Information-theoretic Planck scale cutoff: Predictions for the CMB

Achim Kempf

With: A. Chatwin-Davies (CalTech), R. Martin (U. Cape Town)

Departments of Applied Mathematics and Physics

Institute for Quantum Computing, University of Waterloo

RQI-N Conference, YITP, Kyoto, 6 July 2017

Overview

- Planck length => finite information density, finite bandwidth?
- How to maintain covariance?
- Experimental tests?
- New results in inflationary cosmology: we're lucky!

QM + GR

➔

Planck length + finite info density?

Increase position resolution,

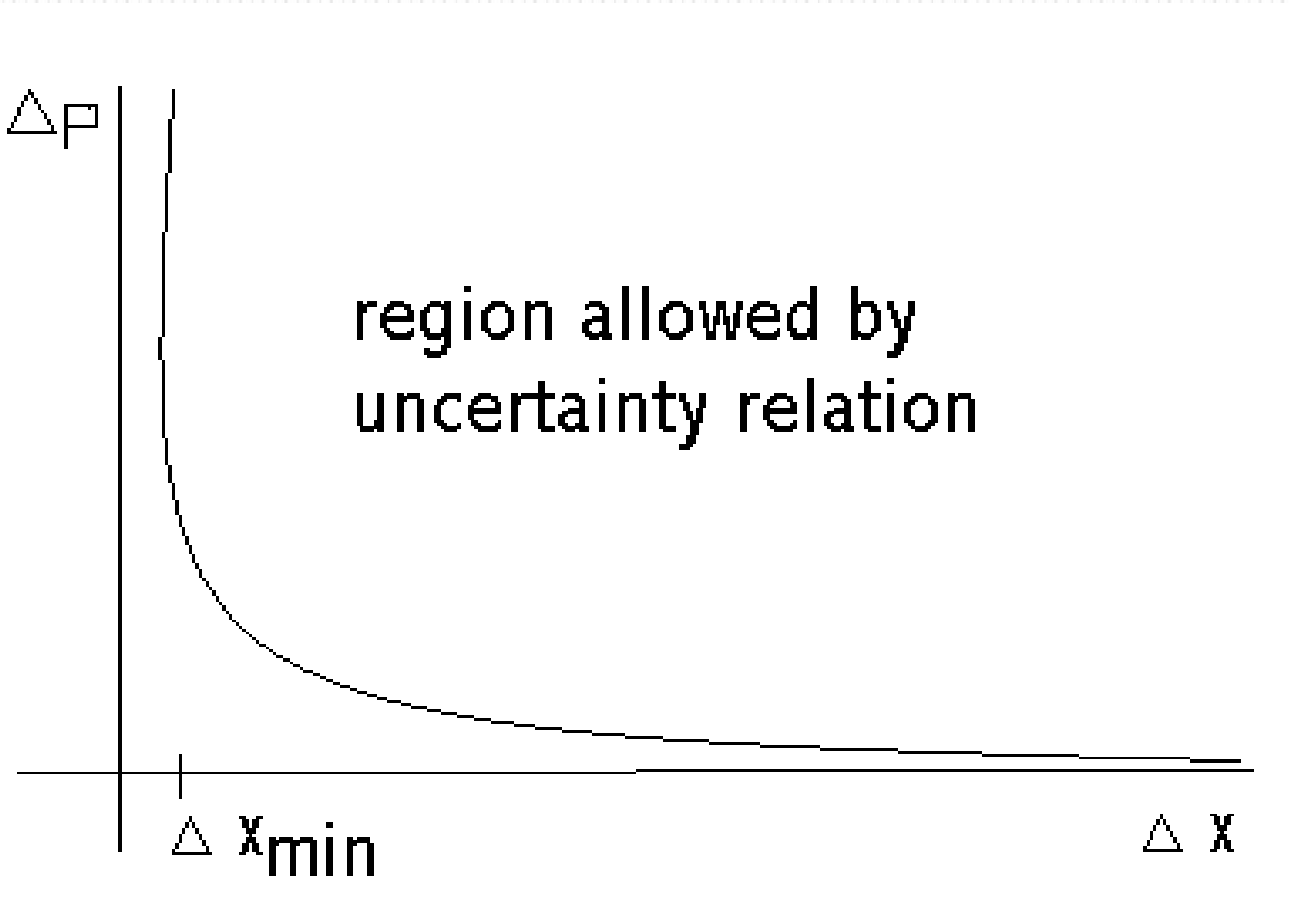
=> momentum / energy uncertainty increases

=> mass / curvature uncertainty increases

=> distance uncertainty increases

➔ **Cannot resolve distances below** 10^(-35)m.

$$\Delta x_{min} = L_{Planck}$$



Information-theoretic meaning?

