原子核物理における
チャネル結合法と核反応研究

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Self-introduction

Current position

- Postdoctoral Fellow at Nuclear Data Center, Japan Atomic Energy Agency (JAEA)
  → Istituto Nazionale di Fisica Nucleare (INFN) Postdoctoral Fellow at Napoli
  (from September 2016)

Ph.D (March 2015)

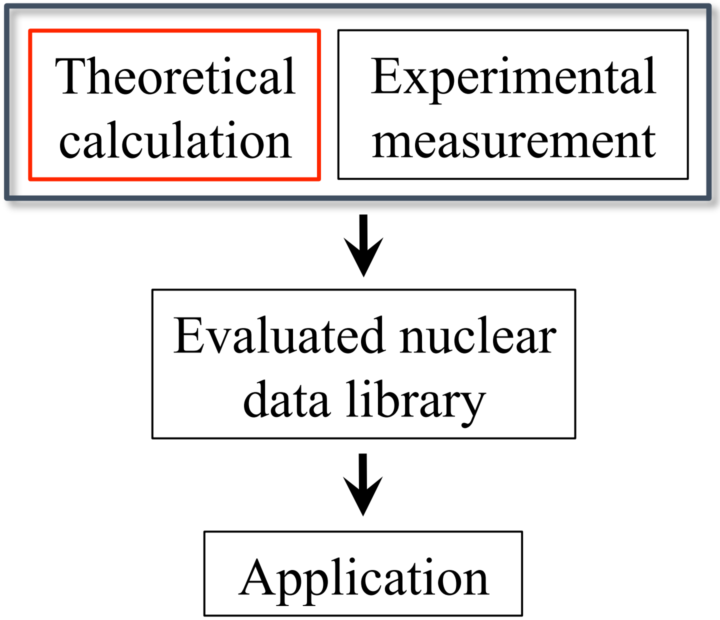
- Graduate School of Science, Osaka University
  (in actual Research Center for Nuclear Physics (RCNP), Osaka University)

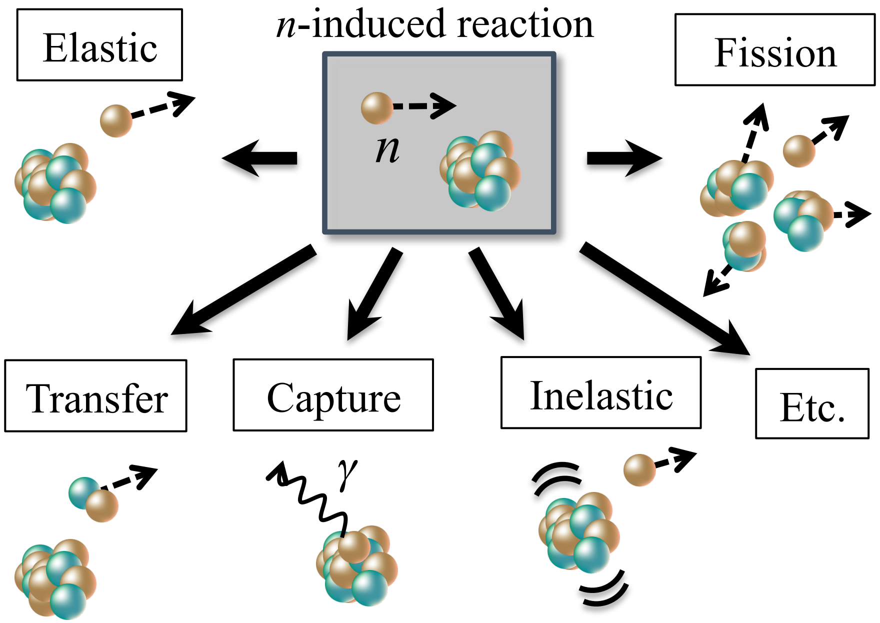
How to become postdoc (D3)

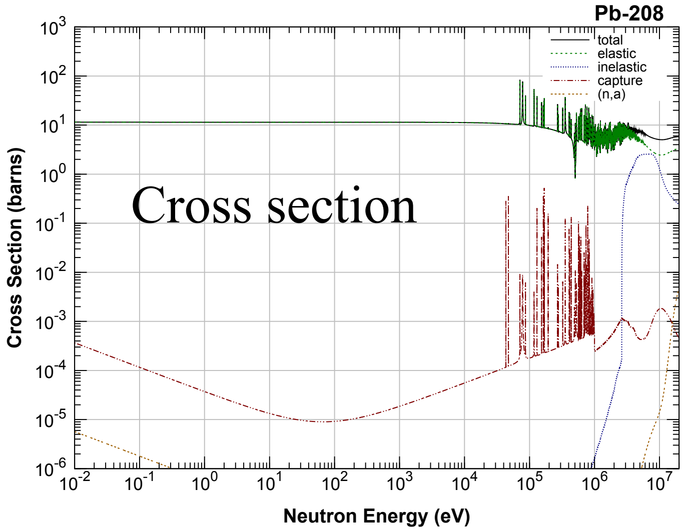
- Regular
  ✗ JSPS Research Fellowship for Young Scientists (学振特別研究員(PD))
  ✗ JSPS Postdoctoral Fellowship for Research Abroad (学振海外特別研究員)
  ✗ RIKEN SPDR (理研基礎特研)
  ✗ TRIUMF Postdoctoral Research Fellow
  ✓ JAEA Postdoctoral Fellow (原子力機構博士研究員)
- Irregular
  ✗ CEA-Saclay Postdoctoral position

Self-introduction

Nuclear Data







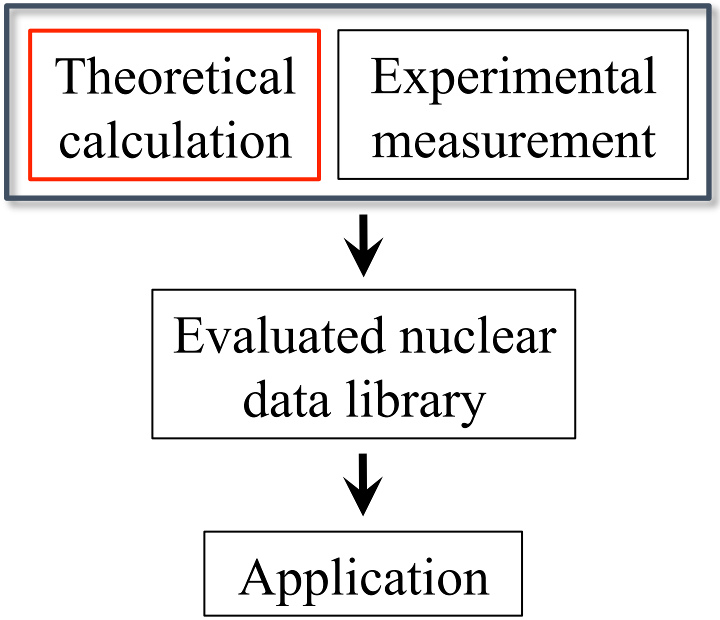
Systematically and globally

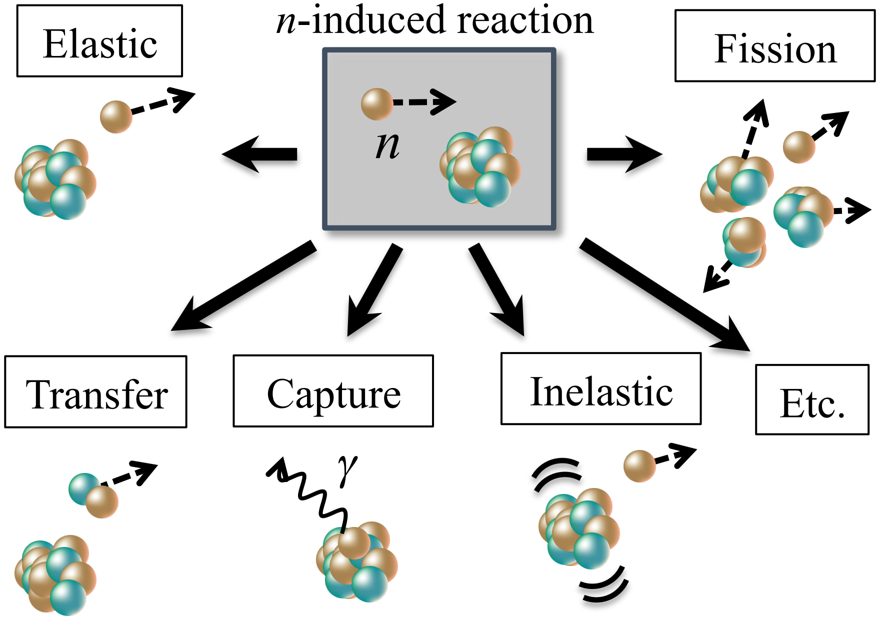
- Nuclear level density
- Mass
- Spin and parity
- Deformation parameter
- *β*-decay data

⋮

Self-introduction

Nuclear Data





Systematically and globally

- Nuclear level density
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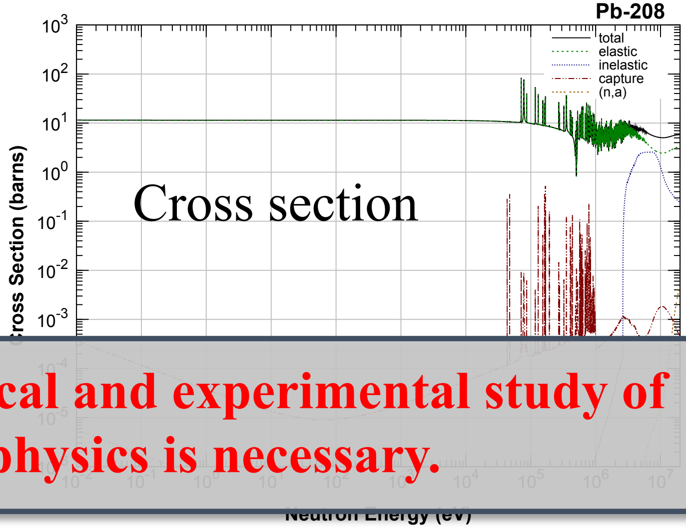


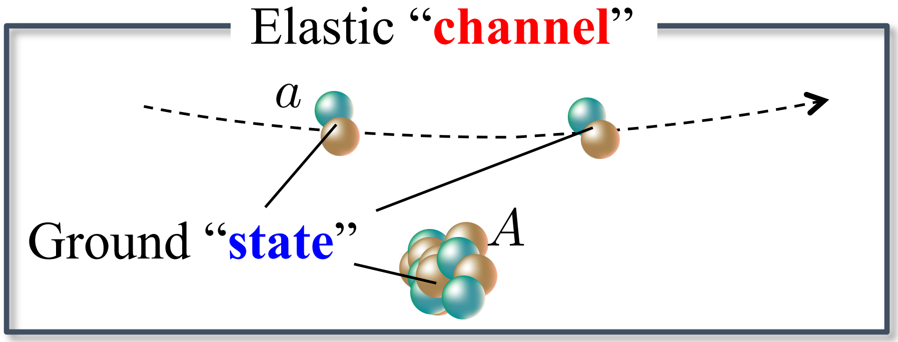
Theoretical and experimental study of nuclear physics is necessary.

1. Coupled channels method

Introduction

“Channel” (terminology)

- e.g. Elastic scattering



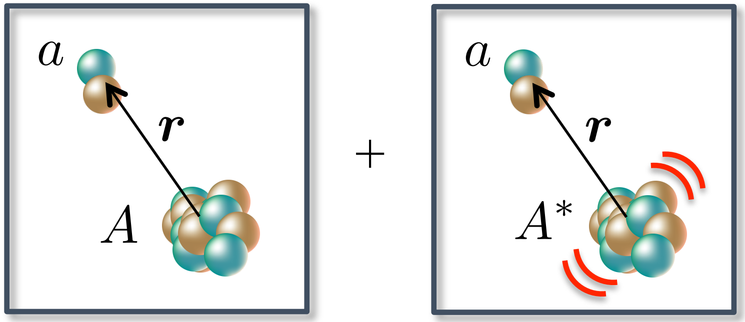
“**Channels**” are classified by **quantum states of nuclei** and **quantum numbers of relative motion** (momenta and angular momenta).

Coupled channels (CC)

- e.g. Inelastic scattering

Elastic channel

Inelastic channel



$\chi_{\mathrm{EL}}(\boldsymbol{r})\,|aA\rangle$ $\qquad$ $\chi_{\mathrm{IE}}(\boldsymbol{r})\,|aA^*\rangle$

Total WF expressed by **superposition**

$$\Psi = \chi_{\mathrm{EL}}(\boldsymbol{r})\,|aA\rangle + \chi_{\mathrm{IE}}(\boldsymbol{r})\,|aA^*\rangle$$

Coupled-channels equations

$$[h + K + V - E]\Psi = 0$$

$$\begin{array}{l} \langle aA| \\ \langle aA^*| \end{array} \quad \begin{cases} [K + \langle aA\,|V|\,aA\rangle - E_{\mathrm{EL}}]\chi_{\mathrm{EL}}(\boldsymbol{r}) = -\,\langle aA\,|V|\,aA^*\rangle\,\chi_{\mathrm{IE}}(\boldsymbol{r}) \\ [K + \langle aA^*\,|V|\,aA^*\rangle - E_{\mathrm{IE}}]\chi_{\mathrm{IE}}(\boldsymbol{r}) = -\,\langle aA^*\,|V|\,aA\rangle\,\chi_{\mathrm{EL}}(\boldsymbol{r}) \end{cases}$$

Introduction

Matrix representation

$$\begin{cases} [K + \langle aA\,|V|\,aA\rangle - E_{\mathrm{EL}}]\chi_{\mathrm{EL}}(\boldsymbol{r}) = -\boxed{\langle aA\,|V|\,aA^*\rangle}\chi_{\mathrm{IE}}(\boldsymbol{r}) \\ [K + \langle aA^*\,|V|\,aA^*\rangle - E_{\mathrm{IE}}]\chi_{\mathrm{IE}}(\boldsymbol{r}) = -\boxed{\langle aA^*\,|V|\,aA\rangle}\chi_{\mathrm{EL}}(\boldsymbol{r}) \end{cases}$$

$$\Downarrow$$

$$\begin{pmatrix} K + \langle aA\,|V|\,aA\rangle - E_{\mathrm{EL}} & \boxed{\langle aA\,|V|\,aA^*\rangle} \\ \boxed{\langle aA^*\,|V|\,aA\rangle} & K + \langle aA^*\,|V|\,aA^*\rangle - E_{\mathrm{IE}} \end{pmatrix} \begin{pmatrix} \chi_{\mathrm{EL}} \\ \chi_{\mathrm{IE}} \end{pmatrix} = 0$$

The off-diagonal components connect both channels.

- How to solve coupled differential equations

Numerically

Modified Numerov method,
Euler’s method,
Störmer’s 6-point method,
Iteration, etc.

M. A. Melkanoff *et al.*, Methods in Computational Physics, Vol. 6 (1966), Academic Press (New York).

Matrix representation

$$\left\{\begin{array}{l}[K + \langle aA\,|V|\,aA\rangle - E_{\mathrm{EL}}]\chi_{\mathrm{EL}}(\boldsymbol{r}) = -\boxed{\langle aA\,|V|\,aA^*\rangle}\,\chi_{\mathrm{IE}}(\boldsymbol{r}) \\ [K + \langle aA^*\,|V|\,aA^*\rangle - E_{\mathrm{IE}}]\chi_{\mathrm{IE}}(\boldsymbol{r}) = -\boxed{\langle aA^*\,|V|\,aA\rangle}\,\chi_{\mathrm{EL}}(\boldsymbol{r})\end{array}\right.$$

$$\Downarrow$$

$$\begin{pmatrix} K + \langle aA\,|V|\,aA\rangle - E_{\mathrm{EL}} & \boxed{\langle aA\,|V|\,aA^*\rangle} \\ \boxed{\langle aA^*\,|V|\,aA\rangle} & K + \langle aA^*\,|V|\,aA^*\rangle - E_{\mathrm{IE}} \end{pmatrix}\begin{pmatrix}\chi_{\mathrm{EL}} \\ \chi_{\mathrm{IE}}\end{pmatrix} = 0$$

The off-diagonal components connect both channels.

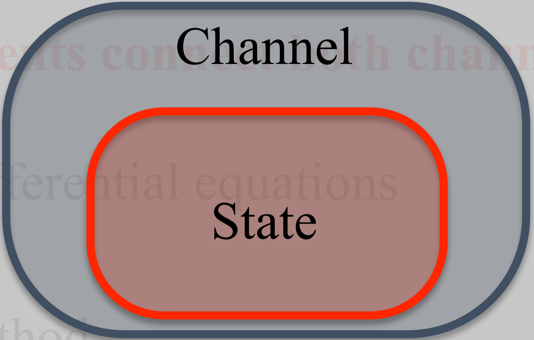
- How to solve coupled differential equations

Numerically

Modified Numerov method,
Euler's method,
Störmer's 6-point method,
Iteration, etc.

- Can the CC method be applied only for **reaction studies**?
(Because "channel" is related to reactions…)

→ **No**. Its essence and technique are same as in **structural studies**.



M. A. Melkanoff *et al.*, Methods in Computational Physics, Vol. 6 (1966), Academic Press (New York).

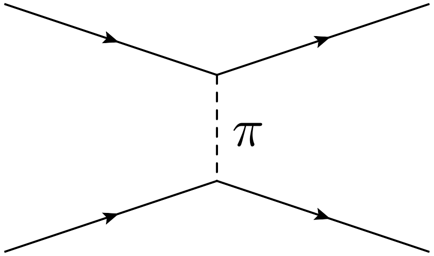
Examples

Deuteron

- One-pion exchange potential (OPEP)

$$V_\pi = f\left(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right)\left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) S_{12}\right] \frac{e^{-\mu r}}{\mu r},$$

$$S_{12} = \frac{3}{r^2}\left(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r}\right)\left(\boldsymbol{\sigma}_2 \cdot \boldsymbol{r}\right) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$



Tensor term gives Δ*l* = 2 component.

- D-state (*l*=2) admixture due to tensor force

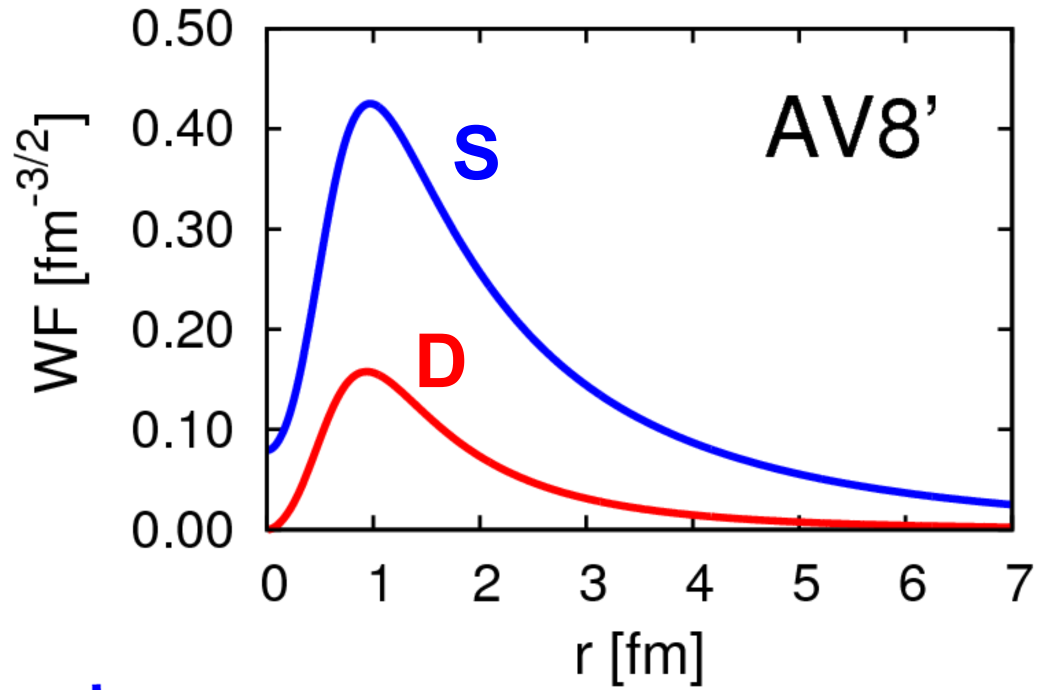
$$\begin{pmatrix} K + V_{00} - \varepsilon & \overset{\textbf{Tensor}}{\boxed{V_{02}}} \\ \underset{\textbf{Tensor}}{\boxed{V_{20}}} & K + V_{22} - \varepsilon \end{pmatrix} \begin{pmatrix} u_0 \\ u_2 \end{pmatrix} = 0$$

V_{00} : Central

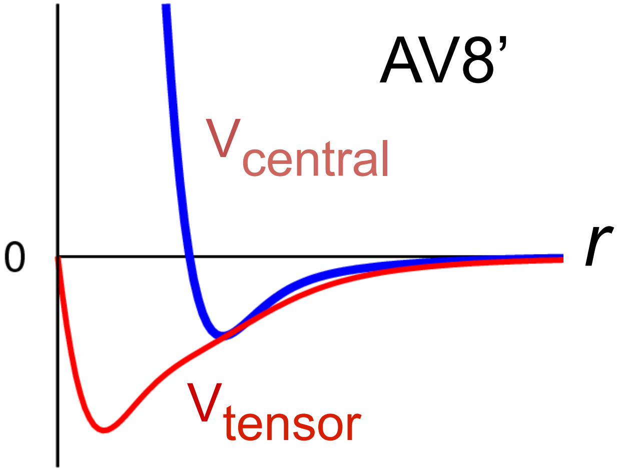
V_{22} : Central + spin-orbit + tensor + quadratic spin-orbit (l^2)

Examples

- Calculation by T. Myo



Energy	-2.24 MeV
Kinetic	19.88
Central	-4.46
Tensor	**-16.64**
LS	-1.02
P(L=2)	5.77%
Radius	1.96 fm



$R_m(s)$=2.00 fm

$R_m(d)$=1.22 fm

d-wave is **"spatially compact"** (high momentum)

Examples

Hartree-Fock (HF) method

- HF equation derived from variation principle

$$-\frac{\hbar^2}{2m}\nabla^2\varphi_i(\boldsymbol{r})+\sum_j\int d\boldsymbol{r}'v(|\boldsymbol{r}-\boldsymbol{r}'|)\varphi_j^*(\boldsymbol{r}')\varphi_j(\boldsymbol{r}')\varphi_i(\boldsymbol{r})$$

$$\underline{-\sum_j\int d\boldsymbol{r}'v(|\boldsymbol{r}-\boldsymbol{r}'|)\varphi_j^*(\boldsymbol{r}')\varphi_j(\boldsymbol{r})\varphi_i(\boldsymbol{r}')}=\varepsilon_i\varphi_i(\boldsymbol{r})$$

$$\rightarrow\left[-\frac{\hbar^2}{2m}\nabla^2+\sum_j\langle j|v|j\rangle-\varepsilon_i\right]\varphi_i(\boldsymbol{r})=\underline{\sum_j\langle j|v|i\rangle\varphi_j(\boldsymbol{r})}$$

Fock (exchange) term connects different s.p. states.

Hartree-Fock-Bogoliubov (HFB) method

- HF + Bogoliubov transformation to explicitly treat paring

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix}\begin{pmatrix} U_k \\ V_k \end{pmatrix}=E_k\begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

→ **Talk by Y. Kobayashi**

Paring potential connects different quasi-particle states.

