Creation of a localised source in quantum field theory

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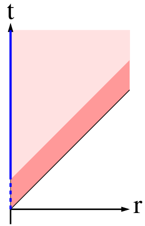
E. G. Brown and JL JHEP **1508**, 061 (2015)
L. J. Zhou, M. E. Carrington, G. Kunstatter, JL PRD **95**, 085007 (2017)
W. M. H. Wan Mokhtar and JL in preparation



Plan

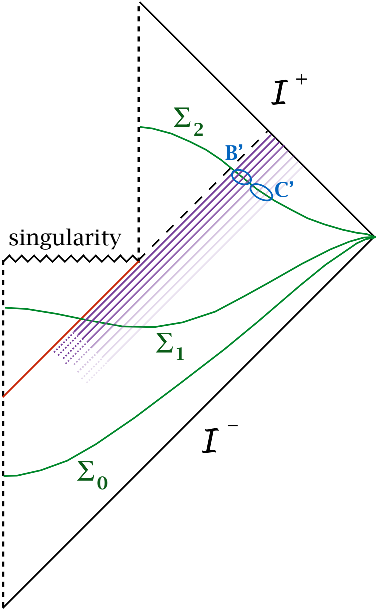
1. **Motivation: Firewalls**
   $\longrightarrow$ **Correlation breakdown in quantum field theory**
2. **Wall for scalar field in $1+1$**
3. **Wall for spinor field in $1+1$**
4. **Pointlike source for scalar field in $3+1$**
5. **Summary**

1. Motivation: Firewall proposal

Almheiri et al 2013

Suppose BH evaporates fully and the process preserves unitarity

- Pure state on $\Sigma_2 \Rightarrow$
  B' and C' strongly correlated



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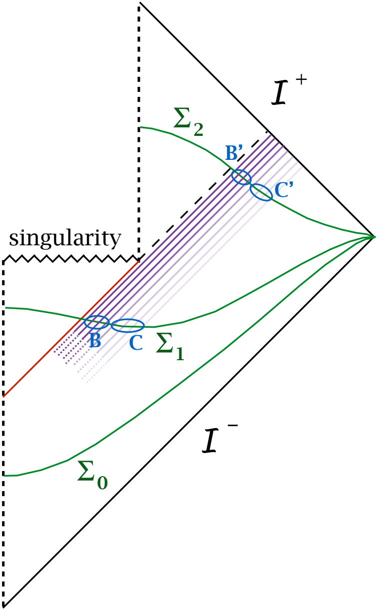

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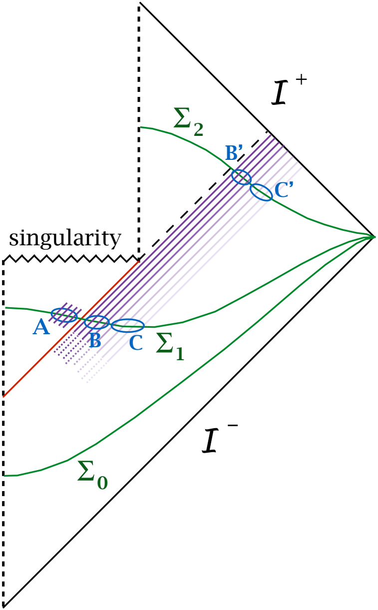

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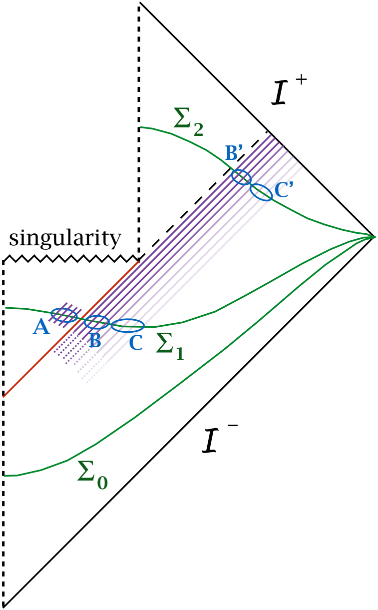