- Wave function have a <u>finite bandwidth</u>:

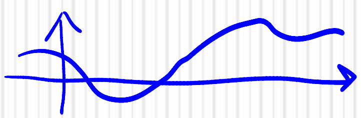
$$\Psi(x) = \int_{-\sigma_{\max}}^{\sigma_{\max}} \tilde{\Psi}(\sigma)\, e^{-2\pi i \sigma x} d\sigma$$

- **Intuition:**
  If there were arbitrarily short wavelengths, δ(x-x') could be obtained, violating the new uncertainty principle.

Role of bandlimitation in information theory?

Central!

Information can be:

- discrete (letters, digits, etc): R 7 2 S B
- continuous (e.g., music): 

Unified in 1949 by Shannon, for bandlimited signals.

Shannon sampling theorem

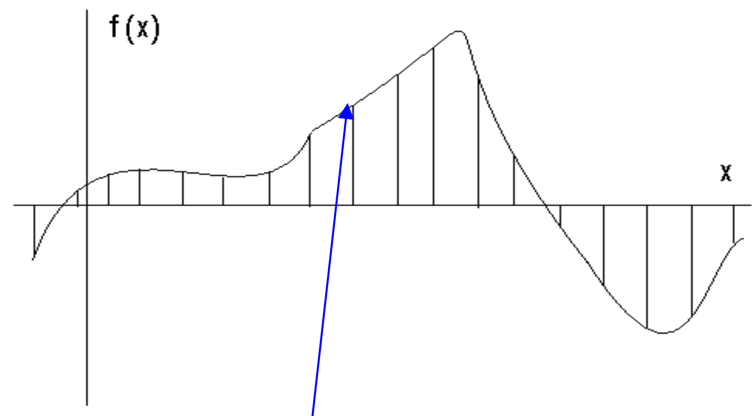
- Assume f is bandlimited, i.e:

$$f(t) = \int_{-\omega_{\max}}^{\omega_{\max}} \tilde{f}(\omega)\, e^{-2\pi i \omega t}\, d\omega$$

- Take samples of f(t) at Nyquist rate:

$$t_{n+1} - t_n = (2\omega_{\max})^{-1}$$

- **Then, exact reconstruction is possible:**



samples

$$f(t) = \sum_n f(t_n)\, \frac{\sin[\,2\pi\,(t - t_n)\,\omega_{\max}\,]}{\pi\,(t - t_n)\,\omega_{\max}}$$

It is one of the most used theorems:

- analog/digital conversion
- communication engineering & signal processing
- scientific data taking, e.g., in astronomy.

Properties of bandlimited functions

- **Differential operators are also finite difference operators.**
- **Differential equations are also finite difference equations.**
- **Integrals are also series:**

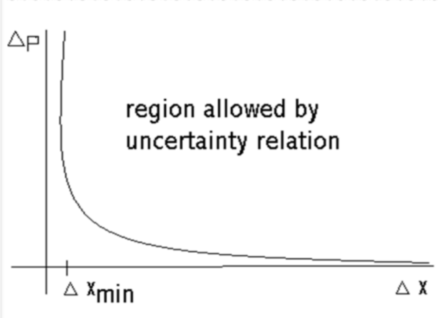
$$\int_{-\infty}^{\infty} f(x)^* g(x)\, dx = \frac{1}{2\omega_{\max}} \sum_{n=-\infty}^{\infty} f(x_n)^* g(x_n)$$

Remark:

Useful also as a summation tool for series

(traditionally used, e.g., in analytic number theory)

What if physical fields are bandlimited?



They possess equivalent representations

- **on a differentiable spacetime manifold**

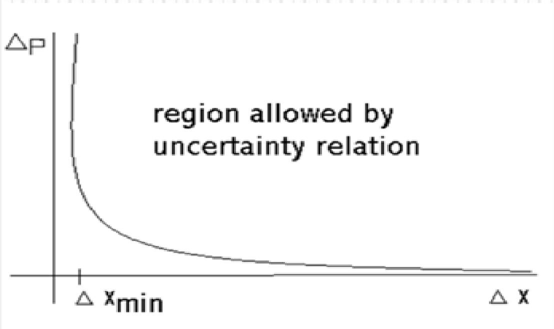
  (which shows preservation of external symmetries)

- **on any lattice of sufficiently dense spacing**

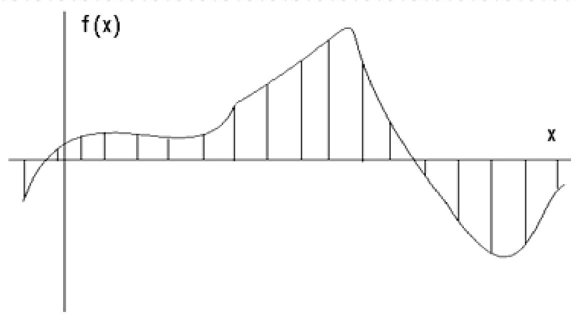
  (which shows UV finiteness of QFTs).

