Study of cluster structure by GCM

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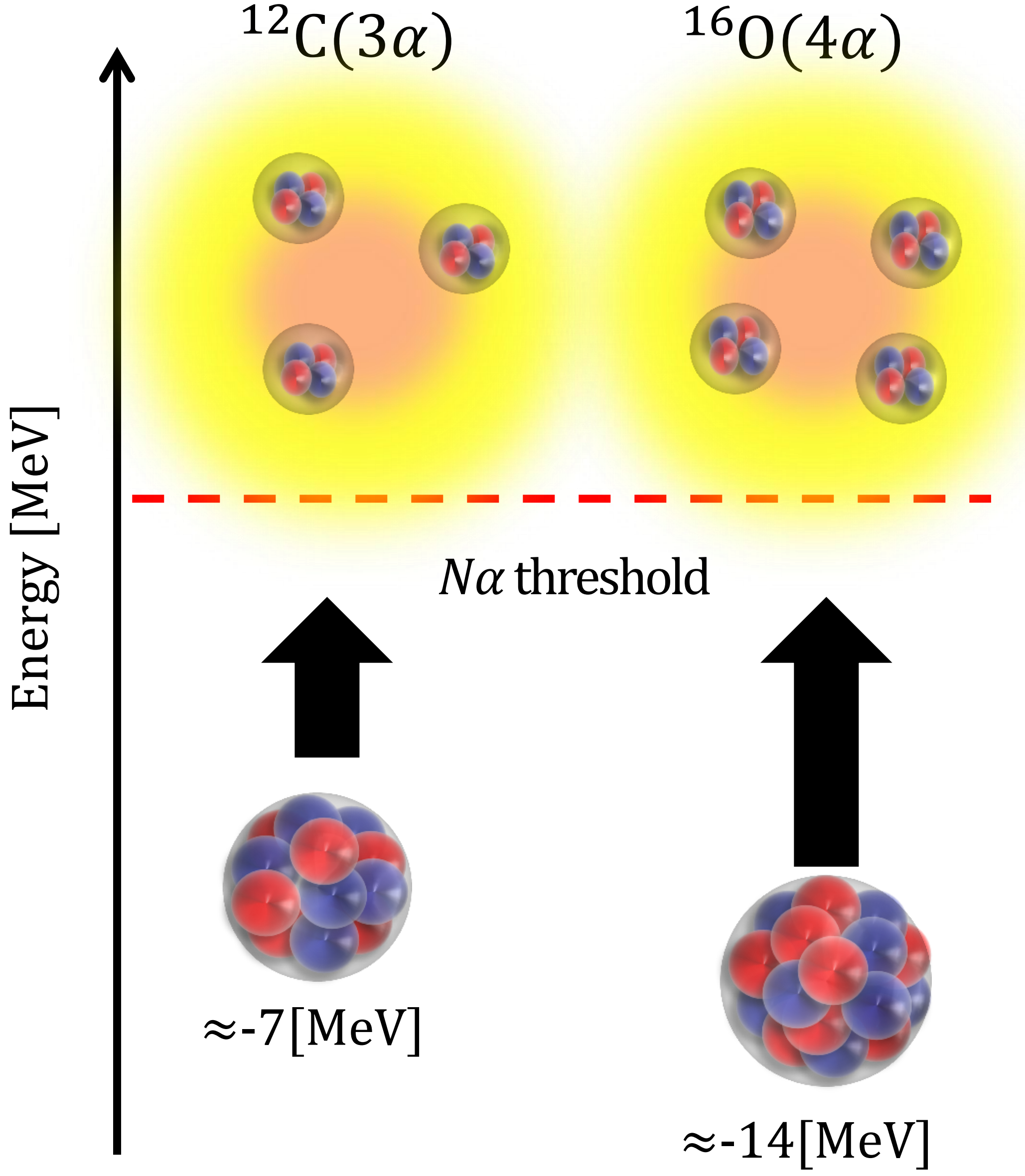
Introduction Cluster structure and α gas-like states



Recent studies for gas-like states.

