YONUPA "Young Nuclear and Particle Physicist Group of Japan"
Summer School 2021: August 6-10, 2021 (via Zoom)
8/8 (Sun) 10:00-11:30 Sekizawa - Lecture 3

時間依存密度汎関数法で探る原子核ダイナミクス: 原子核反応から超流動現象，中性子星まで

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Lecture Plan

Theory (8/7 10:00-11:30)

#1. An introduction to microscopic mean-field approaches and (TD)DFT

Nuclear reactions (8/7 13:00-14:30)

#2. Recent advances in microscopic approaches for heavy-ion reactions

Superheavy element synthesis & deep-inelastic collisions

Neutron stars (8/8 10:00-11:30)

#3. Neutron-star "glitch" and neutron superfluid

Dynamics of quantized vortices in the inner crust of neutron stars

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Dynamics of <u>quantized vortices</u> in the inner crust of neutron stars

Voyage towards the limit of existence

Now we are sailing towards the edge of the nuclear landscape..

Let’s leave the planet of finite nuclei!

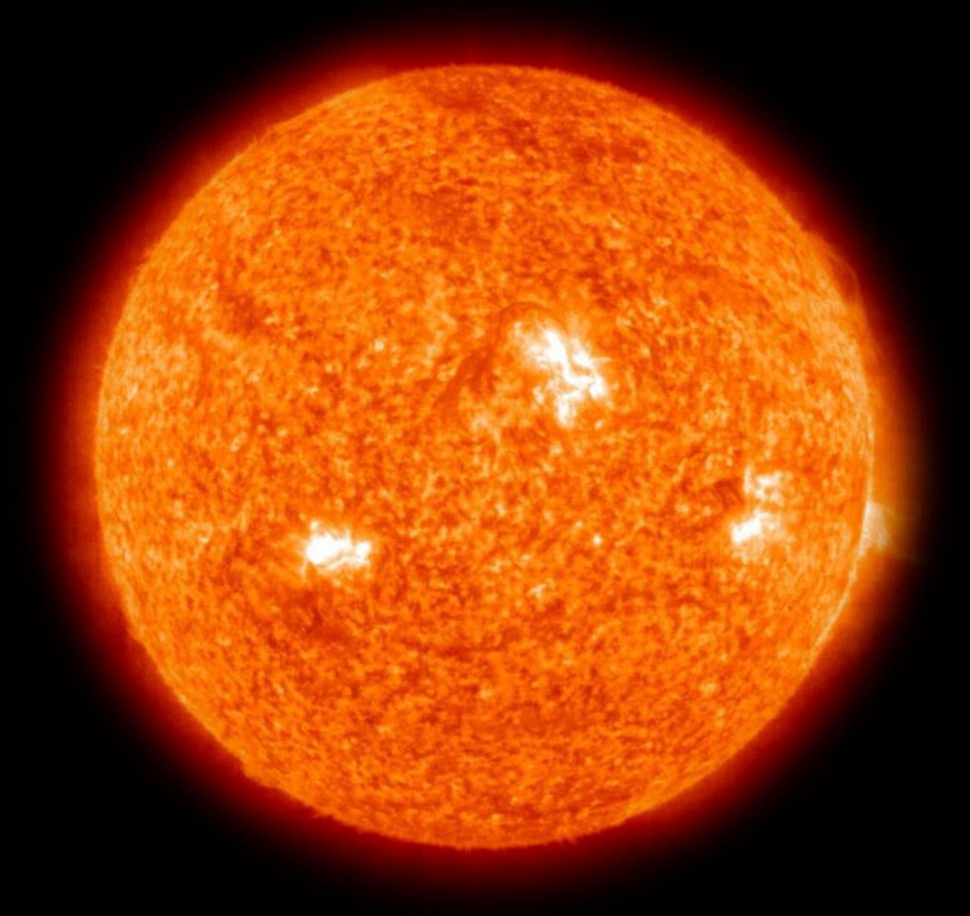
“Flat Earth” by [iStock](); [“Star Wars Spaceship” by Valérian Pierret @Artstation]()

From nuclei to **neutron stars**

“3d hyperspace background with warp tunnel effect Free Photo” @freepik

The Sun

- Radius: ~7×10^{8}m (~109 times bigger than Earth)
- Mass: ~2×10^{30}kg (~330 thousands times heavier than Earth)
- Central temp.: ~10 million ℃
- Surface temp.: ~5000 ℃

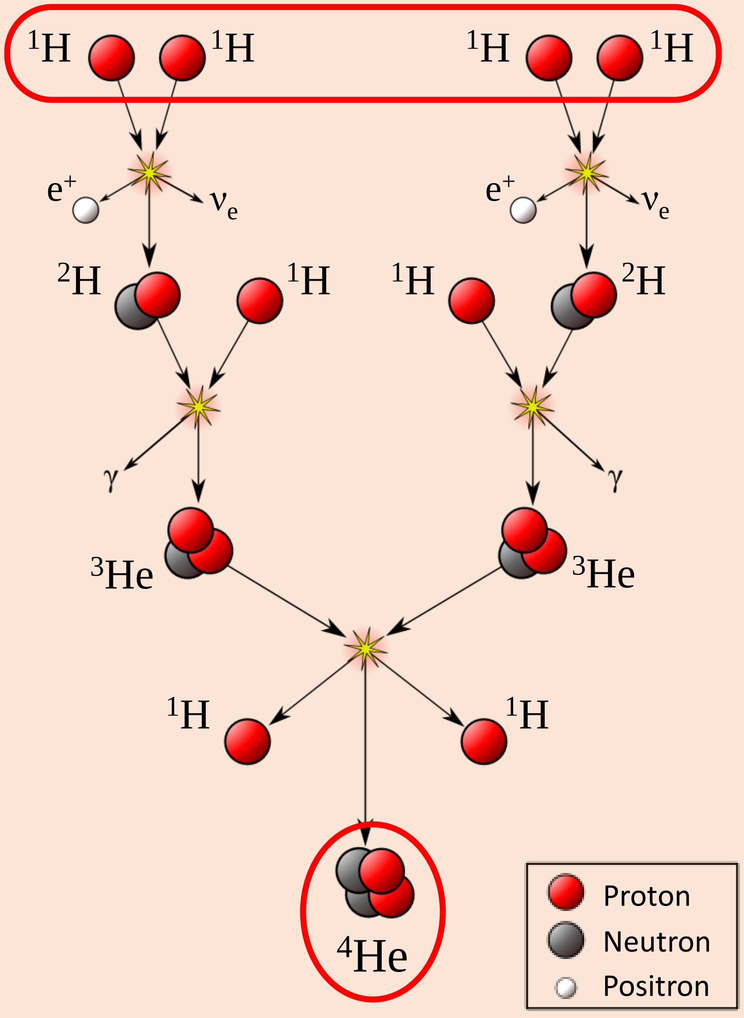


Stars shine due to nuclear fusion reactions

Picture: https://en.wikipedia.org/wiki/Sun

Energy source of the Sun: Nuclear fusion

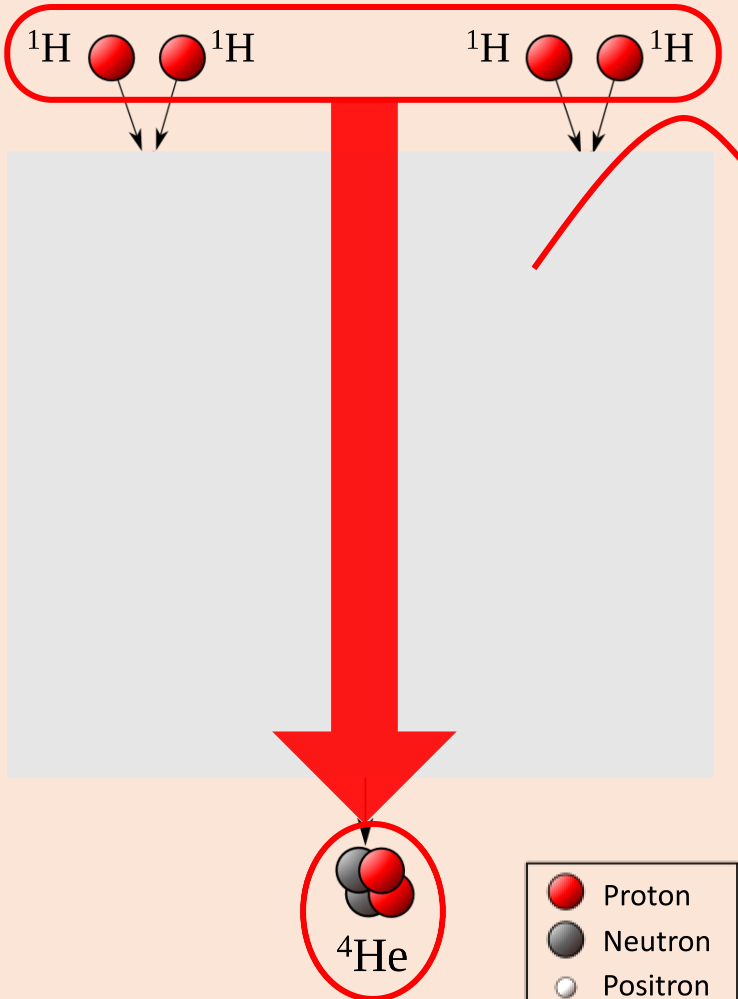
Proton-proton (p-p) chain



$$4\text{p} + 2\text{e}^- \rightarrow {}^4\text{He} + 6\,\gamma + 2\nu_\text{e} + 26.65\,\text{MeV}$$

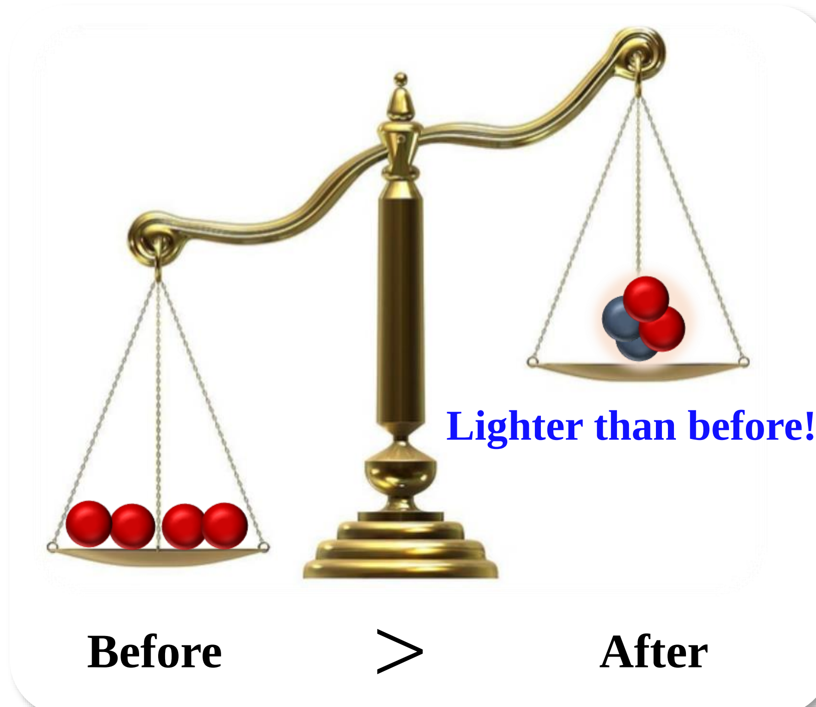
Energy source of the Sun: Nuclear fusion

Proton-proton (p-p) chain



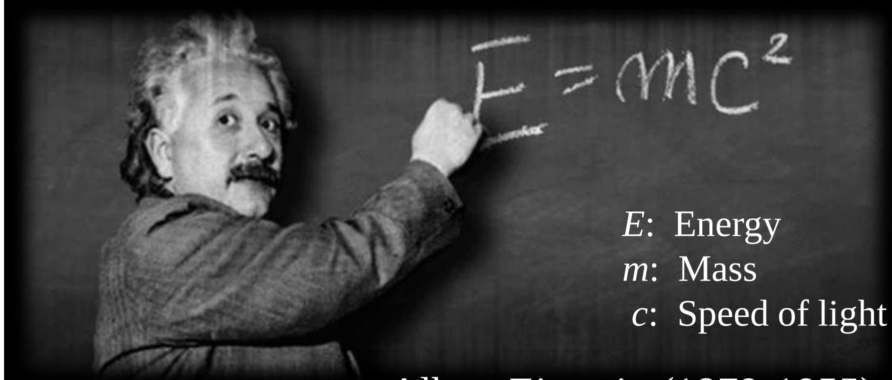
$$4p + 2e^- \rightarrow {}^4\mathrm{He} + 6\,\gamma + 2\nu_e + 26.65\,\mathrm{MeV}$$

What happened:
Four protons became a helium nucleus!



Energy source of the Sun: Nuclear fusion

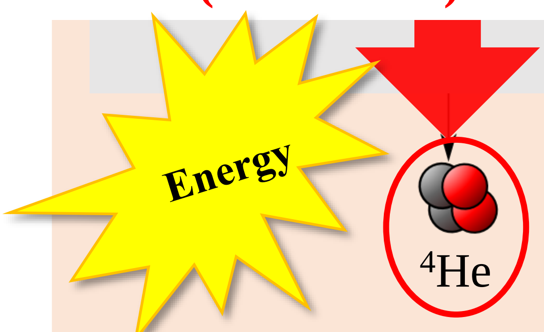
Equivalence between mass and energy



Albert Einstein (1879-1955)

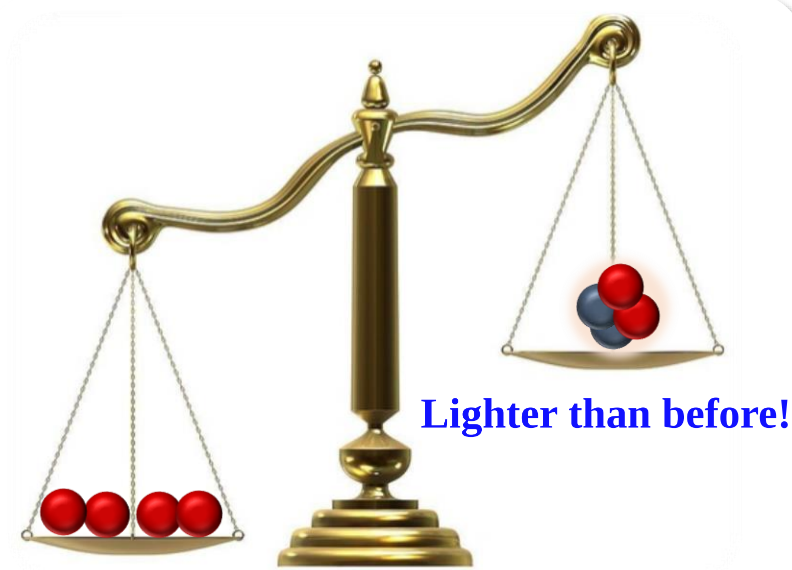
What happened:
Four protons became a helium nucleus!

This reaction generates energy (exothermic)



●	Proton
●	Neutron
○	Positron

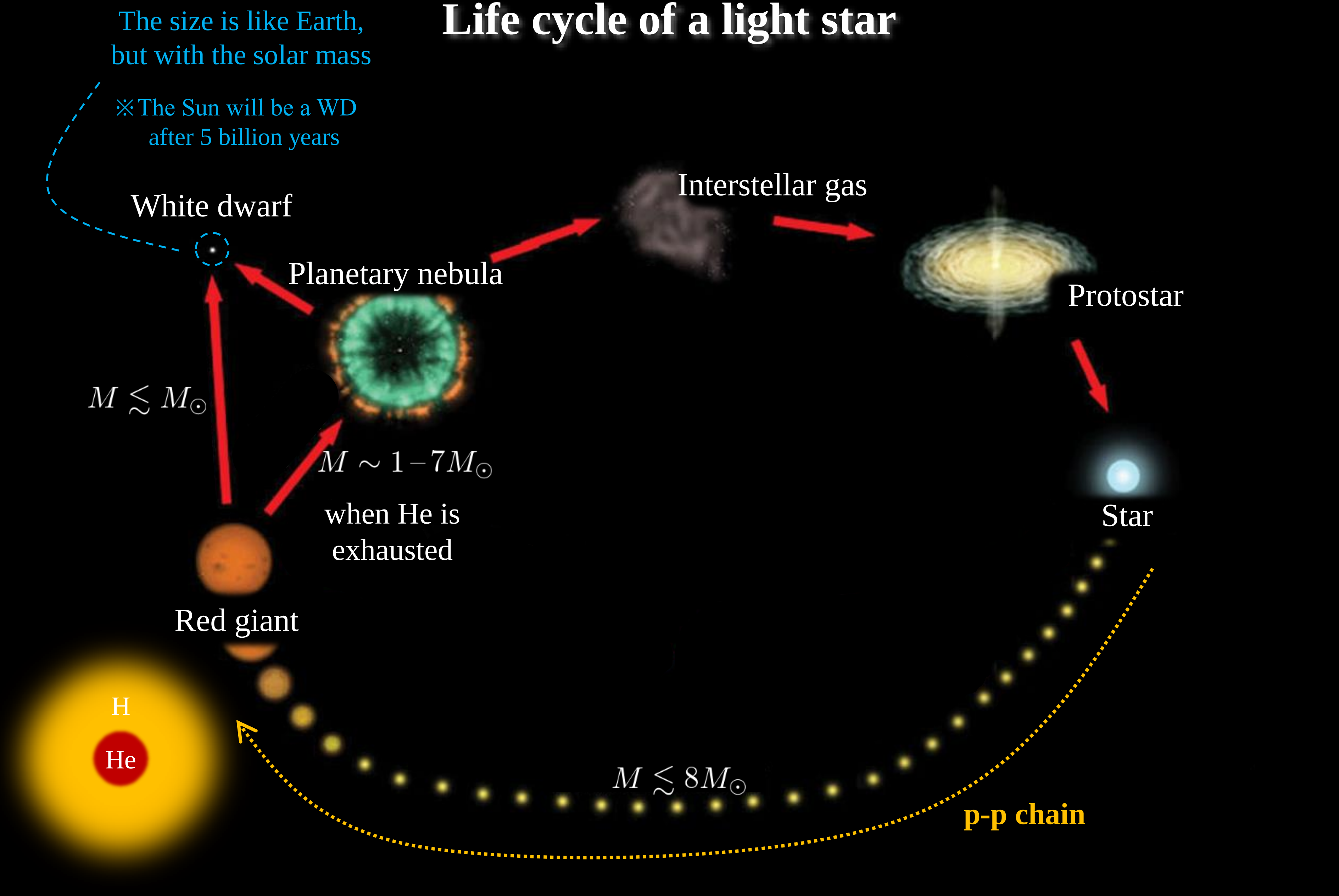
$$4\mathrm{p} + 2\mathrm{e}^- \rightarrow {}^4\mathrm{He} + 6\,\gamma + 2\nu_\mathrm{e} + 26.65\,\mathrm{MeV}$$



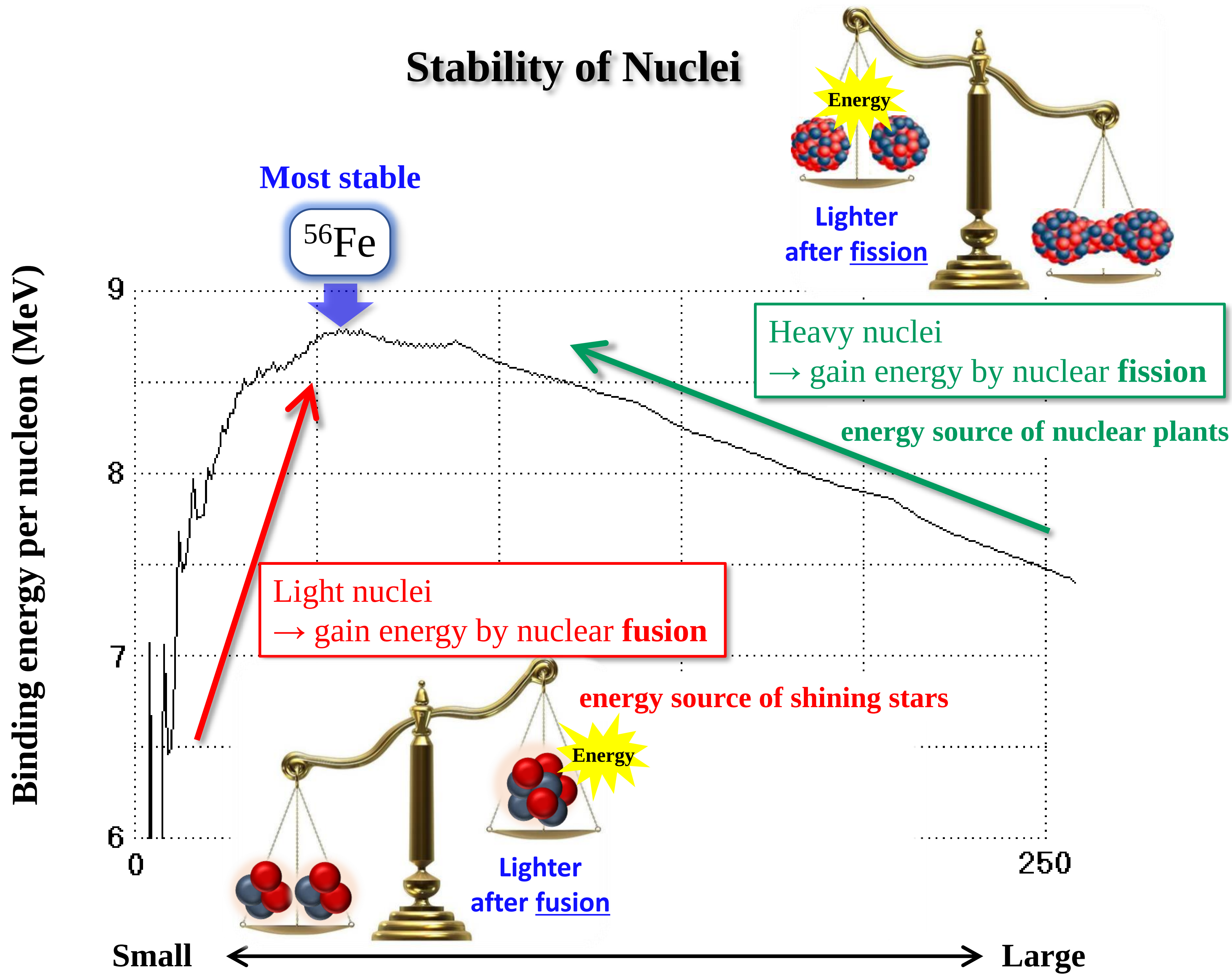
Before > After

Quantum tunneling allows for overcoming the Coulomb barrier!