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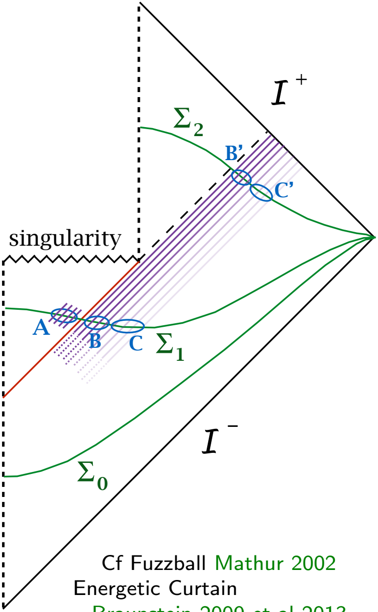
Almheiri at al (AMPS) 2013

resolution proposal:

A–B correlations broken by "drama" at the shrinking horizon even for macroscopic BH

"Firewall"



Cf Fuzzball Mathur 2002
Energetic Curtain
Braunstein 2009 et al 2013

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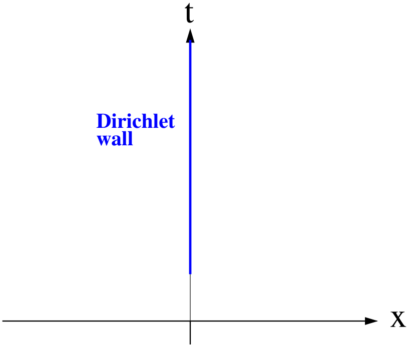
Brown and JL 2015

1+1 Minkowski

$\phi(t, x)$ massless

$$\partial_t^2 \phi - \partial_x^2 \phi = 0$$



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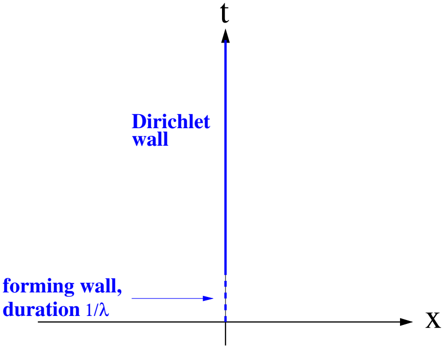

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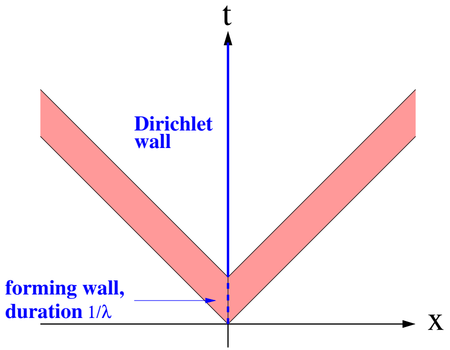

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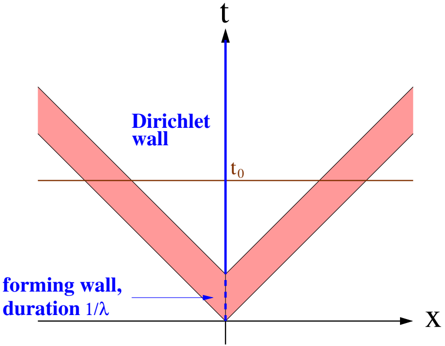

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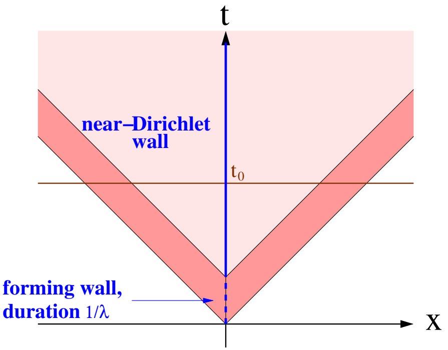

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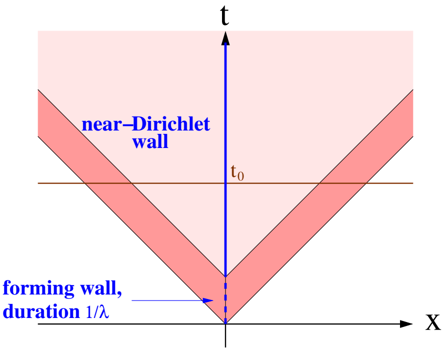

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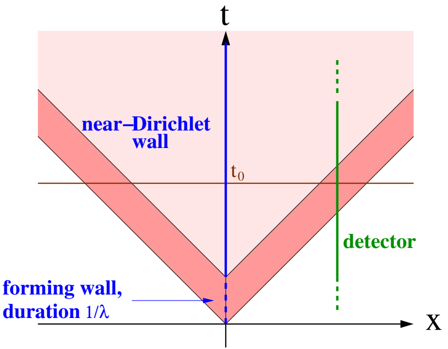

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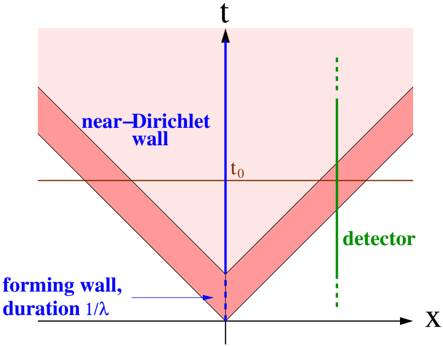

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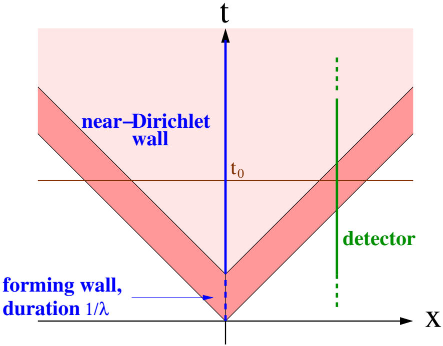

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Moral: sharp wall formation singular gravitationally but nonsingular for a matter coupling

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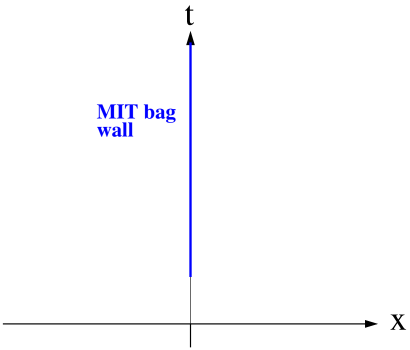

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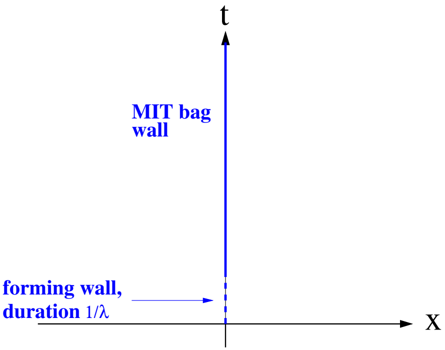

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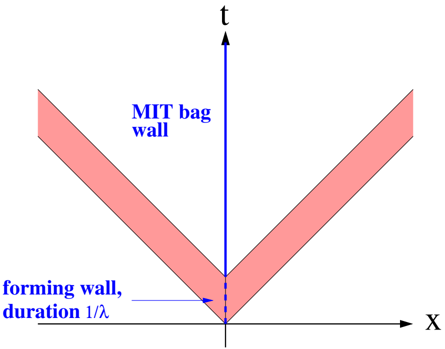