- ^{8}Be(2α), ^{12}C(3α) , ^{16}O(4α) are studied in detail

A. Tohsaki et.al PRL, **87**.192501 (2001)
Y. Funaki et.al PRC, **82**.024312 (2010)
Y. Kanada-En'yo PRC, **89**.024304 (2014)

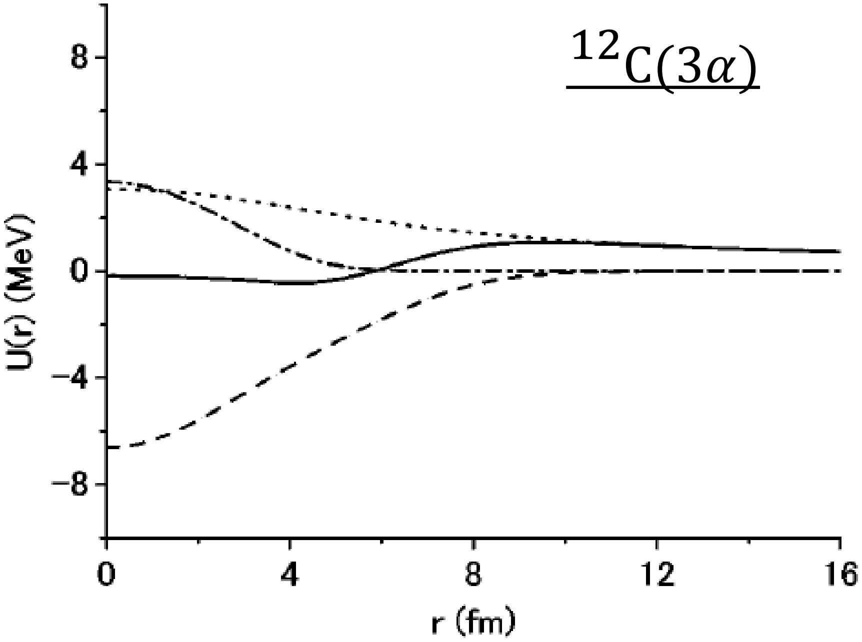
- ^{20}Ne(5α), ^{24}Mg(6α) and heavier systems are also expected to have gas-like state.

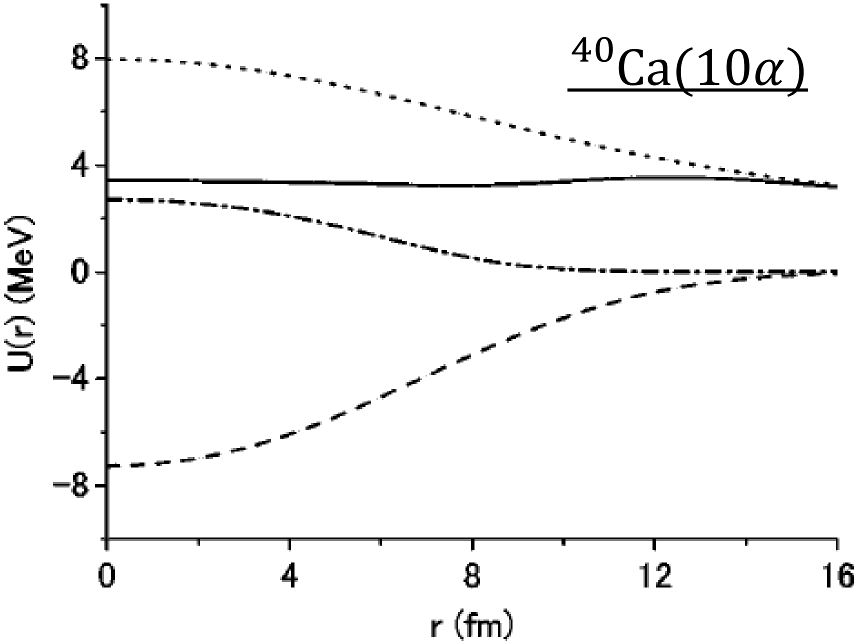
◎ How many α particles can form dilute-gas like state?

Introduction Theoretical study for many α system

T. Yamada and P. Schuck, PRC **69**, 024309 (2004)

◎ Gas-like state possibility exist up to ～10α system .





