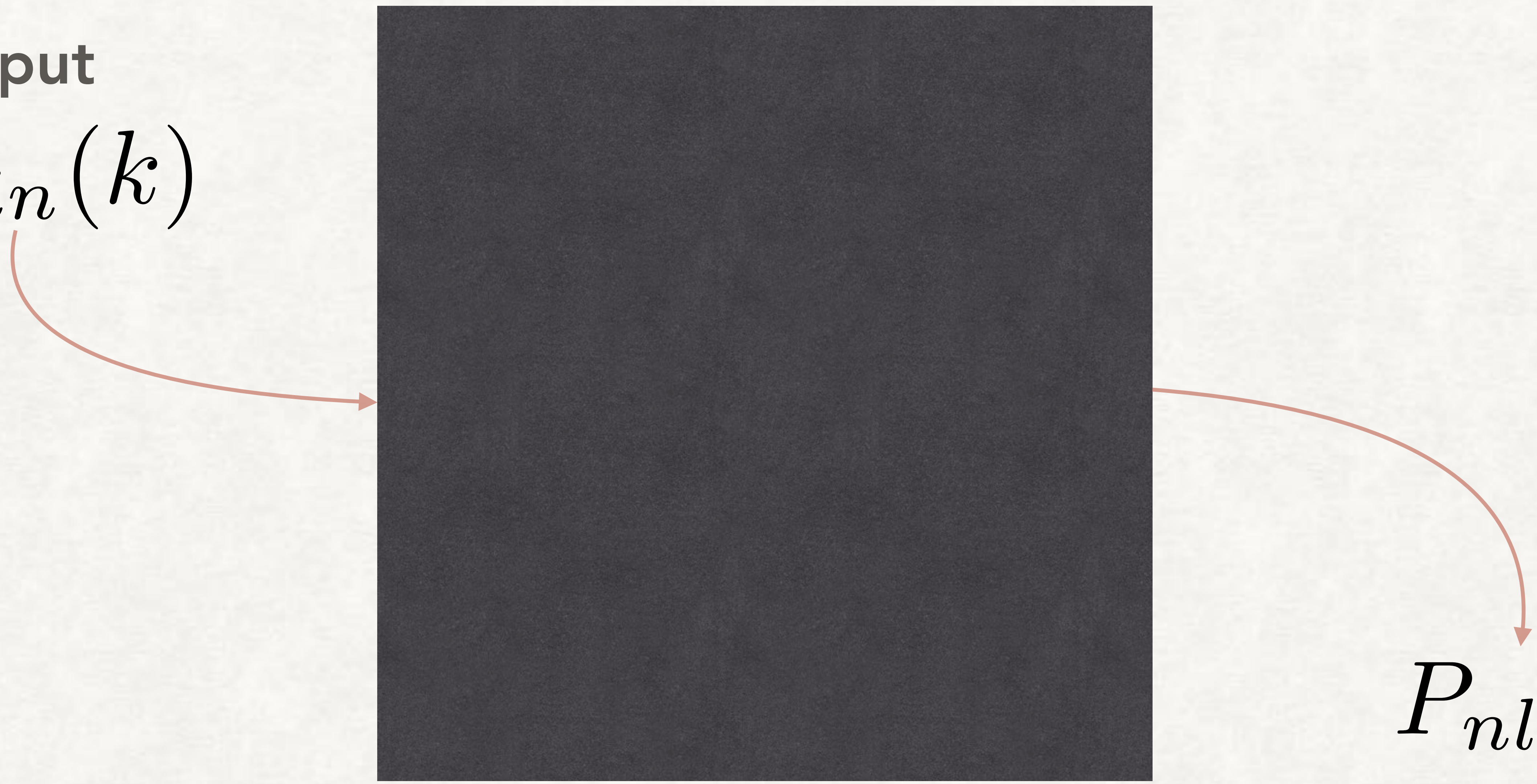
SYSTEM-LEVEL RESPONSE FUNCTION

TN, Bernardeau, Taruya '16

large scale structure gravitational evolution

Input

$$P_{lin}(k)$$



$$P_{nl}(k)$$

Output

$$K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}$$

SYSTEM-LEVEL RESPONSE FUNCTION

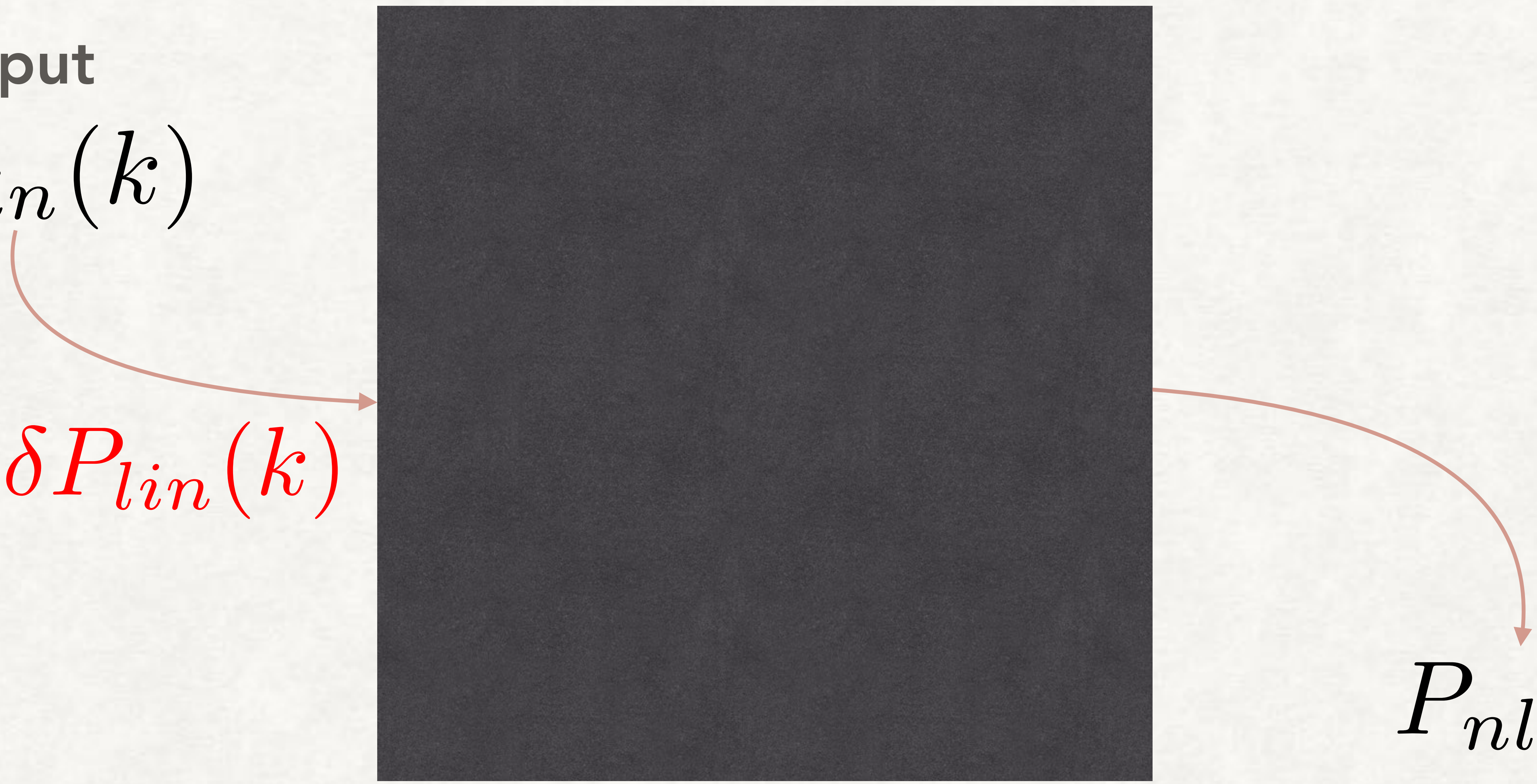
TN, Bernardeau, Taruya '16

large scale structure gravitational evolution

Input

$P_{lin}(k)$

$+\delta P_{lin}(k)$



$P_{nl}(k)$

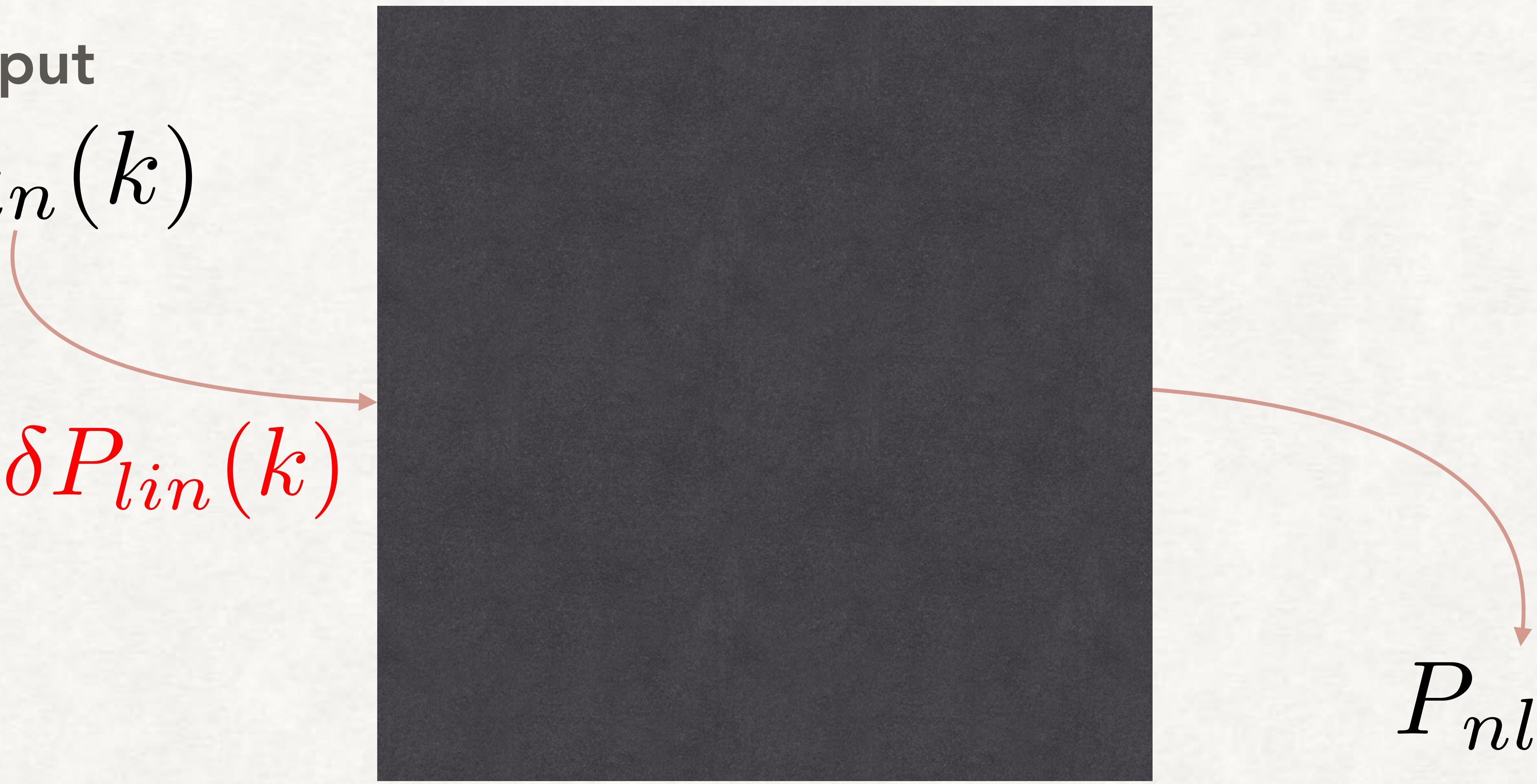
Output

$$K(k, q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}$$

SYSTEM-LEVEL RESPONSE FUNCTION

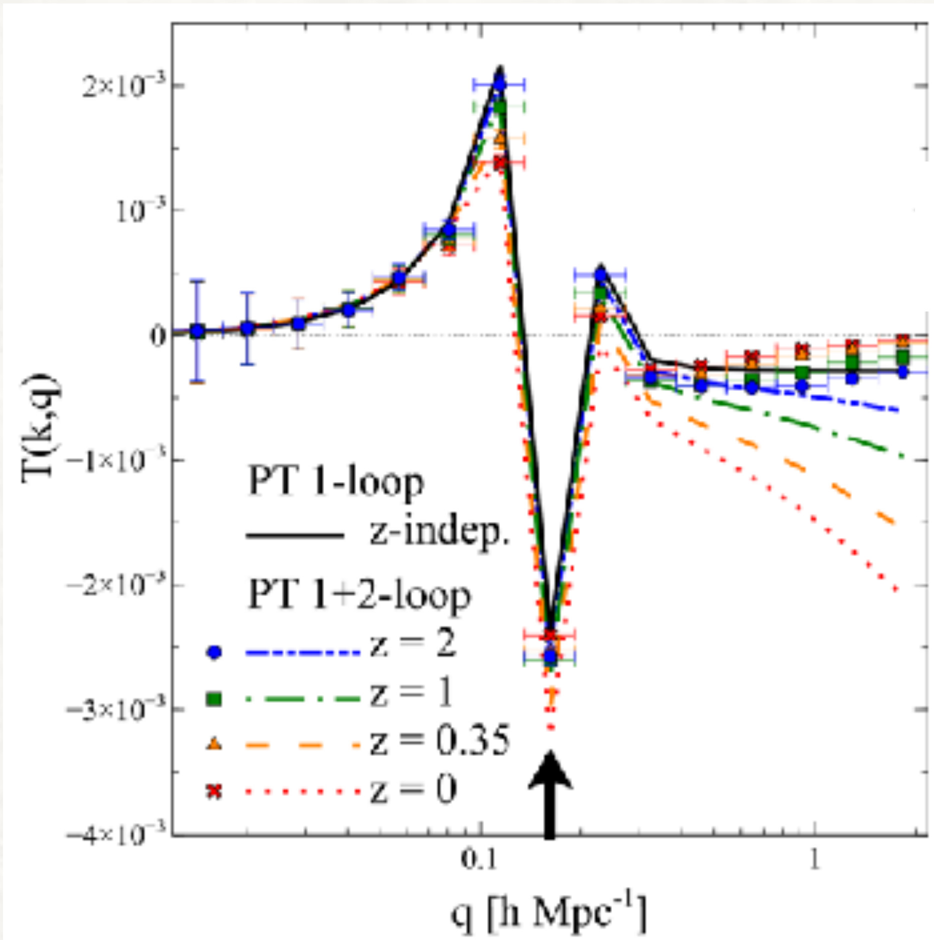
TN, Bernardeau, Taruya '16

large scale structure gravitational evolution



$$K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}$$

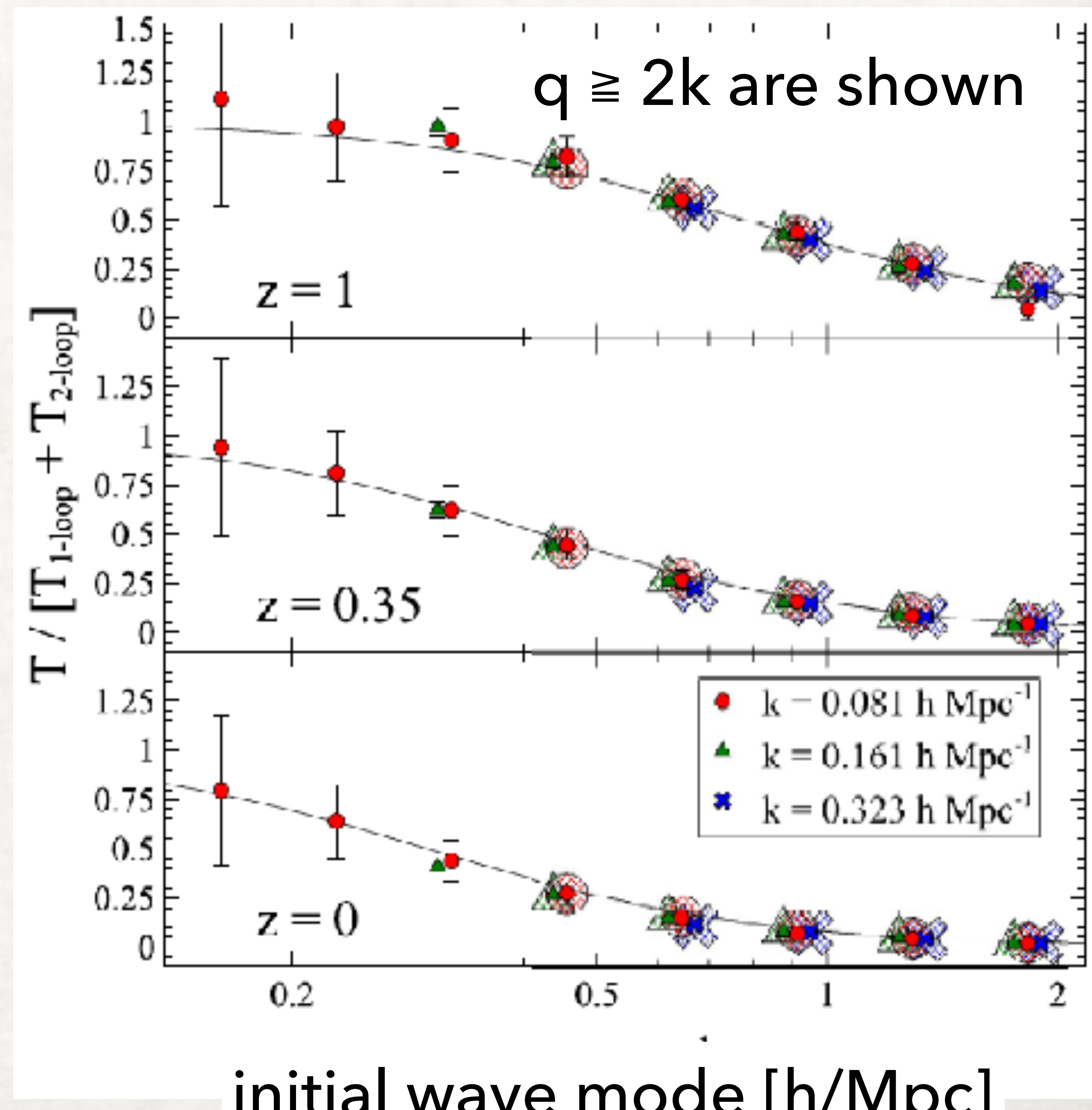
RESPONSE FUNCTION: SIM VS PT



TN, Bernardeau, Taruya '16

Rescaled quantity:

$$T(k,q) \equiv [K(k,q) - K^{\rm lin}(k,q)]/[q P^{\rm lin}(k)]$$



- SPT (2-loop) >> N-body @ high q
- This is exactly where PT breaks down
- What N-body tells us is:

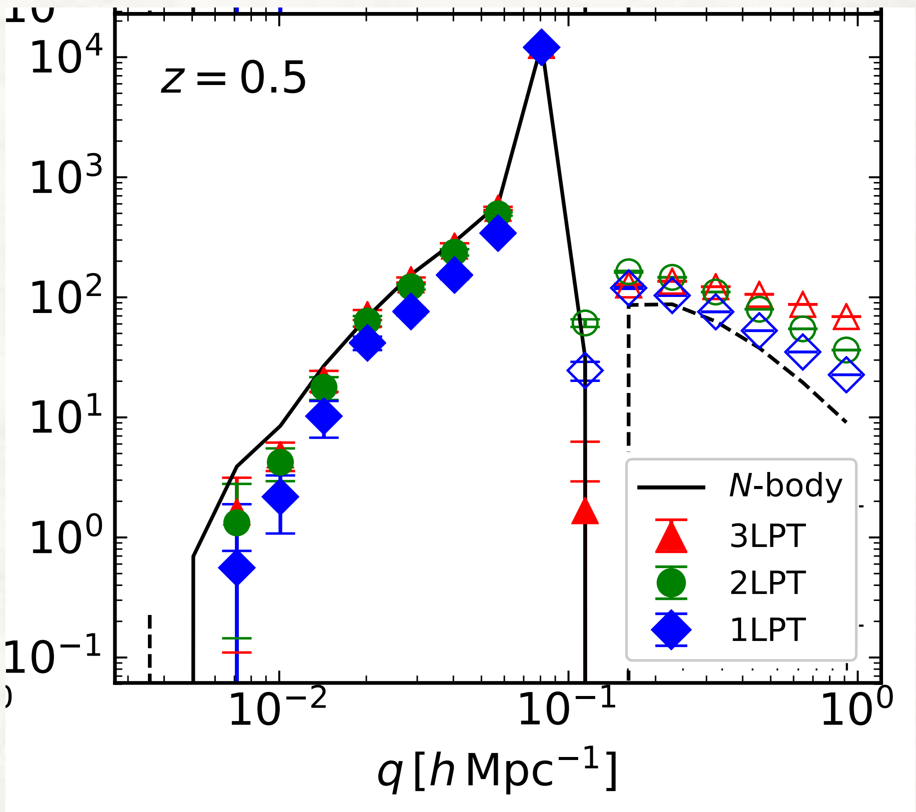
"Physics at strongly nonlinear regime does not propagate to large scales"

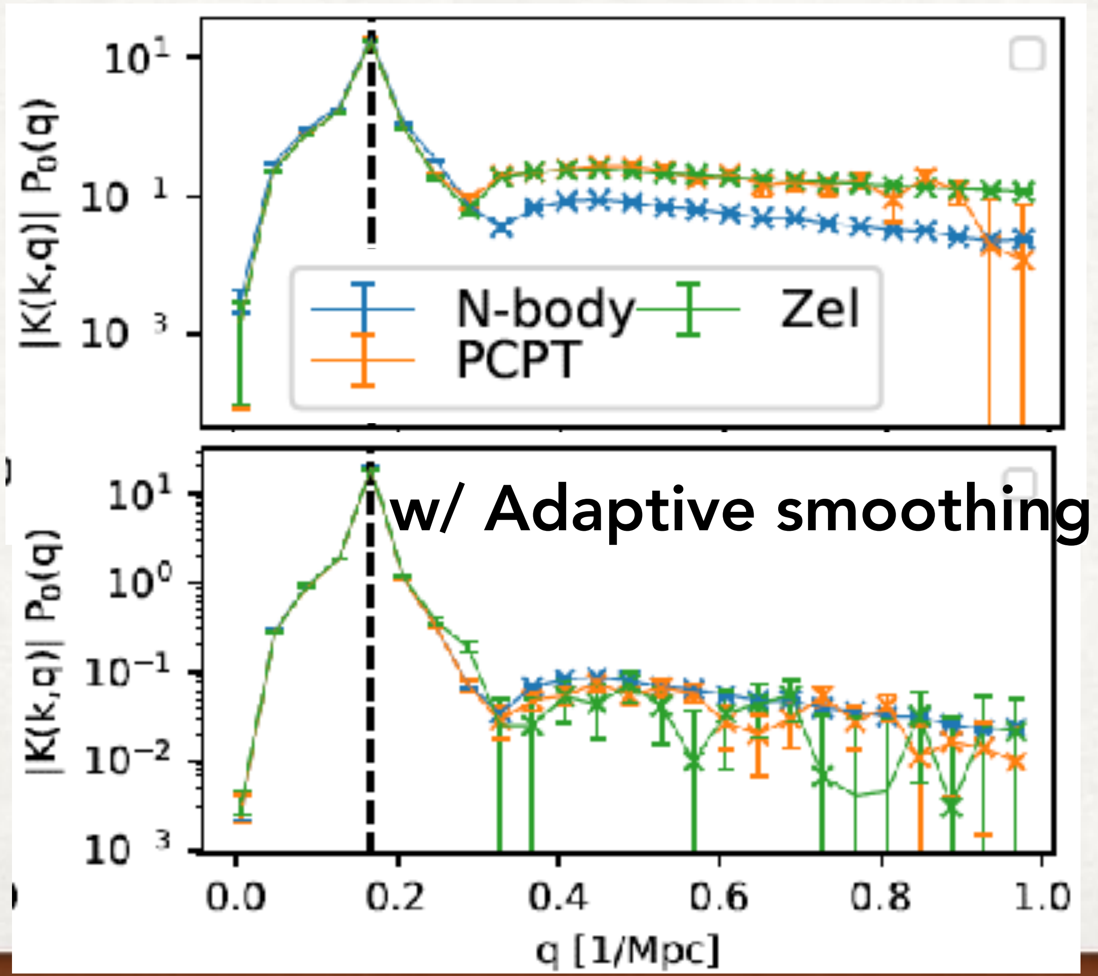
UV SENSITIVITY?