Conclusions so far:

QM + QFT :



=> Fields are bandlimited :



Spacetime can be simultaneously continuous and discrete in the same way that information can.

But this is not covariant!

Lorentz contraction and time dilation:

- How could a minimum length or time ever be covariant?
- How could a bandwidth in space or time ever be covariant?

Are we back to square one?

Recall GR + QM:

Use scattering experiments to resolve distances more and more precisely.

=> momentum / energy fluctuations increase

=> mass fluctuations increase

=> curvature fluctuations increase

=> distance uncertainty increases

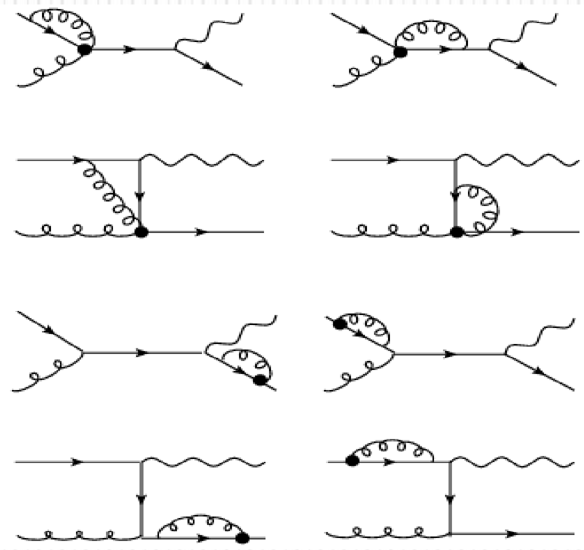
=> **expect that cannot resolve distances below** 10^(-35)m.

QFT + GR

=>

Planck length + info cutoff ?

- Feynman graphs with loops:



- Virtual particles can be arbitrarily far off shell:
  $(p_0)^2 - (\mathbf{p})^2$ can take any value!

- Do virtual particle masses beyond the Planck mass really exist ?

- **Can field fluctuations really be arbitrarily far off shell ?**

Covariant UV cutoff

Cut off spectrum of the d'Alembertian:

$$Z[J] = \int_F e^{iS[\phi]+i\int J\phi \, d^n x} D[\phi]$$

Here, the space of fields, F, is spanned by the eigenfunctions of the d'Alembertian w. eigenvalues:

$$|\,(p_0)^2 - (\mathbf{p})^2\,| < \Lambda_{\mathbf{Planck}}$$

This generalizes covariantly to curved spacetimes.

Relation to spacetime structure?

We cut off extreme virtual masses, i.e., off-shell fluctuations.

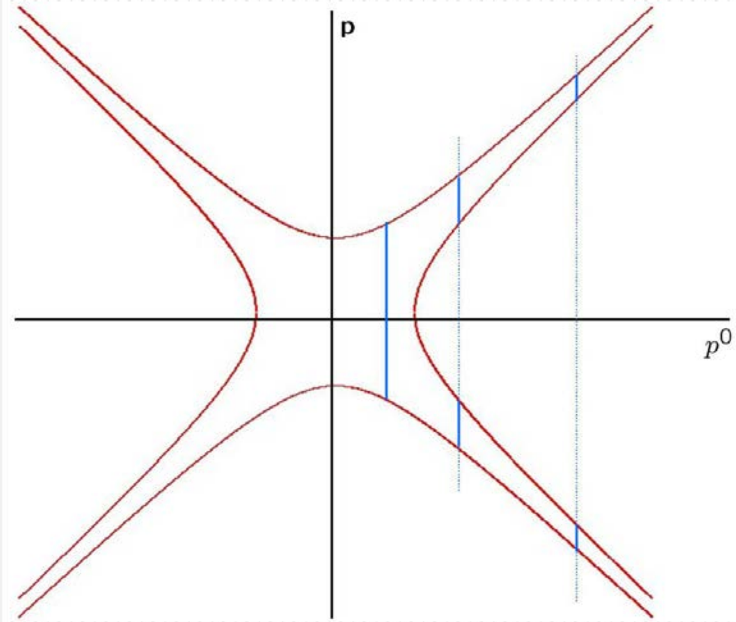
- Does this imply a minimum length or wavelength?
- Does it imply a spatial or temporal bandwidth?