◎ Boson approximation is applied (Gross-Pitaevskii equation)

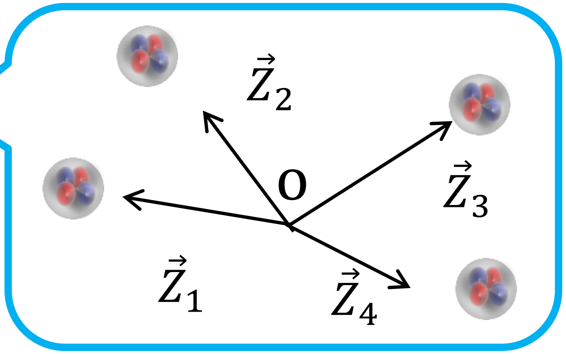
The prediction should be verified by the microscopic cluster model without Boson approximation.

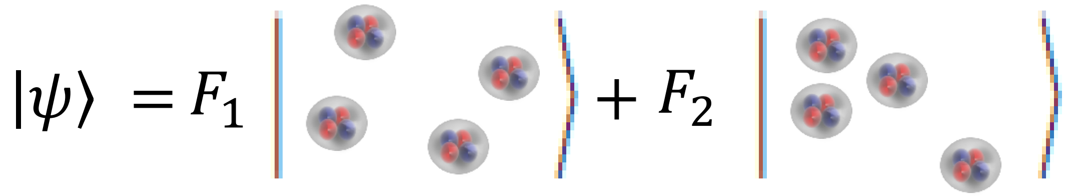
⇒ We need microscopic model that can describe gas-like states of many α particles

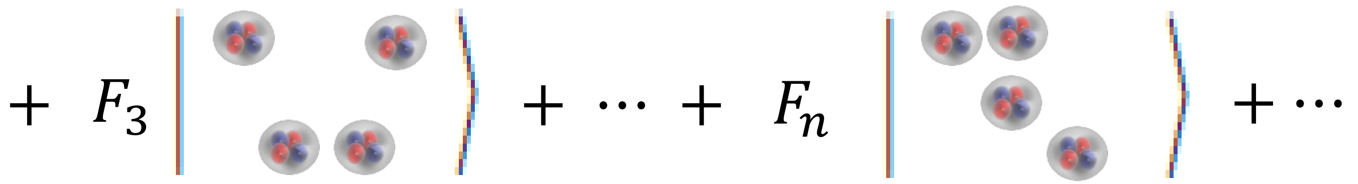
Introduction microscopic description of gas-like state

Gas-like state is described by the superposition of “many” Slater determinants

$$\psi(\vec{r}_1,\cdots,\vec{r}_{16}) = \int d\vec{Z}_1\cdots d\vec{Z}_4 F(\vec{Z}_1,\cdots,\vec{Z}_4)\underline{\phi(\vec{Z}_1,\cdots,\vec{Z}_4)}$$







- ➢ Number of Slater determinants becomes huge as the number of α increases

  ⇒ We need effective way to generate basis wave functions.

Several methods to genarate $N\alpha$ wave functions

➢ Random generation	T. Ichikawa et.al PRC, **83**.061301 (2011)
➢ Imaginary time evolution	Y. Fukuoka et.al PRC, **88**.014321 (2013)
➢ Real time evolution	

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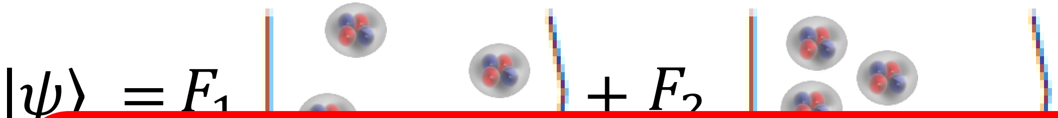