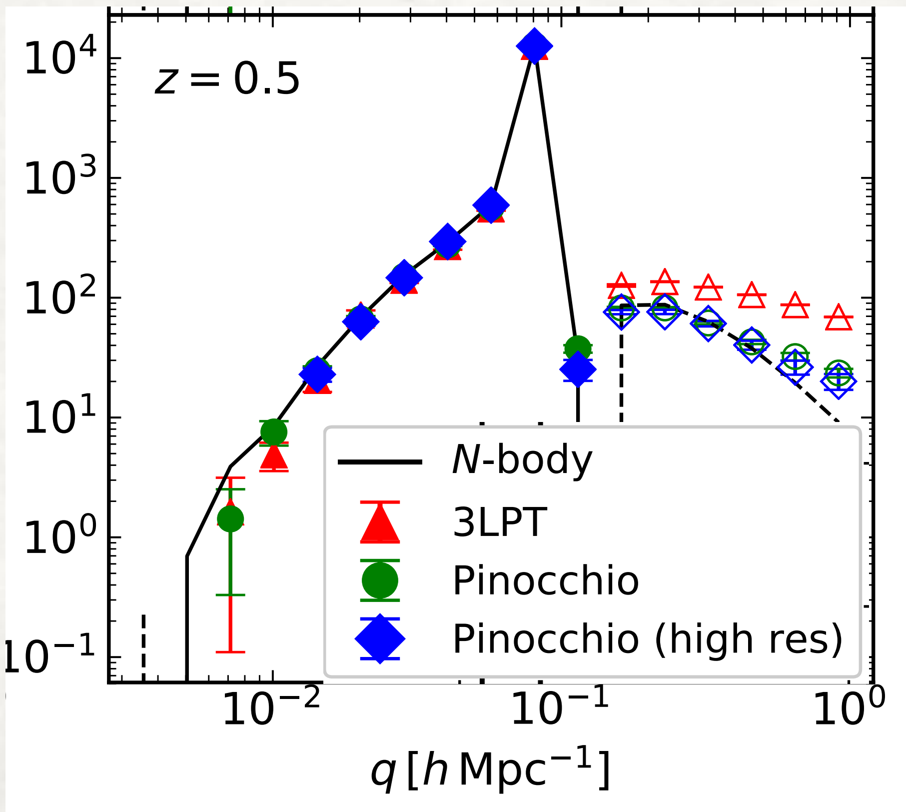
Halle, Colombi, Taruya, TN, in prep

TN, Taruya, Colombi, Halle, in prep

- 1D dynamics: Zel'dovich solution is the exact solution before shell crossing
  - Exhibits too strong UV sensitivity
  - Smoothing out shell crossing regions helps
- 3D LPT dynamics
  - PINpointing Orbit Crossing Collapsed HIerarchical Objects (PINOCCHIO, Monaco+'02) works well







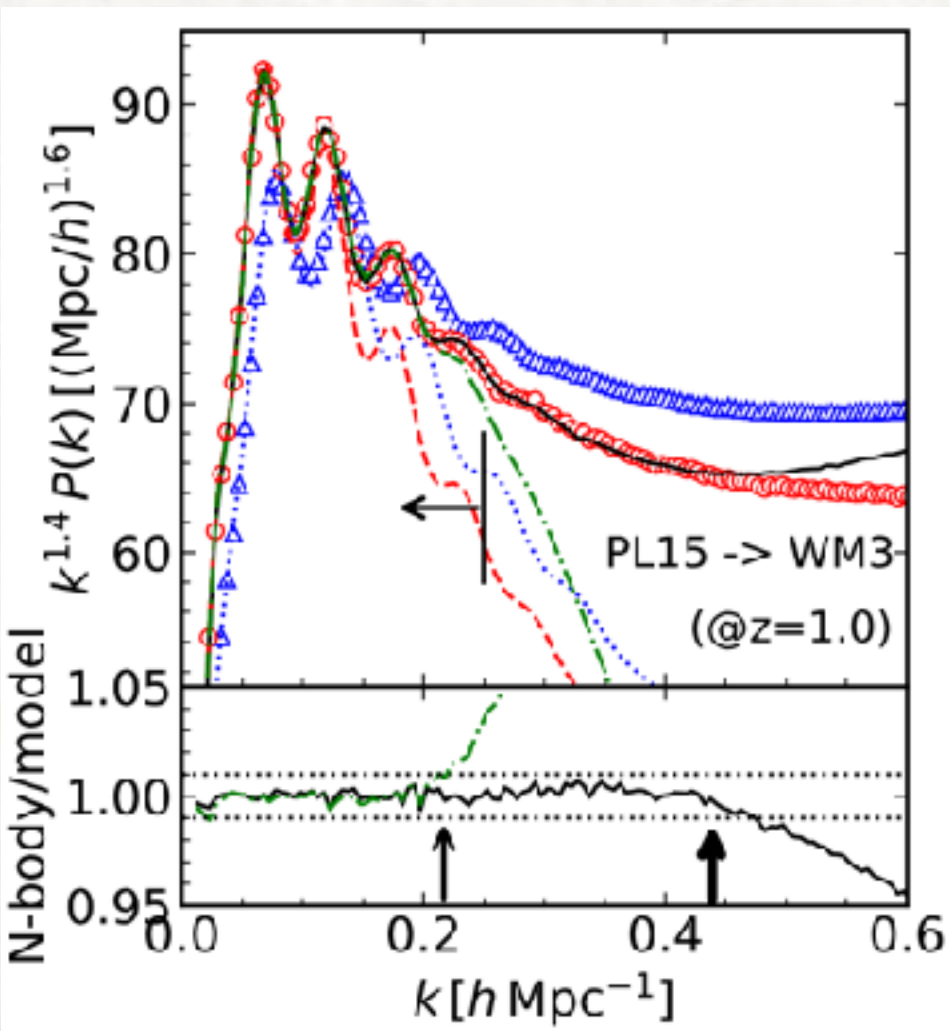
PRACTICAL USAGE? RECONSTRUCTION

- From the definition of a functional derivative

TN, Bernardeau, Taruya '17

$$P_{\rm nl}(k;\boldsymbol{p}_1) \approx P_{\rm nl}(k;\boldsymbol{p}_0) + \int \mathrm{d}\ln q\, K(k,q) \times \left[P_{\rm lin}(q;\boldsymbol{p}_1) - P_{\rm lin}(q;\boldsymbol{p}_0)\right],$$

- Use this to predict P_{nl} for model $\mathbf{p_1}$ given P_{nl} for another model $\mathbf{p_0}$



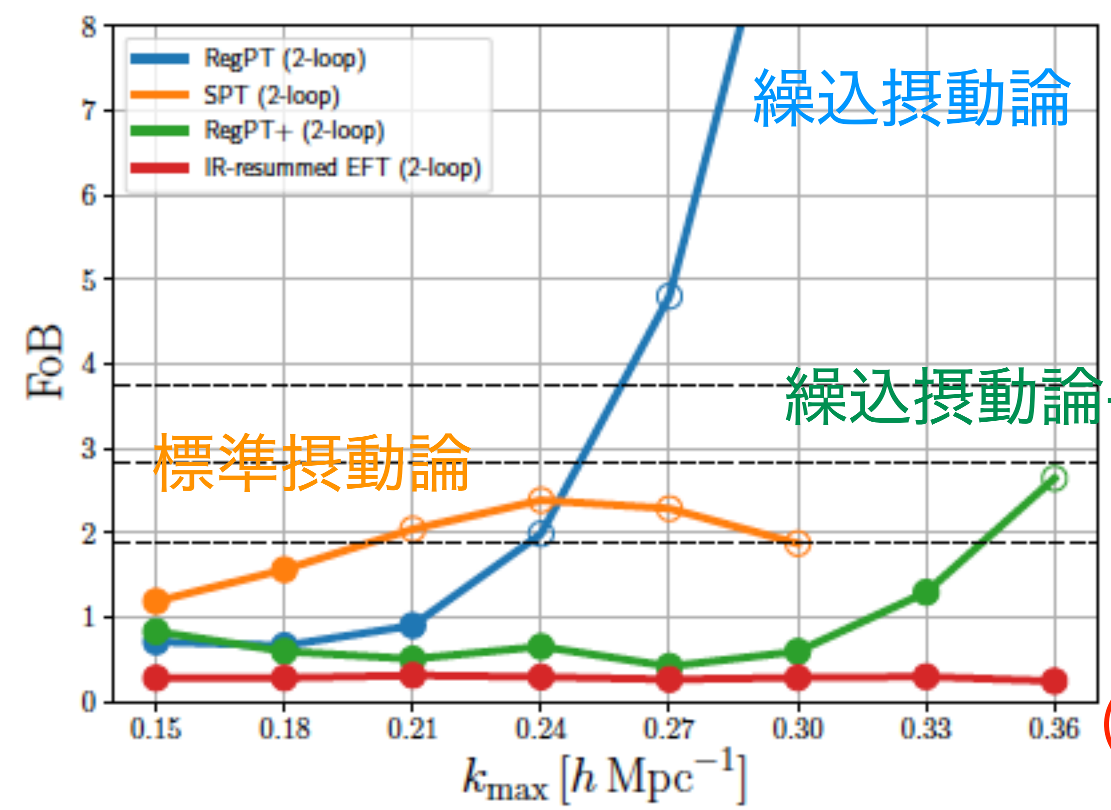
RESPRESSO (Rapid and Efficient SPectrum calculation based on RESponSe functiOn)
python package

http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/public_codes/respresso

PT CHALLENGE

Osato, TN, Bernardeau, Taruya '18

$$\mathrm{FoB} = \left[\sum_{\alpha,\beta} \delta\theta_\alpha \left(\tilde{S}\right)^{-1}_{\alpha\beta} \delta\theta_\beta\right]^{1/2}$$



繰込摂動論

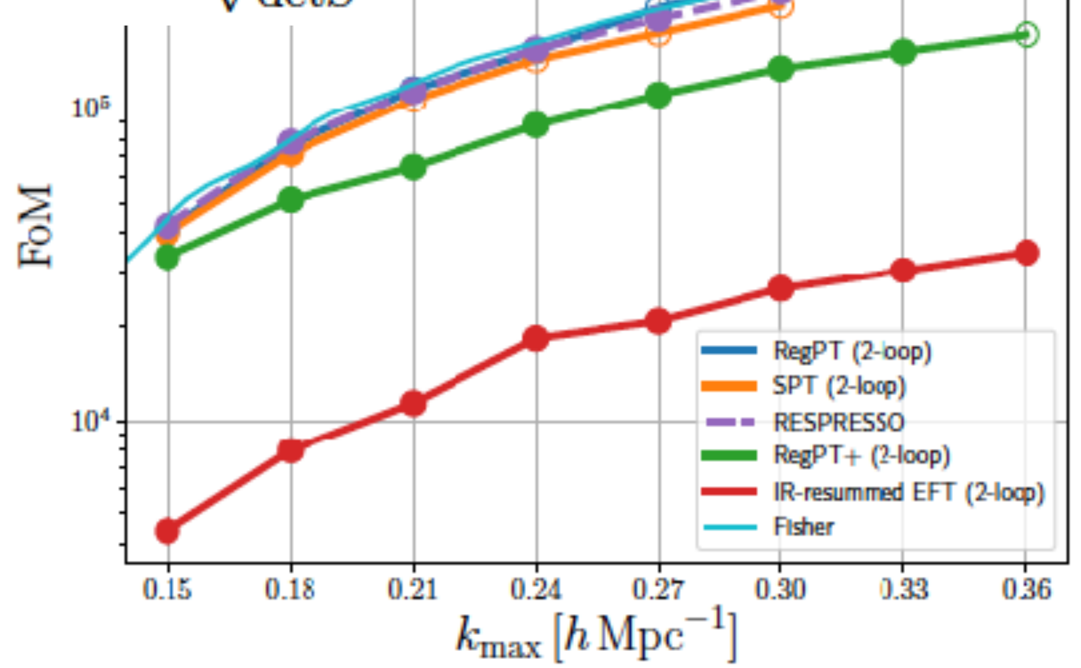
繰込摂動論+ 1 free param

標準摂動論

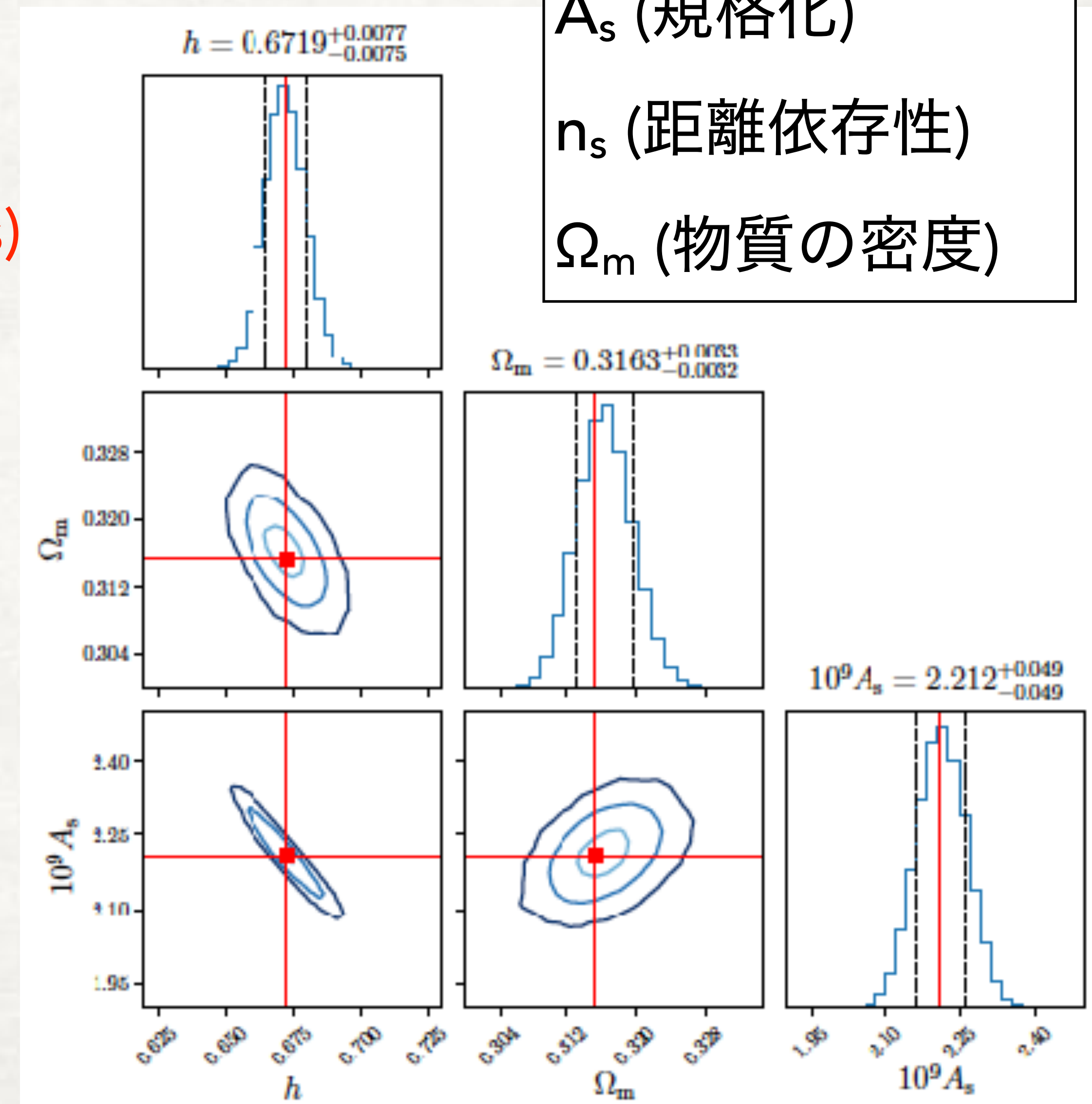
有効場の理論

(3 free params)

$$\mathrm{FoM} \equiv \frac{1}{\sqrt{\det\tilde{S}}}.$$



RESPRESSO



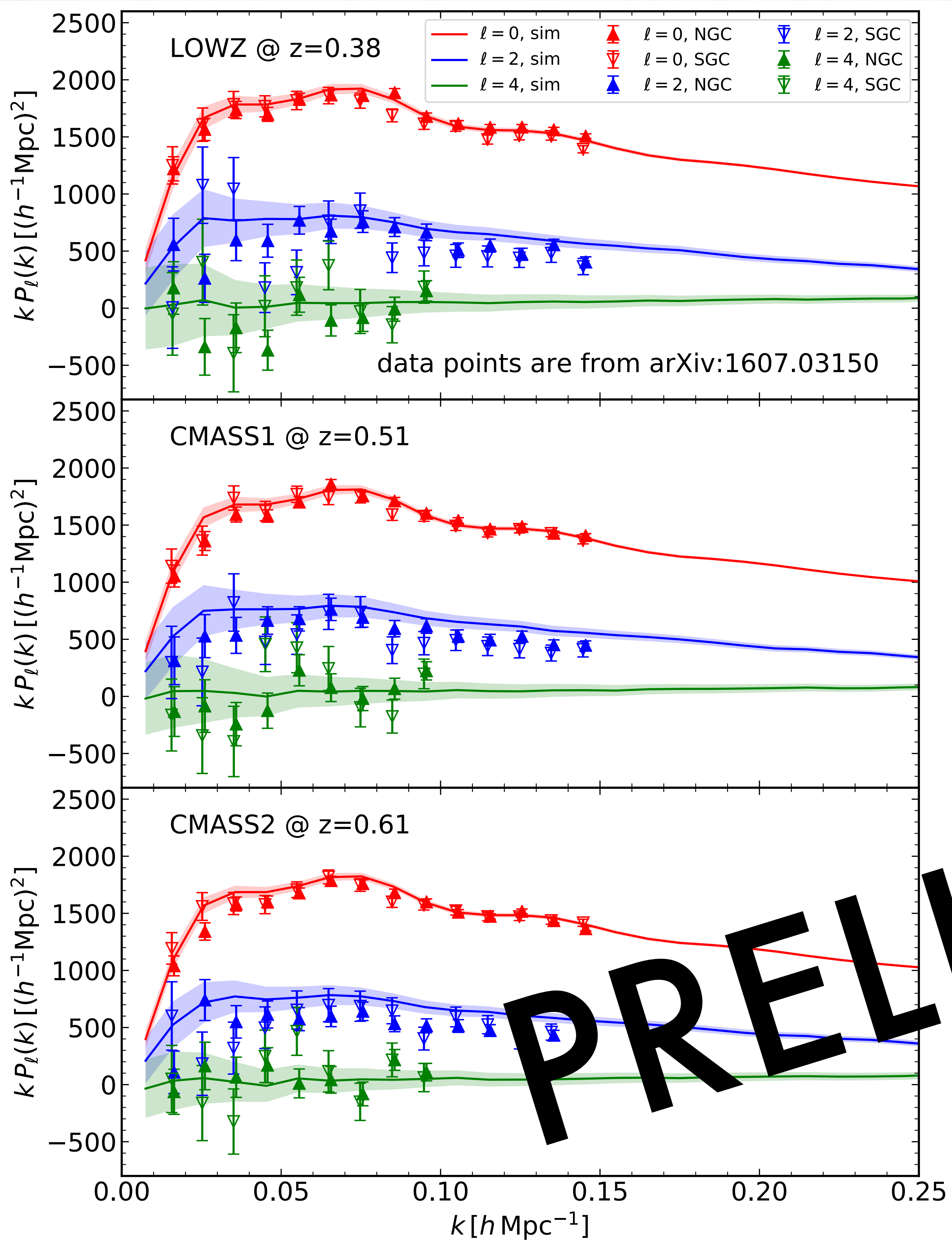
A_s (規格化)

n_s (距離依存性)

Ω_m (物質の密度)

PT CHALLENGE 2

TN, Takada, Senatore, Zaldarriaga++ in prep



- 「全部入り」のデータ
  - http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/data/PTchallenge/

WHERE TO GO FOR A THEORIST

LIMITATIONS OF SIMULATIONS / OUR KNOWLEDGE

		small scale ~1Mpc	10~100Mpc	large scale 100Mpc ~
matter field	analytical	Bad	Good(?)	Very Good(?)
	N-body	?	Very Good	Good
biased tracers	analytical	Bad	?	?
			Just a parameterization…	
halos	N-body	?	Very Good	Good
subhalos	N-body	?	?	?
galaxies	N-body	N/A	N/A	N/A
	Hydro	?	?	?

WHERE TO GO FOR A THEORIST

LIMITATIONS OF SIMULATIONS / OUR KNOWLEDGE

		small scale ~1Mpc	10~100Mpc	large scale 100Mpc ~
matter field	analytical	Bad	Good(?)	Very Good(?)
	N-body	?	Very Good	Good
biased tracers	analytical	Bad	?	?

Just a parameterization…

The Galaxy Power Spectrum and Bispectrum in Redshift Space

Vincent Desjacques, Donghui Jeong, Fabian Schmidt

(Submitted on 11 Jun 2018)

We present the complete expression for the next-to-leading (1-loop) order galaxy power spectrum and the leading-order galaxy bispectrum in redshift space in the general bias expansion, or equivalently the effective field theory of biased tracers. We consistently include all line-of-sight dependent selection effects. These are degenerate with many, but not all, of the redshift-space distortion contributions, and have not been consistently derived before. Moreover, we show that, in the framework of effective field theory, a consistent bias expansion in redshift space must include these selection contributions. Physical arguments about the tracer sample considered and its observational selection have to be used to justify neglecting the selection contributions. In summary, the next-to-leading order galaxy power spectrum and leading-order galaxy bispectrum in the general bias expansion are described by 22 parameters, which reduces to 11 parameters if selection effects can be neglected. All contributions to the power spectrum can be written in terms of 28 independent loop integrals.