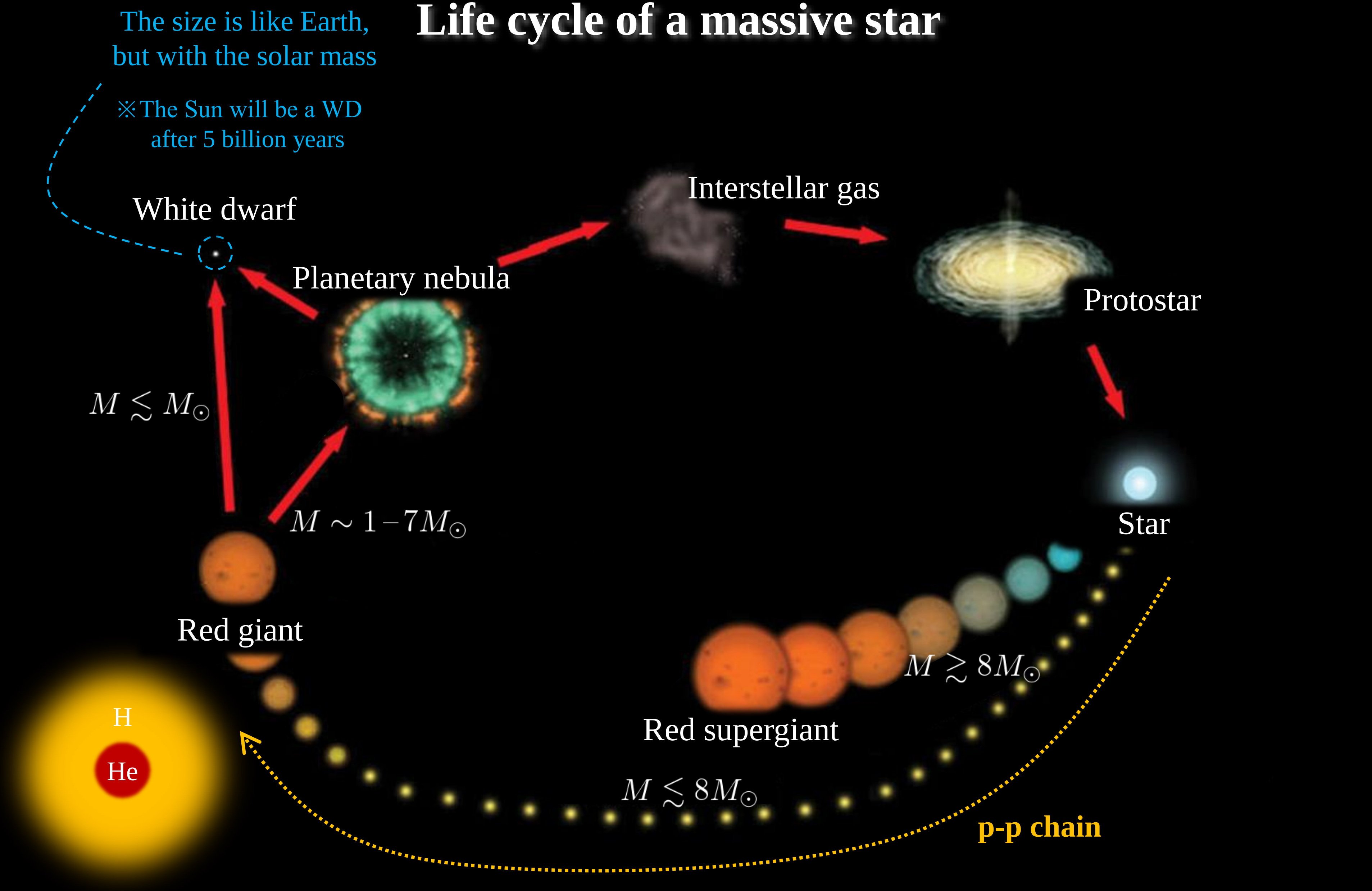
Life cycle of a light star



Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml


Stability of Nuclei
Most stable
56Fe
Energy
Lighter
after fission
Heavy nuclei
→ gain energy by nuclear fission
energy source of nuclear plants
Binding energy per nucleon (MeV)
9
8
7
6
0
250
Light nuclei
→ gain energy by nuclear fusion
energy source of shining stars
Energy
Lighter
after fusion
Small
Large

Life cycle of a massive star



Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml

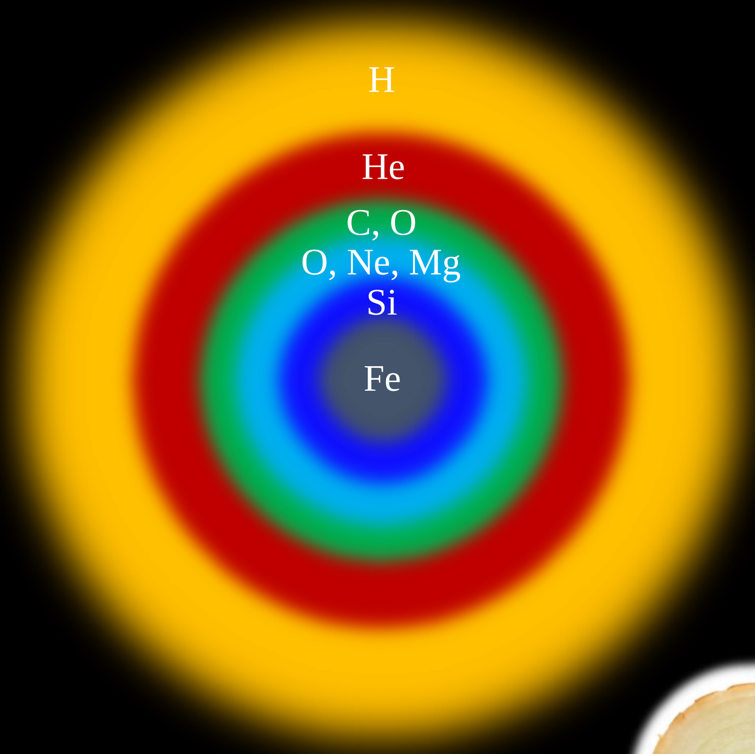
The fate of a massive star

cf. Lectures by Dr. Thomas Chillery

Nuclear reactions:

$^{1}H \rightarrow {}^{4}He \rightarrow {}^{12}C \rightarrow {}^{16}O \rightarrow {}^{20}Ne \rightarrow {}^{24}Mg \rightarrow {}^{28}Si \rightarrow \ldots \rightarrow {}^{56}Fe$

"Onion structure"



After forming the iron core…
→ no more fuel
→ gravitational collapse
→ supernova explosion

The Crab Nebula
Remnant of the SN in 1054

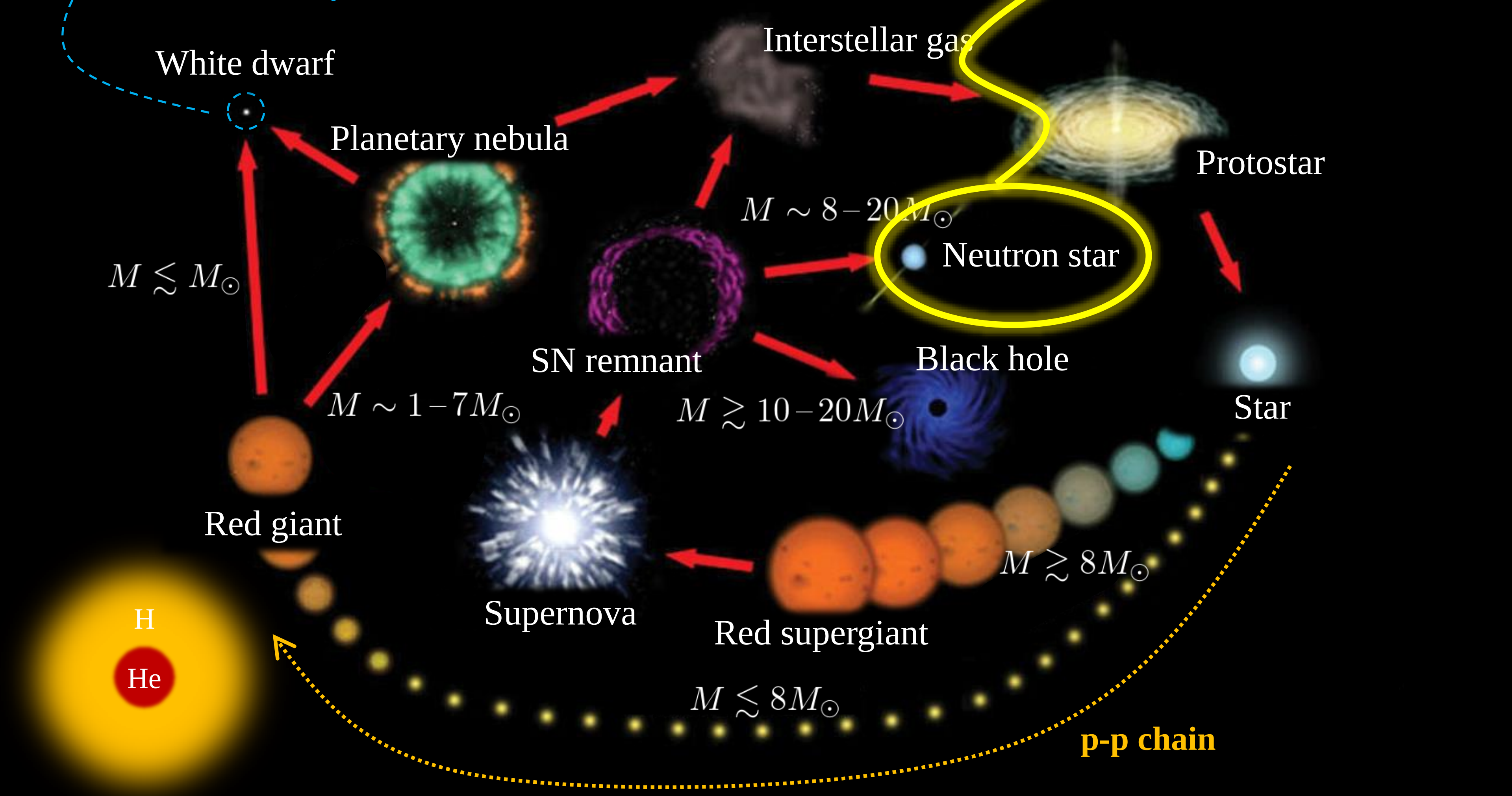
Picture: https://en.wikipedia.org/wiki/Crab_Nebula

Life cyc

An ultra-heavy, super-*N*-rich "nucleus"
→ Paradise for nuclear physicists! :D

The size is like Earth,
but with the solar mass

※The Sun will be a WD
after 5 billion years



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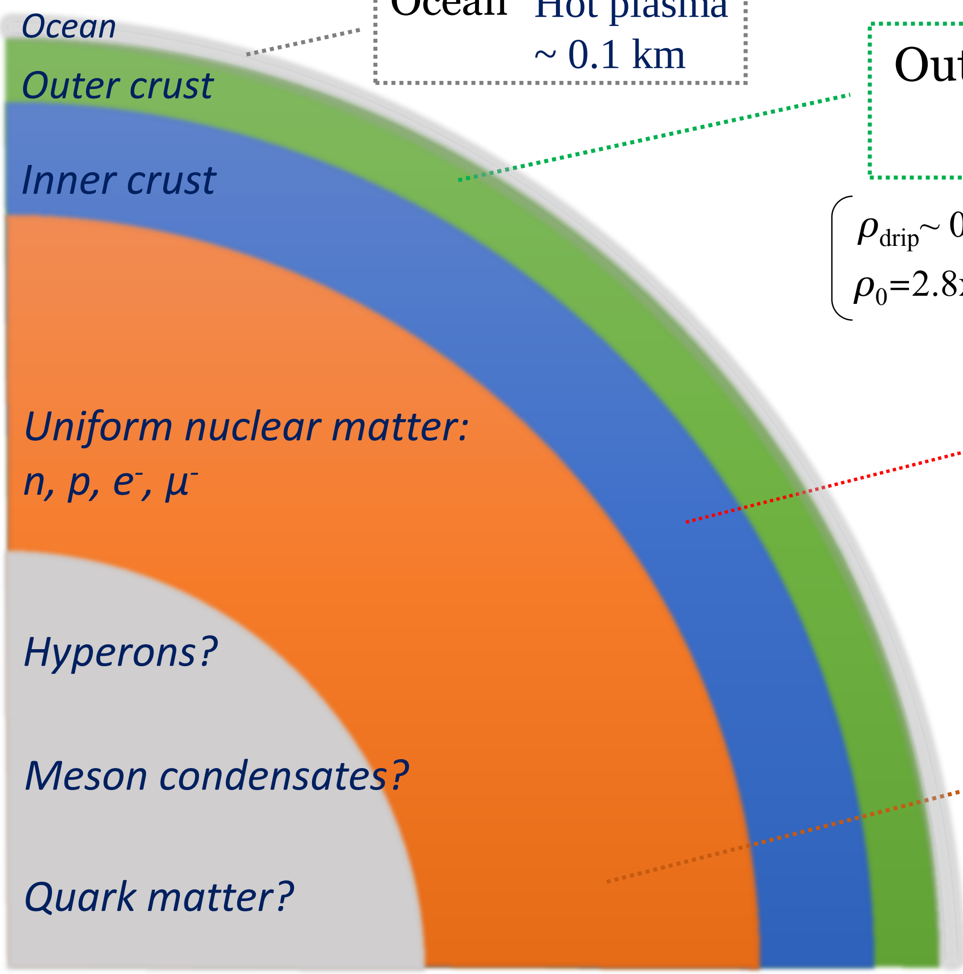
Neutron star is a great playground for nuclear physicists



Ocean Hot plasma ~ 0.1 km

Outer crust Nuclei (Fe), electrons

0.3-0.6 km $\rho < \rho_{drip}$

$\rho_{drip} \sim 0.0014\rho_0$; Above ρ_{drip} unbound neutrons exist outside nuclei

$\rho_0 = 2.8\text{x}10^{14}$ g/cm^3=0.16 fm^{-3} ; Nuclear saturation density

Inner crust Nuclei, electrons, "dripped" neutrons

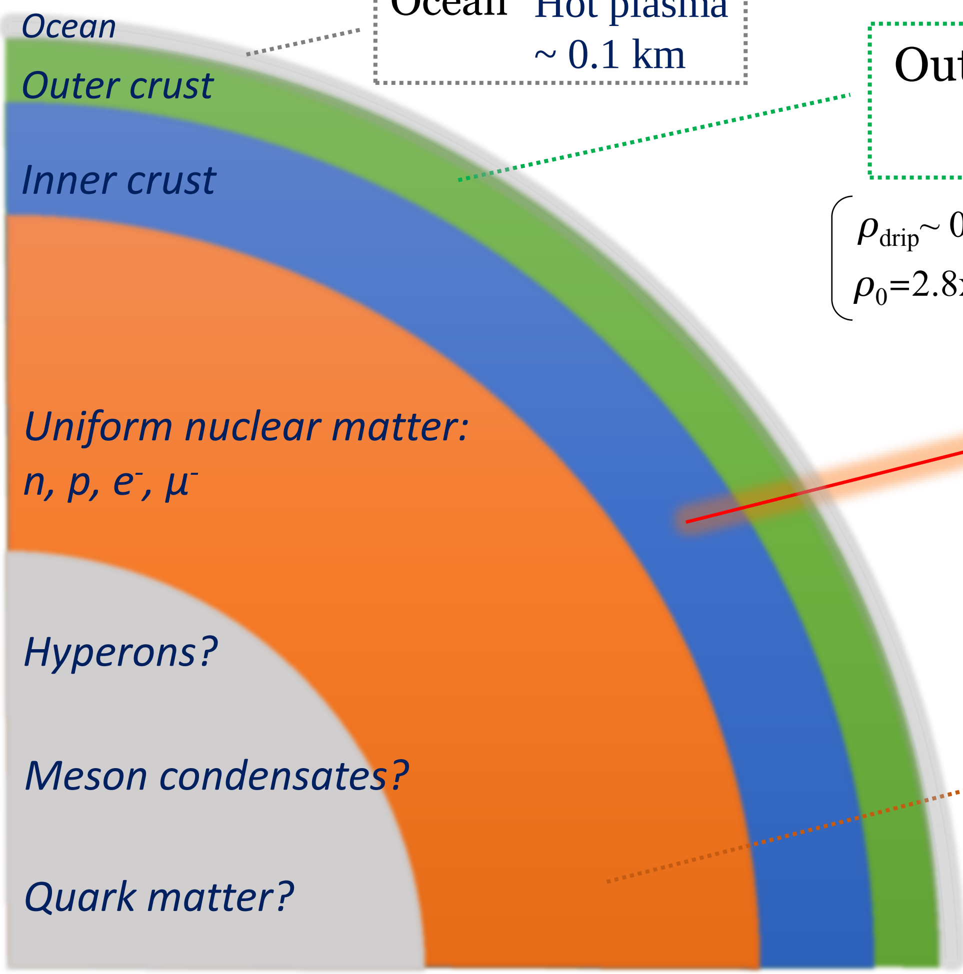
0.5-0.8 km $\rho_{drip} \leq \rho \leq 0.6\rho_0$

Outer/inner core

9-12 km $0.6\rho_0 < \rho \leq 3\text{-}5\rho_0$

Structure of a neutron star

Neutron star is a great playground for nuclear physicists



Ocean Hot plasma
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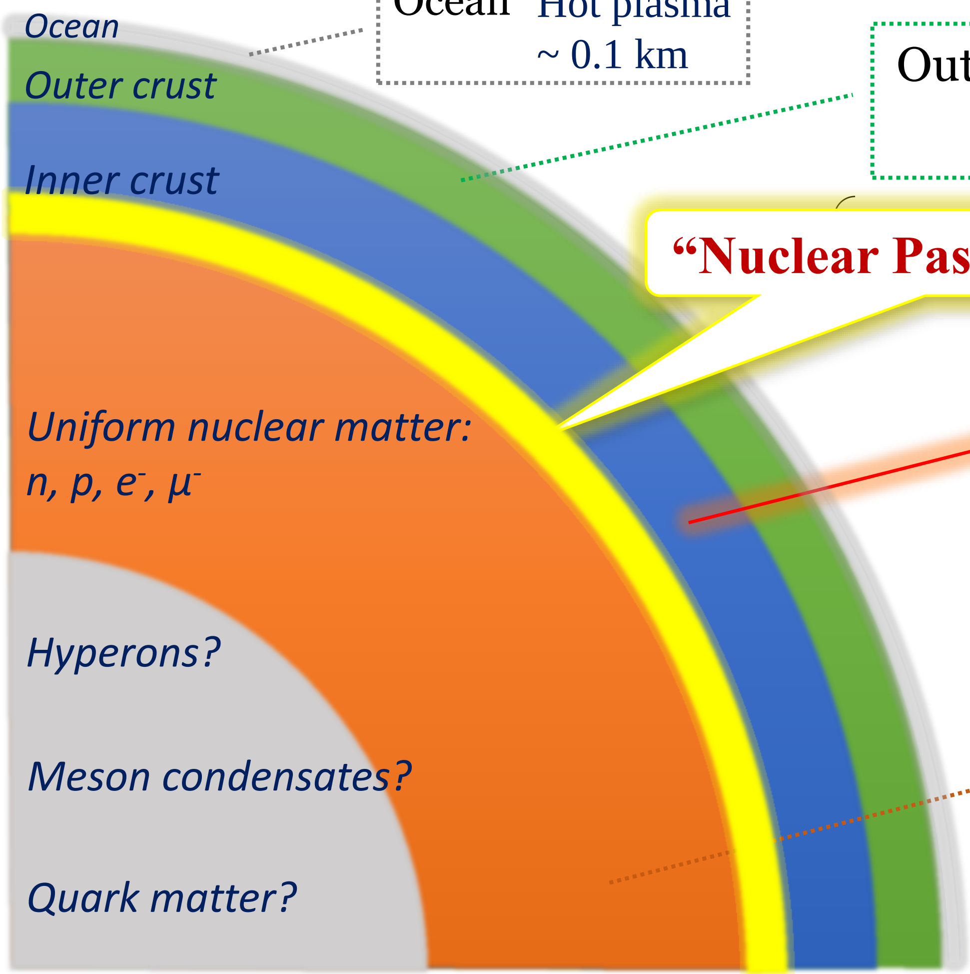


Ocean — Hot plasma ~ 0.1 km

Outer crust — Nuclei (Fe), electrons
0.3-0.6 km $\rho < \rho_{drip}$

ound neutrons exist outside nuclei

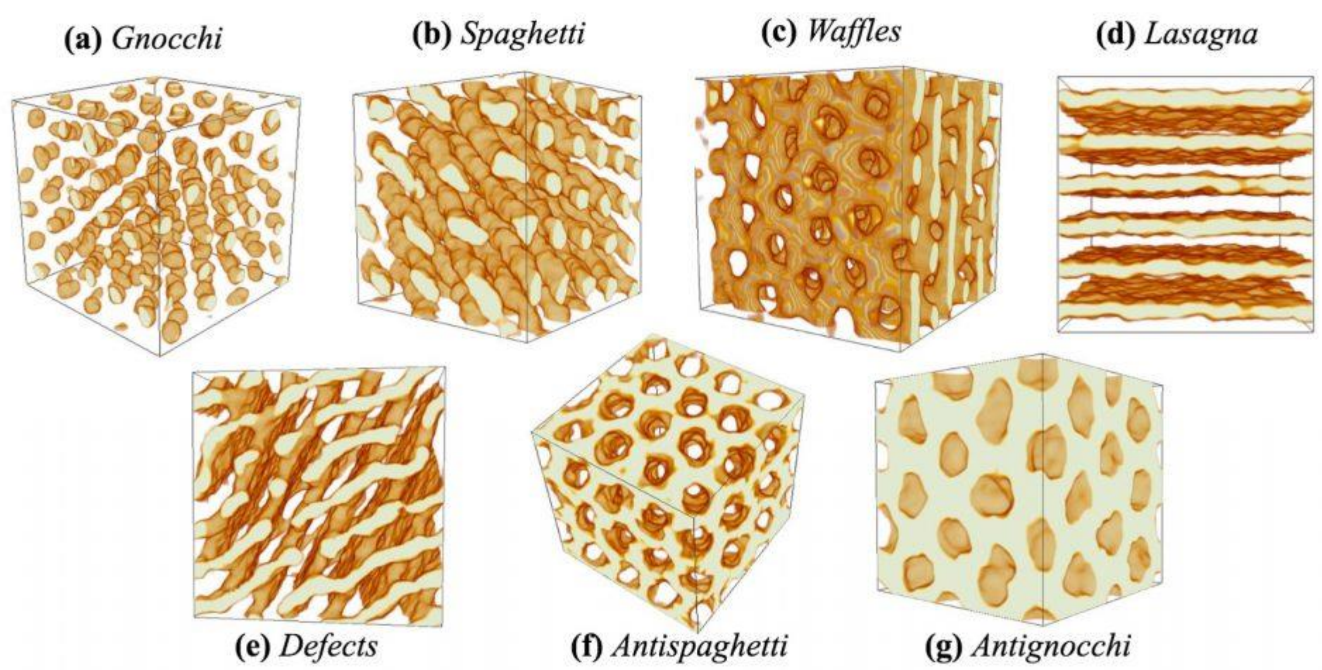
; Nuclear saturation density

"Nuclear Pasta" may exist!

