QCD近藤効果

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Contents

QCD Lagrangian

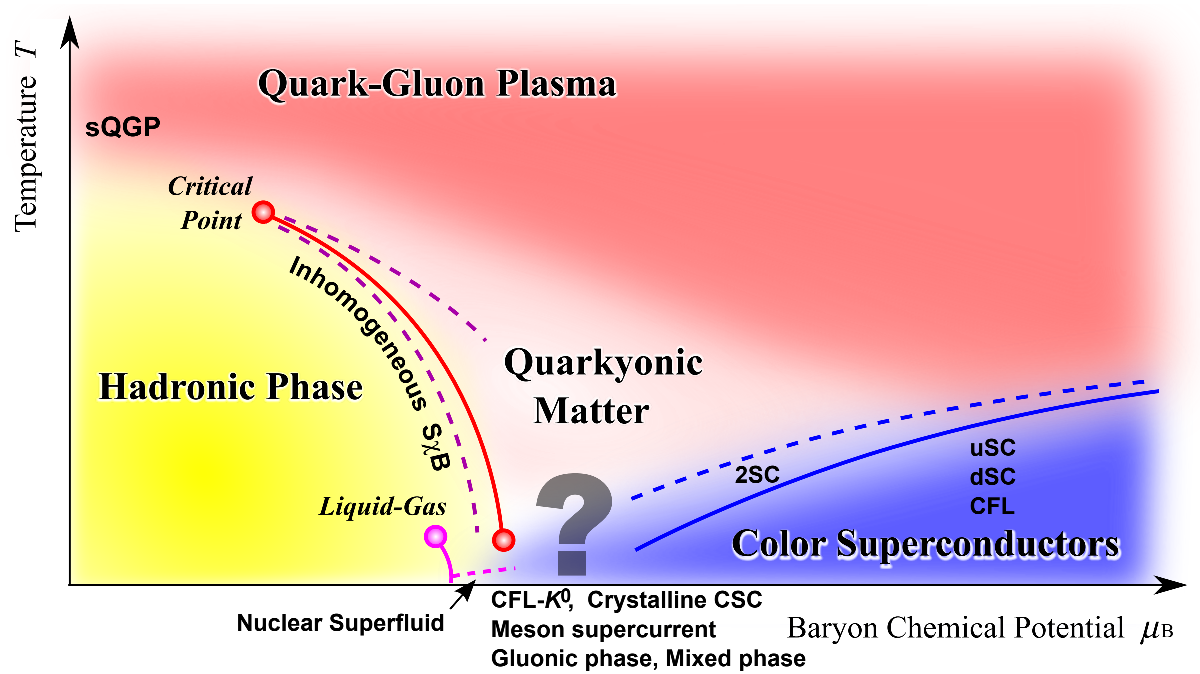
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} + \bar{q}\left(i\gamma_\mu D^\mu - M_q\right) q$$

$$D_\mu = \partial_\mu - igA^A_\mu T^A$$

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + gf^{ABC} A^B_\mu A^C_\nu$$

QCD phase diagram

K. Fukushima and T. Hatsuda, Rept. prog. Phys. (2011)



Quarks

Electron

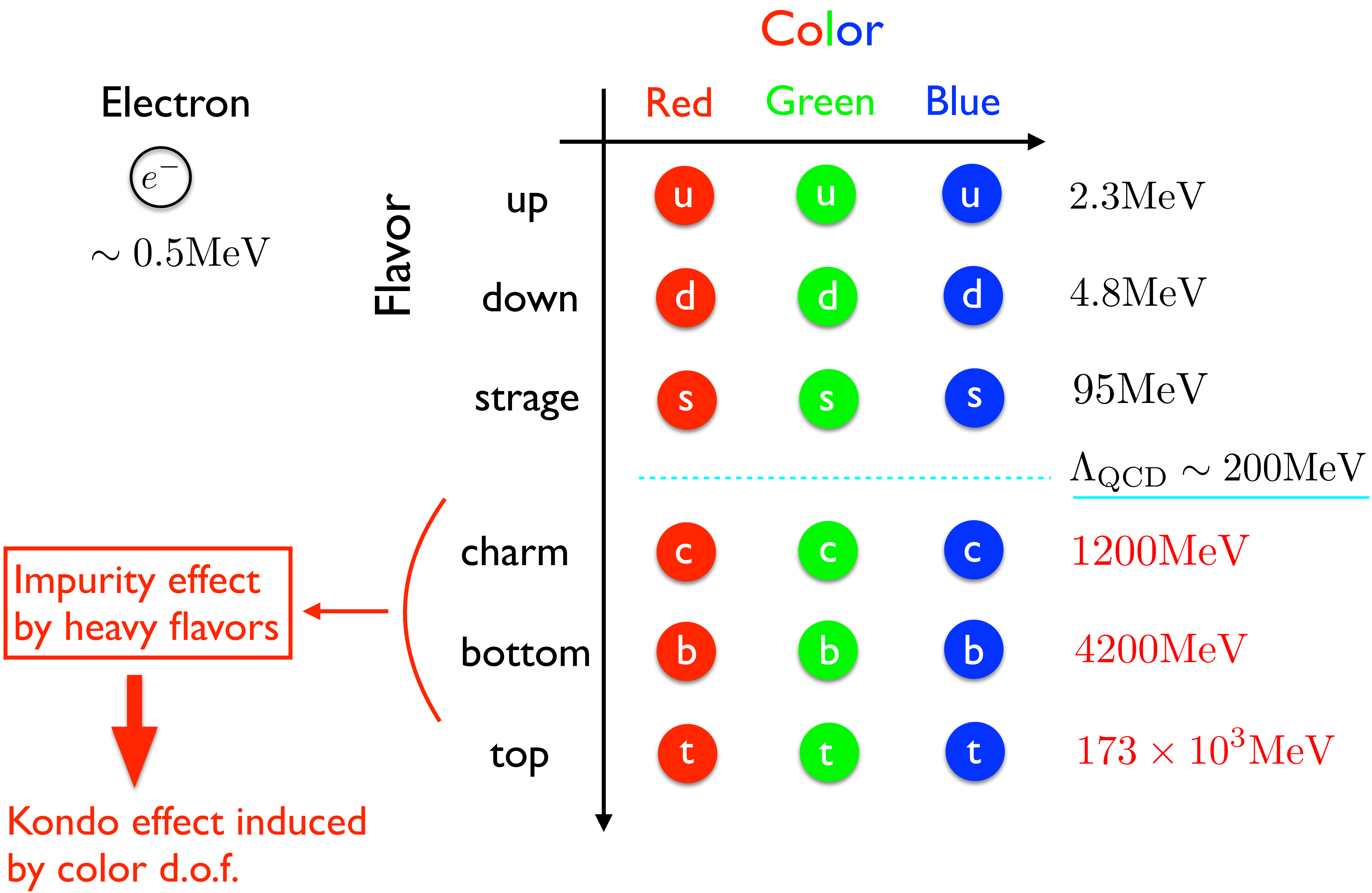
e^-

$\sim 0.5\text{MeV}$

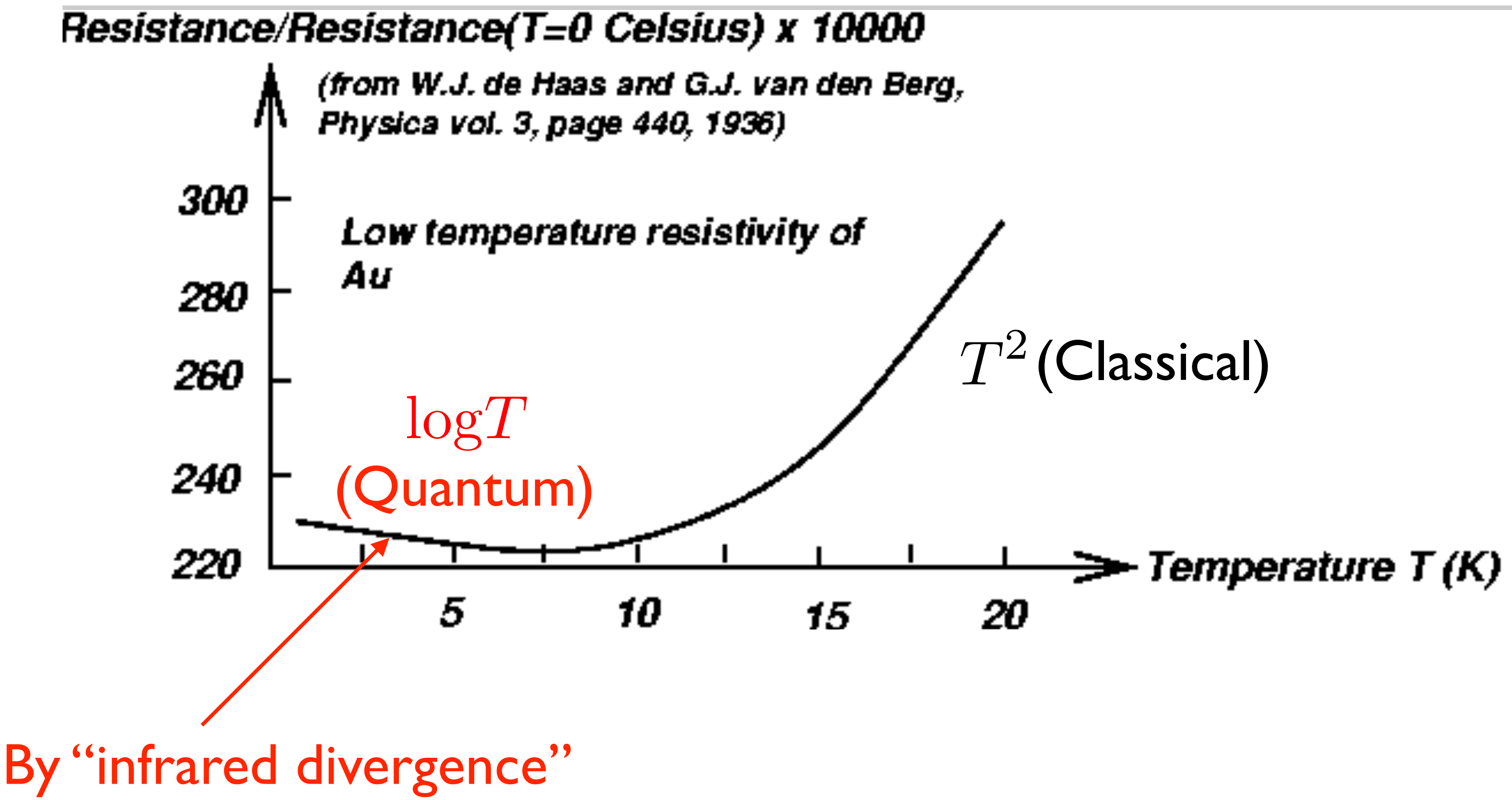
Flavor	Red	Green	Blue	
up	u	u	u	2.3MeV
down	d	d	d	4.8MeV
strage	s	s	s	95MeV
				$\Lambda_{\text{QCD}} \sim 200\text{MeV}$
charm	c	c	c	1200MeV
bottom	b	b	b	4200MeV
top	t	t	t	$173 \times 10^3\text{MeV}$

Color

Quarks


Color
Electron
e^-
$\sim 0.5\mathrm{MeV}$
Red
Green
Blue
Flavor
up
u u u
$2.3\mathrm{MeV}$
down
d d d
$4.8\mathrm{MeV}$
strage
s s s
$95\mathrm{MeV}$
$\Lambda_{\mathrm{QCD}} \sim 200\mathrm{MeV}$
charm
c c c
$1200\mathrm{MeV}$
Impurity effect by heavy flavors
bottom
b b b
$4200\mathrm{MeV}$
top
t t t
$173 \times 10^3\mathrm{MeV}$
Kondo effect induced by color d.o.f.

Kondo effect



Kondo effect is firstly observed in experiment as an enhancement of electrical resistivity of impure metals.

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

Jun Kondo
(1930-)

J. Kondo has explained the phenomenon based on the second order perturbation of interaction between conduction electron and impurity.

Conditions for the appearance of Kondo effect

0) Heavy impurity

i) Fermi surface

ii) Quantum fluctuation (loop effect)

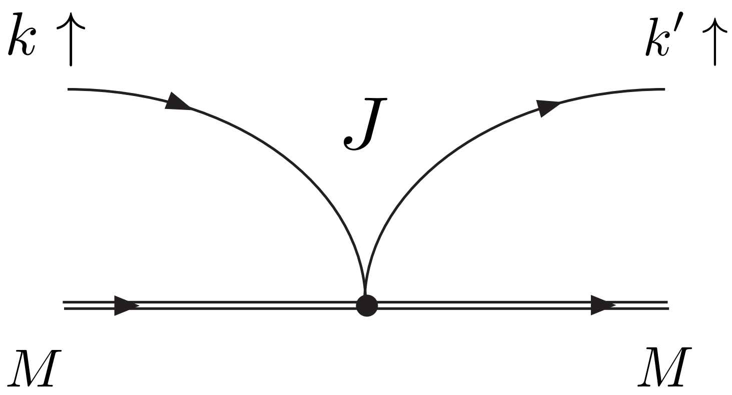
iii) Non-Abelian property of interaction
(spin-flip int.)