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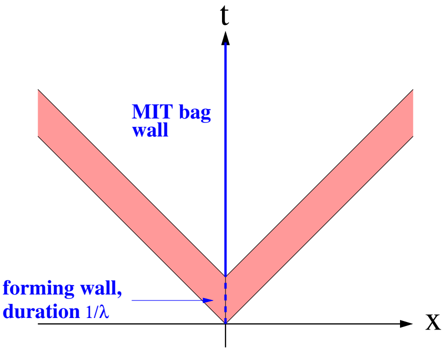

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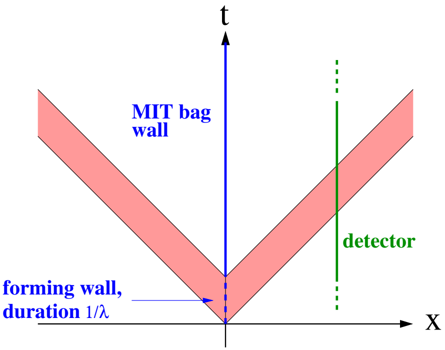

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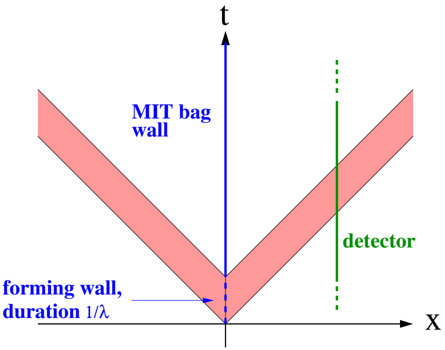

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Zhou et al 2016

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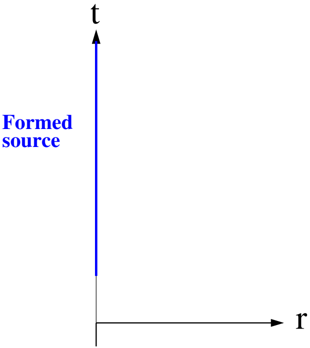

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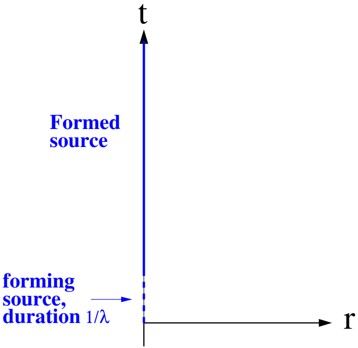
(spherically symmetric sector)



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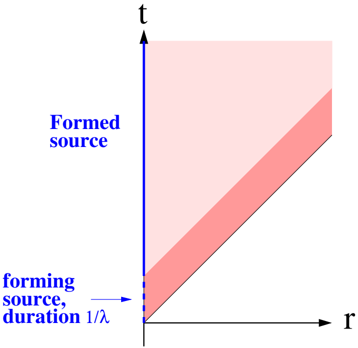

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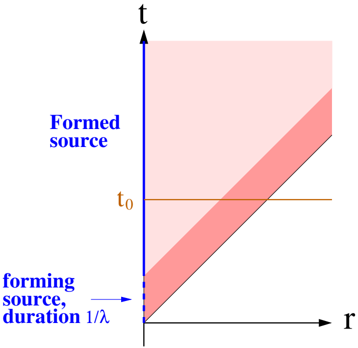

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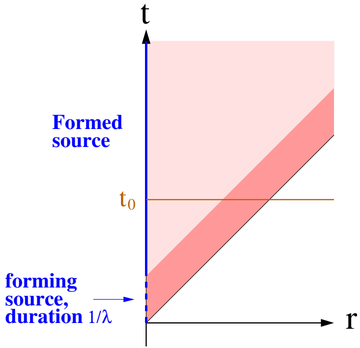