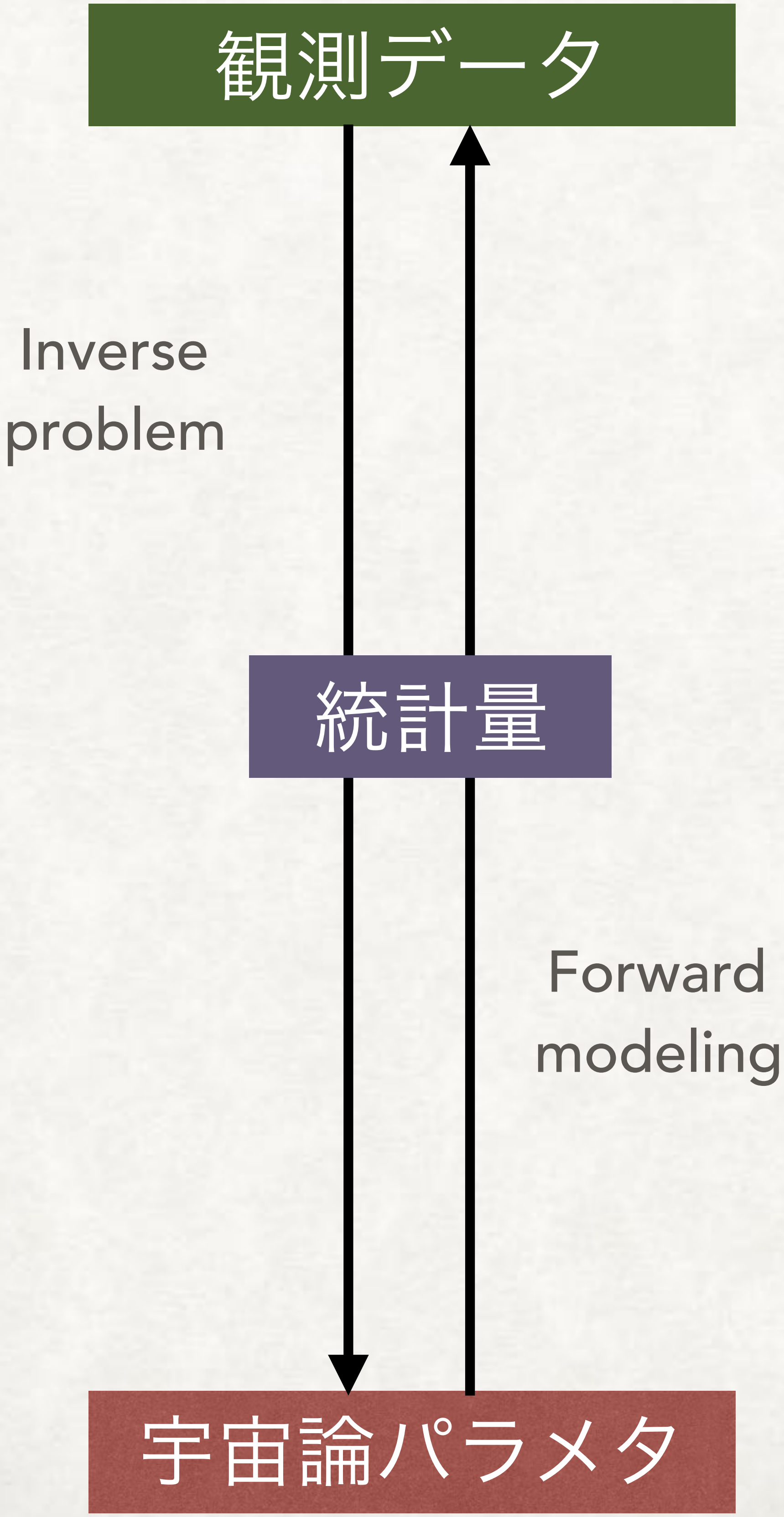
WHERE TO GO FOR A THEORIST

LIMITATIONS OF SIMULATIONS / OUR KNOWLEDGE

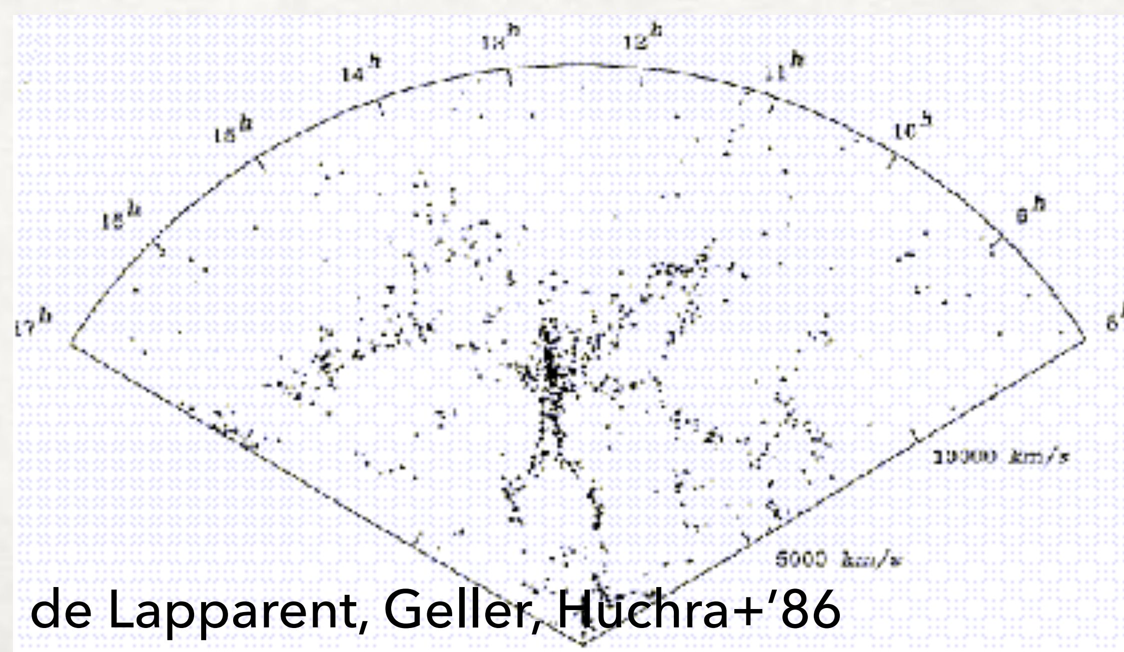
small scale ⟵⟶ large scale

		~1Mpc	10~100Mpc	100Mpc ~
matter field	analytical	Bad	Good(?)	Very Good(?)
	N-body	?	Very Good	Good
biased tracers	analytical	Bad	?	?
			Just a parameterization…	
halos	N-body	?	Very Good	Good
subhalos	N-body	?	?	?
galaxies	N-body	N/A	N/A	N/A
	Hydro	?	?	?

シミュレーションと機械学習



- 問題設定の自由度は大きい
  - どこからどこまでMLに頼るのか？
- 観測データの膨大な自由度を調査し切れるか？
  - 過学習を防ぐ機構？
  - やはり統計量を経由することで、自由度を減らす？



de Lapparent, Geller, Huchra+’86

最近の例

Ravanbakhsh, Oliva, Fromenteau+'17
Mathuriya, Bard, Mendygral+'18

>7,000,000 parameters
69.33 Gflop / mimi-batch (mini-batch size = 1)

観測データ

Inverse problem

宇宙論パラメタ

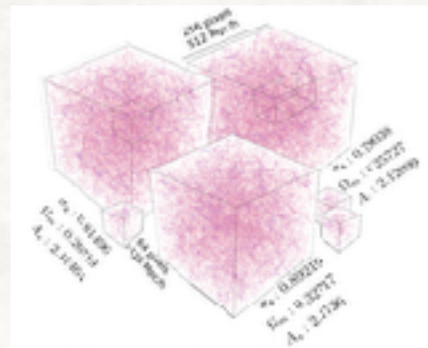


Figure 1. Dark matter distribution in three cubes produced using different sets of parameters. Each cube is divided into small sub-cubes for training and prediction. Note that although cubes in this figure are produced using very different cosmological parameters in our constrained sampled set, the effect is not visually discernible.

Fig. 2. CosmoFlow network topology showing the different network layers and data sizes at each layer.

- L=512Mpc/h, N=512^3
- 12,632 simulations with COLA
  - 150 as validation data, 50 as test data
- 8 x 128^3 voxels (101,056 sub-volumes)
- Deep Convolutional Net

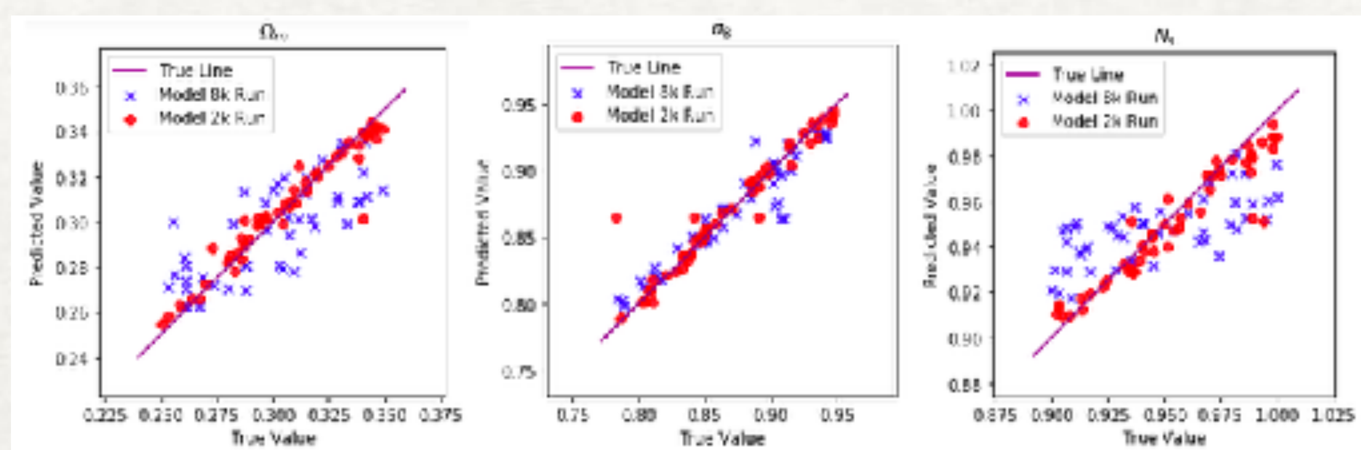


Fig. 6. Estimates of Ω_M, σ_8 and n_s from the 2048- and 8192-node runs.

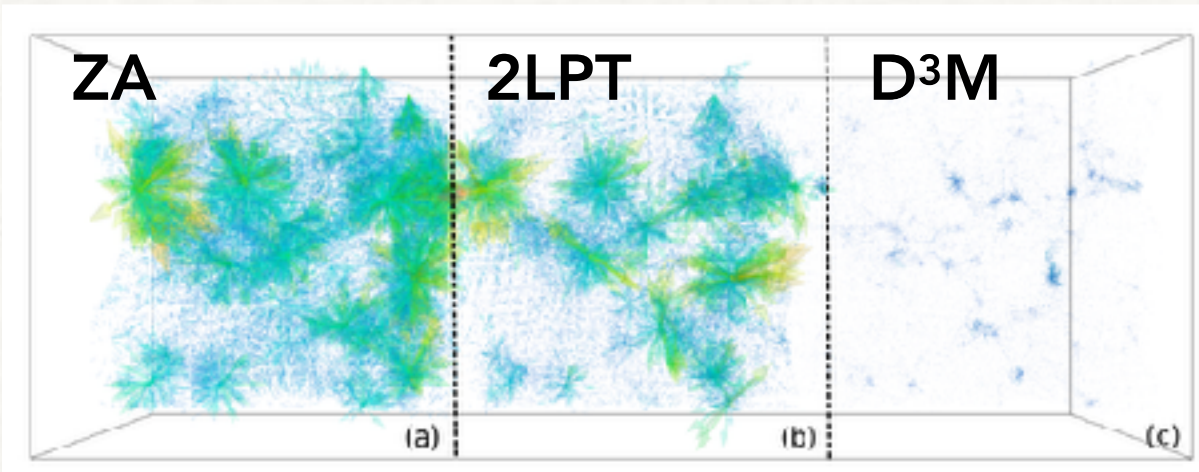
最近の例

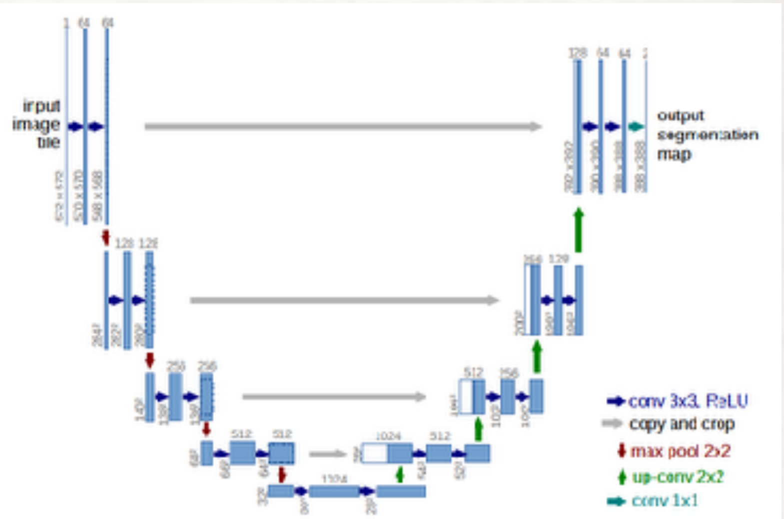
He, Li, Feng+’18

観測データ

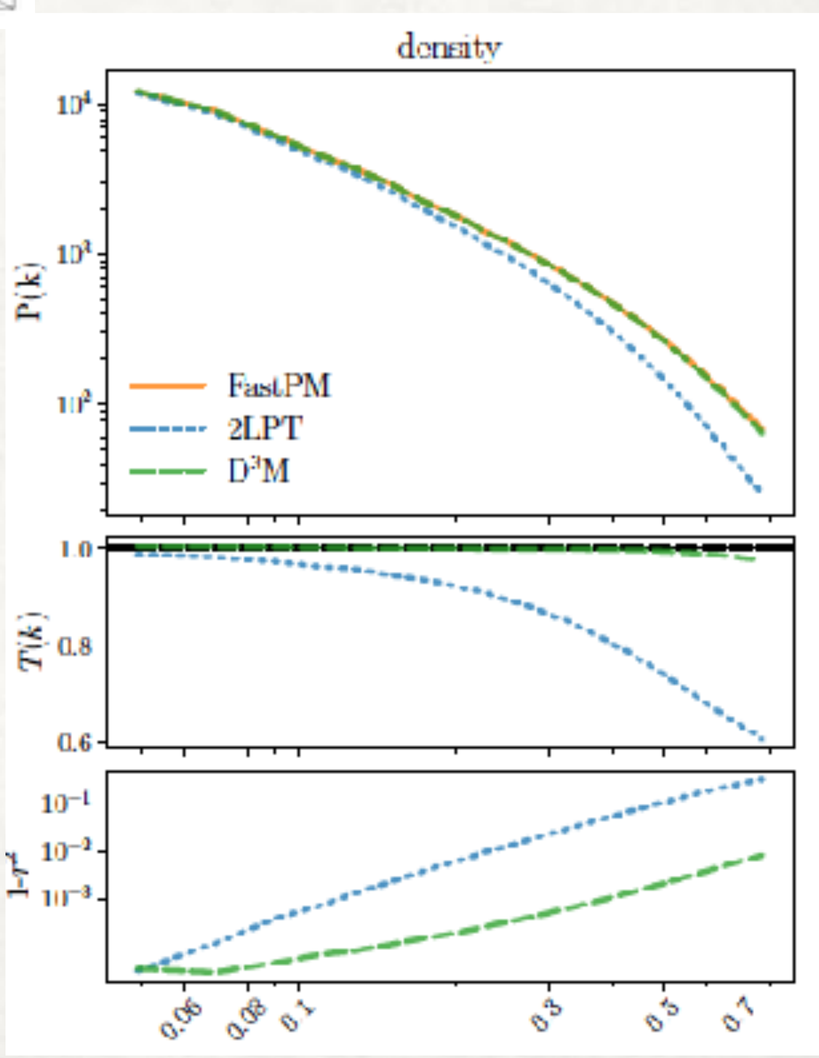
Forward modeling

宇宙論パラメタ

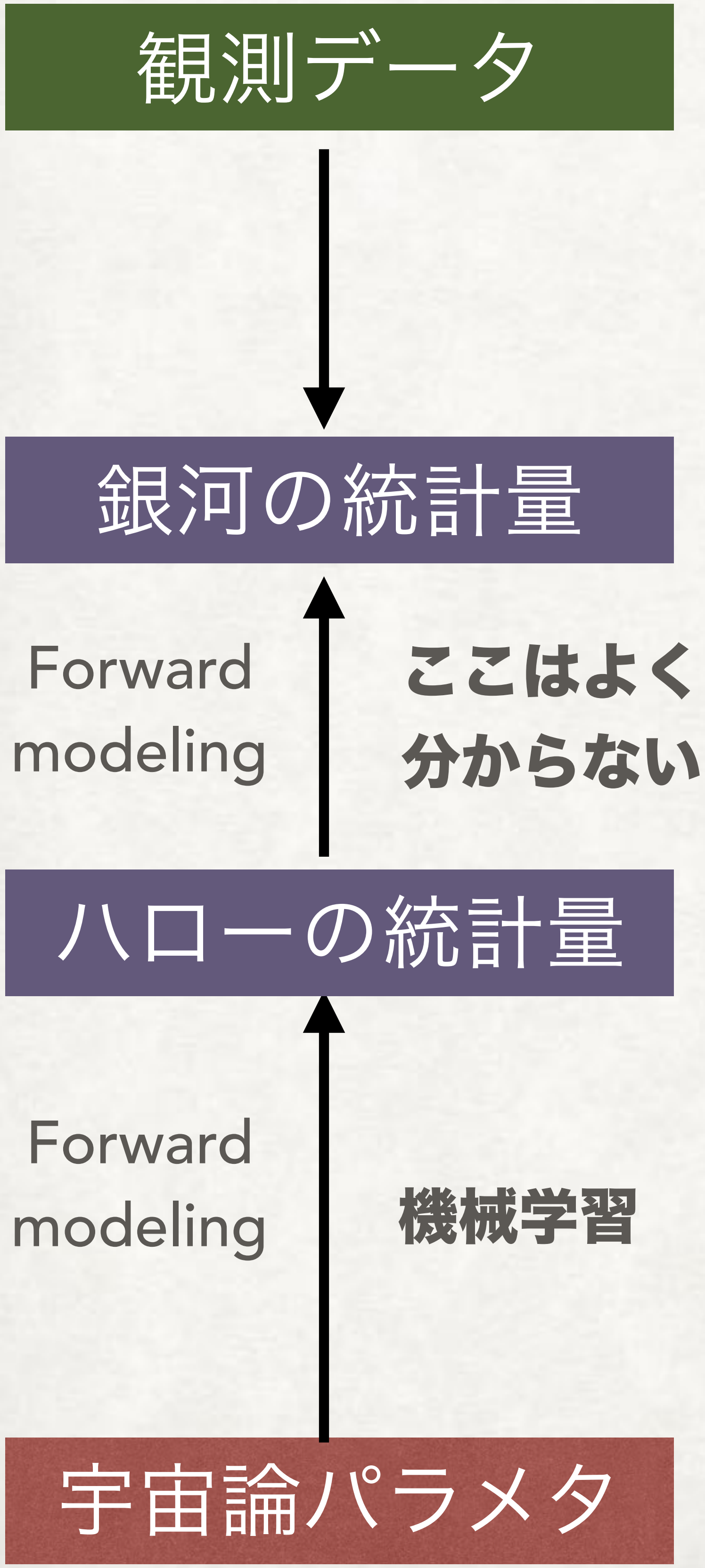




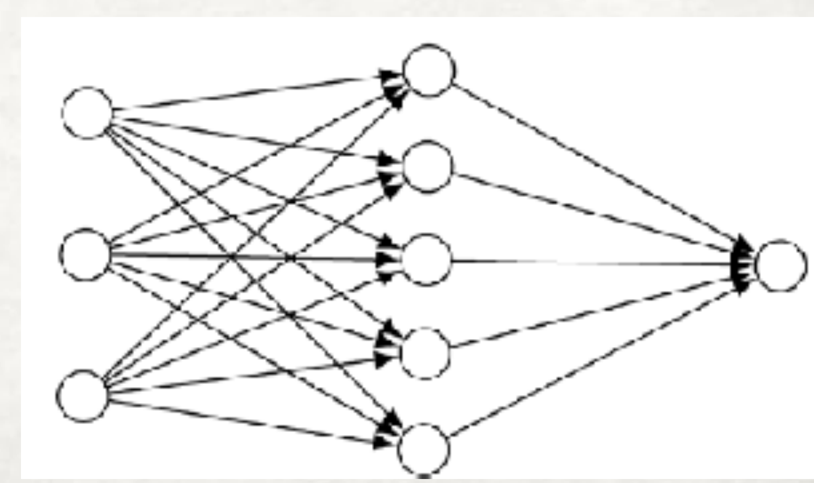
- Pairs of (Zel’dovich Approx. — full N-body simulation)
  - ZA as the input
  - N-body as the output
- Perform 10,000 pairs of particle realizations
  - L = 128 Mpc/h, N = 32^3



HSCでは

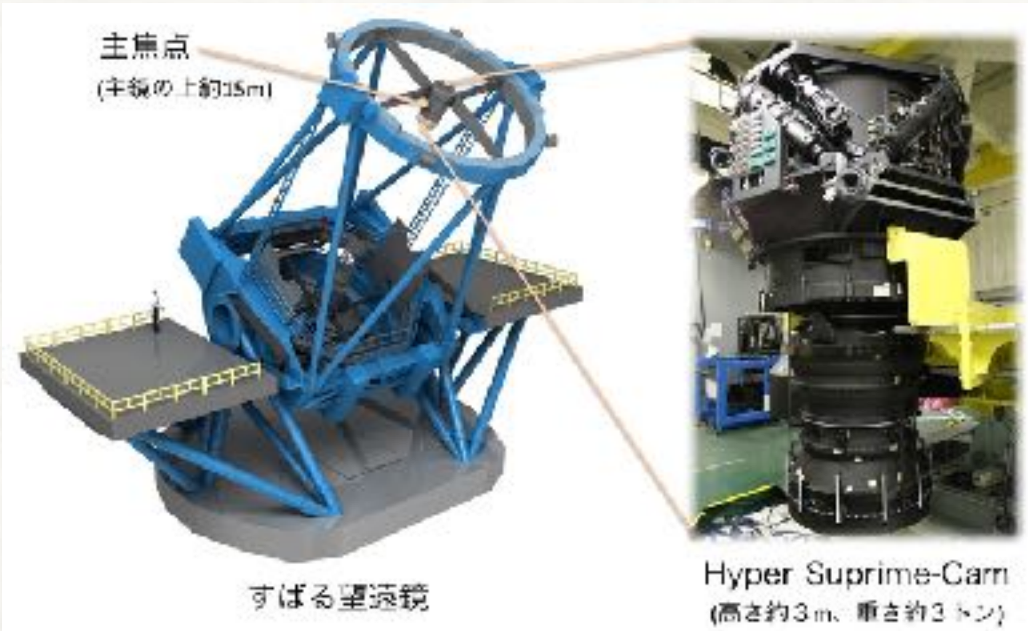


- 全面的に機械学習を導入するのはやや時期尚早
- 統計量ベースの解析
- 従来のforward modeling + MCMCに則った解析
- シミュレーションで予言できることとできないことがある
  - できるところまでやろう
  - できないところは仕方ないので、それらしい処方箋（フリーパラメタ）を入れて、マージナライズする
- **Deep learningは不要**
  - e.x., P(k) の Ω_m 依存性

