Ghost-free theories with arbitrary higher-order time derivatives

Hayato Motohashi (YITP)

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

Gravitational Wave Physics and Astronomy Genesis

科研費 KAKENHI

Most general healthy theories

1850
Ostrogradsky theorem

1971
Lovelock theorem

1974
Horndeski theory

2011
Galileon
Deffayet et al,
Kobayashi et al

2014
Beyond
Horndeski
(GLPV)

2015
Degenerate
theories

2018
Degenerate theories with
arbitrary higher-order
derivatives

MÉMOIRE

SUR

LES ÉQUATIONS DIFFÉRENTIELLES

RELATIVES AU PROBLÈME DES ISOPÉRIMÈTRES.

PAR

M. OSTROGRADSKY.

Lu le 17 (29) novembre 1848.

Nous développons dans ce mémoire des conséquences importantes, jusqu'à présent inaperçues, dérivant de la forme sous laquelle se présente la variation d'une quantité, qui renferme, avec la variable principale ou indépendante, plusieurs fonctions de cette variable et leurs dérivées des différents ordres. Pour faciliter le discours, nous appellerons A la quantité dont il s'agit, et nous donnerons le nom de temps à la variable indépendante. La dernière dénomination se justifie par ce que cette variable joue dans notre mémoire à peu près le même rôle que le temps dans la Dynamique.

On sait que la variation de la quantité A qui dépend du temps, de fonctions quelconques du temps et de leurs dérivées, se résout en deux parties distinctes. La première est une différentielle exacte, quelles que soient les fonctions du temps que A renferme, et quelles que soient les variations renfermeront point les quantités relatives à la direction dont il s'agit, et ne changeront pas, en les reportant à une origine des coordonnées mobile, d'un mouvement rectiligne et uniforme. On en conclura que, parmi les quantités que l'intégration des ces équations introduira, il s'en trouvera neuf relatives: à la direction des axes coordonnés, à la position dans un instant donné de leur origine et au mouvement, rectiligne et uniforme, de cette origine. On ne peut le supposer en mouvement curviligne ou varié, car les forces d'inertie et, par suite, les équations du mouvement en dépendraient.

En supposant successivement que les facteurs P et Π soient les différences partielles des coordonnées, qu'on aura choisies pour déterminer le mouvement, par rapport à chacune des neuf quantités dont nous venons de parler; la formule (91), dans le cas dynamique dont il est question, deviendra intégrable, et nous fournira neuf intégrales avec autant *de constantes arbitraires H*. Or, eu égard aux valeurs des facteurs P et Π, l'équation (92) prendra de suite la forme convenable, et nous donnera et les variations dH des neuf *constantes arbitraires H*, variations produites par les forces perturbatrices; et par suite, nous aurons en même temps un nombre considérable de valeurs du symbole (a_r, a_s).

On déterminera, par un calcul direct, celles des valeurs du symbole dont il s'agit, qui échapperaient aux procédés particuliers, que nous venons d'éclaircir par un exemple.

Ostrogradsky theorem

Ostrogradsky (1850), Pons (1989)
Woodard, astro-ph/0601672; 1506.02210

Nondegenerate L

→ $2n$-th order EL eqs
requiring more initial conditions ($n \geq 2$)

→ Ghost DOF

Example: $L = \ddot{\phi}^2/2$ with $\phi = \phi(t)$

- 4th order EL eq : $\ddddot{\phi} = 0$
  4 initial conditions = 2 DOF.
- Hamiltonian is unbounded
  2 DOF = 1 healthy + 1 ghost (negative energy)

Generalizing Ostrogradsky theorem

What about degenerate L for $(2n-1)$-th order EL eqs?

$$\phi^a = \phi^a(t)$$

- $L = L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$

  4th order EL eqs → Ghost

  3rd order EL eqs → Ghost

  2nd order EL eqs → $L(\dot{\phi}^a, \phi^a)$ after integ. by parts.

- $L = L(\phi^{a(n)}, \cdots \ddot{\phi}^a, \dot{\phi}^a, \phi^a)$

  $2n$ th order EL eqs → Ghost

  $(2n-1)$-th order EL eqs → Ghost

Need no-ghost conditions for all the ghosts.