Covariant cutoff

E.g. in flat spacetime:

$$|p_0^2 - \vec{p}^2| < \Lambda^2$$

No overall bandlimitation!

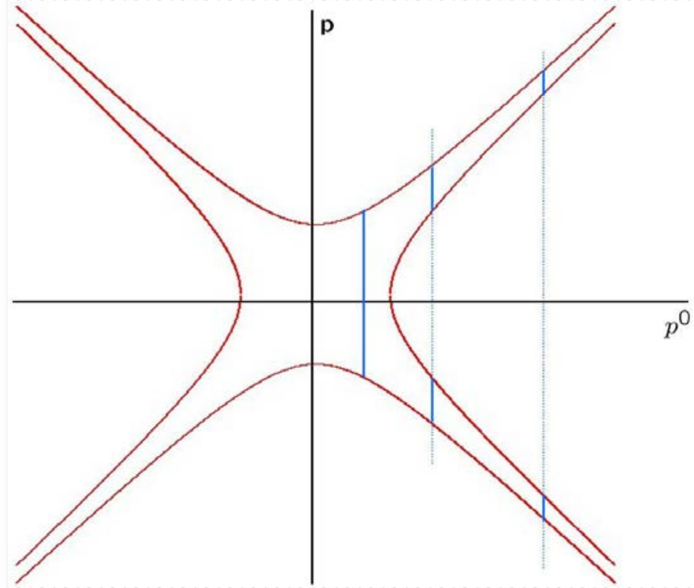


- Every spatial mode (fixed **p**) has a sampling theorem in time.
- Every temporal mode (fixed p_0) has a sampling thm. in space.

Covariant cutoff

E.g. in flat spacetime:

$$|p_0^2 - \vec{p}^2| < \Lambda^2$$



- Sub-Planck wavelengths exist but have negligible bandwidth!
- Sub-Planckian wavelengths freeze out!
- Wavelengths and bandwidths transform together, covariantly!

Conclusions so far

- QM+GR:

  $\Delta x_{min} = L_{Planck}$ and spatial bandlimitation

- QFT+GR:

  Planckian bound on virtual particles' masses

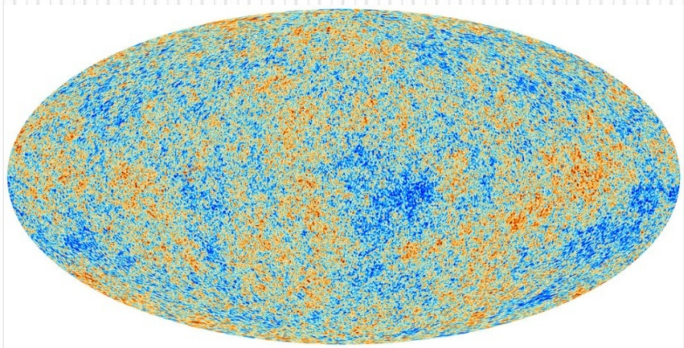
  Planckian bound on off-shell quantum field fluctuations

  Transplankian wavelengths exist but freeze out dynamically.

How could one experimentally test such a Planck scale cutoff?

Any signature visible in the CMB ?

CMB's structure originated close to Planck scale



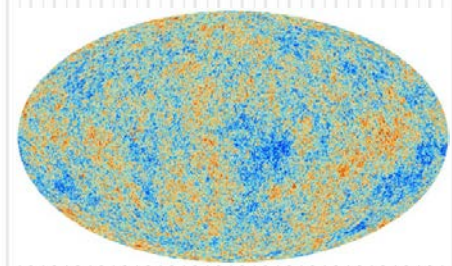
Hubble scale in inflation was likely only about 5 orders from the Planck scale.

Natural UV cutoffs in inflation

Multiple groups have non-covariant predictions for CMB.

- No agreement, if the effect is first or second order in

  Planck length / Hubble length



- I.e., is the effect $O(10^{-5})$ or $O(10^{-10})$?

Problem: hard to separate symmetry breaking from cutoff

Calculate predictions with locally Lorentz covariant UV cutoff !

Calculation of signature in the CMB

1. Calculate the projector onto covariantly bandlimited fields.

2. Apply projector to the Feynman rules (Feynman propagator).

3. Evaluate propagator at equal time, at horizon crossing.

➔primordial fluctuation spectrum ➔ CMB spectrum

New perspectives

Need the projector onto covariantly bandlimited fields.