Examples

Shell model

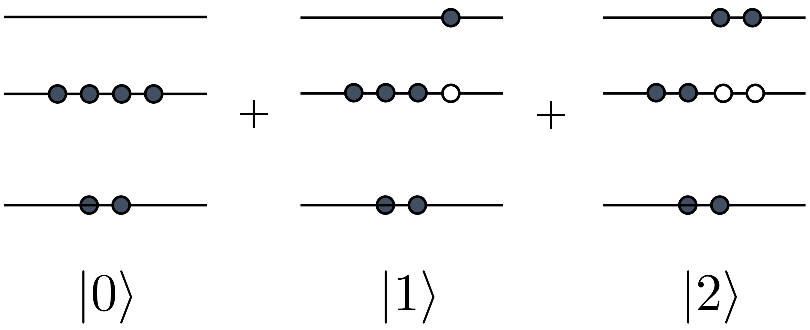
- Configuration mixing

$$|\Psi\rangle = c_0\,|0\rangle + c_1\,|1\rangle + \cdots = \sum_i c_i\,|i\rangle$$

$$H\,|\Psi\rangle = E\,|\Psi\rangle$$

$$\sum_j \langle i\,|H|\,j\rangle\,c_j = Ec_j$$

$$\rightarrow\quad [\langle i\,|H|\,i\rangle - E]\,c_i = -\sum_{j\neq i}\langle i\,|H|\,j\rangle\,c_j$$



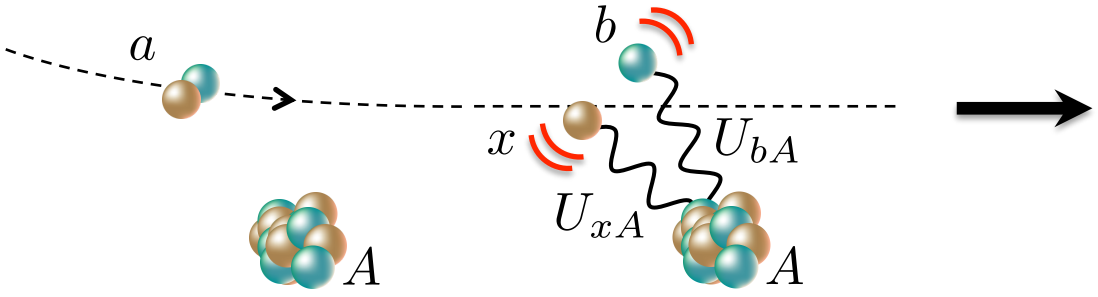
The off-diagonal matrix element provides configuration mixing.

- Superposition or basis expansion
- Matrix elements
  → **Admixture of states = CC**

2. Reaction theory based on the CC method

Reaction model (CDCC)

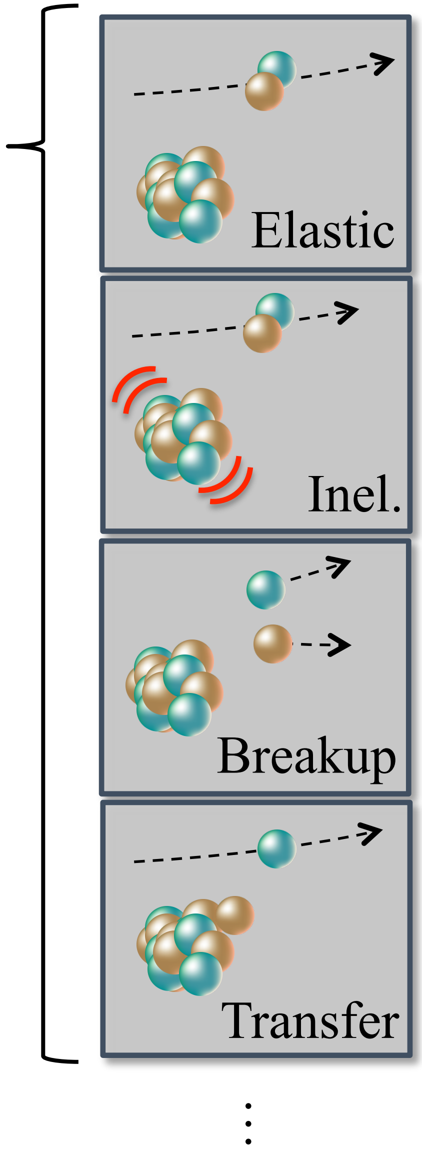
Excitation of projectile into continuum state



The *x*-*b* continuum state

||

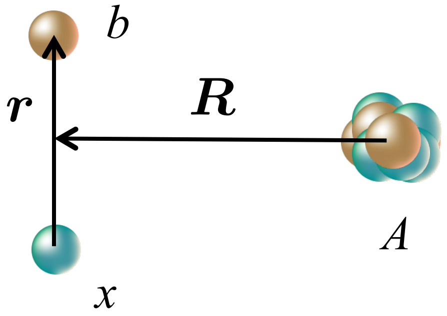
Breakup



Based on the $x + b + A$ three body model

Projectile can breakup in intermediate state
→ Superposition of **elastic** and **breakup** channels

$$\Psi^{(+)}(\boldsymbol{r},\boldsymbol{R}) = \underline{\psi_{xb}(k_0,\boldsymbol{r})\chi_{aA}(K_0,\boldsymbol{R})} + \underline{\int_0^\infty \psi_{xb}(k,\boldsymbol{r})\chi_{aA}(K,\boldsymbol{R})dk}$$



Reaction model (CDCC)

Continuum-discretized coupled-channels method (CDCC)

- How to treat breakup channels

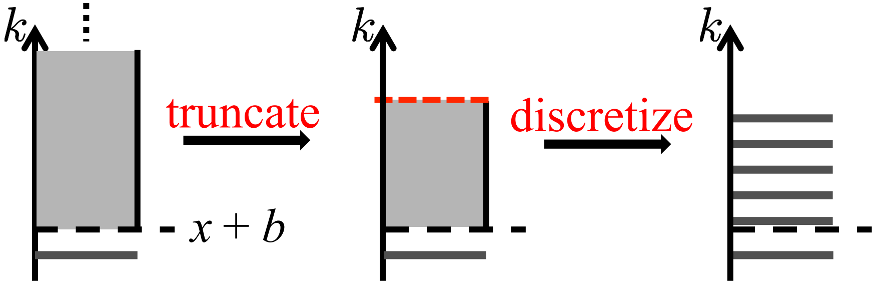
$$\Psi^{(+)}(\boldsymbol{r},\boldsymbol{R}) = \psi_{xb}(k_0,\boldsymbol{r})\chi_{aA}(K_0,\boldsymbol{R}) + \underbrace{\int_0^\infty \psi_{xb}(k,\boldsymbol{r})\chi_{aA}(K,\boldsymbol{R})dk}$$

Infinite number of continuum states

Truncation & discretization

$$\Psi^{(+)}(\boldsymbol{r},\boldsymbol{R}) \approx \sum_i \psi^i_{xb}(\boldsymbol{r})\chi^{ii_0}_{aA}(\boldsymbol{R})$$

M. Kamimura *et al.*, Prog. Theor. Phys. Suppl. No. 89, 1 (1986).
N. Austern *et al.*, Phys. Rep. **154**, 125 (1987) .
M. Yahiro *et al.*, Prog. Theor. Exp. Phys. **2012**, 01A209 (2012).



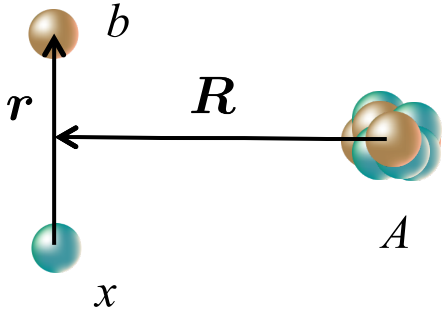
- CDCC equation

$$[h + K + U_{xA}(\boldsymbol{r},\boldsymbol{R}) + U_{bA}(\boldsymbol{r},\boldsymbol{R}) - E]\,\Psi^{(+)}(\boldsymbol{r},\boldsymbol{R}) = 0$$

$\langle\psi^i_{xb}|$

$$[K + U_{ii}(\boldsymbol{R}) - E_i]\,\chi^{ii_0}_{aA}(\boldsymbol{R}) = -\sum_{j\neq i} U_{ij}(\boldsymbol{R})\chi^{ji_0}_{aA}(\boldsymbol{R})$$

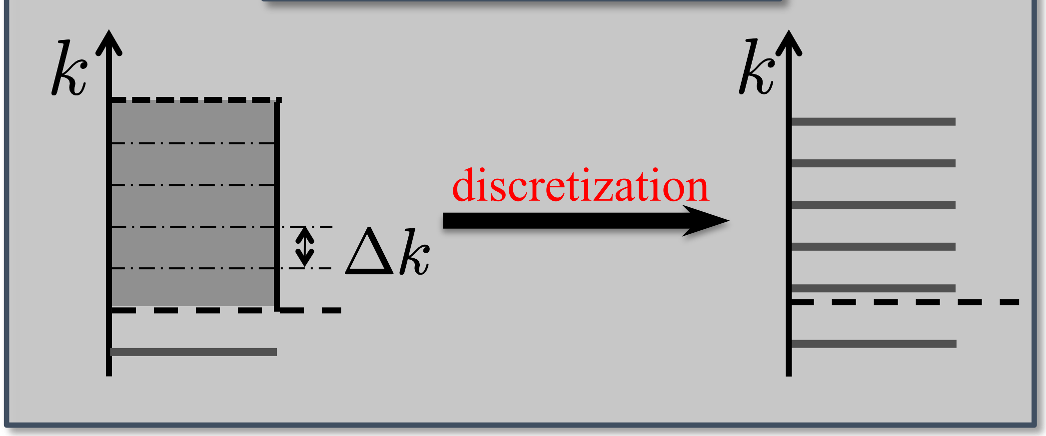
$$U_{ij}(\boldsymbol{R}) = \langle\psi^i_{xb}|\,U_{xA}(\boldsymbol{r},\boldsymbol{R}) + U_{bA}(\boldsymbol{r},\boldsymbol{R})\,|\psi^j_{xb}\rangle$$



Reaction model (CDCC)

How to discretize

bin (average) method



$$\hat{\psi}_{n\ell m}(\boldsymbol{r}) = \frac{1}{\sqrt{\Delta k_n}} \int_{k_{n-1}}^{k_n} \psi_{\ell m}(k, \boldsymbol{r}) dk$$

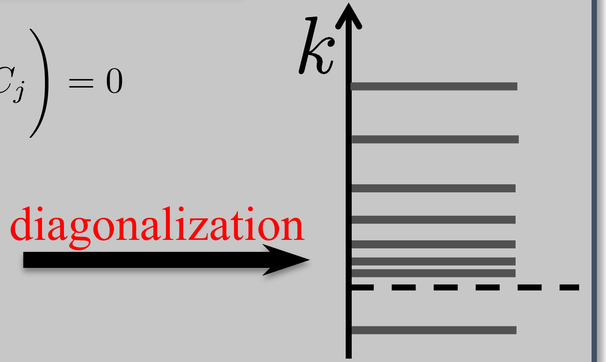
$$\hat{\varepsilon}_n = \frac{\hbar^2}{2\mu_r} \left[\frac{(k_n + k_{n-1})^2}{4} + \frac{(\Delta k_n)^2}{12} \right]$$

pseudo state

$$\left[\begin{pmatrix} H_{ij} \end{pmatrix} - \varepsilon_n \begin{pmatrix} N_{ij} \end{pmatrix} \right] \begin{pmatrix} C_j \end{pmatrix} = 0$$

$$H_{ij} = \left\langle \psi_i \middle| \hat{H} \middle| \psi_j \right\rangle$$

$$N_{ij} = \langle \psi_i | \psi_j \rangle$$



$$\psi_{\ell m}(\boldsymbol{r}) = \phi_\ell(r) i^\ell Y_{\ell m}(\hat{\boldsymbol{r}}),$$

$$\phi_\ell(r) = \sum_i^{i_{\max}} c_i \varphi_{\ell i}(r),$$

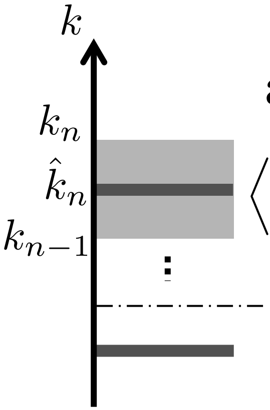
$$\varphi_{\ell i}(r) = N_i r^\ell \exp\left[-\left(\frac{r}{\rho_i} \right)^2 \right]$$

Reaction model (CDCC)

Equivalence of two methods for discretizationtion

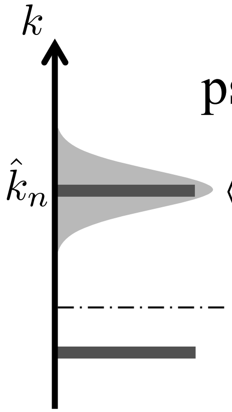
- Overlap with true scattering wave

average method



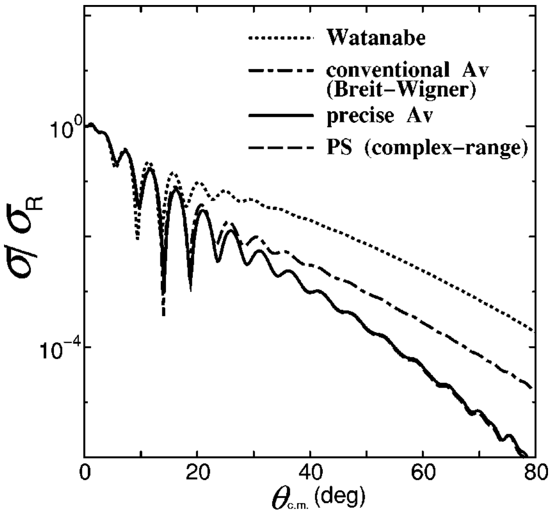
$$\left\langle \phi(k,\boldsymbol{r}) \,\middle|\, \hat{\phi}_n^{(\mathrm{av})}(\boldsymbol{r}) \right\rangle$$

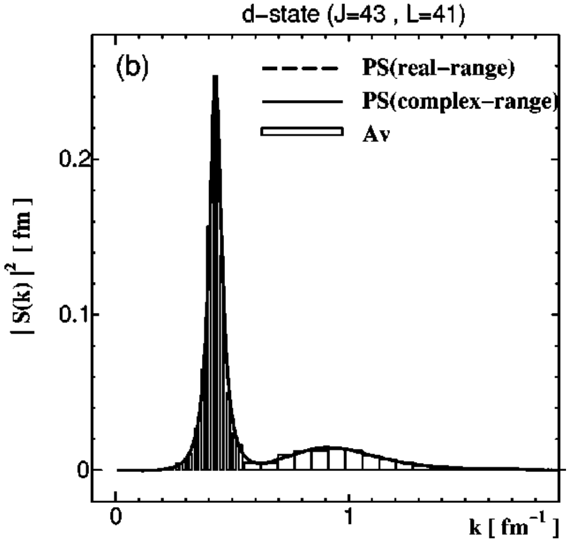
pseudostate method



$$\left\langle \phi(k,\boldsymbol{r}) \,\middle|\, \hat{\phi}_n^{(\mathrm{ps})}(\boldsymbol{r}) \right\rangle$$

- Observables (^{6}Li + ^{40}Ca at 156 MeV) T. Matsumoto *et al.,* Phys. Rev. C **68**, 064607 (2003).





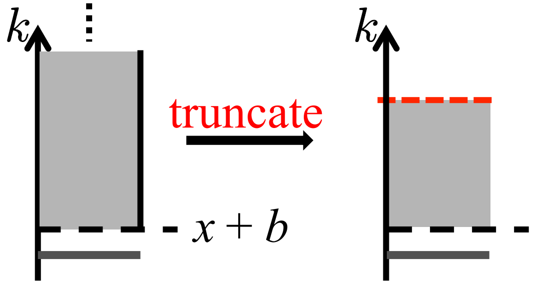
Reaction model (CDCC)

Truncation regarding momentum & angular momentum spaces

- Momentum truncation

$$\Psi_{\mathrm{br}}(\boldsymbol{r},\boldsymbol{R}) \equiv \int_0^\infty \psi_{xb}(k,\boldsymbol{r})\chi_{aA}(K,\boldsymbol{R})dk$$

$$\to \int_0^{k_{max}} \psi_{xb}(k,\boldsymbol{r})\chi_{aA}(K,\boldsymbol{R})$$



- Angular momentum truncation (Austern-Yahiro-Kawai theorem)

N. Austern *et al.*, Phys. Rev. Lett. **63**, 2649 (1989) .

CDCC with ang. mom. truncation

$$P = \int d\hat{\boldsymbol{r}} \sum_{l=0}^{l_m} \sum_m Y_{lm}(\hat{\boldsymbol{r}})\, Y^*_{lm}(\hat{\boldsymbol{r}})\,,$$

$$[E-K-V-PUP]\,\Psi^{\mathrm{CDCC}} = 0.$$

Distorted-Faddeev equations

$$[E-K-V-PUP]\,\hat{\Psi}_a = V\left(\hat{\Psi}_x + \hat{\Psi}_b\right),$$

$$[E-K-U_x-U_b]\left(\hat{\Psi}_x+\hat{\Psi}_b\right) = \underline{(U-PUP)}\,\hat{\Psi}_a.$$

Expected to be small

Ψ_{CDCC} can be a good approximation of $\hat{\Psi}_d$ if l_m is large enough.

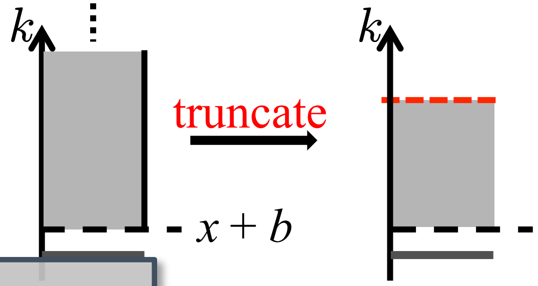
Reaction model (CDCC)

Truncation regarding momentum & angular momentum spaces

- Momentum truncation

$$\Psi_{\rm br}(\boldsymbol{r},\boldsymbol{R}) \equiv \int_0^{\infty} \psi_{xb}(k,\boldsymbol{r})\chi_{aA}(K,\boldsymbol{R})dk$$

$$\to \int_0^{k_{max}} \psi_{xb}(k,\boldsymbol{r})\chi_{aA}(K,\boldsymbol{R})$$



- Angular momentum truncation (Austern-Yahiro-Kawai theorem)

N. Austern *et al.*, Phys. Rev. Lett. **63**, 2649 (1989) .

Model space should be set so that observables we want to see can be described properly.

CDCC with ang. mom. truncation

$$P = \int d\hat{\boldsymbol{r}} \sum_{l=0}^{l_m} \sum_{m} Y_{lm}\left(\hat{\boldsymbol{r}}\right) Y_{lm}^{*}\left(\hat{\boldsymbol{r}}\right),$$

$$\left[E - K - V - PUP\right] \Psi^{\rm CDCC} = 0.$$

Distorted-Faddeev equations

$$\left[E - K - V - PUP\right] \hat{\Psi}_a = V\left(\hat{\Psi}_x + \hat{\Psi}_b\right),$$

$$\left[E - K - U_x - U_b\right]\left(\hat{\Psi}_x + \hat{\Psi}_b\right) = \underline{\left(U - PUP\right)}\,\hat{\Psi}_a.$$

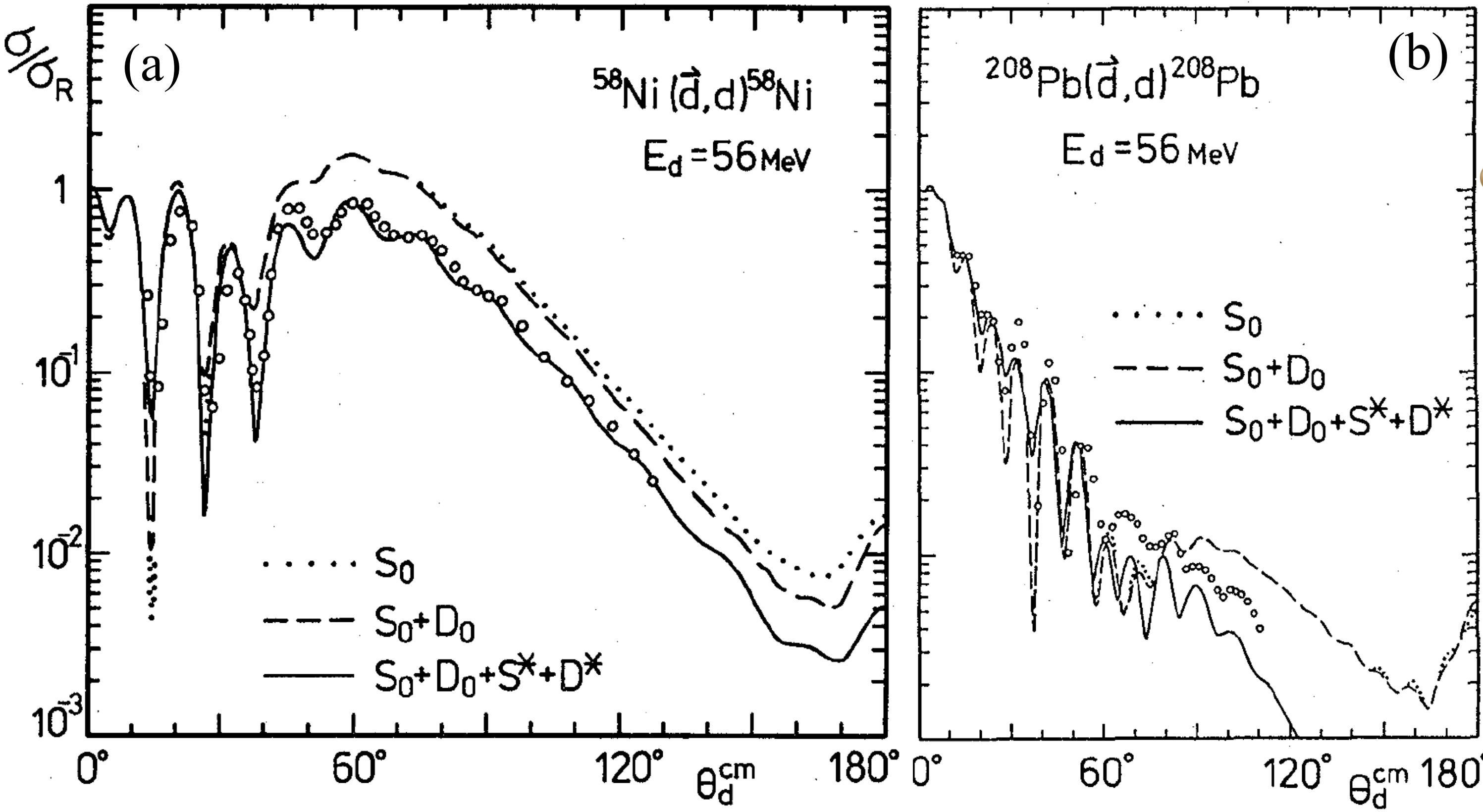
Expected to be small

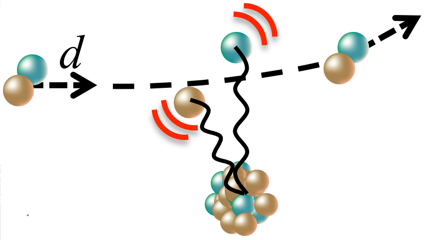
$\Psi_{\rm CDCC}$ can be a good approximation of $\hat{\Psi}_d$ if l_m is large enough.