Inner crust — Nuclei, electrons, "dripped" neutrons
0.5-0.8 km $\rho_{drip} \le \rho \le 0.6\rho_0$

Outer/inner core
9-12 km $0.6\rho_0 < \rho \le 3\text{-}5\rho_0$

What is Nuclear Pasta?



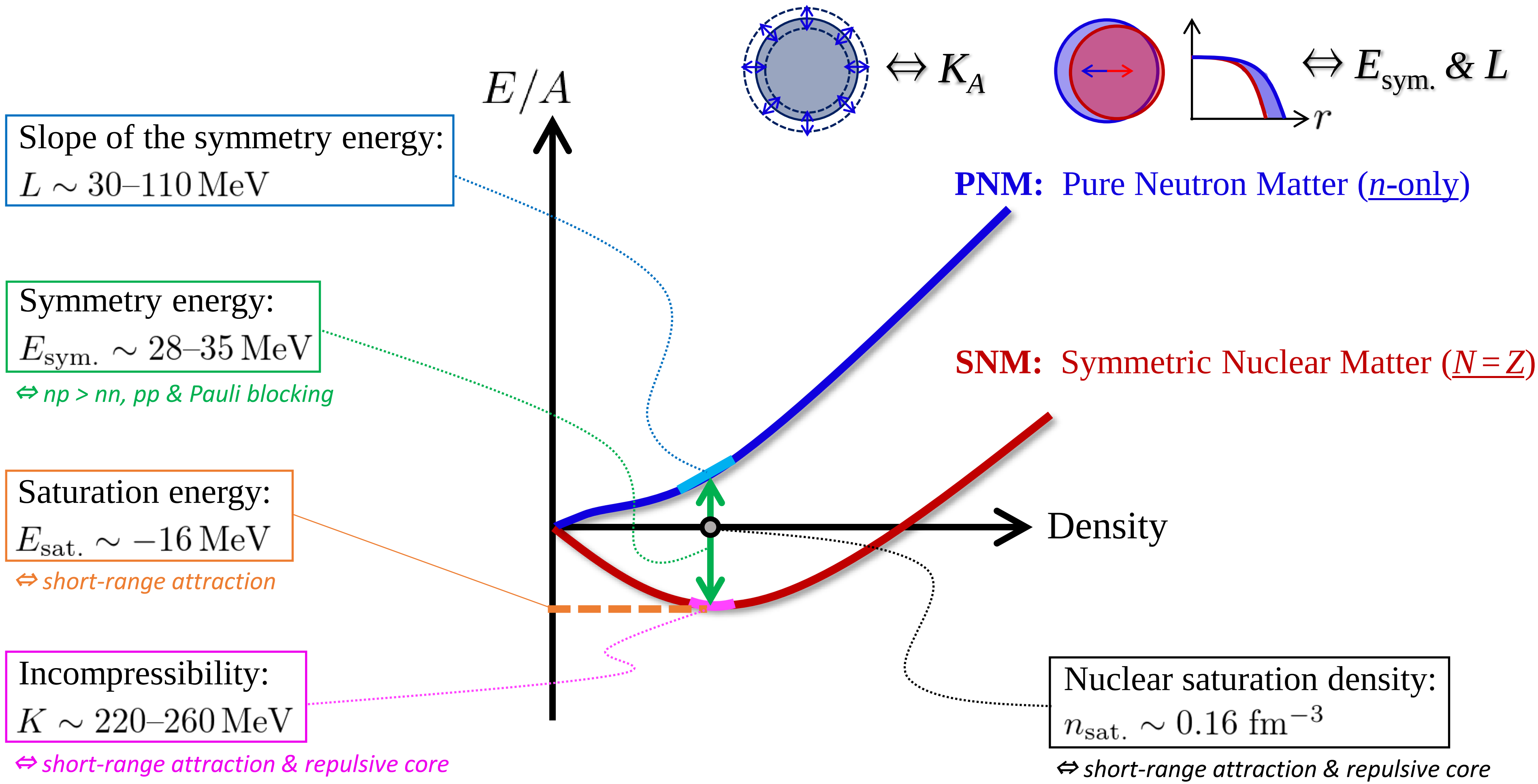
M. E. Caplan and C. J. Horowitz, Rev. Mod. Phys. **89**, 041002 (2017)

Equation of state (EoS) of nuclear matter

➢ EoS characterizes the nuclear matter properties

✓ From low-energy nuclear experiments, one may extract information in the vicinity of the saturation density

E.g.) ISGMR: U. Grag and G. Colò, PPNP**101**(2018)55; Electric dipole (E1) polarizability → skin thickness: A. Tamii *et al.*, EPJA**50**(2014)28



cf. Frontiers in Physics Vol. 21, Understanding the World of Supernova Explosion from Atomic Nuclei (原子核から読み解く超新星爆発の世界), K. Sumiyoshi (Kyoritsu printing Co., Ltd.)

➢ An EoS defines a Mass-Radius relation of neutron stars

□ Equation of state, $E(n)$

→ A relation between pressure and density:

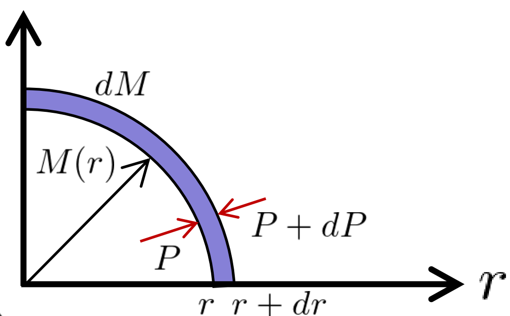
$$P(n) = -\frac{dE}{dV} = n^2 \frac{dE/A}{dn}$$

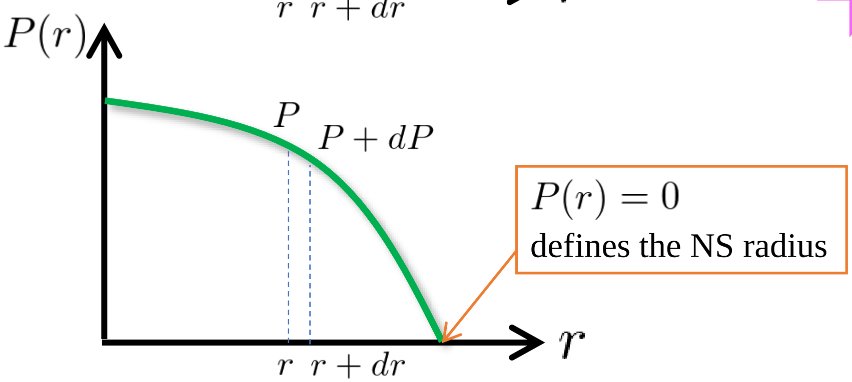
□ Tolman-Oppenheimer-Volkoff (TOV) equation:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} \underbrace{\left(1+\frac{P(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{rc^2}\right)^{-1}}_{\text{correction from General Relativity (GR)}}$$

where $\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$ $M(r)$: Integrated mass within radius r

Equilibrium condition:
Gravitational force (inward) + Pressure (outward) = 0





Mass-radius relation

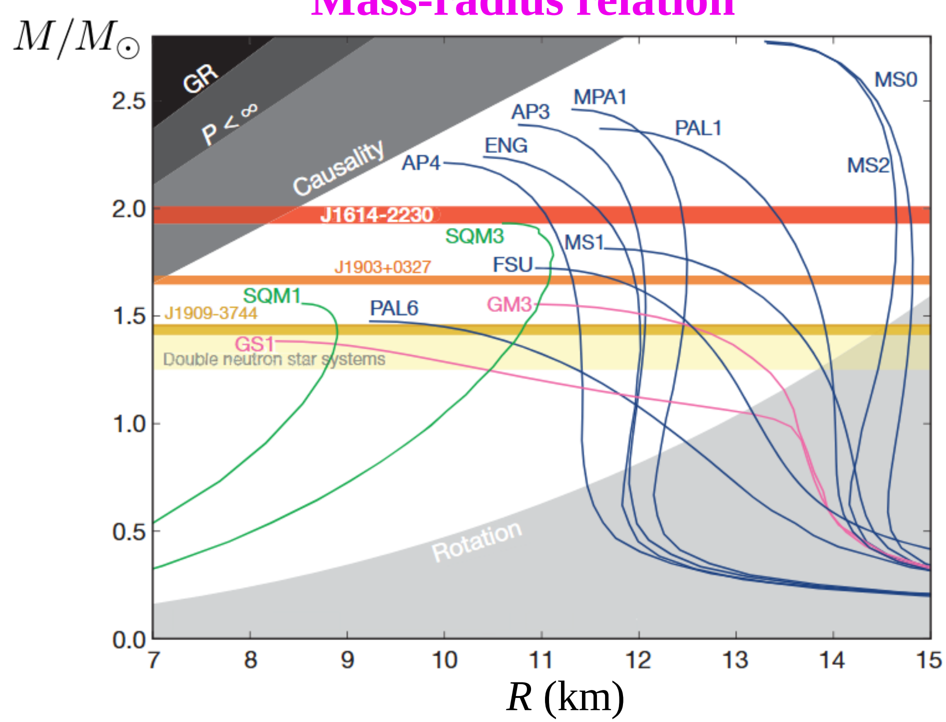


Figure: P.B. Demorest et al., Nature **467**, 1081 (2010)

How to solve the TOV equation - Ingredients

TOV eq.: $$\boxed{\frac{dn_B}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{rc^2}\right)^{-1}\left(\frac{dP}{dn_B}\right)^{-1}} \quad \because \frac{dP}{dr} = \frac{dP}{dn_B}\frac{dn_B}{dr}$$

Energy & Pressure

Skyrme EDF:

Energy density

$$E[n] = \langle\Phi|\hat{H}_{\text{Skyrme}}|\Phi\rangle = \int \boxed{\varepsilon_{\text{N}}(\boldsymbol{r})}\, d\boldsymbol{r}$$

$$\frac{E(n_n,n_p)}{A} = \frac{\varepsilon_{\text{N}}(n_n,n_p)}{n_B}$$: Total energy par particle - often referred to as the equation of state (EOS)

$$P(n_B) = -\left.\frac{\partial E}{\partial V}\right|_A = -\frac{\partial n_B}{\partial V}\left.\frac{\partial E}{\partial n_B}\right|_A = n_B^2\left.\frac{\partial (E/A)}{\partial n_B}\right|_A$$: Pressure; and its derivative $\frac{dP}{dn_B}$ $\quad \because n_B = \frac{A}{V}$

Other definitions

Baryon (nucleon) number density: $n_B = n_n + n_p$ $\quad n_p = Y_p n_B \quad n_n = n_B - n_p$

Proton fraction: $Y_p = n_p/n_B$

Energy density for *npeμ* matter: $\varepsilon(n_n,n_p,n_e,n_\mu) = n_n m_n c^2 + n_p m_p c^2 + \varepsilon_{\text{N}}(n_n,n_p) + \varepsilon_e(n_e) + \varepsilon_\mu(n_\mu)$

treated as *relativistic* ideal Fermi gases

Mass density: $\rho = \frac{\varepsilon}{c^2}$

Chemical potential: $\mu_i = \frac{\partial\varepsilon}{\partial n_i}$

A recipe for solving the TOV equation

TOV eq.: $$\frac{dn_B}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{rc^2}\right)^{-1}\left(\frac{dP}{dn_B}\right)^{-1}$$

*We know:
$$\frac{E}{A}(n_n,n_p)=\frac{\varepsilon_{\mathrm{N}}(n_n,n_p)}{n_B}, P(n_B), \frac{dP}{dn_B}, \mu_q(n_n,n_p)=m_q+\frac{\partial\varepsilon_{\mathrm{N}}}{\partial n_q}$$

$\boxed{n_c}$ Central baryon density (initial condition at $r = 0$)

If $P(n_B(R))=0$
$M_{\mathrm{NS}}=M(R)$

Determine Y_p via the β-equilibrium condition (for *npeμ* matter):

Y_p

$$n_p = Y_p n_B,\ n_n = n_B - n_p \Rightarrow \mu_n, \mu_p$$

$$\mu_e = \mu_n - \mu_p \qquad \mu_e = \mu_\mu$$: β equilibrium

Change Y_p

$$\mu_l = \sqrt{m_l^2 c^4 + (\hbar c)^2 k_{F,l}^2} \qquad (l = e, \mu)$$

$$k_{F,l} = (3\pi^2 n_l)^{1/3} \Rightarrow n_e, n_\mu$$

No

$$n_p \overset{?}{=} n_e + n_\mu$$: Charge neutrality

Yes

Integrate TOV eq. by, e.g., Runge-Kutta
$r \to r + dr$
$n_B(r+dr)$

$$\rho = \frac{\varepsilon}{c^2} = n_n m_n + n_p m_p + \frac{\varepsilon_{\mathrm{N}}(n_n,n_p)}{c^2} + \frac{\varepsilon_e(n_e)}{c^2} + \frac{\varepsilon_\mu(n_\mu)}{c^2}$$: Mass density

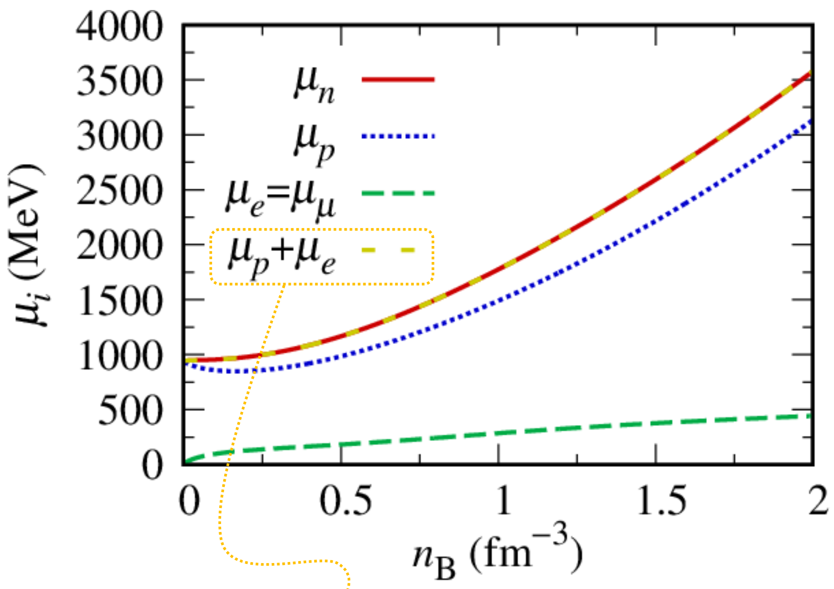
$$M(r) = \int_0^r 4\pi r^2 \rho(r) dr$$: Gravitational mass within radius r

Baryon density:
$n_B \equiv n_n + n_p$

Proton fraction:
$Y_p \equiv n_p / n_B$

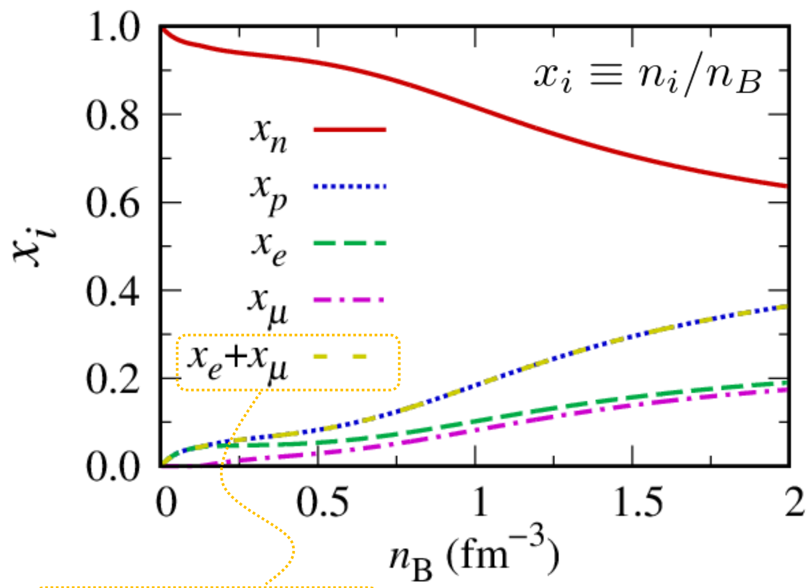
Note that:
$n_\mu < n_e$
$m_e \simeq 0.5$ MeV
$m_\mu \simeq 105.7$ MeV

Chemical potentials vs. central density



β-equilibrium condition:
$$\mu_n = \mu_p + \mu_e$$

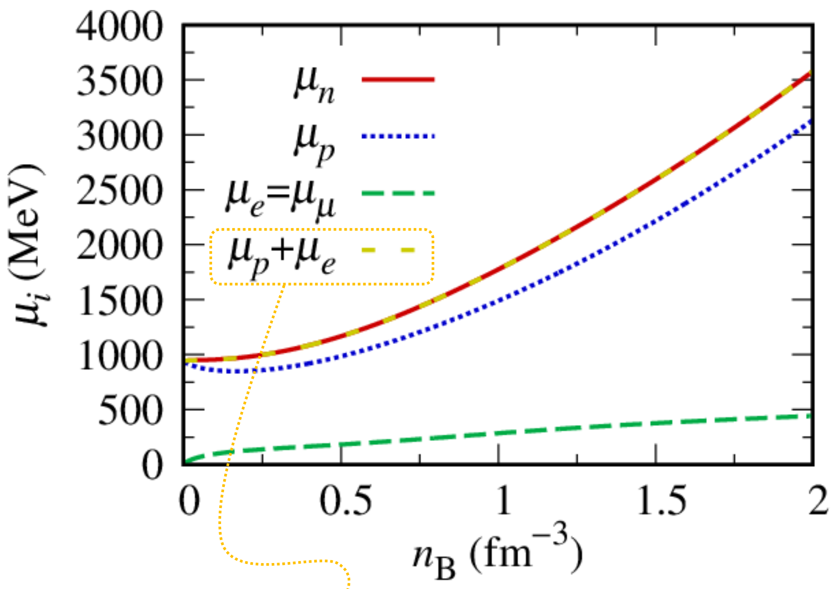
Particle fractions vs. central density



Charge neutrality:
$$n_p = n_e + n_\mu$$

Particle fractions of npeμ matter

Chemical potentials vs. central density



β-equilibrium condition:
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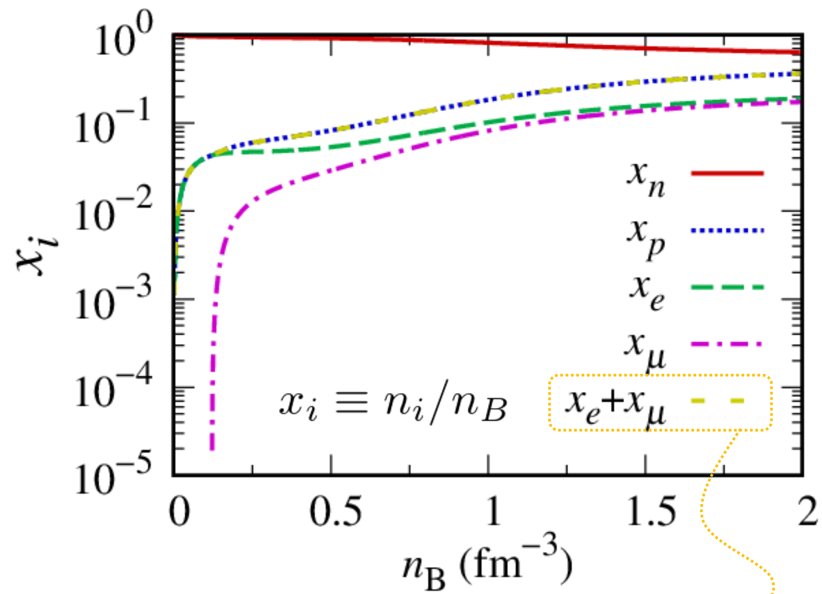
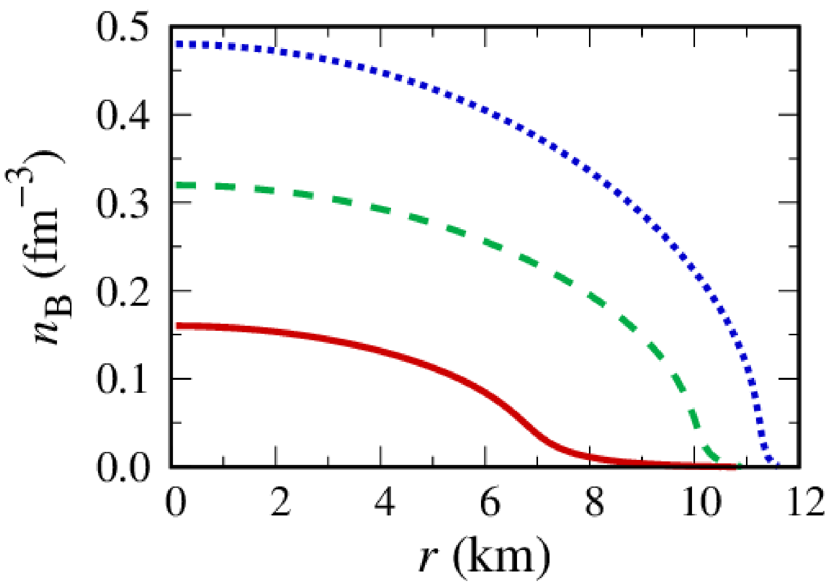
Particle fractions vs. central density



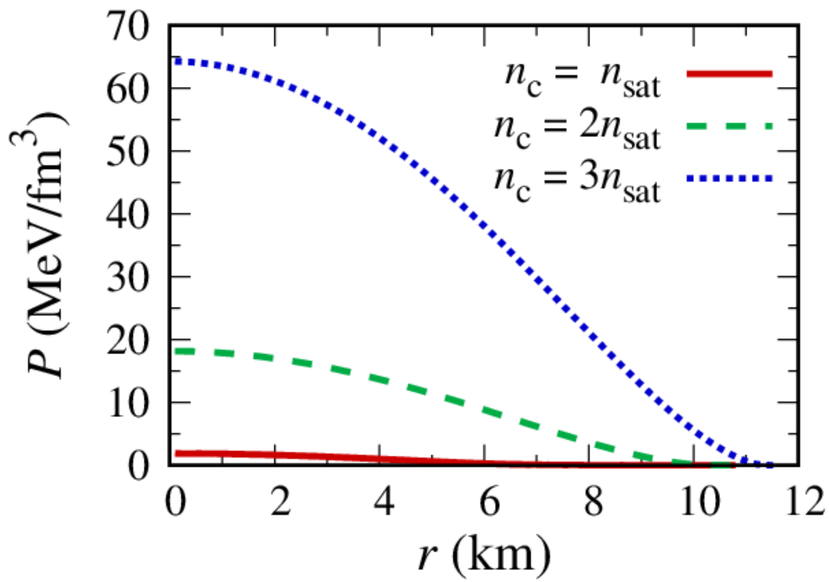
Charge neutrality:
$$n_p = n_e + n_\mu$$

✓ For various central densities, the TOV equation has been solved.

Baryon density vs. radius



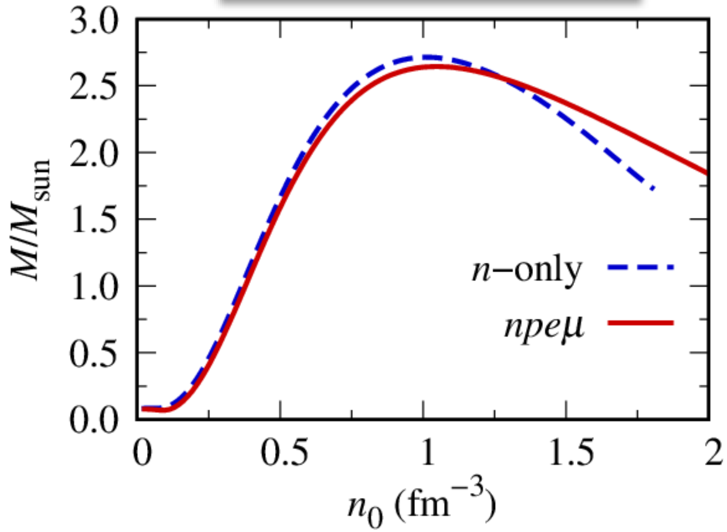
Pressure vs. radius



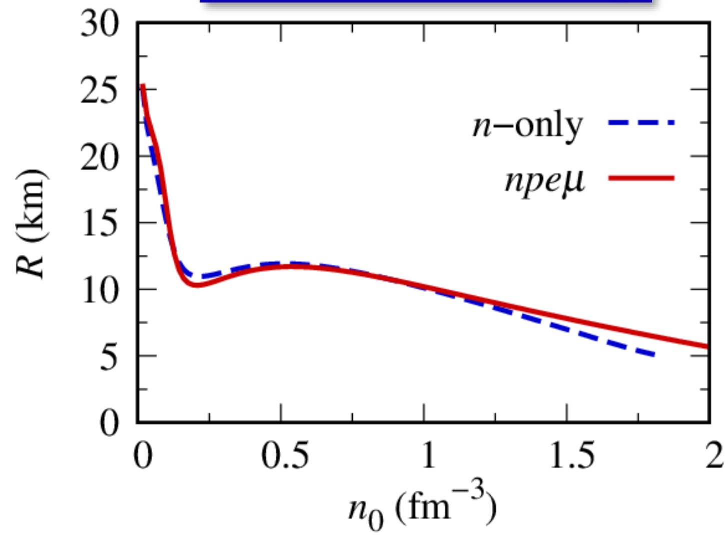
Neutron star properties

*Something went wrong - it disagrees with published results.
Take it just as an example.

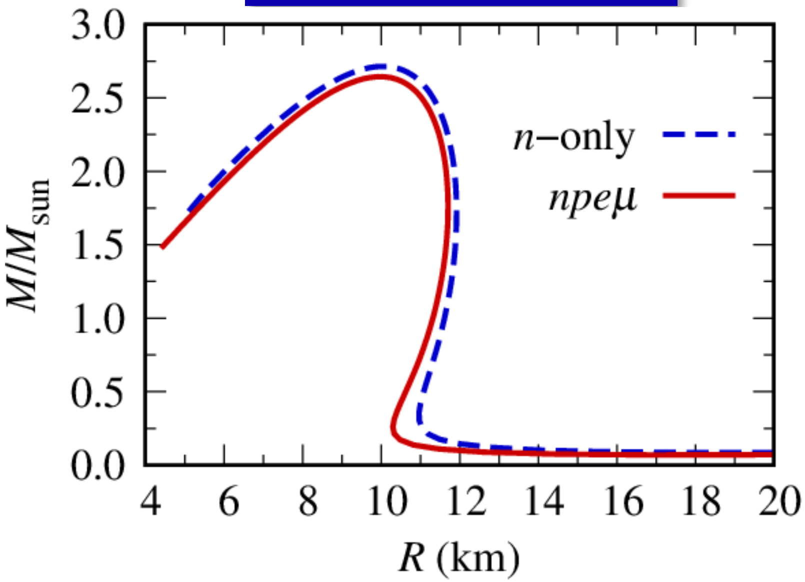
Mass vs. central density



Radius vs. central density



Mass-Radius relation

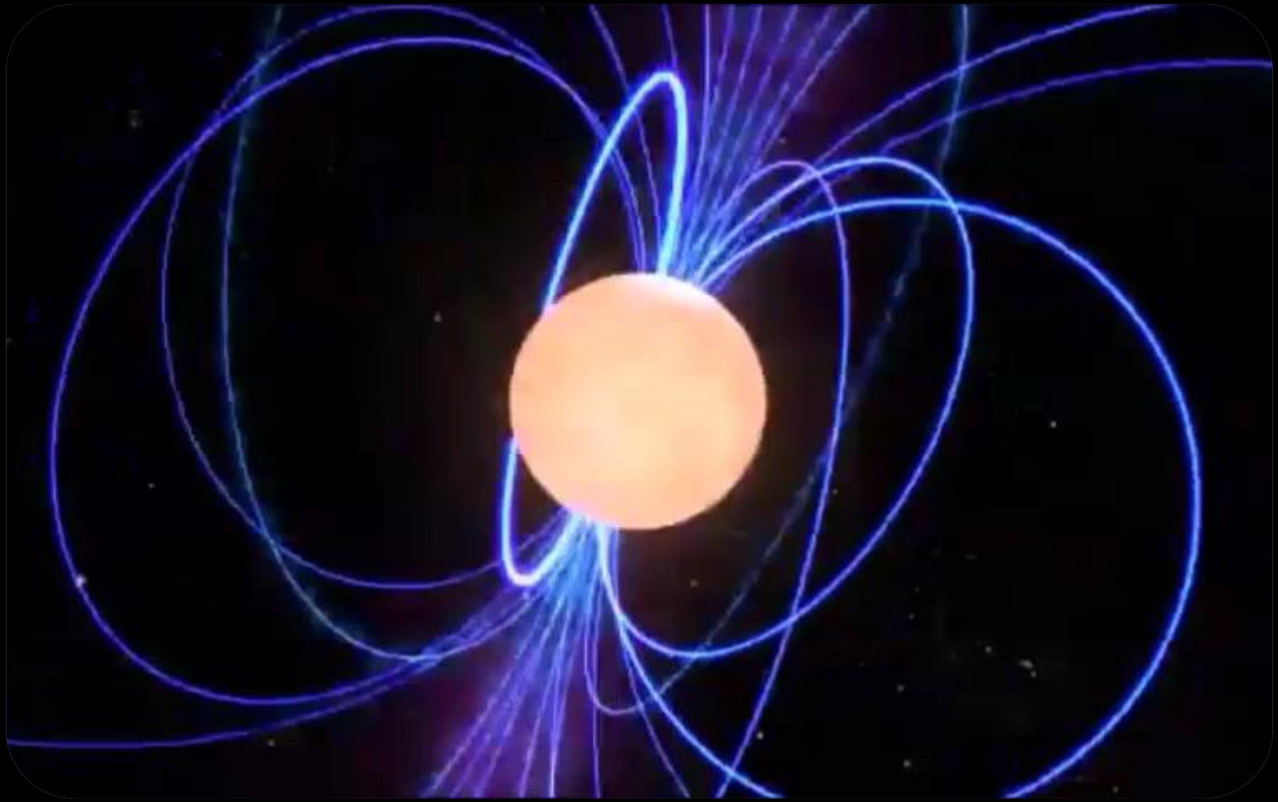


Neutron-star
“glitch”

Picture: https://astronomy.com/magazine/ask-astro/2017/12/stellar-magnets

Pulsar - a rotating neutron star

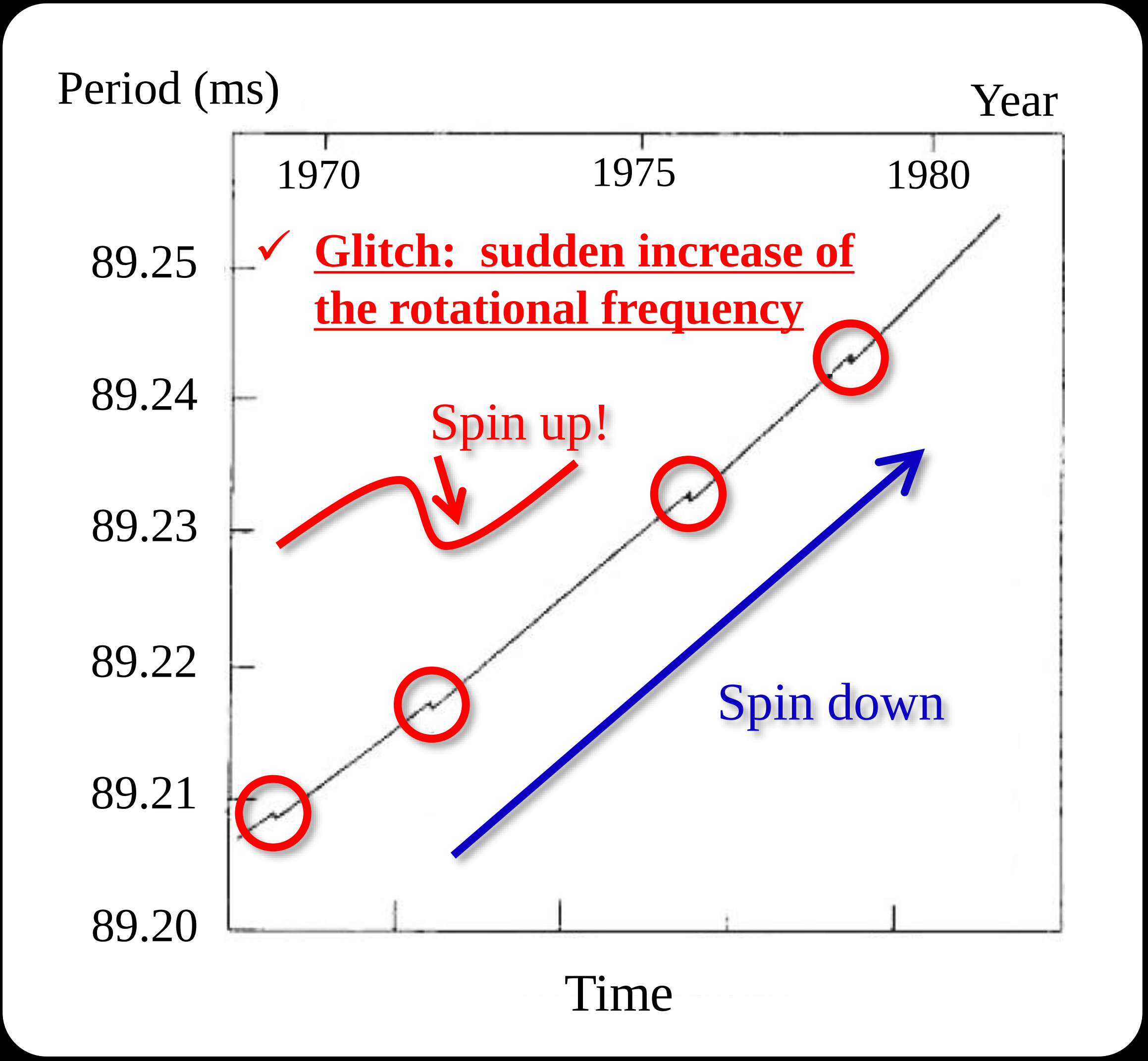
- ✓ First discovery in August 1967 → “Little Green Man” LGM-1 → PSR B1919+21
- ✓ Since then, <u>more than 2650 pulsars</u> have been observed
- ✓ It gradually <u>spins down</u> due to the EM radiation



What is the glitch?

Typical example: the Vela pulsar

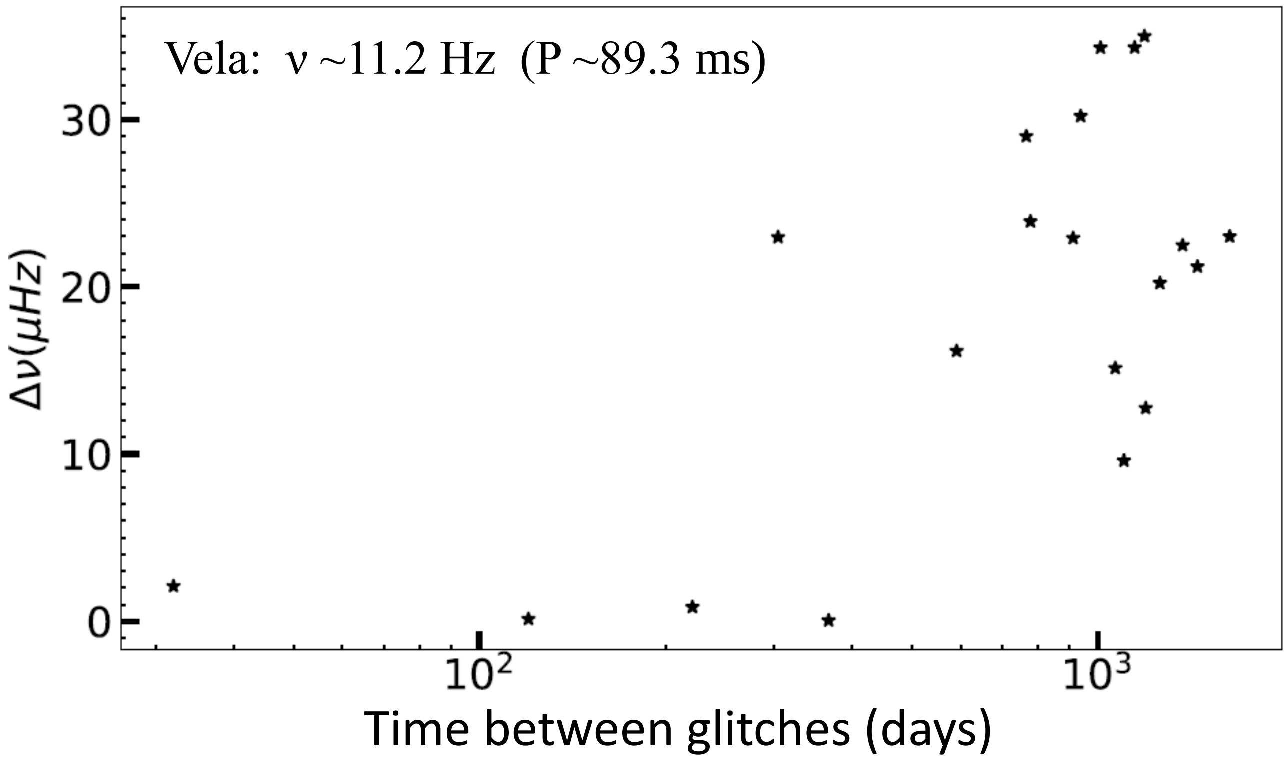
➢ *Irregularity* has been observed from continuous monitoring of the pulsation period



Typical example: the Vela pulsar

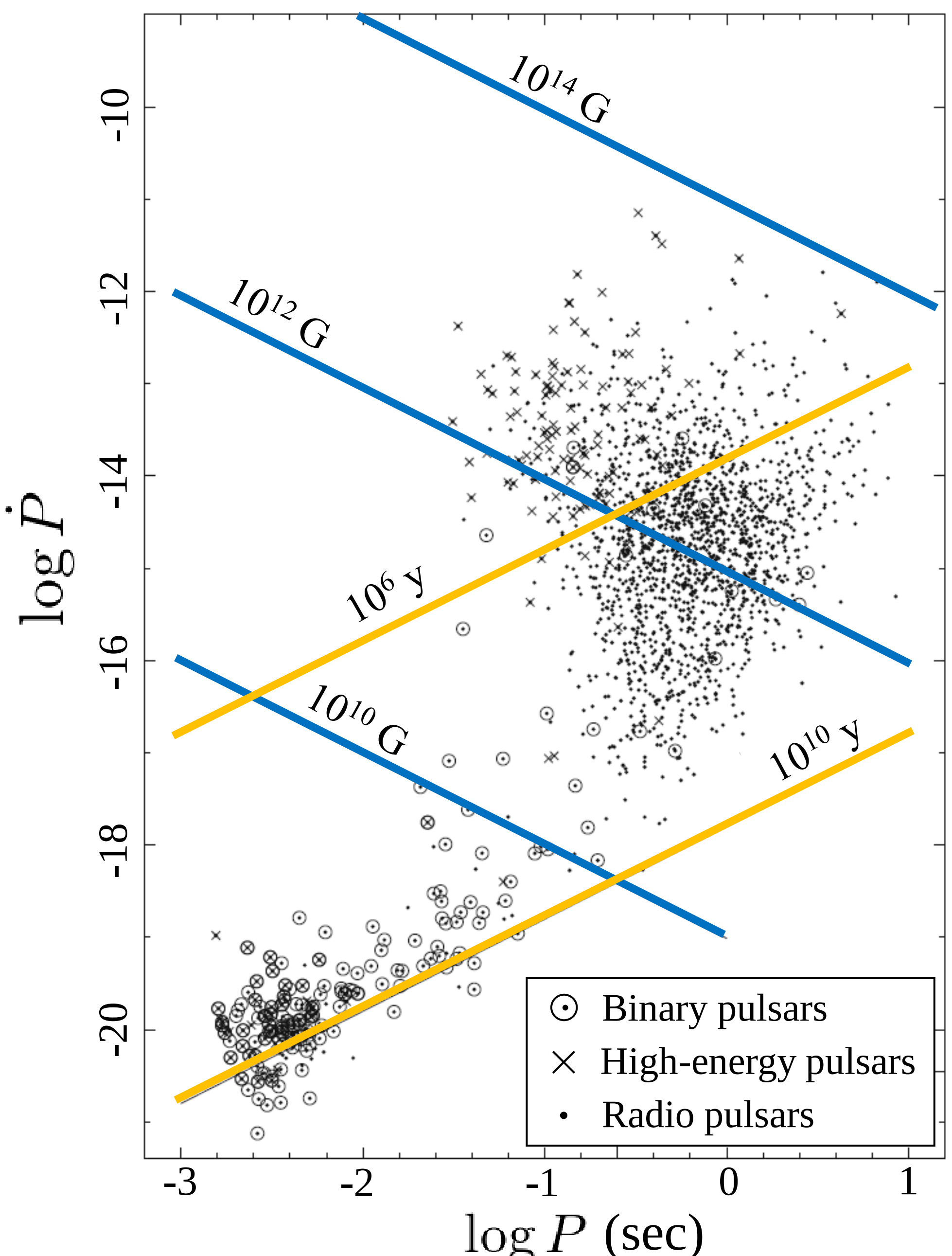
- ✓ Vela glitches roughly every 3 years (quasi-periodic)
- ✓ 21 glitches have been observed since its discovery in 1969
- ✓ Most of them are "large" ($\Delta\nu/\nu \sim 10^{-6}$), but there are a few small glitches
- ✓ Increase of spin down rate have also been observed: $\Delta\dot{\nu}/\dot{\nu} \sim 10^{-3}$-$10^{-1}$