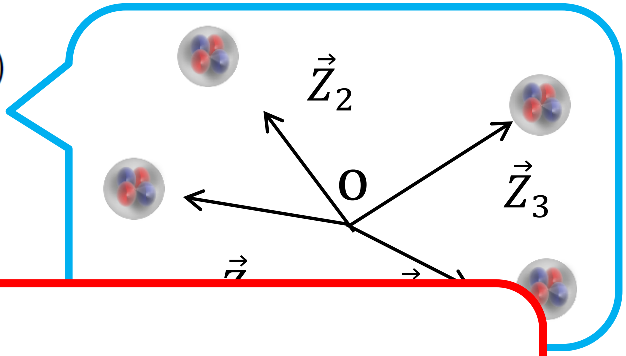

- In this talk, we introduce a method based on the real time evolution
- The new method is applied to 3α, 4α systems

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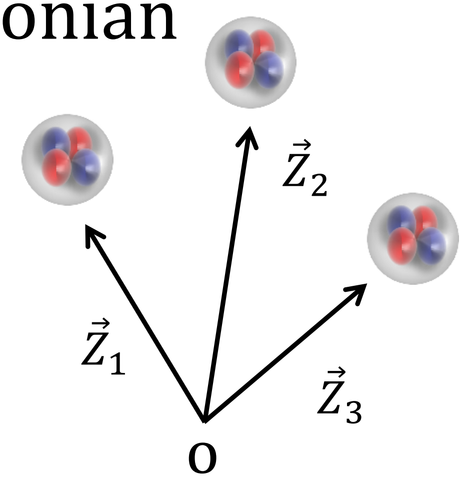
Several methods to genarate $N\alpha$ wave functions

- Random generation T. Ichikawa et.al PRC, **83**.061301 (2011)
- Imaginary time evolution Y. Fukuoka et.al PRC, **88**.014321 (2013)
- Real time evolution ⇐ Our method

Framework Wave function and Hamiltonian



Hamiltonian

$$H = \sum_{i} T_i - t_{c.m} + \sum_{i<j} V_{ij} + \sum_{\text{proton}} V_c$$

Volkov No. 2 Y.Funaki PRC.**92** 021302(2015)

Basis wave function (Brink-wave function)

$$\phi(\vec{Z}_1,\cdots,\vec{Z}_N) = \mathcal{A}\left\{\phi_\alpha(\vec{Z}_1)\phi_\alpha(\vec{Z}_2)\cdots\phi_\alpha(\vec{Z}_N)\right\}$$

$\phi_\alpha(\vec{Z}_i)$; α particle wave function (wave packet) located at $\vec{Z}_i$

Equation of motion for wave packet

$$\delta\frac{\langle\phi \mid i\frac{\partial}{\partial t} - H \mid \phi\rangle}{\langle\phi \mid \phi\rangle} = 0$$

➡

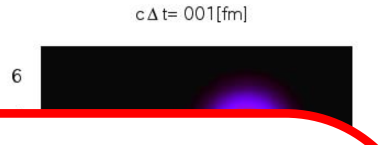
$$i\hbar\sum_{j,\sigma} C_{i\rho j\sigma}\frac{\partial \vec{Z}_{j\sigma}}{\partial t} = \frac{\partial H}{\partial \vec{Z}^{*}_{i\rho}}$$

Time dependent variational principle

Equation of motion for wave packets

Framework Time evolution of wave packets

$$i\hbar\sum C_{i\rho j\sigma}\frac{\partial \vec{Z}_{j\sigma}}{\partial t} = \frac{\partial H}{\partial \vec{Z}^{*}}$$



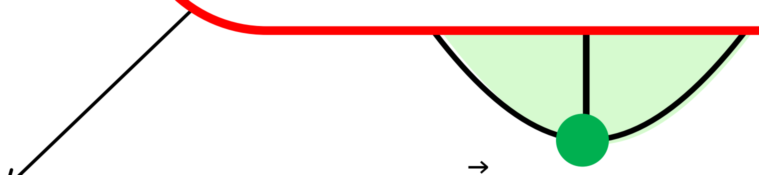
◎Many body wave function represented by wave packet is ⋯

- ○ Ergodic nature. J. Schnack , H. Feldmeier NPA, **601**. (1996)
- ○ Obeying quantum statics. A. Ono , H. Horiuchi PRC.**53**. (1996)

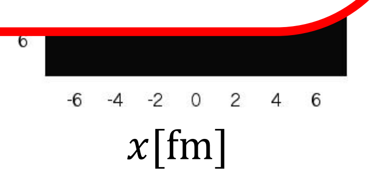
⬇

➢ If the system is evolved long enough, all of important configurations of α particles appear automatically .

⇒ Basis wave functions to describe gas-like state are automatically generated !



Position of center $\vec{Z}$



- $E^{*} \approx 5[\mathrm{MeV}]$

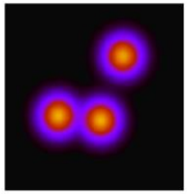
Framework New generator coordinate method

- The time evolution generates all important configurations

Superposition of the wave functions obtained by the time evolution should give a good description for the gas-like state

In other words, we perform GCM calculation using the real time *t* as the generator coordinate

Step1: Time evolution to generate basis wave function



$$i\hbar \sum_{j,\sigma} C_{i\rho j\sigma} \frac{\partial \vec{Z}_{j\sigma}}{\partial t} = \frac{\partial H}{\partial \vec{Z}^{*}_{i\rho}}$$

Step2: Superpose basis wave function and solve GCM equation

$$\psi(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4N}) = \int_0^\infty dt\, \underline{F^{J\pi}_{MK}(t)}\, P^{J\pi}_{MK} \phi(\vec{Z}_1(t), \vec{Z}_2(t), \cdots, \vec{Z}_N(t))$$

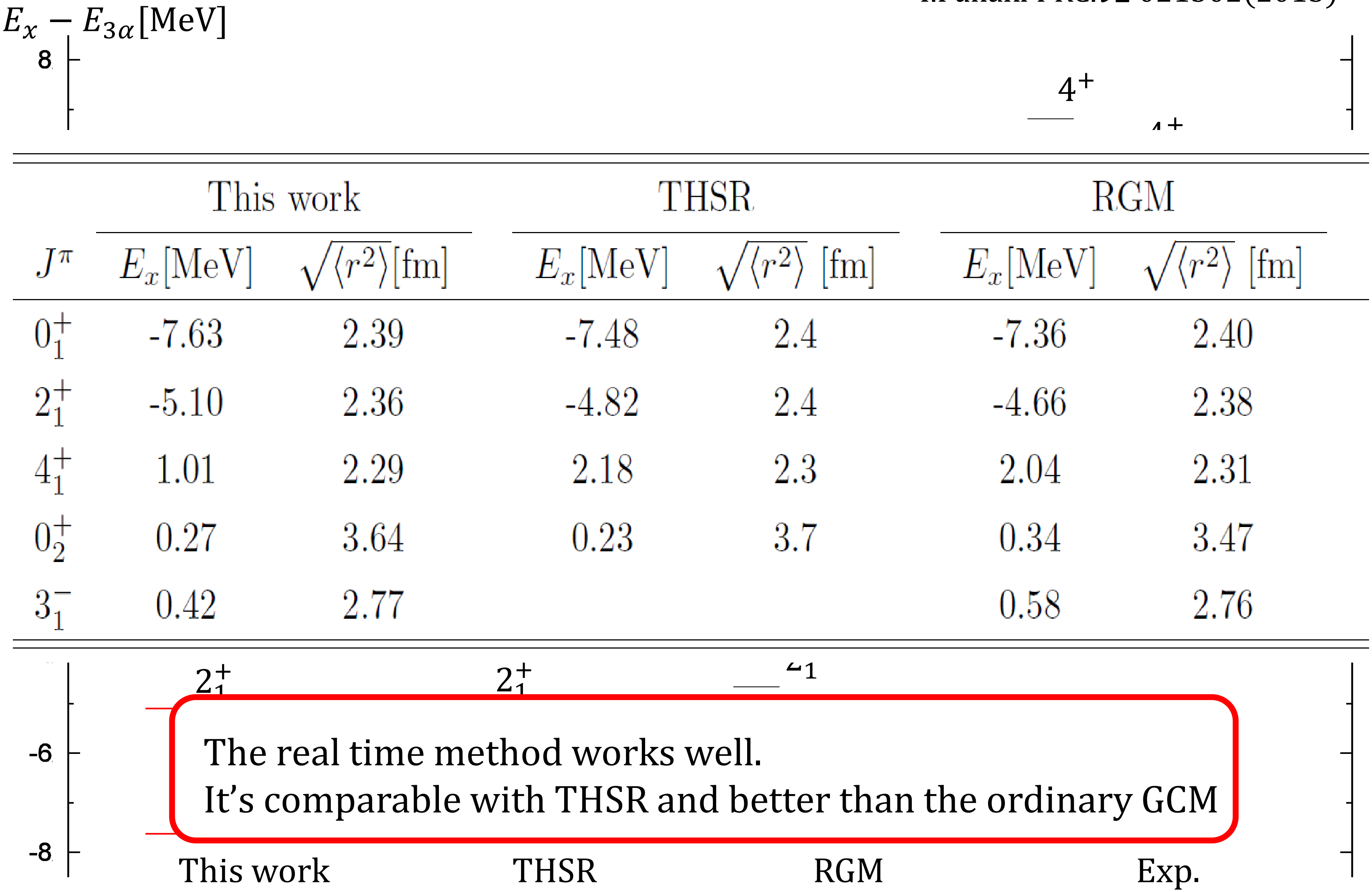
Determined by diagonalization of Hamiltonian