Reaction model (CDCC)

Breakup effects on elastic scattering (1)

M. Kamimura et al., Prog. Theor. Phys. Suppl. **89**, 1 (1986).

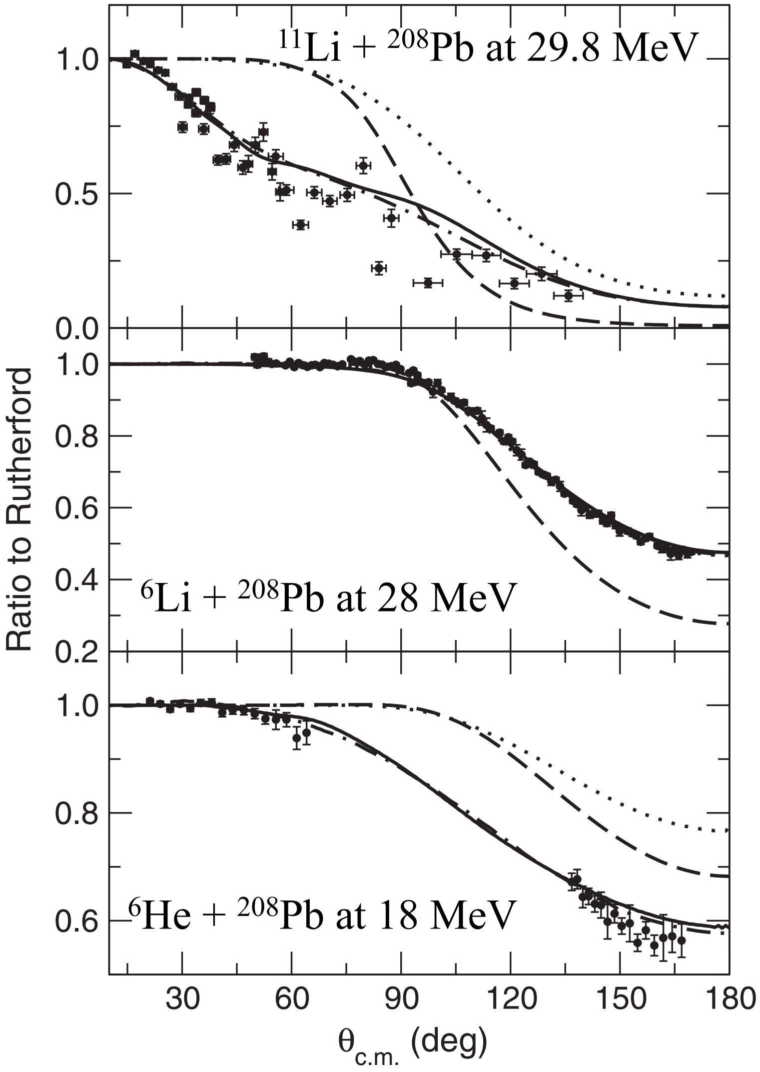




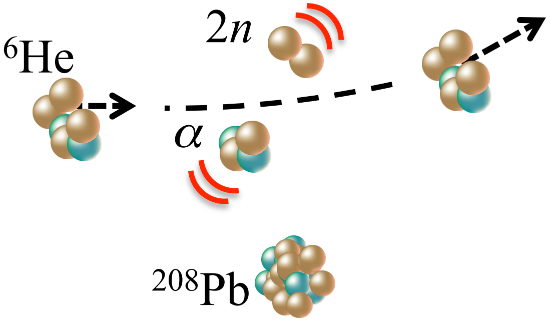
Breakup states of *d* is essential to reproduce experimental data.

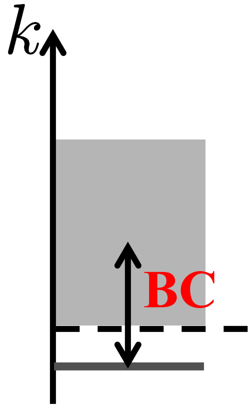
Reaction model (CDCC)

Breakup effects on elastic scattering (2)



N. Keeley, K. Kemper, and K. Rusek, Phys. Rev. C **88**, 017602 (2013).





It is necessary to include the continuum states of the projectiles to reproduce measured CS.

$$\chi_{\rm CDCC}(\boldsymbol{r}_i) = \underset{\textbf{Elastic}}{\underline{\chi_0(\boldsymbol{r}_i)}} + \underset{\textbf{Breakup}}{\underline{\chi_c(\boldsymbol{r}_i)}}$$

$$\begin{pmatrix} K_i + U_{00} - E_0 & U_{0c} \\ U_{c0} & K_i + U_{cc} - E_c \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_c \end{pmatrix} = 0$$

$$\sigma_{\rm EL} \propto \left|\chi_0(\boldsymbol{r}_{\rm asy})\right|^2 \quad (\boldsymbol{r}_{\rm asy} \gg \boldsymbol{r}_N)$$

The coupling back to the elastic channel (back coupling; BC) is essential.

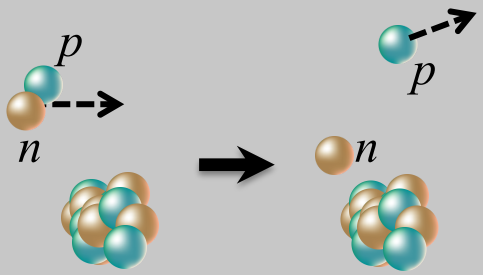
3. Transfer reaction with the CC method

Background

Physics through transfer reactions

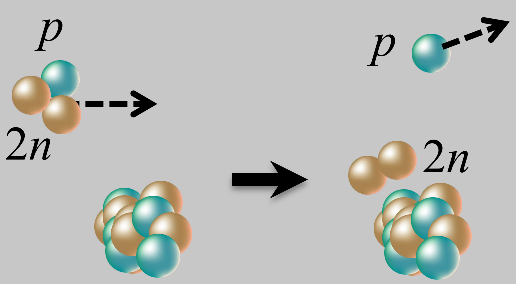
Transfer reaction: is sensitive to nuclear states in the initial and final channels.
useful to generate states selectively due to matching condition.
→ Probe single-particle structures

One-nucleon transfers
Ex. (d,p)



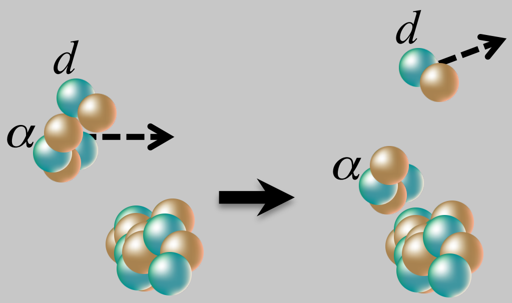
single-particle states

Two-nucleon transfers
Ex. (t,p)



nucleon-nucleon correlations

α transfers
Ex. (^{6}Li,d)



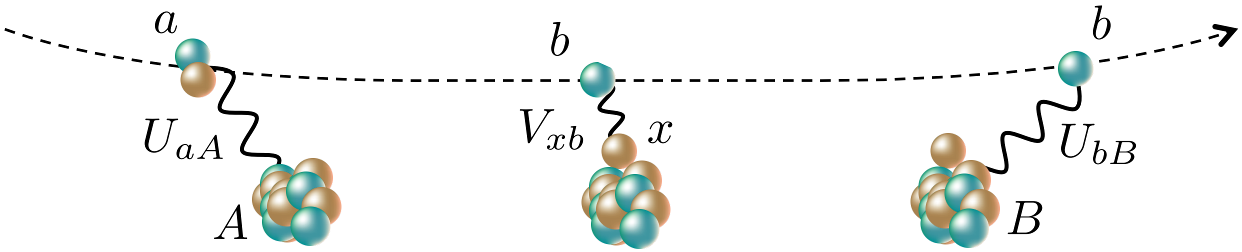
clustering

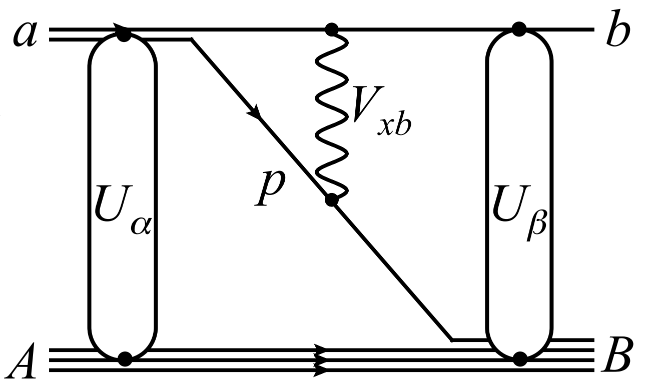
Background

Description of transfer reactions (conventional approach)

- The transition matrix for the $A(a, b)B$ reaction within the **distorted-wave Born approximation (DWBA)**.

$$T_{\mathrm{DWBA}} = \left\langle \Psi_{\beta}^{(-)} \middle| V_{xb} \middle| \Psi_{\alpha}^{(+)} \right\rangle$$



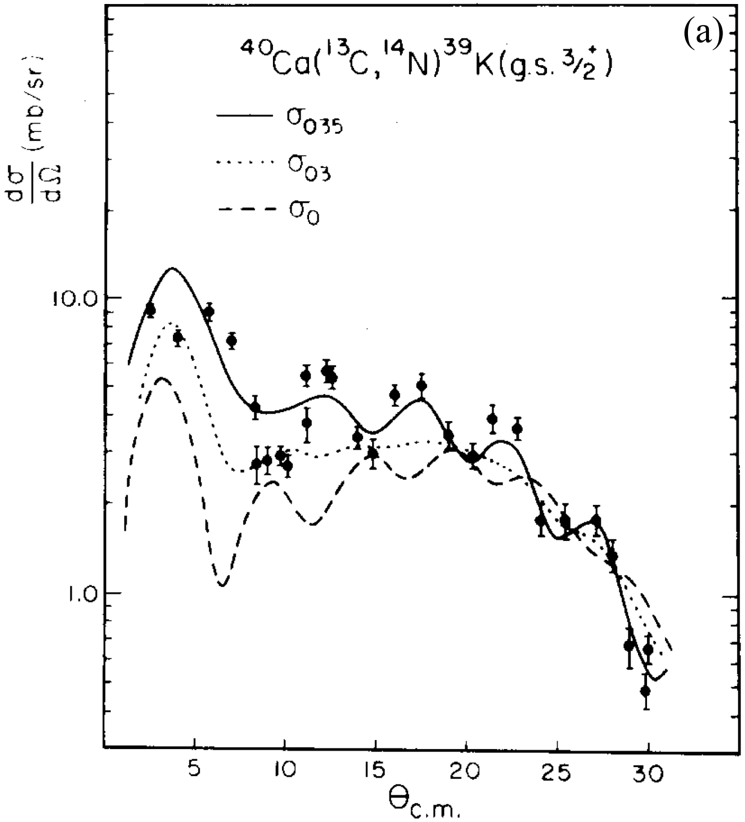


- **The optical potential** U_{aA} (U_{bB}) for the $a + A$ ($b + B$) **2-body system** generates the distorted wave.
- **One-step transition** induced by the residual interaction V_{xb} (V_{xA}) for the post (prior) form is assumed.

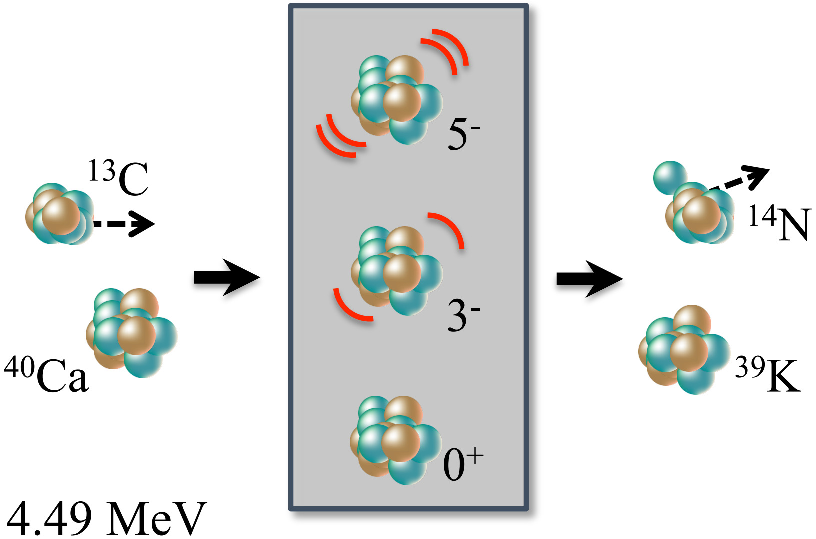
Beyond DWBA (CC on transfer reactions)

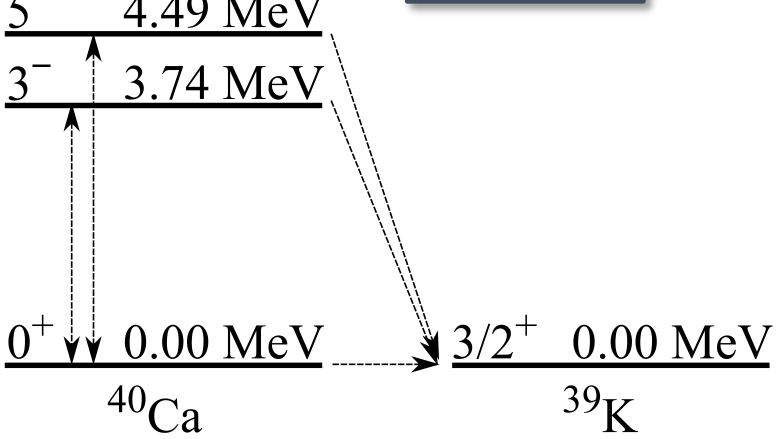
- To take into account channel-couplings due to the three-body dynamics, the **coupled-channels Born approximation (CCBA)** was proposed.

S. K. Penny and G. R. Satchler, Nucl. Phys. **53**, 145 (1964).
P. J. Iano and N. Austern, Phys. Rev. **151**, 853 (1966).



K. Low, T. Tamura, and T. Udagawa, Phys. Lett. **B67**, 5 (1977).





Background

Beyond DWBA (CC on transfer reactions)

- To take into account channel-couplings due to the three-body dynamics, the **coupled-channels Born approximation (CCBA)** was proposed.

S. K. Penny and G. R. Satchler, Nucl. Phys. **53**, 145 (1964).
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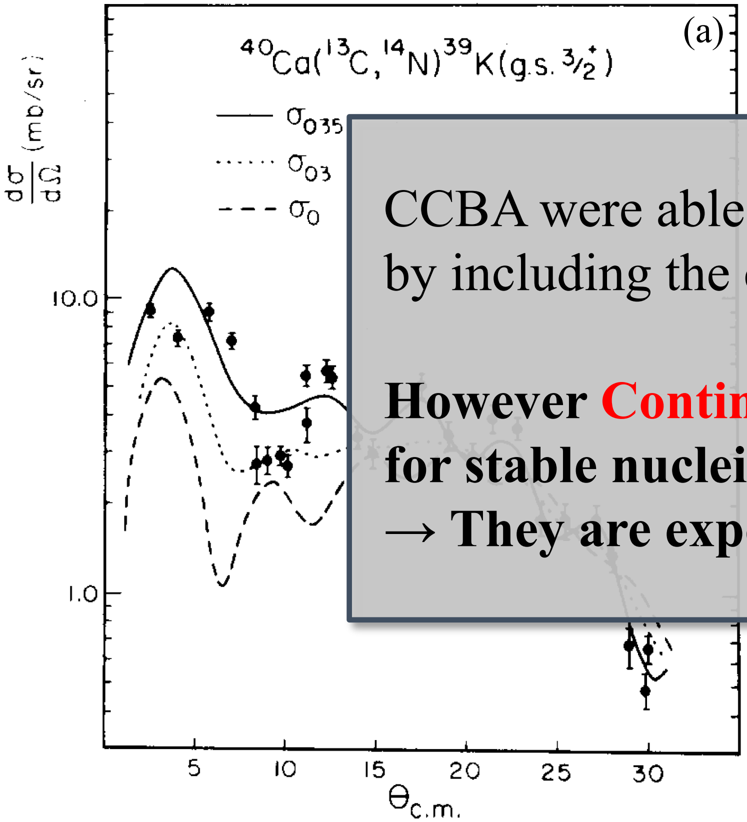

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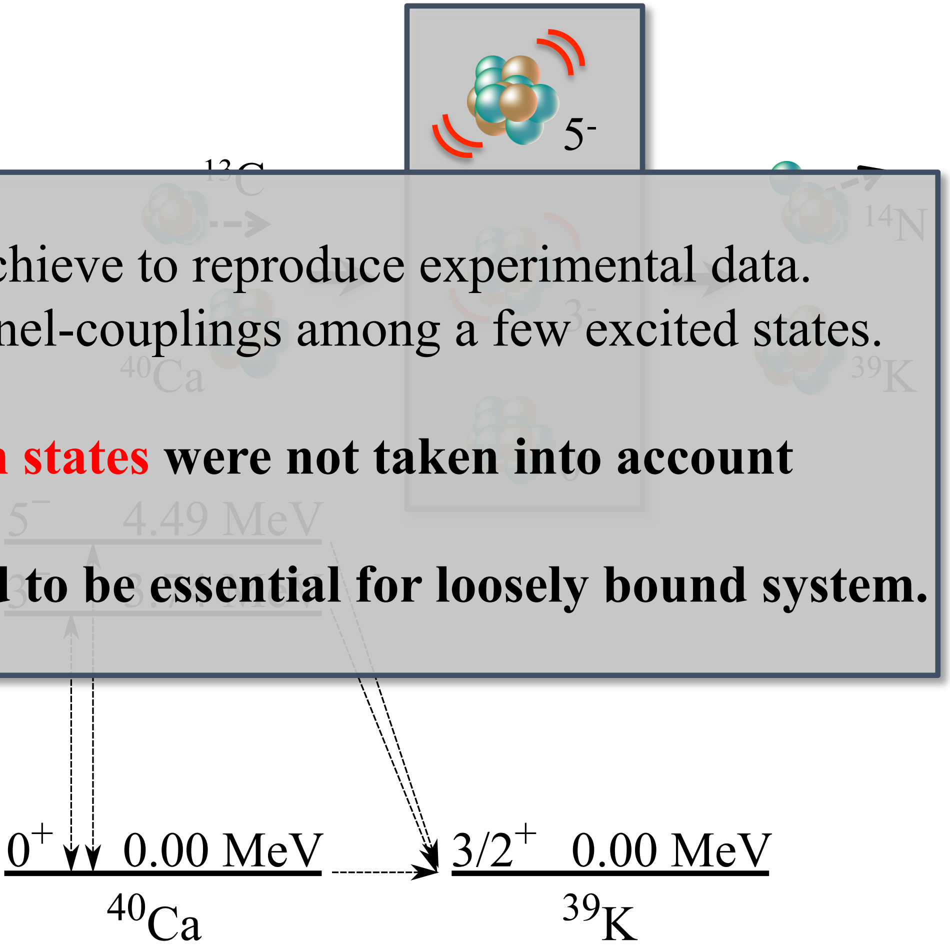


CCBA were able to achieve to reproduce experimental data.
by including the channel-couplings among a few excited states.

However Continuum states were not taken into account for stable nuclei.
→ They are expected to be essential for loosely bound system.

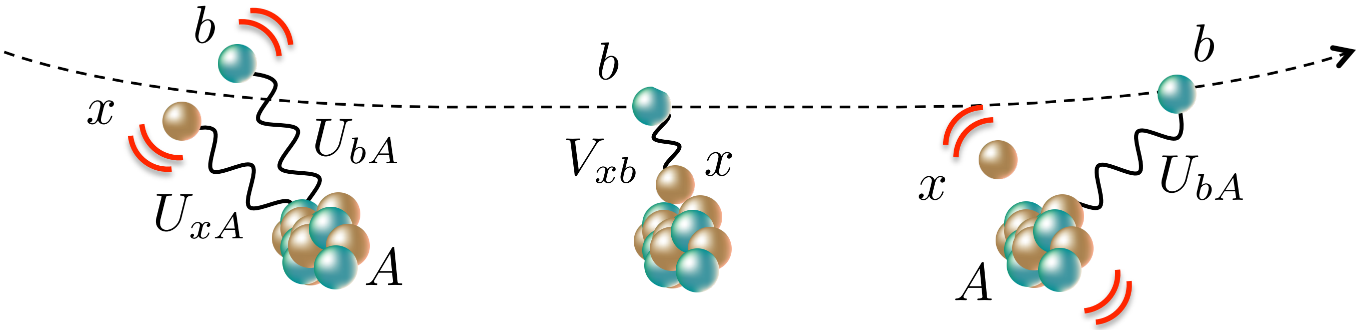
Model (CCBA)

Beyond DWBA

M. Kamimura *et al.*, Prog. Theor. Phys. Suppl. No. 89, 1 (1986).
N. Austern *et al.*, Phys. Rep. **154**, 125 (1987) .
M. Yahiro *et al.*, Prog. Theor. Exp. Phys. **2012**, 01A209 (2012).

- **Coupled-channels Born approximation (CCBA)** with **the continuum-discretized coupled-channels (CDCC)** method.

$$T_{\mathrm{CCBA}} = \left\langle \Psi^{(-)}_{\beta(\mathrm{CDCC})} \middle| V_{xb} \middle| \Psi^{(+)}_{\alpha(\mathrm{CDCC})} \right\rangle$$



- **The optical potential** U_{xA} (U_{bA}) for the subsystem $x + A$ ($b + A$) generates the distorted wave based on the **3-body model**.
- **The CDCC wave functions both in the initial and final channels**.
  → **Remnant term** is canceled out exactly.
  → **Rearrangement component** is involved implicitly.