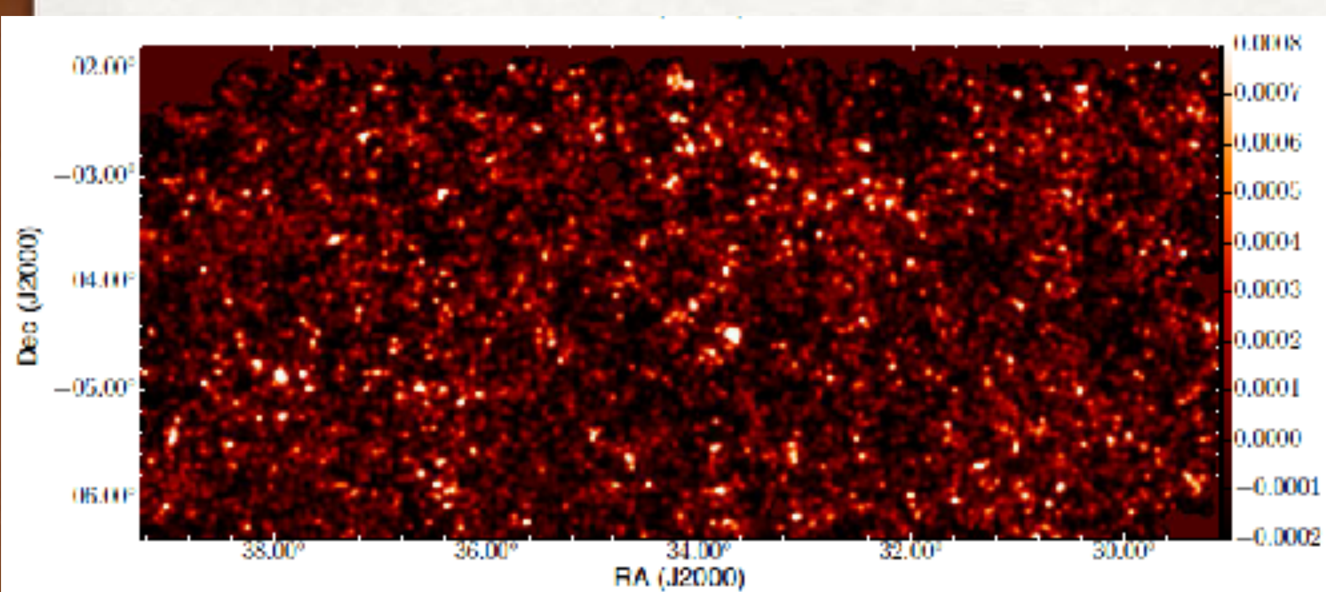


WHAT WE WANT TO (HAVE TO) DO

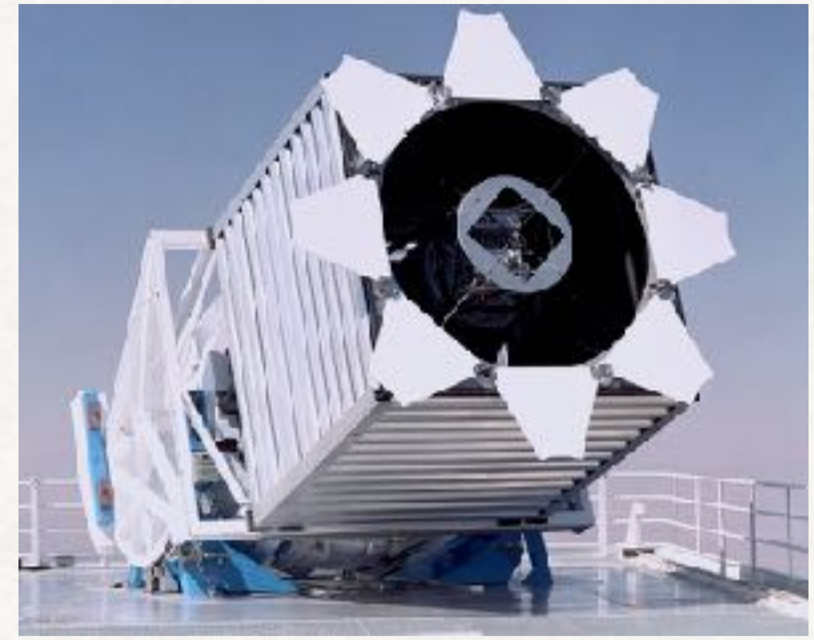
Hyper Suprime Cam (HSC)



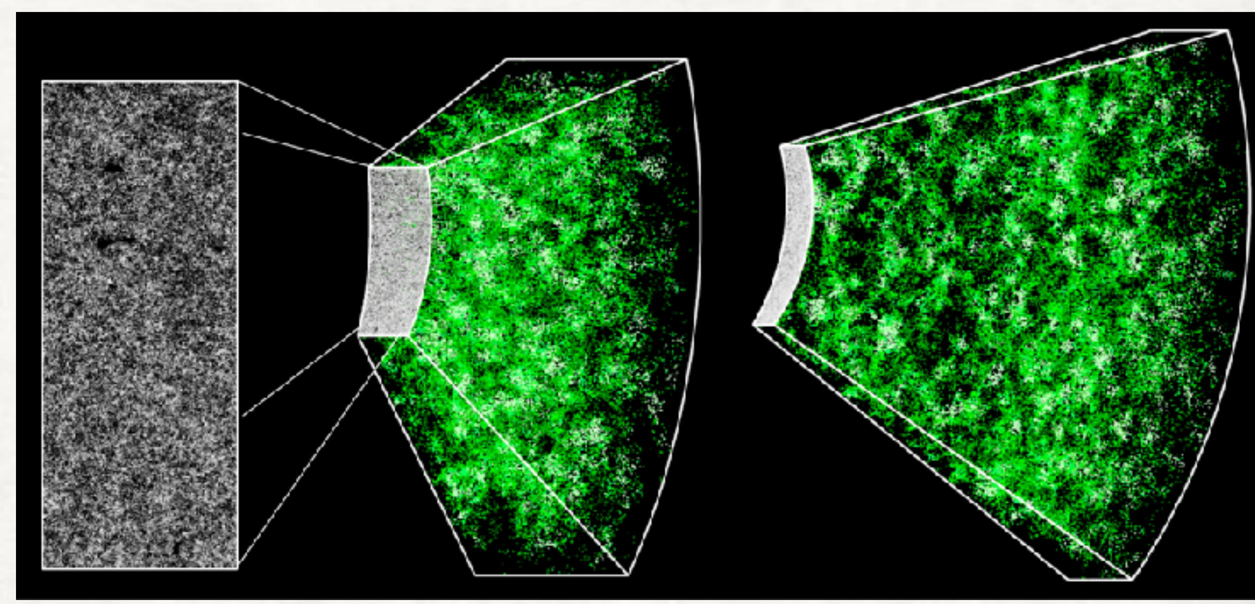
弱重力レンズ: convergence κ (Oguri+'17)



Sloan Digital Sky Survey



銀河分布: overdensity δ_g



<κ κ>

銀河バイアス無し —> Hikage+'18

$<\delta_g\ \kappa>$ $<\delta_g\ \delta_g>$

バイアスと宇宙論の縮退は解けるか？

—> Miyatake, TN+ in prep

DARK EMULATOR: WHAT IT CAN DO

TN+’18

OVERVIEW

```
In [18]: import darkemu

In [19]: emu = darkemu.base_class()

initialize cosmo_class
Initialize xlin emulator
initialize xinl emulator
Initialize pklin emulator
initialize propagator emulator
Initialize sigma_d emulator
initialize cross-correlation emulator
initialize auto-correlation emulator
Initialize hmf emulator
Initialize sigmaM emulator
```

$$(\omega_{\rm b}, \omega_{\rm c}, \Omega_{\rm de}, \ln(10^{10}A_{\rm s}), n_{\rm s}, w)$$

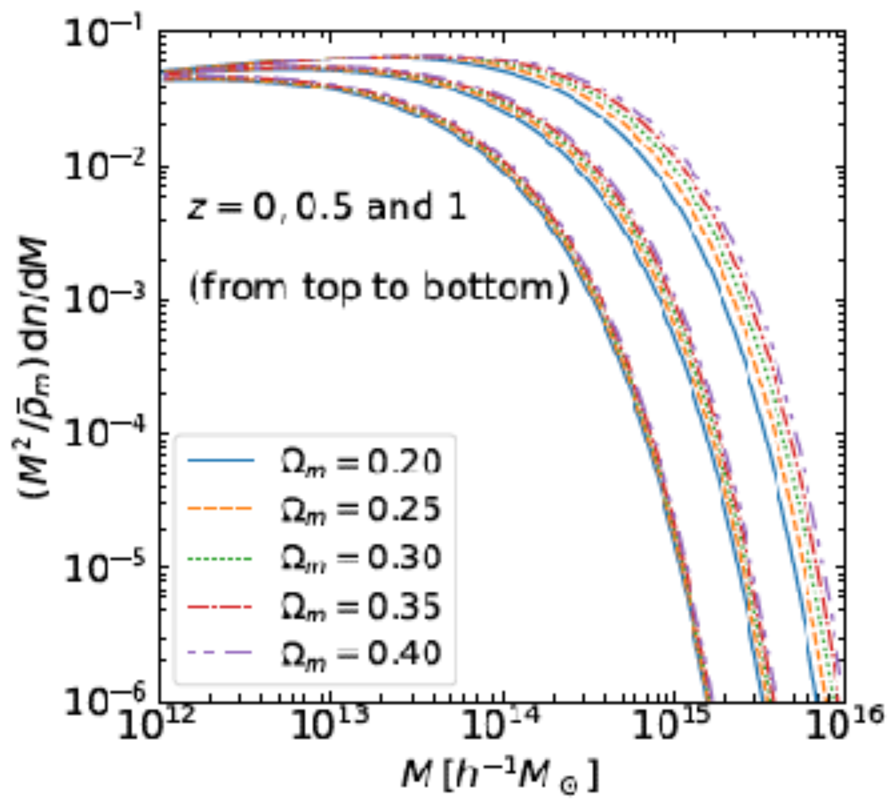
```
In [14]: cparam = np.array([0.02225,0.1198,0.6844,3.094,0.9645,-1.])
         emu.set_cosmology(cparam)
```

```
emu.get_nhalo(massbins[ii],massbins[ii+1],1.,z)
```

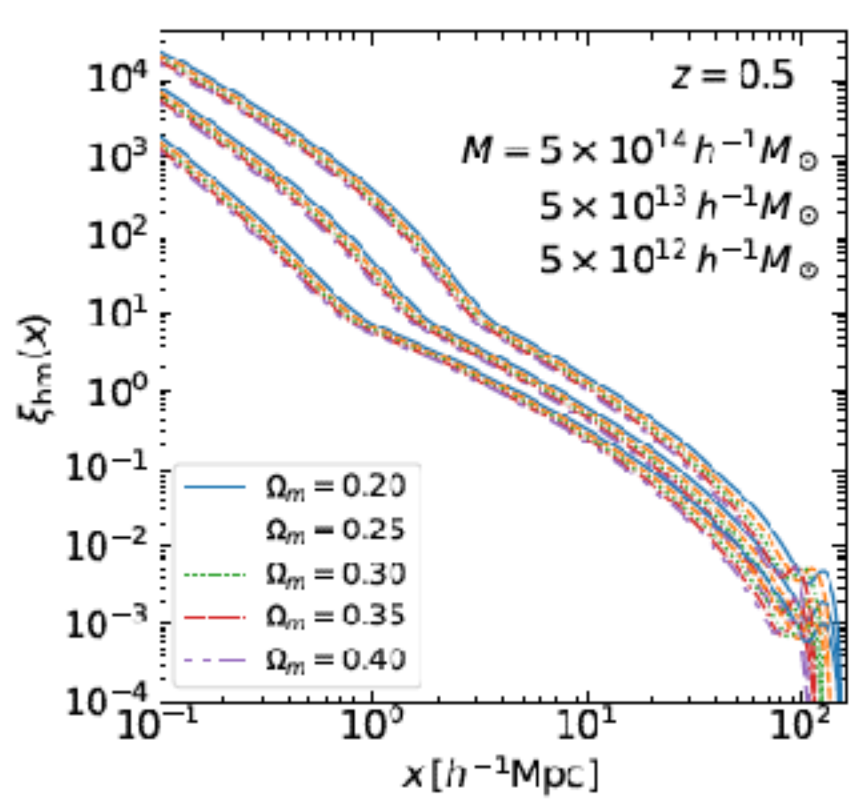
```
emu.get_xicross_mass(rs,Mh,z)
```

```
emu.get_xiauto_mass(rs,Mh,Mh,z)
```

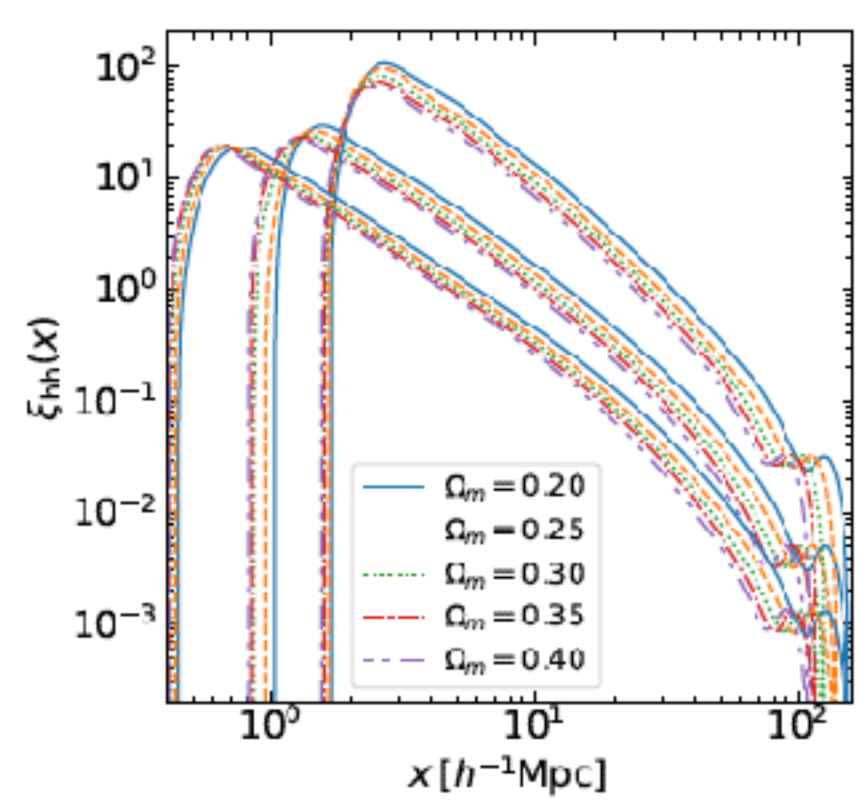
Halo mass function



Halo-Matter Cross CF



Halo-Halo Auto CF



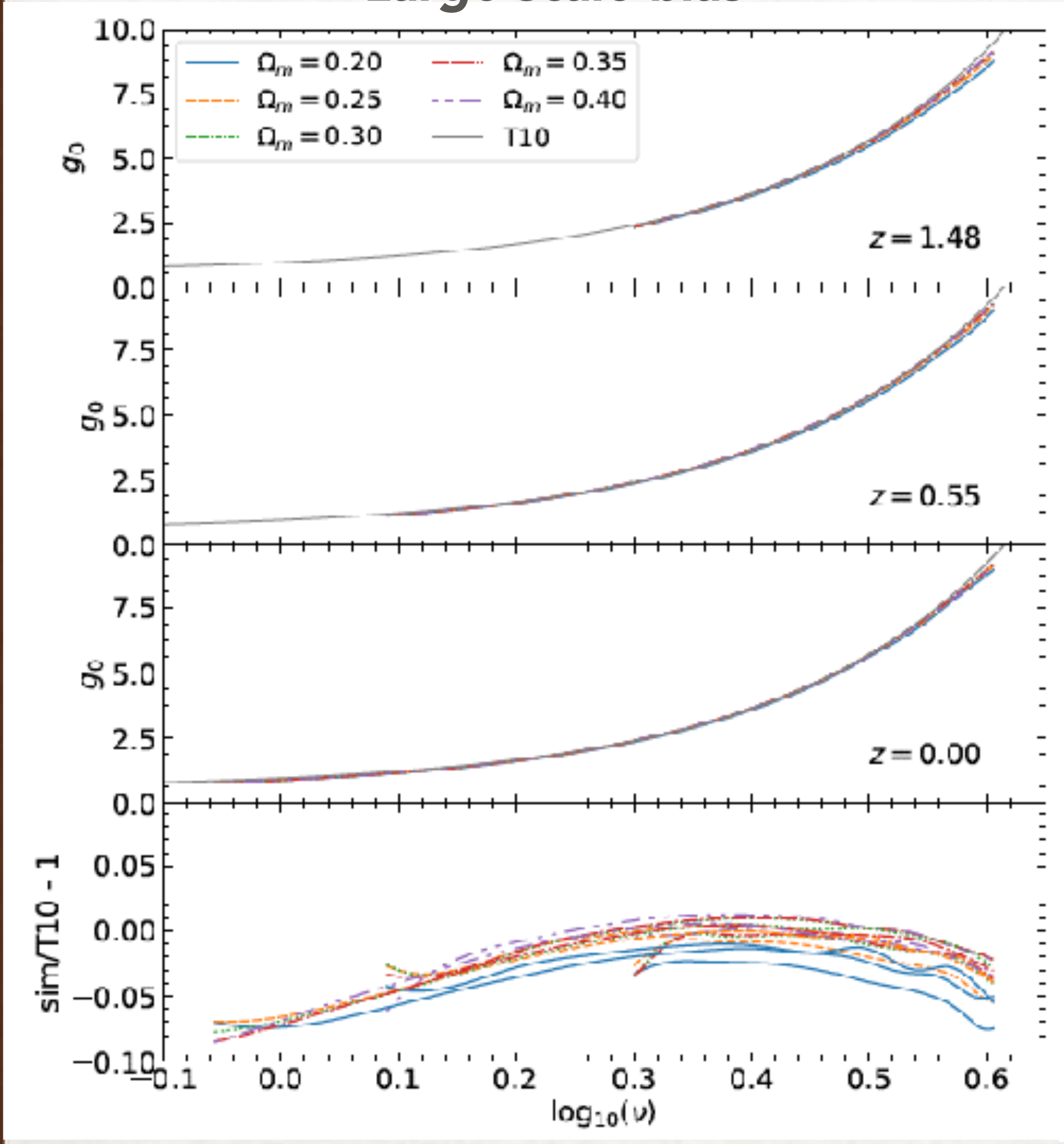
1 curve ~ 100 mili secs on a typical laptop computer