$P^{J\pi}_{MK}$; Projection operator of parity and angler momentum

Results & Discussions for 3α and 4α systems

Result for 3α system (^{12}C)

M.Kamimura NPA.**351,** 456 (1981)
Y.Funaki PRC.**92** 021302(2015)

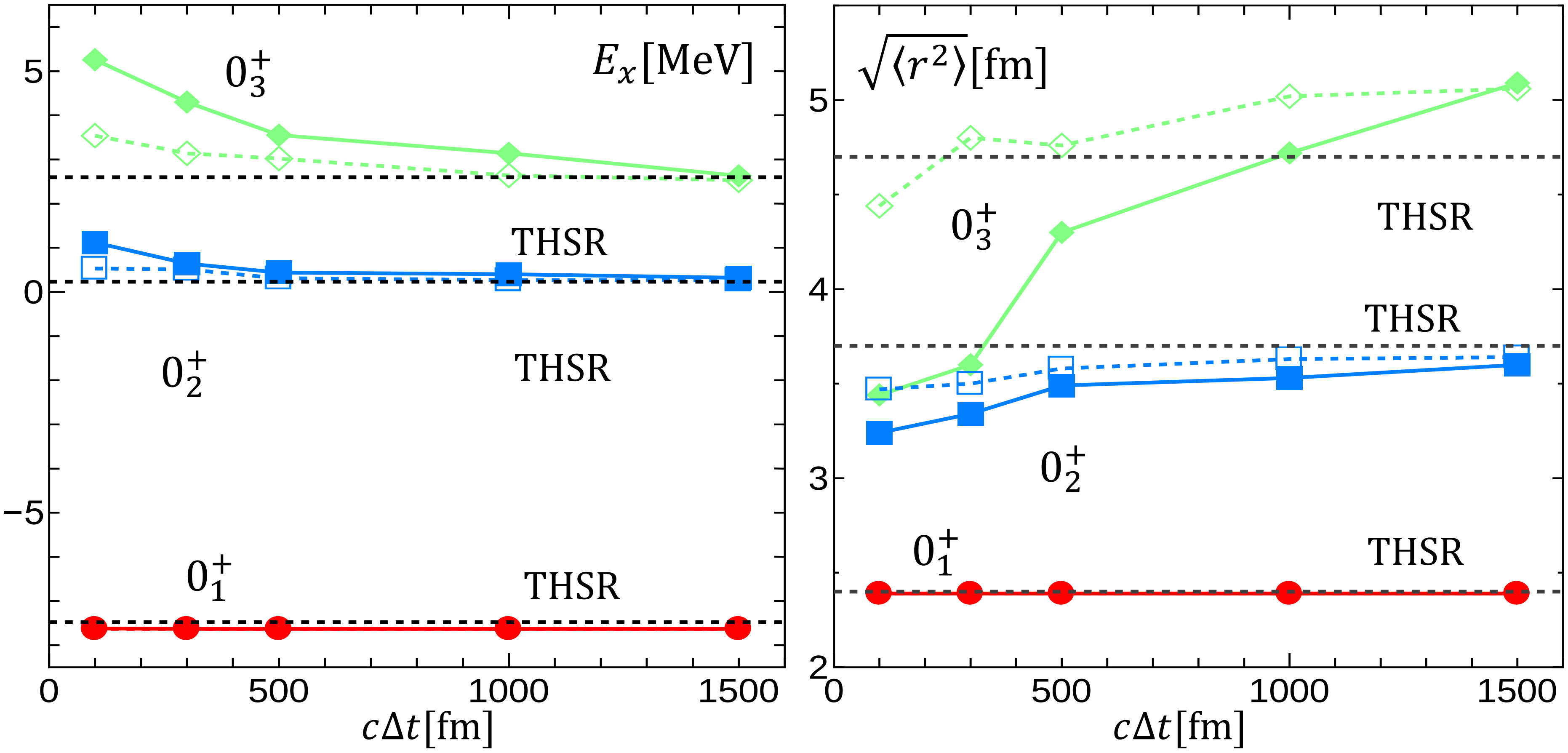


	This work		THSR		RGM	
J^π	E_x[MeV]	$\sqrt{\langle r^2\rangle}$[fm]	E_x[MeV]	$\sqrt{\langle r^2\rangle}$ [fm]	E_x[MeV]	$\sqrt{\langle r^2\rangle}$ [fm]