- s-d model (Kondo model)

$$H_{sd} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_{k,k'} J \vec{s}_{k'k} \cdot \vec{S}$$

- Scattering amplitude $T(k\uparrow \rightarrow k'\uparrow)$

Born term



$$T_{Born} = \langle k'\uparrow | J\vec{s}_{k'k} \cdot \vec{S} | k\uparrow\rangle = JS_z$$

with

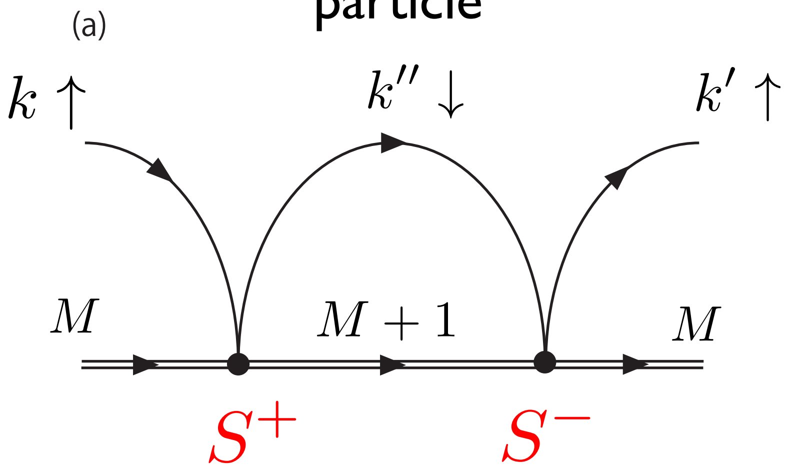
$$|k\uparrow\rangle = c^\dagger_{k\uparrow}|FS\rangle$$

$$\langle FS|c^\dagger_{k'\uparrow} c_{k\uparrow}|FS\rangle = f_F(\epsilon_k)\delta(k'-k)$$

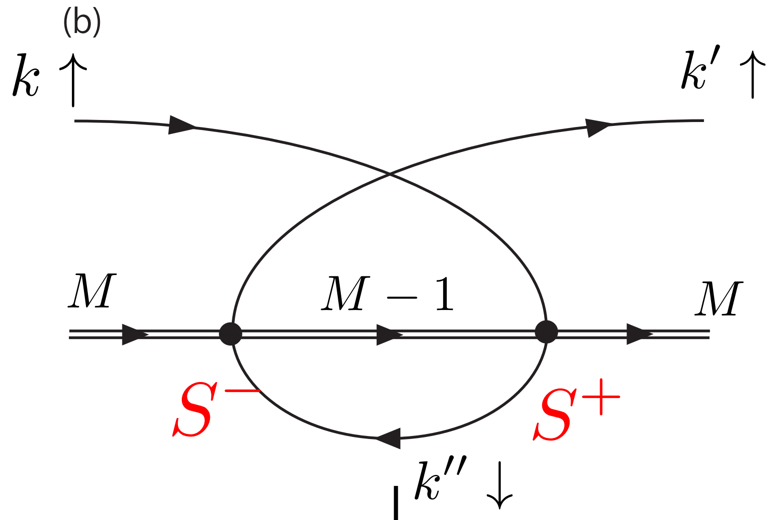
Second order perturbation theory

- only spin flip processes -

particle



hole



$$T_a^{(2)} = \langle k'\uparrow | \sum_{k''} \frac{\left(\frac{J}{2} S^- c^\dagger_{k'\uparrow} c_{k''\downarrow}\right)\left(\frac{J}{2} S^+ c^\dagger_{k''\downarrow} c_{k\uparrow}\right)}{\epsilon - \epsilon_{k''} + i\eta} | k\uparrow\rangle$$

$$T_b^{(2)} = \langle k'\uparrow | \sum_{k''} \frac{\left(\frac{J}{2} S^+ c^\dagger_{k''\downarrow} c_{k\uparrow}\right)\left(\frac{J}{2} S^- c^\dagger_{k'\uparrow} c_{k''\downarrow}\right)}{\epsilon_{k''} - \epsilon - i\eta} | k\uparrow\rangle$$

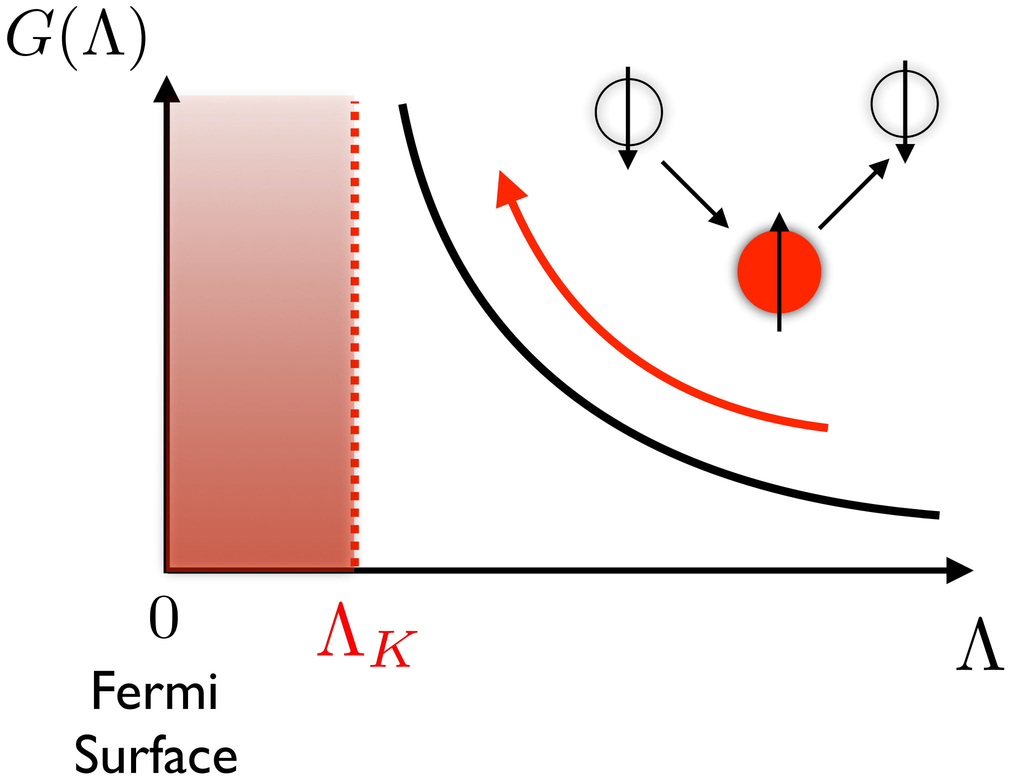
$$\epsilon = \epsilon_{k'} = \epsilon_k$$

$$\longrightarrow \quad T^{(2)}_{a+b} = \frac{J^2}{4}\left[S^+, S^-\right] \rho_F \log\left(\frac{D}{T}\right)$$

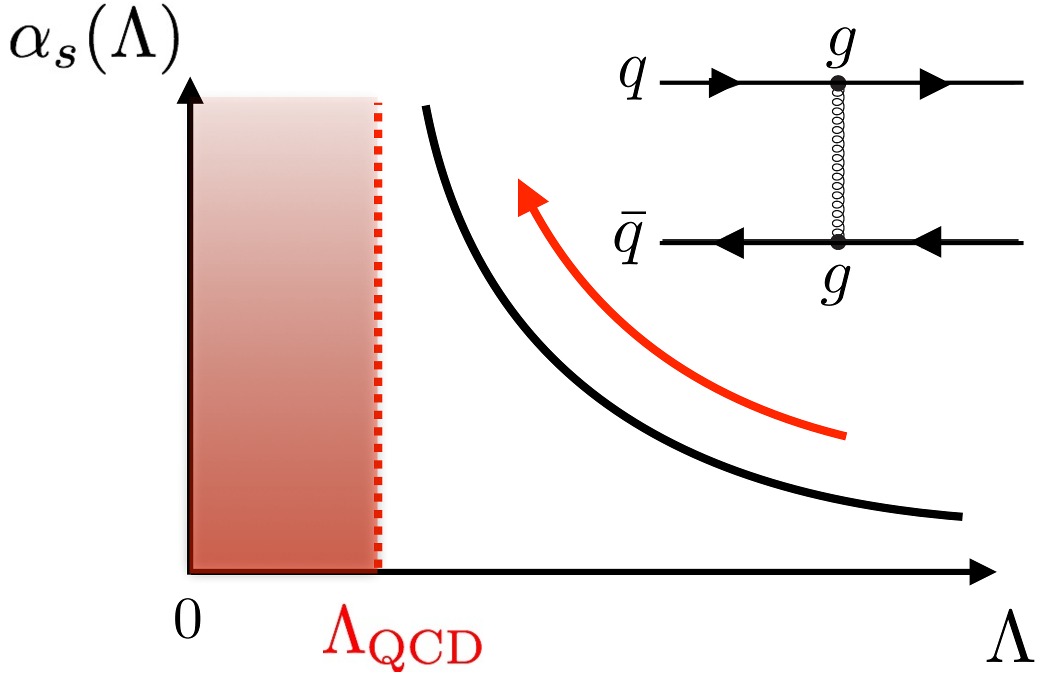
ρ_F : density of state on the Fermi surface, D : Bandwidth