*Δν: change of the rotation frequency due to a glitch



cf. Glitch Table and Glitch Catalogue

P-$\dot{P}$ diagram for pulsars (*not necessarily glitchers)



- ✓ Period (P): milliseconds to seconds
- ✓ Gradually spins down ($\dot{P} > 0$) due to the EM radiations
- ✓ Very stable "clock", especially for millisecond pulsars, i.e. $\dot{P} \sim 10^{-20}$

➢ Characteristic age:

$$\tau_c = P/(2\dot{P})$$

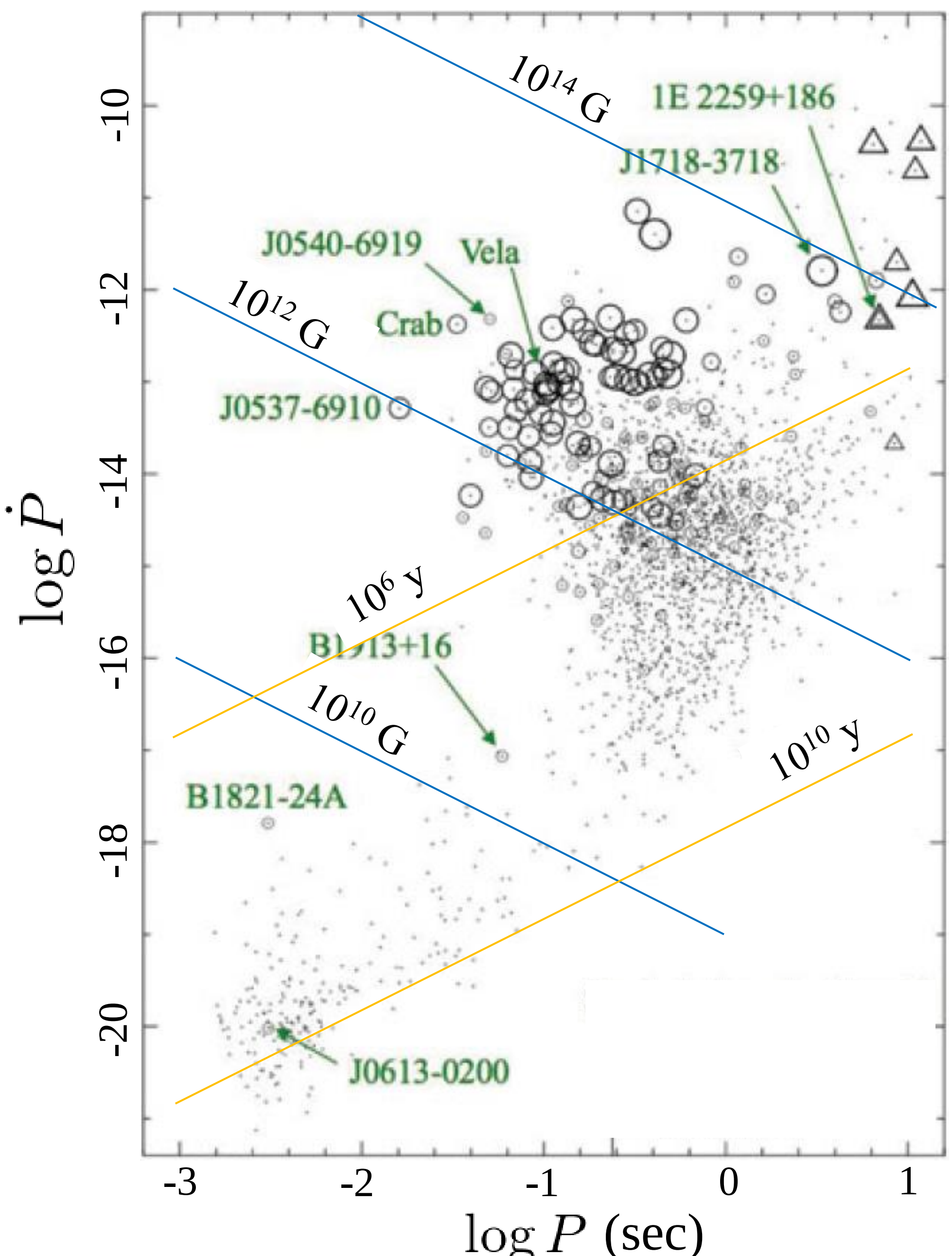
➢ Surface dipole magnetic field strength:

$$B = 3.2 \times 10^{19} (P\dot{P}) \text{ G}$$

Figure taken from:
R.N. Manchester, J. Astrophys. Astr. **38**, 42 (2017)

cf. Glitch Table and Glitch Catalogue

P-$\dot{P}$ diagram for pulsars (*not necessarily glitchers)



- ✓ **More than 548 glitches** have been observed **in more than 180 pulsars**
- ✓ Symbol size: glitch size (typically, log(Δν/ν) ~ 10^{-10}-10^{-5})
- ✓ Young pulsars (including magnetars) exhibit larger glitches than older ones

- ➢ Characteristic age:

$$\tau_c = P/(2\dot{P})$$

- ➢ Surface dipole magnetic field strength:

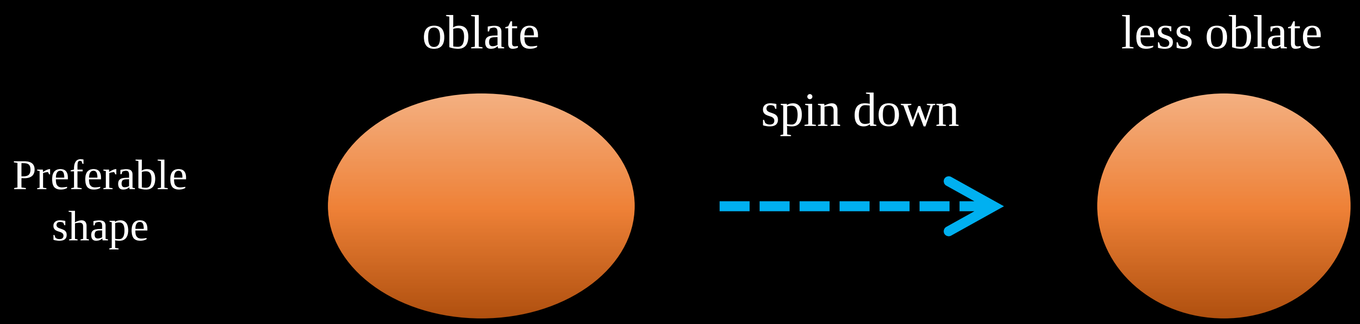
$$B = 3.2 \times 10^{19} (P\dot{P}) \text{ G}$$

Figure taken from:
R.N. Manchester, Proc. IAU Symp. **337**, 197 (2017)

What happened?

Something must happen inside the neutron star!

- ✓ "Starquake" model [G. Baym et al., Nature **224**, 872 (1969)]



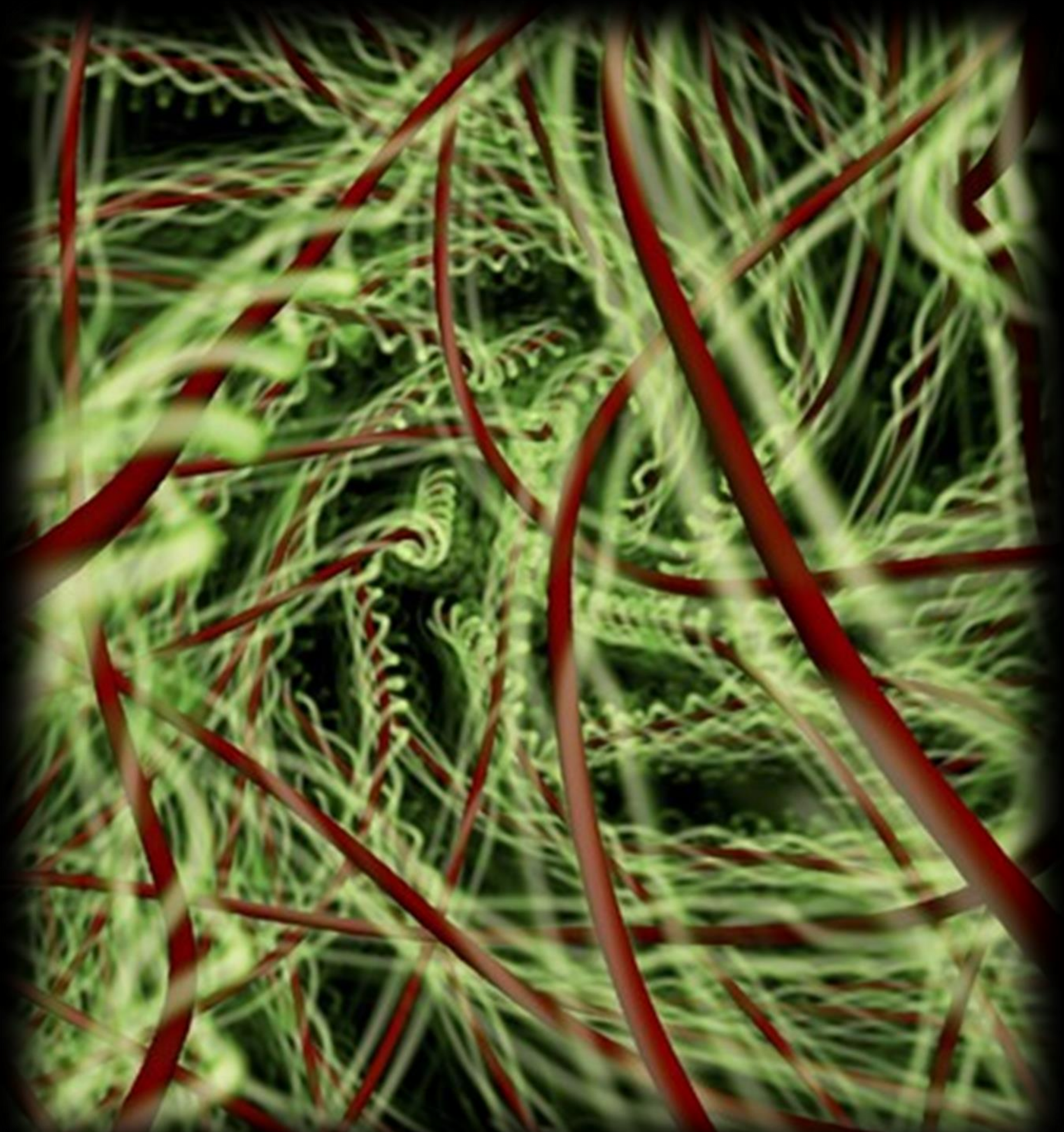
- ✓ Sudden change of moment of inertia → Glitch?

It requires <u>hundreds of years</u> to have a next glitch..

- ✓ Vortex mediated glitch [P.W. Anderson and N. Itoh, Nature **256**, 25 (1975)]

➢ <u>Dynamics of superfluid *"quantized vortices"* play a key role!</u> **!?**

Quantum vortices



Picture: N.P. Müller and G. Krstulovic, PRB**102**(2020)134513

In superfluid, vortices are quantized!

Superfluid order parameter:

$$\Delta(\boldsymbol{r},t) = |\Delta(\boldsymbol{r},t)|e^{i\phi(\boldsymbol{r},t)}$$

Superfluid velocity:

$$\boldsymbol{v}_s(\boldsymbol{r},t) = \frac{\hbar}{m}\nabla\phi(\boldsymbol{r},t)$$

Vorticity:

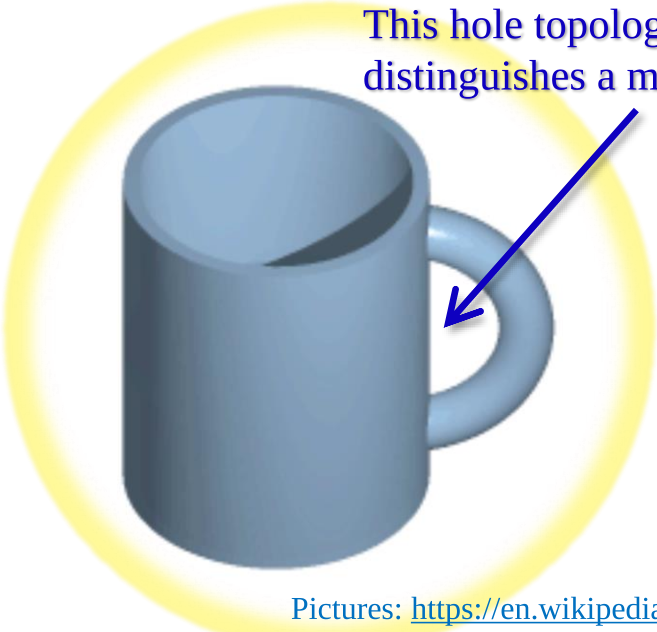
$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v}_s = 0$$

superfluid is irrotational

Circulation:

$$\kappa = \int_S (\nabla \times \boldsymbol{v}_s)\cdot d\boldsymbol{S} = 0$$

*Unless, there is no topological defect



This hole topologically distinguishes a mag from a cow

Pictures: https://en.wikipedia.org/wiki/Topology

In superfluid, vortices are quantized!

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*Unless, there is no topological defect

If there is a defect:

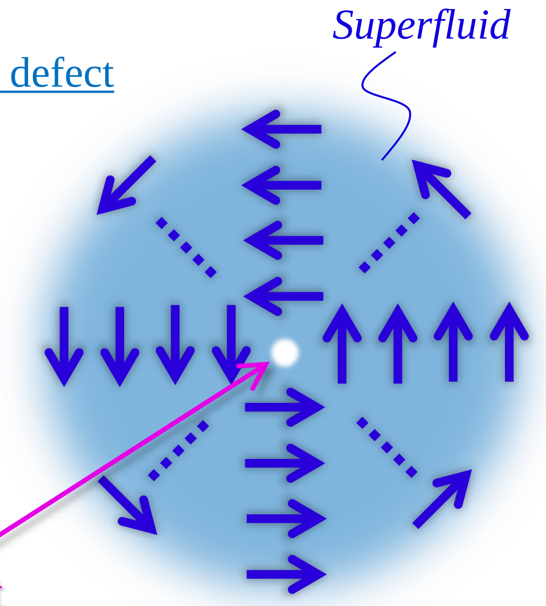
$$\kappa = \oint_C \boldsymbol{v}_s \cdot d\boldsymbol{l} = \frac{\hbar}{m}\oint_C \nabla\phi\cdot d\boldsymbol{l} = \frac{\hbar}{m}2\pi n$$

Quantization of circulation

*the phase $\phi(\boldsymbol{r})$ at the same point must be equivalent!

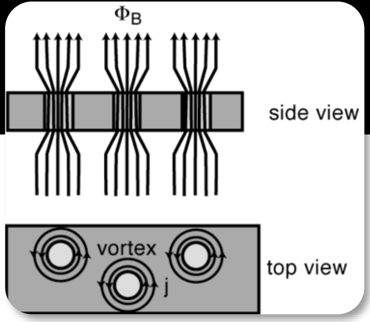
Flow velocity of rotation shall be quantized!

A hole at which superfluidity is lost



Quantum vortex

What is a quantum vortex?



In superconductor, magnetic flux is quantized!

Magnetic flux:

Meissner effect

$$\Phi = \int_S \boldsymbol{B}\cdot d\boldsymbol{S} = \int_S (\nabla\times\boldsymbol{A})\cdot d\boldsymbol{S} = 0$$

$$\boldsymbol{j}_s = -\frac{n_s e_s^2}{m_s}\boldsymbol{A} + \frac{n_s e_s \hbar}{m_s}\nabla\phi$$: the London equation

If there is a defect:

$$\Phi = \oint_C \boldsymbol{A}\cdot d\boldsymbol{l} \approx \frac{\hbar}{e_s}\oint_C \nabla\phi\cdot d\boldsymbol{l} = \frac{\hbar}{e_s} 2\pi n$$

n_s, m_s, e_s: density, mass, and charge of a carrier (Cooper pair)

<u>Quantization of magnetic flux</u> (fluxtube, fluxoid, or fluxon)

Circulation:

$$\kappa = \int_S (\nabla\times\boldsymbol{v}_s)\cdot d\boldsymbol{S} = 0$$

*Unless, there is no <u>topological defect</u>

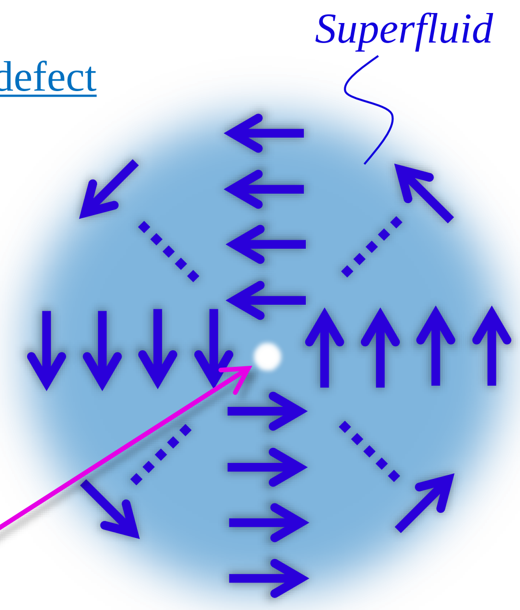
If there is a defect:

$$\kappa = \oint_C \boldsymbol{v}_s\cdot d\boldsymbol{l} = \frac{\hbar}{m}\oint_C \nabla\phi\cdot d\boldsymbol{l} = \frac{\hbar}{m} 2\pi n$$

<u>Quantization of circulation</u>

*the phase $\phi(\boldsymbol{r})$ at the same point must be equivalent!

Flow velocity of rotation shall be quantized!

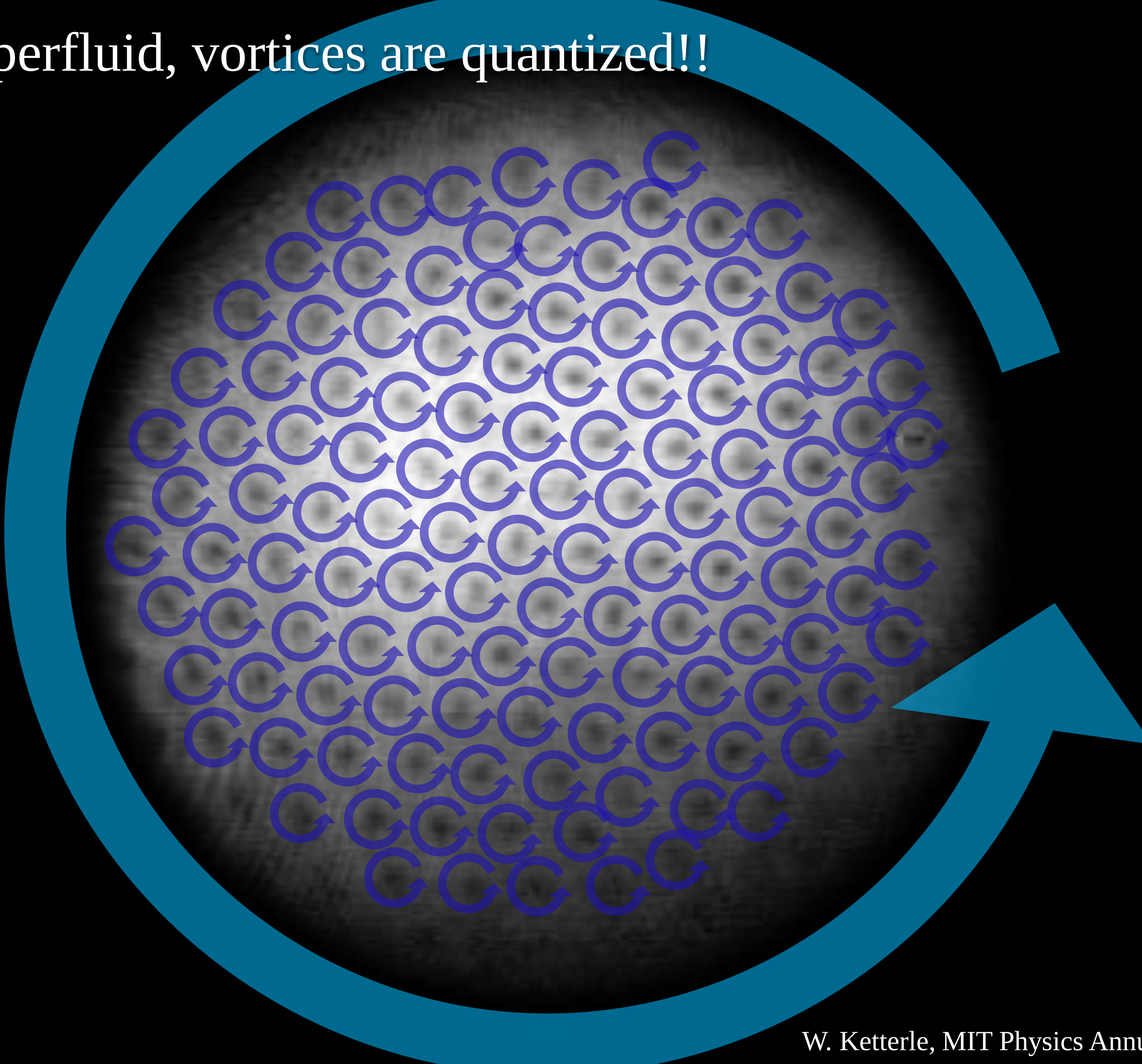


A hole at which superfluidity is lost

Quantum vortex

In daily life, a vortex is continuous..

In superfluid, vortices are quantized!!



W. Ketterle, MIT Physics Annual. 2001

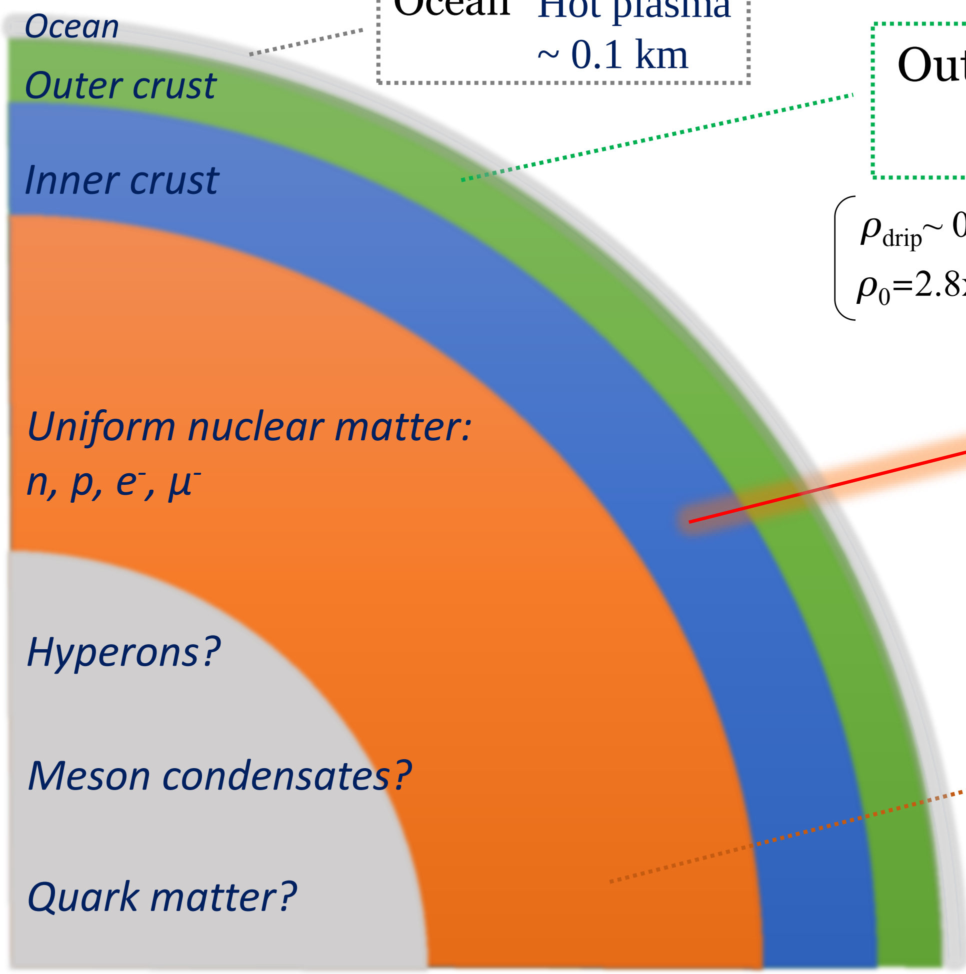
G.P. Bewley, D.P. Lathrop, and K.R. Sreenivasan, Nature **441**, 588 (2006)

A movie from a talk by W. Guo (available from https://youtu.be/P2ckefSAN20) at
INT Program 19-1a "Quantum Turbulence: Cold Atoms, Heavy Ions, and Neutron Stars"
March 18 - April 19, 2019

Direct visualization of quantized vortices

Hydrogen particles were trapped in the vortex core, then worked as a tracer

Neutron star is a great playground for nuclear physicists



Ocean Hot plasma ~ 0.1 km

Outer crust Nuclei (Fe), electrons
0.3-0.6 km $\rho < \rho_{\rm drip}$

$\rho_{\rm drip} \sim 0.0014\rho_0$; Above $\rho_{\rm drip}$ unbound neutrons exist outside nuclei
$\rho_0 = 2.8\text{x}10^{14}$ g/cm^3=0.16 fm^{-3} ; Nuclear saturation density

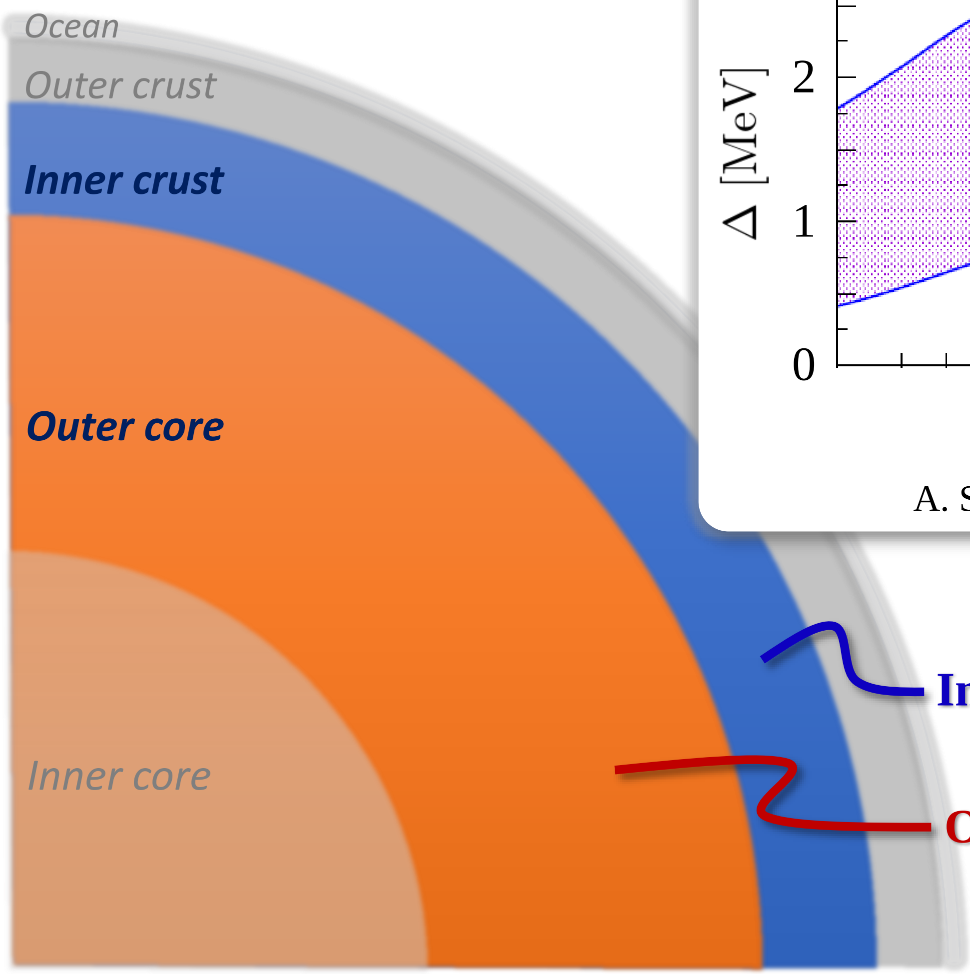
Inner crust Nuclei, electrons, "dripped" neutrons
0.5-0.8 km $\rho_{\rm drip} \le \rho \le 0.6\rho_0$

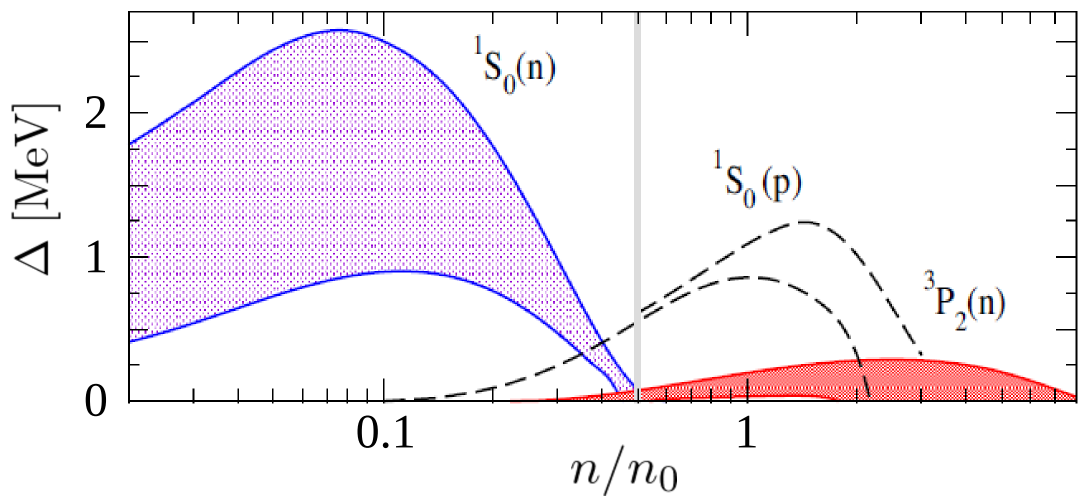
Outer/inner core
9-12 km $0.6\rho_0 < \rho \le 3\text{-}5\rho_0$

Structure of a neutron star

$T < T_c \sim 10^{10}\,\mathrm{K} \quad B < B_c \sim 10^{17}\,\mathrm{G}$

Neutrons (protons) are superfluid (superconducting) in neutron stars!





A. Sedrakian and J.W. Clark, Eur. Phys. J. A **55**, 167 (2019)

Inner crust: S-wave neutron superfluid

Outer core: S-wave proton superconductor (+ P-wave neutron superfluid)

A lattice of neutron-rich nuclei are immersed in a neutron superfluid

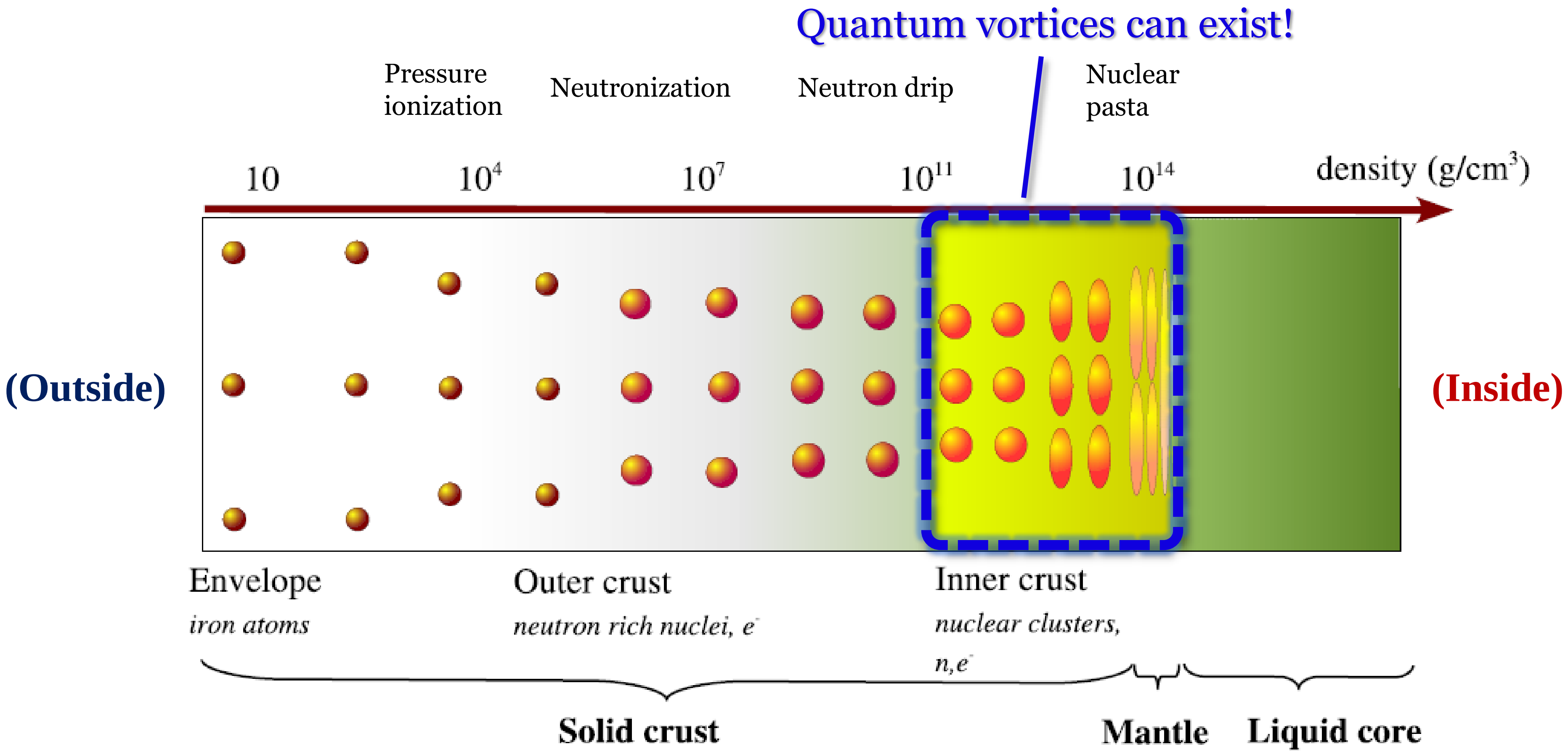


Fig.4 in N. Chamel and P. Haensel, Living Rev. Relativity **11**, 10 (2008)

In rotating superfluid, an array of quantum vortices is generated

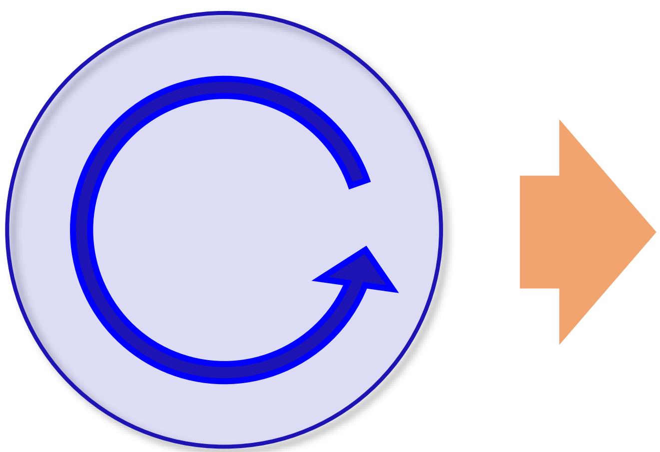
- Observation in ultra-cold atomic gases

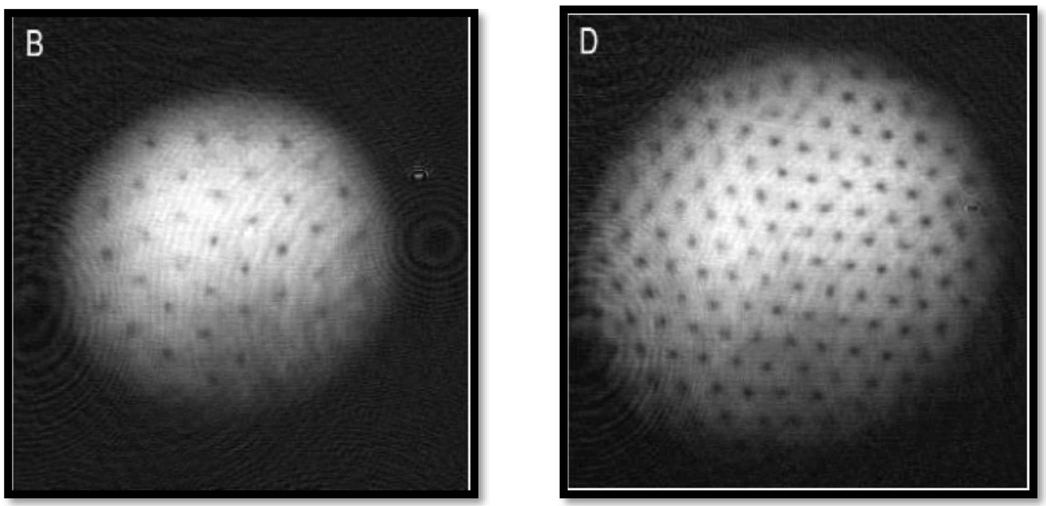
Vortex-number density

$\propto$

Rotation speed

Low ⟷ High





W. Ketterle, MIT Physics Annual. 2001

In rotating superfluid, an array of quantum vortices is generated

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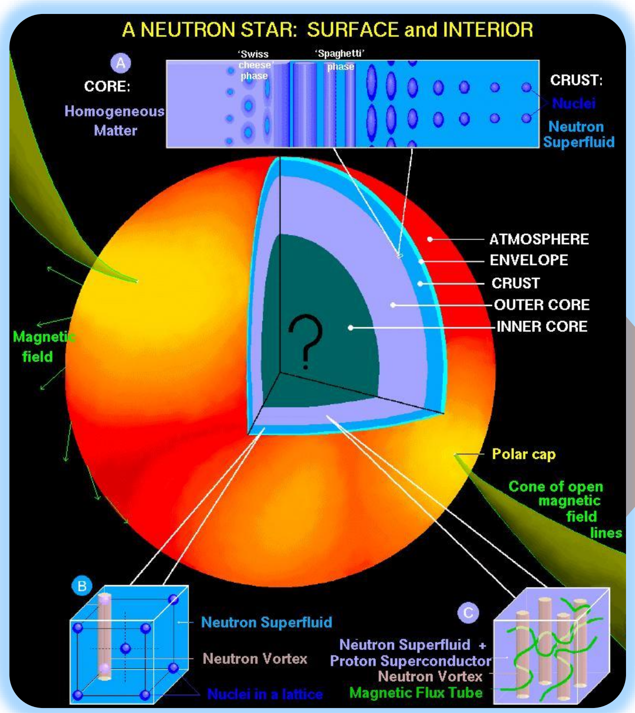
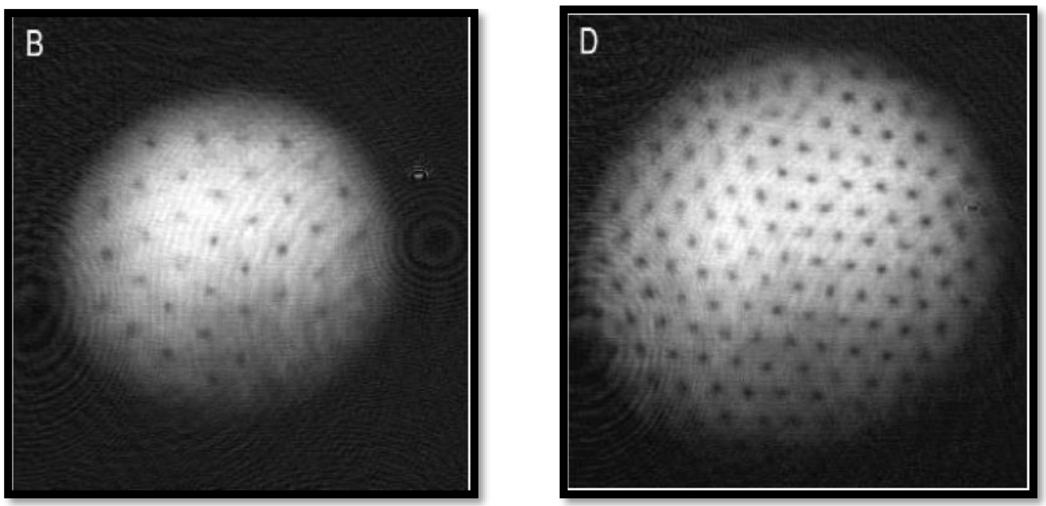


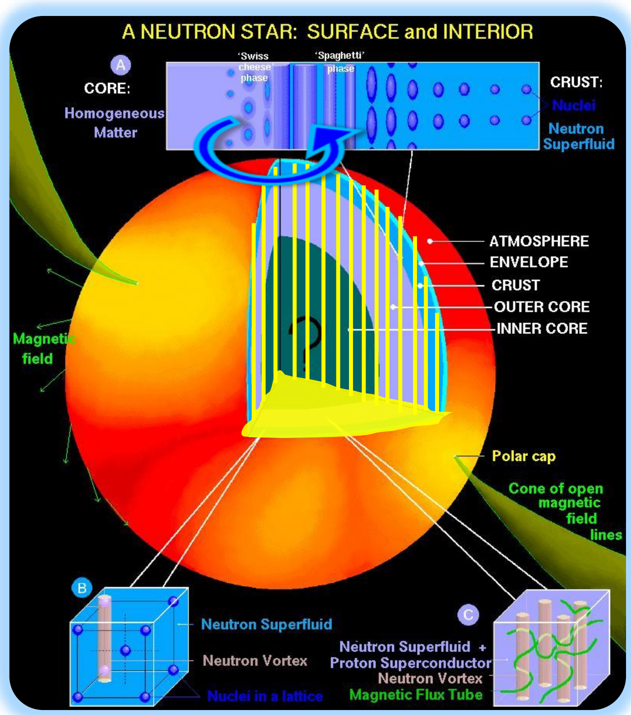
Vortex-number density $\propto$ Rotation speed

Low ⟷ High



W. Ketterle, MIT Physics Annual. 2001

There must be a huge number (~10^{18}) of vortices inside a neutron star!!

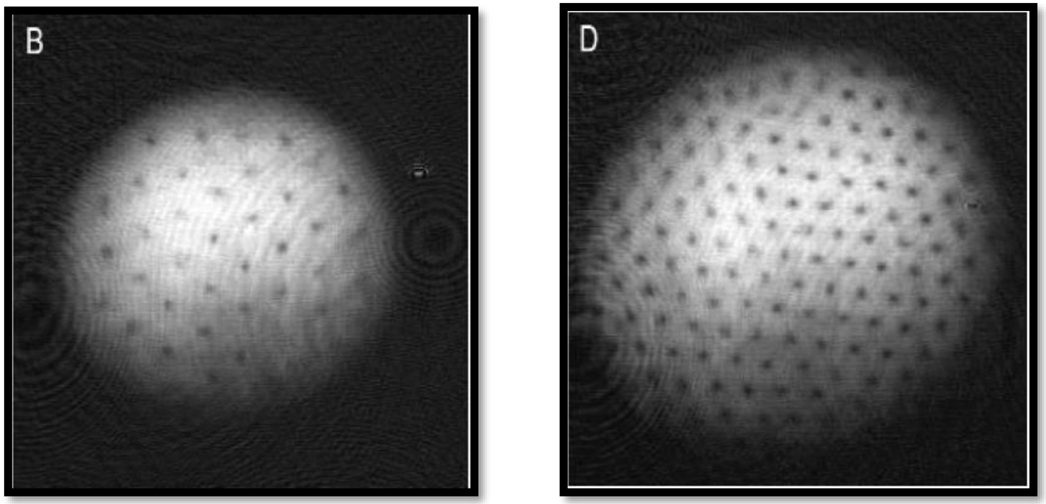


Vortex-number density

$\propto$

Rotation speed

Low High



W. Ketterle, MIT Physics Annual. 2001

How many vortices may be present inside a neutron star?

cf. N. Chamel, J. Astrophys. Astr. **38**, 43 (2017)

- <u>For neutron vortices:</u>

$$n_v \sim \frac{m\Omega}{\pi\hbar} \sim 6\times 10^5 \left(\frac{10\,\mathrm{ms}}{P}\right)\,\mathrm{cm}^{-2}$$: number density

$$d_v \sim n_v^{-1/2} \sim 10^{-3}\left(\frac{P}{10\,\mathrm{ms}}\right)^{1/2}\,\mathrm{cm}$$: average distance

- <u>For flux tubes</u> (proton vortices):

$$n_\Phi \sim \frac{2eB}{hc} \sim 5\times 10^{18}\left(\frac{B}{10^{12}\,\mathrm{G}}\right)\,\mathrm{cm}^{-2}$$: number density

$$d_\Phi \sim n_\Phi^{-1/2} \sim 5\times 10^{-10}\left(\frac{10^{12}\,\mathrm{G}}{B}\right)^{1/2}\,\mathrm{cm}$$: average distance

For a typical neutron star (<u>*R*=10 km, *B*=10^{12} G, *P*=10 ms</u>), one finds:

- ✓ Number of neutron vortices: $n_v \pi R^2 \sim 10^{18}$ $(d_v \sim 10^{-5}\ \mathrm{m})$
- ✓ Number of flux tubes: $n_\Phi \pi R^2 \sim 10^{30}$ $(d_\Phi \sim 10^{-11}\ \mathrm{m})$

Why the core may not be important?

✓ <u>Neutron vortices are **magnetized** in the core</u>

*Protons are dragged by neutrons via nuclear force

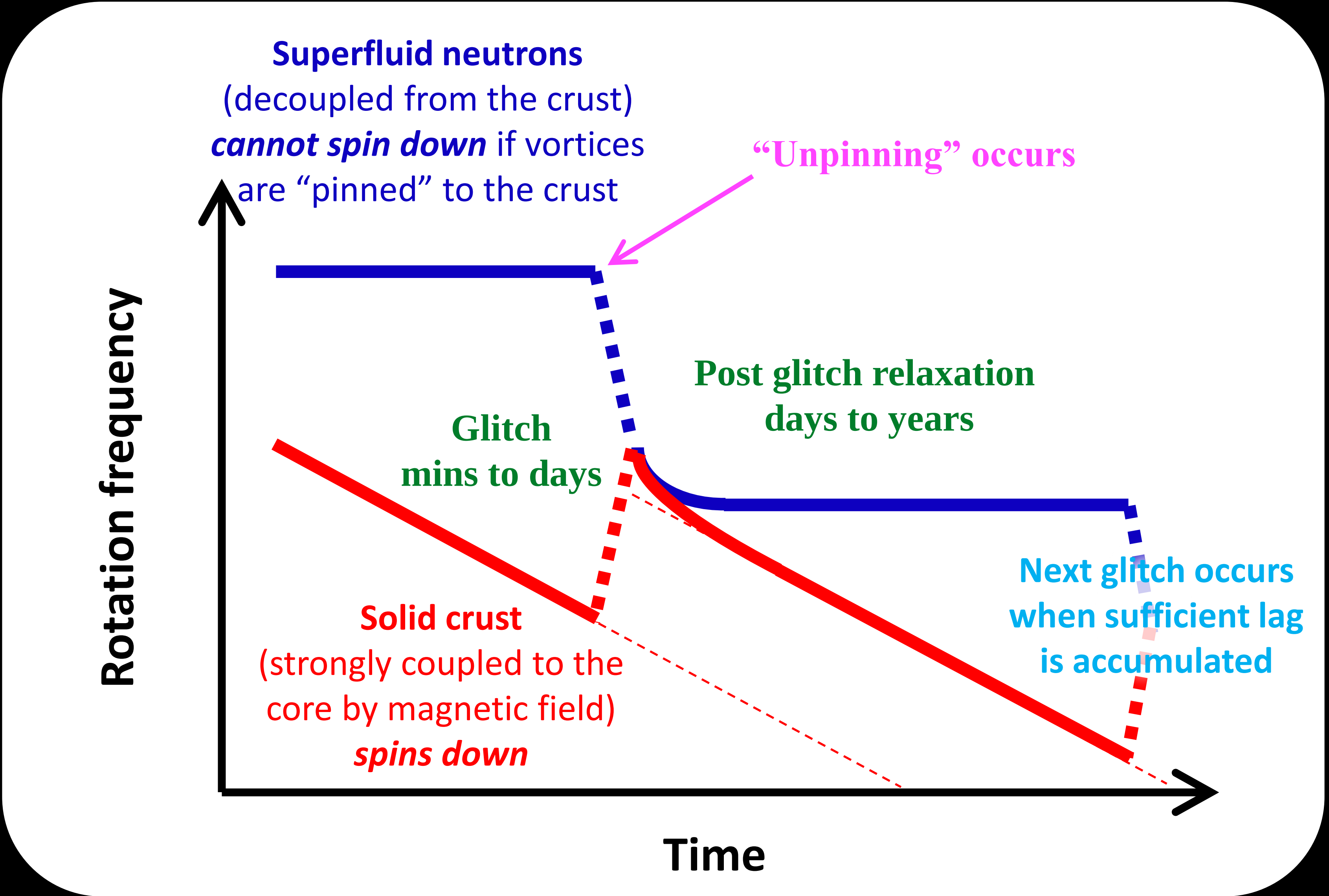
Entrainment may increase neutron effective mass
→ inner crust may (or may not) be enough, see:
- N. Chamel, PRC**85**(2012)035801
- N. Anderson et al., PRL**109**(2012)241103
- N. Chamel, PRL**110**(2013)011101
- G. Watanabe and C.J. Pethick, PRL**119**(2017)062701
- Yu Kashiwaba and T. Nakatsukasa, arXiv:1904.10712

✓ <u>Then, the neutron superfluid is **strongly coupled** with the core due to:</u>

- magnetic field
- scattering of electrons

➢ If so, a "lag" can not be developed, and thus, the core may not work as an angular momentum reservoir

The vortex mediated glitch: Naive picture



To fully understand the glitches, we need to clarify:

Glitch dynamics

How do vortices move?

and, of course,
details of NS matter..

Pinning mechanism

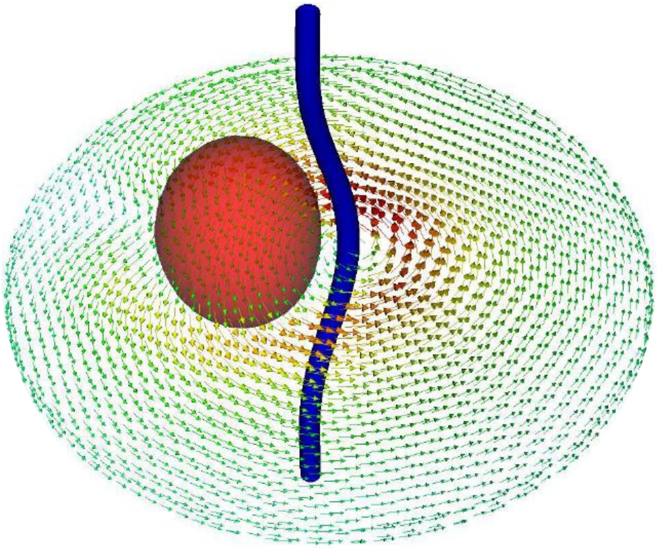
How are vortices pinned?

Trigger mechanism

How are vortices unpinned?

**We attacked this problem using
the state-of-the-art microscopic nuclear theory**

Vortex-nucleus dynamics within TDSLDA



G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes,
Phys. Rev. Lett. **117**, 232701 (2016)

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

- TDSLDA equations (formally equivalent to TDHFB or TD-BdG equations)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

$h_\sigma = \dfrac{\delta E}{\delta n_\sigma}$: s.p. Hamiltonian

$\Delta = -\dfrac{\delta E}{\delta \nu^*}$: pairing field

$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$: number density

$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$: anomalous density

$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar\sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla}v_{k,\sigma}(\boldsymbol{r},t)]$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

# of qp orbitals ~ # of grid points

TDSLDA: TDDFT with local treatment of pairing

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Supercomputing!!

$$h_\sigma = \frac{\delta E}{\delta n_\sigma}$$: s.p. Hamiltonian

$$\Delta = -\frac{\delta E}{\delta \nu^*}$$: pairing field

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$$: number density

$$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$$: anomalous density

$$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar\sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla}v_{k,\sigma}(\boldsymbol{r},t)]$$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

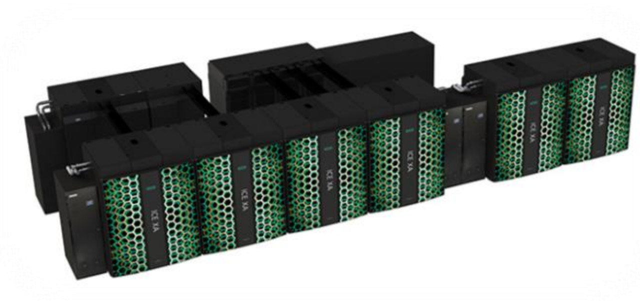
# of qp orbitals ~ # of grid points

*The number indicates the rank according to the TOP500 list (June 28, 2021)

Piz Daint, CSCS, Switzerland (No. 15)

TITAN, ORNL, USA

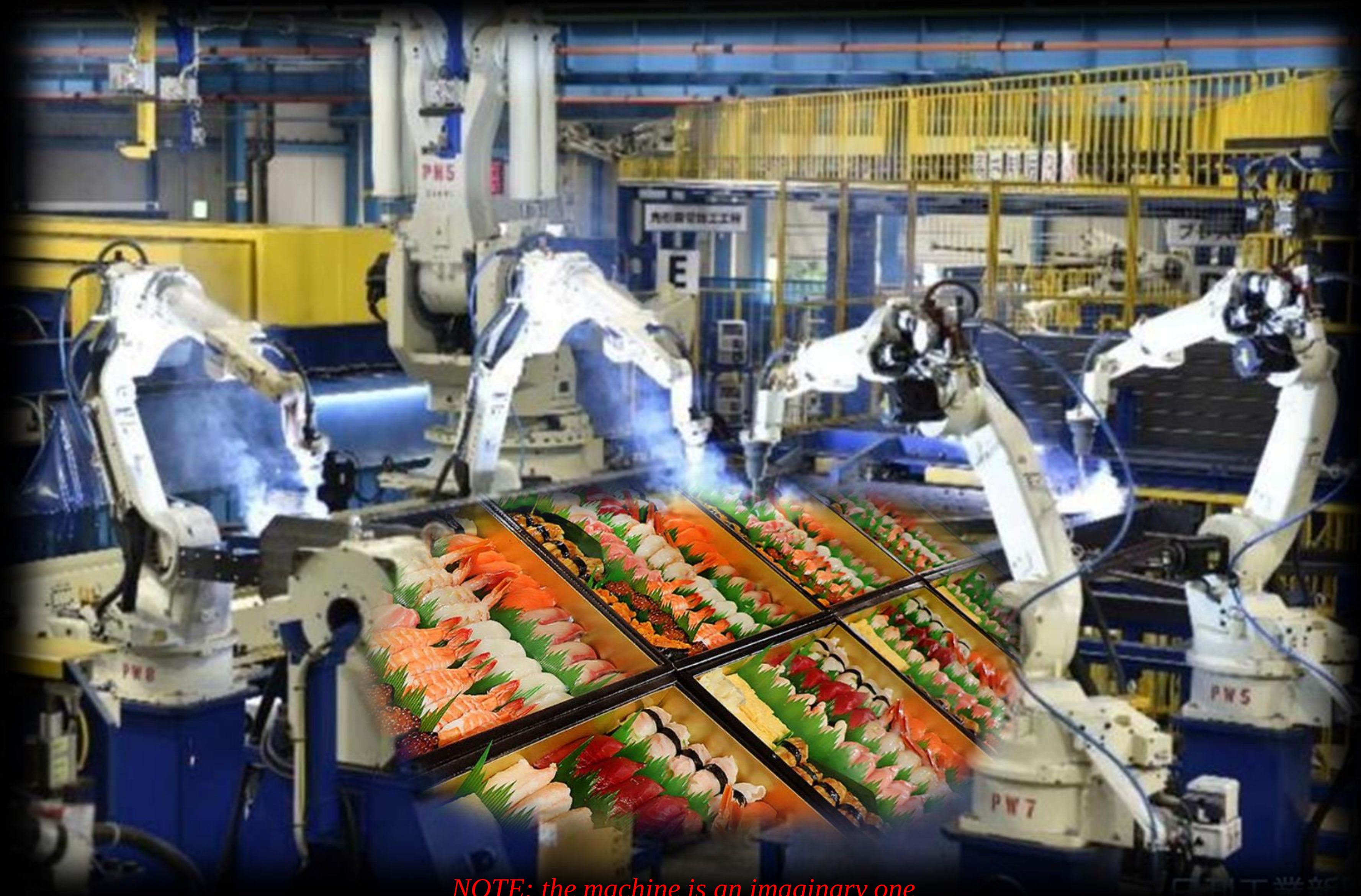
TSUBAME3.0, Japan (No. 50)



Summit, ORNL, USA (No. 2)
GPU, 200 PFlops/s

Why GPUs?

CPU is like a very fast “sushi-production” machine



NOTE: the machine is an imaginary one

Pictures: Nikkan Kogyo Shimbun on Sep. 15, 2020 (https://newswitch.jp/p/23807); Kyouzan Sushi (https://www.kyouzan.jp/menu/)

CPU is like a very fast "sushi-production" machine

The speed is out of reach by a human power :(

Pictures: PAKUTAS; Nikkan Kogyo Shimbun on Sep. 15, 2020 (https://newswitch.jp/p/23807); Kyouzan Sushi (https://www.kyouzan.jp/menu/)

What if we gather manpower?

Picture: 4.7” iPhone 6 Production to Start Next Week, July 18, 2014 (https://www.iphonefaq.org/archives/973661)

If a huge number of simple calculations are the bottleneck, and memory fits, a GPU may surpass a CPU!

- ✓ A single GPU can launch <u>thousands of computing units</u> in parallel
- ✓ Memory (~16 GB/GPU) and data transfer cost (GPU-CPU) limit the applicability

CPU

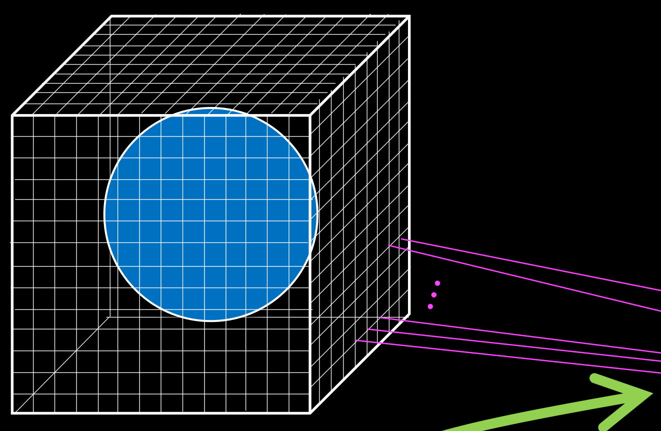
GPU

Notice: TDSLDA equation is "local" in space

- ✓ Each thread takes care of <u>each spatial grid point</u>.
- ✓ <u>For spatial derivatives</u>, we employ <u>cuFFT</u> which requires GPU-CPU data transfers.

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

GPU



Data transfer for cuFFT

- Orbitals are distributed to tens to thousands of GPUs

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Piz Daint, CSCS, Switzerland (No. 15)

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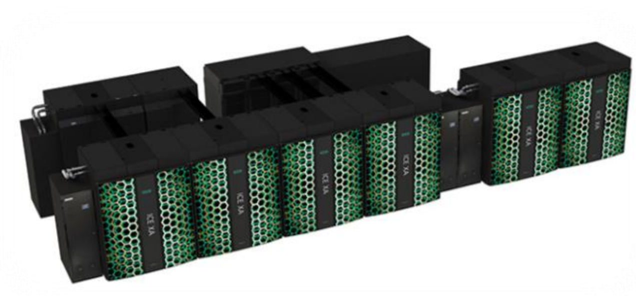
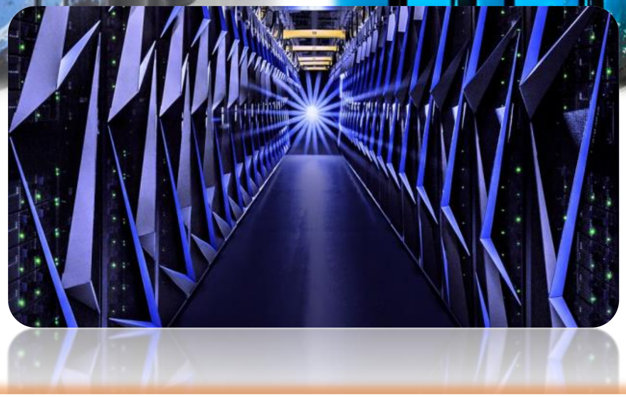
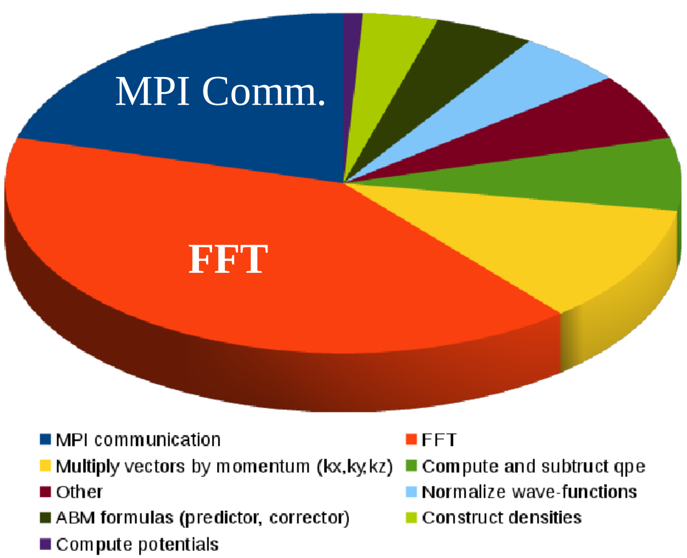




Summit, ORNL, USA (No. 2)
GPU, 200 PFlops/s

Present computing capabilities:

- ✓ Full 3D (w/o symmetry restrictions)
- ✓ Volume as large as 100^3 lattice points
- ✓ Evolution up to 10^6 time steps (as long as 10^{-19} sec)



TDSLDA has been successfully applied for both UFG and nuclear systems

Unitary Fermi Gas (UFG)

$$\left(k_{\mathrm{F}} a \to \infty,\ k_{\mathrm{F}} r_{\mathrm{eff}} \to 0 \qquad \begin{array}{r l} a & : \text{s-wave scattering length} \\ r_{\mathrm{eff}} & : \text{effective range} \end{array} \right)$$

A. Bulgac and S. Yoon, PRL**102**(2009)085302.	Large-amplitude pairing field dynamics
A. Bulgac *et al.*, Science **332**(2011)1288.	Dynamics of quantum vortices
A. Bulgac *et al.*, PRL**108**(2012)150401.	Quantum shock waves and domain walls
A. Bulgac *et al.*, PRL**112**(2014)025301.	Dynamics of vortex rings
G. Wlazłowski *et al.*, PRA**91**(2015)031602(R).	Dynamics of vortices and quantum turbulence
G. Wlazłowski *et al.*, PRL**120**(2018)253002.	Solitonic cascades in spin-polarized UFG
P. Magierski et al., PRA**100**(2019)033613.	Spin-polarized droplet in UFG

Nuclear systems

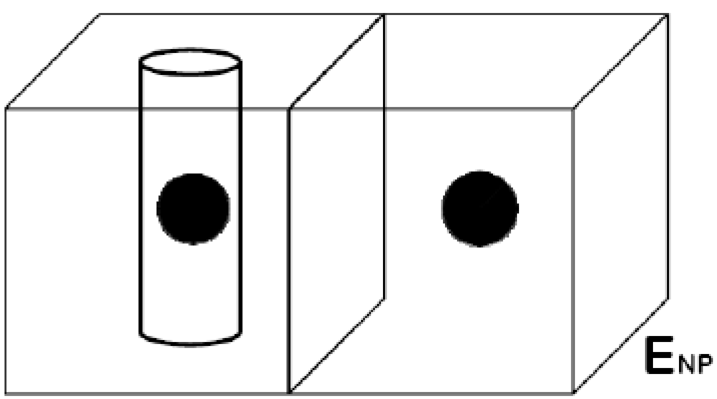
I. Stetcu *et al.*, PRC**84**(2011)051309(R).	Isovector giant dipole resonance (IVGDR)
I. Stetcu *et al.*, PRL**114**(2015)012701.	Relativistic coulomb excitation
A. Bulgac *et al.*, PRL**116**(2016)122504; PRC**100**(2019)034615.	Induced fission of ^{240}Pu
G. Wlazłowski *et al.*, PRL**117**(2016)232701.	Vortex-nucleus interaction

A key to understand the glitches is:

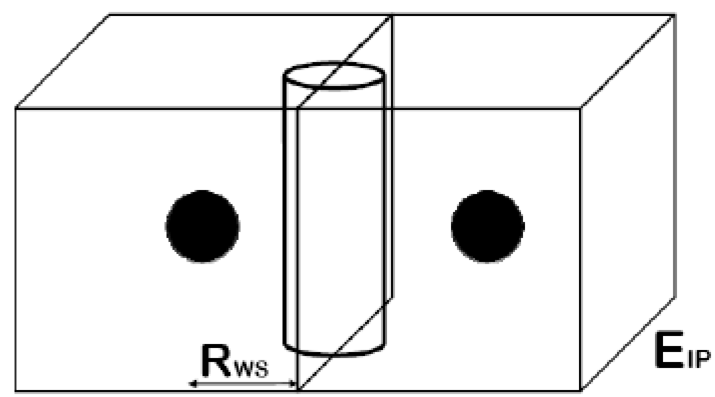
Vortex pinning mechanism in the inner crust of neutron stars

Q. Is the vortex-nucleus interaction

Attractive? or Repulsive?