DARK EMULATOR: WHAT IT CAN DO

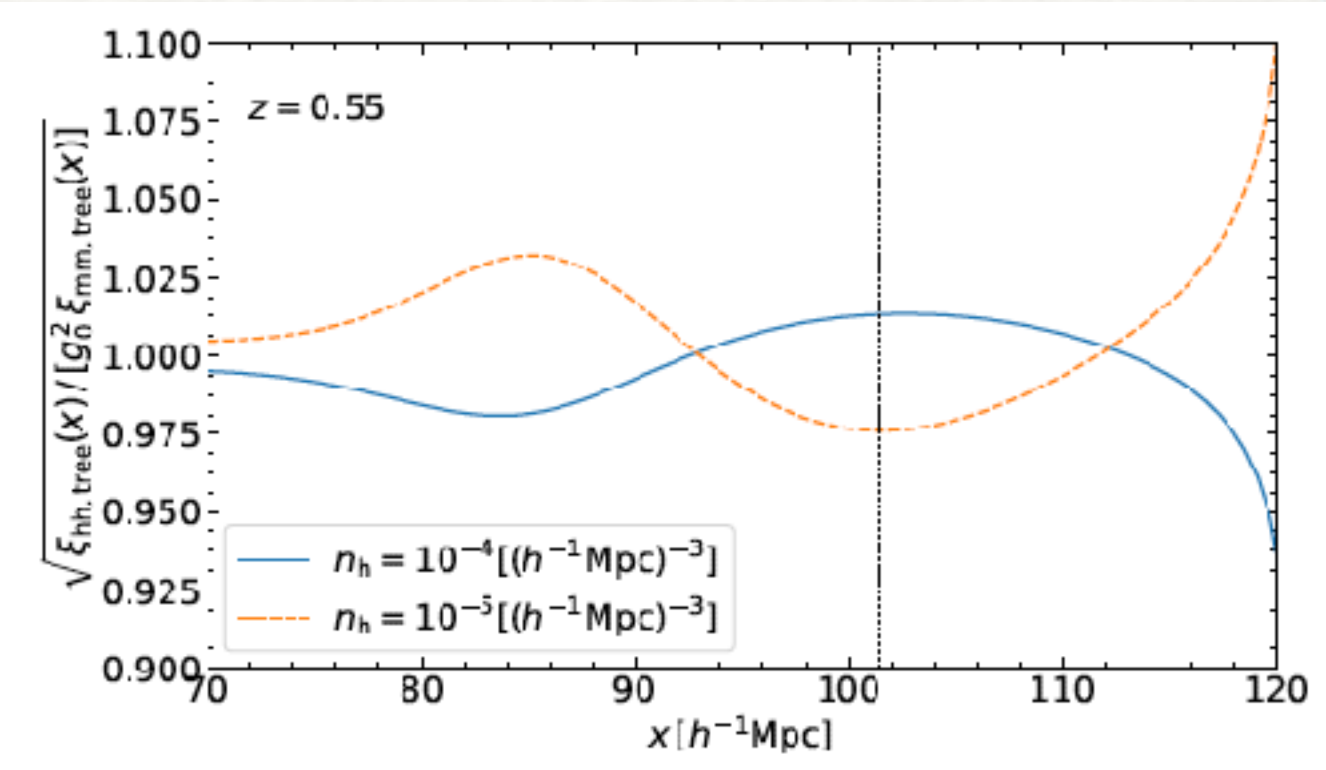
TN+'18

LARGE SCALES

Large-scale bias



Scale-dependent bias around BAO

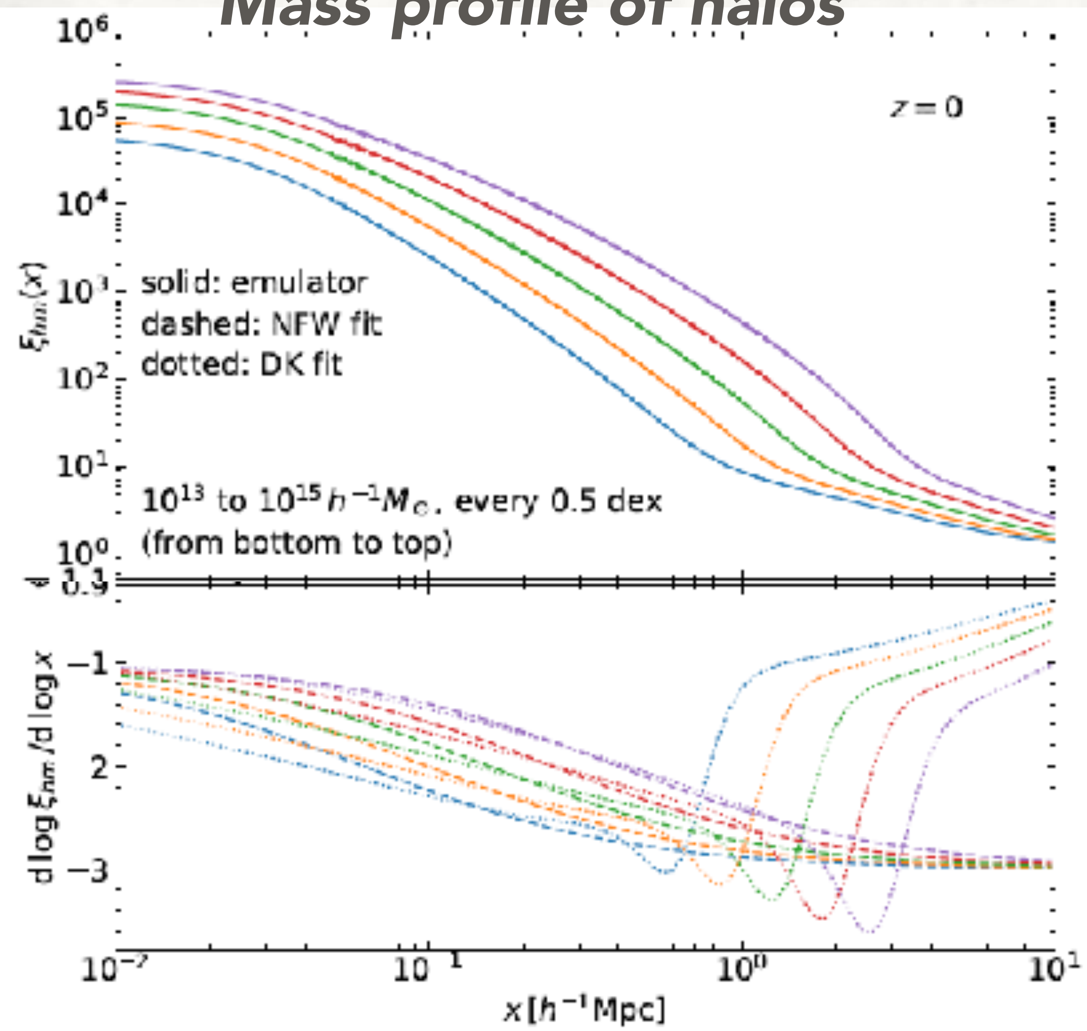


TN+’18

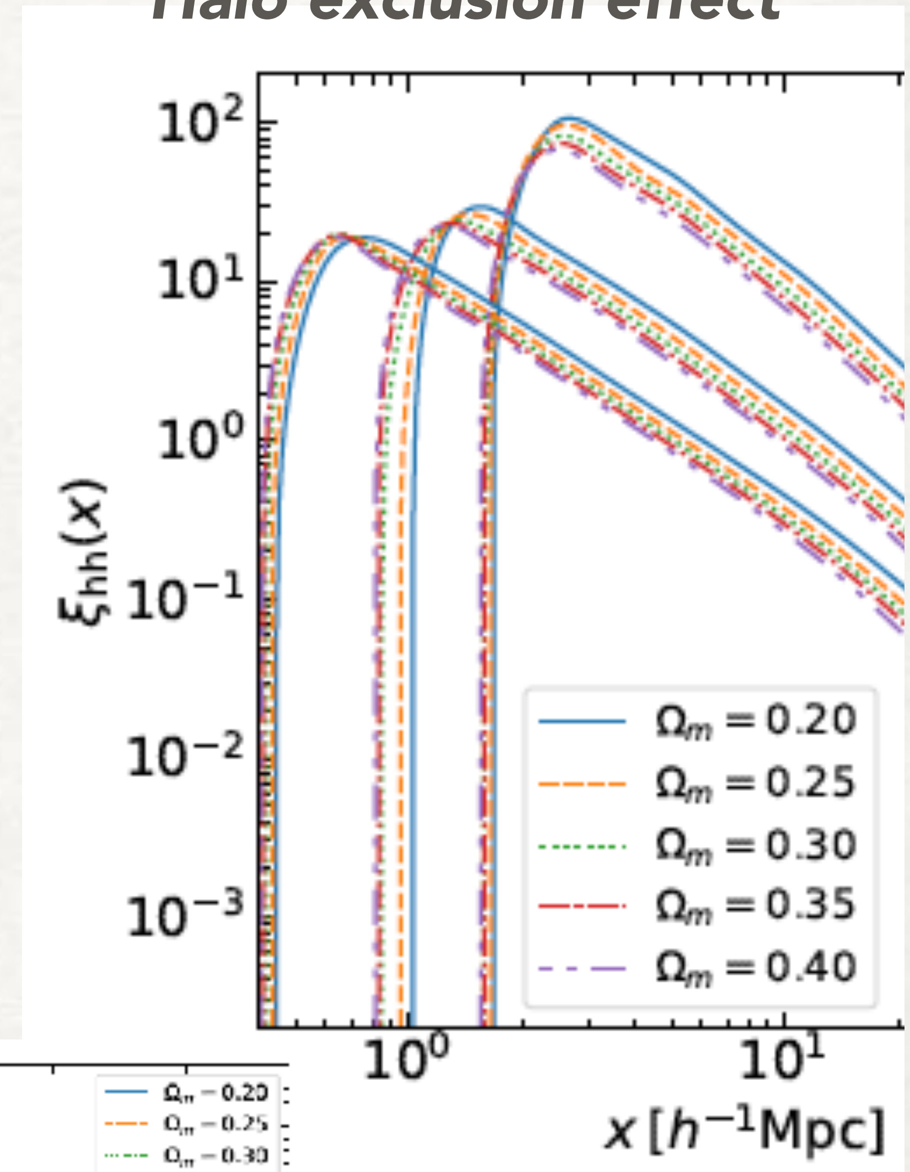
DARK EMULATOR: WHAT IT CAN DO

SMALL SCALES

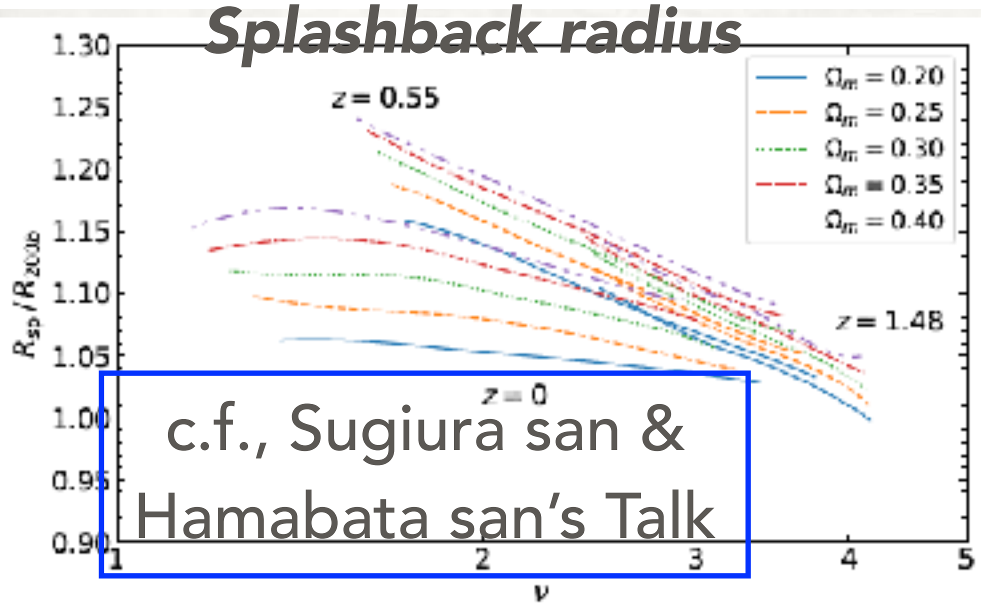
Mass profile of halos



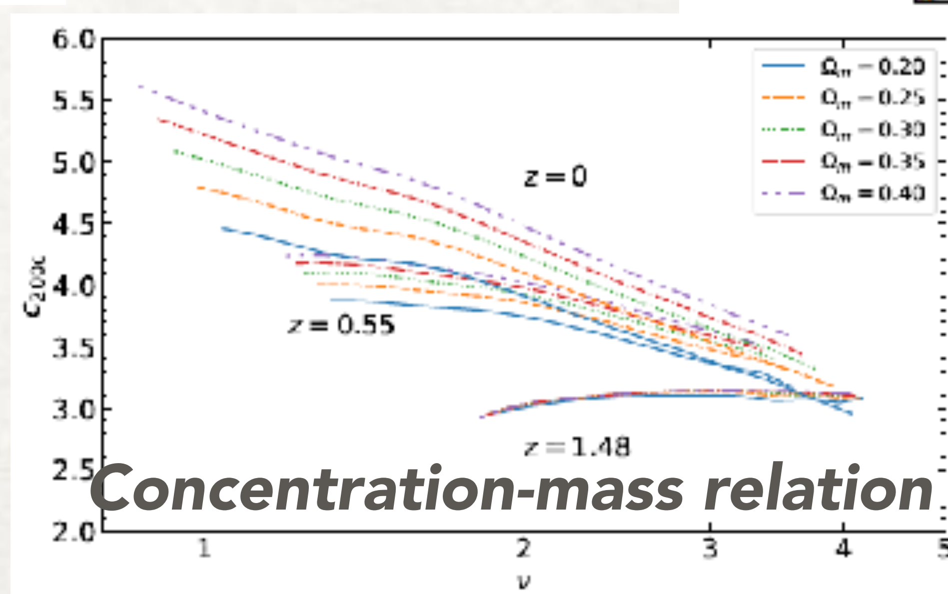
Halo exclusion effect



Splashback radius



c.f., Sugiura san & Hamabata san’s Talk

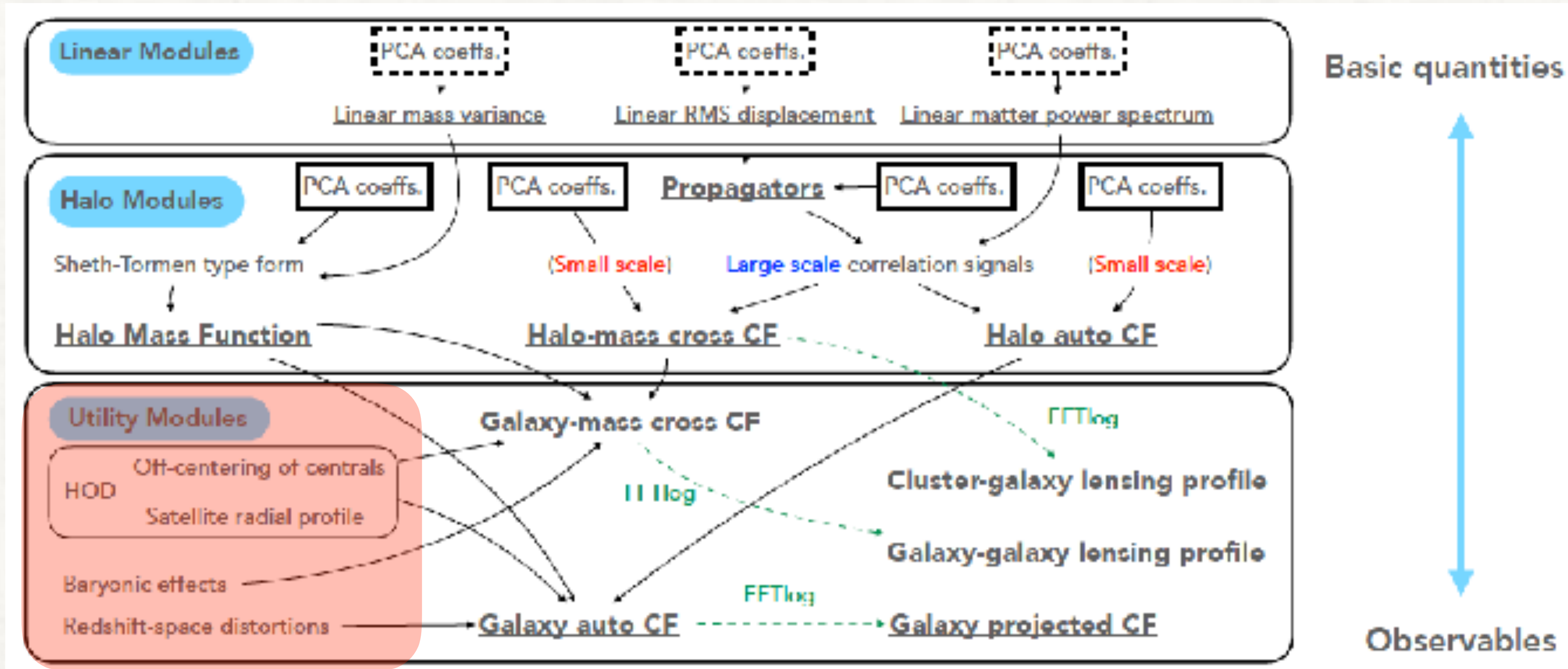


Concentration-mass relation

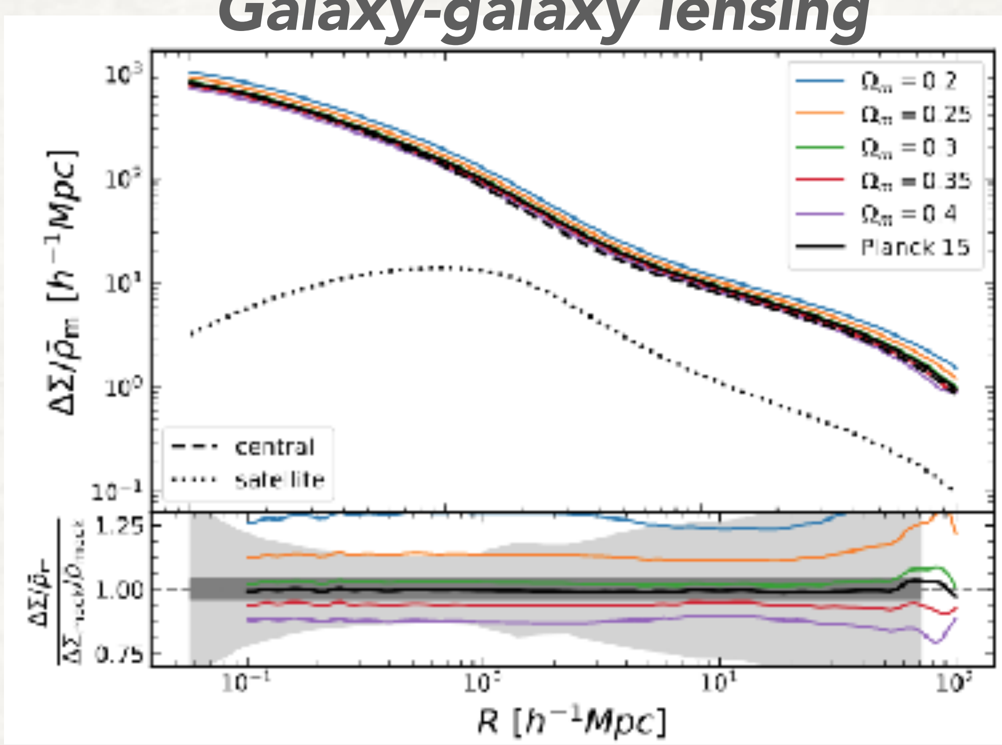
SMALL-SCALE UNCERTAINTIES

TN+'18

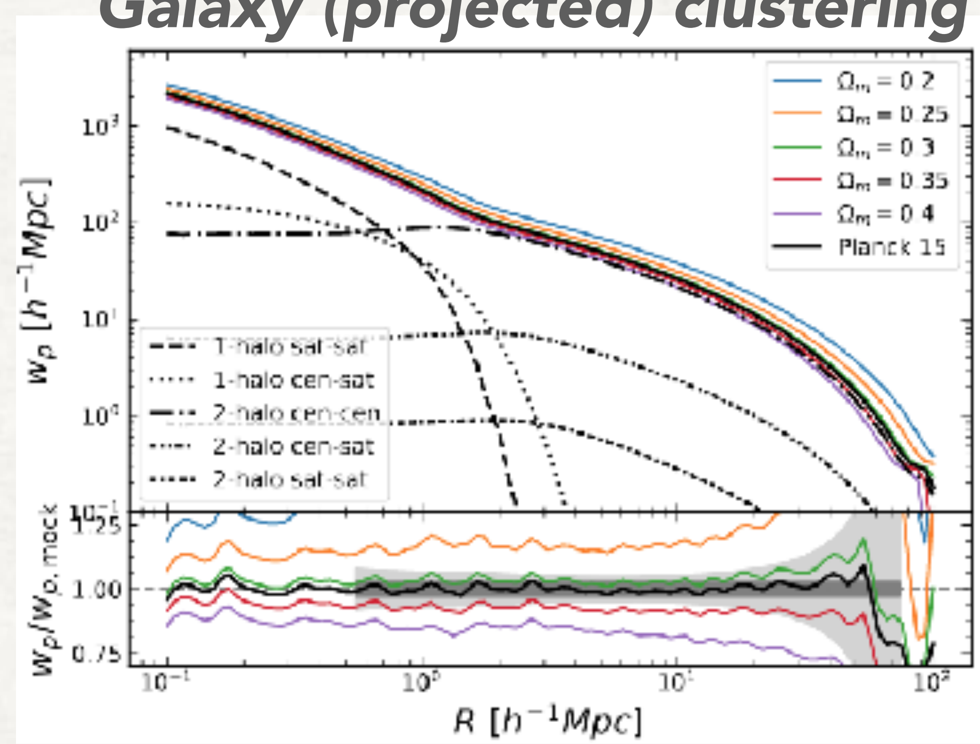
Put here (as) many (as you want) nuisance parameters to account for unknowns