A. M. Moro *et al*., Phys. Rev. C **80**, 064606 (2009).

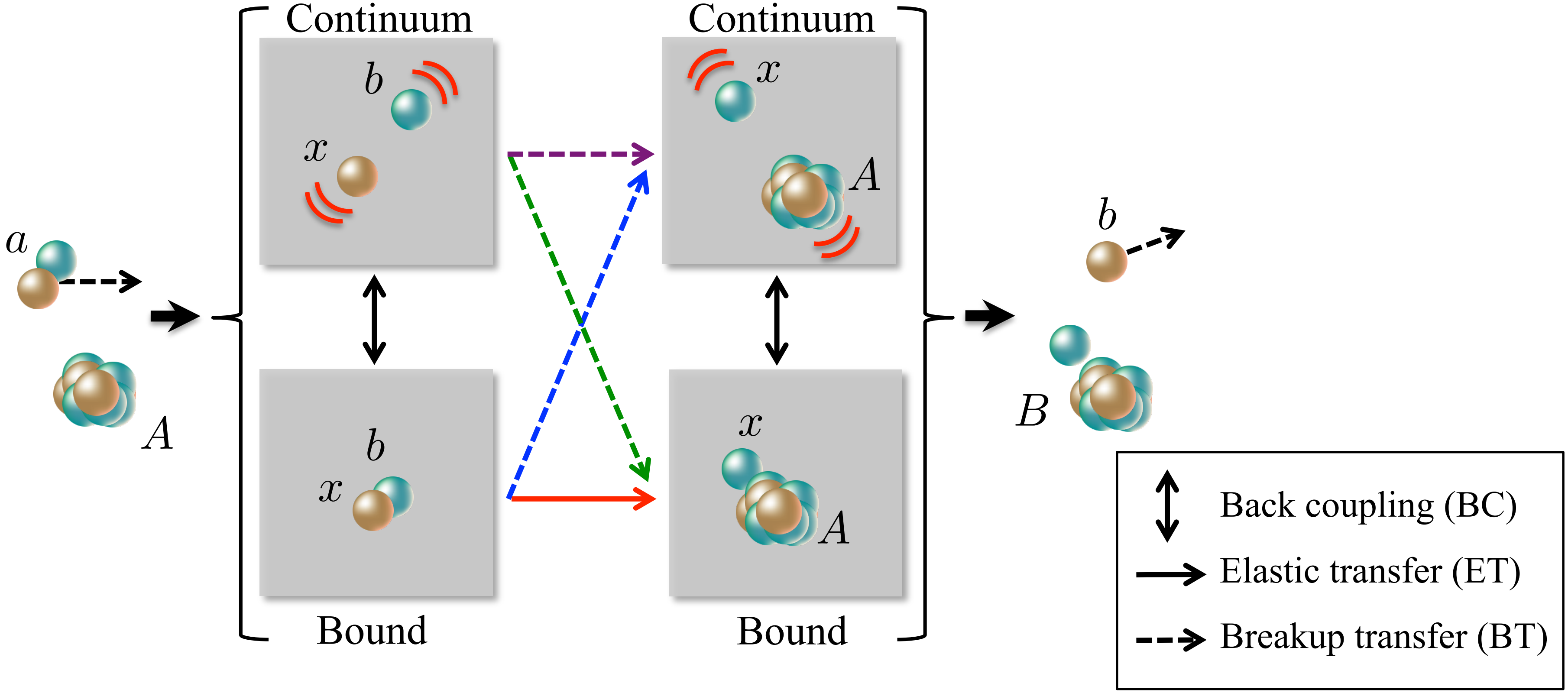
Model (CCBA)

Breakup process

- Decomposition of the transition matrix

$$T_{\rm CCBA} = \left\langle \Psi^{(-)}_{\beta({\rm el})} + \Psi^{(-)}_{\beta({\rm br})} \middle| V_{xb} \middle| \Psi^{(+)}_{\alpha({\rm el})} + \Psi^{(+)}_{\alpha({\rm br})} \right\rangle$$

$$= \underline{T_{\beta({\rm el}),\alpha({\rm el})}} + \underline{T_{\beta({\rm el}),\alpha({\rm br})}} + \underline{T_{\beta({\rm br}),\alpha({\rm el})}} + \underline{T_{\beta({\rm br}),\alpha({\rm br})}}$$



↕ Back coupling (BC)

→ Elastic transfer (ET)

⇢ Breakup transfer (BT)

Model (CCBA)

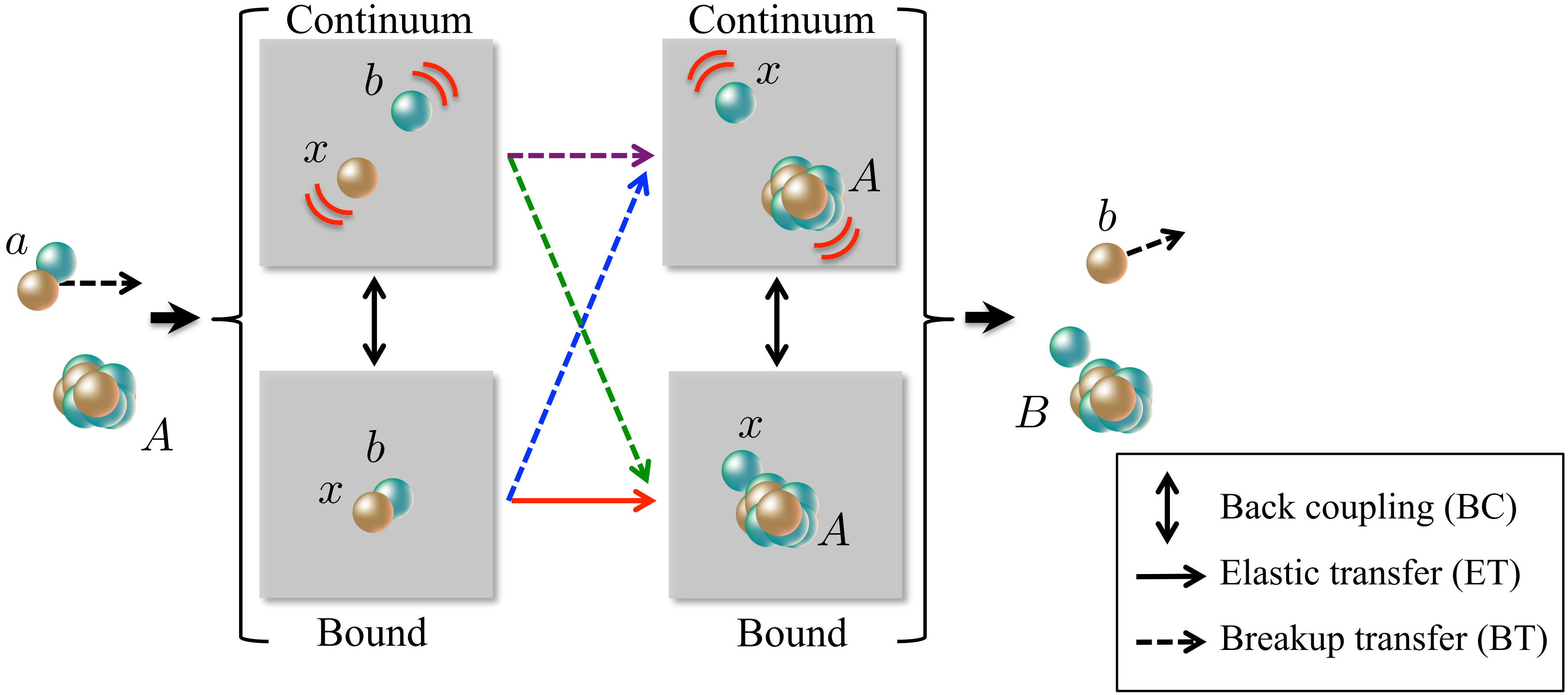
- ✓ **BC is implicitly taken into account in DWBA as "absorption".**
- ✓ **BT is never involved in DWBA.**

Breakup process

- Decomposition of the transition matrix

$$T_{\rm CCBA} = \left\langle \Psi^{(-)}_{\beta({\rm el})} + \Psi^{(-)}_{\beta({\rm br})} \middle| V_{xb} \middle| \Psi^{(+)}_{\alpha({\rm el})} + \Psi^{(+)}_{\alpha({\rm br})} \right\rangle$$

$$= \underline{T_{\beta({\rm el}),\alpha({\rm el})}} + \underline{T_{\beta({\rm el}),\alpha({\rm br})}} + \underline{T_{\beta({\rm br}),\alpha({\rm el})}} + \underline{T_{\beta({\rm br}),\alpha({\rm br})}}$$



3. 1. ^{8}B(d, n)^{9}C

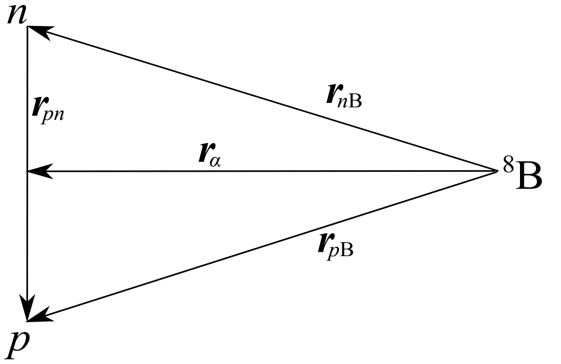
Model (numerical setup)

Numerical setting

- Initial channel

$V_{pn}(r_{pn})$: 1 range Gaussian (Ohmura potential)

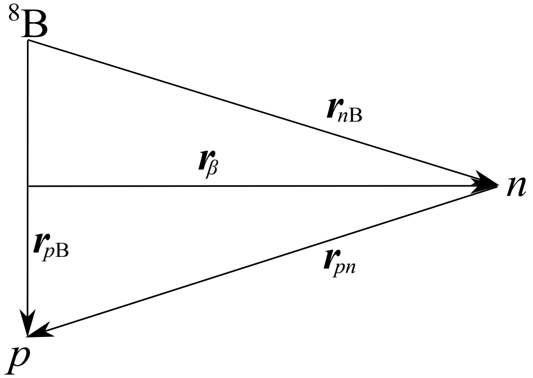
$U_{x\mathrm{B}}^{(\alpha)}(r_{x\mathrm{B}})$: Global optical potentials (Woods-Saxon)



- Final channel

$U_{p\mathrm{B}}^{(\beta)}(r_{p\mathrm{B}})$: Woods-Saxon potential (reproduces the ground state energy of ^{9}C)

$U_{n\mathrm{B}}^{(\beta)}(r_{n\mathrm{B}})$: Same as that in the initial channel



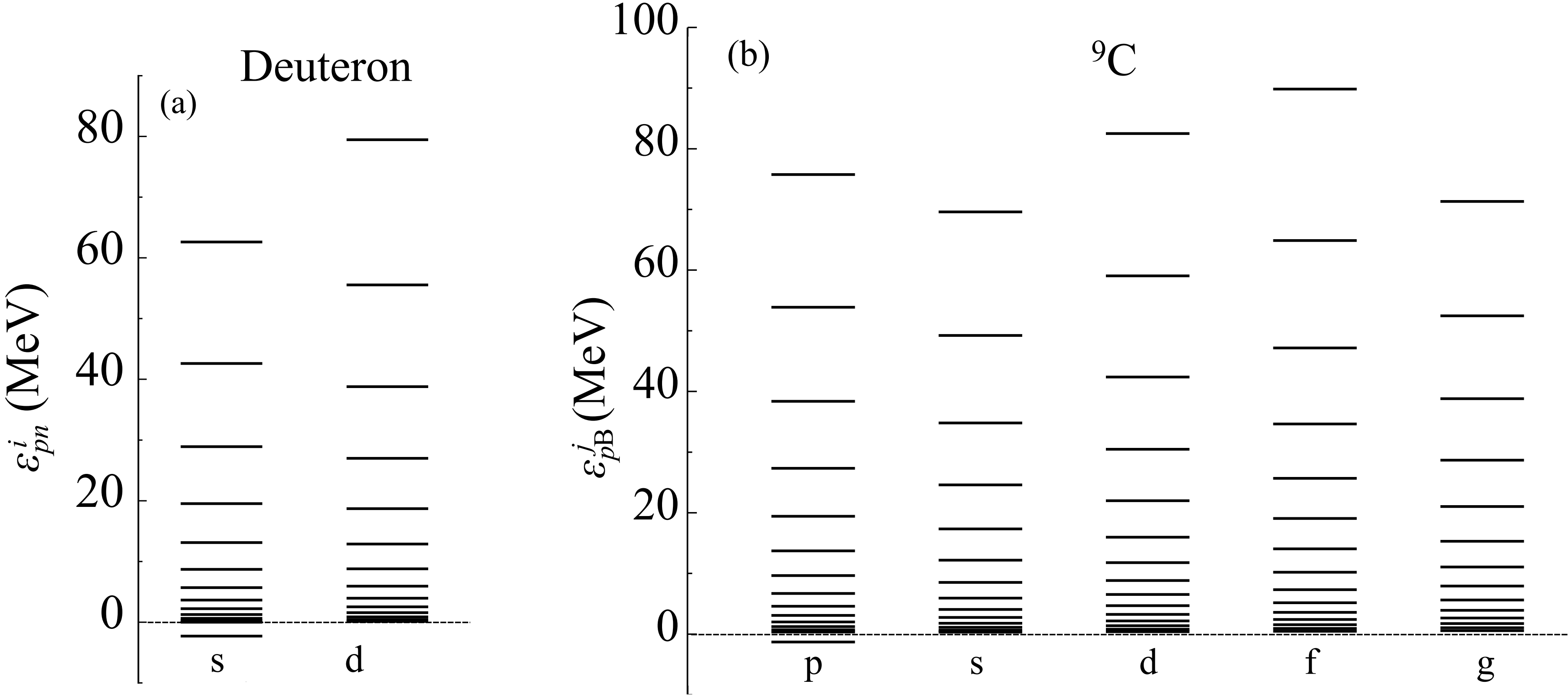
T. Ohmura *et al*., Prog. Theor. Phys. **43**, 347 (1970).
B. A. Watson *et al*., Phys. Rev. **182**, 997 (1969).
J. H. Dave and C. R. Gould, Phys. Rev. C **28**, 2212 (1983).

Interactions are phenomenologically determined.

Result 1 T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

- Discretization (pseudostate method)
  → The internal Hamiltonians are diagonalized with Gaussian basis functions.

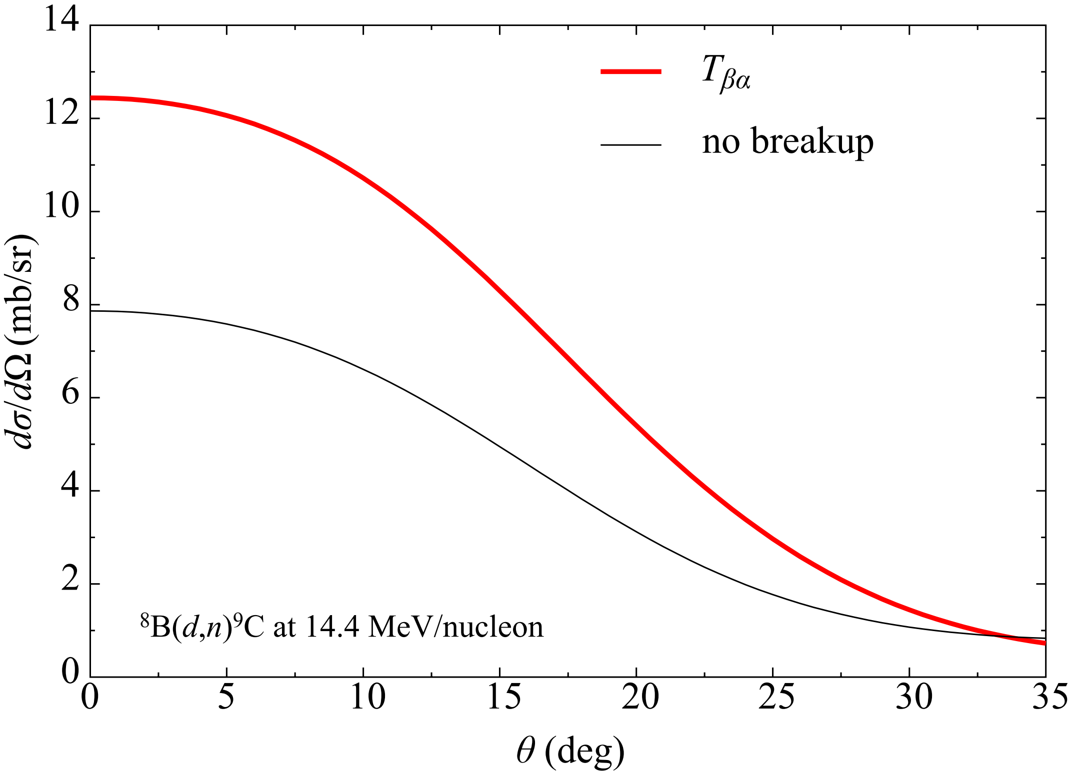
$$\left(h_{pn}-\varepsilon^{i}_{pn}\right)\psi^{i}_{pn}(\boldsymbol{r}_{pn})=0 \qquad \left(h_{p\mathrm{B}}-\varepsilon^{j}_{p\mathrm{B}}\right)\psi^{j}_{p\mathrm{B}}(\boldsymbol{r}_{p\mathrm{B}})=0.$$



Result 1

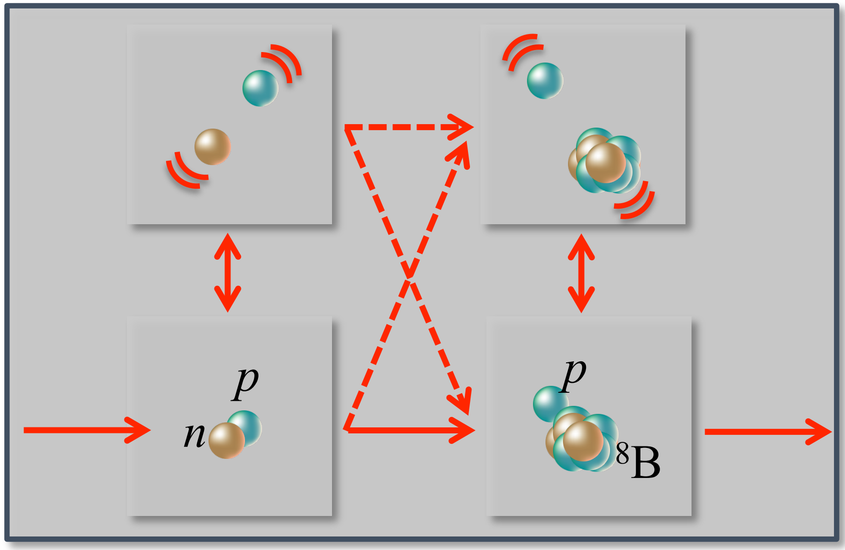
T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

Breakup effect on ^{8}B(d, n)^{9}C

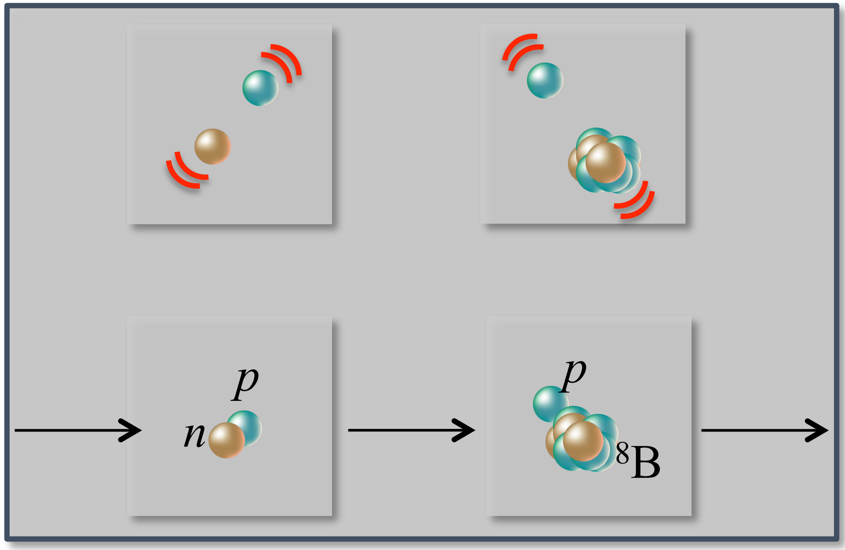


- Significant breakup effect (**58%**) can be seen at the forward angles of the angular distribution of the cross section.

T_{CCBA}



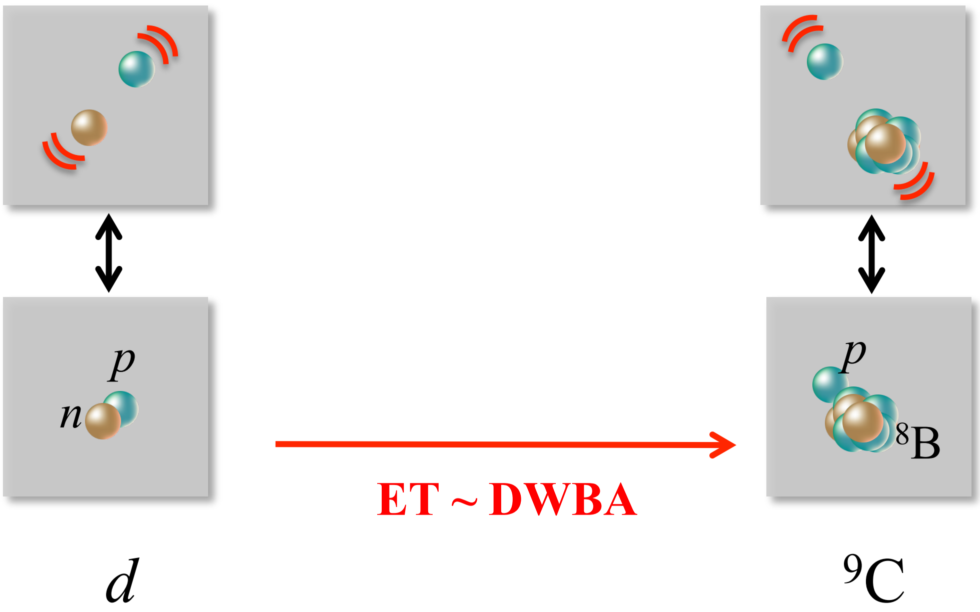
No breakup

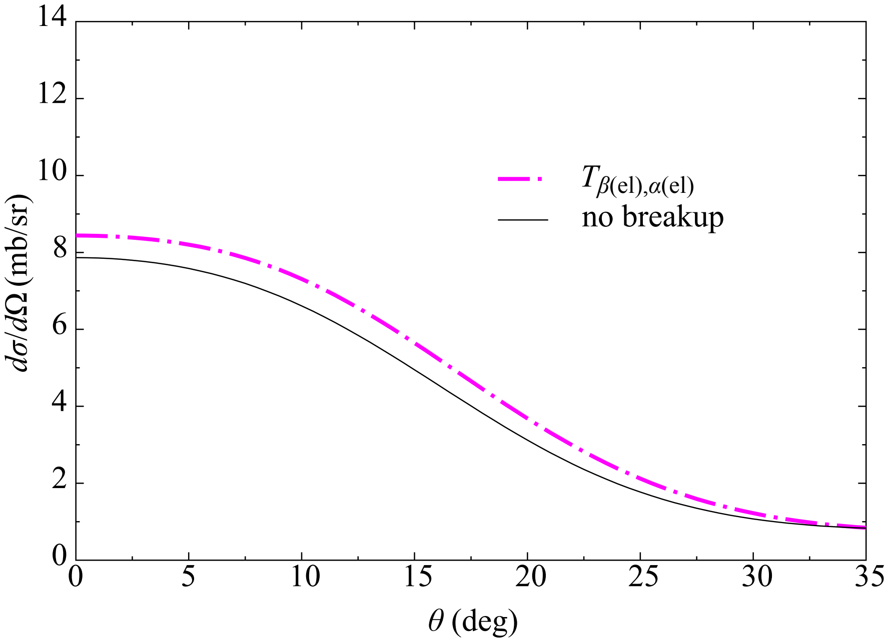


Result 1

T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

Breakup effects of each path



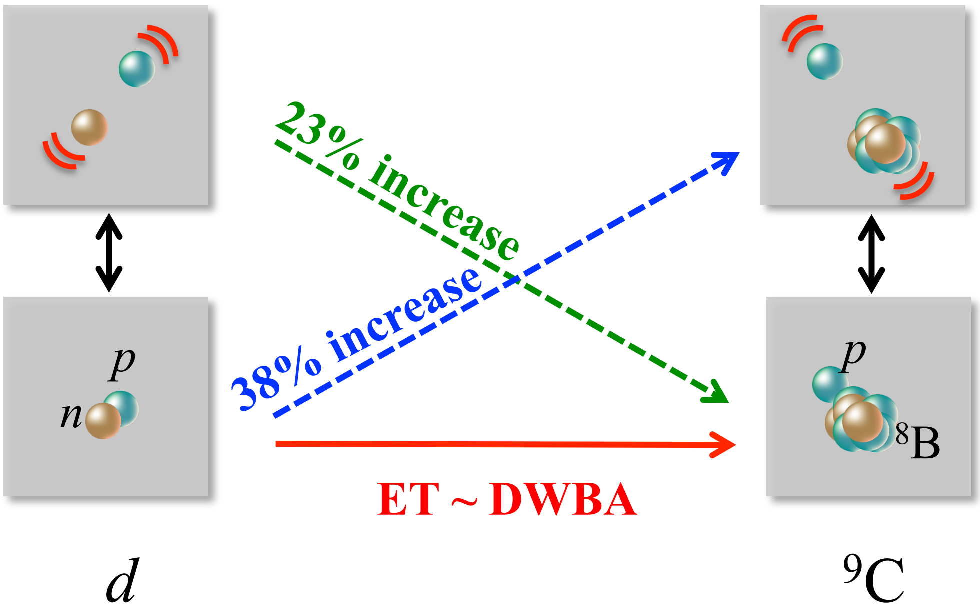


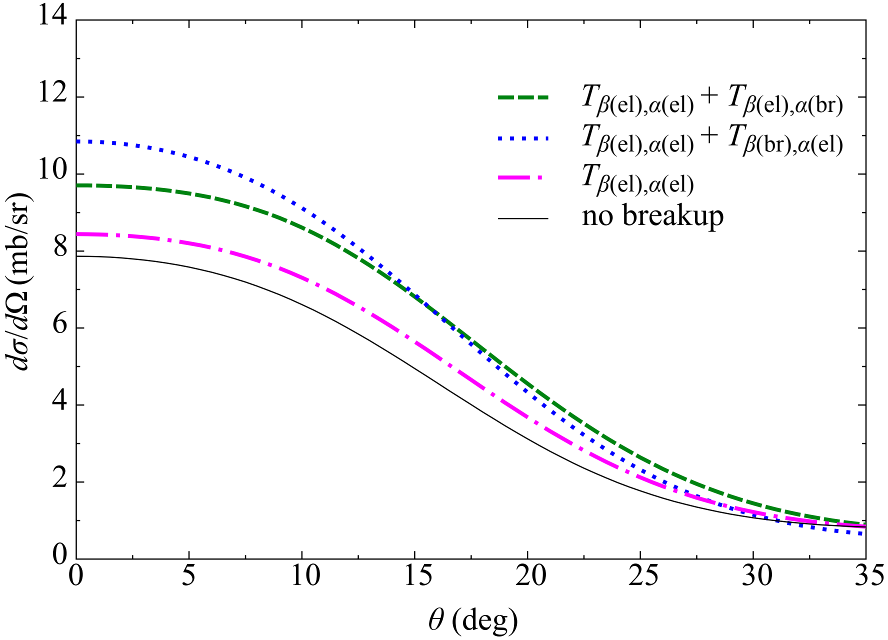
- The BC is weak and the ET result can be regarded as that of DWBA.