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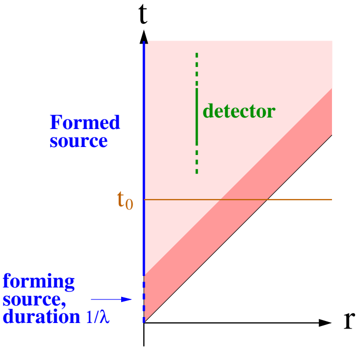

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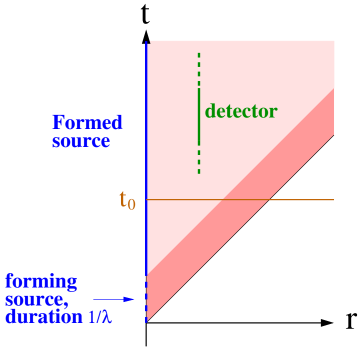

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Moral: sharp source formation (quite) singular both gravitationally and for a matter coupling

Summary

- **Rapid creation of a localised source tends to be singular!**
  - Both gravitationally and for a model atom's response
  - 1+1 scalar field exceptional (and needs an infrared cutoff)
- **Model for a black hole firewall?**
  - Spacetime will react. How?
  - $G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$? May or may not suffice...
- **Fully-developed firewall?**
  - **Quantum theory of spacetime needed**

Appendix: pointlike detector in quantum field theory

(Unruh-DeWitt)

Quantum field

D	spacetime dimension
ϕ	real scalar field
$\|0\rangle$	(initial) state

Two-state detector (atom)

$\|\|0\rangle\rangle$	state with energy 0
$\|\|1\rangle\rangle$	state with energy ω
$\mathsf{x}(\tau)$	detector worldline, τ proper time

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(Unruh-DeWitt)

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Interaction: one of

$$H^{(0)}_{\text{int}}(\tau) = c\chi(\tau)\mu(\tau)\phi\big(\mathsf{x}(\tau)\big) \qquad \longleftarrow \text{usual UDW}$$

$$H^{(1)}_{\text{int}}(\tau) = c\chi(\tau)\mu(\tau)\frac{\mathrm{d}}{\mathrm{d}\tau}\phi\big(\mathsf{x}(\tau)\big) \qquad \longleftarrow \text{derivative-coupling}$$

c	coupling constant
χ	switching function, C_0^∞
μ	detector's monopole moment operator

Probability of transition

$$\|0\rangle\rangle \otimes |0\rangle \longrightarrow \|1\rangle\rangle \otimes |\text{anything}\rangle$$

in first-order perturbation theory:

$$P(\omega) = c^2 \underbrace{\left|\langle\langle 0\|\mu(0)\|1\rangle\rangle\right|^2}_{\substack{\text{detector internals only:}\\ \text{drop!}}} \times \underbrace{F(\omega)}_{\substack{\text{trajectory and } |0\rangle\text{:}\\ \text{response function}}}$$

$$F^{(0)}(\omega) = \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau''\, e^{-i\omega(\tau'-\tau'')}\, \chi(\tau')\chi(\tau'')\, W(\tau', \tau'')$$

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$$W(\tau', \tau'') = \langle 0|\phi\big(x(\tau')\big)\phi\big(x(\tau'')\big)|0\rangle \qquad \text{Wightman function}$$

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$$F^{(0)}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\tau' \int_{-\infty}^{\infty} \mathrm{d}\tau''\, \mathrm{e}^{-i\omega(\tau'-\tau'')}\, \chi(\tau')\chi(\tau'')\, W(\tau',\tau'')$$

$$F^{(1)}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\tau' \int_{-\infty}^{\infty} \mathrm{d}\tau''\, \mathrm{e}^{-i\omega(\tau'-\tau'')}\, \chi(\tau')\chi(\tau'')\, \partial_{\tau'}\partial_{\tau''} W(\tau',\tau'')$$

$$W(\tau',\tau'') = \langle 0|\phi\big(\mathsf{x}(\tau')\big)\phi\big(\mathsf{x}(\tau'')\big)|0\rangle$$

Wightman function

(distribution!)