0_1^+	-7.63	2.39	-7.48	2.4	-7.36	2.40
2_1^+	-5.10	2.36	-4.82	2.4	-4.66	2.38
4_1^+	1.01	2.29	2.18	2.3	2.04	2.31
0_2^+	0.27	3.64	0.23	3.7	0.34	3.47
3_1^-	0.42	2.77			0.58	2.76

The real time method works well.
It's comparable with THSR and better than the ordinary GCM

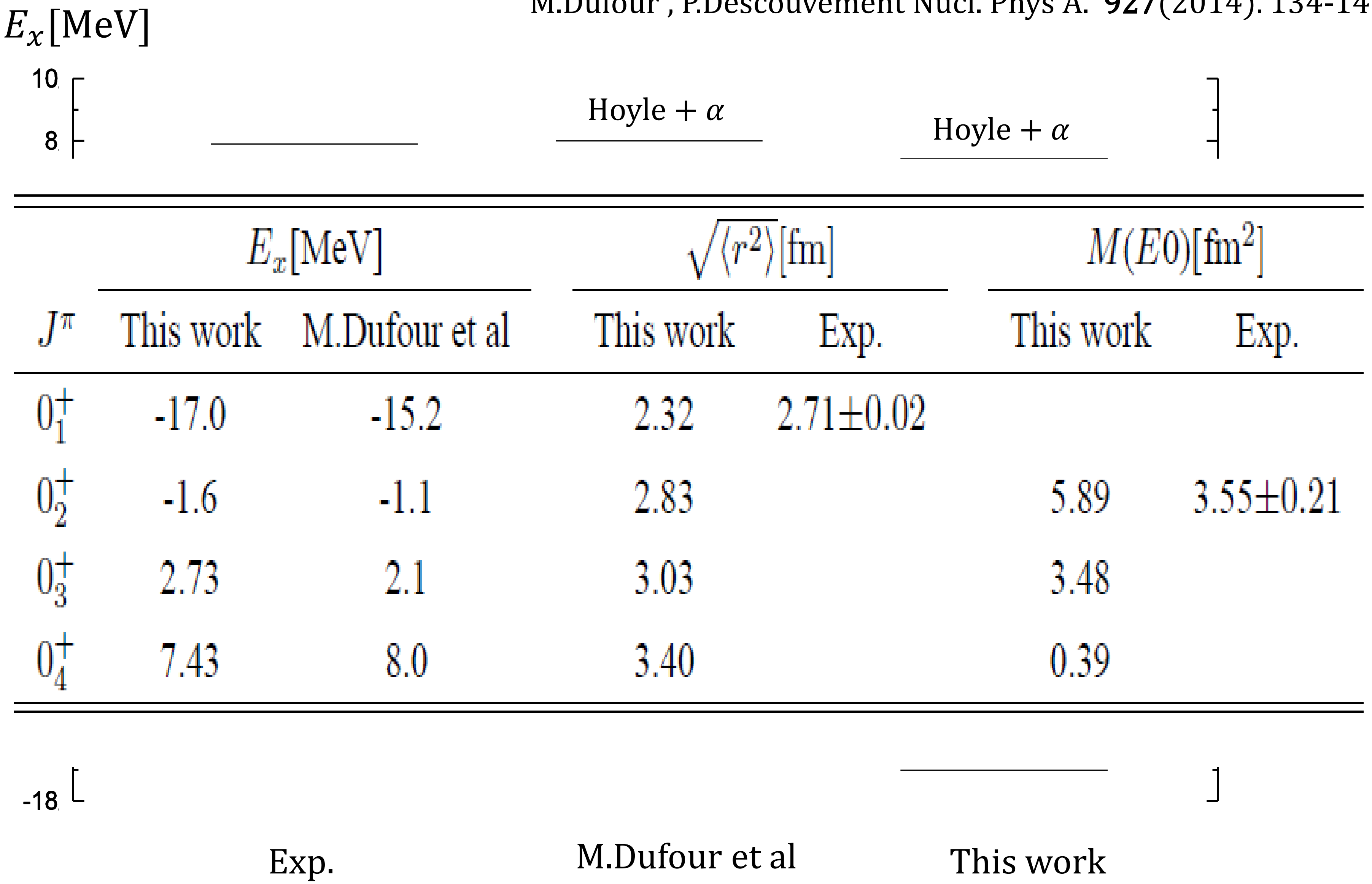
Discussion The convergence energy and r.m.s