Galaxy-galaxy lensing



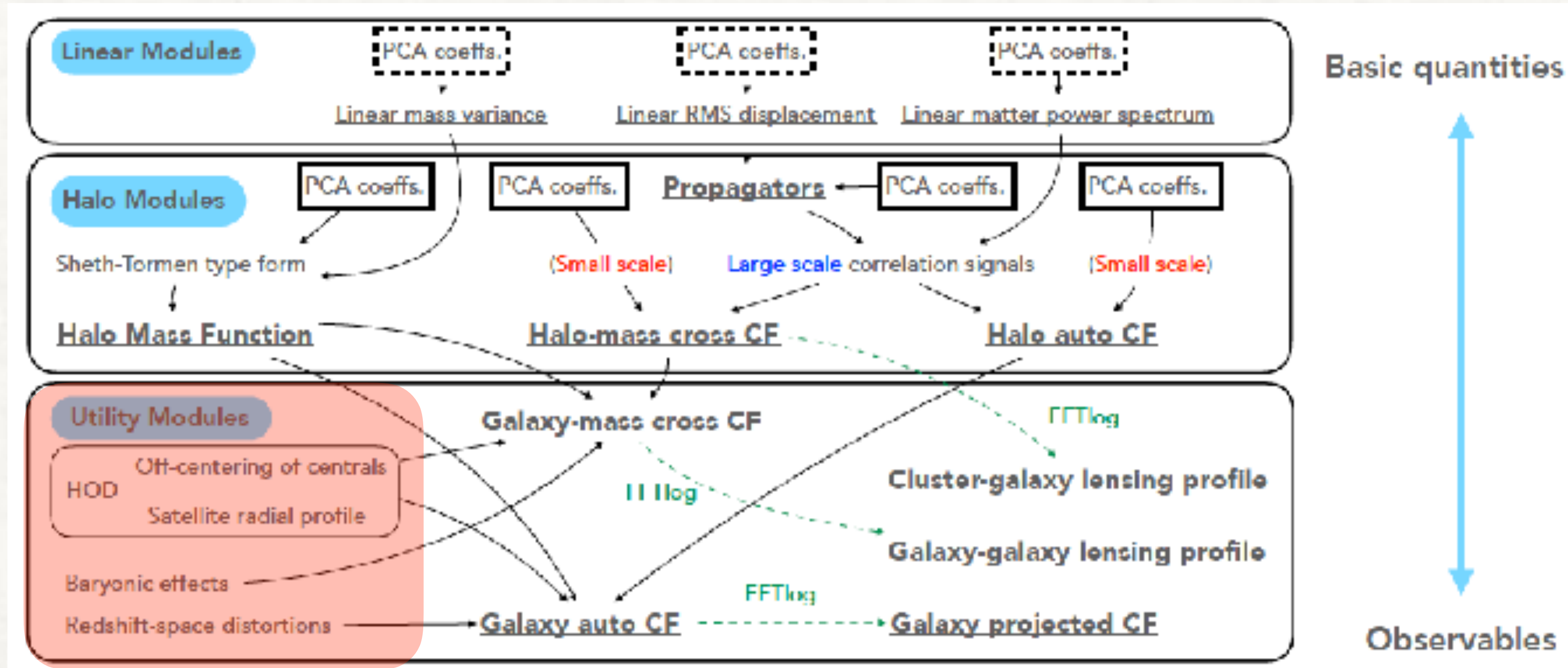
Galaxy (projected) clustering



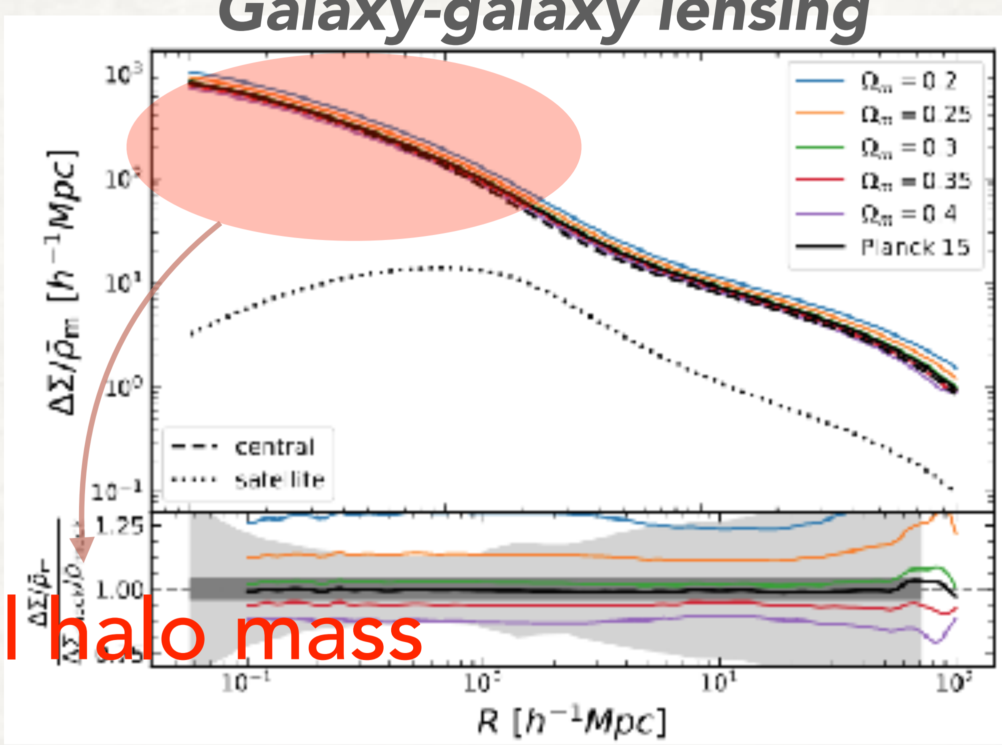
SMALL-SCALE UNCERTAINTIES

TN+'18

Put here (as) many (as you want) nuisance parameters to account for unknowns

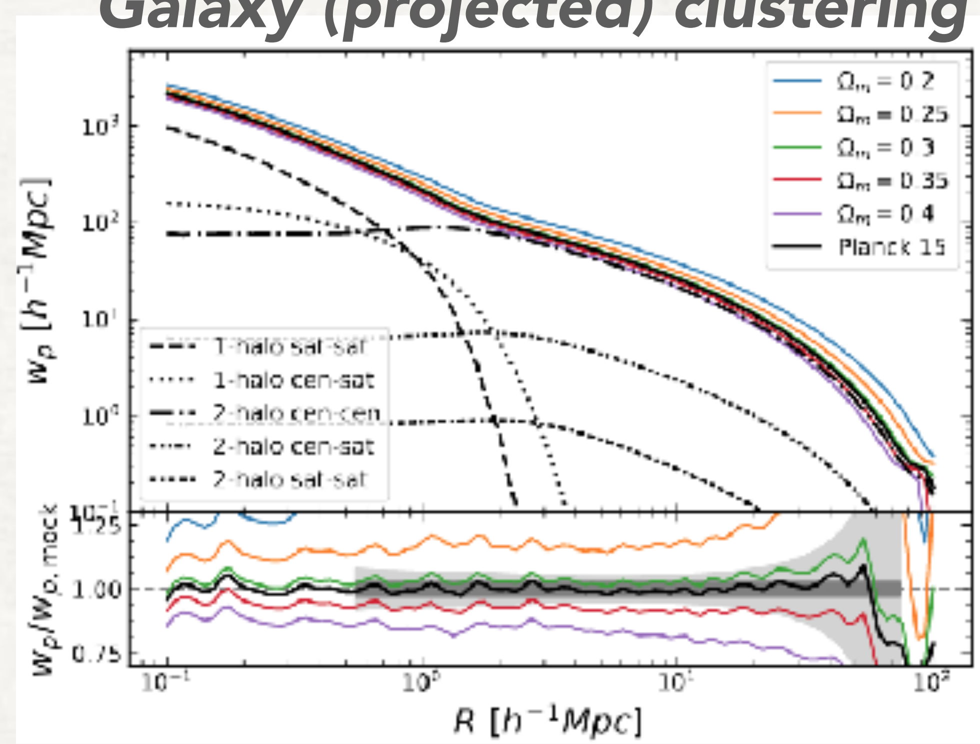


Galaxy-galaxy lensing



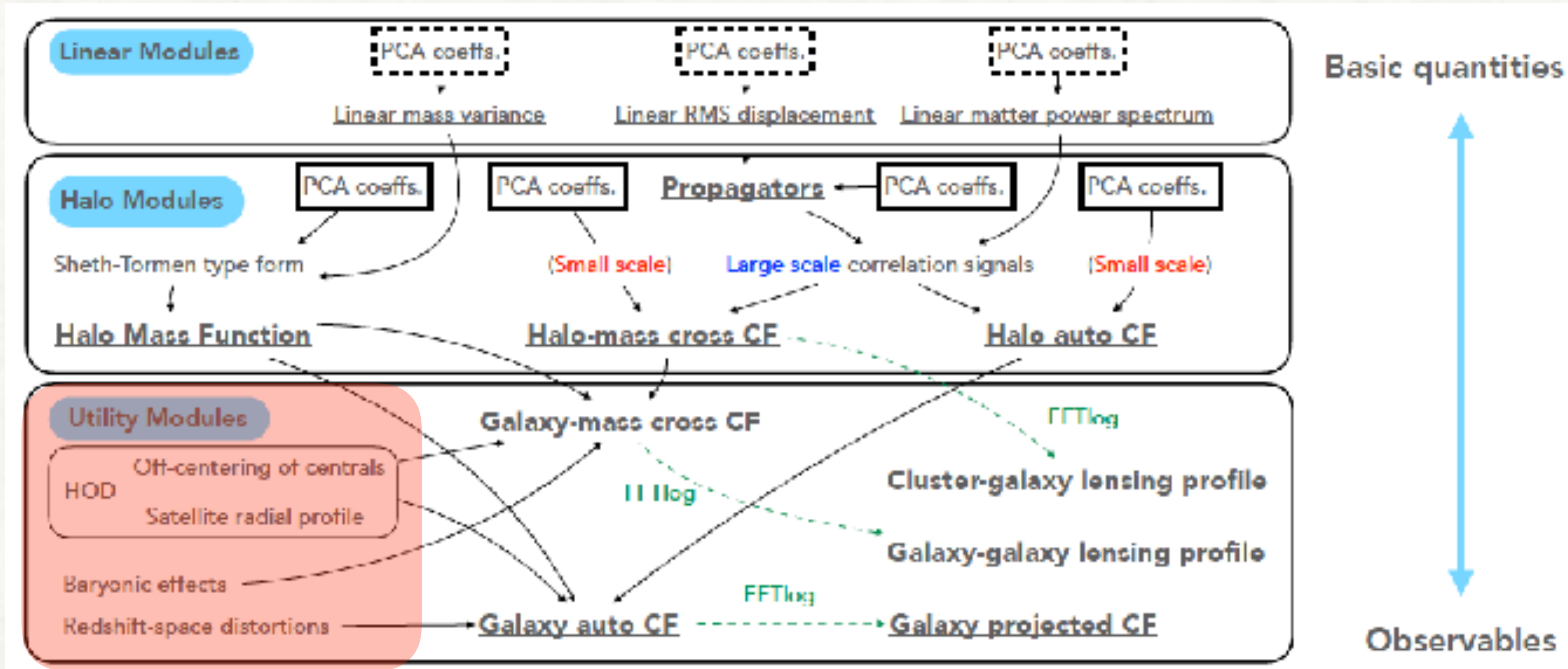
Typical halo mass

Galaxy (projected) clustering



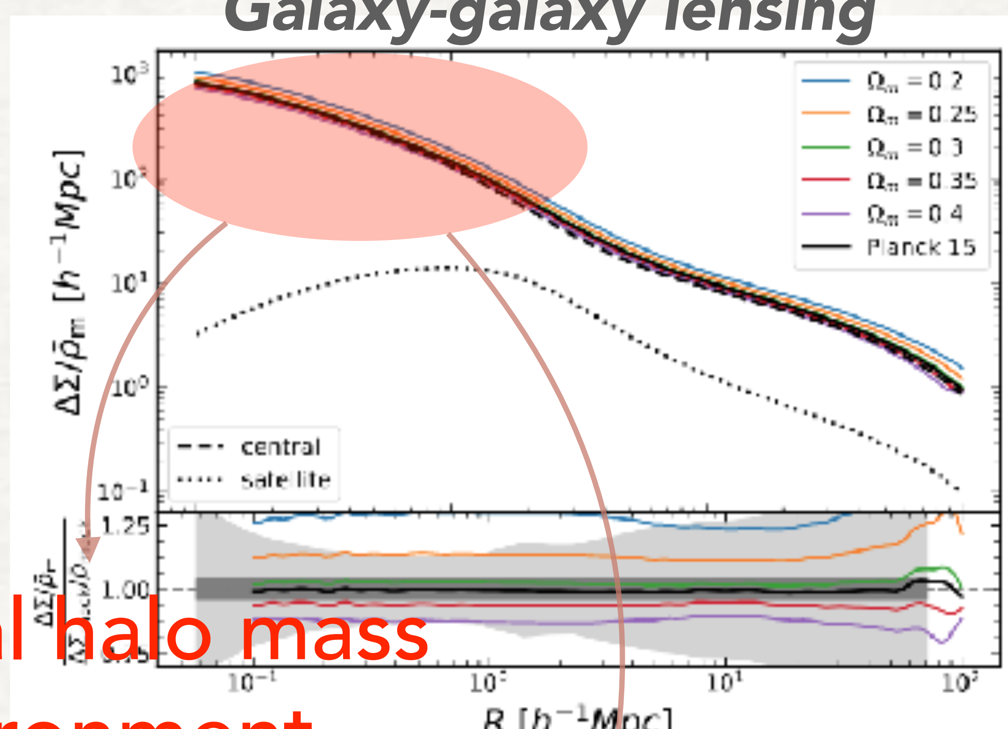
SMALL-SCALE UNCERTAINTIES

TN+'18

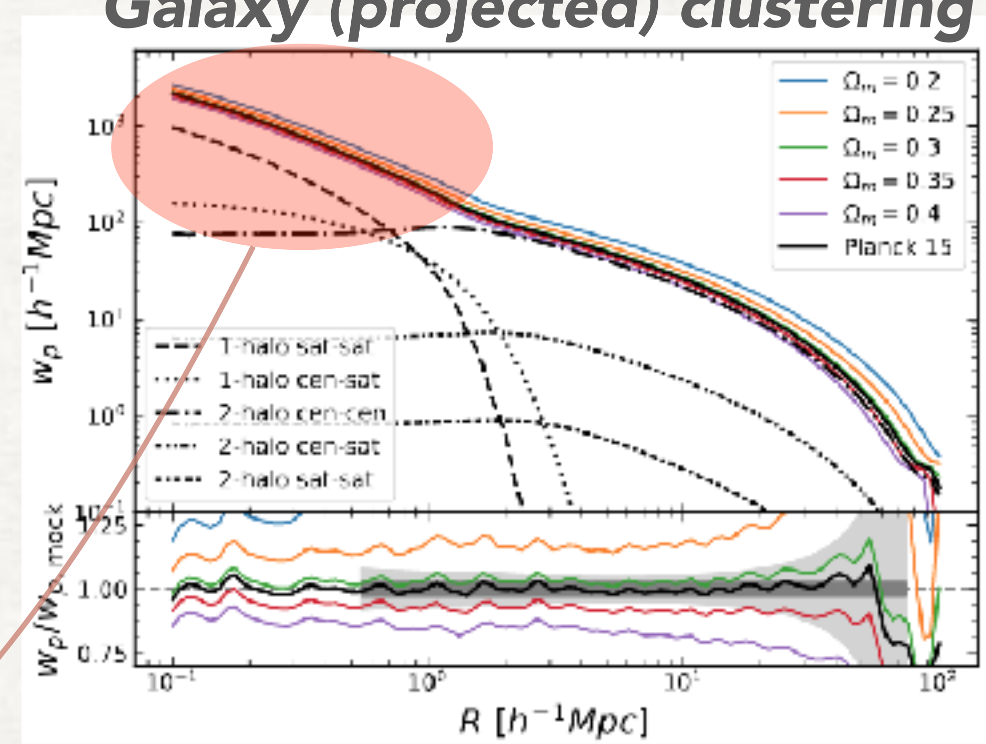


Put here (as) many (as you want) nuisance parameters to account for unknowns

Galaxy-galaxy lensing



Galaxy (projected) clustering



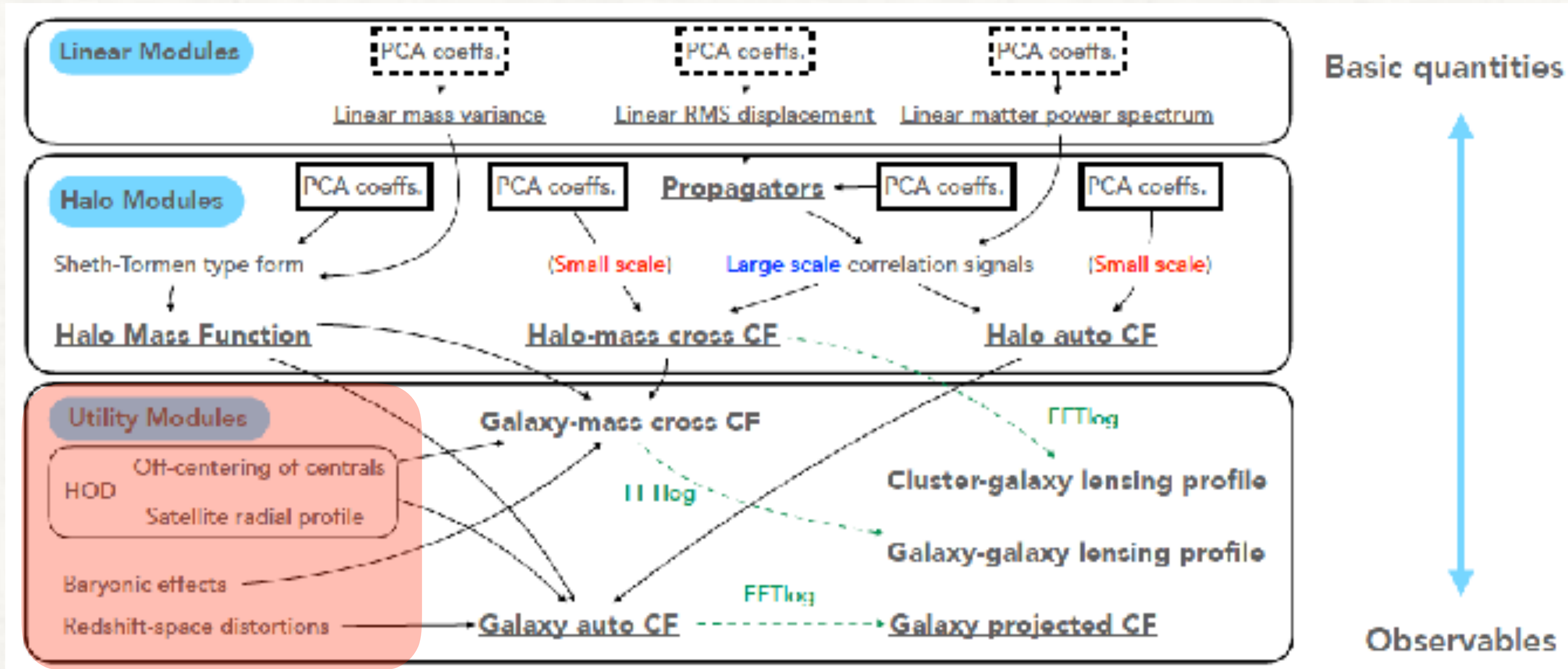
Typical halo mass
Environment

Distribution of galaxies around cluster center

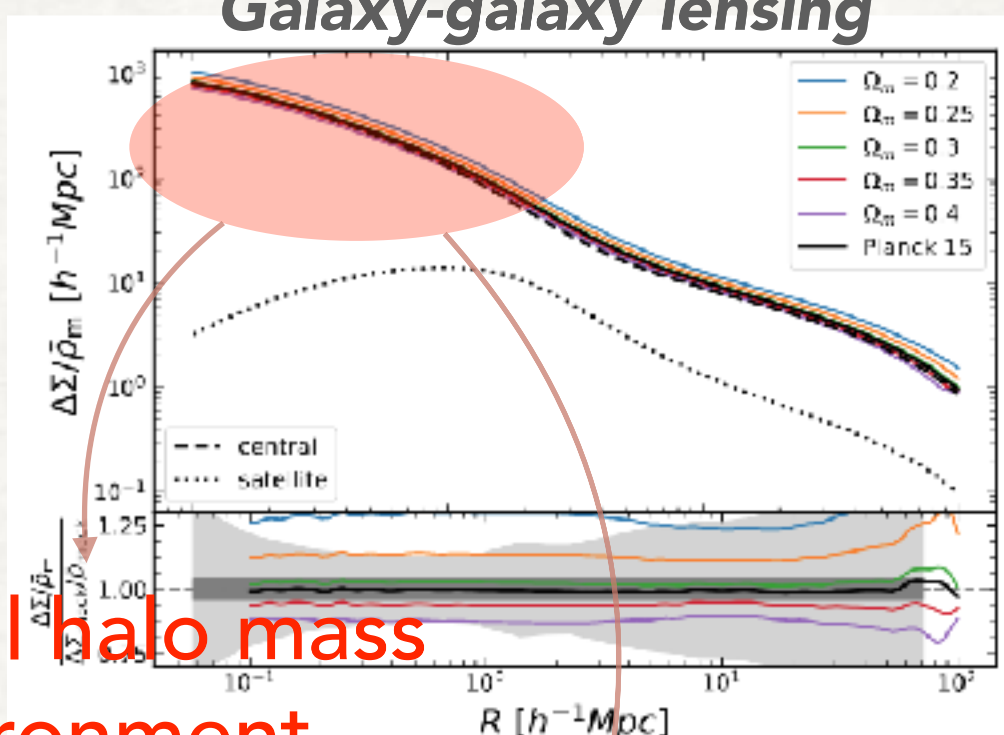
SMALL-SCALE UNCERTAINTIES

TN+'18

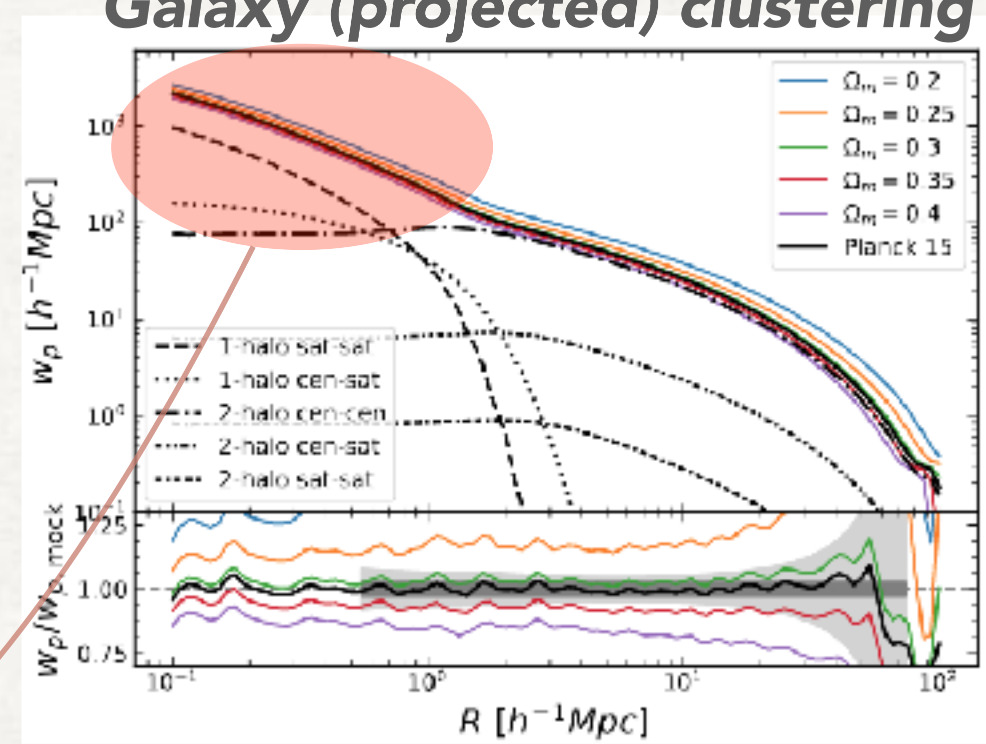
Put here (as) many (as you want) nuisance parameters to account for unknowns



Galaxy-galaxy lensing



Galaxy (projected) clustering



Typical halo mass
Environment

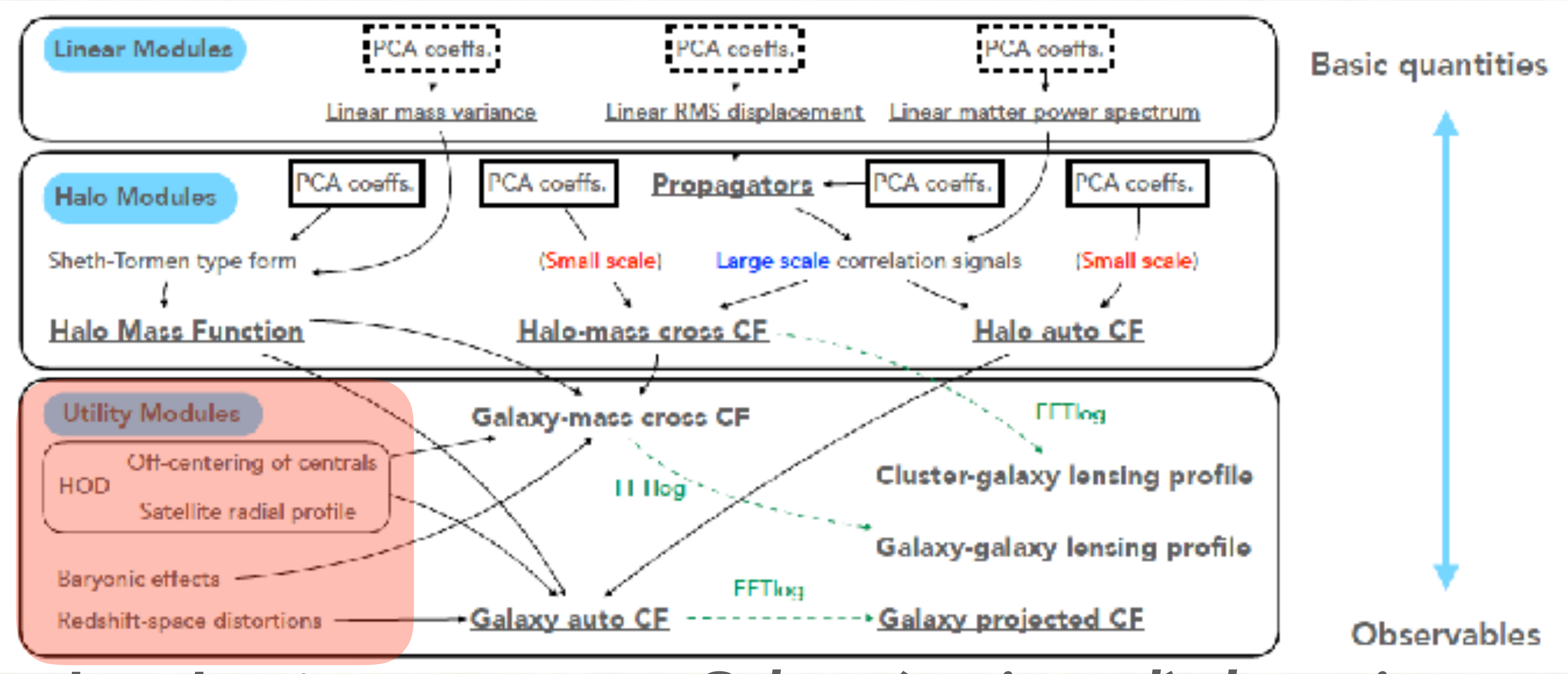
Distribution of galaxies around cluster center

(Theory) → Large-scale bias

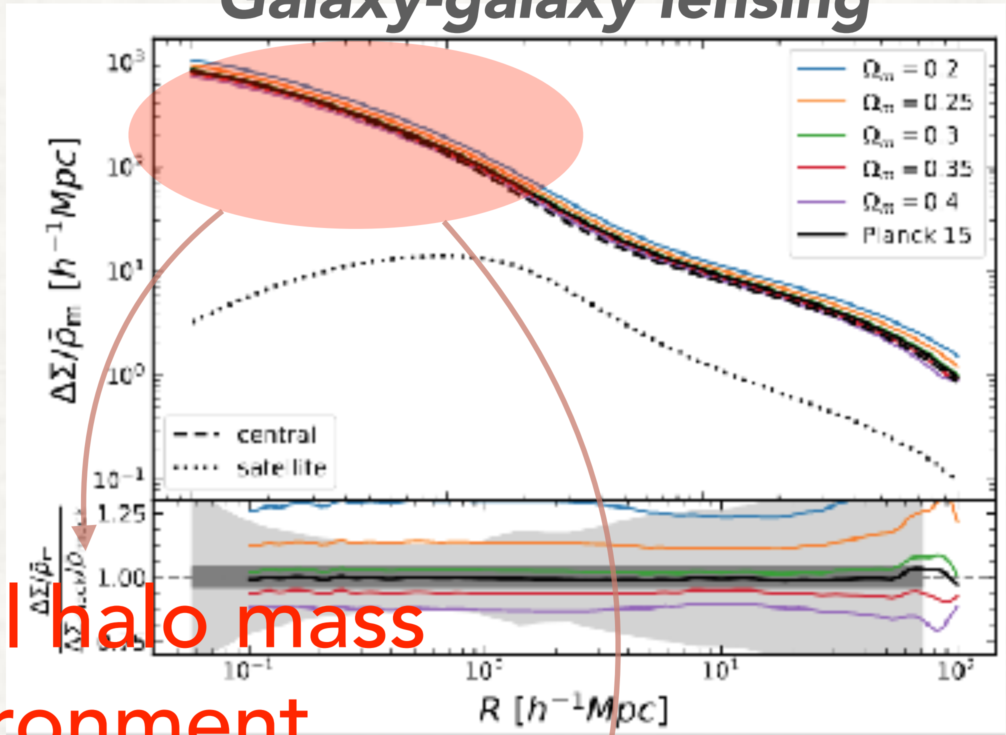
SMALL-SCALE UNCERTAINTIES

TN+'18

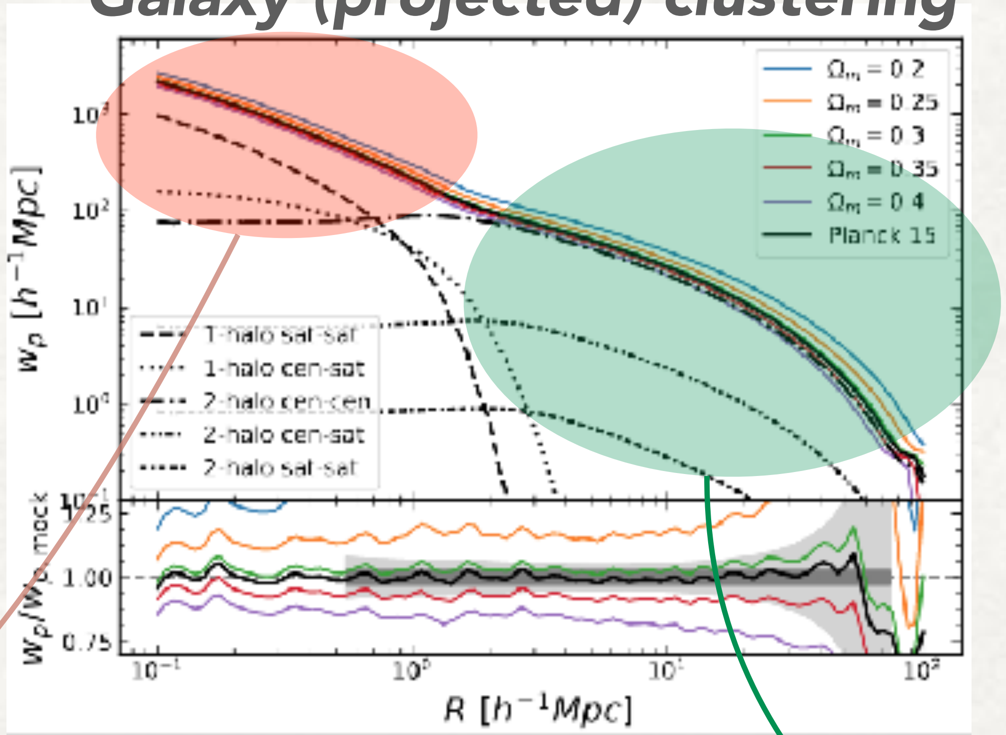
Put here (as) many (as you want) nuisance parameters to account for unknowns



Galaxy-galaxy lensing



Galaxy (projected) clustering



Typical halo mass
Environment

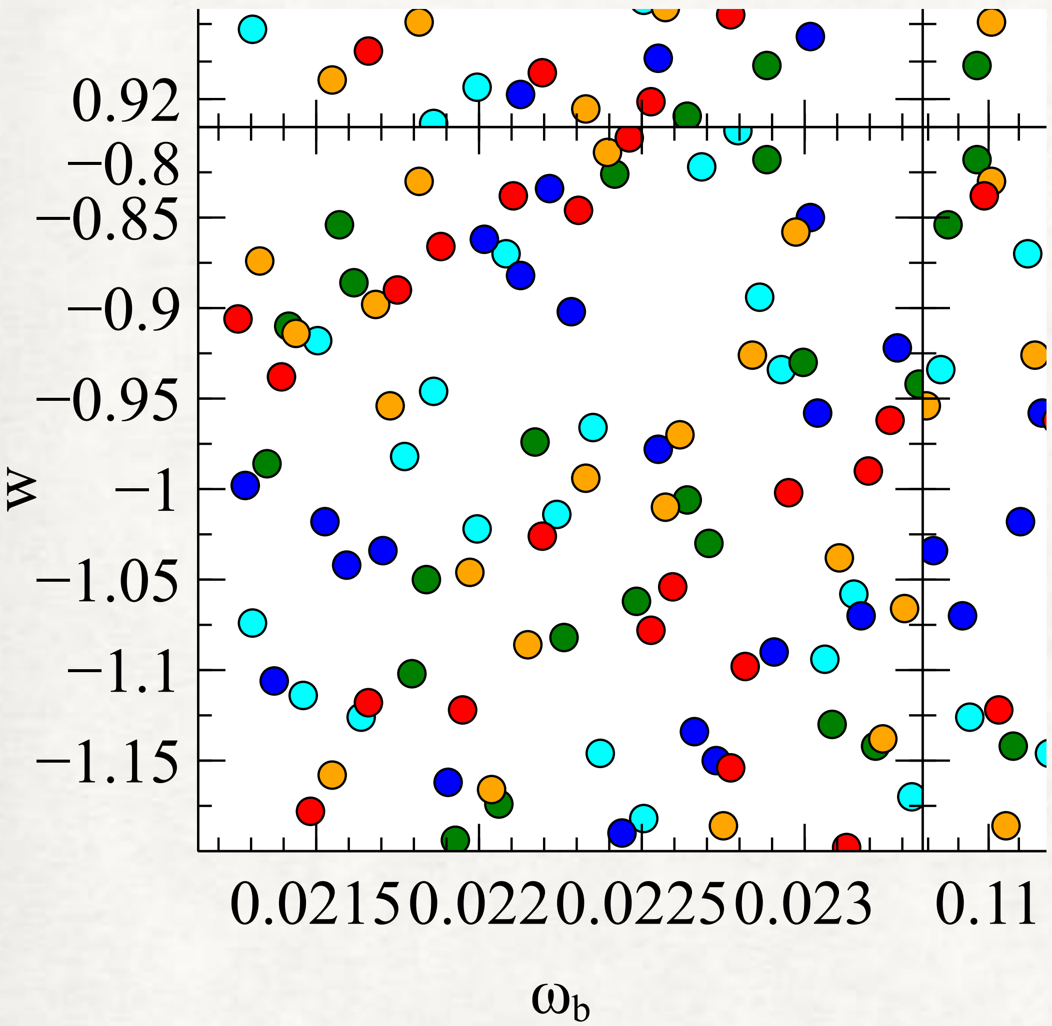
Distribution of galaxies around cluster center

(Theory) → Large-scale bias → Cosmology!

DARK QUEST SIMULATIONS

TN+'18

DQ 1: to be made public after HSC cosmology analyses (~early next year)



Circles: 100 parameter sets to be covered

Centered at Planck 2015

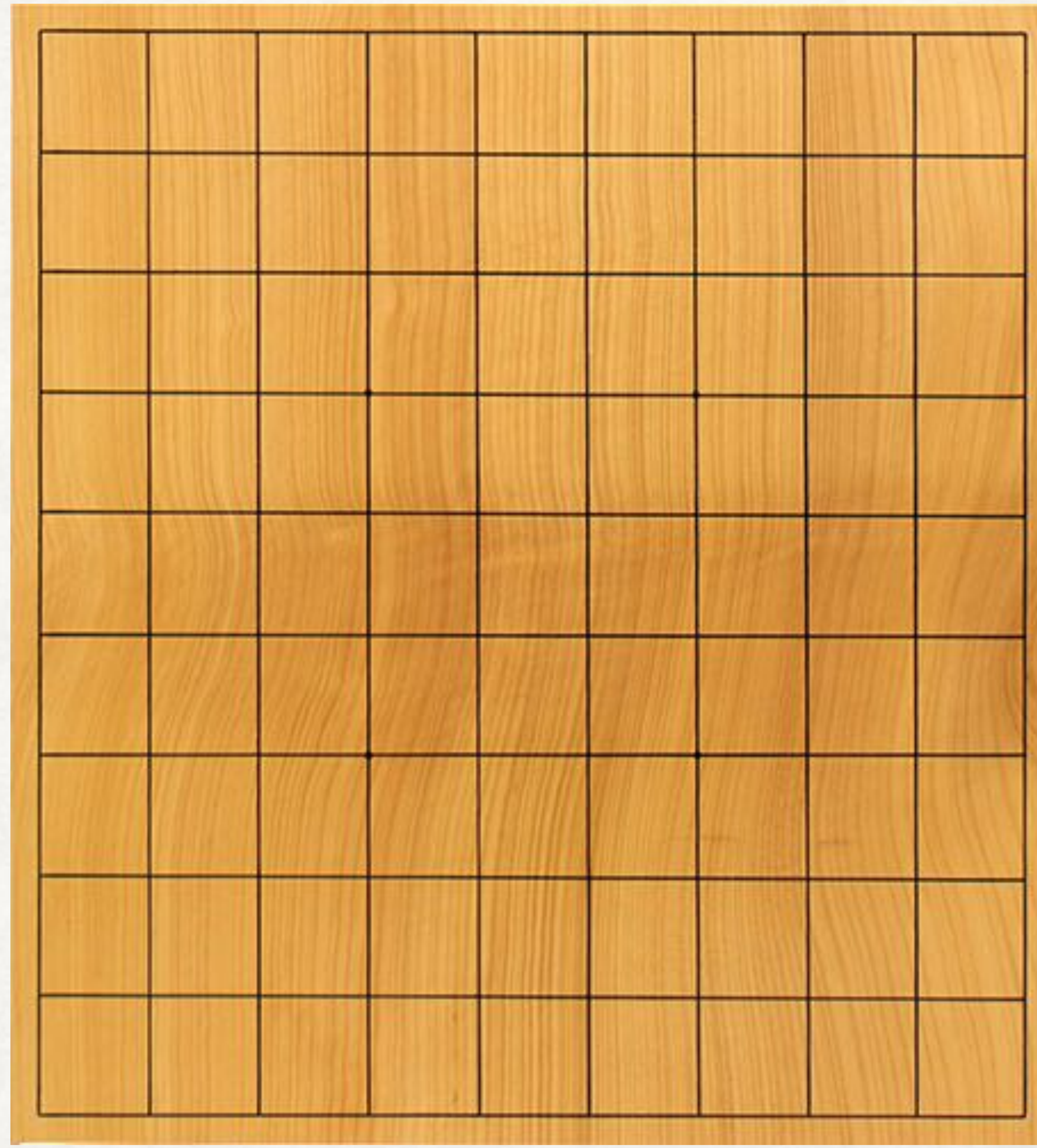
- Ensemble of N=2048^3 sims
  - 2 base resolutions
    - L = 1 Gpc/h and 2 Gpc/h
  - 100+1 6D-wCDM models
    - 28 HR (14 LR) fiducial runs
    - 1 run at every 100 LHD sample
  - density on 1024^3 grid points
  - Rockstar halos + postprocess
    - Subhalos excluded
    - Spherical density profile (40 bins from 10kpc/h to 5Mpc/h)

$\omega_b = \Omega_b h^2$: ±5%	$\ln(10^{10} A_s)$: ±20%
$\omega_c = \Omega_c h^2$: ±10%	n_s: ±5%
Ω_Λ: ±20%	w: ±20%

LATIN HYPERCUBE DESIGN

- Curse of dimensionality
  - Regular lattice is not tractable in high dimensions
- Latin hypercube
  - Definition: each sample point is the only one both on the row and the column
  - Can tell if one input parameter has the dominant effect

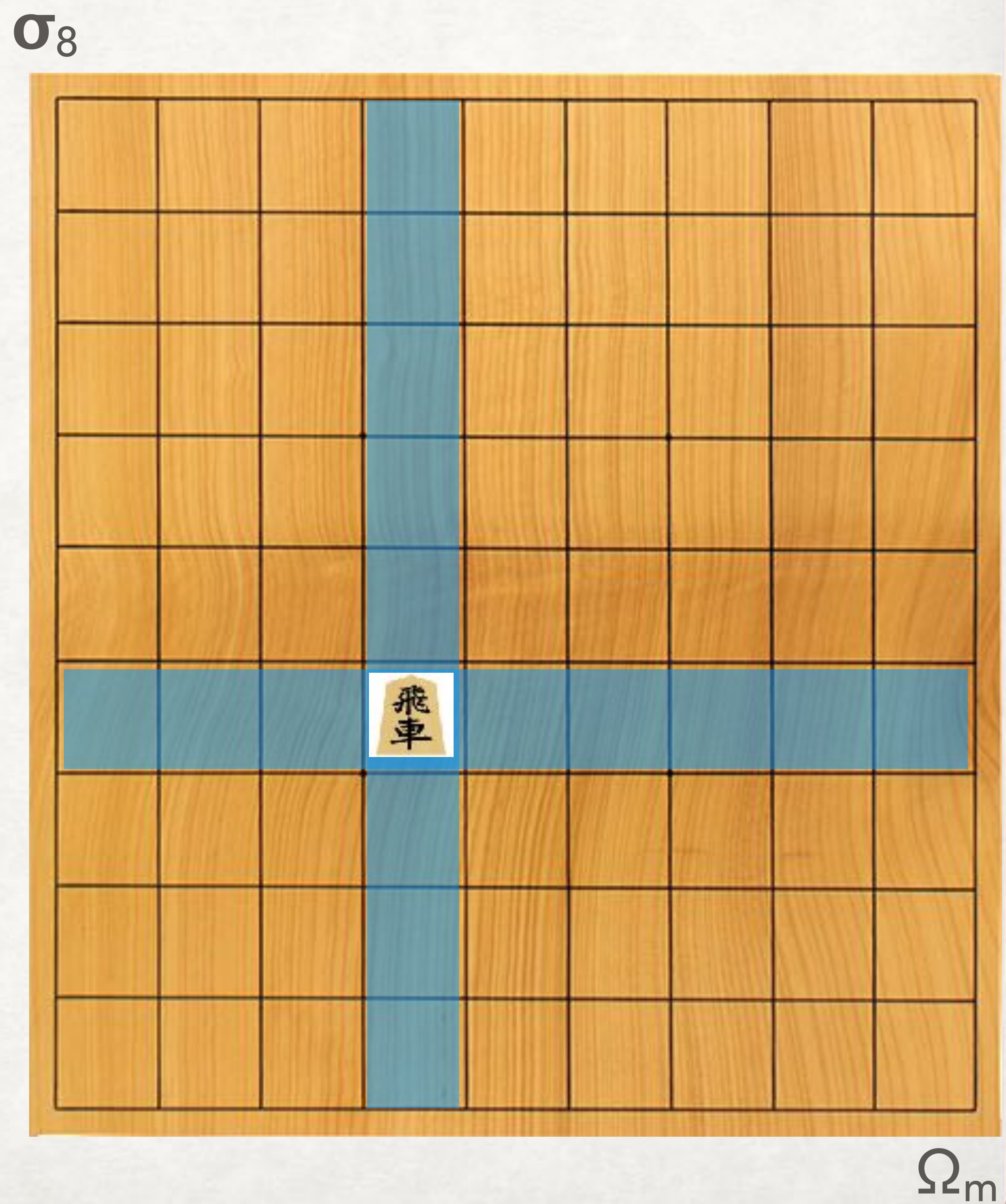
σ_8



Ω_m

LATIN HYPERCUBE DESIGN

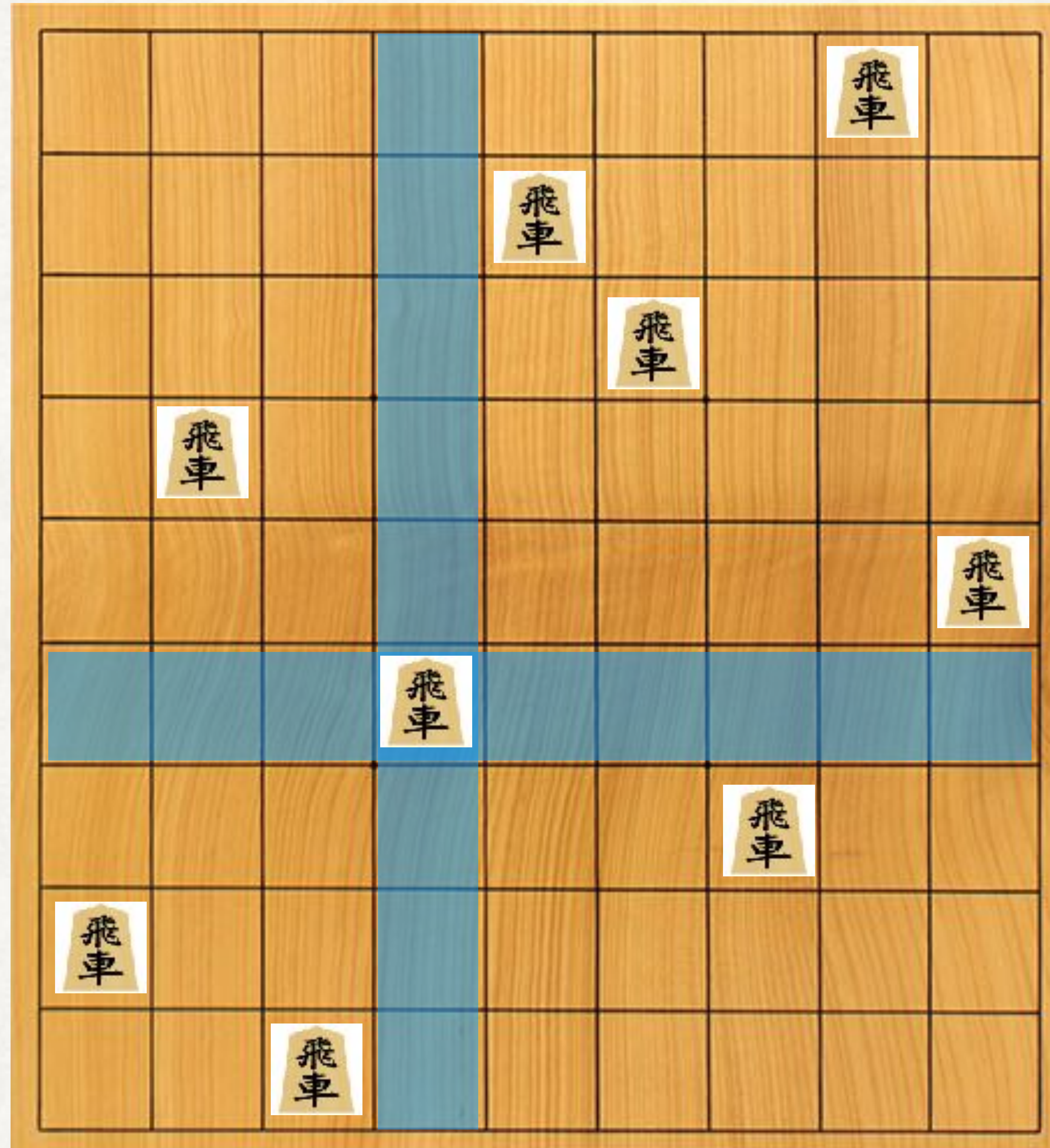
- Curse of dimensionality
  - Regular lattice is not tractable in high dimensions
- Latin hypercube
  - Definition: each sample point is the only one both on the row and the column
  - Can tell if one input parameter has the dominant effect