Result 1

T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

Breakup effects of each path



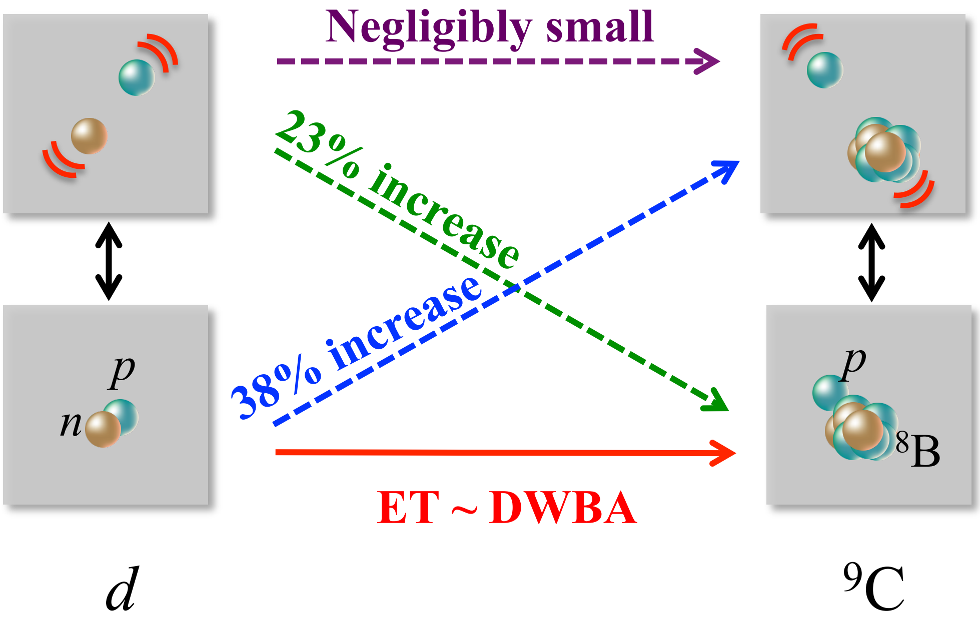


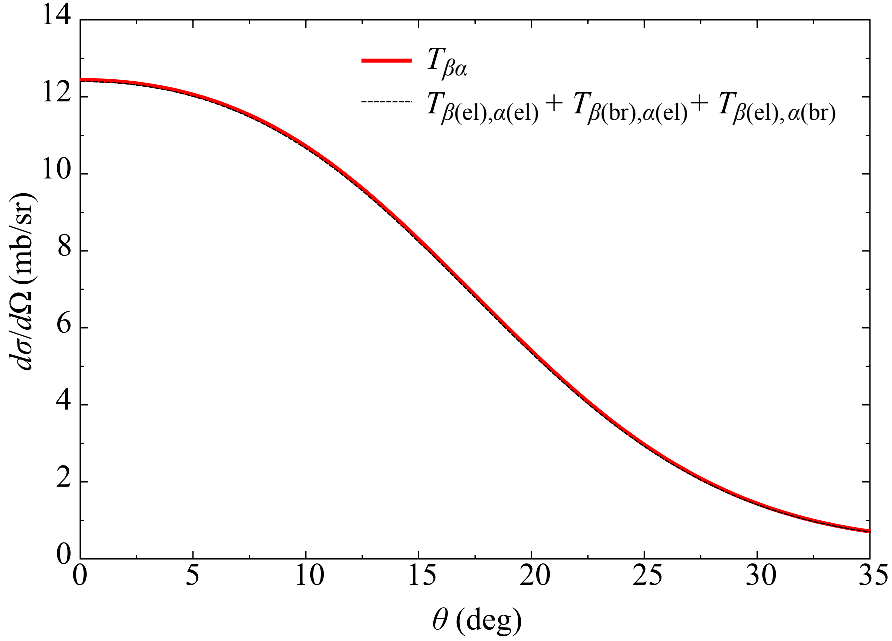
- The BC is weak and the ET result can be regarded as that of DWBA.
- Strong interferences between the ET and the BT in each channel enhance the cross section. → **Never involved in DWBA**.

Result 1

T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

Breakup effects of each path



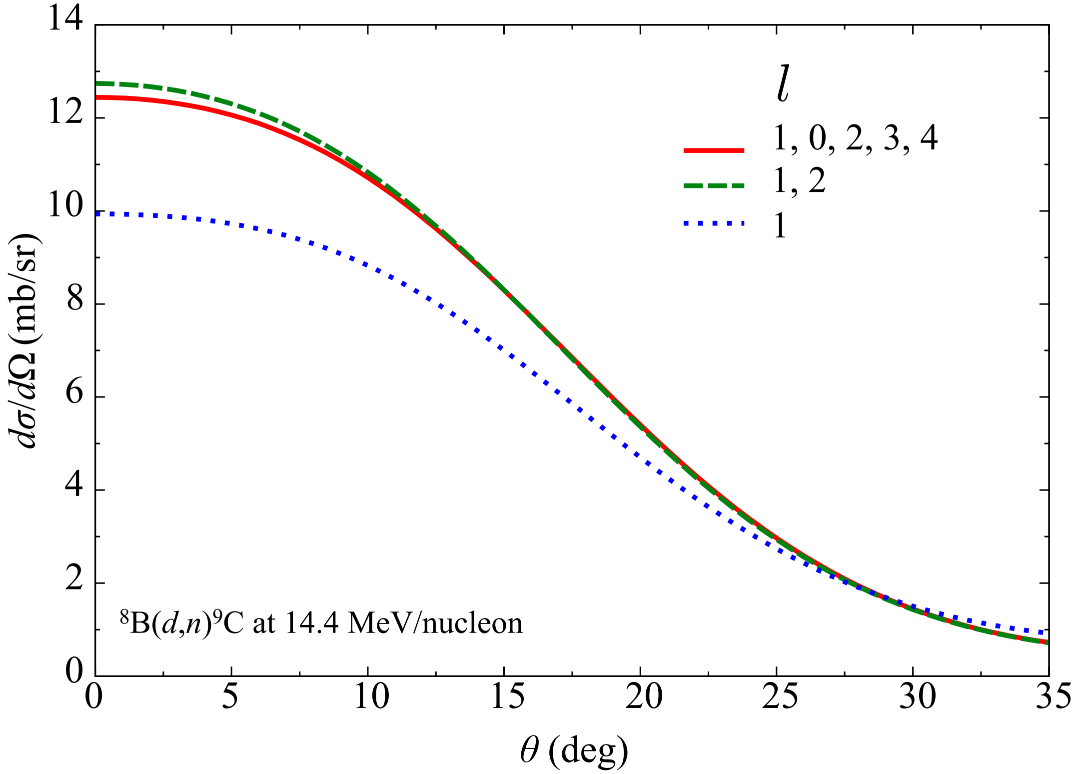


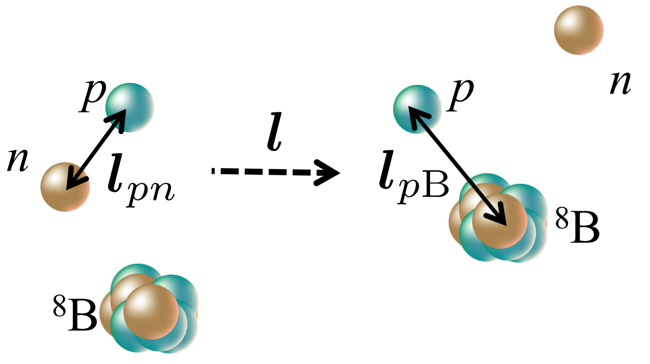
- The BC is weak and the ET result can be regarded as that of DWBA.
- Strong interferences between the ET and the BT in each channel enhance the cross section. → **Never involved in DWBA**.
- The BT among continuum states is negligible.

Result 1

T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

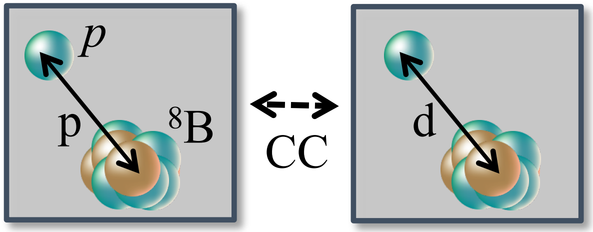
Dynamical change of transferred angular momentum l





$$l = l_{pB} \quad (l_{pn} = 0)$$

DWBA: $l = 1$ (unique)
CCBA: l can dynamically change



- A **25%** increase due to CC with the d-wave of ^9C is confirmed.

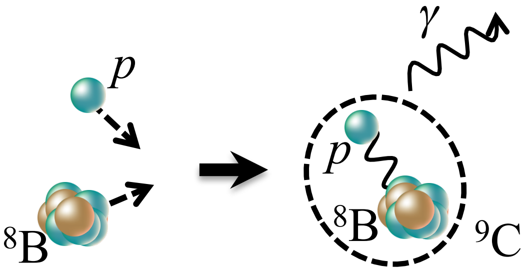
Nuclear astrophysics

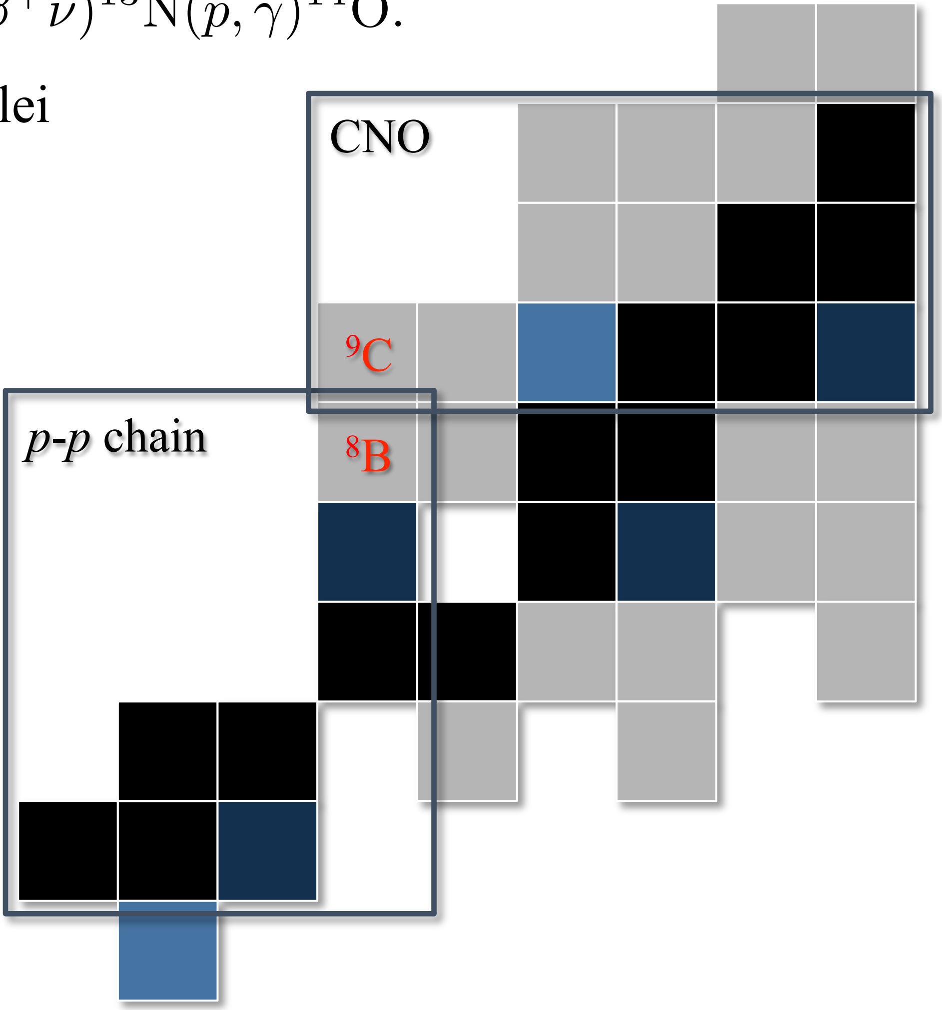
Determination of the astrophysical reaction rate

- Ignition of the hot *pp* chain   M. Wiescher *et al.*, Astrophys. J. **343**, 352 (1989).

$$^{8}\mathrm{B}(p,\gamma)^{9}\mathrm{C}(\alpha,p)^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}(\beta^{+}\nu)^{13}\mathrm{N}(p,\gamma)^{14}\mathrm{O}.$$

→ Important process to produce nuclei heavier than A=8.

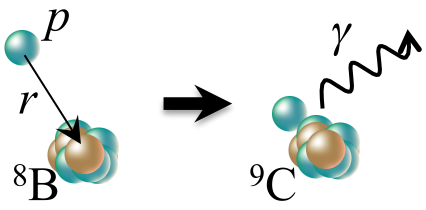




Nuclear astrophysics

Determination of the astrophysical reaction rate

- Transition matrix for the radiative capture

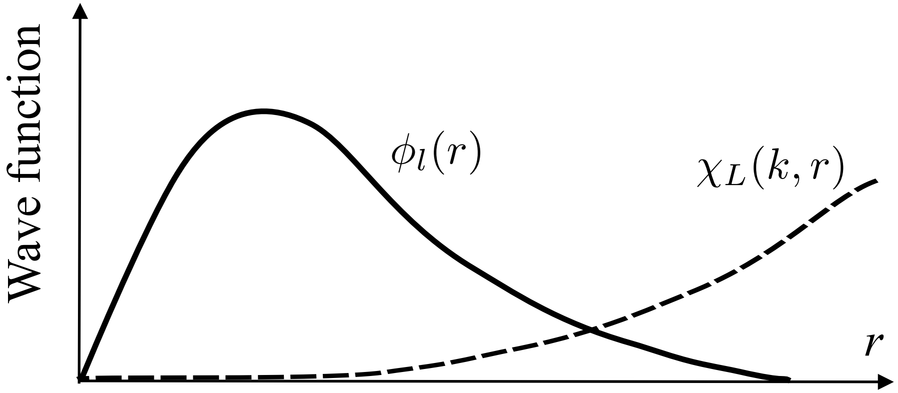


$$T = \left\langle \psi_{p\mathrm{B}} \middle| \hat{O}_{\mathrm{EM}} \middle| \chi^{(+)}_{p\mathrm{B}} \right\rangle$$

$$\begin{aligned} \psi_{p\mathrm{B}}(\boldsymbol{r}) &= \langle \Phi_p \Phi_{\mathrm{B}} \,|\, \Phi_{\mathrm{C}} \rangle \\ &= \phi_l(r) Y_{lm}(\hat{\boldsymbol{r}}) \\ &\xrightarrow{\text{asy.}} C_l \frac{W_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \end{aligned}$$

$$\chi^{(+)}_{p\mathrm{B}}(\boldsymbol{k}, \boldsymbol{r}) = \frac{4\pi}{kr} \sum_{LM} \chi_L(k,r) Y^*_{LM}(\hat{\boldsymbol{k}}) Y_{LM}(\hat{\boldsymbol{r}})$$

- Astrophysical reactions are a low-energy scattering.
  → Scattering wave is suppressed in interior region.
  → **Only the surface of ϕ_l (ANC)** contributes on the cross section and determine the reaction rate.



Nuclear astrophysics

Asymptotic normalization coefficient from observables

A. M. Mukhamedzhanov and N. K. Timofeyuk, Yad. Fiz. **51**, 679 (1990) [Sov. J. Nucl. Phys. **51**, 431 (1990)].

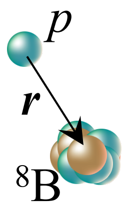
$$\phi_l(r) \xrightarrow{\text{asy.}} C_l \frac{W_l(r)}{r}$$

In the model $\phi^l_{p\mathrm{B}}$ is calculated
as a single particle wave function.

$$\phi_l(r) \approx \sqrt{S_l}\frac{u_l(r)}{r}$$

$$\xrightarrow{\text{asy.}} \sqrt{S_l} b_l^{(\mathrm{sp})} \frac{W_l(r)}{r}$$

S_l: spectroscopic factor
$b_l^{(\mathrm{sp})}$: single particle ANC



→

$$(C_l)^2 = \underline{S_l}\,\underline{(b_l^{(\mathrm{sp})})}^2$$

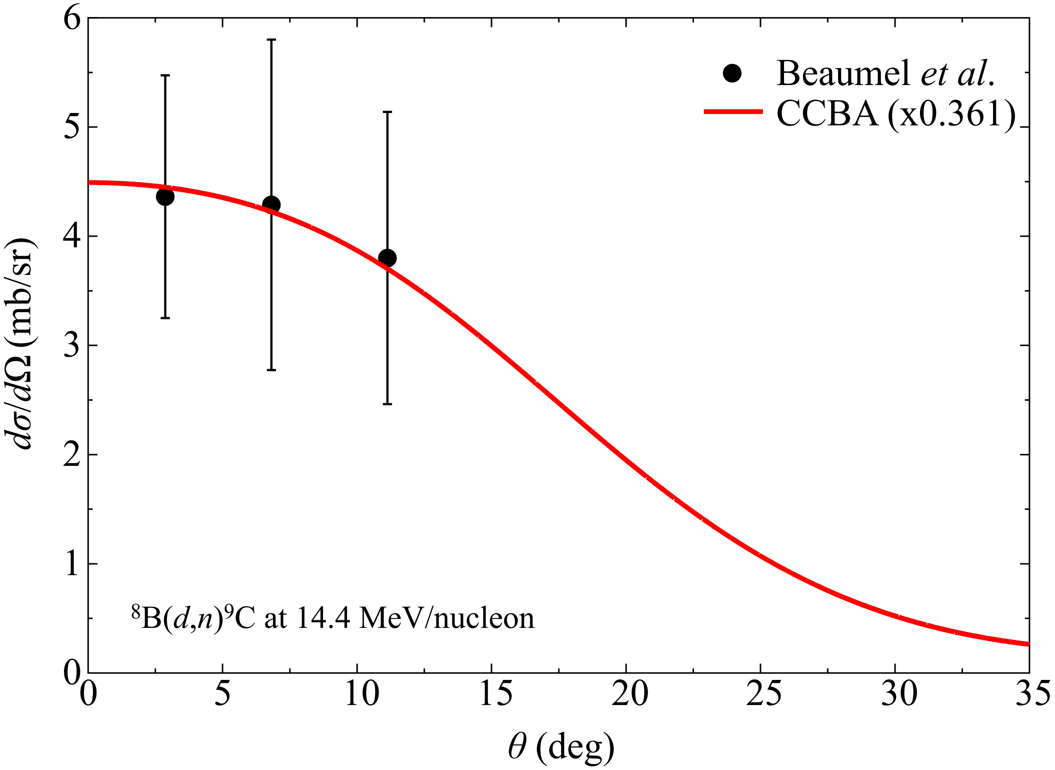
determined
from observables

calculated with
the single particle w. f.

Result 1

T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

ANC from transfer cross section



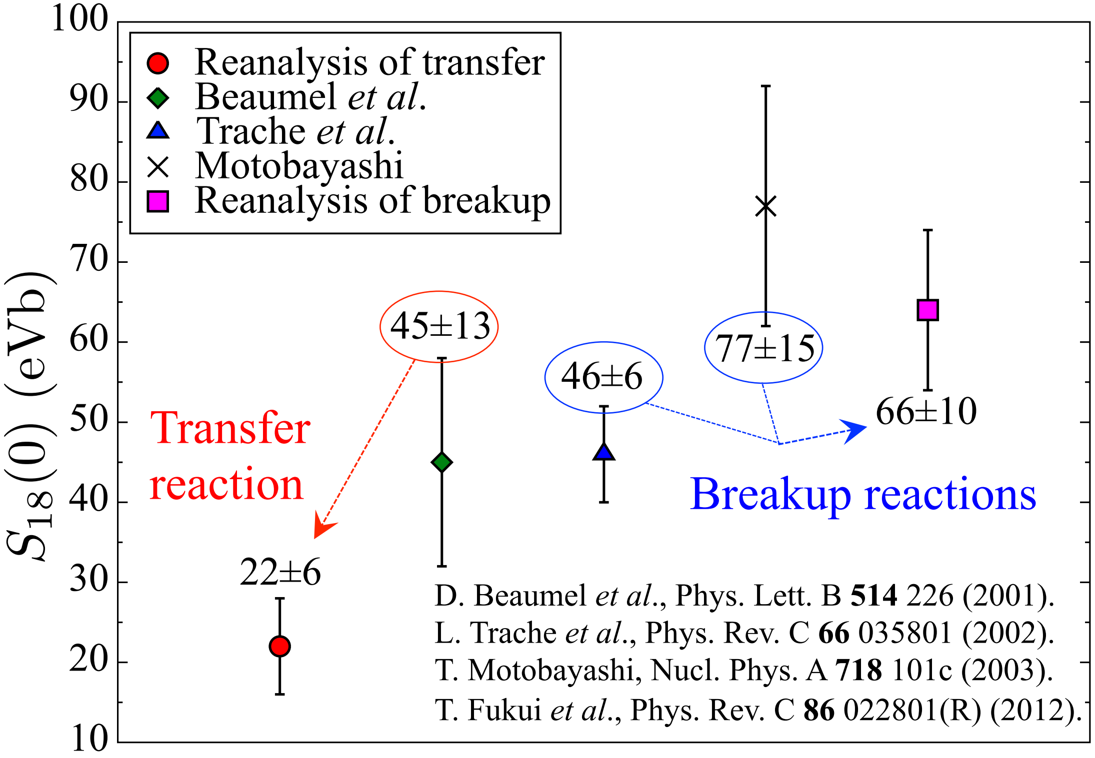
- We obtain the value of ANC,
  $(C_l)^2 = 0.59$
  $\pm$ 0.02 (theor.)
  $\pm$ 0.13 (exp.) fm^{-1},
  which is about **51% smaller** than that of the previous DWBA result.

D. Beaumel *et al.*, Phys. Lett. **B514**, 226 (2001).

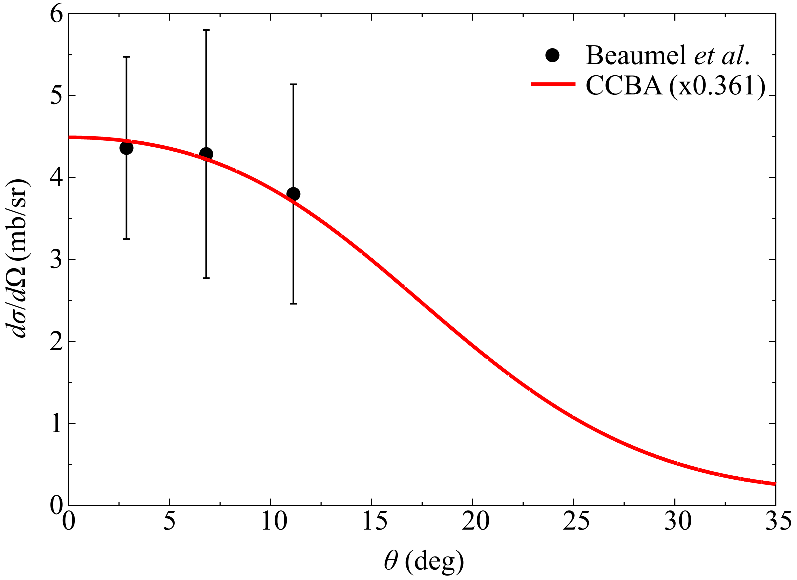
Result 1 T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

Breakup effect on S_{18} of ^{8}B(p, γ)^{9}C

$$S_{18}(\varepsilon_{pB}) = \sigma_{p\gamma}(\varepsilon_{pB})\varepsilon_{pB}\exp[2\pi\eta]$$



D. Beaumel *et al.*, Phys. Lett. B **514** 226 (2001).
L. Trache *et al.*, Phys. Rev. C **66** 035801 (2002).
T. Motobayashi, Nucl. Phys. A **718** 101c (2003).
T. Fukui *et al.*, Phys. Rev. C **86** 022801(R) (2012).



What we can say is that the breakup effect enhances the transfer cross section.

Future work

(1) Inclusion of the 3-body configuration in ^{9}C ($p + p + {}^7$Be).
(2) The CCBA analysis of the mirror reaction ^{8}Li(d, p)^{9}Li.

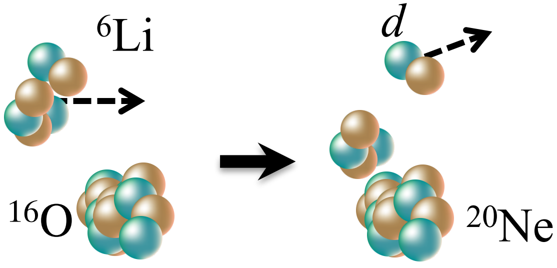
[12] B. Guo *et al.*, Nucl. Phys. **A761**, 162 (2005).

3. 2. ^{16}O(^{6}Li, d)^{20}Ne

α-transfer reaction to investigate clustering in ^{20}Ne (g.s.)

- There is NO direct evidence of the clustering (**surface manifestation**) in a **ground state** of nuclei.
- Measurements and their analyses with the Distorted-wave Born Approximation (DWBA of the (^{6}Li,d) or (d,^{6}Li) reaction have been done.
- Unphysical normalizations (spectroscopic factor (SF) $S_\alpha > 1$) are needed to fit calculated cross sections to the data.

^{16}O(^{6}Li, d)^{20}Ne



[1] N. Anantaraman *et al.*, Nucl. Phys. **A313**, 445 (1979).
[2] F. D. Becchetti *et al.*, Nucl. Phys. **A303**, 313 (1978).
[3] T. Tanabe *et al.*, Phys. Rev. C **24**, 2556 (1981).
[4] W. Oelert *et al.*, Phys. Rev. C **20**, 459 (1979).

E_{Li} (MeV)	S_α
20 [1]	2.7
32 [1]	10.3
38 [1]	7.4
42 [2]	2.59
75 [3]	0.24
95 [4]	0.23

These are due to the ambiguities of

(1) **the optical model potential (OMP) of ^{6}Li**
(2) **the α-^{16}O wave function (WF)**.

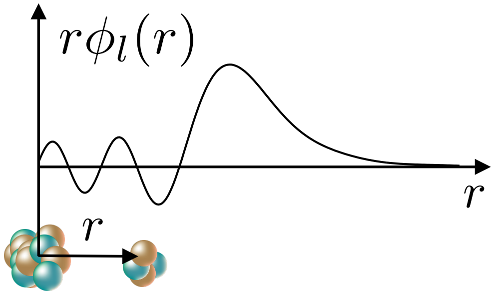
Background

SF is NOT suitable to discuss surface manifestation

(1) The SF is defined as a norm of the cluster-overlap function.

$$S_\alpha \equiv \int dr r^2 \left|\phi_l(r)\right|^2$$

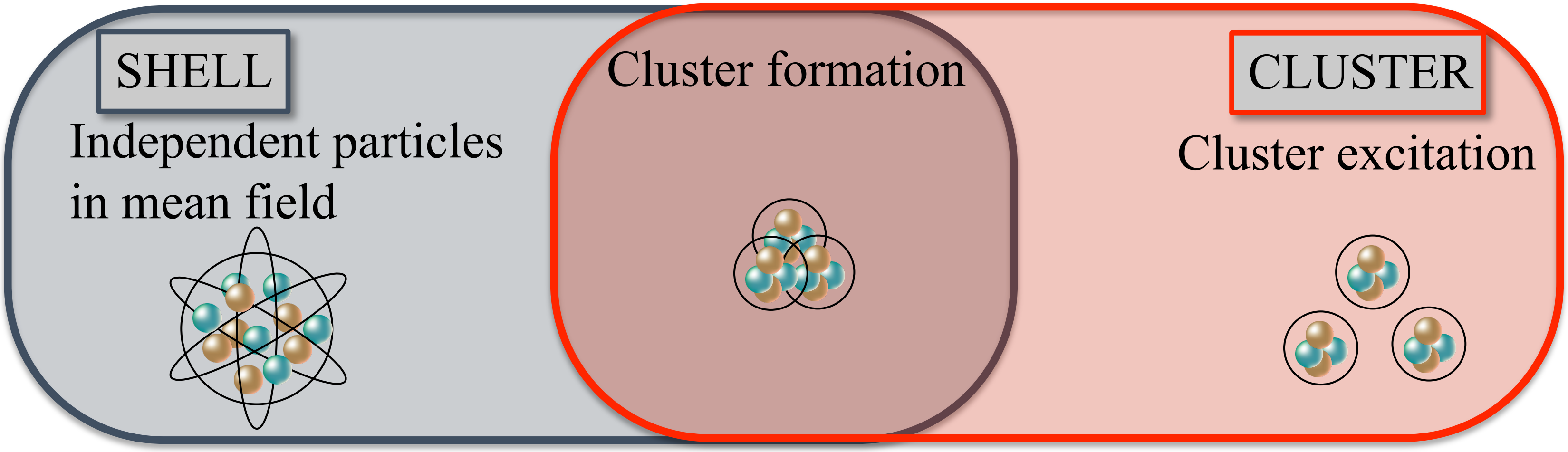
→ It involves the information of $\phi_l(r)$ at the interior region.



(2) Shell-cluster duality (Bayman-Bohr theorem) B. F. Bayman and A. Bohr, Nucl. Phys. **9**, 596 (1958/1959).

Even if there is no spatial manifestation, S_α can reach unity.

→ Shell model wave function is equivalent to that of cluster model in ground state.



SHELL

Independent particles in mean field

Cluster formation

CLUSTER

Cluster excitation

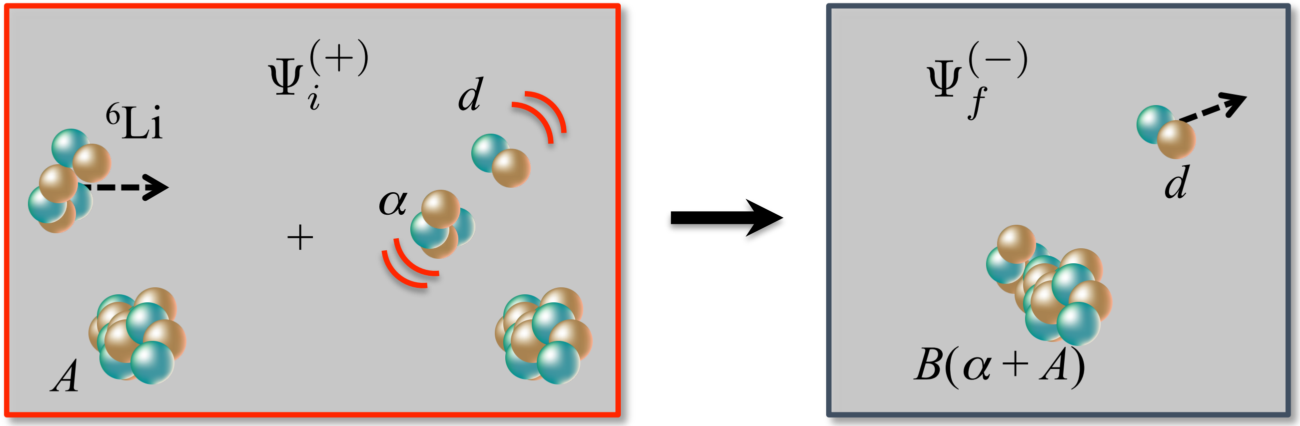
Model

Reaction model

- **CCBA** with **CDCC only in the initial channel**

$$T_{\mathrm{CCBA}} = \left\langle \Psi_f^{(-)} \left| V_{\mathrm{tr}} \right| \Psi_i^{(+)} \right\rangle$$

M. Kamimura *et al.,* Prog. Theor. Phys. Suppl. No. 89, 1 (1986).
N. Austern *et al.,* Phys. Rep. **154**, 125 (1987) .
M. Yahiro *et al.,* Prog. Theor. Exp. Phys. **2012**, 01A209 (2012).



CDCC

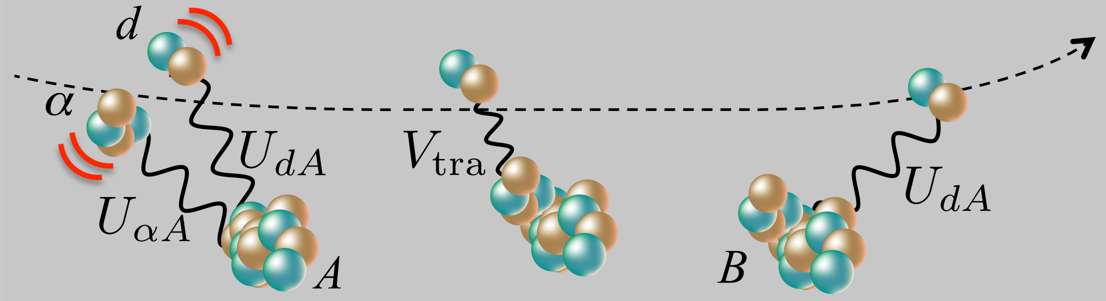
1ch potential model

- The CC among bound and discretized-continuum (DC) states of the projectile is explicitly taken into account.

Model

Difference between CCBA and DWBA

CCBA (three-body model, present work)



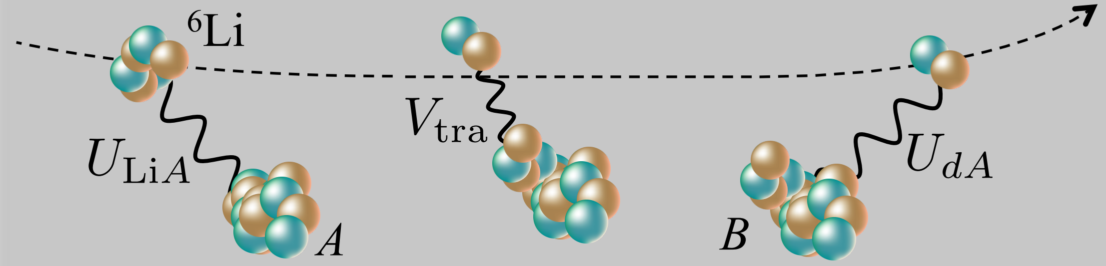
OMPs (phenomenological)

$U_{\alpha A}$ U_{dA}

F. Michel *et al*., Phys. Rev. C **28**, 1904 (1983).
J. H. Dave and C. R. Gould, Phys. Rev. C **28**, 2212 (1983).
Y. Han *et al*., Phys. Rev. C **74**, 044615 (2006).

^{6}Li-OMP is needless.

DWBA (two-body model, conventional approach)



OMPs (phenomenological)

$U_{{\rm Li}A}$ U_{dA}

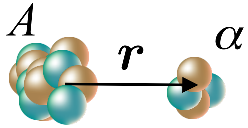
$U_{{\rm Li}A}$ has large ambiguity.

- A part of the CC effect is **implicitly** taken into account as **an imaginary part** of $U_{{\rm Li}A}$.

Model

Structure model

- **Microscopic cluster model (MCM)** with **GCM**



$B = \alpha + A$

$$|\Phi_{\rm GCM}\rangle = \left|\sqrt{\frac{M_\alpha!M_A!}{M_B!}}\mathcal{A}\left[\phi_l^{\rm (GCM)}(r)\,Y_{l0}(\hat{\boldsymbol{r}})\,\varphi_\alpha\varphi_A\varphi_{\rm c.m.}\right]\right\rangle$$

$$\phi_l^{\rm (GCM)}(r) \xrightarrow[\text{Antisym.}]{} \boxed{\phi_l^{\rm (MCM)}(r)}$$

Input of the reaction calculation.
Norm is unity.

- **The Volkov No. 2 effective interaction** of the Majorana para. $m = 0.62$ with the width para. $\nu = 0.16\ \mathrm{fm}^{-1}$ is adopted.

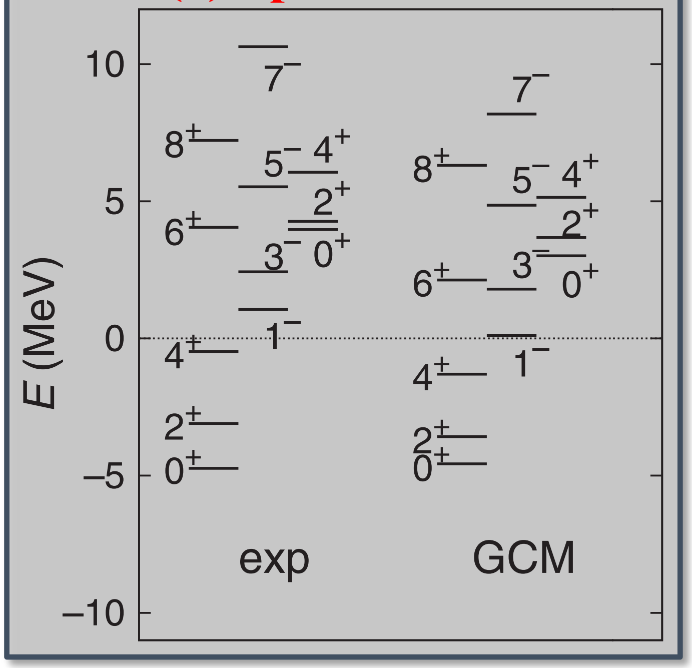
A. B. Volkov, Nucl. Phys. **74**, 33 (1965).
T. Matsue *et al*., Prog. Theor. Phys. **53**, 706 (1975).

- Consistency of the calculated quantities with the measured ones:

(1) Root-mean-square radius of ^{16}O

Y. Kanada-En'yo *et al*., Prog. Theor. Exp. Phys. **2014**, 073D02 (2014).
D. R. Tilley *et al*., Nucl. Phys. **A636**, 249 (1993).

(2) Spectra of ^{20}Ne

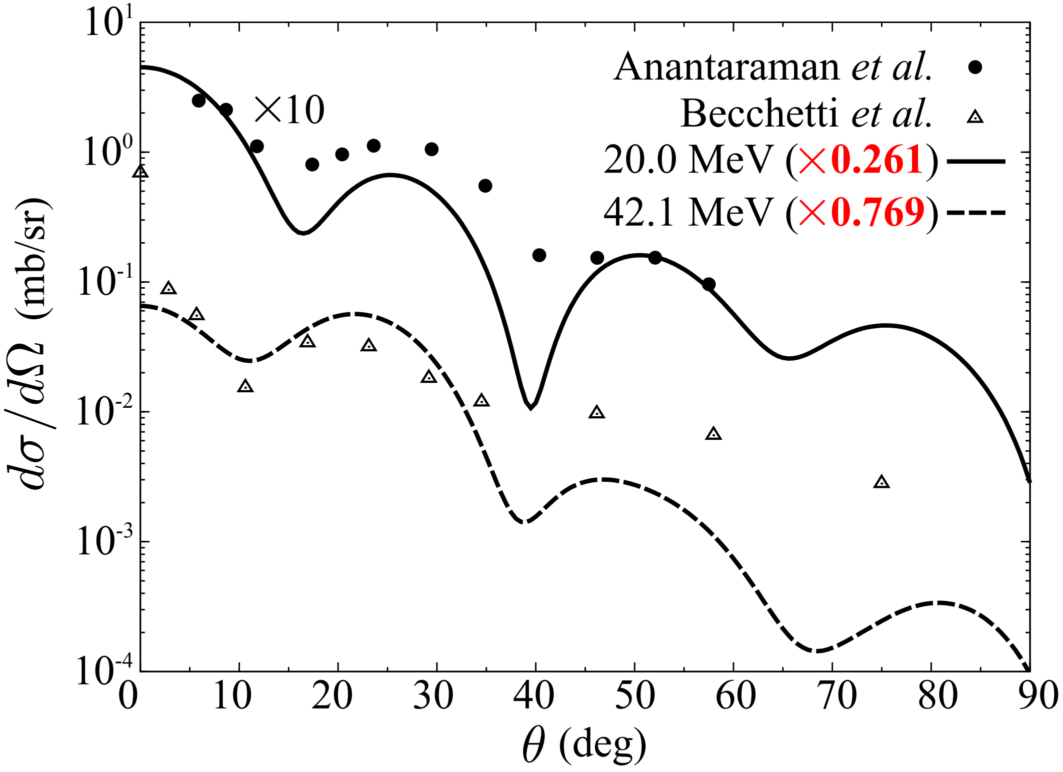


Result 2

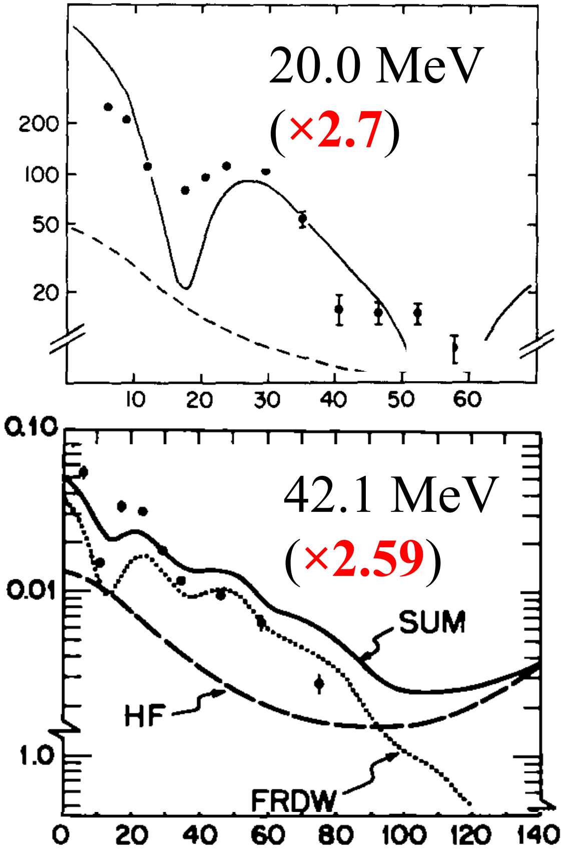
T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

^{16}O(^{6}Li, *d*)^{20}Ne(g.s.) to search surface manifestation of cluster

Present CCBA



Previous DWBA



Improvement

(1) Diffraction pattern of the 1st and 2nd peaks

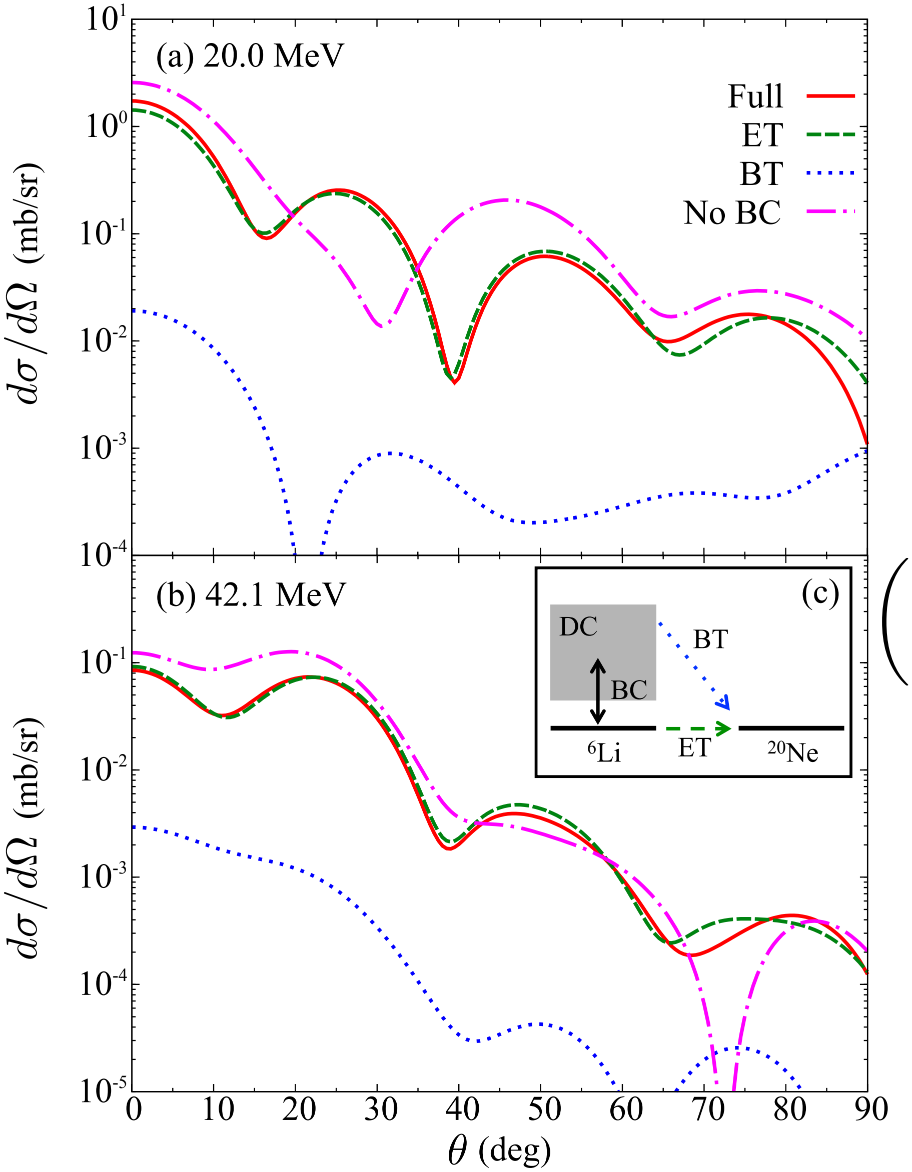
(2) Reasonable values of the normalization factors

→ Governed by reliabilities of both **the α-^{16}O WF** and **OMP**

N. Anantaraman *et al.,* Nucl. Phys. **A313**, 445 (1979).
F. D. Becchetti *et al.,* Nucl. Phys. **A303**, 313 (1978).

Result 2 T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

Breakup effects of ^{6}Li



- Decomposition of the CDCC distorted wave into **elastic** and **breakup** channels.

$$\chi_{\rm CDCC}(\boldsymbol{r}_i) = \underline{\chi_0(\boldsymbol{r}_i)} + \underline{\chi_c(\boldsymbol{r}_i)}$$

- **Full** ~ **Elastic transfer (ET)**
  ≠ **No back coupling (BC)**

→ **Breakup transfer (BT)** is negligible.
Only the BC (CC due to off-diagonal potentials) is essential.

$$\begin{pmatrix} K_i + U_{00} - E_0 & \boxed{U_{0c}} \\ \boxed{U_{c0}} & K_i + U_{cc} - E_c \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_c \end{pmatrix} = 0$$

→ DWBA can provide reasonable results, if an appropriate ^{6}Li-OMP, in which BC is implicitly taken into account as its imaginary part, is given.

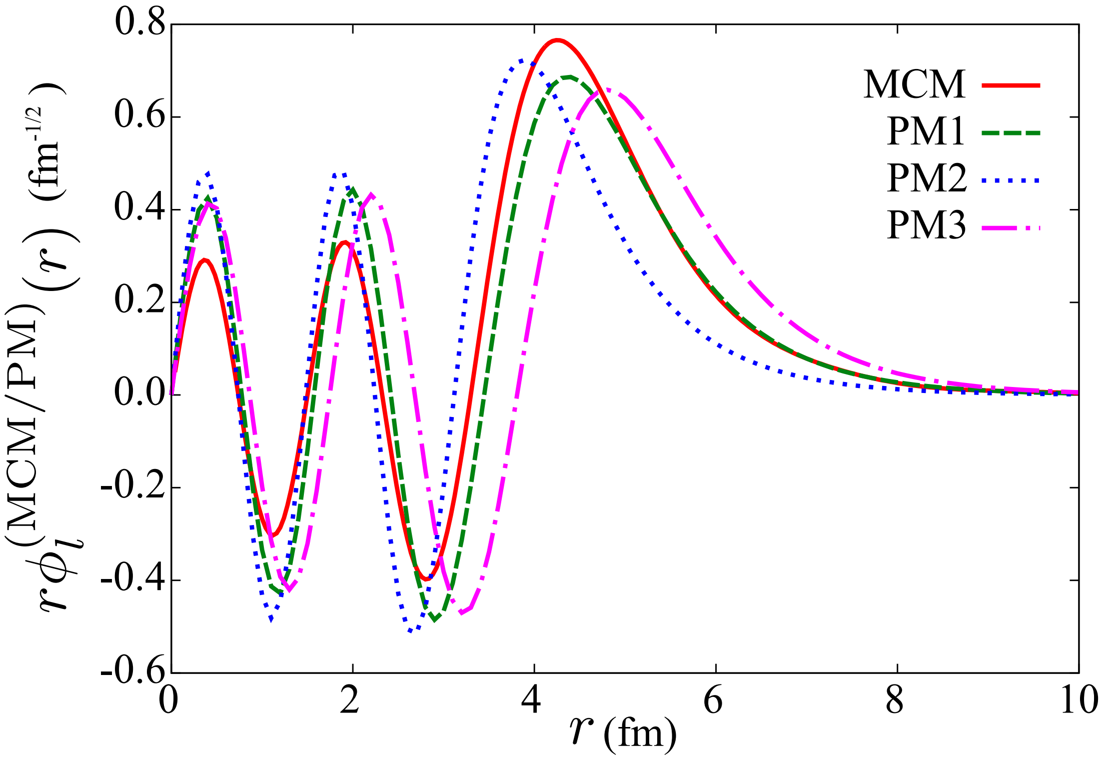
T. Fukui *et al.*, Prog. Theor. Phys. 125, 1193 (2011)
T. Fukui *et al.*, Phys. Rev. C **91**, 014604 (2015).

Result 2 T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

Potential model (PM) to investigate radial distribution of WF

$$\left[K_{\alpha A} + V_{\alpha A}^{(\mathrm{N+C})}(r) - \varepsilon_f\right] \phi_l^{(\mathrm{PM})}(r) = 0$$

- **PM1** describes the tail behavior of the **MCM** WF (**PM2** and **PM3** shift it to inside and outside respectively) with the Woods-Saxon potential.
- All of the WFs are **normalized to be unity**.



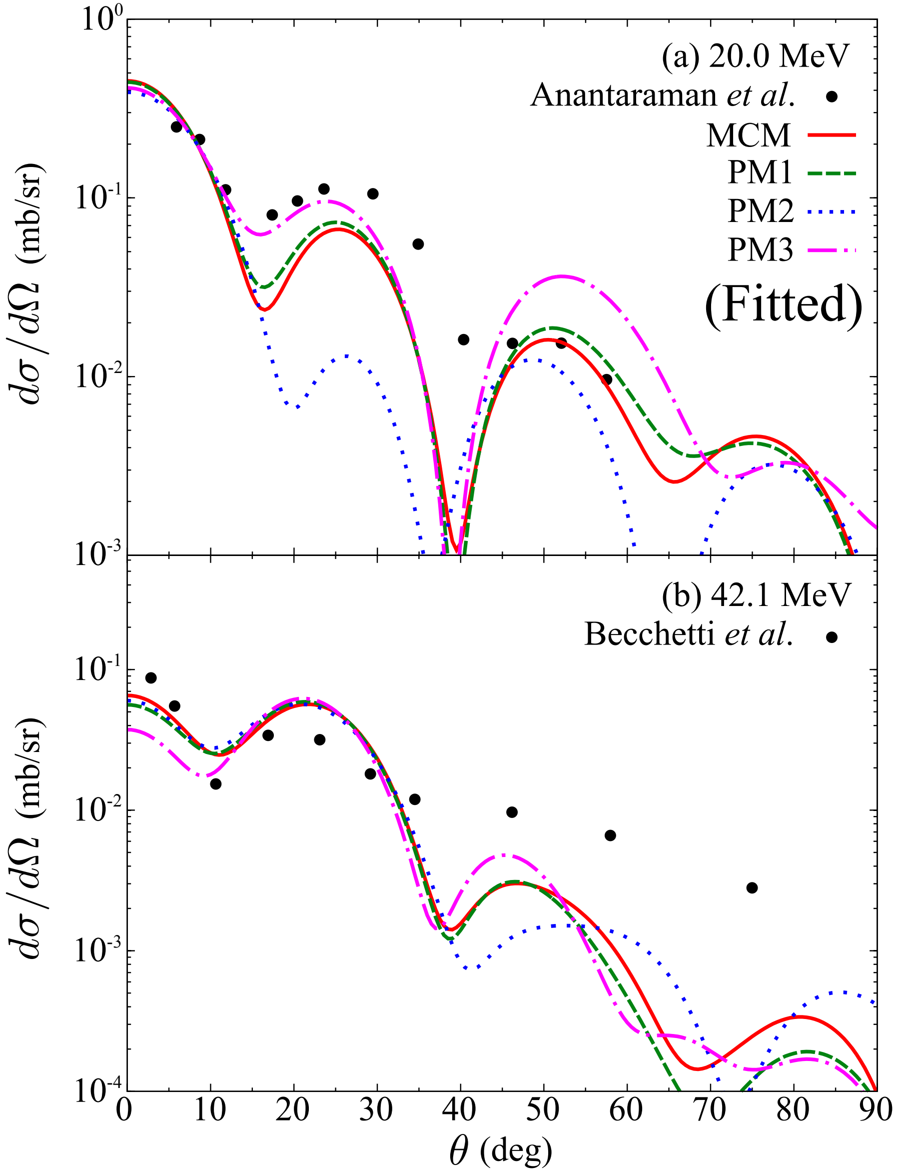
$$V_{\alpha A}^{(\mathrm{N})}(r) = -\frac{V_0}{1 + \exp\left(\frac{r - r_0}{a_0}\right)}$$

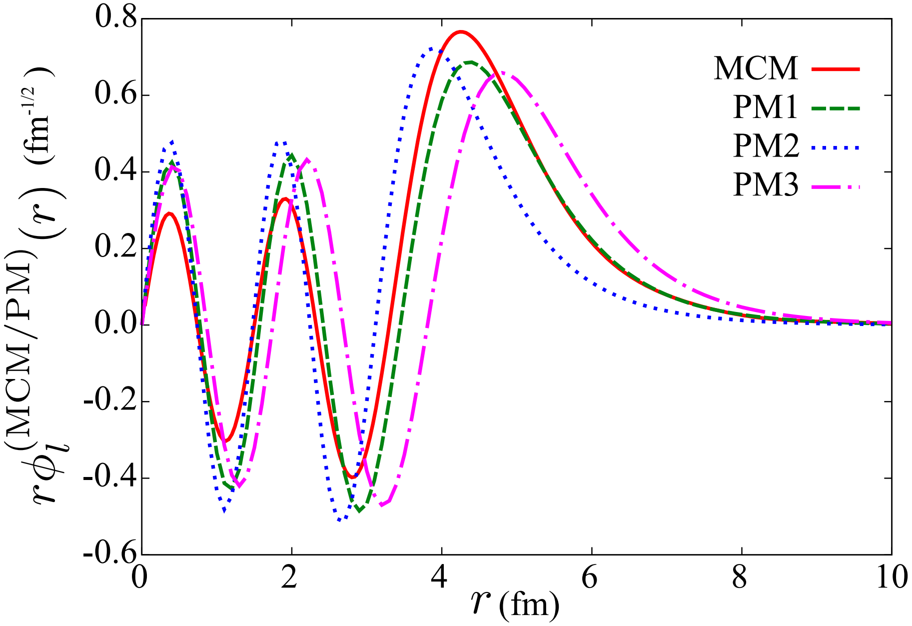
	r_0 (fm)	a_0 (fm)
PM1	$1.25 \times (16)^{1/3}$	0.76
PM2	$1.25 \times (16)^{1/3}$	0.52
PM3	$1.40 \times (16)^{1/3}$	0.85

V_0: adjusted to reproduce the binding energy of 4.73 MeV.

Result 2 T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

Transfer CS with PM



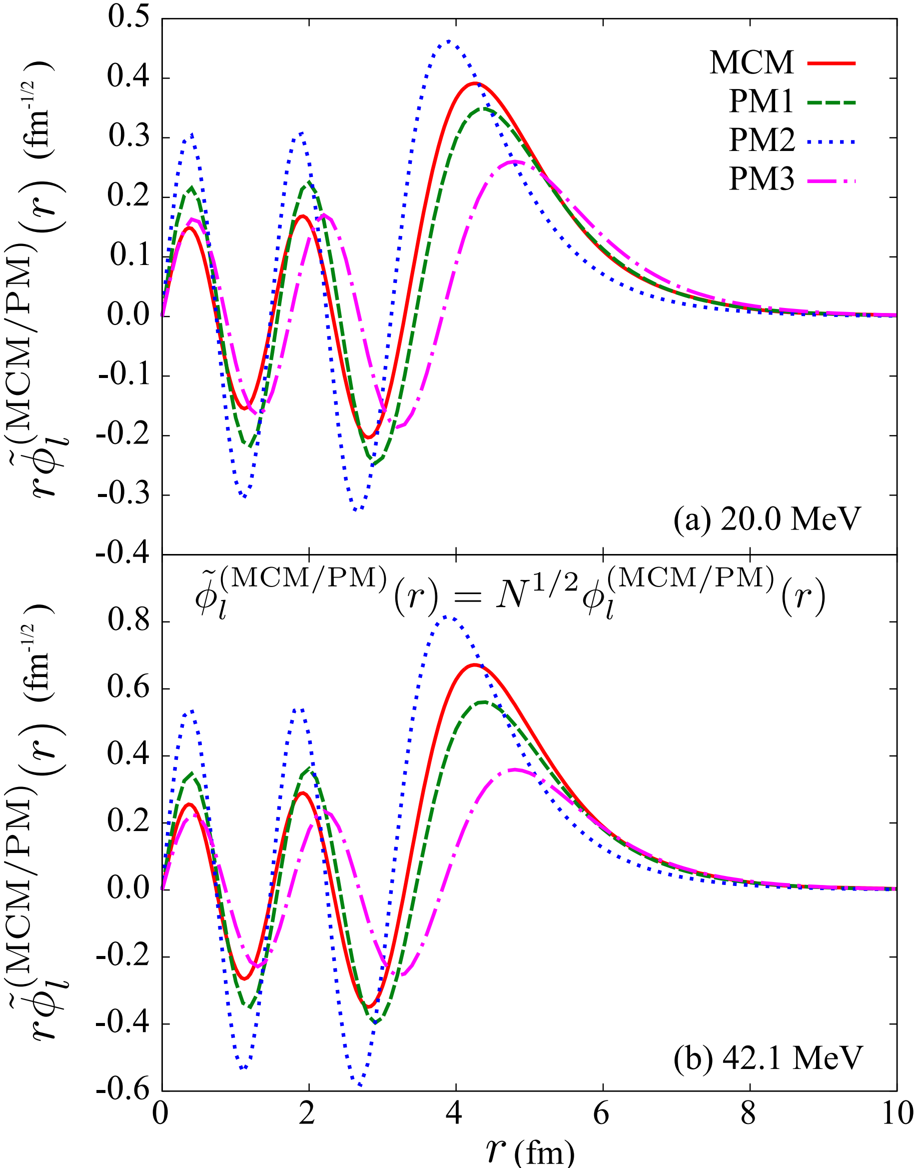


- **PM1** provides the CS similar to **MCM**'s.
- Failure of the result with **PM2** and **PM3**.

N. Anantaraman *et al.,* Nucl. Phys. **A313**, 445 (1979).
F. D. Becchetti *et al.,* Nucl. Phys. **A303**, 313 (1978).

Result 2 T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

WF with normalization



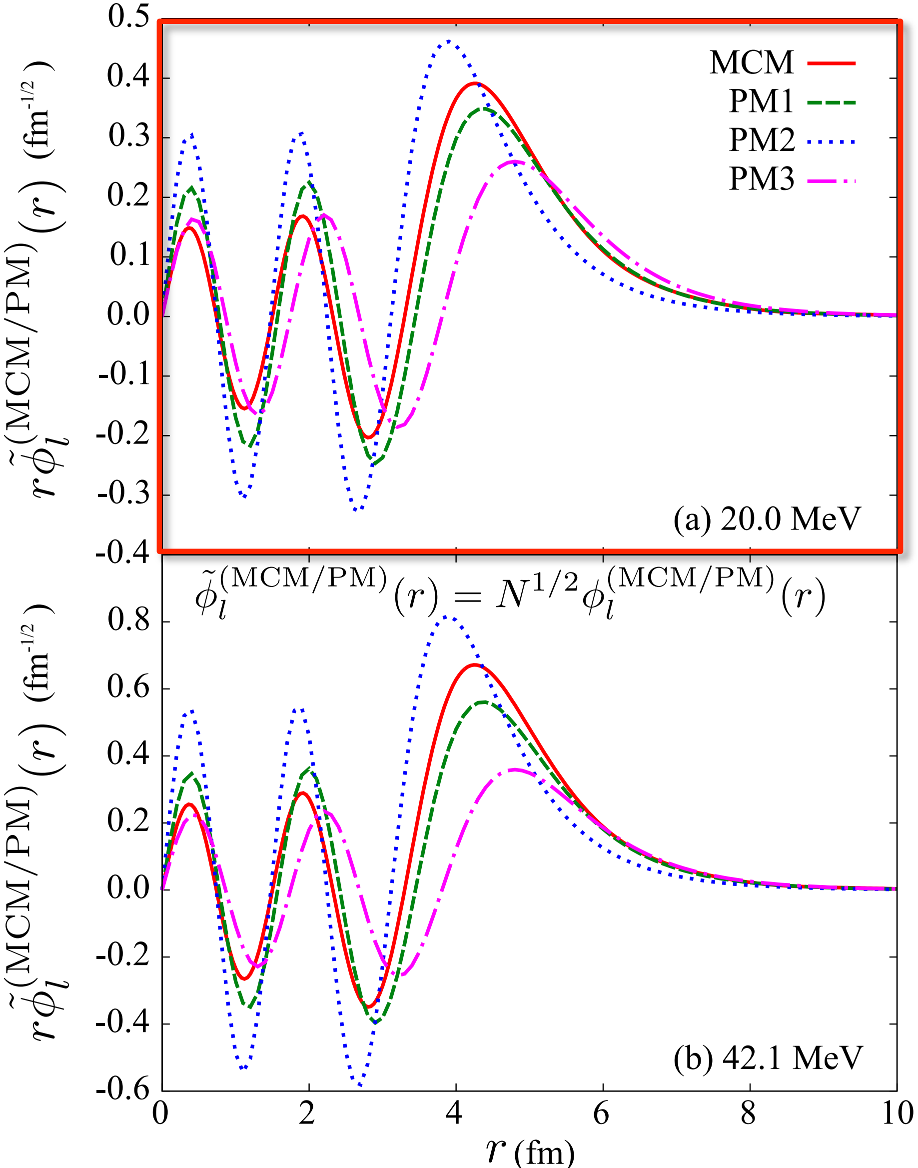
$$\tilde{\phi}_l^{(\mathrm{MCM})}(r) = (N_{\mathrm{MCM}})^{1/2}\,\phi_l^{(\mathrm{MCM})}(r),$$
$$\tilde{\phi}_l^{(\mathrm{PM})}(r) = (N_{\mathrm{PM}})^{1/2}\,\phi_l^{(\mathrm{PM})}(r)$$

- WFs are multiplied by the normalization factors.
- Similar behaviors at **the surface region**.

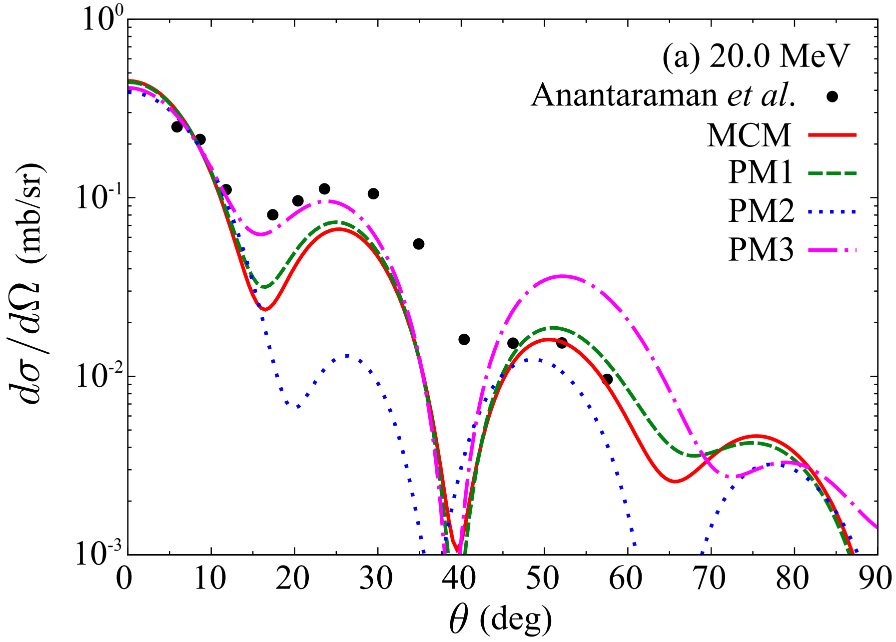
Result 2

T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

WF with normalization



$$\tilde{\phi}_l^{(\mathrm{MCM/PM})}(r) = N^{1/2}\phi_l^{(\mathrm{MCM/PM})}(r)$$



- ■ **PM1** and **PM3** give the CSs which are consistent with the measured data ($\theta \lesssim 40°$).
- ■ **PM2** gives the small CS and the diffraction pattern different from others.

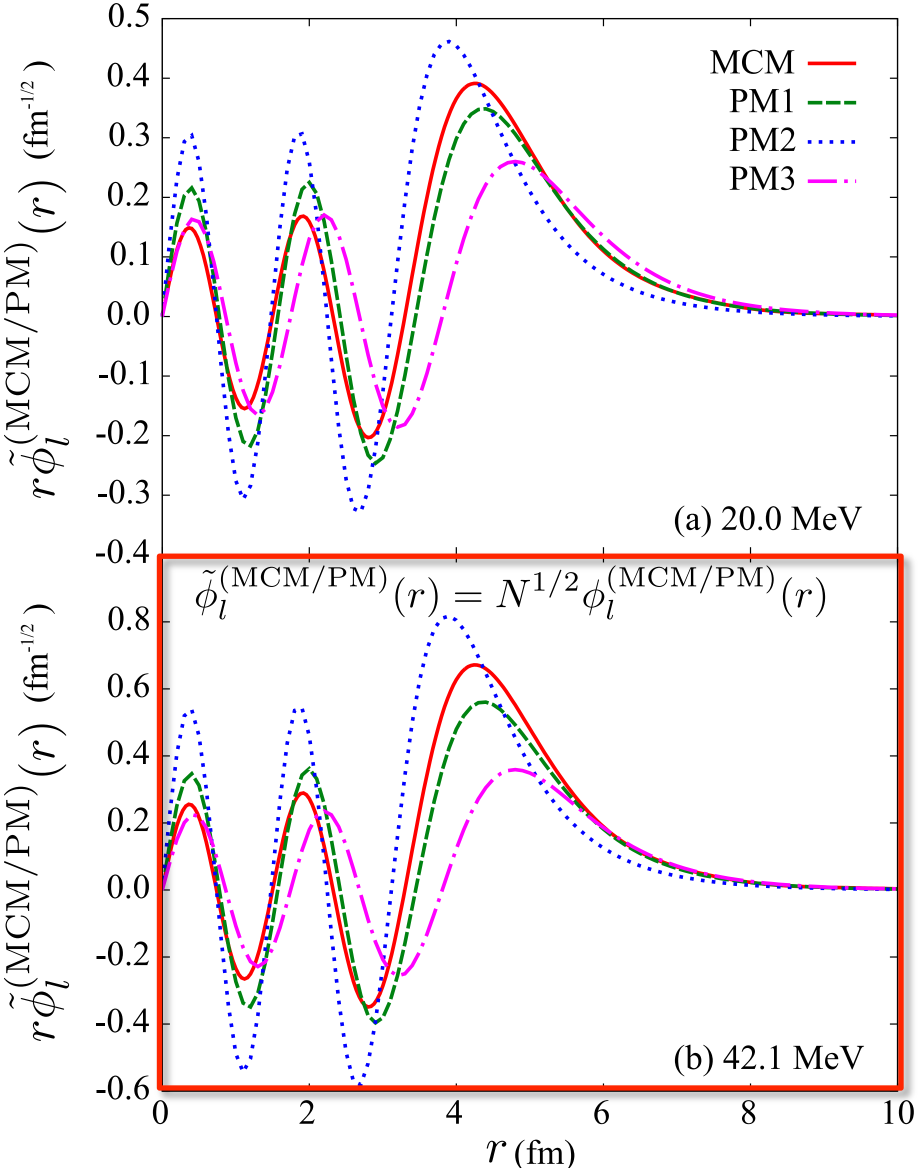
→ **CS probes WF at $r \gtrsim 5$ fm**, in which **PM1** and **PM3** behave similarly, whereas **PM2** has the small amplitude.

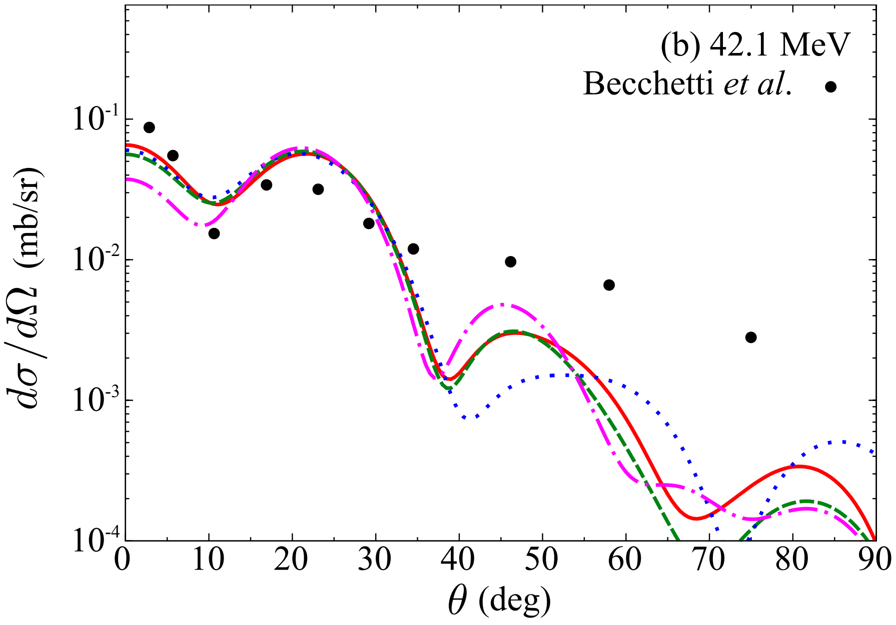
Result 2 T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

WF with normalization

- **PM1** and **PM2** give CSs consistent with the measured data ($\theta \lesssim 40^\circ$).
- **PM3** gives the small CS at $\theta = 0^\circ$.

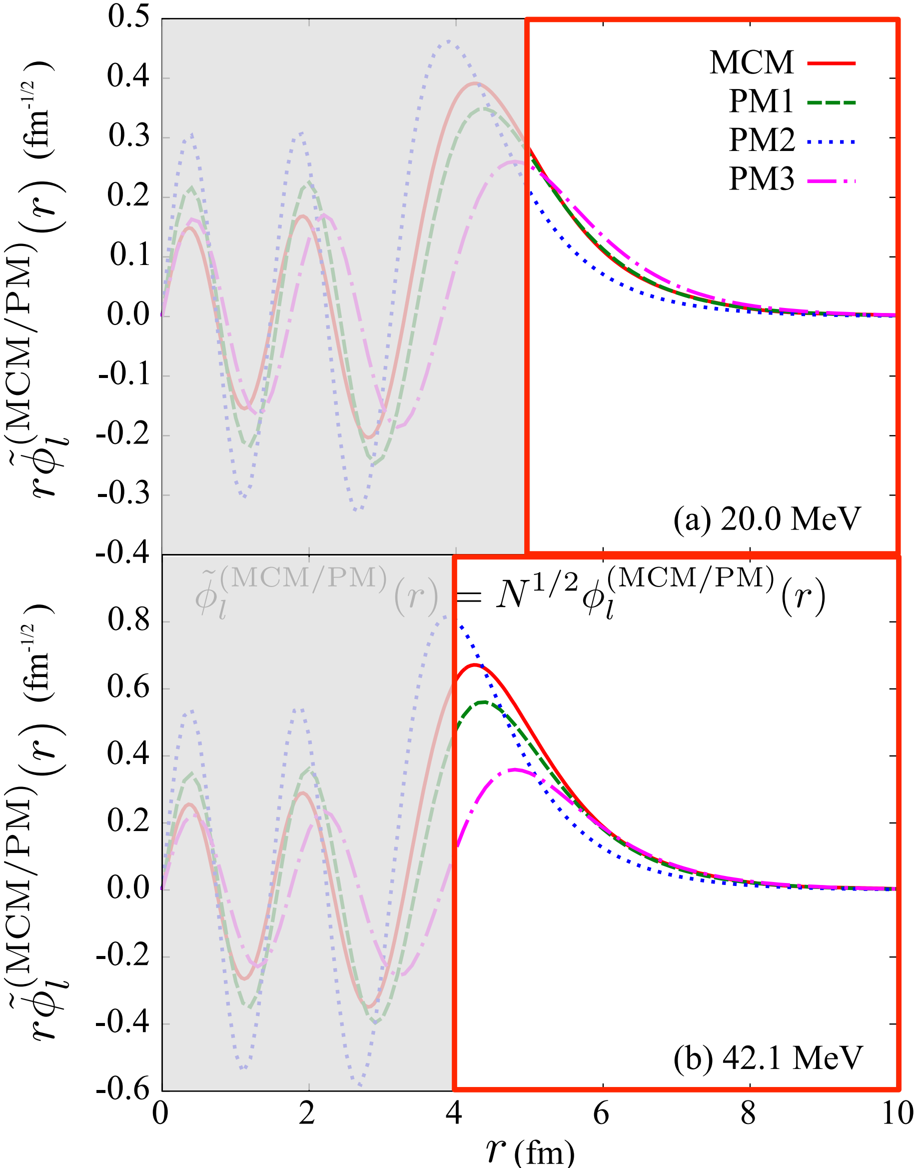
→ **CS probes WF at $r \gtrsim 4$ fm**, in which the integrated values of WF for **PM1** and **PM2** are consistent with each other, whereas that for **PM3** is significantly small.





Result 2 T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

WF with normalization

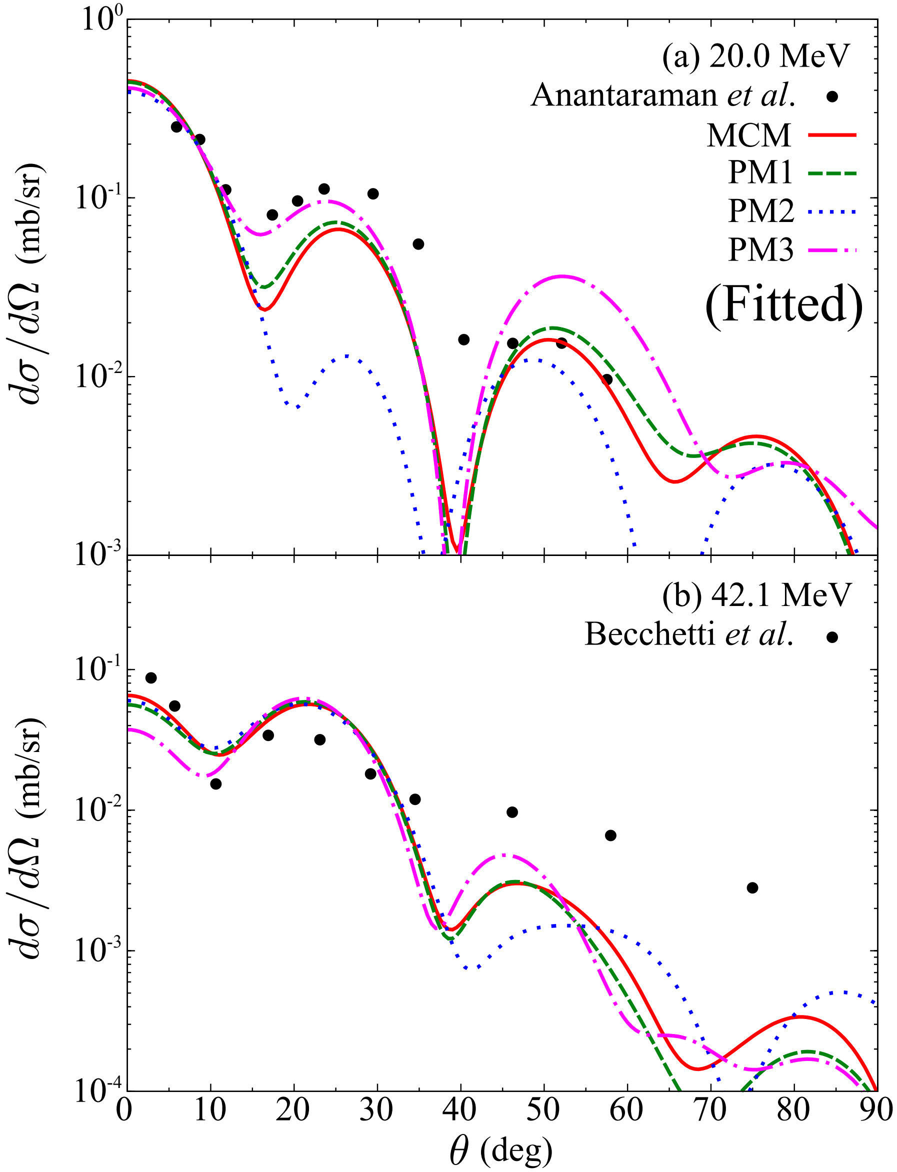


- Angular distributed CS at the forward angles ($\theta \lesssim 40^\circ$) can extract **NOT SF but only the surface manifestation of the WF**.

- The normalization factor for the improper WFs (**PM2** and **PM3**) involves **an artificial renormalization**, even if it has correct asymptotic behavior.

Result 2 T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

Normalization of cross section



$E_{\mathrm{Li}} = 20.0$ MeV

	MCM	PM1	PM2	PM3
N	0.261	0.258	0.407	0.156

※ 2.7 from DWBA analysis

N. Anantaraman *et al.*, Nucl. Phys. **A313**, 445 (1979).

$E_{\mathrm{Li}} = 42.1$ MeV

	MCM	PM1	PM2	PM3
N	0.769	0.667	1.276	0.297

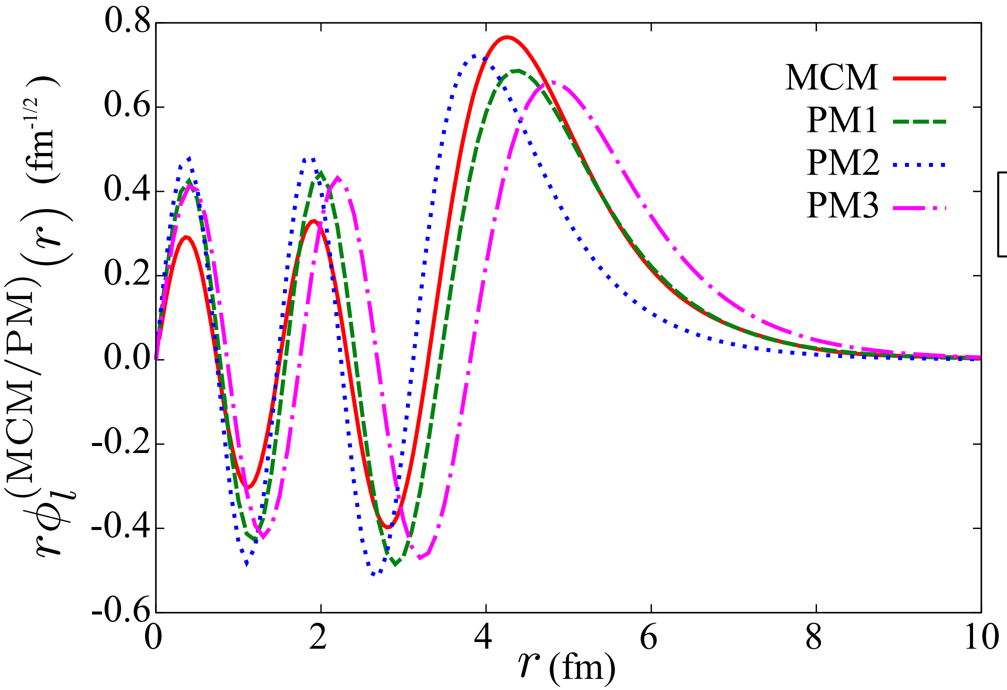
※ 2.59 from DWBA analysis

F. D. Becchetti *et al.*, Nucl. Phys. **A303**, 313 (1978).

Result 2

T. Fukui *et al.*, Phys. Rev. C **93**, 034606 (2016).

Normalization of cross section



$E_{\mathrm{Li}} = 20.0$ MeV

	MCM	PM1	PM2	PM3
N	0.261	0.258	0.407	0.156

※ 2.7 from DWBA analysis

N. Anantaraman *et al.*, Nucl. Phys. **A313**, 445 (1979).

Consistent

Artificial **enhancement** (**decrease**) due to improper behavior of the w.f.

$E_{\mathrm{Li}} = 42.1$ MeV

	MCM	PM1	PM2	PM3
N	0.769	0.667	1.276	0.297

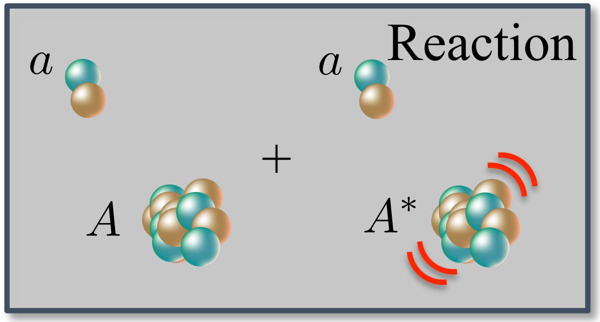
※ 2.59 from DWBA analysis

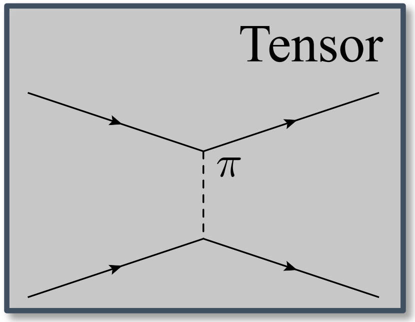
F. D. Becchetti *et al.*, Nucl. Phys. **A303**, 313 (1978).

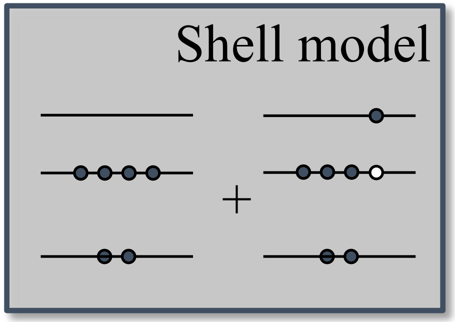
Summary

CC method

- A common concept in nuclear physics.

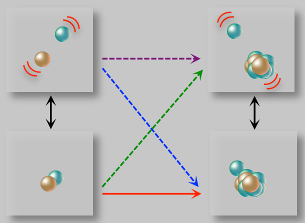






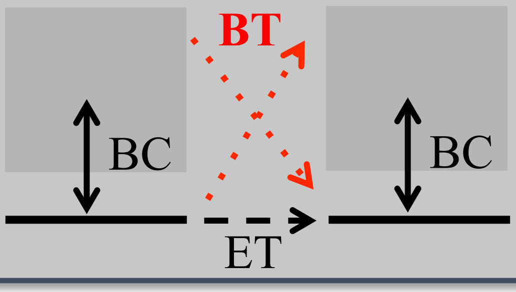
CCBA analyses

Analyses of $^{8}\mathrm{B}(d, n)^{9}\mathrm{C}$ and $^{16}\mathrm{O}(^{6}\mathrm{Li}, d)^{20}\mathrm{Ne}$ with **CCBA**



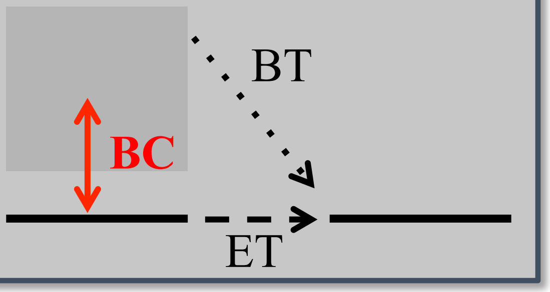
$^{8}\mathrm{B}(d, n)^{9}\mathrm{C}$

Small BC effect.
The BT is important.



$^{16}\mathrm{O}(^{6}\mathrm{Li}, d)^{20}\mathrm{Ne}$

Only the BC plays an important role.



Why is the breakup effect large?

Why opposite?

→ **Explained in detail in T. Fukui *et al.*, Phys. Rev. C 91, 014604 (2015).**

3. 3. Future work
(transfer to unbound state)

Future work

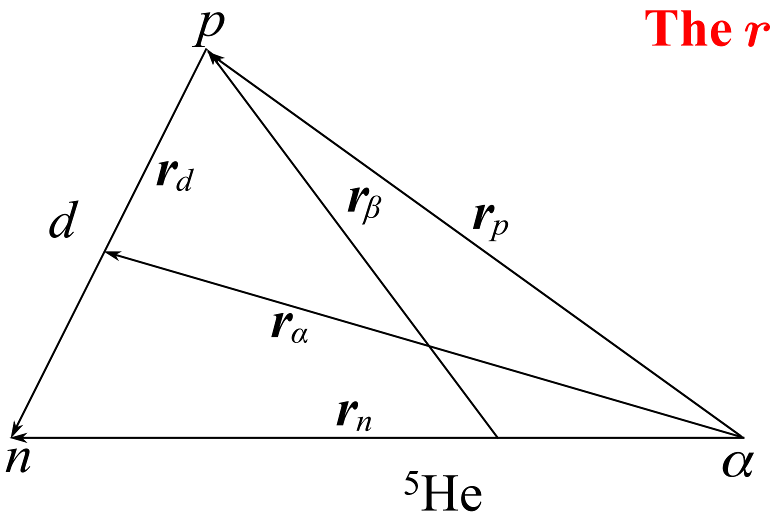
Transfer reaction to unbound state (ex. ^{4}He(*d*, *p*)^{5}He)

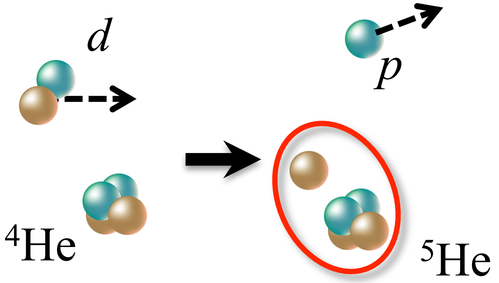
- The transition matrix of the post-form representation for (*d*, *p*) reaction

$$T^{(\mathrm{post})}_{\mathrm{DWBA}} = \left\langle \chi^{(-)}_{\beta}\psi_n \middle| V_{pn} \middle| \psi_d \chi^{(+)}_{\alpha} \right\rangle$$

$$= \int d\boldsymbol{r}_\alpha \int d\boldsymbol{r}_d \chi^{*(-)}_{\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_d) \underbrace{\psi^*_n(\boldsymbol{r}_\alpha, \boldsymbol{r}_d)}_{\text{oscillate}} \underbrace{V_{pn}(\boldsymbol{r}_d)\psi_d(\boldsymbol{r}_d)}_{\text{attenuate}} \chi^{(+)}_{\alpha}(\boldsymbol{r}_\alpha).$$

The r_α integration does not converge!





Future work

Previous approaches

- Some treatments have been suggested under some approximations:

ZR

(1) Introduce “convergence factor” $e^{-\gamma r}$, and then take $\gamma \to 0$.

R. Huby and J. R. Mines, Rev. Mod. Phys. **37**, 406 (1965).

(2) Integrate in the complex plane with $e^{-\gamma r}$.

C. M. Vincent and H. T. Fortune, Phys. Rev. C **2**, 782 (1970).

(3) Divide *T*-matrix into three parts with an channel radius.

G. Baur and D. Trautmann, Phys. Rep. **25**, 293 (1976).

(4) Approximate it as a bound state.

- More precise treatments

(5) Reduce the dimension to surface integration with an channel radius.

V. E. Bunakov, Nucl. Phys. **A140**, 241 (1970).

(6) Modification of (5) with CDCC framework.

A. M. Mukhamedzhanov, Phys. Rev. C **84**, 044616 (2011).

Future work

New approach

- The transition matrix of the post-form representation for (*d*, *p*) reaction

$$T^{(\mathrm{post})}_{\mathrm{DWBA}} = \left\langle \chi^{(-)}_{\beta}\psi_n \middle| V_{pn} \middle| \psi_d\chi^{(+)}_{\alpha} \right\rangle$$

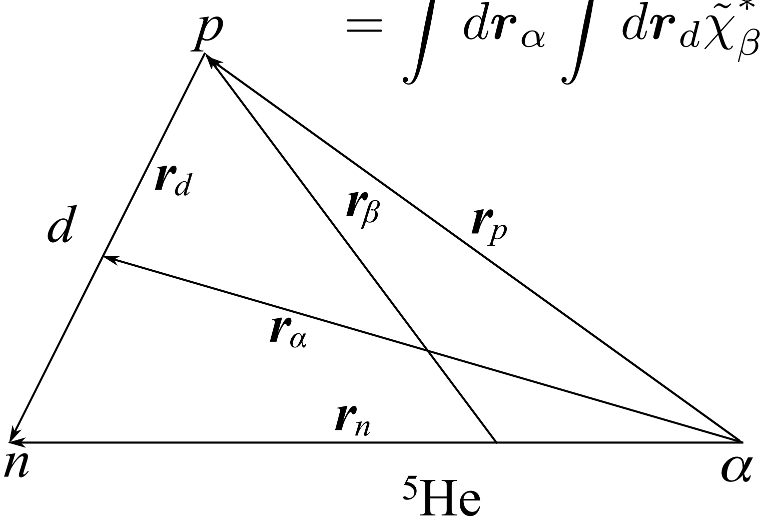
$$= \int d\boldsymbol{r}_\alpha \int d\boldsymbol{r}_d \chi^{*(-)}_{\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_d) \underset{\text{oscillate}}{\underline{\psi^*_n(\boldsymbol{r}_\alpha, \boldsymbol{r}_d)}} \underset{\text{attenuate}}{\underline{V_{pn}(\boldsymbol{r}_d)\psi_d(\boldsymbol{r}_d)}} \chi^{(+)}_{\alpha}(\boldsymbol{r}_\alpha).$$

The r_α integration does not converge!

- **The prior form**

$$T^{(\mathrm{prior})}_{\mathrm{DWBA}} = \left\langle \tilde{\chi}^{(-)}_{\beta}\psi_n \middle| V_{n\alpha} \middle| \psi_d\chi^{(+)}_{\alpha} \right\rangle$$

$$= \int d\boldsymbol{r}_\alpha \int d\boldsymbol{r}_d \tilde{\chi}^{*(-)}_{\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_d) \underset{\text{oscillate}}{\underline{\psi^*_n(\boldsymbol{r}_\alpha, \boldsymbol{r}_d)}} \underset{\text{attenuate}}{\underline{V_{n\alpha}(\boldsymbol{r}_\alpha, \boldsymbol{r}_d)}} \underset{\text{attenuate}}{\underline{\psi_d(\boldsymbol{r}_d)}} \chi^{(+)}_{\alpha}(\boldsymbol{r}_\alpha).$$



These respectively attenuate for two independent coordinates.
→ **The integration does converge.**

Future work

New approach

- The transition matrix of the post-form representation for (d, p) reaction

$$T^{(\mathrm{post})}_{\mathrm{DWBA}} = \left\langle \chi^{(-)}_{\beta} \psi_n \middle| V_{pn} \middle| \psi_d \chi^{(+)}_{\alpha} \right\rangle$$

The r_α integration does not converge.

$$= \int d\boldsymbol{r}_\alpha \int d\boldsymbol{r}_d \chi^{*(-)}_{\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_d) \underbrace{\psi^*_n(\boldsymbol{r}_\alpha, \boldsymbol{r}_d)}_{\text{oscillate}} \underbrace{V_{pn}(\boldsymbol{r}_d)\psi_d(\boldsymbol{r}_d)}_{\text{attenuate}} \chi^{(+)}_{\alpha}(\boldsymbol{r}_\alpha).$$

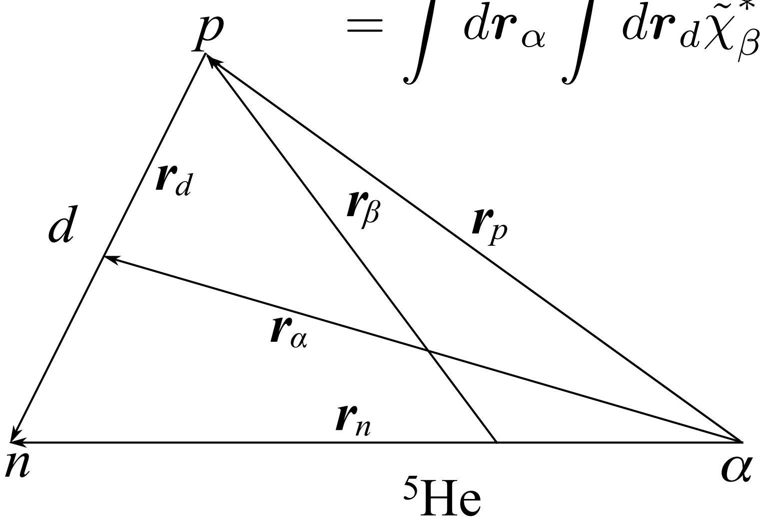
The distorted wave $\tilde{\chi}^{(-)}_{\beta}$ should be exact.

→ **The CCBA approach is necessary** for the final channel.

$$T^{(\mathrm{prior})}_{\mathrm{DWBA}} \to T^{(\mathrm{prior})}_{\mathrm{CCBA}}$$

- **The prior form**

$$T^{(\mathrm{prior})}_{\mathrm{DWBA}} = \left\langle \tilde{\chi}^{(-)}_{\beta} \psi_n \middle| V_{n\alpha} \middle| \psi_d \chi^{(+)}_{\alpha} \right\rangle$$



$$= \int d\boldsymbol{r}_\alpha \int d\boldsymbol{r}_d \tilde{\chi}^{*(-)}_{\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_d) \underbrace{\psi^*_n(\boldsymbol{r}_\alpha, \boldsymbol{r}_d)}_{\text{oscillate}} \underbrace{V_{n\alpha}(\boldsymbol{r}_\alpha, \boldsymbol{r}_d)}_{\text{attenuate}} \underbrace{\psi_d(\boldsymbol{r}_d)}_{\text{attenuate}} \chi^{(+)}_{\alpha}(\boldsymbol{r}_\alpha).$$

These respectively attenuate for two independent coordinates.

→ **The integration does converge.**