HM, Suyama, 1411.3721

✓Ostrogradsky theorem

$$\det K \neq 0 \Longrightarrow H \text{ is unbounded}$$

✓No-ghost condition (DC1 & DC2)

$$K_{ab} = 0 \ \& \ M_{ab} = 0 \Longrightarrow H \text{ is bounded}$$

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b} \qquad \text{sym}$$

$$M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a} \qquad \text{anti-sym}$$

HM, Suyama, 1411.3721

✓Ostrogradsky theorem updated

$\det K \neq 0$ or $\det M \neq 0 \Longrightarrow H$ is unbounded

✓No-ghost condition (DC1 & DC2)

$K_{ab} = 0$ & $M_{ab} = 0 \Longrightarrow H$ is bounded

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b} \qquad \text{sym}$$

$$M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a} \qquad \text{anti-sym}$$

HM, Suyama, 1411.3721

✓ Ostrogradsky theorem updated

$\det K \neq 0$ or $\det M \neq 0 \Longrightarrow H$ is unbounded

✓ No-ghost condition (DC1 & DC2)

$K_{ab} = 0$ & $M_{ab} = 0 \Longrightarrow H$ is bounded

Highest   Next-highest

✓ EL eq

$$K_{ab}\ddddot{\phi}^b + (\dot{K}_{ab} + M_{ab})\dddot{\phi}^b = (\text{terms up to } \ddot{\phi}^a)$$

$\Longrightarrow$ 2nd-order system

Eliminating Ostrogradsky ghost

HM, Suyama, 1411.3721

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

No ghost condition

$K_{ab} = 0$ & $M_{ab} = 0$

$$\phi^a = \phi^a(t)$$

Eliminating Ostrogradsky ghost

HM, Suyama, 1411.3721

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

No ghost condition

$K_{ab} = 0$ & $M_{ab} = 0$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

$$\phi^a = \phi^a(t)$$
$$q^i = q^i(t)$$

Healthy scalar-tensor theories

Horndeski

- Second order EOM (sufficient for no ghost)

GLPV

- Found by heuristic method

quadratic DHOST / EST

- Degeneracy structure found for a toy model

$$L = \frac{1}{2} a \ddot{\phi}^2 + \frac{1}{2} k_0 \dot{\phi}^2 + \frac{1}{2} k_{ij} \dot{q}^i \dot{q}^j + b_i \ddot{\phi} \dot{q}^i + c_i \dot{\phi} \dot{q}^i - V(\phi, q)$$

which applies to theory up to $(\nabla\nabla\phi)^2$.

More general analysis is required beyond $(\nabla\nabla\phi)^2$.

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

$$L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$$

$$\dot{\phi}^a$$
$$||$$
$$(\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \boldsymbol{q^i}, \lambda_a)$$
$$\updownarrow$$
$$(\boldsymbol{P_a}, \boldsymbol{\pi_a}, \boldsymbol{p_i}, \rho^a)$$

- Hamiltonian analysis

$$L_{eq} = L(\dot{Q}^a, Q^a, \phi^a; \dot{q}^i, q^i) + \lambda_a(\dot{\phi}^a - Q^a)$$

Canonical momenta

$$\begin{cases} P_a = L_{\dot{Q}^a} \\ p_i = L_{\dot{q}^i} \\ \pi_a = \lambda_a \\ \rho^a = 0 \end{cases}$$

$\pi_a = \lambda_a$, $\rho^a = 0$ → Primary constraints (C1)

Hamiltonian

$$H = H_0(P, Q, \phi, p, q) + \pi_a Q^a$$

π_a shows up only linearly. In general H is unbounded.

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

$$L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$$

$$\dot{\phi}^a \; || \; (\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \boldsymbol{q^i}, \lambda_a)$$

$$\updownarrow$$

$$(P_a, \pi_a, \boldsymbol{p_i}, \rho^a)$$

$$H = H_0 + \pi_a Q^a \qquad \det L_{ij} \neq 0$$

- DC1: $K_{ab} \equiv L_{ab} - L_{ia} L_{ij}^{-1} L_{jb} = 0$

$\Longrightarrow$ Additional C1: $\Psi_a \equiv P_a - F_a(Q, \phi, p, q) = 0$

✓ Fixed

- DC2: $M_{ab} \equiv \{\Psi_a, \Psi_b\} = 0$

$\Longrightarrow$ C2: $\Upsilon_a \equiv \pi_a - G_a(Q, \phi, p, q) = 0$

✓ Fixed

✓ We eliminated all the ghosts. H is bounded.