➔Need to diagonalize the d'Alembertian.

Technical challenges:

* Families of self-adjoint extensions
* Kernel of d'Alembertian non-vanishing
* Propagator is non-self-adjoint ambiguous right inverse

Offers new perspective on:

* Big bang initial conditions
* Identification of the vacuum state

Numerical challenge

Need the projector onto covariantly bandlimited fields.

➔ Need inner product of eigenfunctions of d'Alembertian.

Computationally hard problem:

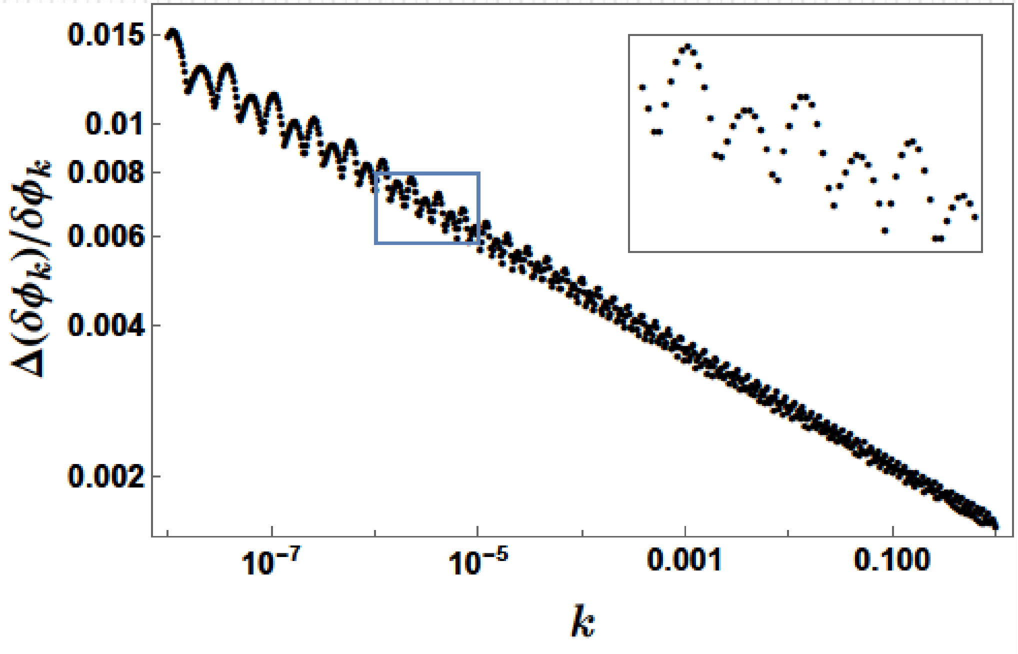
Similar to calculating the inner product of two plane waves numerically.

$$\int_{-\infty}^{\infty} e^{itw} e^{-itw'} \, dt = 2\pi\delta(w - w')$$

- Here, not plane waves but at best hypergeometric functions.

Results for the covariant UV cutoff

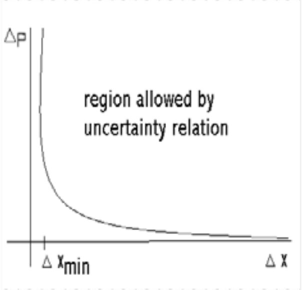
Predicted relative change in CMB spectrum (power law inflation):



The predicted oscillations' amplitude is linear in (Planck length/Hubble length)!

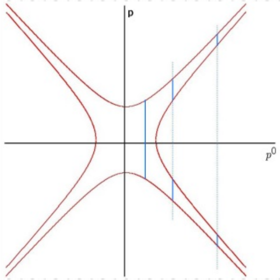
Conclusions

- QM+GR:



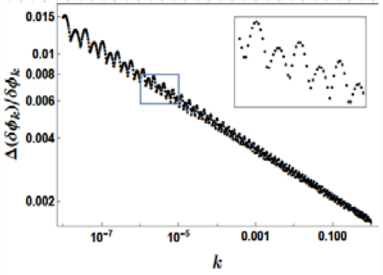
$\Delta x_{min} = L_{Planck}$ and spatial bandlimitation.

- QFT+GR:



Transplankian wavelengths: vanishing bandwidth.

- In inflationary cosmology:



Predict oscillatory 10^{-5} effect in the CMB.

Outlook

Impact of covariant UV cutoff on:

- Hawking radiation?
- Proton decay?

Minkowski space:

- Impact on the equal time fluctuation spectrum in 3+1 dim: