Charmed baryon における
2体系の研究へ向けて

半澤　光平 (総研大)

Contents

- Introduction
  - General property of charmed baryon

- Calculation for deuteron (as an example)
  - Boson exchange model in NN interaction
  - Numerical calculation

- Summary

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考えているモデルから

ポテンシャルを導出

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求めたポテンシャルを用いて

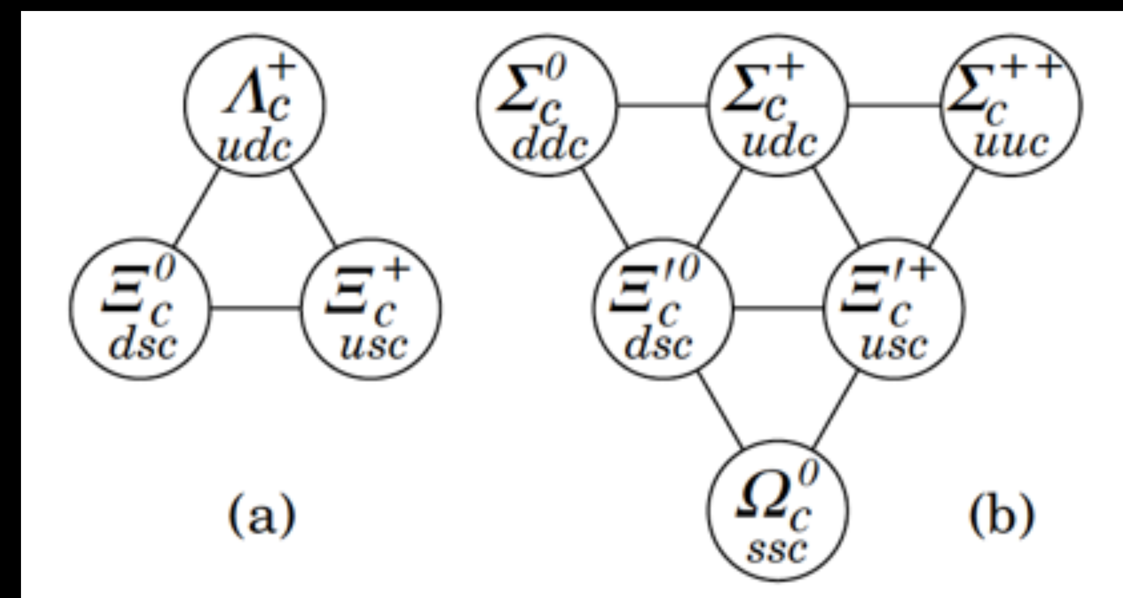
数値計算

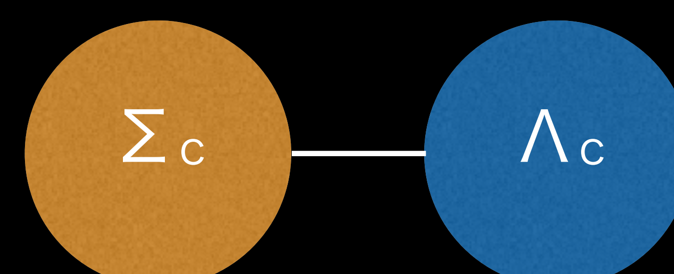
Introduction

General property of charmed baryon

Motivation

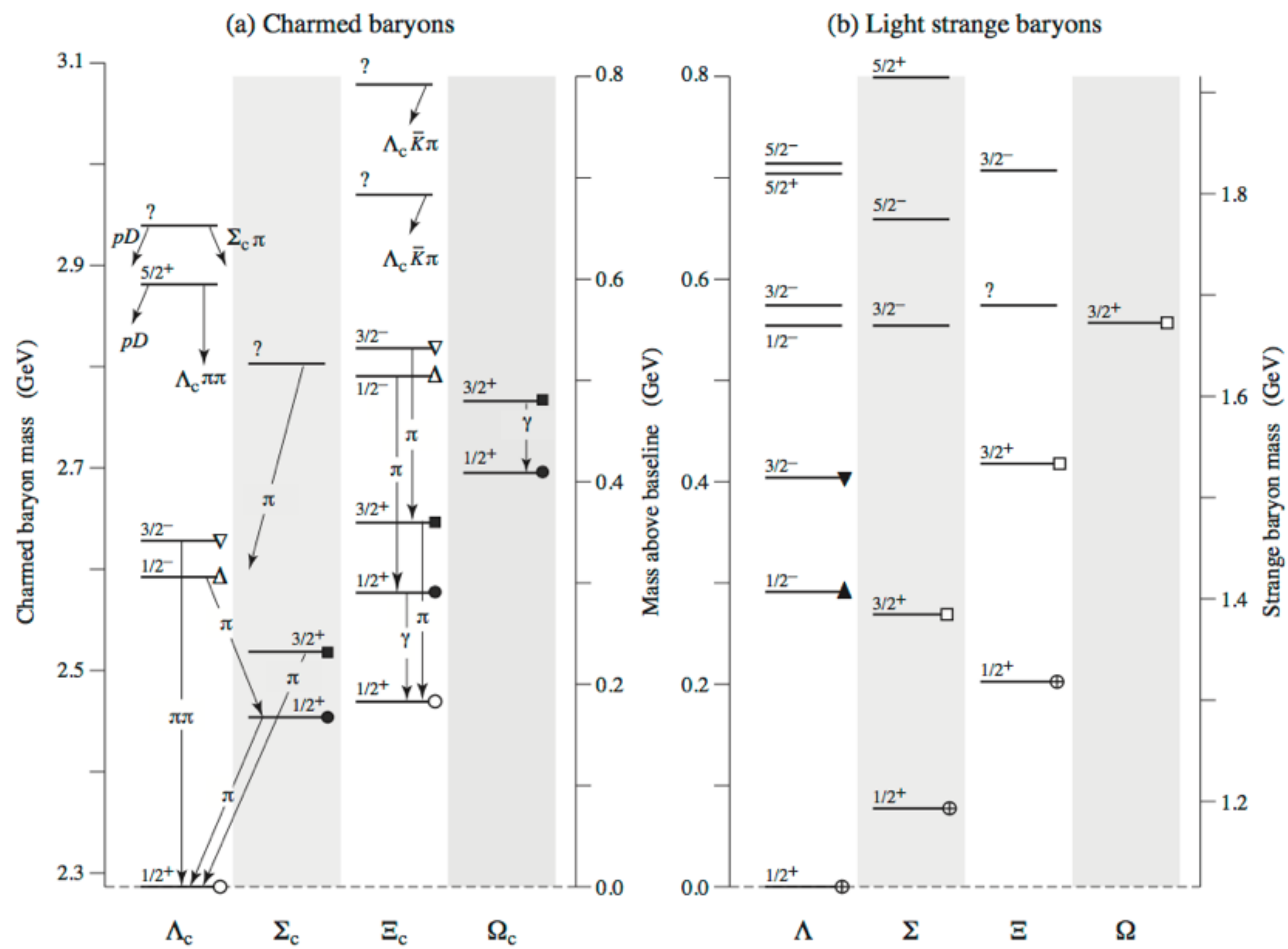
- フレーバーの拡張
- light quark で構成されるバリオン間相互作用との比較
- charmed baryon の2体束縛状態の存在可能性が軽いバリオンに比べ高い可能性がある
- チャームセクターでは結合チャネルがより重要になる可能性





束縛状態 ???

Known states



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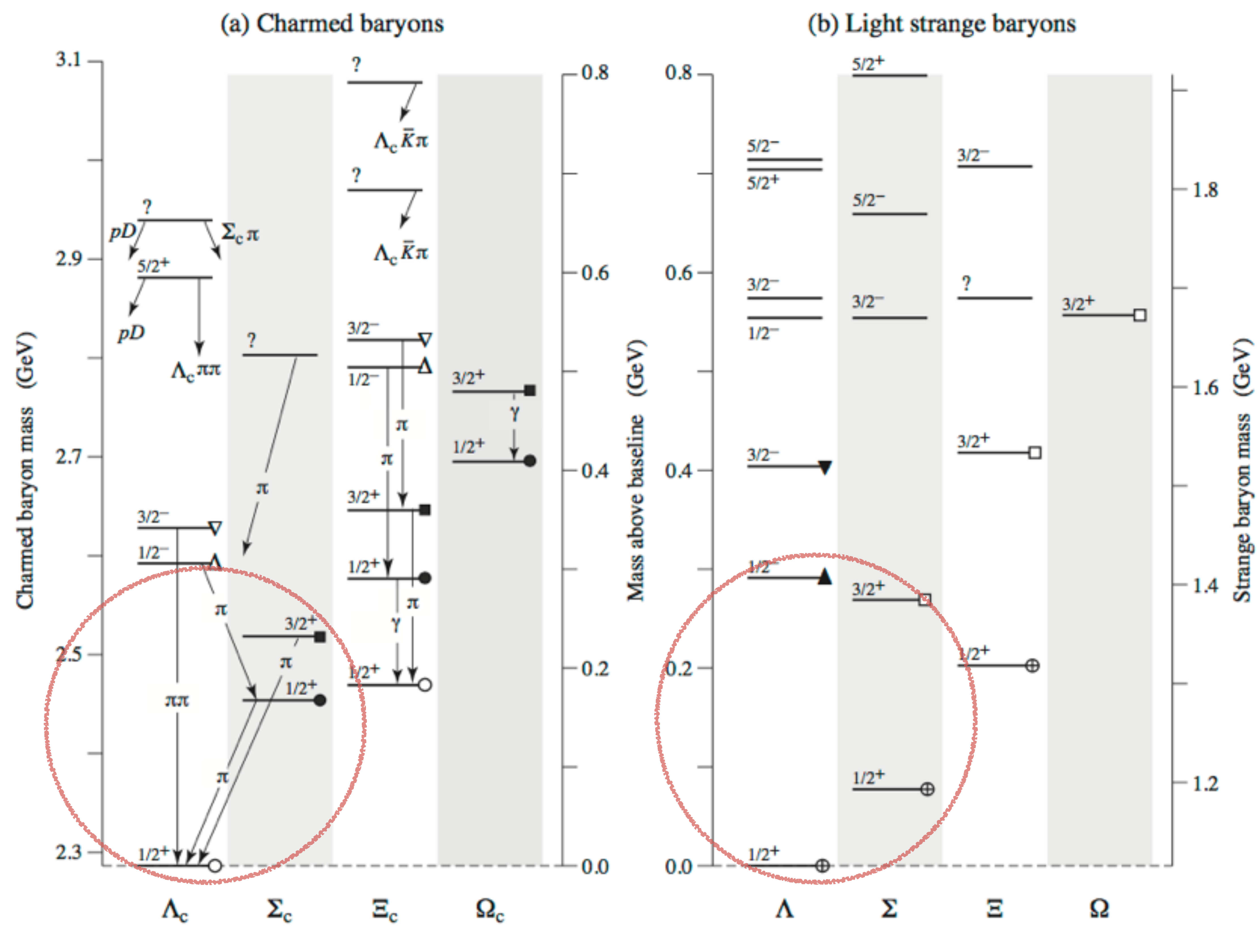

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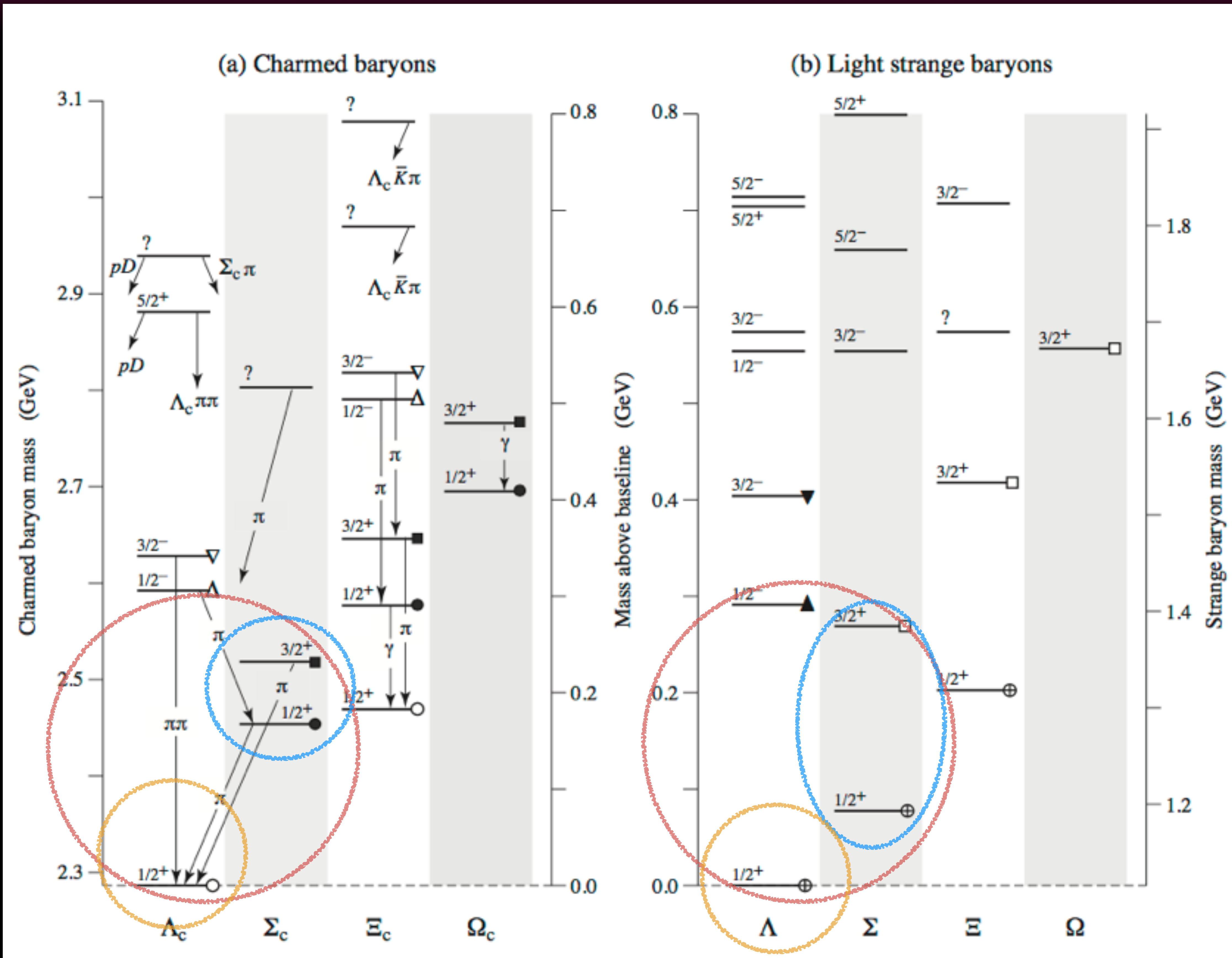

Known states

$\Lambda_c N$ system

$J^p = 0^+$

$^1S_0$ $^1S_0$ $^5D_0$

$\Lambda_c N - \Sigma_c N - \Sigma_c^* N$

3 channels

ΛN system

$J^p = 0^+$

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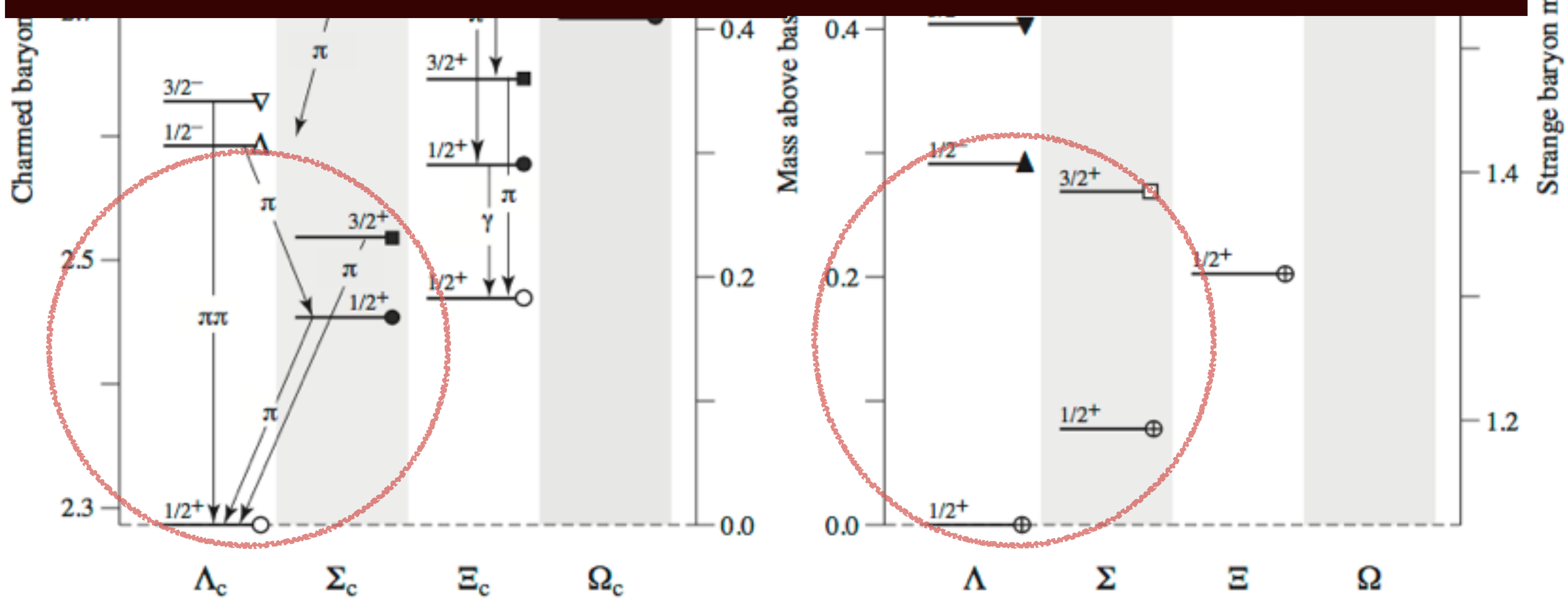
$\Lambda N - \Sigma N$

2 channels



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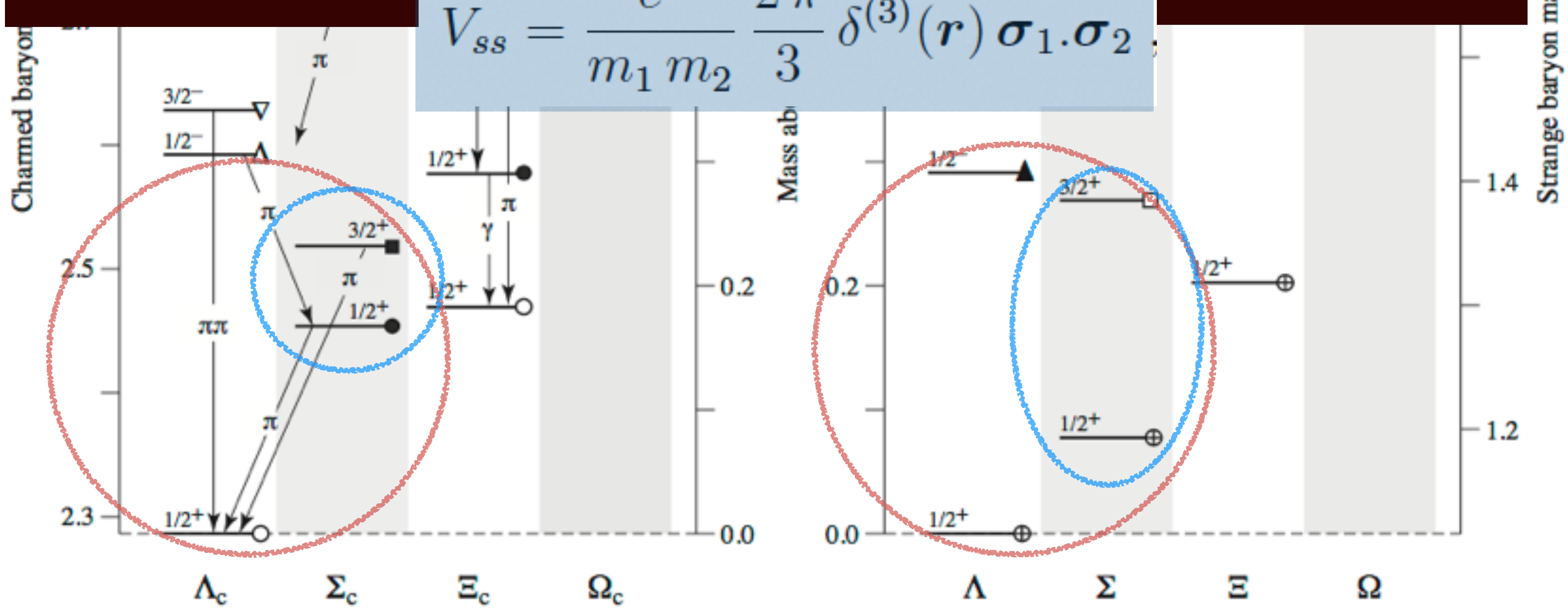
2 channels

$$V_{ss} = \frac{e^2}{m_1\, m_2}\,\frac{2\,\pi}{3}\,\delta^{(3)}(\boldsymbol{r})\,\boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2$$



Calculation for "deuteron" as an example

Boson Exchange Model in NN interaction

One Boson Exchange Model による2体系計算　手順

Boson exchange modelをベースに様々な対称性や仮定を用いて、モデルを構築

$$\mathcal{L} = \bar{\psi}_N(i\,\partial\!\!\!/ - m_N)\psi_N - g_{N\pi N}\bar{\psi}_N\gamma^5\boldsymbol{\tau}\psi_N\cdot\boldsymbol{\phi}_\pi - \frac{1}{2}(\partial_\mu\phi_\pi)(\partial^\mu\phi_\pi) + \frac{1}{2}m_\pi^2\phi_\pi^2$$

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バリオン二体散乱振幅を計算し、r-space のポテンシャルを導出

$$V(\boldsymbol{r}) = -\frac{\alpha_{N\pi N}}{4m_N^2}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla})\frac{1}{r}e^{-m_\pi r}$$

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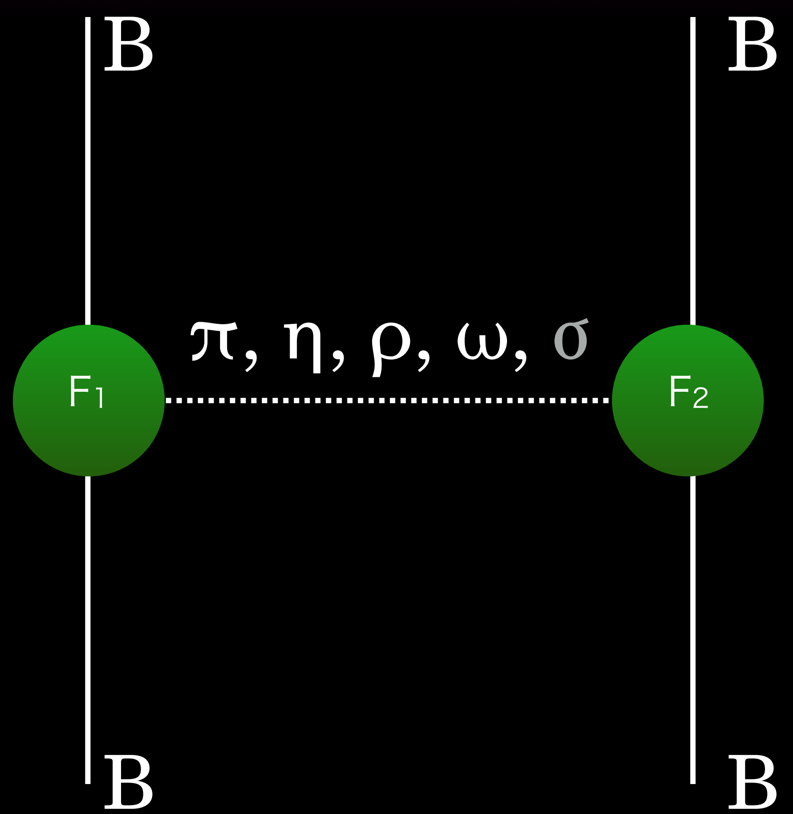
Gauss基底展開法を用いてbaryon二体系のシュレディンガー方程式を数値的に解く

One Boson Exchange Model



$\pi^{\pm}$ $I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57018 \pm 0.00035$ MeV (S = 1.2)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

π^0 $I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9766 \pm 0.0006$ MeV (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.52 \pm 0.18) \times 10^{-17}$ s (S = 1.2)
$c\tau = 25.5$ nm

η $I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 547.862 \pm 0.017$ MeV
Full width $\Gamma = 1.31 \pm 0.05$ keV

$\rho(770)$ [h] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV

$\omega(782)$ $I^G(J^{PC}) = 0^-(1^{--})$

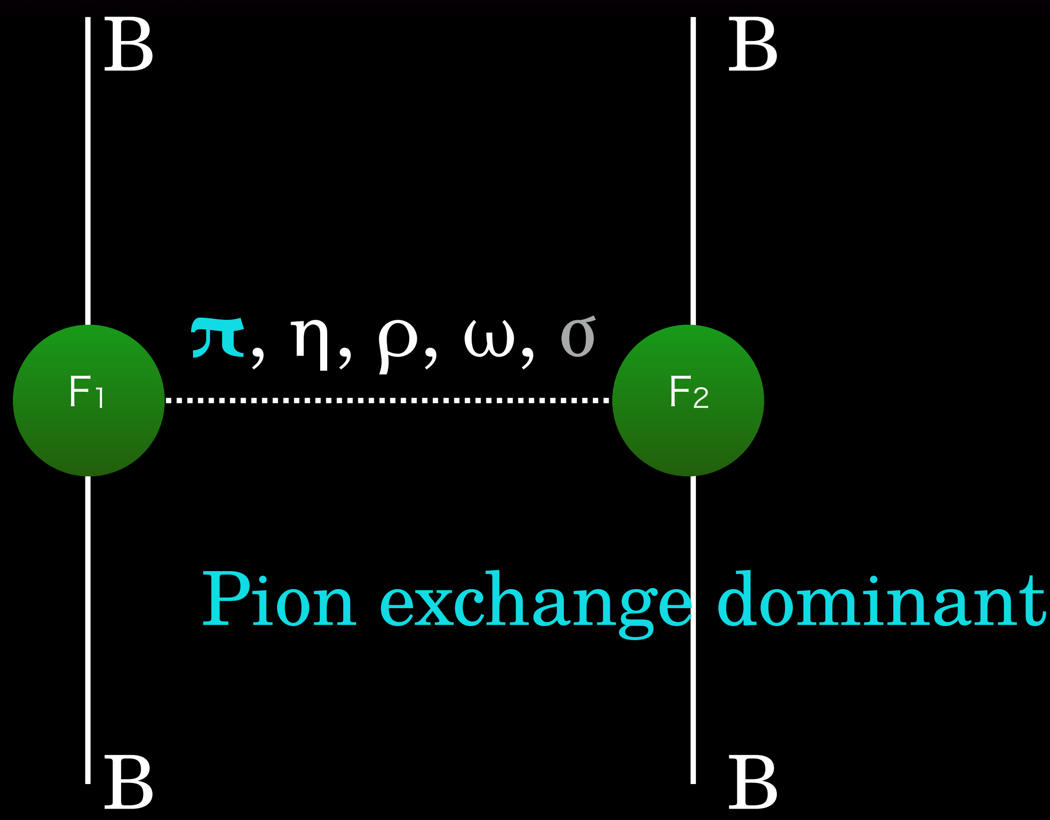
Mass $m = 782.65 \pm 0.12$ MeV (S = 1.9)
Full width $\Gamma = 8.49 \pm 0.08$ MeV
$\Gamma_{ee} = 0.60 \pm 0.02$ keV

$f_0(500)$ or σ [g] **was $f_0(600)$** $I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400\text{–}550)$ MeV
Full width $\Gamma = (400\text{–}700)$ MeV

K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update

One Boson Exchange Model



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One Pion Exchange

Lagrangian density :

$$\mathcal{L} = \bar{\psi}_N(i\,\partial\!\!\!/ - m_N)\psi_N - g_{N\pi N}\bar{\psi}_N\gamma^5\boldsymbol{\tau}\psi_N\cdot\boldsymbol{\phi}_\pi - \frac{1}{2}(\partial_\mu\phi_\pi)(\partial^\mu\phi_\pi) + \frac{1}{2}m_\pi^2\phi_\pi^2$$

Where,

$$\psi_N = \begin{bmatrix}\psi_n\\ \psi_p\end{bmatrix},\quad \boldsymbol{\phi_\pi} = \begin{bmatrix}\phi_{\pi+}\\ \phi_{\pi 0}\\ \phi_{\pi-}\end{bmatrix},\quad \boldsymbol{\tau} = \begin{bmatrix}\tau_+\\ \tau_0\\ \tau_-\end{bmatrix}$$

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Amplitude :

$$\mathcal{M} = -i\frac{g_{N\pi N}^2}{4m_N^2}((\boldsymbol{\tau})_{I_{1f},I_{1i}}\cdot(\boldsymbol{\tau})_{I_{2f},I_{2i}})\frac{((\boldsymbol{\sigma})_{S_{1f},S_{1i}}\cdot\boldsymbol{q})((\boldsymbol{\sigma})_{S_{2f},S_{2i}}\cdot\boldsymbol{q})}{\boldsymbol{q}^2+m_\pi^2}$$

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In the matrix representation,

$$\mathcal{M} = -i\frac{g_{N\pi N}^2}{4m_N^2}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})}{\boldsymbol{q}^2 + m_\pi^2}$$

One Pion Exchange

The potential in r-space : $V(\boldsymbol{r}) = -i \int d^3q \; e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \mathcal{M}$

$$V(\boldsymbol{r}) = -\frac{\alpha_{N\pi N}}{4m_N^2}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla})\frac{1}{\boldsymbol{r}}e^{-m_\pi \boldsymbol{r}}$$

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finally,

$$V(\boldsymbol{r}) = -\frac{\alpha_{N\pi N}}{12}\frac{m_\pi^2}{m_N^2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+S_{12}(\boldsymbol{r})(\frac{3}{(m_\pi r)^2}+\frac{3}{m_\pi r}+1))\frac{1}{r}e^{-m_\pi r}$$

$$S_{12} = 3\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})}{r^2} - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

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Tenser force

s-wave & d-wave mixing

Calculation for “deuteron”

Numerical Calculation

```
program main
  use definition
  use true_mat
  use expectation_value
  implicit none
  complex:: info
  integer,parameter:: lwork=3*nbase
  double precision:: work(lwork)

  call def_bb
  call def_nng
 call def_hhg
  print*, "frag1"
  ff=nng
  call dsyev('v', 'u', nbase, ff, nbase, mu, work, lwork, info)
  call true_hh
  print*, "frag2"
  dd=hh
  call dsyev('v', 'u', nbase, dd, nbase, energy, work, lwork, info)
  print*, "frag3"
  call def_cc
  call def_psi
  call def_rr2g

  call exv_tt
  call ex
  call exv
  call ex_rr2
  call probability
  call information

  call file_output
  return
end program main
```

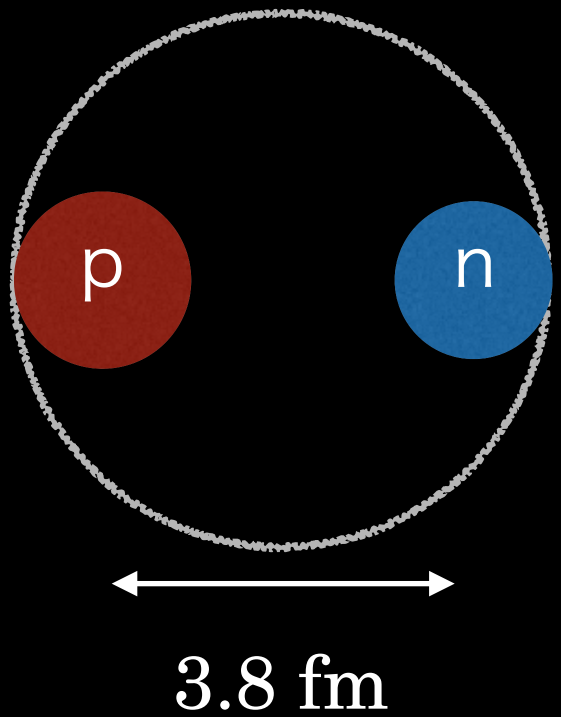
```
m1=   939.00000000000000       m2=   939.00000000000000
reduced mass =   469.50000000000000
number of bases =          40
total number of bases =          80
n space =        1000
bb(1)=  0.10000000000000001       bb(nmax)=   20.000000000000000
------------------------------
------------------------------
B.E.(          1 )=  -2.2825443086333439
<kinetic energy> (           1 )=   16.497245555281836
<central potential> (           1 )=  -21.306251328589600
<tensor potential>(           1 )=  -115.71413422948001
rms(          1 )=   3.9509722468371553
uu(~0)=  -8.9142700417922993E-002
ww(~0)=  -1.7097100283732005E-005
prob_S(          1 )=  0.95212784433943387
prob_D(          1 )=   4.7872155660566039E-002
```

Deuteron

$J^p=1^+$, T=0

B.E. = 2.22 MeV

Electric Quadrupole moment Q_d = 0.286 fm^2



$$\hat{H} = \hat{T} + \hat{V}_c + \hat{V}_T + \hat{V}_{LS}$$

- Central force
  - Attractive potential pocket
  - Repulsive core
- Tensor force
  - Spin triplet (S=1) のみ寄与
  - S-wave & D-wave の混合
- LS force

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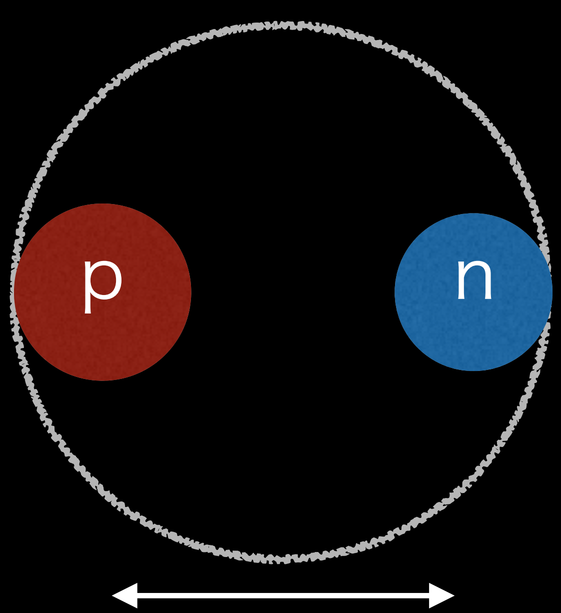

3.8 fm

$$\hat{H} = \hat{T} + \hat{V}_c + \hat{V}_T + \hat{V}_{LS}$$

Omit in this time

- <u>Central force</u>
  - Attractive potential pocket
  - Repulsive core
- <u>Tensor force</u>
  - Spin triplet (S=1) のみ寄与
  - S-wave & D-wave の混合
- <u>LS force</u>

数値計算　過程

Gauss 基底展開法

$$|\psi\rangle = \sum_{L=0,2} \sum_n C_n^{(L)} g_{L,n}(r) \, |^3L_1\rangle$$

$$g_{0,n}(r) = \frac{2}{\sqrt{\pi}} b_n^{1/2} \exp\left[-b_n r^2\right]$$

$$g_{2,n}(r) = \frac{4}{\sqrt{\pi}} b_n^{3/2} \exp\left[-b_n r^2\right]$$

$$b_n = b_1 \left(\frac{b_{N_{max}}}{b_1}\right)^{\frac{n-1}{N_{max}-1}}$$

Central force :

$$V_c(r) = \sum_i V_{c,i}^0 \exp\left[-(r/\eta_{c,i})^2\right]$$

Tensor force :

$$V_T(r) = S_{12} \sum_i V_{T,i}^0 \exp\left[-(r/\eta_{T,i})^2\right]$$

波動関数を
Gauss 基底で展開

Gauss 基底を用いて
直交基底を生成

得られた直交基底を用いて
Hamiltonian を対角化

得られた固有ベクトルを
用いて、各物理量の期待値
を計算

Tamagaki Potential (G3RS-1)

G3RS: Gaussian soft core potential with three ranges

Central force :

$$V_c(r) = \sum_i V^0_{c,i} \exp\left[-(r/\eta_{c,i})^2\right]$$

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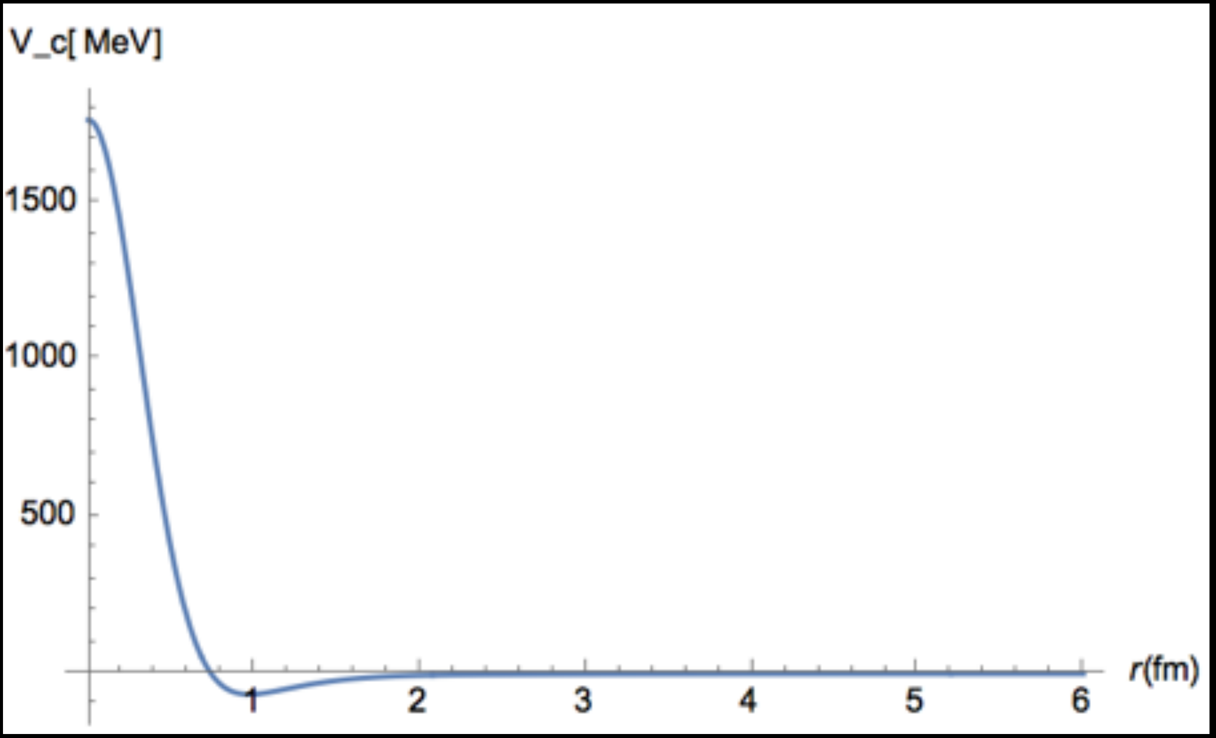
i	1	2	3	
$V_{c,i}$	−5	−230	2000	[MeV]
$\eta_{c,i}$	2.5	0.942	0.447	[fm]

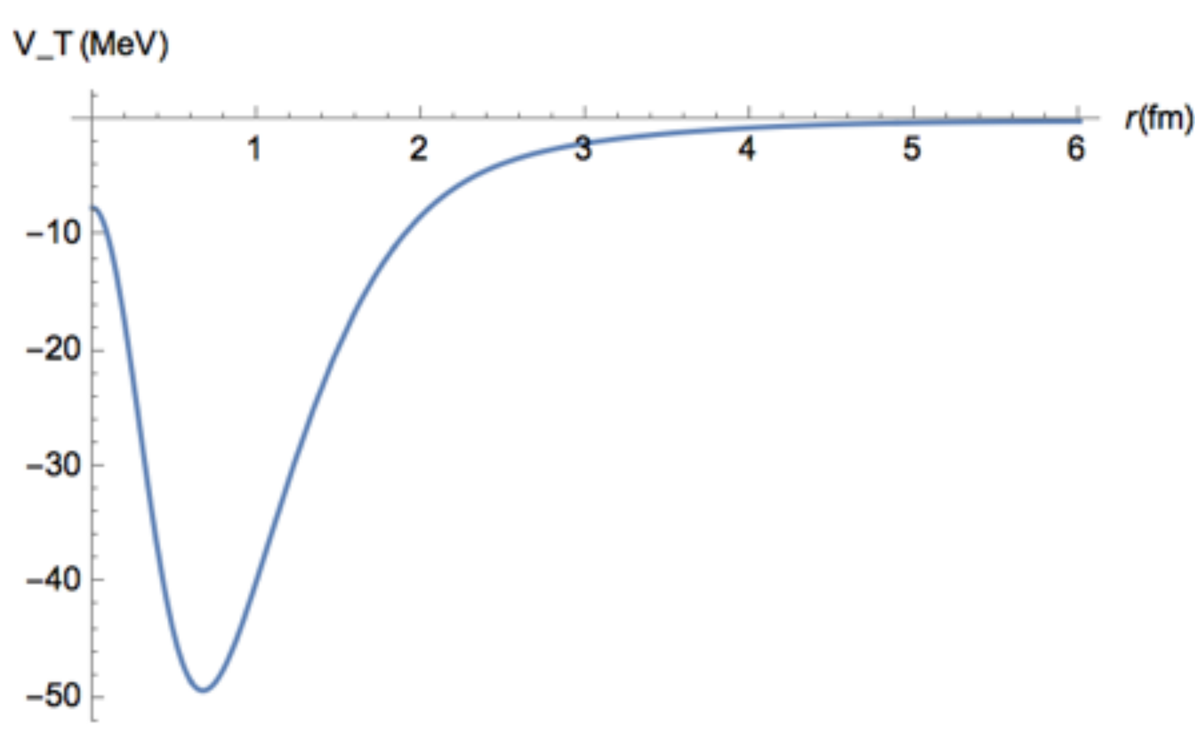
i	1	2	3	
$V_{T,i}$	−7.5	−67.5	67.5	[MeV]
$\eta_{T,i}$	2.5	1.2	0.447	[fm]

R. Tamagaki, Prog. Theor. Phys. 38, 91 (1968)

V_c [MeV]



V_t [MeV]



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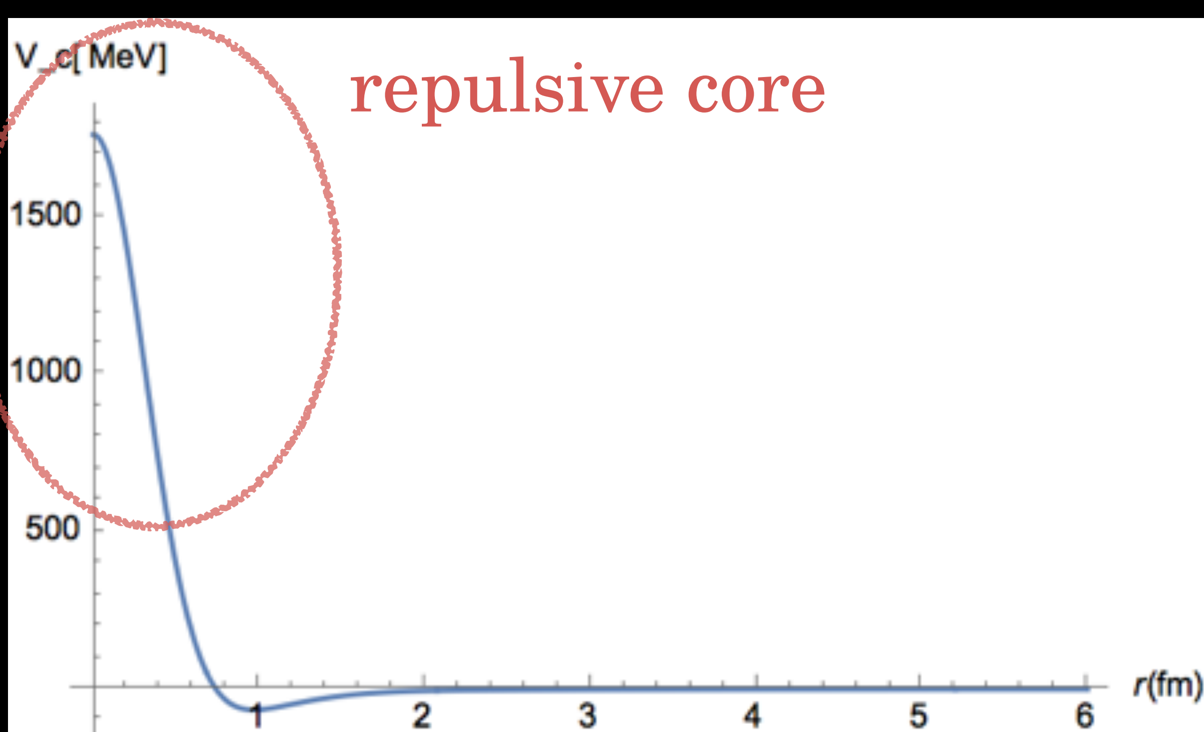

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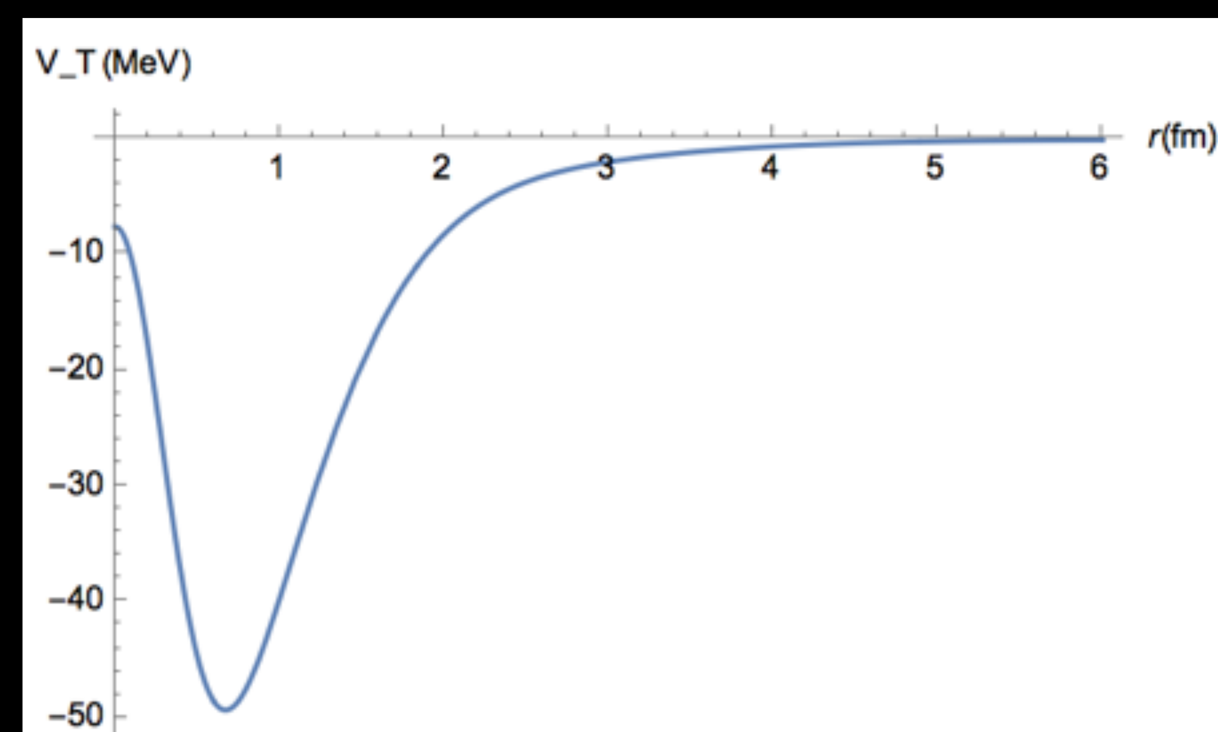

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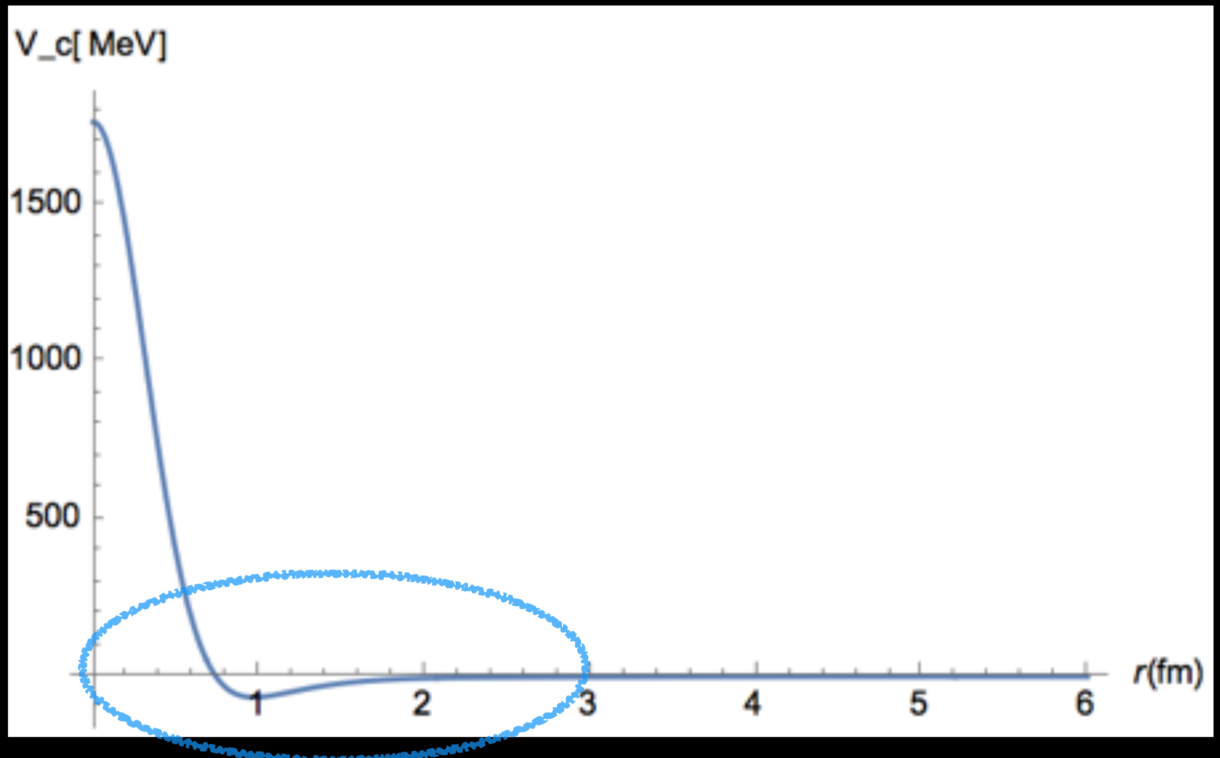

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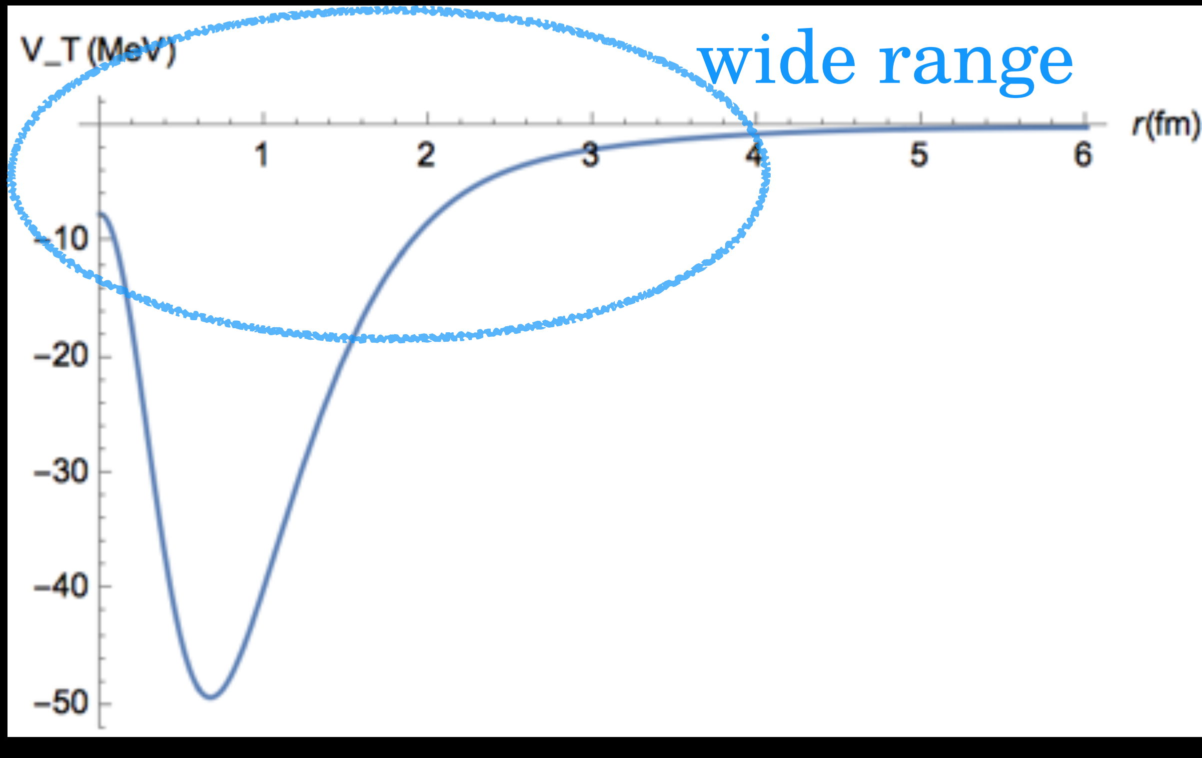

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G3RS: Gaussian soft core potential with three ranges

Central force :

$$V_c(r) = \sum_i V^0_{c,i} \exp\left[-(r/\eta_{c,i})^2\right]$$

Tensor force :

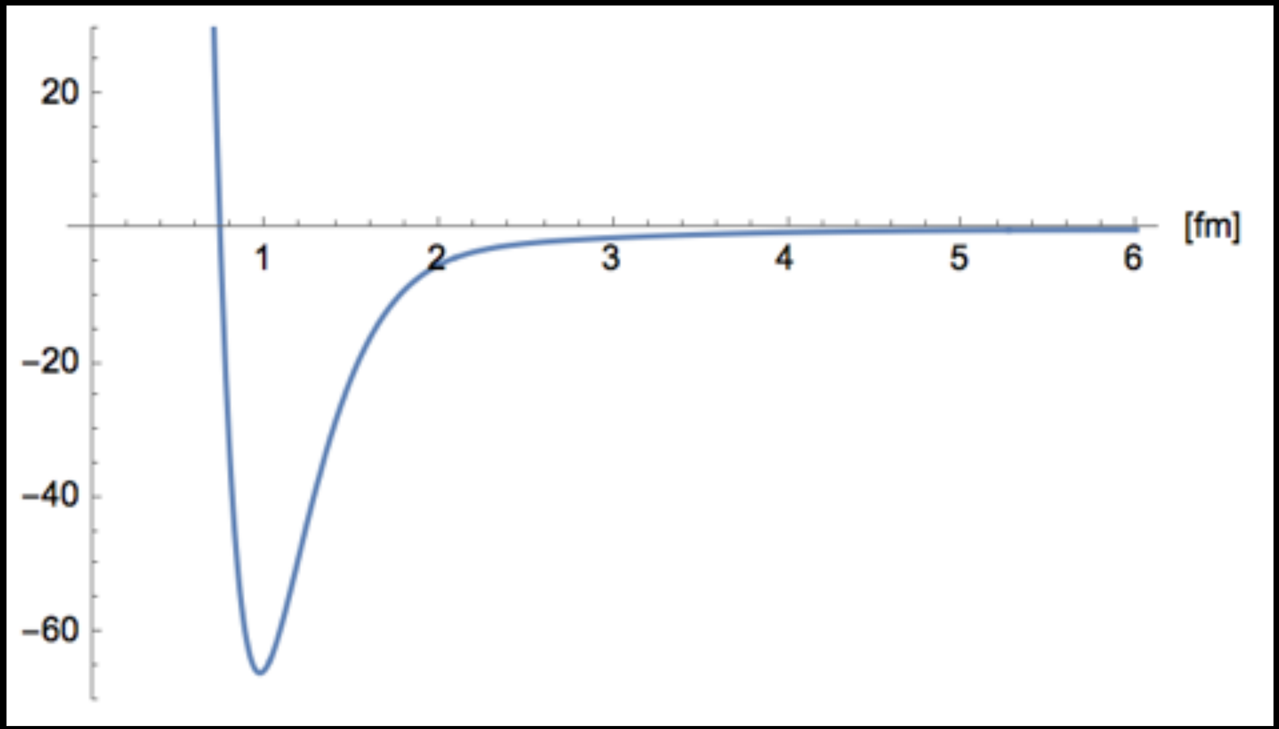
$$V_T(r) = S_{12} \sum_i V^0_{T,i} \exp\left[-(r/\eta_{T,i})^2\right]$$

i	1	2	3	
$V_{c,i}$	−5	−230	2000	[MeV]
$\eta_{c,i}$	2.5	0.942	0.447	[fm]

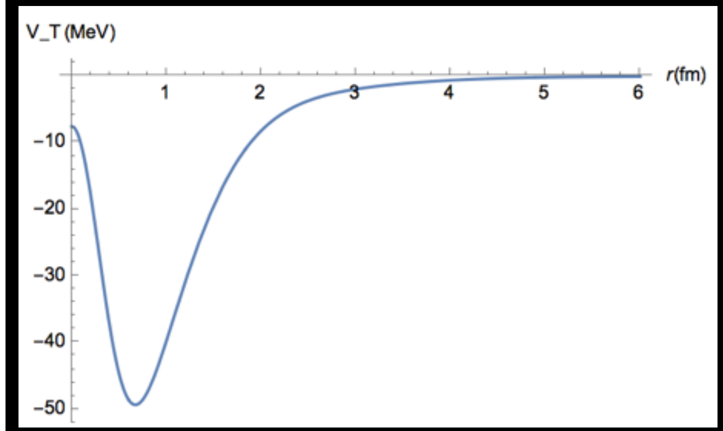
i	1	2	3	
$V_{T,i}$	−7.5	−67.5	67.5	[MeV]
$\eta_{T,i}$	2.5	1.2	0.447	[fm]

R. Tamagaki, Prog. Theor. Phys. 38, 91 (1968)

V_c [MeV]



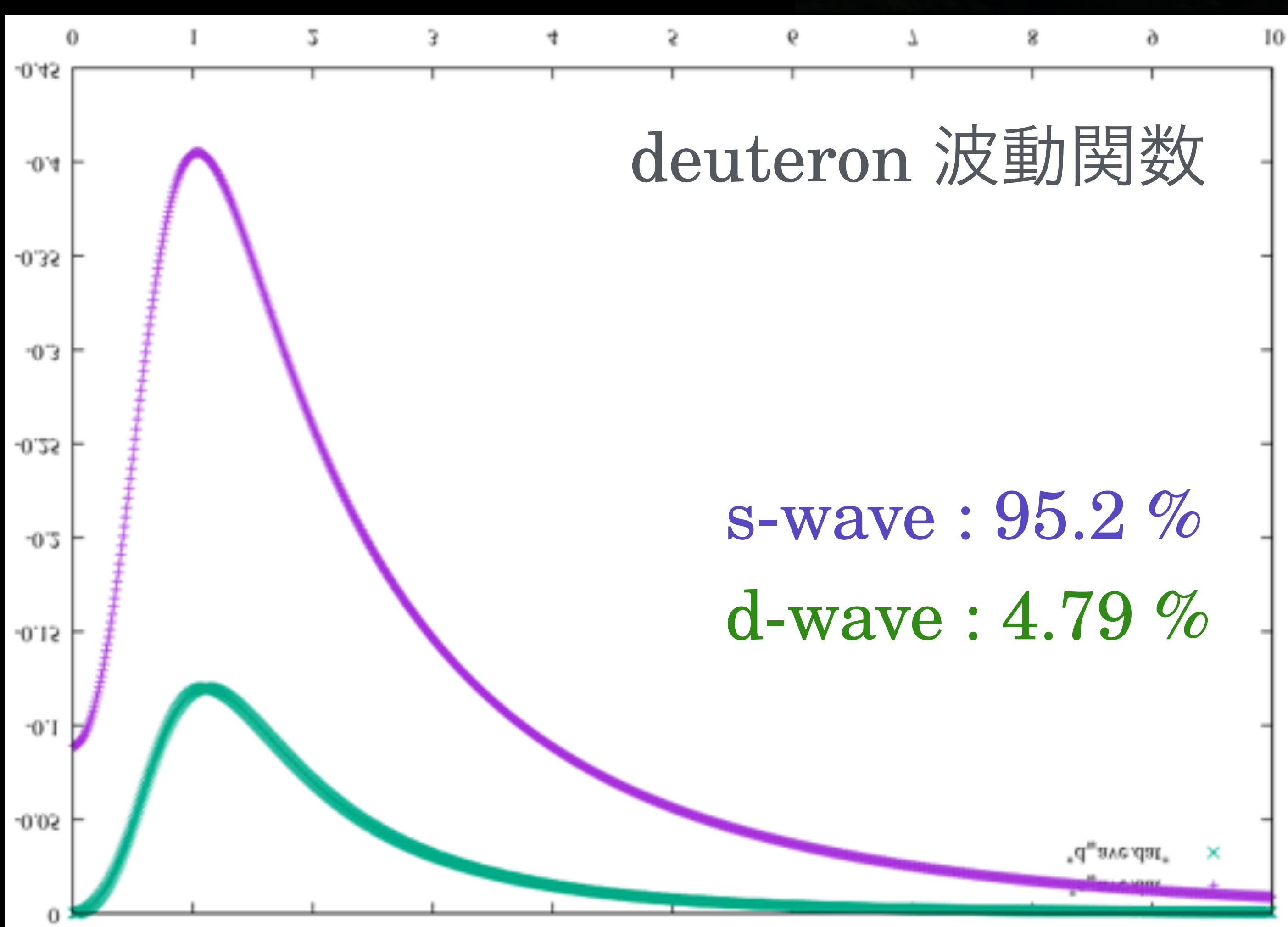
V_t [MeV]



Result


```
m1=    939.00000000000000        m2=    939.00000000000000
reduced mass =    469.50000000000000
number of bases =           40
total number of bases =           80
n space =         1000
bb(1)=   0.10000000000000001       bb(nmax)=    20.000000000000000
--------------------------------
--------------------------------
B.E.(           1 )=   -2.2825443086718691
<kinetic energy> (           1 )=    16.497245555272343
<central potential> (           1 )=   -7.3037026710165378
<tensor potential>(           1 )=   -11.476087192930956
rms(           1 )=    3.9509722468456099
uu(~0)=    8.9142700408849057E-002
ww(~0)=    1.7097100306832829E-005
prob_S(           1 )=  0.95212784433949849
prob_D(           1 )=    4.7872155660503443E-002
```

[fm$^{-3/2}$]



[fm]

B.E. = 2.28 MeV

<T> = 16.5 MeV

<V_c> = -7.30 MeV

<V_t> = -11.5 MeV

“N-N” mean distance = 3.95 fm

Summary

興味

- heavy quark の自由度と light quark の自由度では、バリオン2体系の振る舞いにどのような違いが出るか

手段

- Charm baryon 2体系をBoson Exchange potential model を用いて計算
- シュレディンガー方程式をGauss 基底展開法を用いて数値計算
- Deuteron における計算方法と同様にcharm sector でも結合しうるチャネル分の空間を生成し、結合チャネルを計算する

現段階の進捗

- Deuteron を用いて、研究手法を確立
- Charm sector のバリオン間相互作用を幾つか導出済

目標

- 未だ検証されていないバリオン二体系を計算
- light quark sector と heavy quark sector における振る舞いの違いを検討

Back up

Charm quark

$$\frac{m_c}{(m_u+m_d)/2} \sim 370$$

$$\frac{m_c}{m_s} \sim 13$$

QUARKS

The *u*-, *d*-, and *s*-quark masses are estimates of so-called "current-quark masses," in a mass-independent subtraction scheme such as $\overline{\text{MS}}$ at a scale $\mu \approx 2$ GeV. The *c*- and *b*-quark masses are the "running" masses in the $\overline{\text{MS}}$ scheme. For the *b*-quark we also quote the 1S mass. These can be different from the heavy quark masses obtained in potential models.

u $\qquad I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_u = 2.3^{+0.7}_{-0.5}$ MeV $\qquad$ Charge $= \frac{2}{3}\,e$ $\qquad I_z = +\frac{1}{2}$
$m_u/m_d = 0.38$–0.58

d $\qquad I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_d = 4.8^{+0.5}_{-0.3}$ MeV $\qquad$ Charge $= -\frac{1}{3}\,e$ $\qquad I_z = -\frac{1}{2}$
$m_s/m_d = 17$–22
$\overline{m} = (m_u+m_d)/2 = 3.5^{+0.7}_{-0.2}$ MeV

s $\qquad I(J^P) = 0(\frac{1}{2}^+)$

$m_s = 95 \pm 5$ MeV $\qquad$ Charge $= -\frac{1}{3}\,e$ $\qquad$ Strangeness $= -1$
$m_s \,/\, ((m_u + m_d)/2) = 27.5 \pm 1.0$

c $\qquad I(J^P) = 0(\frac{1}{2}^+)$

$m_c = 1.275 \pm 0.025$ GeV $\qquad$ Charge $= \frac{2}{3}\,e$ $\qquad$ Charm $= +1$

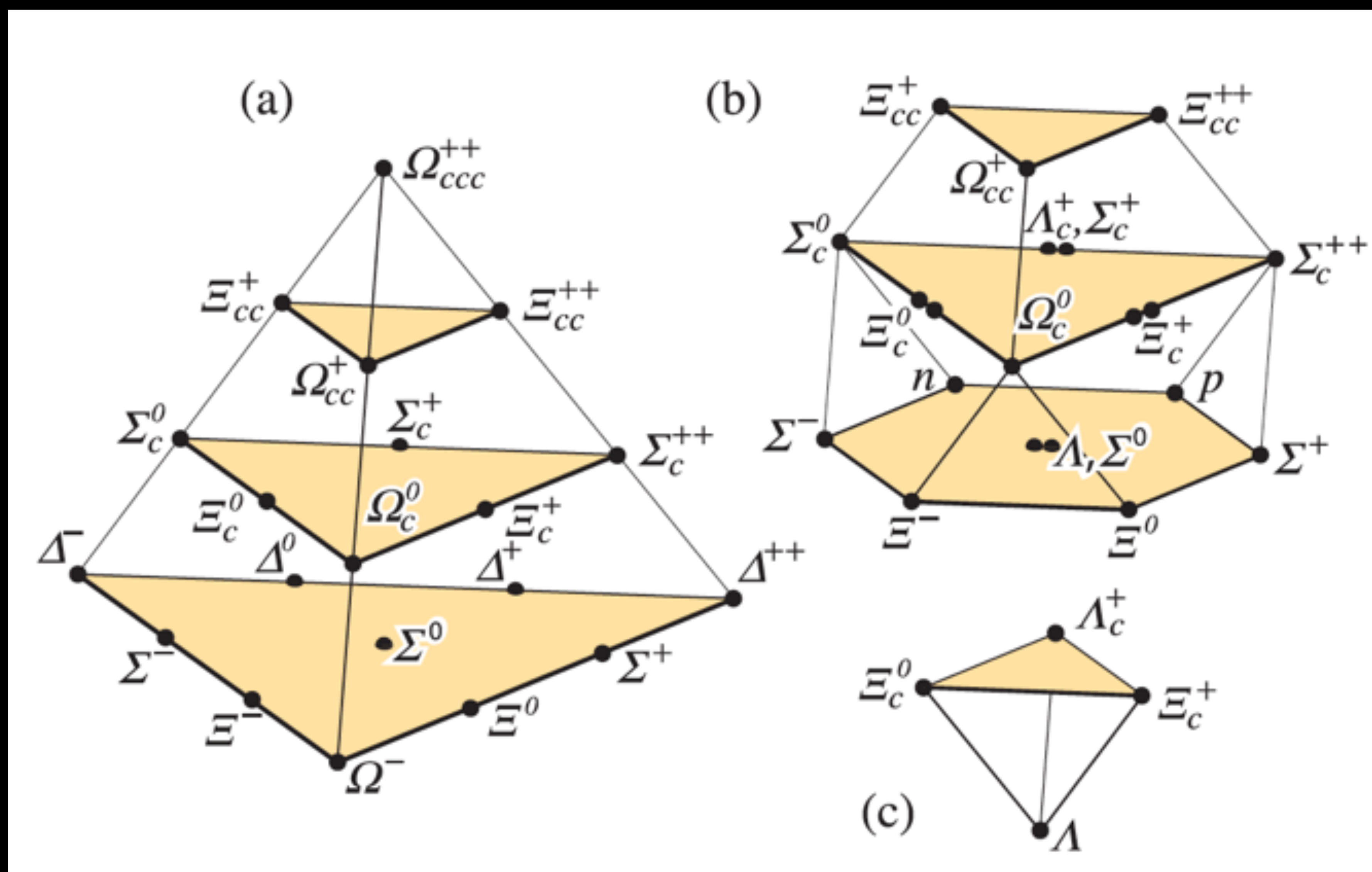
K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014)

SU(4) multiplets

CHARMED BARYONS
$(C = +1)$

$\Lambda_c^+ = udc,\quad \Sigma_c^{++} = uuc,\quad \Sigma_c^+ = udc,\quad \Sigma_c^0 = ddc,$
$\Xi_c^+ = usc,\quad \Xi_c^0 = dsc,\quad \Omega_c^0 = ssc$

$$4 \times 4 \times 4 = 20 + 20'_1 + 20'_2 + \bar{4}$$



K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014)

One ρ meson Exchange

Lagrangian density :

$$\mathcal{L} = \bar{\psi}_N(i\,\partial\!\!\!/ - m_N)\psi_N - g_{N\rho N}\bar{\psi}_N\gamma_\mu\tau^a\psi_N A_\rho^{a\mu} - \frac{1}{4}F^a_{\rho\,\mu\nu}F_\rho^{a\mu\nu} + \frac{1}{2}m_\rho^2 A^a_{\rho_\mu}A_\rho^{a\mu}$$

Amplitude :

$$\mathcal{M} = -ig^2_{N\rho N}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{1 - \frac{1}{4m_N^2}(i\boldsymbol{\sigma}\cdot(\boldsymbol{k}\times\boldsymbol{q}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{q}^2 + (\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}))}{\boldsymbol{q}^2 + m_\rho^2}$$

Amplitudes of One Boson Exchange

- Feynman amplitudes

$$\mathcal{M}_a = -ig^2 \bar{u}_{1f} \Gamma_\alpha u_{1i} D^{\alpha\beta}(q_a = p_{1f} - p_{1i}) \bar{u}_{2f} \Gamma_\beta u_{2i}$$

$$\mathcal{M}_b = ig^2 \bar{u}_{2f} \Gamma_\alpha u_{1i} D^{\alpha\beta}(q_b = p_{2i} - p_{1f}) \bar{u}_{1f} \Gamma_\beta u_{2i}$$

$$\Gamma = \{1, \gamma^5, \gamma^\mu, \gamma^\mu \gamma^5\}$$

At "center of mass system" and "static limit",

i.e.

$$\boldsymbol{p}_{2i} = -\boldsymbol{p}_{1i} = \boldsymbol{p}_i, \quad \boldsymbol{p}_{2f} = -\boldsymbol{p}_{1f} = \boldsymbol{p}_f$$
$$p^0_{1i} = p^0_{1f} = p^0_{2i} = p^0_{2f} = p^0, \quad p^0_2 - p^0_1 = q^0 = 0$$

$$\mathcal{M}_a = -ig^2 \bar{u}_{1f}(\boldsymbol{p}_f) \Gamma_\alpha u_{1i}(\boldsymbol{p}_i) D^{\alpha\beta}(\boldsymbol{q}_a = \boldsymbol{p}_f - \boldsymbol{p}_i \equiv \boldsymbol{q}) \bar{u}_{2f}(-\boldsymbol{p}_f) \Gamma_\beta u_{2i}(-\boldsymbol{p}_i)$$

$$\mathcal{M}_b = ig^2 \bar{u}_{2f}(-\boldsymbol{p}_f) \Gamma_\alpha u_{1i}(\boldsymbol{p}_i) D^{\alpha\beta}(\boldsymbol{q}_b = \boldsymbol{p}_f + \boldsymbol{p}_i \equiv 2\boldsymbol{k}) \bar{u}_{1f}(\boldsymbol{p}_f) \Gamma_\beta u_{2i}(-\boldsymbol{p}_i)$$

Λ_c N bound states

Yan-Rui Liu and Makoto Oka
Phys. Rev. D85, 014015 (2012)

$$\mathcal{L}_{\mathcal{B}} = \mathcal{L}_{B_{\bar{3}}} + \mathcal{L}_S + \mathcal{L}_{\text{int}}, \quad (3)$$

$$\mathcal{L}_{B_{\bar{3}}} = \frac{1}{2}\text{tr}[\bar{B}_{\bar{3}}(iv \cdot D)B_{\bar{3}}] + i\beta_B \text{tr}[\bar{B}_{\bar{3}} v^\mu (\Gamma_\mu - V_\mu) B_{\bar{3}}] + \ell_B \text{tr}[\bar{B}_{\bar{3}} \sigma B_{\bar{3}}] \quad (4)$$

$$\mathcal{L}_S = -\text{tr}[\bar{S}^\alpha (iv \cdot D - \Delta_B) S_\alpha] + \frac{3}{2} g_1 (iv_\kappa) \epsilon^{\mu\nu\lambda\kappa} \text{tr}[\bar{S}_\mu A_\nu S_\lambda] + i\beta_S \text{tr}[\bar{S}_\mu v_\alpha (\Gamma^\alpha - V^\alpha) S^\mu] + \lambda_S \text{tr}[\bar{S}_\mu F^{\mu\nu} S_\nu] + \ell_S \text{tr}[\bar{S}_\mu \sigma S^\mu] \quad (5)$$

$$\mathcal{L}_{\text{int}} = g_4 \text{tr}[\bar{S}^\mu A_\mu B_{\bar{3}}] + i\lambda_I \epsilon^{\mu\nu\lambda\kappa} v_\mu \text{tr}[\bar{S}_\nu F_{\lambda\kappa} B_{\bar{3}}] + \text{H.c.},$$

<u>Assumptions</u>

- One boson exchange potential model
- Heavy quark symmetry
- Chiral symmetry for pions
- Hidden symmetry for vector mesons
- Quark model
- Chiral multiplets
- Vector meson dominance
- QCD sum rule

$$B_{\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix},$$

$$B_6 = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c^{\prime +} \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c^{\prime 0} \\ \frac{1}{\sqrt{2}}\Xi_c^{\prime +} & \frac{1}{\sqrt{2}}\Xi_c^{\prime 0} & \Omega_c^0 \end{pmatrix},$$

$$B_6^* = \begin{pmatrix} \Sigma_c^{*++} & \frac{1}{\sqrt{2}}\Sigma_c^{*+} & \frac{1}{\sqrt{2}}\Xi_c^{*+} \\ \frac{1}{\sqrt{2}}\Sigma_c^{*+} & \Sigma_c^{*0} & \frac{1}{\sqrt{2}}\Xi_c^{*0} \\ \frac{1}{\sqrt{2}}\Xi_c^{*+} & \frac{1}{\sqrt{2}}\Xi_c^{*0} & \Omega_c^{*0} \end{pmatrix},$$

$$\Pi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

$$V^\mu = i\frac{g_V}{\sqrt{2}}\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}^\mu,$$

$$A_\mu = \frac{i}{2}[\xi^\dagger(\partial_\mu \xi) + (\partial_\mu \xi)\xi^\dagger],$$

$$\Gamma_\mu = \frac{1}{2}[\xi^\dagger(\partial_\mu \xi) - (\partial_\mu \xi)\xi^\dagger],$$

$$\xi = \exp\left[\frac{i\Pi}{2f}\right],$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu],$$

$$D_\mu B_{\bar{3}} = \partial_\mu B_{\bar{3}} + \Gamma_\mu B_{\bar{3}} + B_{\bar{3}}\Gamma_\mu^T,$$

$$D_\mu S_\nu = \partial_\mu S_\nu + \Gamma_\mu S_\nu + S_\nu \Gamma_\mu^T.$$

Λ_c N bound states

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TABLE XV. $J = 0$ and $J = 1$ binding energies (MeV) corresponding to the variation of only one coupling constant. The unchanged coupling constants are the same as in Eq. (31). The units of $(\lambda_S g_V)$ and h_T are GeV^{-1}. The common cutoff $\Lambda_{\mathrm{com}} = 0.9$ GeV is used.

g_2	B.E.	g_4	B.E.	ℓ_B	B.E.	$(\beta_B g_V)$	B.E.
−0.50	13.17, 11.36	0.7	6.93, 9.54	−2.0	2.09, 4.15	−3.0	14.75, 14.67
−0.55	13.38, 12.38	0.8	8.77, 10.67	−2.5	6.24, 9.15	−4.0	14.36, 14.27
−0.60	13.61, 13.54	0.9	10.99, 11.99	−3.0	12.21, 15.75	−5.0	13.98, 13.88
−0.65	13.83, 14.84	1.0	13.62, 13.51	−3.5	19.75, 23.76	−6.0	13.60, 13.49
−0.70	14.07, 16.29	1.1	16.68, 15.26	−4.0	28.68, 33.03	−7.0	13.22, 13.11
$(\lambda_S g_V)$	B.E.	h_σ	B.E.	h_V	B.E.	h_T	B.E.
13.0	13.03, 13.21	8.0	4.04, 4.10	2.0	13.92, 13.89	5.0	13.48, 13.53
15.0	13.20, 13.29	10.0	10.00, 9.96	2.5	13.76, 13.69	6.0	13.56, 13.50
17.0	13.39, 13.39	12.0	18.08, 17.90	3.0	13.60, 13.49	7.0	13.66, 13.48
19.0	13.58, 13.48	14.0	27.98, 27.63	3.5	13.44, 13.30	8.0	13.77, 13.47
21.0	13.78, 13.59	16.0	39.46, 38.93	4.0	13.28, 13.10	9.0	13.89, 13.47

シュレディンガー方程式

$$\hat{H} = \hat{T} + \hat{V}_c + \hat{V}_T + \hat{V}_{LS}$$

$$\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$$

$$\mu = \frac{m_N}{2}$$

$$r = |\boldsymbol{r}|$$

$$\hat{T} = -\frac{\bar{h}^2}{2\mu}\nabla^2$$: 運動項

$$\hat{V}_c = V_c(r)$$: 中心力項

$$\hat{V}_T = V_T(r)S_{12}$$: テンソル力項

$$S_{12} = 3\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})}{r^2} - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

~~$\hat{V}_{LS} = V_{LS}(r)\boldsymbol{l} \cdot \boldsymbol{s}$: LS力項~~