LATIN HYPERCUBE DESIGN

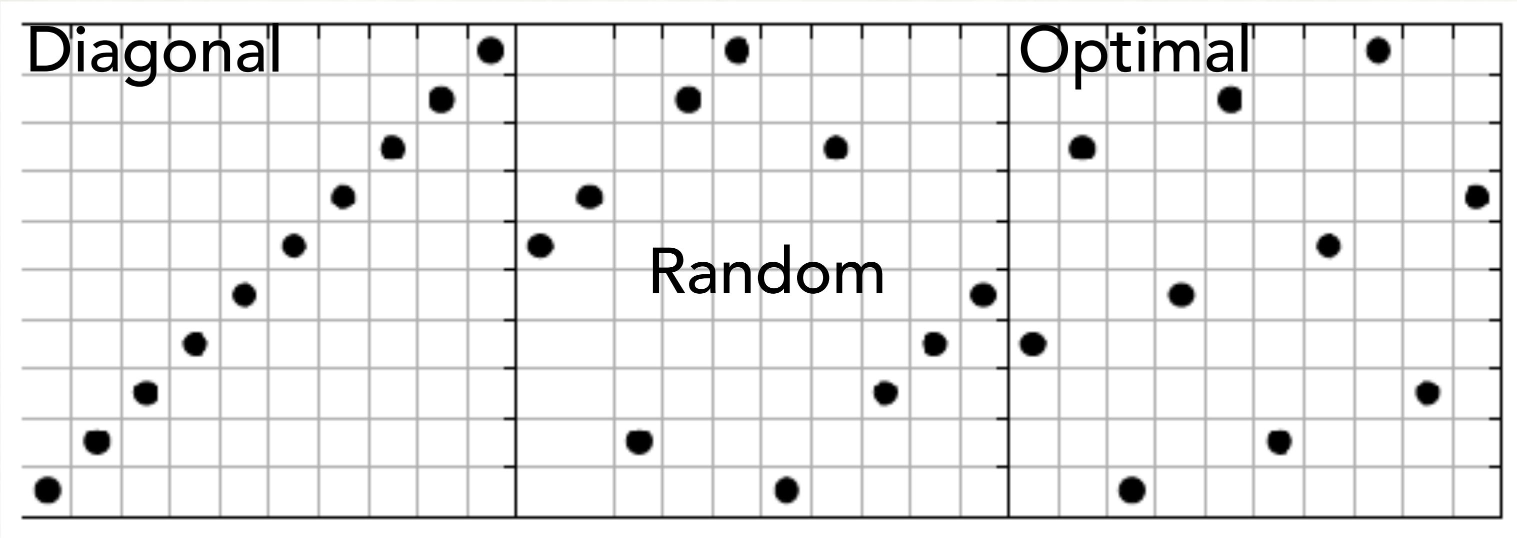
- Curse of dimensionality
  - Regular lattice is not tractable in high dimensions
- Latin hypercube
  - Definition: each sample point is the only one both on the row and the column
  - Can tell if one input parameter has the dominant effect

σ_8



Ω_m

LATIN HYPERCUBE DESIGN



- LHD is not unique
  - Some look nicer, others bad.
- One more to add: space filling property
  - It's all about how to cover your space
  - The closest neighbor should be far (= maximin distance design)

Taken from FIFA webpage

Minimize $$\phi(\mathbf{X}_N) = \left(\frac{2}{N(N-1)} \sum_{i \neq j} \frac{1}{d^r(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})} \right)^{1/r}$$ with some large r

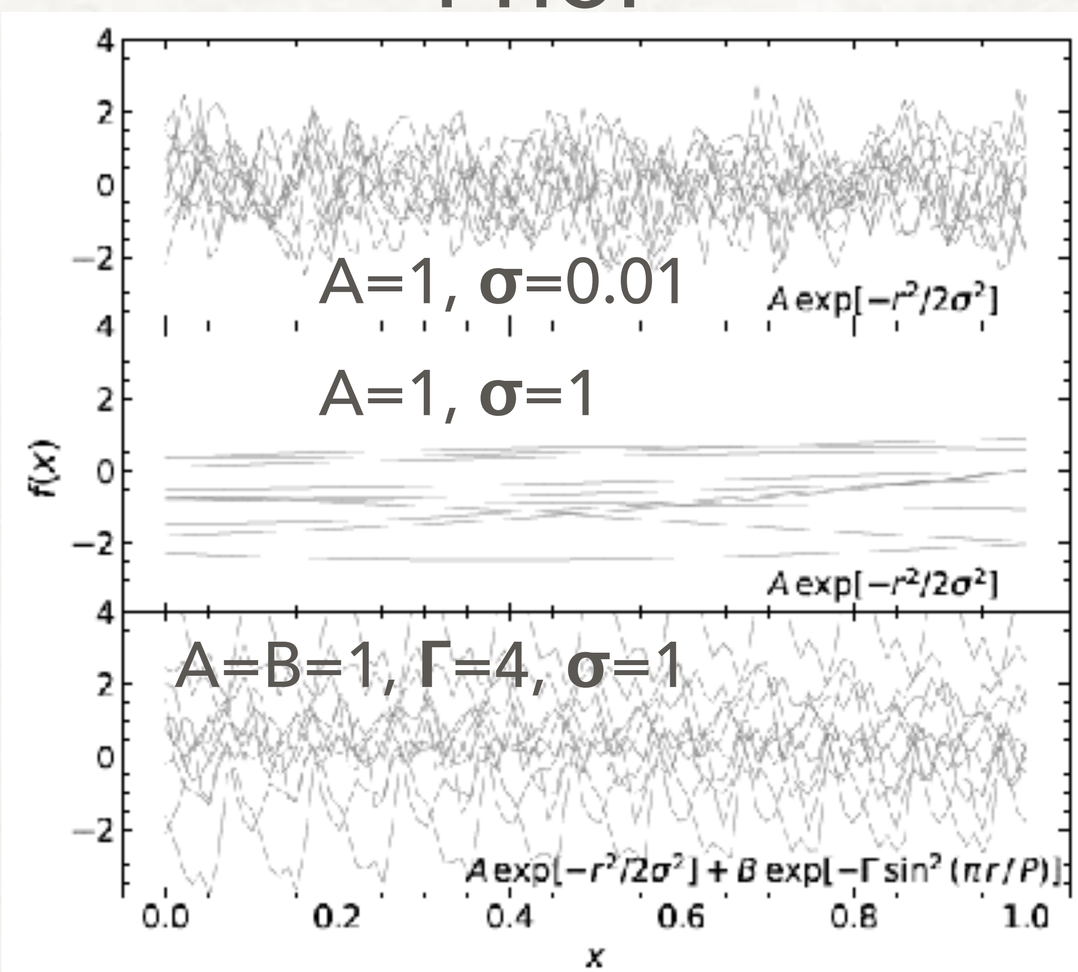
DIMENSION REDUCTION

- Mass function fitting $f(\sigma) = A[\sigma^{-a} + b] \exp\left[-\frac{c}{\sigma^2}\right]$
  - Sheth-Tormen type functional form
  - b and c from Tinker
  - (A, a) at 21 redshifts = 42 component vector -> 6 PCs
- Halo-matter cross correlation
  - (r-bin, n-bin, z-bin) = (66, 13, 21)
  - 18,018 components -> 5 PCs
- Halo auto correlation
  - (r-bin, n_1-bin, n_2-bin, z-bin) = (21, 8, 8, 21)
  - 28,224 components -> 8 PCs
- Propagator fitting $G(k) = [c_0 + c_2 k^2 + c_4 k^4] \exp\left[-k^2 \sigma_d^2 / 2\right]$
  - 3 parameters at 21 redshifts = 63 components -> 3PCs

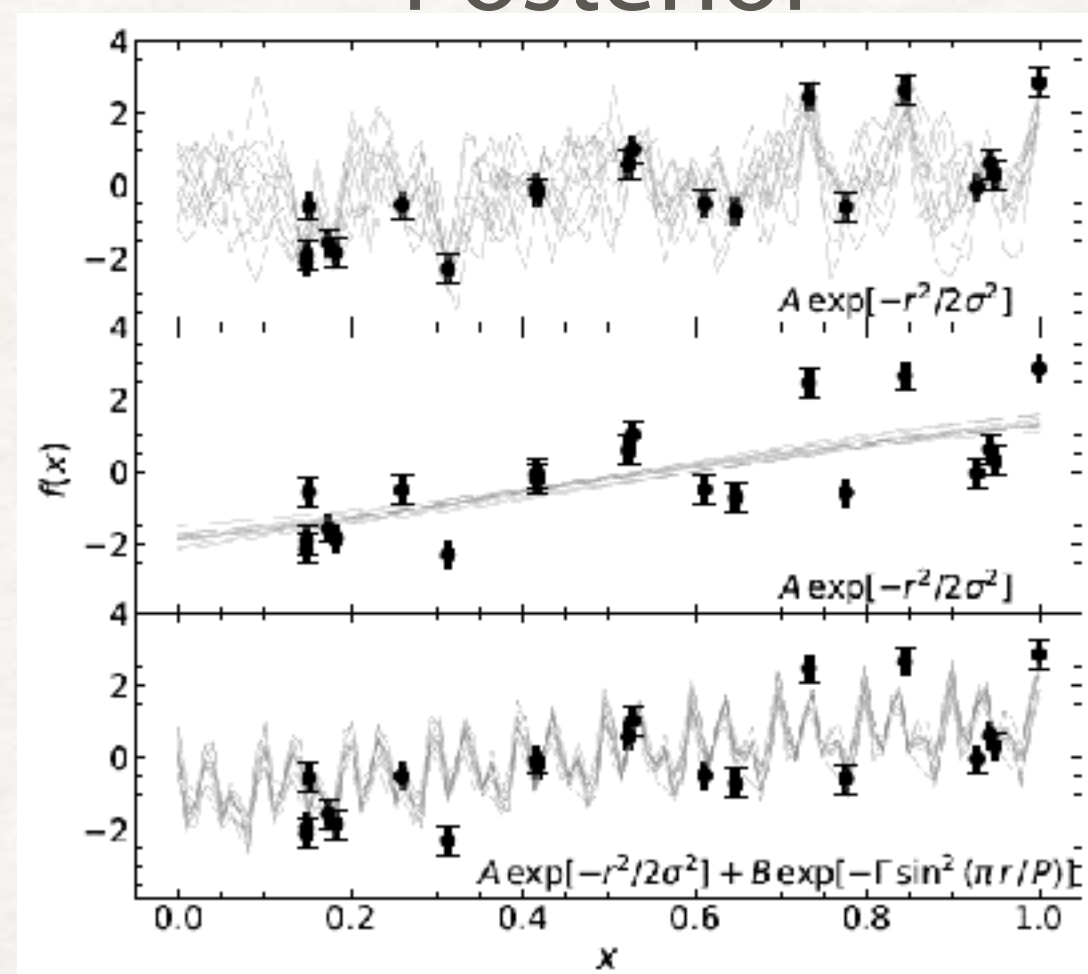
GAUSSIAN PROCESS

- **Gaussian Process**
  - is a prior in a function space
    - Specified by the mean function and covariance function (kernel)
- works as a non-parametric regressor or classifier
- Can be "trained" by tuning the hyperparameters

Prior



Posterior



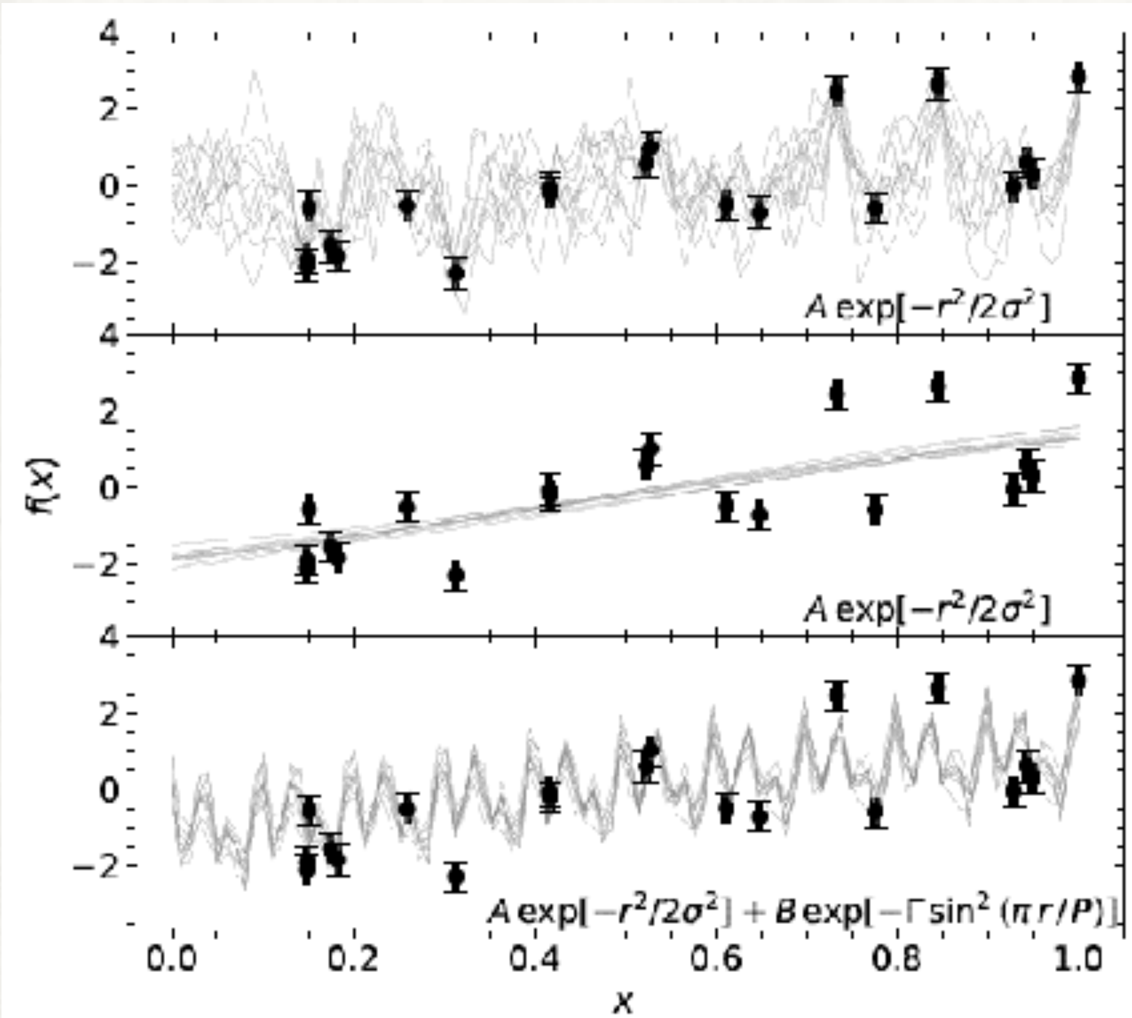
Analytic expression for the posterior

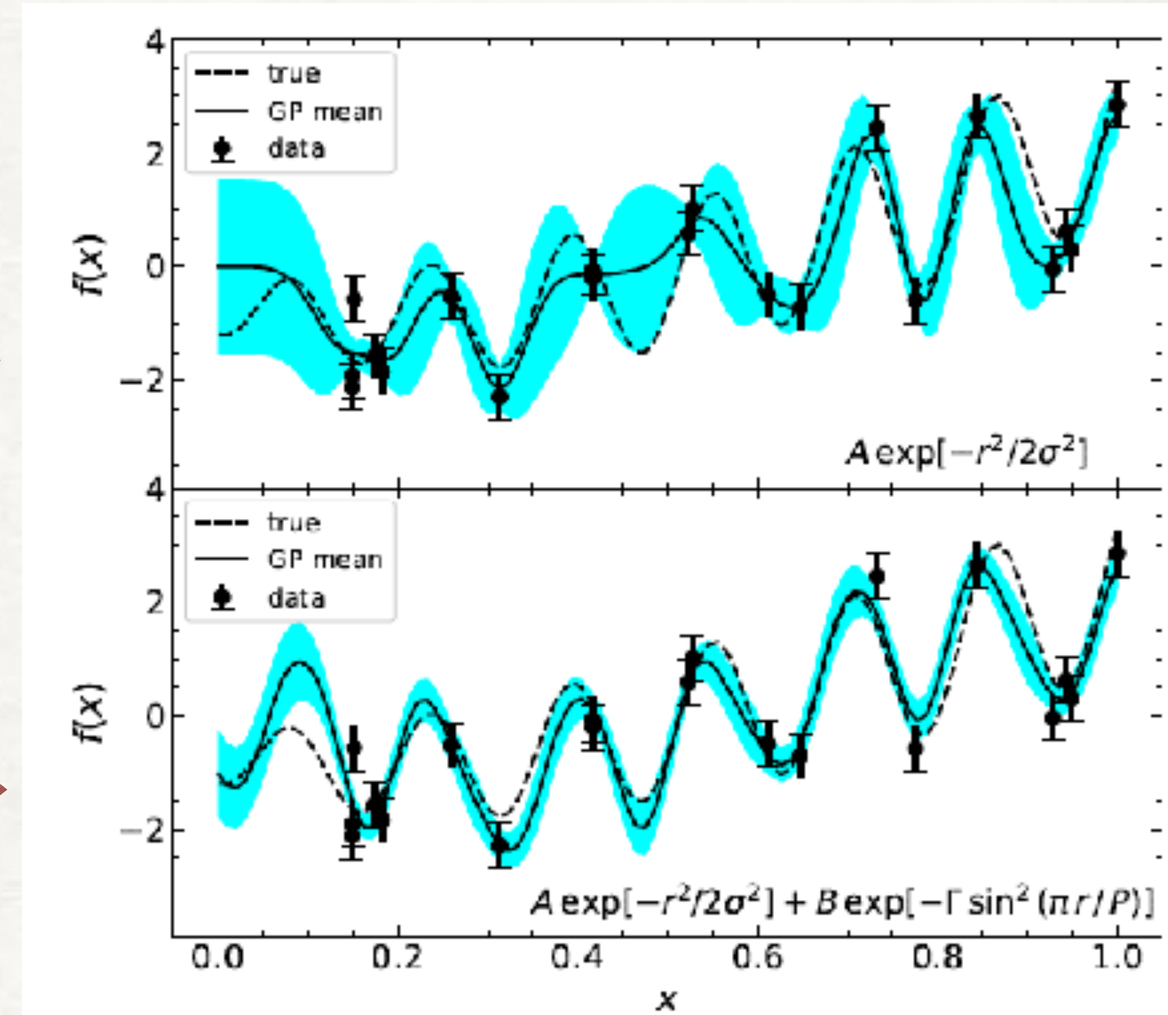
$$P(t_{N+1} \mid \mathbf{t}_N) \propto \exp\left[-\frac{1}{2}\left[\mathbf{t}_N \; t_{N+1}\right] \mathbf{C}_{N+1}^{-1} \begin{bmatrix} \mathbf{t}_N \\ t_{N+1} \end{bmatrix}\right].$$

$$\begin{aligned} \hat{t}_{N+1} &= \mathbf{k}^T \mathbf{C}_N^{-1} \mathbf{t}_N \\ \sigma^2_{\hat{t}_{N+1}} &= \kappa - \mathbf{k}^T \mathbf{C}_N^{-1} \mathbf{k}. \end{aligned}$$

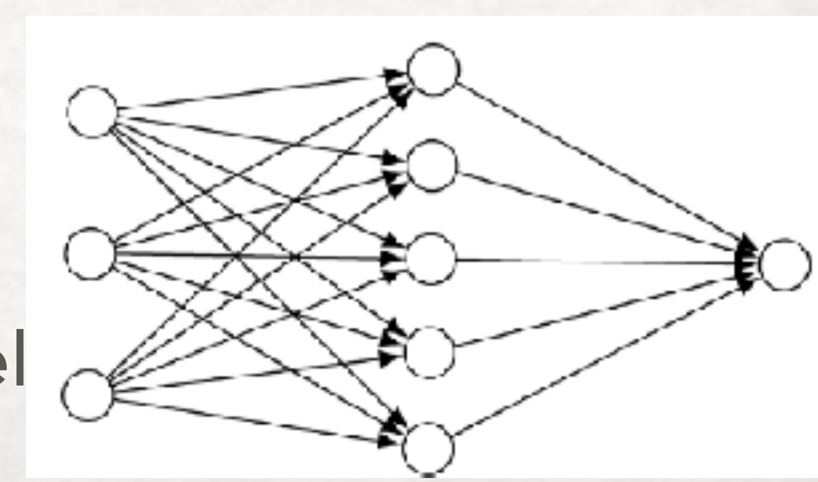
GAUSSIAN PROCESS

Posterior (before optimization) Posterior (after optimization)





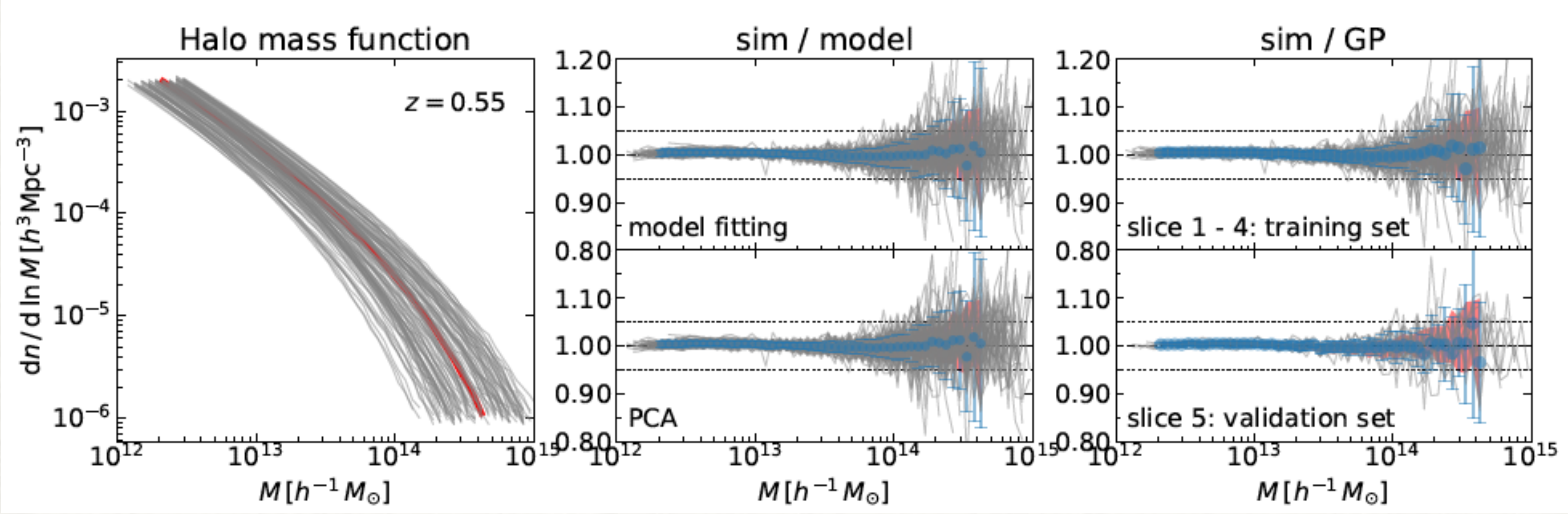
GP = 隠れ層のノード数∞



- We can estimate the uncertainty from GP
  - But we cannot always trust it
- Performance depends on the choice of the kernel

ACCURACY: HALO MASS FUNCTION

Example plot at z = 0.55



Spread in HMF among the 100 models

Gaussian Process Regression

Upper: Model fitting w/ Sheth-Tormen type function (2 free parameters)

Training set

Lower: Compress the 42 (=2 x 21 redshifts) coefficients into 6 PCs

Validation set

Red shades: scatter of 28 fiducial runs

SUMMARY

- 摂動展開に基づく揺らぎの発展
  - 限界が見えてきた: shell crossing
  - シミュレーションとのハイブリッド: response function approach
- N体シミュレーション + 機械学習
  - 従来のforward modeling + MCMCに代わる方法論
    - ２点統計以上の情報を取り出す可能性
    - まだまだアイディア段階だが、いろいろな応用がありうる
  - Emulator
    - 効率的なパラメタサンプリング + 次元削減 + ガウス過程
    - HSCのデータ解析へ実用段階に