✓ EL eqs $\Longrightarrow$ a priori higher-order but can be reduced to 2nd-order system ← Beyond Horndeski

✓ Applies for a wide class of theories

Applications

Crisostomi, Klein, Roest, 1703.01623

- ✓ Field theory in flat spacetime
- ✓ SU(2) — Allys, Peter, Rodriguez, 1609.05870
- ✓ Boson-Fermion — Kimura, Sakakihara, Yamaguchi, 1704.02717
- ✓ Scalar-tensor theories (DHOST)

Langlois, Noui, 1510.06930, 1512.06820
Crisostomi, Koyama, Tasinato, 1602.03119
Achour, Langlois, Noui, 1602.08398
Achour, Crisostomi, Koyama, Langlois, Noui, Tasinato, 1608.08135

- ✓ Vector-tensor theories

Kimura, Naruko, Yoshida, 1608.07066

- ✓ Tensor theories

Crisostomi, Noui, Charmousis, Langlois, 1710.04531

Eliminating Ostrogradsky ghost

HM, Suyama, 1411.3721

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

No ghost condition

$K_{ab} = 0$ & $M_{ab} = 0$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

No ghost condition

$K_{ab} = 0$ & $M_{ab} = 0$

Eliminating Ostrogradsky ghost

HM, Suyama, 1411.3721

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

No ghost condition

$K_{ab} = 0$ & $M_{ab} = 0$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \;\sim\; \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \;\sim\; \phi^a + g_{\mu\nu}$

No ghost condition

$K_{ab} = 0$ & $M_{ab} = 0$

HM, Suyama, Yamaguchi, 1711.08125

1804.07990

- $L(\dddot{\psi}, \ddot{\psi}, \dot{\psi}, \psi; \dot{q}^i, q^i)$
- $L(\dddot{\psi}^n, \ddot{\psi}^n, \dot{\psi}^n, \psi^n; \ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$
- $L(\phi^{i_d(d+1)}, \dots; \phi^{i_{d-1}(d)}, \dots; \dots; \dot{\phi}^{i_0}, \phi^{i_0})$

3rd-order derivatives in L

HM, Suyama, Yamaguchi, 1711.08125

A ghost-free quadratic Lagrangian involving 3rd-order derivatives.

Crucial difference:

Ostrogradsky ghosts appear as

- $L \ni$ 2nd-order time derivatives $\Longrightarrow H$: linear in P
- $L \ni$ 3rd-order time derivatives $\Longrightarrow H$: linear in P & Q

Beyond 2nd-order derivatives,
ghosts are hidden in a nontrivial way.

nth-order derivatives in L

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives.

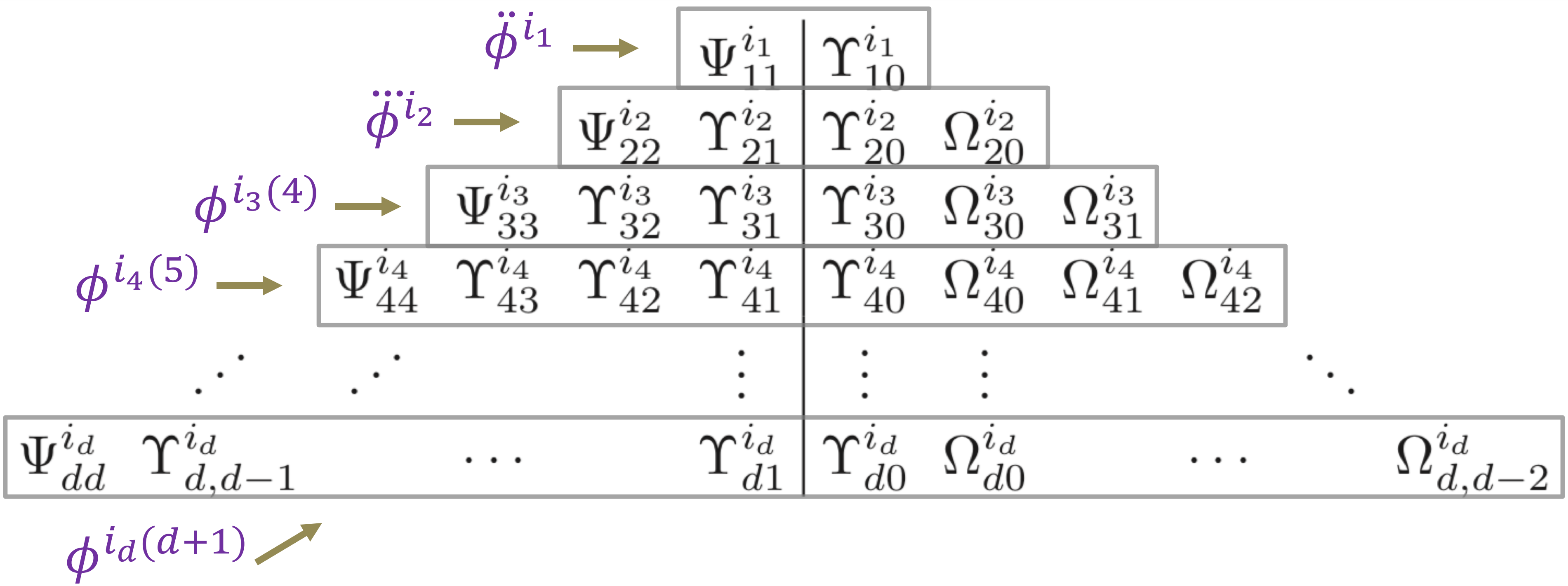
All constraints = 0

$$
\begin{array}{cccccc|ccccc}
 & & & & & \Psi^{i_1}_{11} & \Upsilon^{i_1}_{10} & & & & \\
 & & & & \Psi^{i_2}_{22} & \Upsilon^{i_2}_{21} & \Upsilon^{i_2}_{20} & \Omega^{i_2}_{20} & & & \\
 & & & \Psi^{i_3}_{33} & \Upsilon^{i_3}_{32} & \Upsilon^{i_3}_{31} & \Upsilon^{i_3}_{30} & \Omega^{i_3}_{30} & \Omega^{i_3}_{31} & & \\
 & & \Psi^{i_4}_{44} & \Upsilon^{i_4}_{43} & \Upsilon^{i_4}_{42} & \Upsilon^{i_4}_{41} & \Upsilon^{i_4}_{40} & \Omega^{i_4}_{40} & \Omega^{i_4}_{41} & \Omega^{i_4}_{42} & \\
 & \cdot^{\cdot^{\cdot}} & \cdot^{\cdot^{\cdot}} & & & \vdots & \vdots & \vdots & & & \ddots \\
\Psi^{i_d}_{dd} & \Upsilon^{i_d}_{d,d-1} & & \cdots & & \Upsilon^{i_d}_{d1} & \Upsilon^{i_d}_{d0} & \Omega^{i_d}_{d0} & & \cdots & \Omega^{i_d}_{d,d-2}
\end{array}
$$

nth-order derivatives in the action

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives.

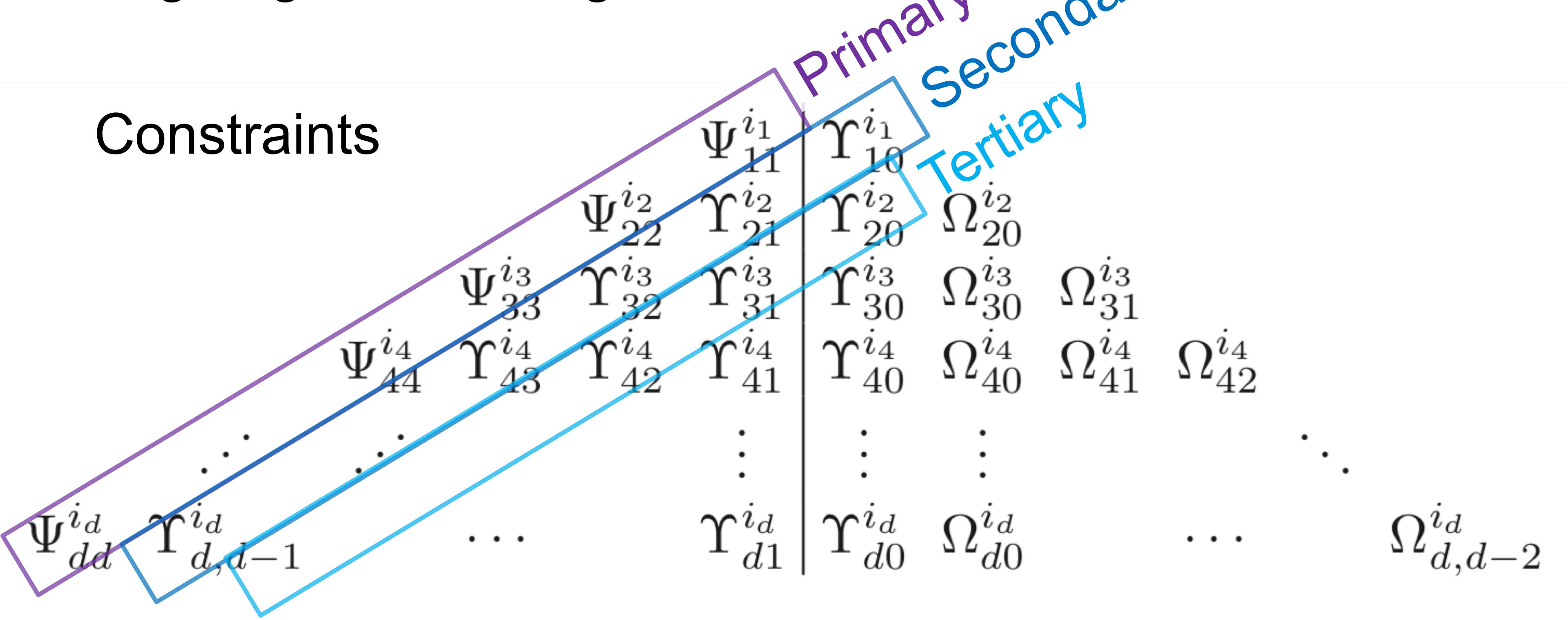


nth-order derivatives in the action

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives

Constraints



nth-order derivatives in the action

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives.

Ostrogradsky theorem

$\det K_{i_1 j_1} \neq 0 \Longrightarrow$ Ghost

$$
\begin{array}{cccccc|cccccc}
 & & & & & \boxed{\Psi^{i_1}_{11}} & \Upsilon^{i_1}_{10} & & & & & \\
 & & & & \Psi^{i_2}_{22} & \Upsilon^{i_2}_{21} & \Upsilon^{i_2}_{20} & \Omega^{i_2}_{20} & & & & \\
 & & & \Psi^{i_3}_{33} & \Upsilon^{i_3}_{32} & \Upsilon^{i_3}_{31} & \Upsilon^{i_3}_{30} & \Omega^{i_3}_{30} & \Omega^{i_3}_{31} & & & \\
 & & \Psi^{i_4}_{44} & \Upsilon^{i_4}_{43} & \Upsilon^{i_4}_{42} & \Upsilon^{i_4}_{41} & \Upsilon^{i_4}_{40} & \Omega^{i_4}_{40} & \Omega^{i_4}_{41} & \Omega^{i_4}_{42} & & \\
 & \iddots & \iddots & & & \vdots & \vdots & \vdots & & \ddots & & \\
\Psi^{i_d}_{dd} & \Upsilon^{i_d}_{d,d-1} & \cdots & & & \Upsilon^{i_d}_{d1} & \Upsilon^{i_d}_{d0} & \Omega^{i_d}_{d0} & & \cdots & & \Omega^{i_d}_{d,d-2}
\end{array}
$$

$\neq 0$

nth-order derivatives in the action

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives.

$$K_{i_1 j_1} = 0 \ \& \ M_{i_1 j_1} = 0 \Longrightarrow \boxed{\Psi^{i_1}_{11} \ \Big| \ \Upsilon^{i_1}_{10}}$$

$$\Psi^{i_1}_{11} = 0 \qquad \Upsilon^{i_1}_{10} = 0$$

$$\begin{array}{ccccc|cccc}
 & & & \Psi^{i_2}_{22} & \Upsilon^{i_2}_{21} & \Upsilon^{i_2}_{20} & \Omega^{i_2}_{20} & & \\
 & & \Psi^{i_3}_{33} & \Upsilon^{i_3}_{32} & \Upsilon^{i_3}_{31} & \Upsilon^{i_3}_{30} & \Omega^{i_3}_{30} & \Omega^{i_3}_{31} & \\
 & \Psi^{i_4}_{44} & \Upsilon^{i_4}_{43} & \Upsilon^{i_4}_{42} & \Upsilon^{i_4}_{41} & \Upsilon^{i_4}_{40} & \Omega^{i_4}_{40} & \Omega^{i_4}_{41} & \Omega^{i_4}_{42} \\
 & \vdots & & & \vdots & \vdots & \vdots & & \vdots \\
\Psi^{i_d}_{dd} & \Upsilon^{i_d}_{d,d-1} & \cdots & & \Upsilon^{i_d}_{d1} & \Upsilon^{i_d}_{d0} & \Omega^{i_d}_{d0} & \cdots & \Omega^{i_d}_{d,d-2}
\end{array}$$

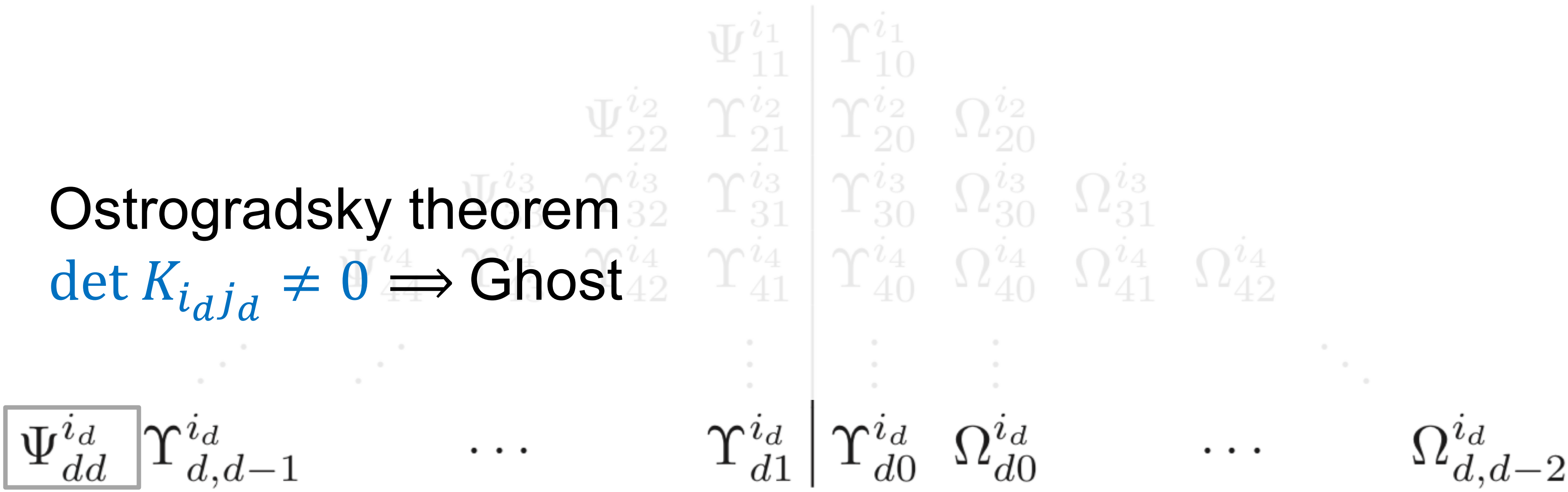
nth-order derivatives in the action

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives.

Ostrogradsky theorem

$\det K_{i_d j_d} \neq 0 \Longrightarrow$ Ghost



$\neq 0$

nth-order derivatives in the action

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives.

$$
\begin{array}{ccccc|ccccc}
 & & & & \Psi^{i_1}_{11} & \Upsilon^{i_1}_{10} & & & & \\
 & & & \Psi^{i_2}_{22} & \Upsilon^{i_2}_{21} & \Upsilon^{i_2}_{20} & \Omega^{i_2}_{20} & & & \\
 & & \Psi^{i_3}_{33} & \Upsilon^{i_3}_{32} & \Upsilon^{i_3}_{31} & \Upsilon^{i_3}_{30} & \Omega^{i_3}_{30} & \Omega^{i_3}_{31} & & \\
 & \Psi^{i_4}_{44} & \Upsilon^{i_4}_{43} & \Upsilon^{i_4}_{42} & \Upsilon^{i_4}_{41} & \Upsilon^{i_4}_{40} & \Omega^{i_4}_{40} & \Omega^{i_4}_{41} & \Omega^{i_4}_{42} & \\
 & & & & \vdots & \vdots & \vdots & & & \\
\Psi^{i_d}_{dd} \quad \Upsilon^{i_d}_{d,d-1} & \cdots & & & \Upsilon^{i_d}_{d1} & \Upsilon^{i_d}_{d0} & \Omega^{i_d}_{d0} & \cdots & & \Omega^{i_d}_{d,d-2}
\end{array}
$$

$K_{i_d j_d} = 0$ & $M_{i_d j_d} = 0$

$\Downarrow$

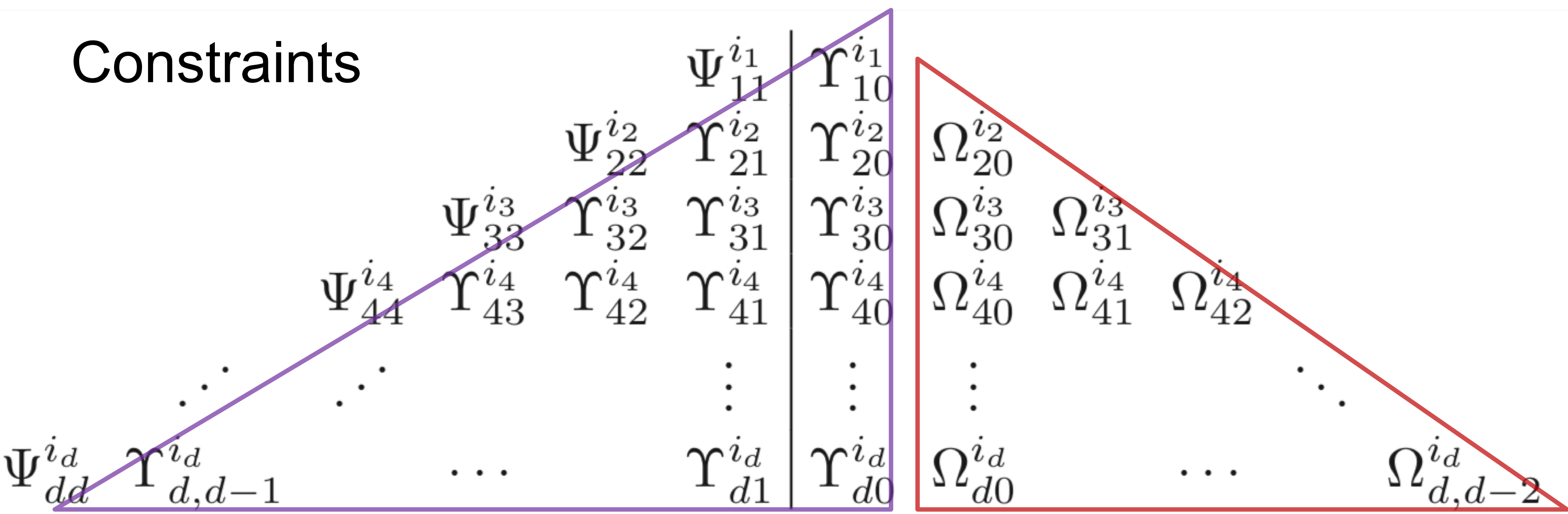
$\Psi^{i_d}_{dd} = 0 \quad \Upsilon^{i_d}_{d,d-1} = 0$

nth-order derivatives in the action

HM, Suyama, Yamaguchi, 1804.07990

Systematic construction of general ghost-free Lagrangian involving nth-order derivatives.