Asymptotic freedom in Kondo effect and QCD

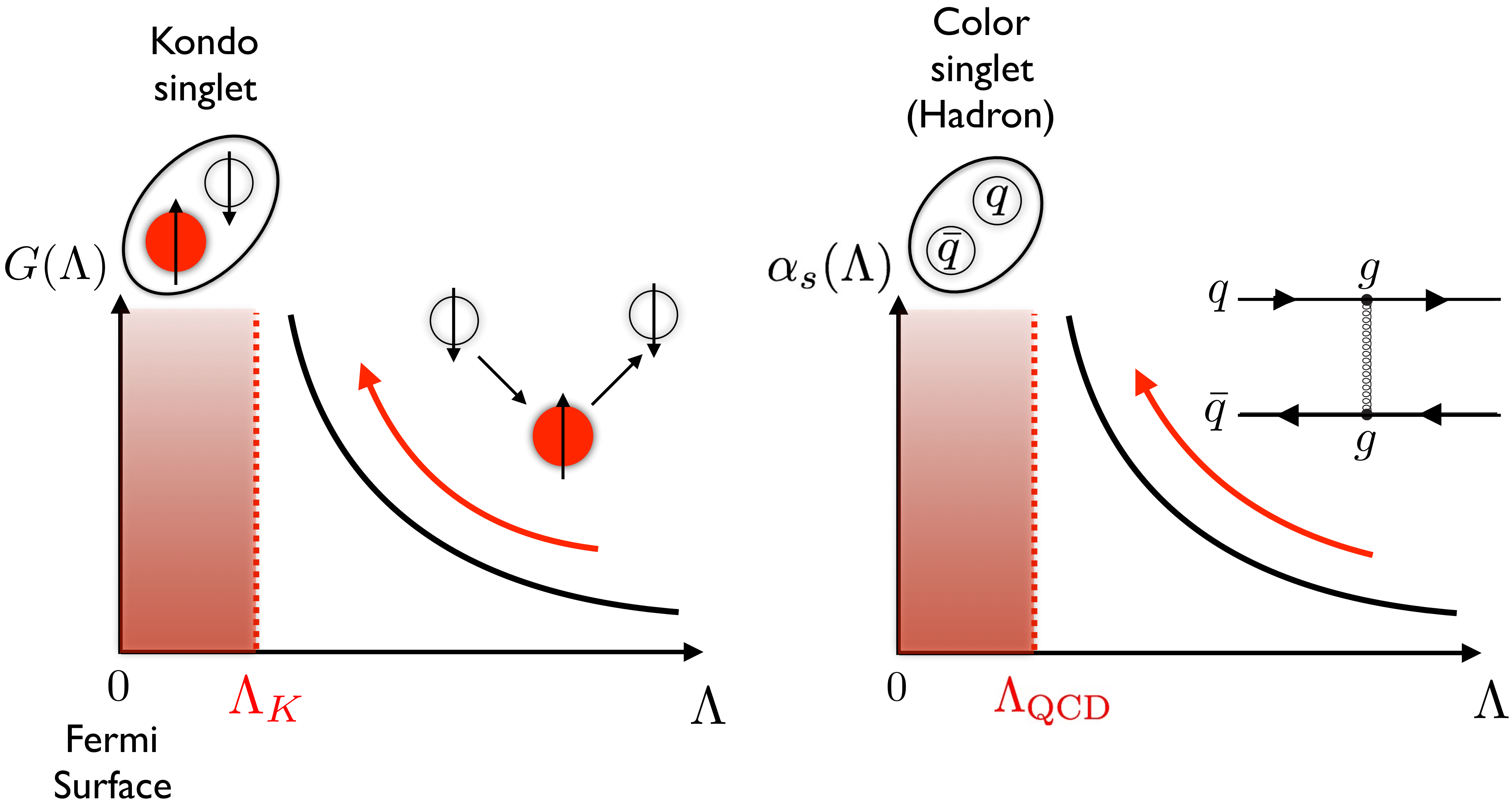
Kondo effect



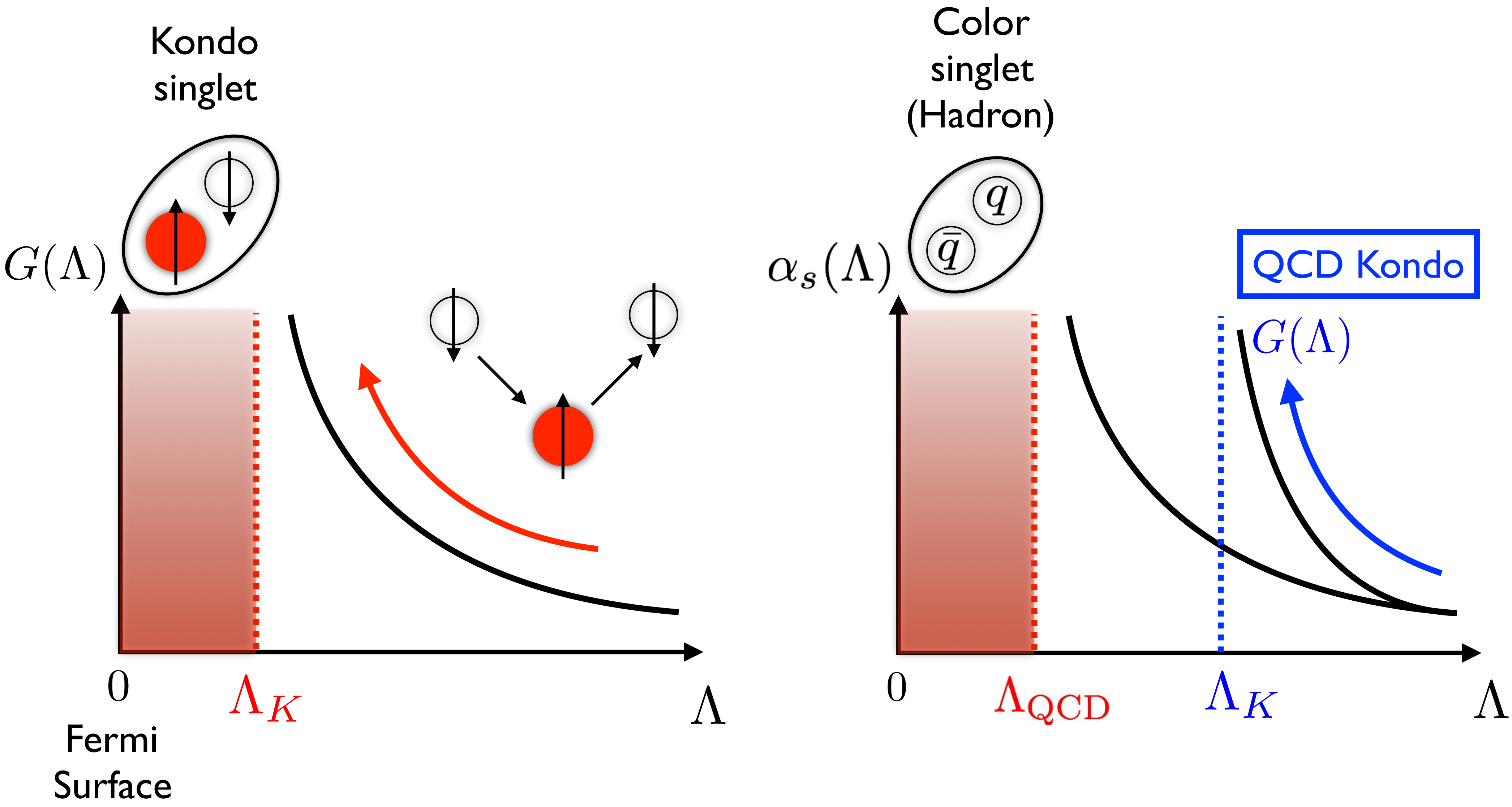
Running coupling of QCD



Asymptotic freedom in Kondo effect and QCD



Asymptotic freedom in Kondo effect and QCD



Conditions for the appearance of Kondo effect

0) Heavy impurity

i) Fermi surface

ii) Quantum fluctuation (loop effect)

iii) Non-Abelian property of interaction
(spin-flip int.)

Conditions for the appearance of QCD Kondo effect

0) Heavy quark impurity

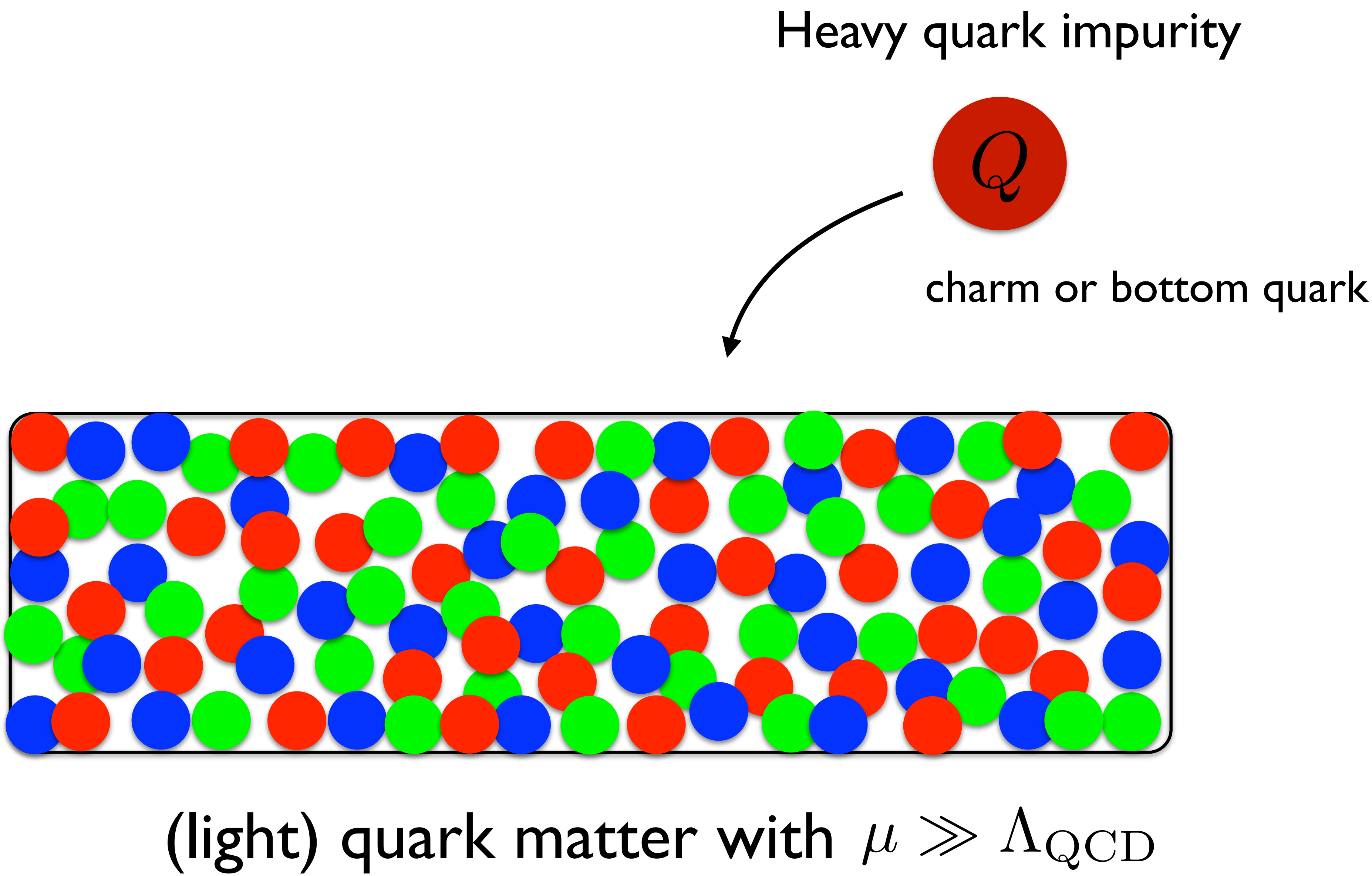
i) Fermi surface of light quarks

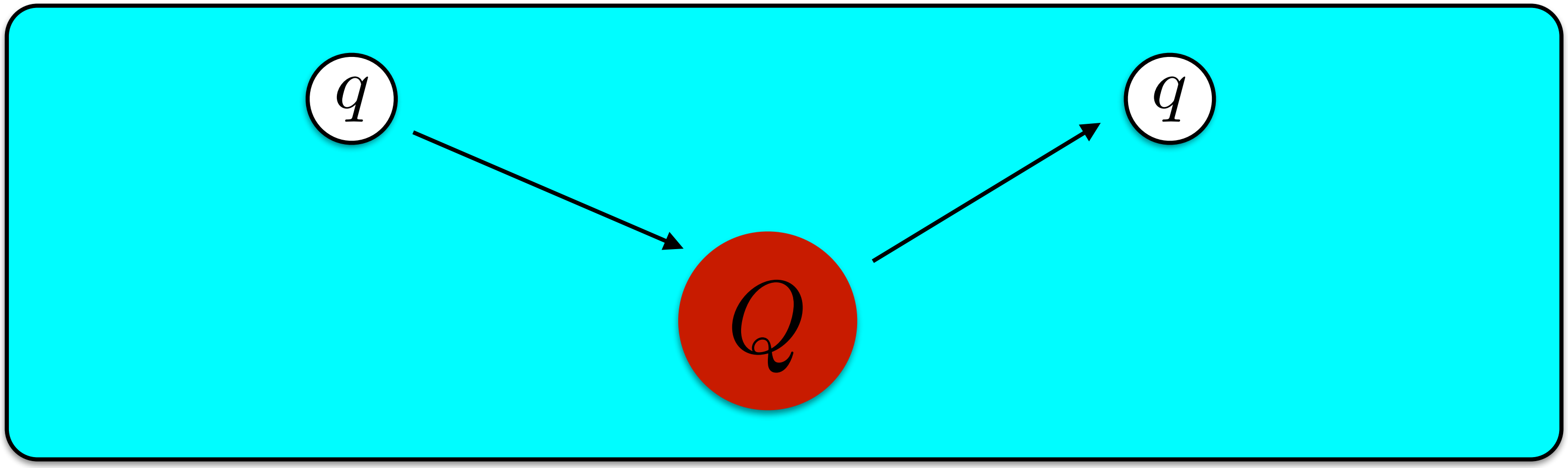
ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

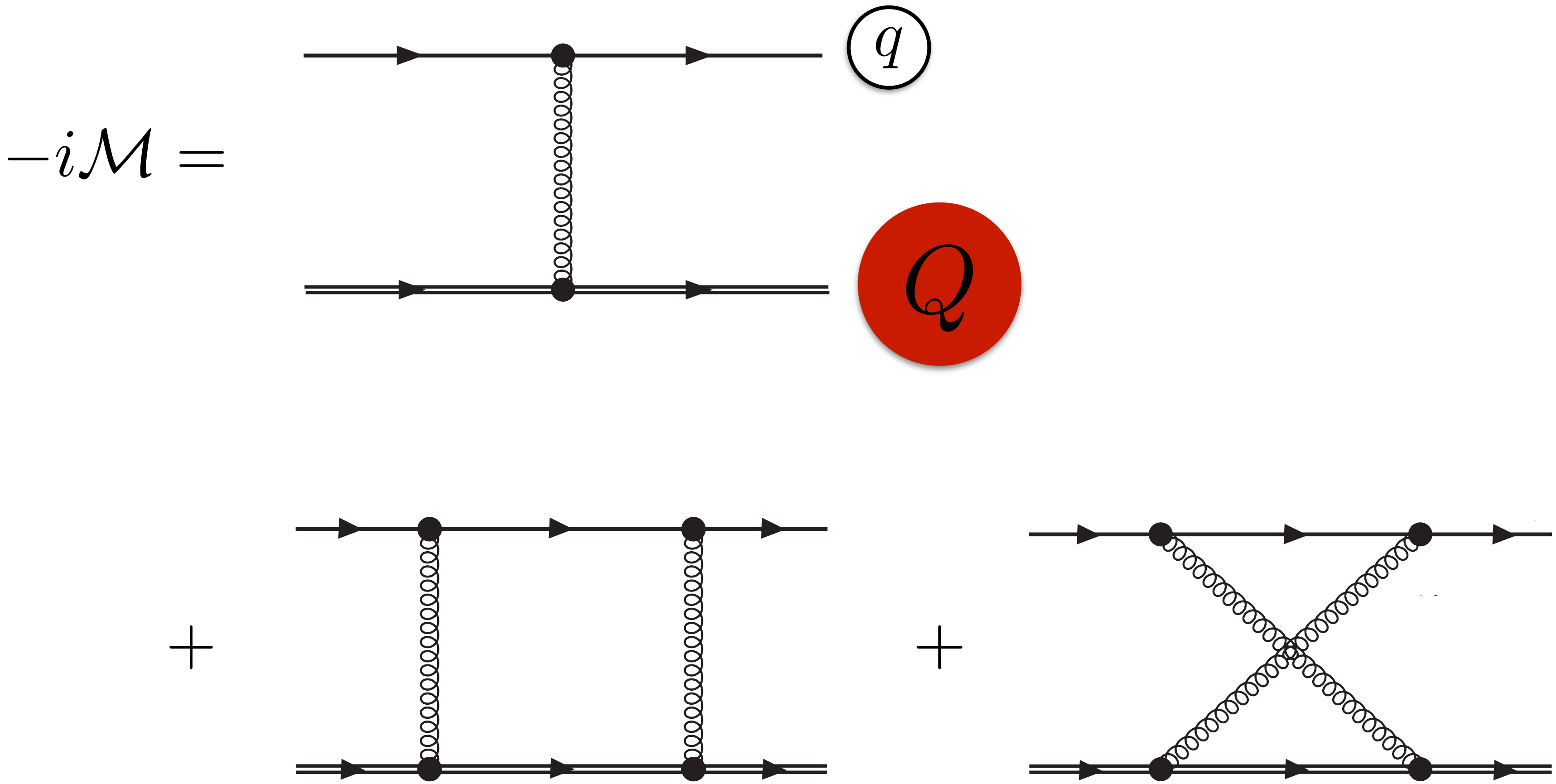
QCD Kondo effect

K. Hattori, K. Itakura, S. O. and S. Yasui, PRD92 (2015) 065003


Heavy quark impurity
Q
charm or bottom quark
(light) quark matter with $\mu \gg \Lambda_{\rm QCD}$



(light) quark matter with $\mu \gg \Lambda_{\rm QCD}$



Heavy quark: $M_Q \to \mathrm{large}$

Quark propagator at finite density (massless quark)

particle — hole

$$iS(q;\mu) = \frac{i\not{q}}{2\epsilon_q}\left(\underline{\theta(|\vec{q}| - k_F)\frac{1}{q^0 - \epsilon_q^+ + i\varepsilon}} + \underline{\theta(k_F - |\vec{q}|)\frac{1}{q^0 - \epsilon_q^+ - i\varepsilon}} \right.$$

$$\left. \underline{- \frac{1}{q^0 - \epsilon_q^- - i\varepsilon}}\right)$$

anti-particle

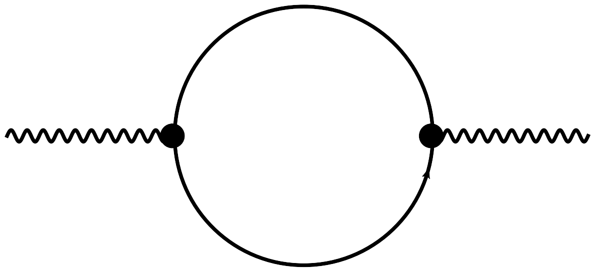
$$\epsilon_q = |\vec{q}|$$

$$\epsilon_q^{\pm} = \pm|\vec{q}| - \mu$$

Gluon propagator at finite density

- Screening effect

Vacuum polarization



$$= G P_T^{\mu\nu} + F P_L^{\mu\nu}$$

$$P_T^{00} = P_T^{0i} = 0$$

$$P_T^{ij} = \delta^{ij} - q^i q^j / |\vec{q}|^2$$

$$P_L^{\mu\nu} = q^\mu q^\nu / q^2 - g^{\mu\nu} - P_T^{\mu\nu}$$

Screening masses

$$G(q) = i \frac{\pi q^0}{2|\vec{q}|} m_D^2 \,, \qquad F(q) = m_D^2 = \frac{g^2 \mu^2}{2\pi^2}$$

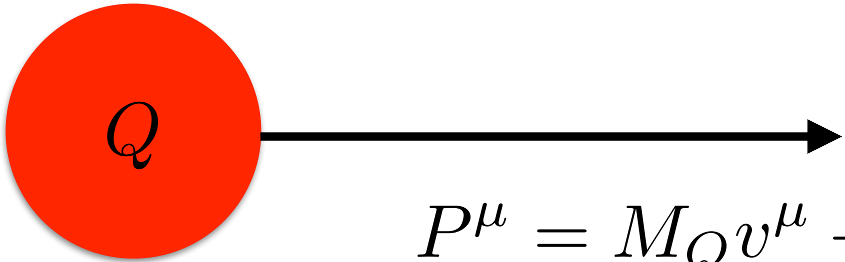
- Gluon propagator

$$i\mathcal{D}^{00} = \frac{i}{q^2 - m_D^2}$$

$$i\mathcal{D}^{ij} = \frac{i(\delta^{ij} - \hat{q}^i \hat{q}^j)}{(q^0)^2 - |\vec{q}|^2 - i\frac{\pi}{2} m_D^2 |q^0| / |\vec{q}|}$$

Heavy quark propagator and vertex

Heavy quark: $M_Q \to \text{large}$

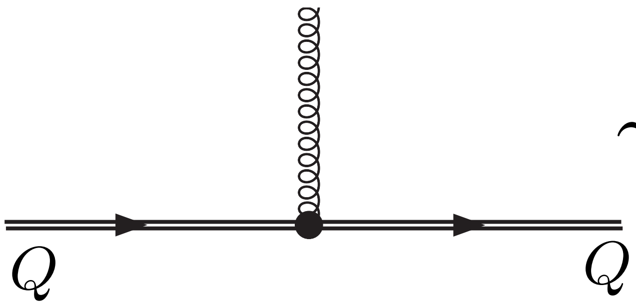


$$P^\mu = \underbrace{M_Q v^\mu}_{\text{on-shell}} + \underbrace{k^\mu}_{\text{off-shell}} \qquad v^\mu = (\sqrt{1+|\vec{v}|^2}, \vec{v})$$

- **Heavy quark propagator**

$$i\frac{\not{P} + M_Q}{P^2 - M_Q^2} \to i\frac{1}{v\cdot k}\frac{1+\not{v}}{2}$$

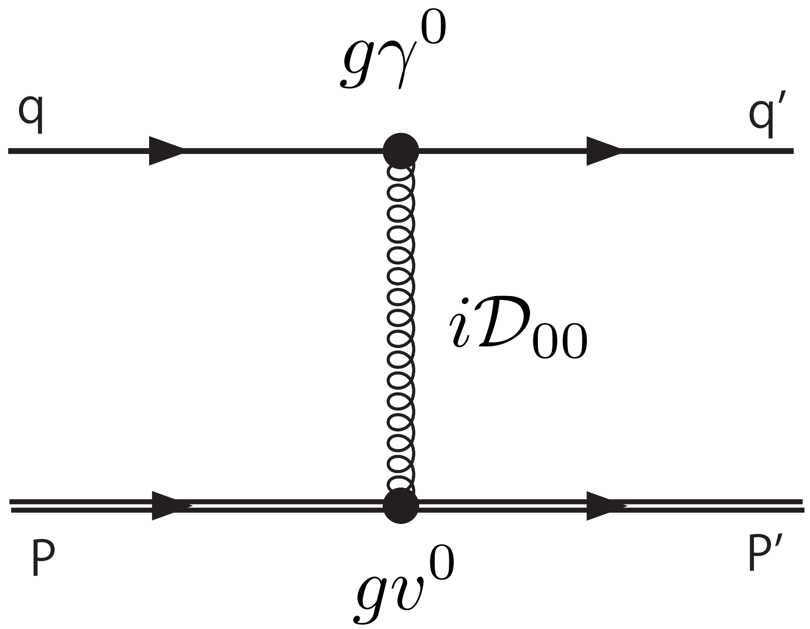
- **Vertex**



$$\gamma^\mu \to \frac{1+\not{v}}{2}\gamma^\mu\frac{1+\not{v}}{2} = v^\mu\frac{1+\not{v}}{2} \to \underbrace{v^\mu}_{\text{Spin-indep.}}$$

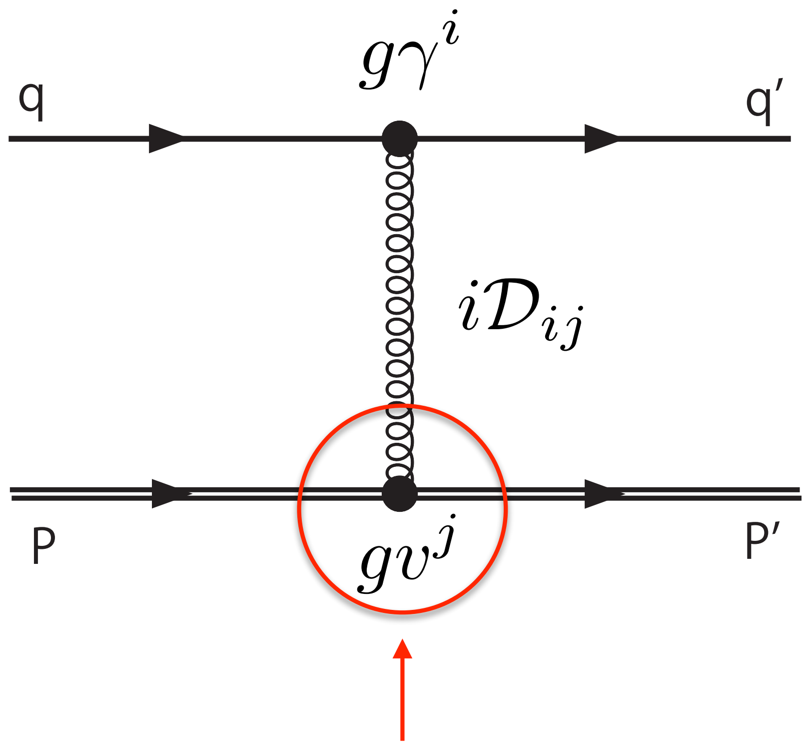
Gluon exchange interactions

Color electric interaction



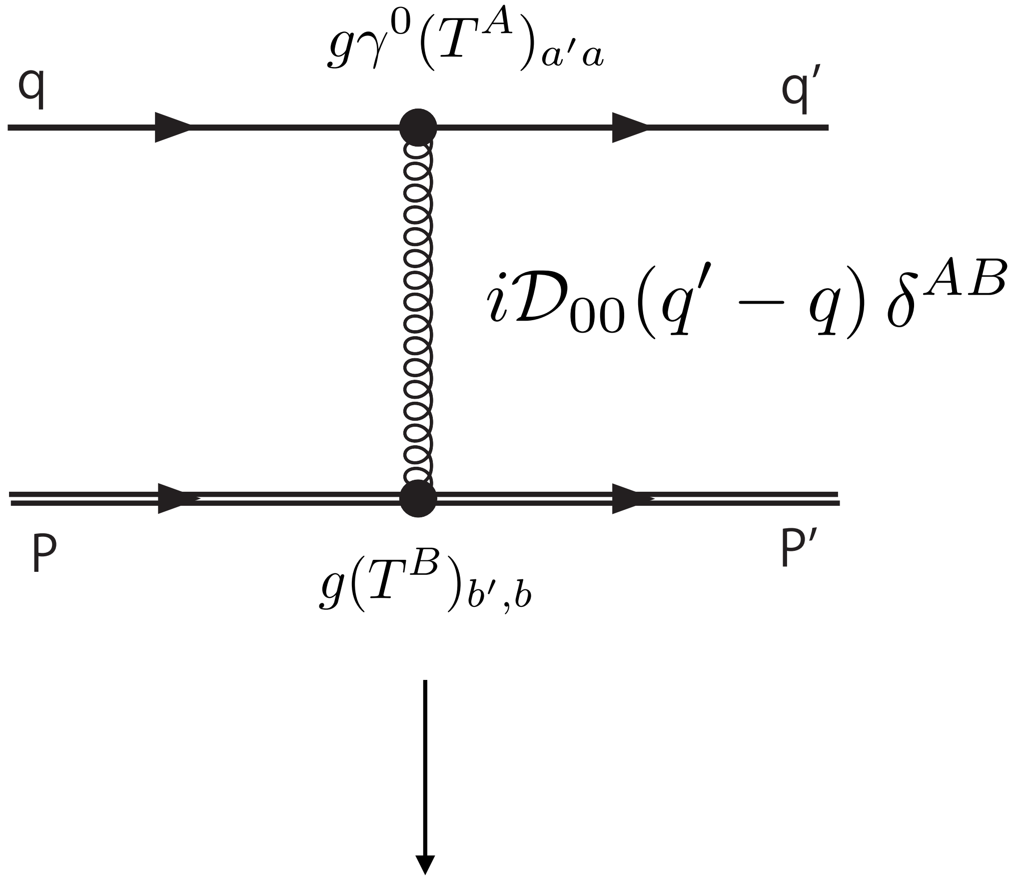
Dominant contribution

Color magnetic interaction



Suppressed by $1/M_Q$

Tree amplitude



$$-i\mathcal{M}_{Born} = -ig^2\mathcal{D}_{00}(q'-q)(T^A)_{a'a}(T^A)_{b'b}\gamma^0 \otimes \frac{1+\gamma^0}{2}$$

S-wave projection (partial wave decomposition)

$$-i\mathcal{M}_{Born}^{S-\text{wave}} = \frac{1}{2}\int_{-1}^{1} d(\cos\theta) P_{l=0}(\cos\theta)\left(-i\mathcal{M}_{Born}\right)$$

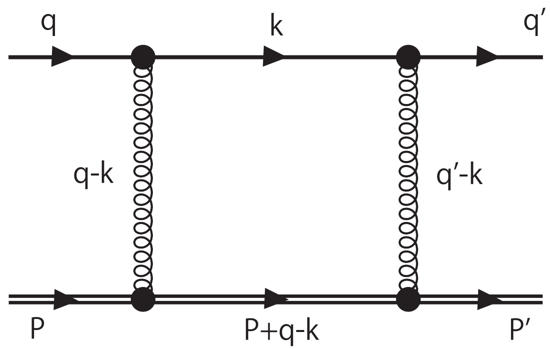
S-wave projected gluon exchange int.

$$\begin{aligned} G &\equiv \frac{1}{2}\int_{-1}^{1} d(\cos\theta) P_{l=0}(\cos\theta) g^2 i\mathcal{D}_{00}(q'-q) \\ &= \frac{1}{2}\int_{-1}^{1} d(\cos\theta) P_{l=0}(\cos\theta)\frac{-g^2}{(q'-q)^2 - m_D^2} \\ &= \frac{g^2}{4\mu^2}\log\frac{4\mu^2}{m_D^2} \end{aligned}$$

$$\underline{-i\mathcal{M}_{Born}^{S-\text{wave}} = -iG(T^A)_{a'a}(T^A)_{b'b}}$$

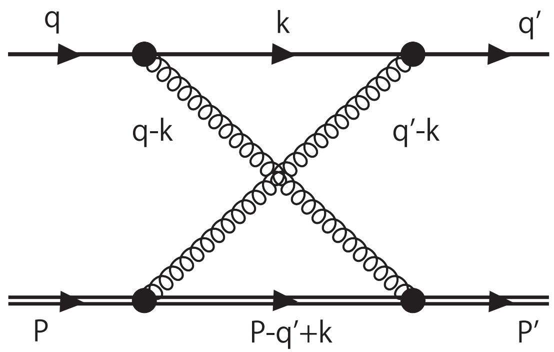
1-loop amplitudes (S-wave projected)

(a) particle



$$i\,G^2\rho_F\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

(b) hole

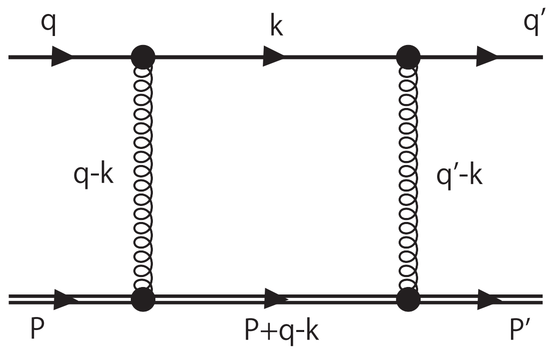


$$-i\,G^2\rho_F\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

$\rho_F = \dfrac{k_F^2}{(2\pi)^2}$: density of state on Fermi surface

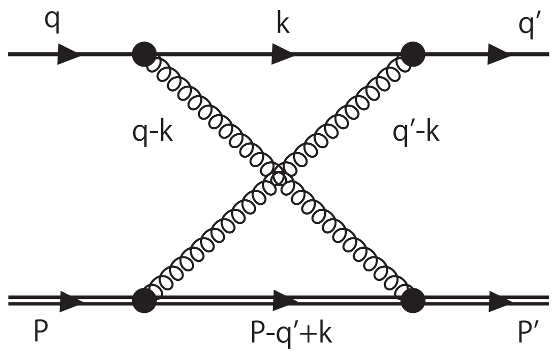
1-loop amplitudes (S-wave projected)

(a) particle



$$i\,G^2\rho_F\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

(b) hole



$$-i\,G^2\rho_F\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

Color factors (Non-abelian property of the QCD interaction)

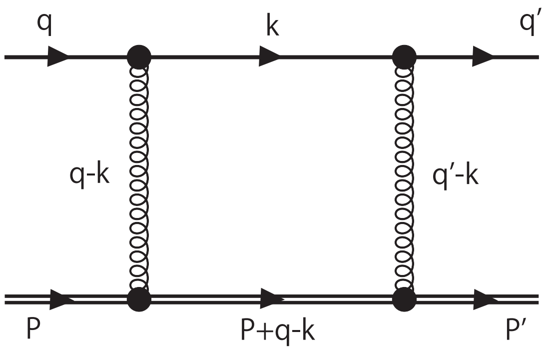
$$\mathcal{T}^{(a)}_{a'a;b'b}=(T^A)_{a'a''}(T^B)_{a''a}(T^A)_{b'b''}(T^B)_{b''b}=\frac{N_c^2-1}{4N_c^2}\delta_{a'a}\delta_{b'b}-\frac{1}{N_c}(T^A)_{a'a}(T^A)_{b'b}$$

$$\mathcal{T}^{(b)}_{a'a;b'b}=(T^A)_{a'a''}(T^B)_{a''a}(T^B)_{b'b''}(T^A)_{b''b}=\frac{N_c^2-1}{4N_c^2}\delta_{a'a}\delta_{b'b}-\left(\frac{1}{N_c}-\underline{\frac{N_c}{2}}\right)(T^A)_{a'a}(T^A)_{b'b}$$

$$\rho_F=\frac{k_F^2}{(2\pi)^2}$$: density of state on Fermi surface

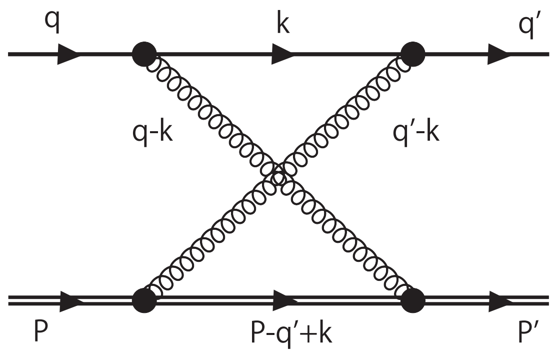
1-loop amplitudes (S-wave projected)

(a) particle



$$iG^2\rho_F\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

(b) hole



$$-iG^2\rho_F\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

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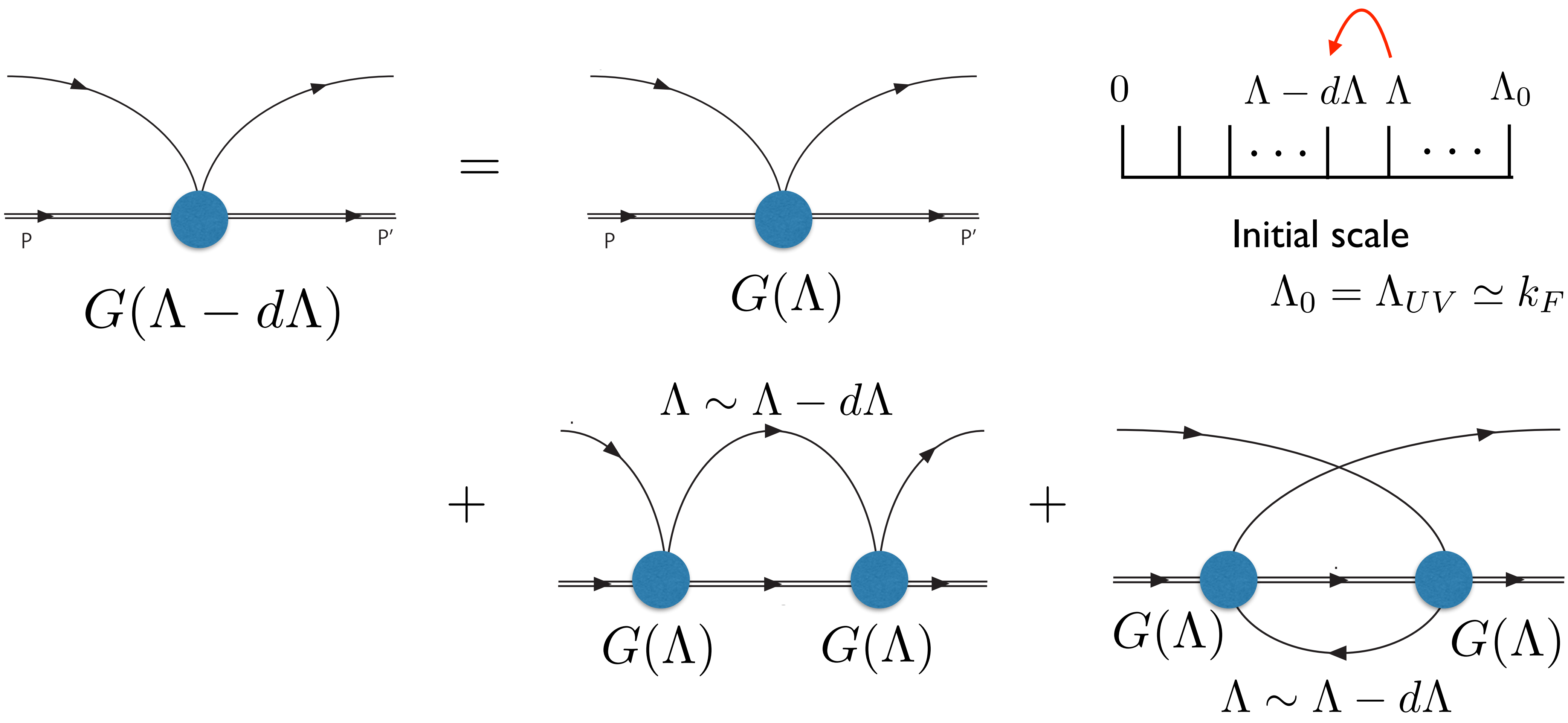
$$\mathcal{T}^{(b)}_{a'a;b'b}=(T^A)_{a'a''}(T^B)_{a''a}(T^B)_{b'b''}(T^A)_{b''b}=\frac{N_c^2-1}{4N_c^2}\delta_{a'a}\delta_{b'b}-\left(\frac{1}{N_c}-\frac{N_c}{2}\right)(T^A)_{a'a}(T^A)_{b'b}$$

$$\longrightarrow\ -i\frac{N_c}{2}G^2\rho_F\log\frac{\Lambda_{UV}}{\Lambda}(T^A)_{a'a}(T^A)_{b'b},\qquad \rho_F=\frac{k_F^2}{(2\pi)^2}$$

: density of state on Fermi surface

Renormalization group equation of scattering amplitude

~poor man's scaling~



Renormalization group equation of scattering amplitude

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{N_c}{2} \rho_F G^2(\Lambda)$$

$\xrightarrow{\times \rho_F}$ $$\Lambda \frac{d\bar{G}(\Lambda)}{d\Lambda} = -\frac{N_c}{2} \bar{G}^2(\Lambda),$$ Dimensionless coupling: $\bar{G} = \rho_F G$

Solution $\longrightarrow$

$$\bar{G}(\Lambda) = \frac{\bar{G}(\Lambda_0)}{1 + \frac{N_c}{2}\bar{G}(\Lambda_0)\log(\Lambda/\Lambda_0)}$$

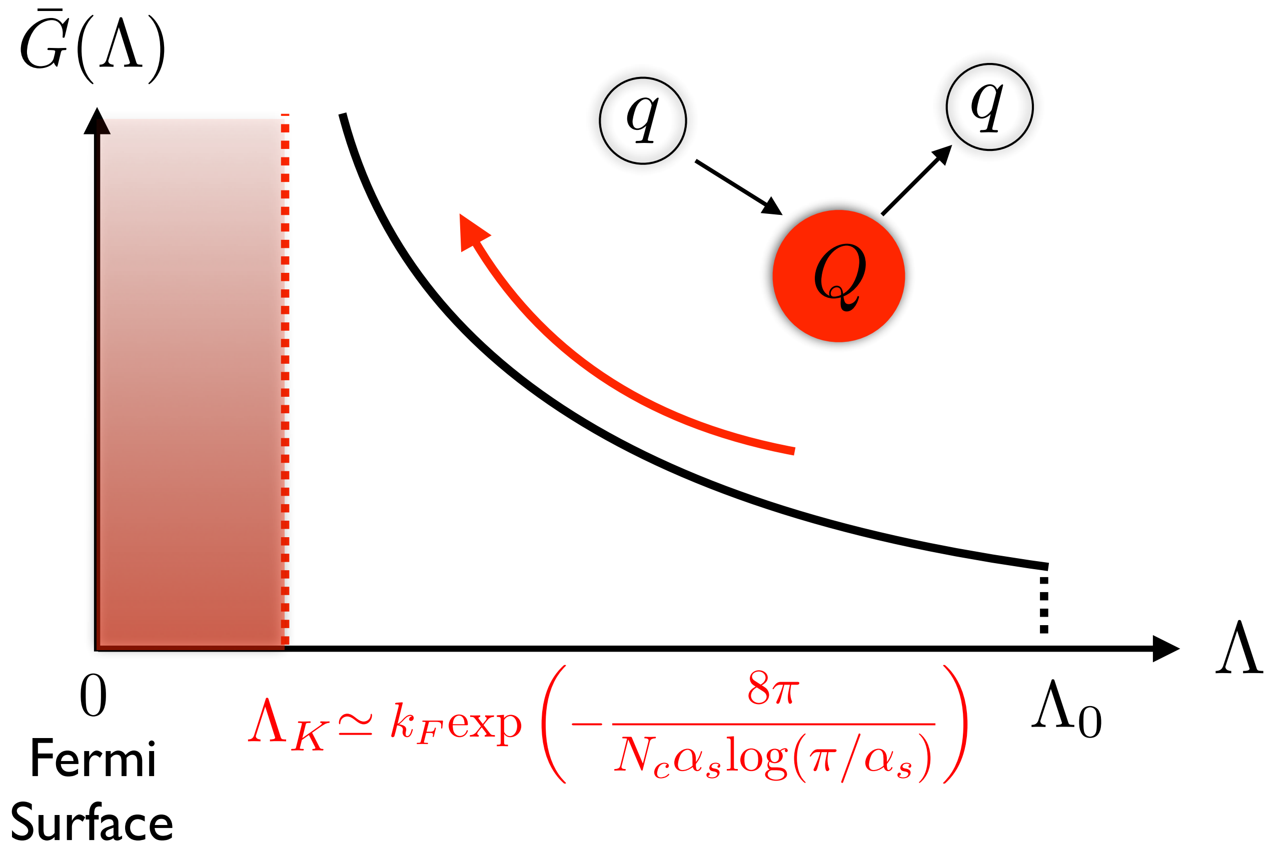
Initial scale

$\Lambda_0 = \Lambda_{UV} \simeq k_F$

Kondo scale (from the Landau pole)

$$\Lambda_K \simeq k_F \exp\left(-\frac{8\pi}{N_c \alpha_s \log(\pi/\alpha_s)}\right)$$

QCD Kondo effect



- The strength of the q-Q interaction increases as the energy scale decreases, and the system becomes non-perturbative one below the Kondo scale.
- This indicates a change of mobility of light quarks.

⟶ Several transport coefficients will be largely affected by QCD Konde effect.

Magnetically induced QCD Kondo effect

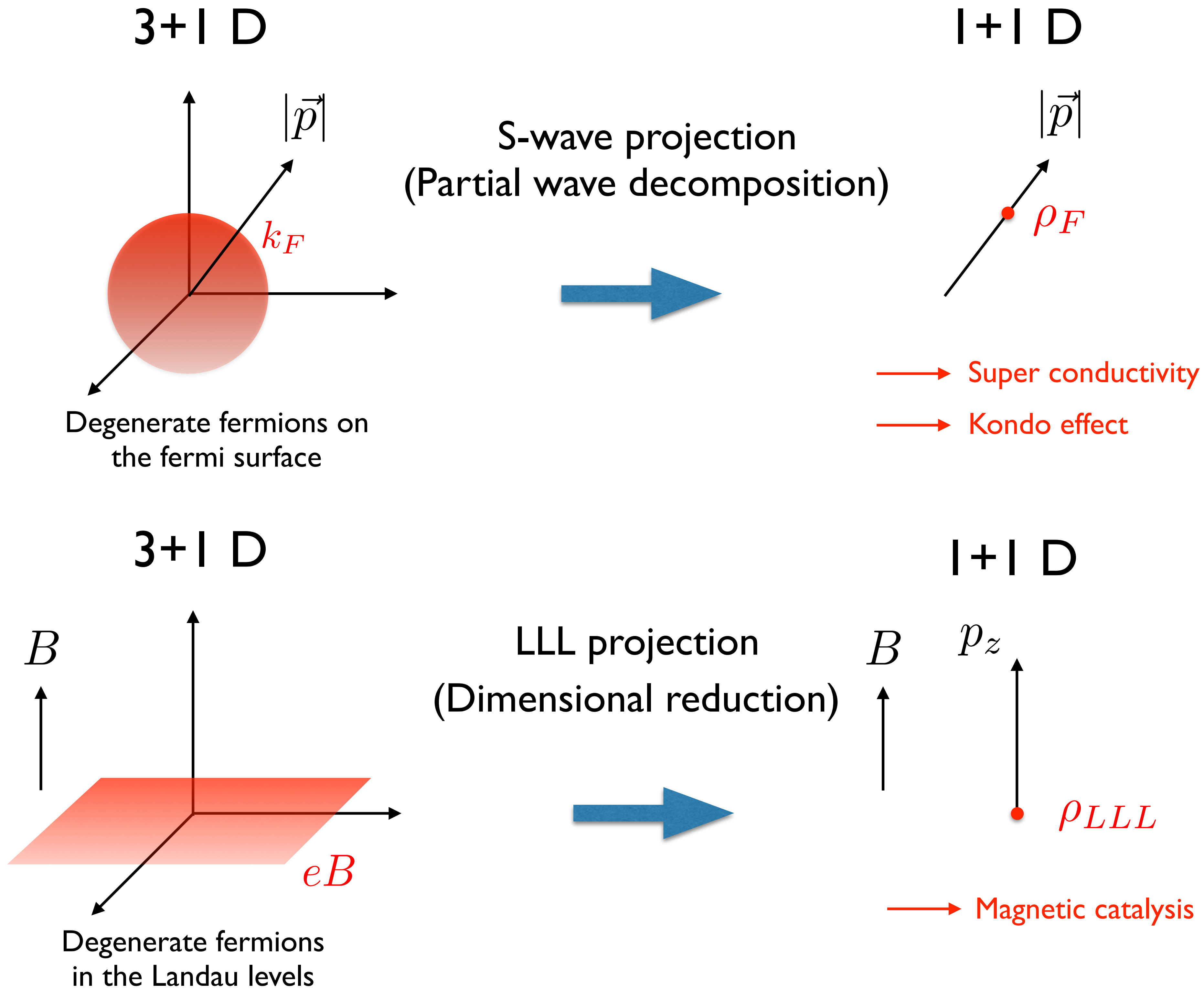
S. O., K. Itakura and Y. Kuramoto, arXiv:1509.06966

Why can Kondo effect occur in magnetic fields?

Why can Kondo effect occur in magnetic fields?

↓

Because the magnetic field can play the same role as the chemical potential which makes Fermi surface.


3+1 D
1+1 D
$|\vec{p}|$
S-wave projection
(Partial wave decomposition)
k_F
ρ_F
Degenerate fermions on
the fermi surface
Super conductivity
Kondo effect
3+1 D
1+1 D
B
LLL projection
(Dimensional reduction)
B
p_z
eB
ρ_{LLL}
Degenerate fermions
in the Landau levels
Magnetic catalysis

Conditions for the appearance of QCD Kondo effect

0) Heavy quark impurity

i) Fermi surface of light quarks

ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

Conditions for the appearance of

"Magnetically induced QCD Kondo effect"

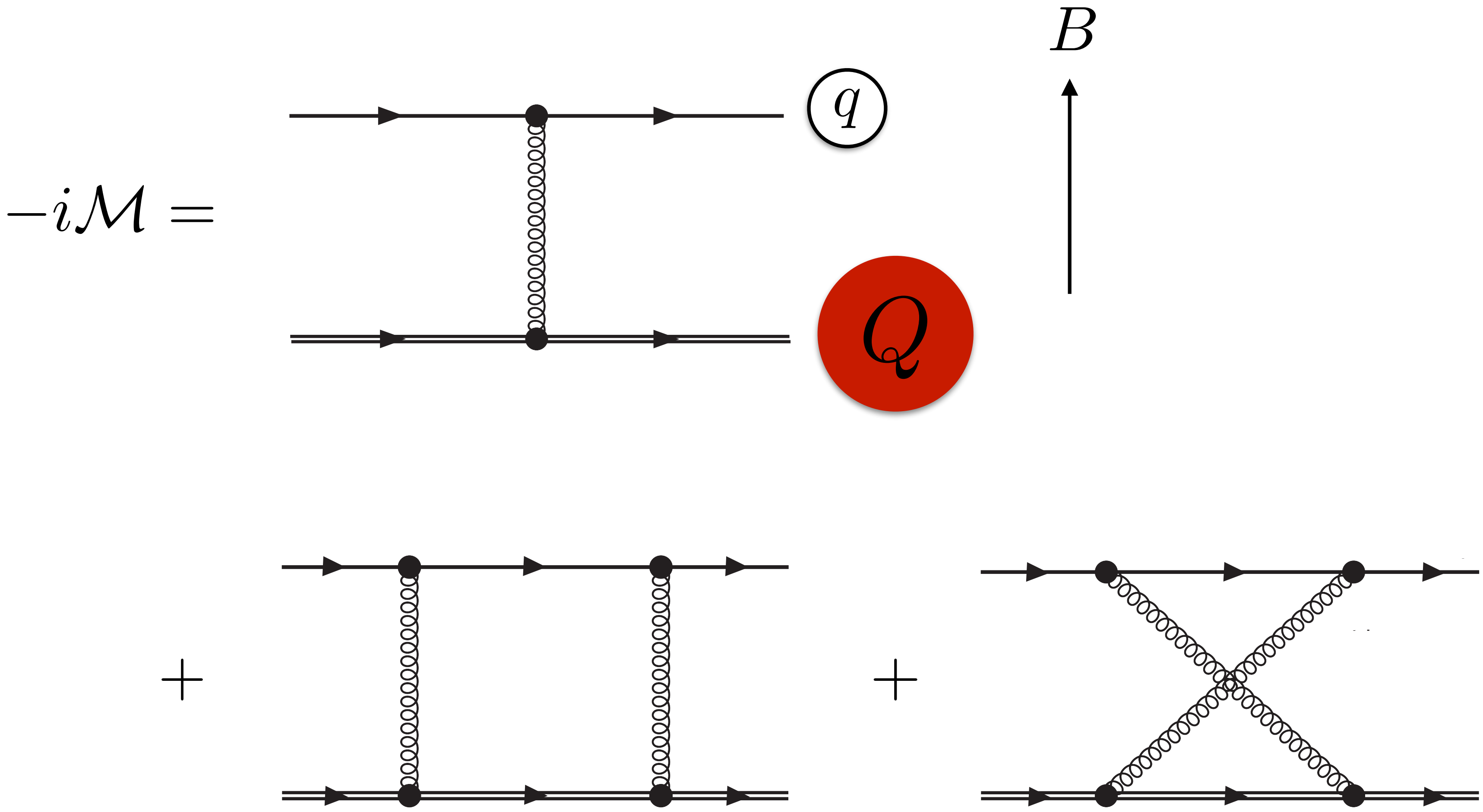
0) Heavy quark impurity

i) Strong magnetic field

ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

The magnetic field does not affect color degrees of freedom.


$-i\mathcal{M} =$
q
Q
B
$+$
$+$

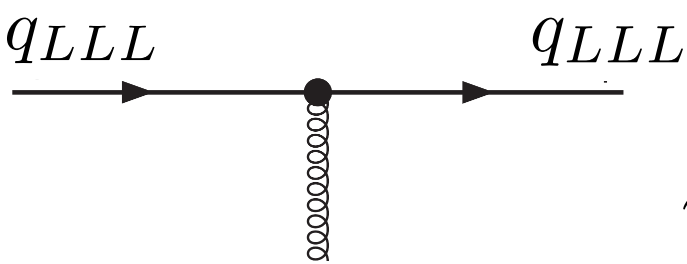
Quark propagator of the Lowest Landau Level (LLL)

$$iS_{LLL}(k;\mu|e_qB) = e^{-\frac{k_\perp^2}{e_qB}}\frac{i}{\epsilon_k}\left\{\frac{\theta(k^3-k_F)}{k^0-\epsilon_k^+ + i\varepsilon} + \frac{\theta(k_F-k^3)\theta(k^3)}{k^0-\epsilon_k^+ - i\varepsilon} - \frac{\theta(-k^3)}{k^0-\epsilon_k^- - i\varepsilon}\right\}\left(k^0\gamma^0 - k^3\gamma^3\right)\mathcal{P}_0$$

with spin projection operator

$$\mathcal{P}_0 = \frac{1+i\gamma^1\gamma^2}{2}$$

Vertex

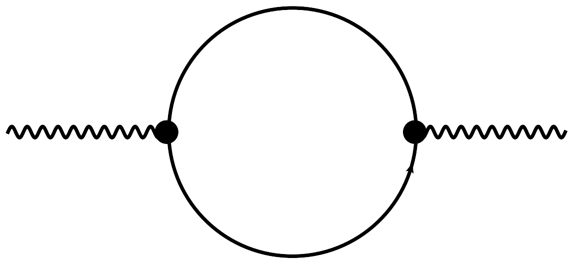


$$\gamma^\mu \to \mathcal{P}_0\gamma^\mu\mathcal{P}_0 = \mathcal{P}_0\gamma^{\bar{\mu}}\mathcal{P}_0\,, \qquad \bar{\mu}=0,3$$

Gluon propagator in strong magnetic fields

- Magnetic screening effect

Vacuum polarization



$$= (p_\parallel^2 g_{\mu\nu}^\parallel - p_\mu^\parallel p_\nu^\parallel)\Pi(p_\perp^2, p_\parallel^2)$$

with

$$\Pi(p_\perp^2, p_\parallel^2) = -\exp\left(-\frac{p_\perp^2}{2e_q B}\right)\frac{m_g^2}{p_\parallel^2}$$

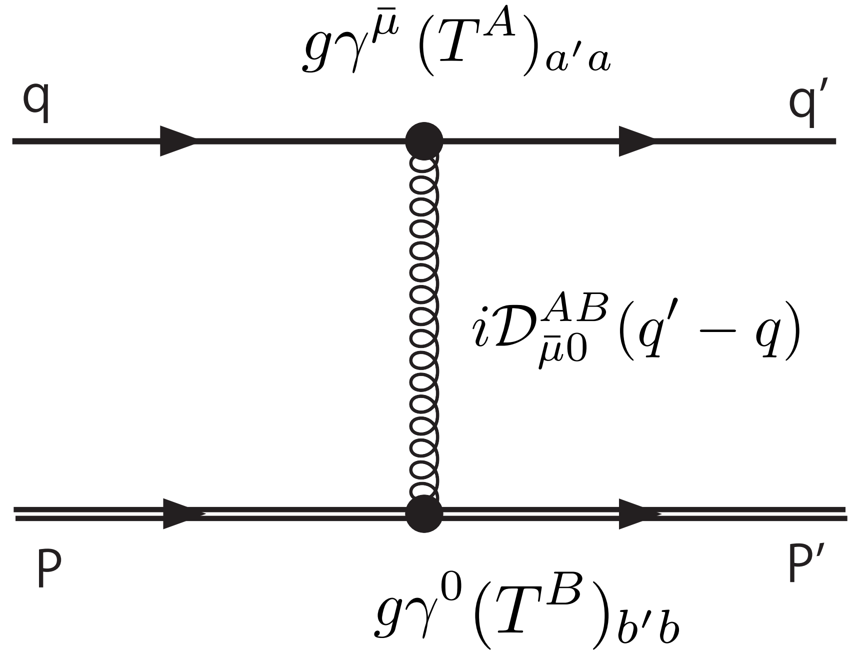
Gluon mass

$$m_g^2 = \frac{\alpha_s}{\pi} e_q B$$

- Gluon propagator V. P. Gusynin, V.A. Miransky and I.A. Shovkovy, NPB 563 (1999)

$$i\mathcal{D}_{\mu\nu}^{AB}(p) = -i\left(\frac{g_{\mu\nu}^\parallel}{p^2 + p_\parallel^2\Pi(p_\perp^2, p_\parallel^2)} + \frac{g_{\mu\nu}^\perp}{p^2} - \frac{p_\mu^\perp p_\nu^\perp + p_\mu^\perp p_\nu^\parallel + p_\mu^\parallel p_\nu^\perp}{p^4}\right)\delta^{AB}$$

QCD interaction in strong magnetic fields



The 1+1 dimensional gluon exchange int.

$$G(q'_{\parallel}-q_{\parallel})\delta^{AB} \equiv \int \frac{d^2Q_\perp}{(2\pi)^2} \mathrm{e}^{-Q_\perp^2/4e_qB} \left[(ig)^2 i\mathcal{D}_{00}^{AB}(q'_{\parallel}-q_{\parallel}, Q_\perp)\right]$$

$$\simeq -\frac{g^2\delta^{AB}}{(2\pi)^2} \int d^2Q_\perp \frac{\mathrm{e}^{-Q_\perp^2/4e_qB}}{(q'_{\parallel}-q_{\parallel})^2 - Q_\perp^2 - m_g^2}$$

$$q'_0 = q_0 = \epsilon_F$$

$$q'_3 - q_3 \simeq \Lambda$$

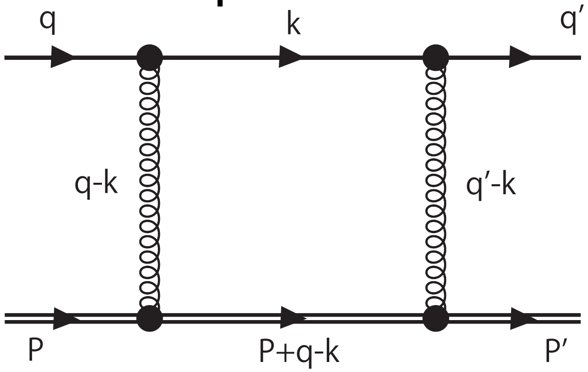
After integrating the transverse momentum, we get

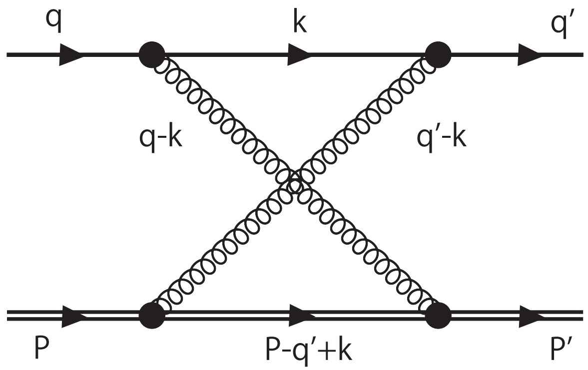
$$\bar{G} \simeq \begin{cases} \alpha_s \log\left(\frac{4e_qB}{\Lambda^2}\right) & \text{(I)} \ \Lambda > m_g \\ \alpha_s \log\left(\frac{4e_qB}{m_g^2}\right) & \text{(II)} \ \Lambda < m_g \end{cases}$$

Leading order amplitude

$$\longrightarrow \quad -i\bar{G}(T^A)_{a'a}(T^A)_{b'b}$$

NLO (1+1dimensional 1-loop amplitudes)

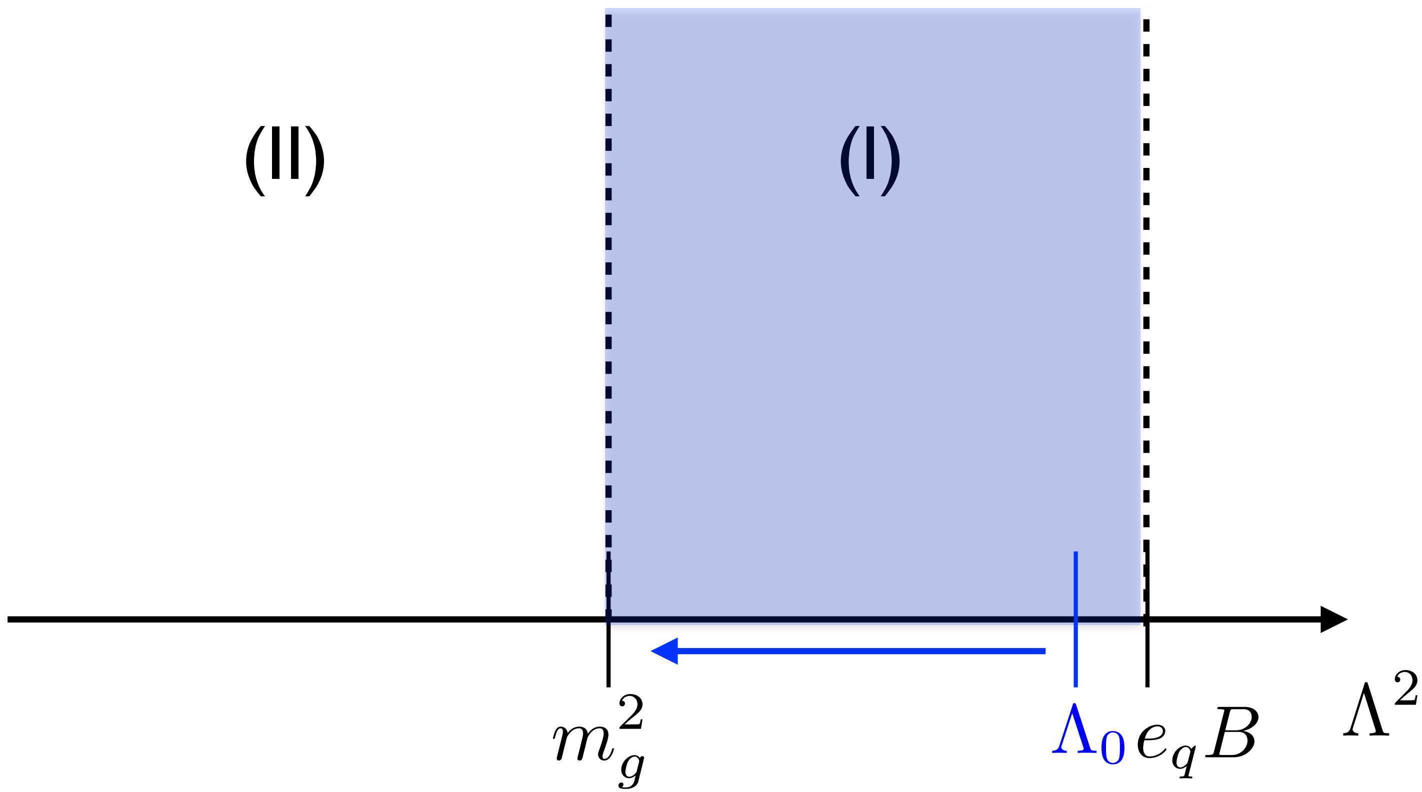




$$i\bar{G}^2\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE \qquad\qquad -\, i\bar{G}^2\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

$$\longrightarrow \quad \underline{-i\bar{G}^2\frac{N_c}{2}\log\frac{\Lambda_{UV}}{\Lambda}(T^A)_{a'a}(T^B)_{b'b}}$$

Scales in QCD in a strong magnetic field



Renormalization group equation in region (I)

$$\Lambda \frac{d}{d\Lambda}\bar{G}(\Lambda) = \underbrace{-2\alpha_s}_{\text{tree}} \underbrace{- \frac{N_c}{4\pi}\bar{G}^2(\Lambda)}_{\text{1-loop}}$$

solution

$$\longrightarrow \quad \bar{G}(\Lambda) = \sqrt{\frac{N_c}{8\pi\alpha_s}}\tan\left[\tan^{-1}\left(\sqrt{\frac{N_c}{8\pi\alpha_s}}\bar{G}(\Lambda_0)\right) - \sqrt{\frac{N_c\alpha_s}{8\pi}}\log\frac{\Lambda^2}{\Lambda_0^2}\right]$$

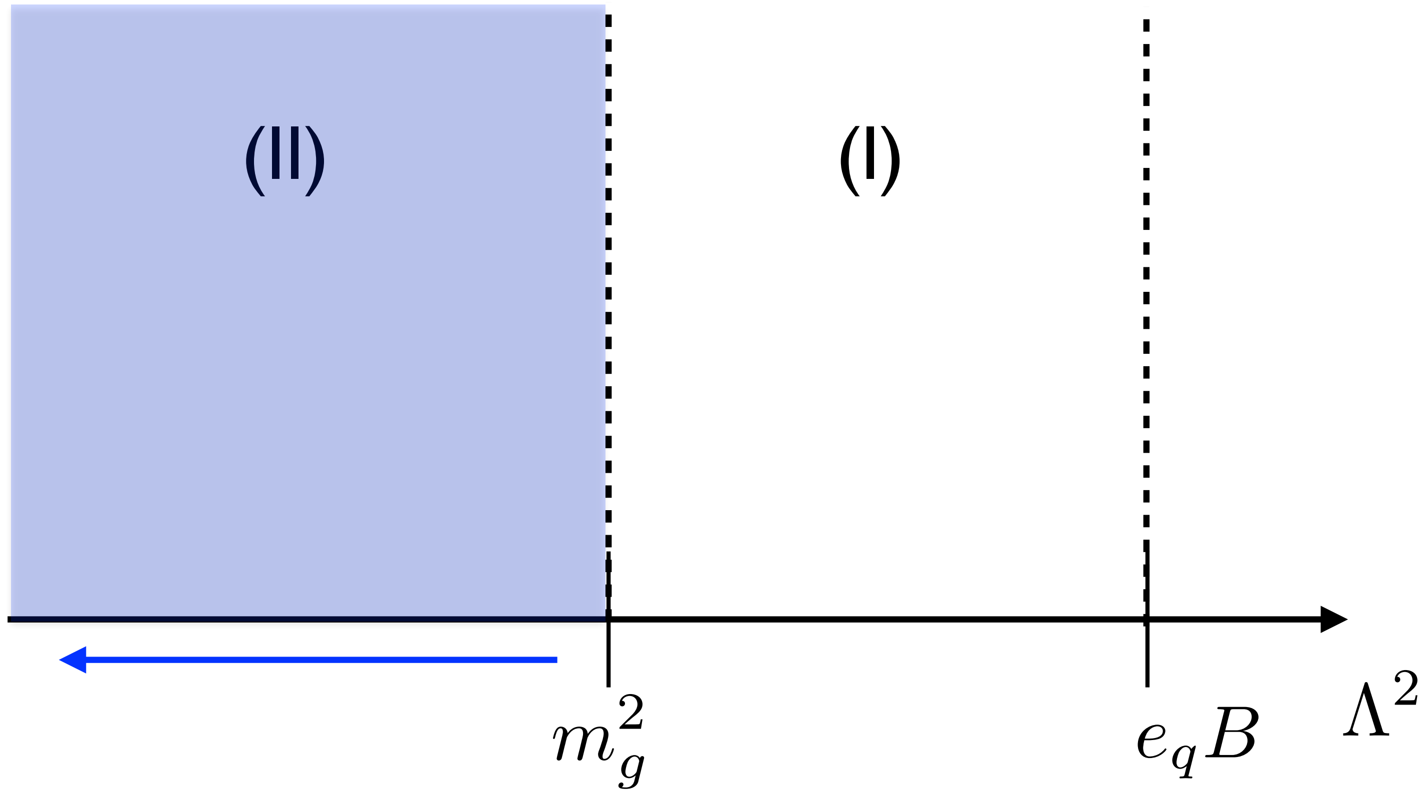
with the initial coupling at $\Lambda = \Lambda_0$

$$\bar{G}(\Lambda_0) = \alpha_s \log\left(\frac{4e_q B}{\Lambda_0^2}\right)$$

At lower limit of region (I), $\Lambda = m_g$

$$\bar{G}(m_g) = \alpha_s \log\frac{4e_q B}{m_g^2}\left\{1 + \frac{1}{3}\cdot\frac{N_c\alpha_s}{8\pi}\log^2\frac{4e_q B}{m_g^2} + \cdots\right\}$$