- ➢ For bound states or sharp resonances, real time method works well.
- ➢ For broad resonances, the results were not converged.



Result for 4α system (^{16}O)

M.Dufour , P.Descouvement Nucl. Phys A. **927**(2014). 134-146.



	E_x[MeV]		$\sqrt{\langle r^2 \rangle}$[fm]		$M(E0)$[fm^2]	
J^π	This work	M.Dufour et al	This work	Exp.	This work	Exp.
0_1^+	-17.0	-15.2	2.32	2.71±0.02		
0_2^+	-1.6	-1.1	2.83		5.89	3.55±0.21
0_3^+	2.73	2.1	3.03		3.48	
0_4^+	7.43	8.0	3.40		0.39	

Summary

- For ^{8}Be(2α), ^{12}C(3α) , ^{16}O(4α), gas state is well known but those of ^{20}Ne(5α), ^{24}Mg(6α) $\cdots$ are not known.
- To solve the difficulty for generating wave function increasing number of α particles, we introduced real time evolution using time t as a generator coordinate.
- For ^{12}C(3α), ^{16}O(4α) nuclei, this method gives consistent result with previous research.

Future work

Y.Funaki PRC.**92** 021302(2015)

- Separate from continuum state.

$$\sum_{t}\langle\phi^{J\pi}_{i,M,K}(t)\mid\frac{1}{4N}\sum_{i=1}^{4N}(\vec{r_i}-\vec{X_G})^2-R^{(\gamma)^2}\mid\phi^{J\pi}_{j,M',K'}(t)\rangle c^{J\pi}_{M,K}(t)=0$$

  - For ^{8}Be(2α) in Brink wave function, this method goes well , so we try to adopt it to ^{12}C(3α) system.

Thank you for your attention.

Discussion   Check of the convergence of calc.

Our method is based on the ergodic nature
⇒ Following two pointes should be checked.

1. When the time evolution is long enough,
   the result should be converged.

2. The result should be independent of
   the initial configuration of α particles at $t = 0$.

Discussion The convergence energy and r.m.s

1. When the time evolution is long enough, the result should be converged.

$$\psi(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4N}) = \int_0^{\Delta t} dt F^{J\pi}_{MK}(t) P^{J\pi}_{MK} \phi(\vec{Z}_1(t), \vec{Z}_2(t), \cdots, \vec{Z}_N(t))$$