Constraints



Eliminate (ordinary) Ostrogradsky ghosts

Eliminate hidden ghosts

Eliminating Ostrogradsky ghost

✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$

✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721

✓ $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$

✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$

✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

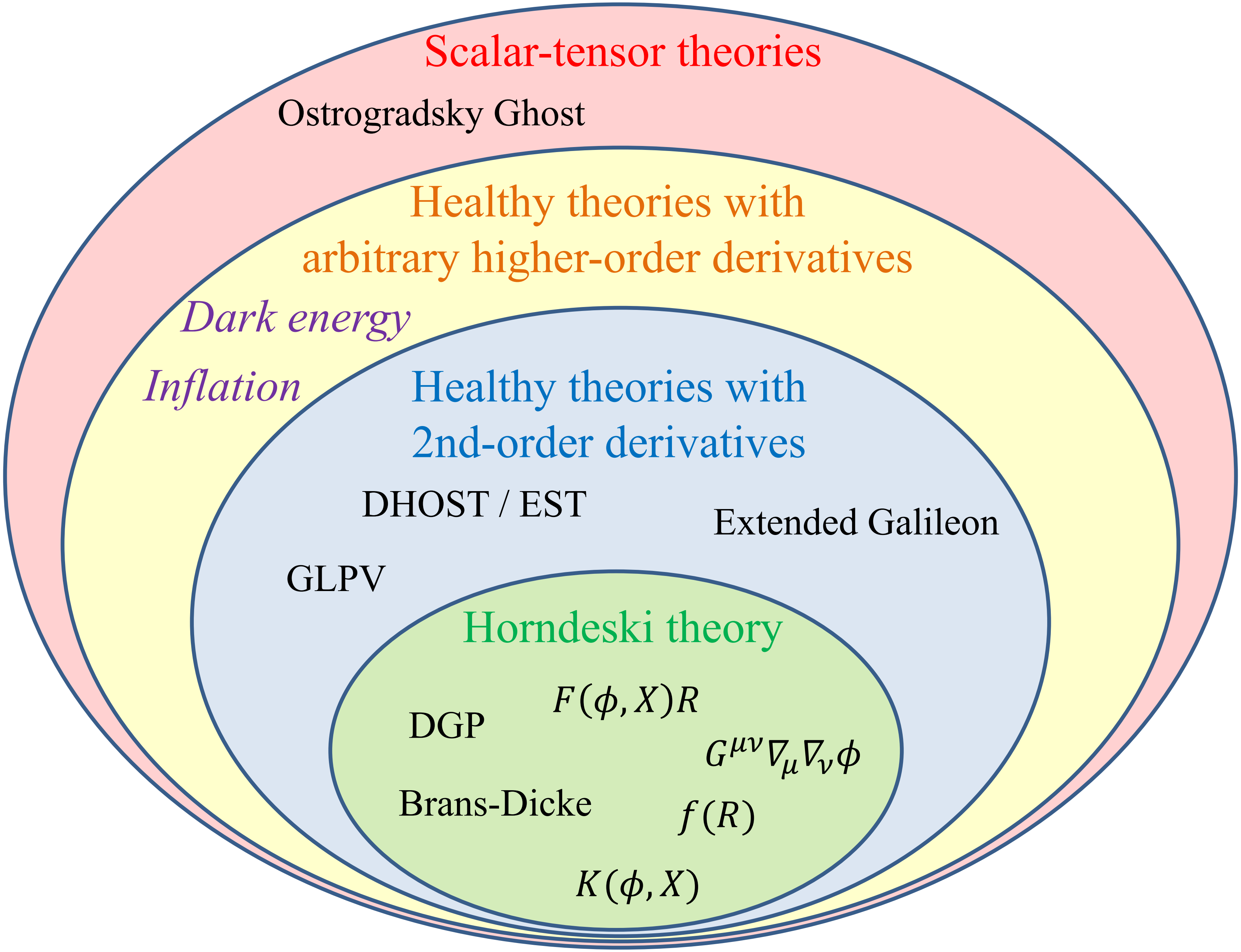
HM, Suyama, Yamaguchi, 1711.08125

1804.07990

✓ $L(\dddot{\psi}, \ddot{\psi}, \dot{\psi}, \psi; \dot{q}^i, q^i)$

✓ $L(\dddot{\psi}^n, \ddot{\psi}^n, \dot{\psi}^n, \psi^n; \ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$

✓ $L(\phi^{i_d(d+1)}, \dots; \phi^{i_{d-1}(d)}, \dots; \dots; \dot{\phi}^{i_0}, \phi^{i_0})$


Scalar-tensor theories
Ostrogradsky Ghost
Healthy theories with
arbitrary higher-order derivatives
Dark energy
Inflation
Healthy theories with
2nd-order derivatives
DHOST / EST
Extended Galileon
GLPV
Horndeski theory
$F(\phi, X)R$
DGP
$G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$
Brans-Dicke
$f(R)$
$K(\phi, X)$

Summary

Ostrogradsky ghosts appear as

- $L \ni$ 2nd-order time derivatives $\Longrightarrow H$: linear in P

which can be removed by degeneracy conditions.

The analysis of $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$ applies to a wide class of model buildings.

We found that for quadratic model with $\dddot{\psi}^n, \dot{q}^i$

- $L \ni$ 3rd-order time derivatives $\Longrightarrow H$: linear in P, Q

We constructed the first ghost-free model with 3rd-order time derivatives in L.

We established a systematic construction process for

- $L \ni$ Arbitrary higher-order time derivatives