Scales in QCD in a strong magnetic field



Renormalization group equation in region (II)

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{N_c}{4\pi}\bar{G}^2(\Lambda)$$

$\xrightarrow{\text{solution}}$

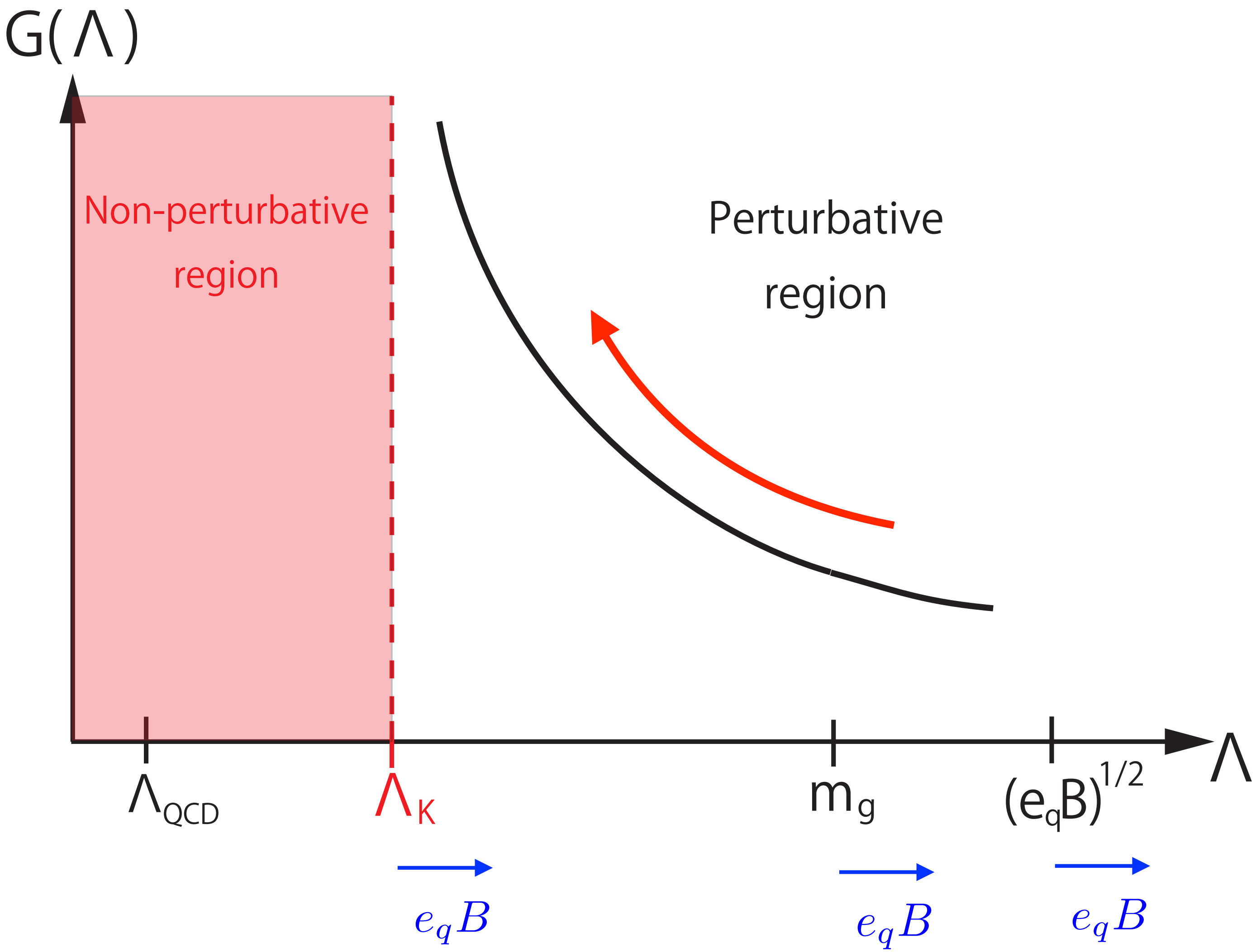
$$\bar{G}(\Lambda) = \frac{\bar{G}(m_g)}{1 + \frac{1}{4\pi}N_c\bar{G}(m_g)\log\frac{\Lambda}{m_g}}$$

Kondo scale (from the Landau pole)

$$\Lambda_K \simeq \sqrt{e_q B}\alpha_s^{1/2}\exp\left\{-\frac{4\pi}{N_c\alpha_s\log(4\pi/\alpha_s)} + \log\left(\frac{4\pi}{\alpha_s}\right)^{1/6}\right\}$$

$$\simeq \sqrt{e_q B}\alpha_s^{1/3}\exp\left\{-\frac{4\pi}{N_c\alpha_s\log(4\pi/\alpha_s)}\right\}$$

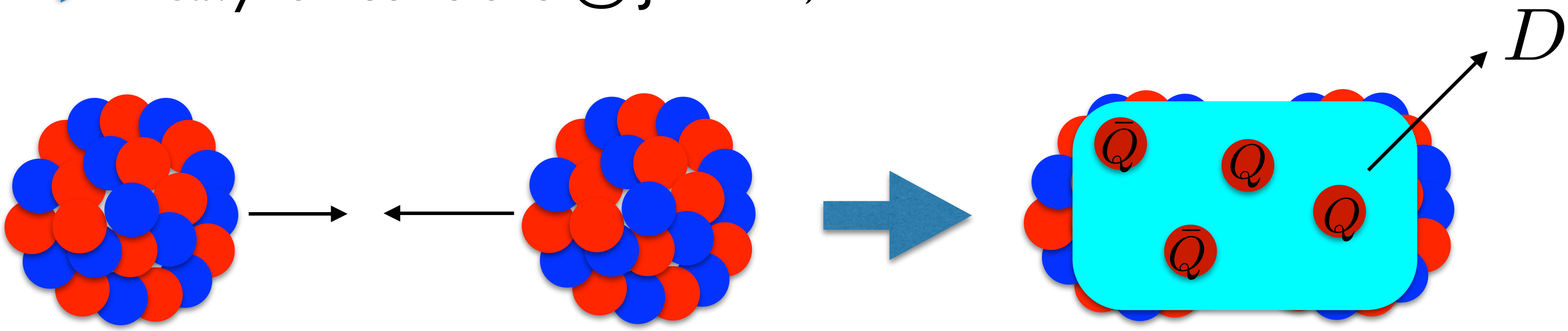
Magnetically induced QCD Kondo effect



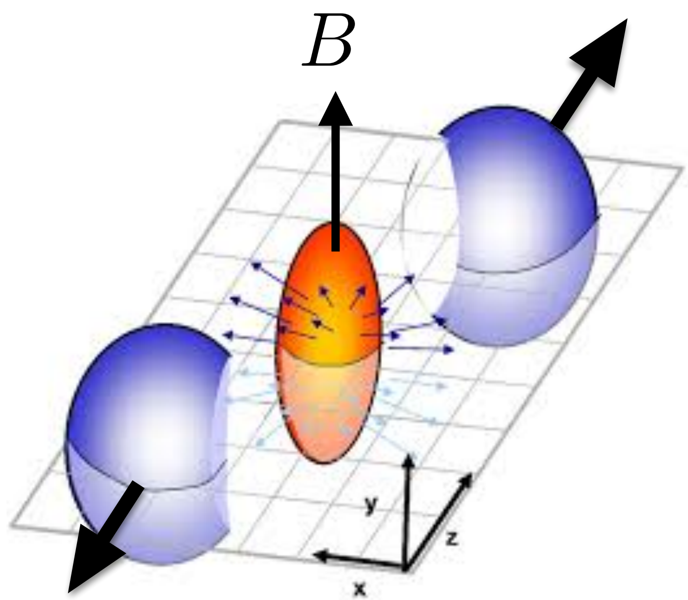
- We have found the QCD Kondo effect induced by strong magnetic fields.
- The Kondo scale slowly but monotonically increases as eB increases, so the Kondo dynamics appears in high energy region with sufficiently large B.

Where can we observe QCD Kondo effect?

- Heavy ion collisions @ J-PARC, GSI-FAIR

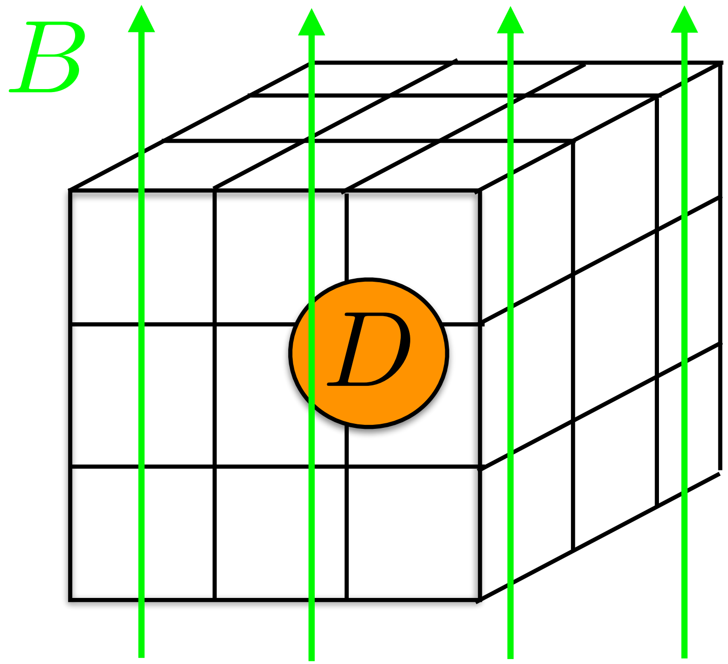


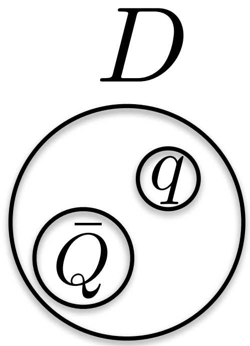
- Non-central heavy ion collisions @ RHIC, LHC



$$|eB| \gtrsim \Lambda_{\rm QCD}^2$$

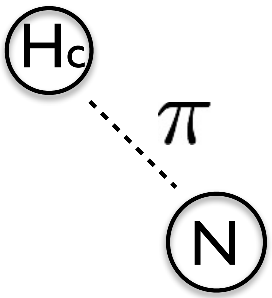
- Lattice QCD simulation (Numerical experiment of QCD)

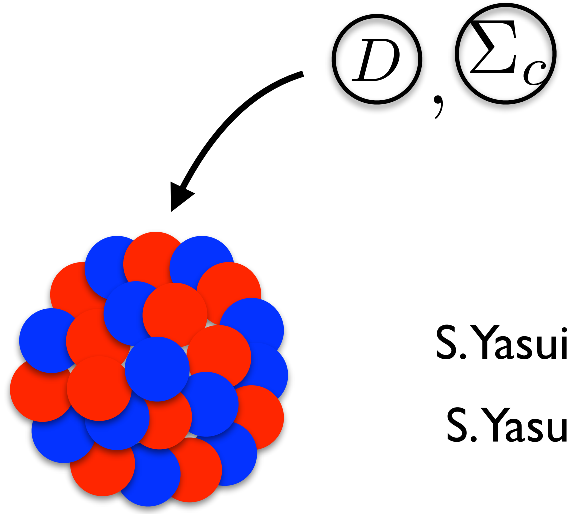




- Charmed Nuclei (Isospin exchange int.)

Isospin exchange int.
mediated by pion





S.Yasui and K.Sudoh, PRC88(2013) 015201

S.Yasui, arXiv:1602.00227

Summary

- We found the characteristic behavior of Kondo effect, a logarithmic enhancement of the scattering amplitude of a light quark off a heavy quark impurity near the Fermi surface.

  $\longrightarrow$ QCD Kondo effect

- We also found that QCD Kondo effect induced by strong strong magnetic fields.

  $\longrightarrow$ Magnetically induced QCD Kondo effect

Outlook

- Non-perturbative analysis below the Kondo scale from conformal field theory (with Taro Kimura @Keio Univ.)

  $\longrightarrow$ Several observables near the IR fixed point.