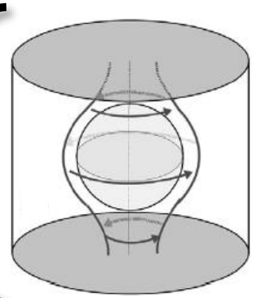
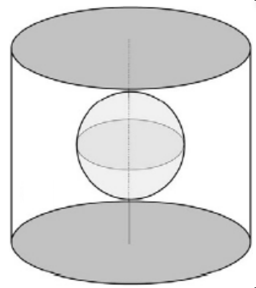
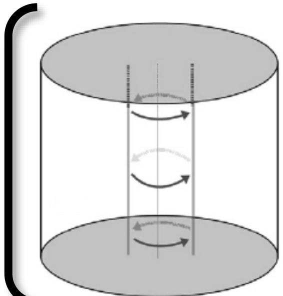
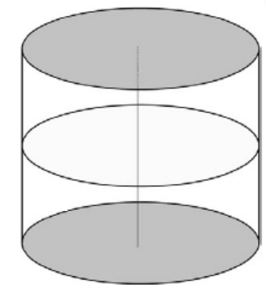


"Nuclear pinning"



"Interstitial pinning"

Microscopic, static HFB calculations were performed assuming axial symmetry

$E_{\mathrm{pin}} = E\Big[$  $-$  $\Big] - E\Big[$  $-$  $\Big]$

Energy to create a vortex line on a nuclear impurity

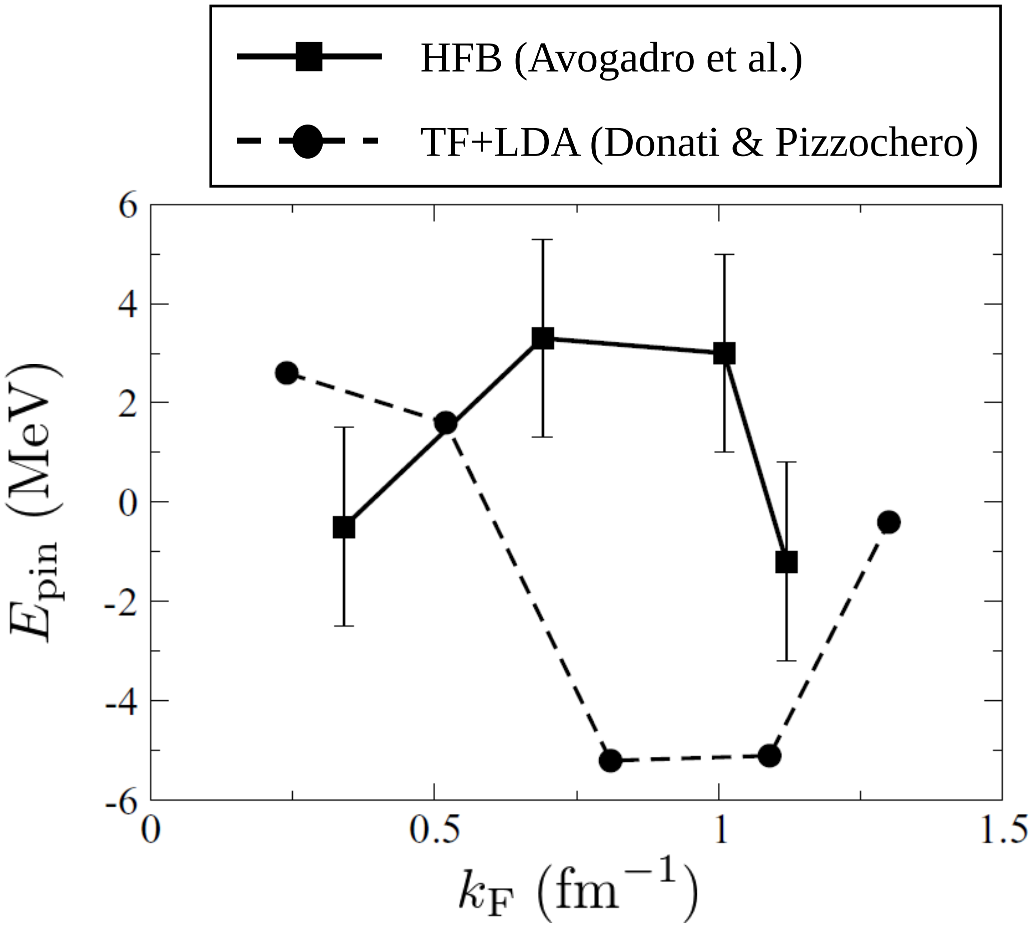
Energy to create a vortex line in a uniform matter

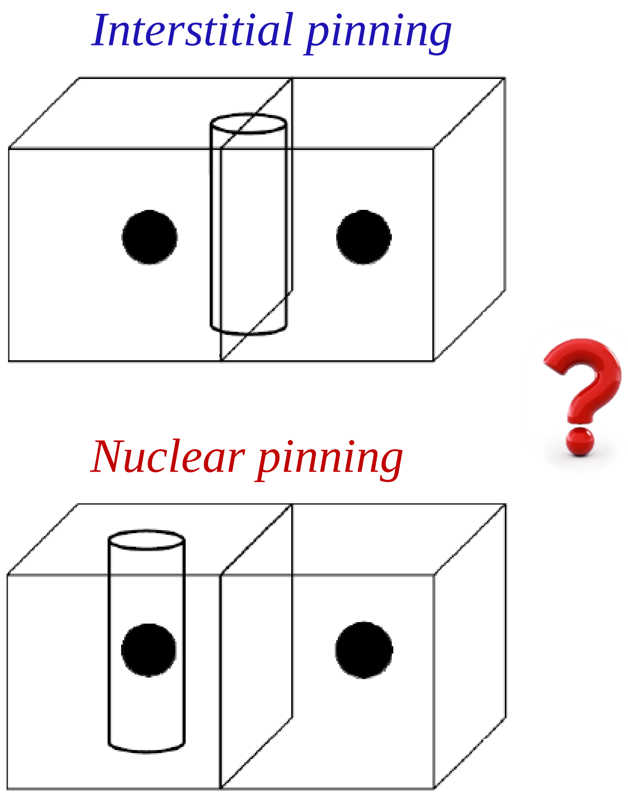
E.g.) 0.026 fm^{-3} (SLy4)

6.19 MeV	13058.04	12954.02	13714.88	13617.05

P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NPA**788**(2007)130; NP**A811**(2008)378

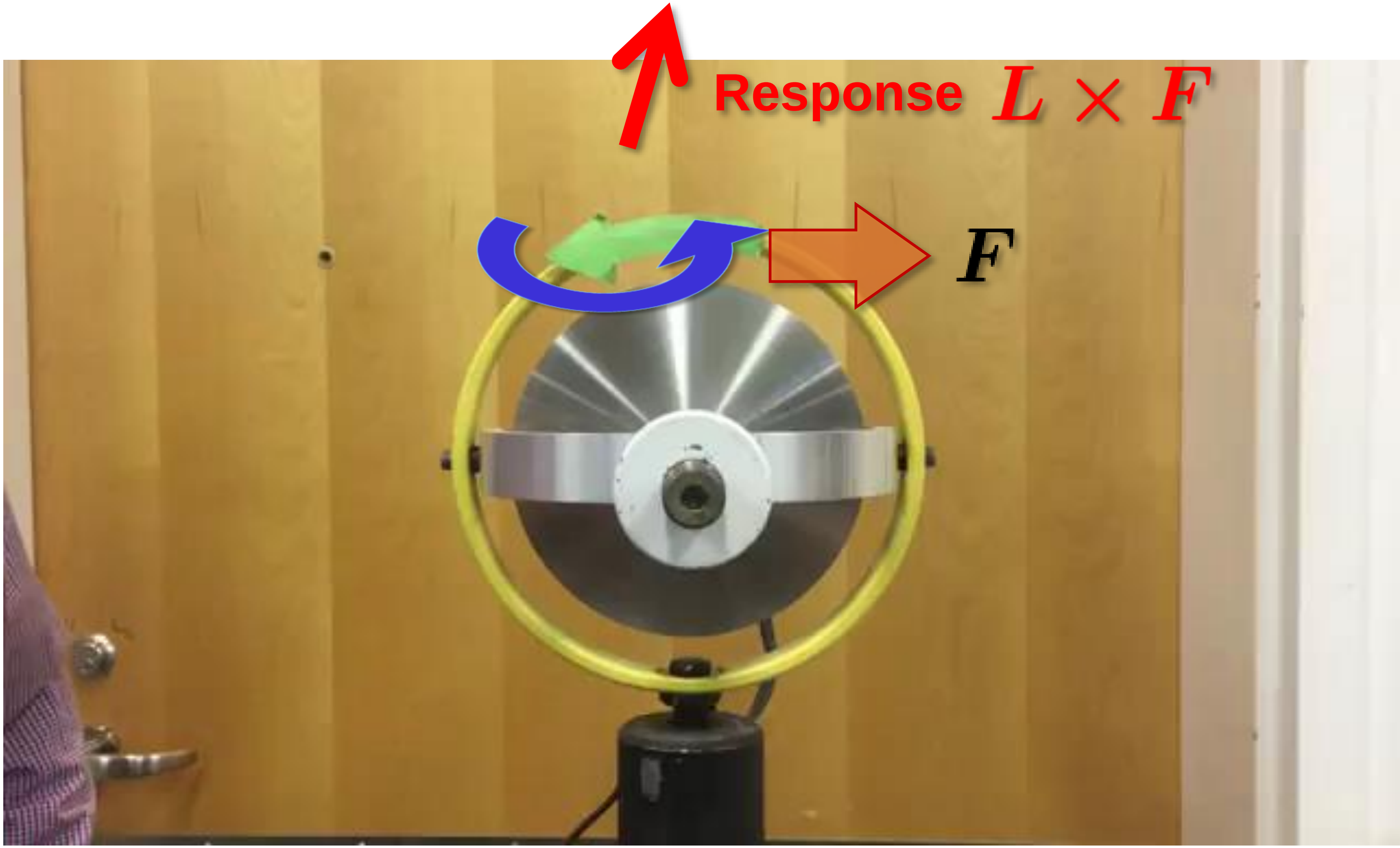
Property of the pinning force was unclear





P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NP**A811**(2008)378

Response of a spinning gyroscope when pushed



We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

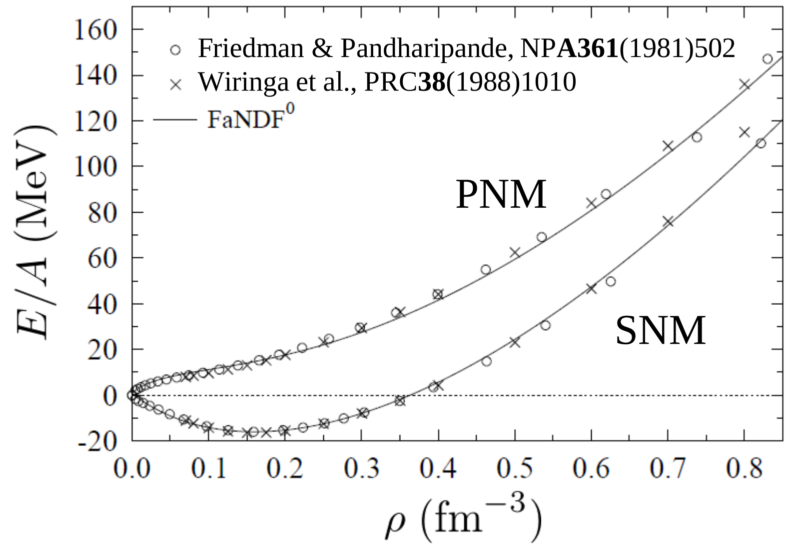
Energy density functional (EDF)

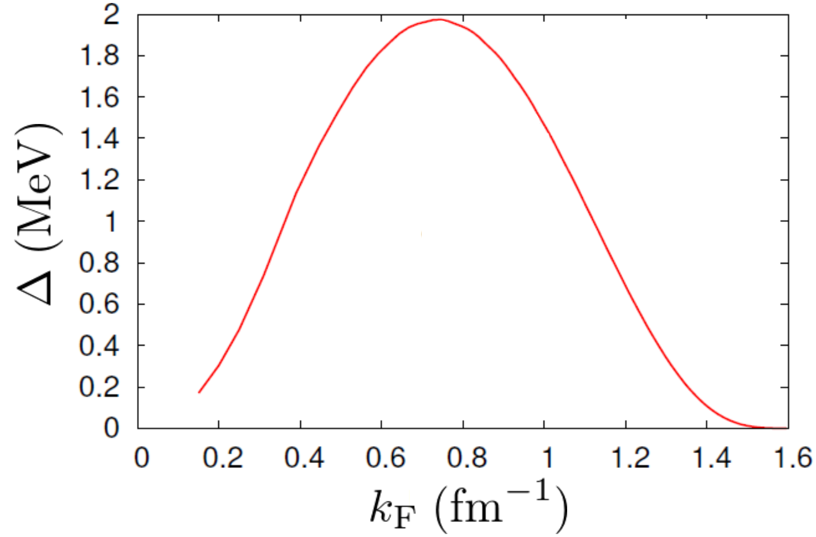
$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\mathrm{pair}}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r}) = \sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





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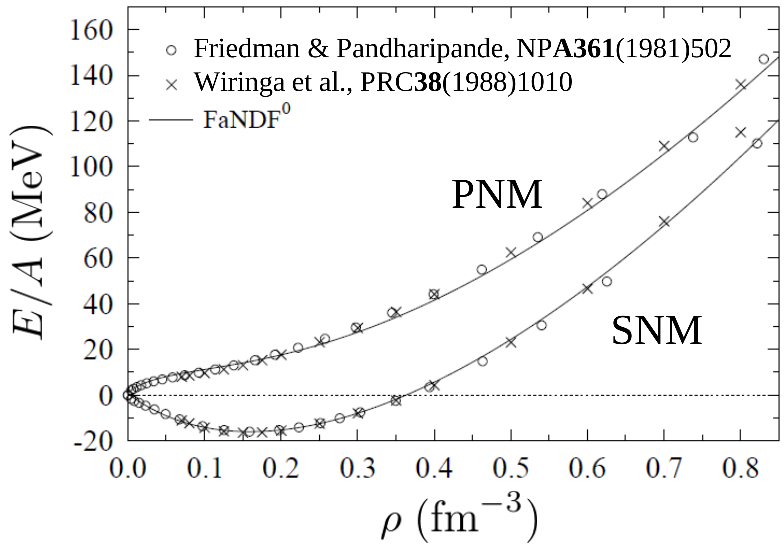

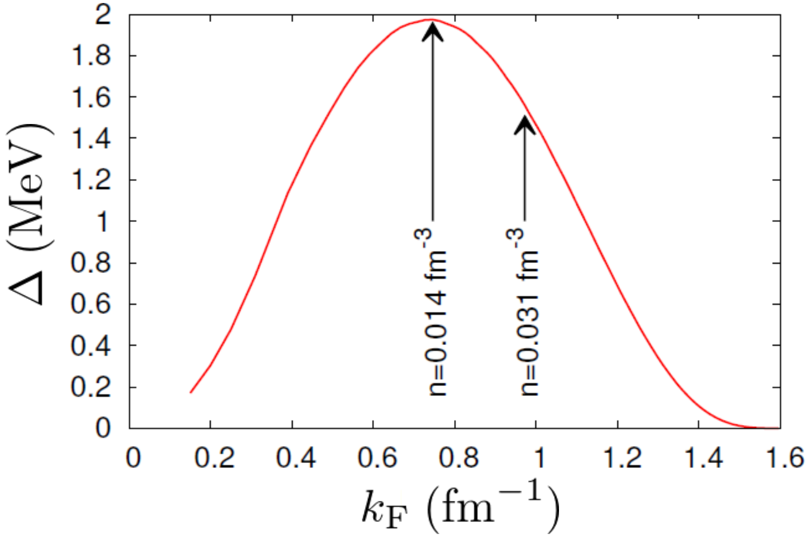

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Computational details

$75\ \mathrm{fm}\times 75\ \mathrm{fm}\times 60\ \mathrm{fm}$

$(50\times 50\times 40,\ \Delta x=1.5\ \mathrm{fm})$

$k_\mathrm{c}=\pi/\Delta x>k_\mathrm{F} \qquad k_\mathrm{F}=(3\pi^2\rho_n)^{1/3}$

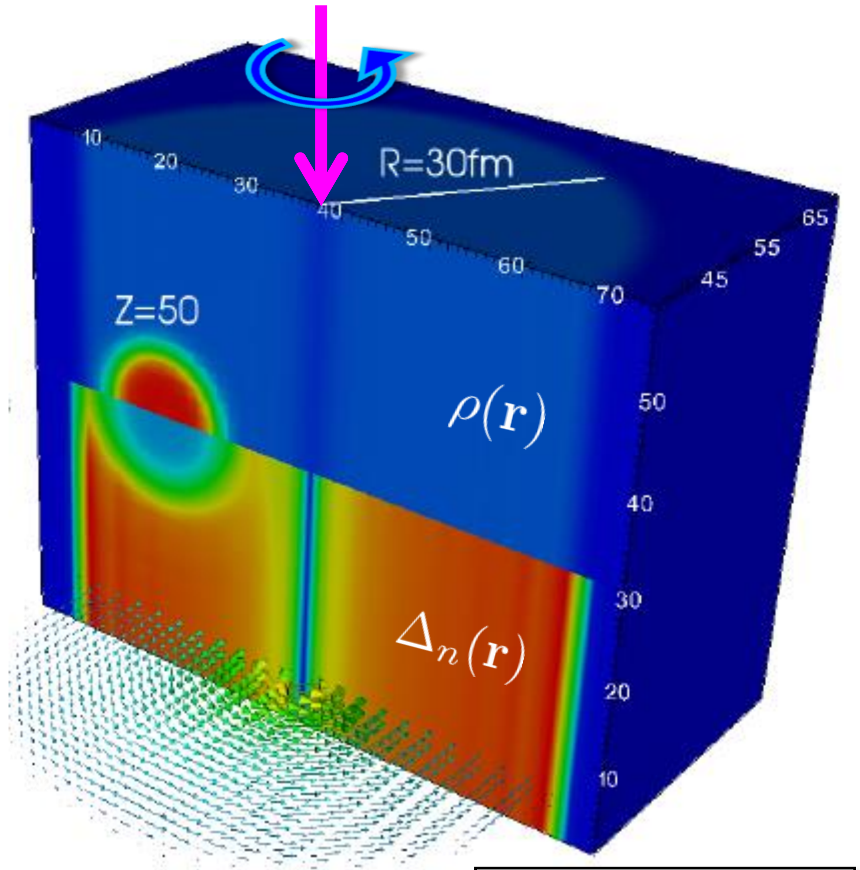
Nuclear impurity: $Z=50$

$\rho_n\simeq 0.014\ \mathrm{fm}^{-3}\ (N\simeq 2{,}530)$

$\rho_n\simeq 0.031\ \mathrm{fm}^{-3}\ (N\simeq 5{,}714)$

\# of quasi-particle w.f. $\approx 100,000$

a vortex line exists here



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# of quasi-particle w.f. $\approx 100{,}000$

MPI+GPU
→ 48h w/ 200GPUs
for 10,000 fm/c


TITAN, Oak Ridge

NERSC Edison, Berkeley

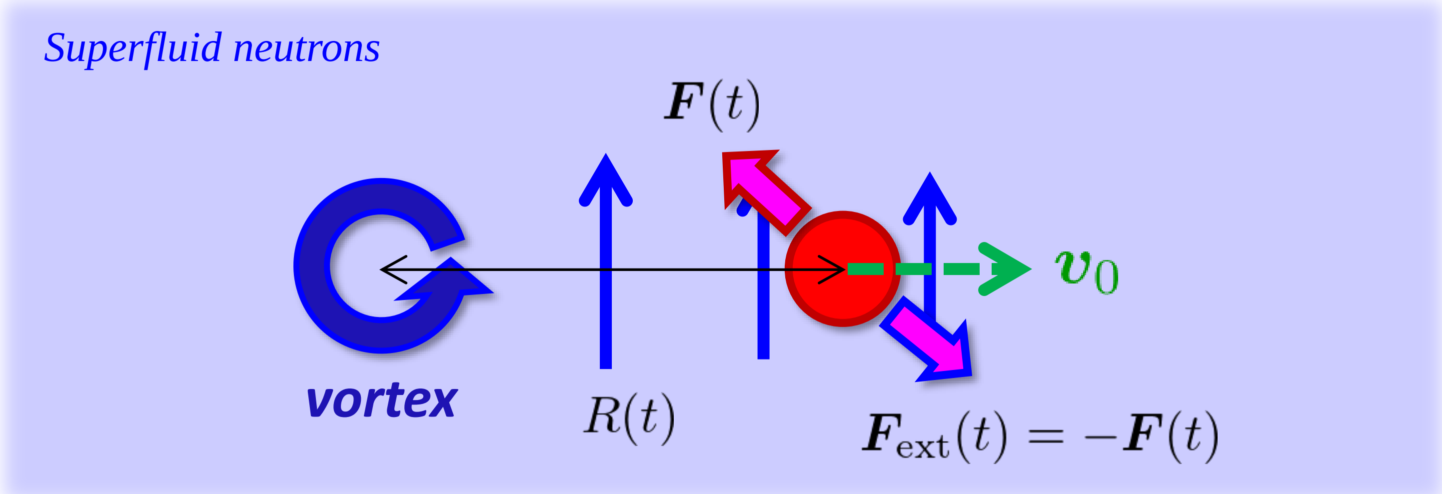
HA-PACS, Tsukuba

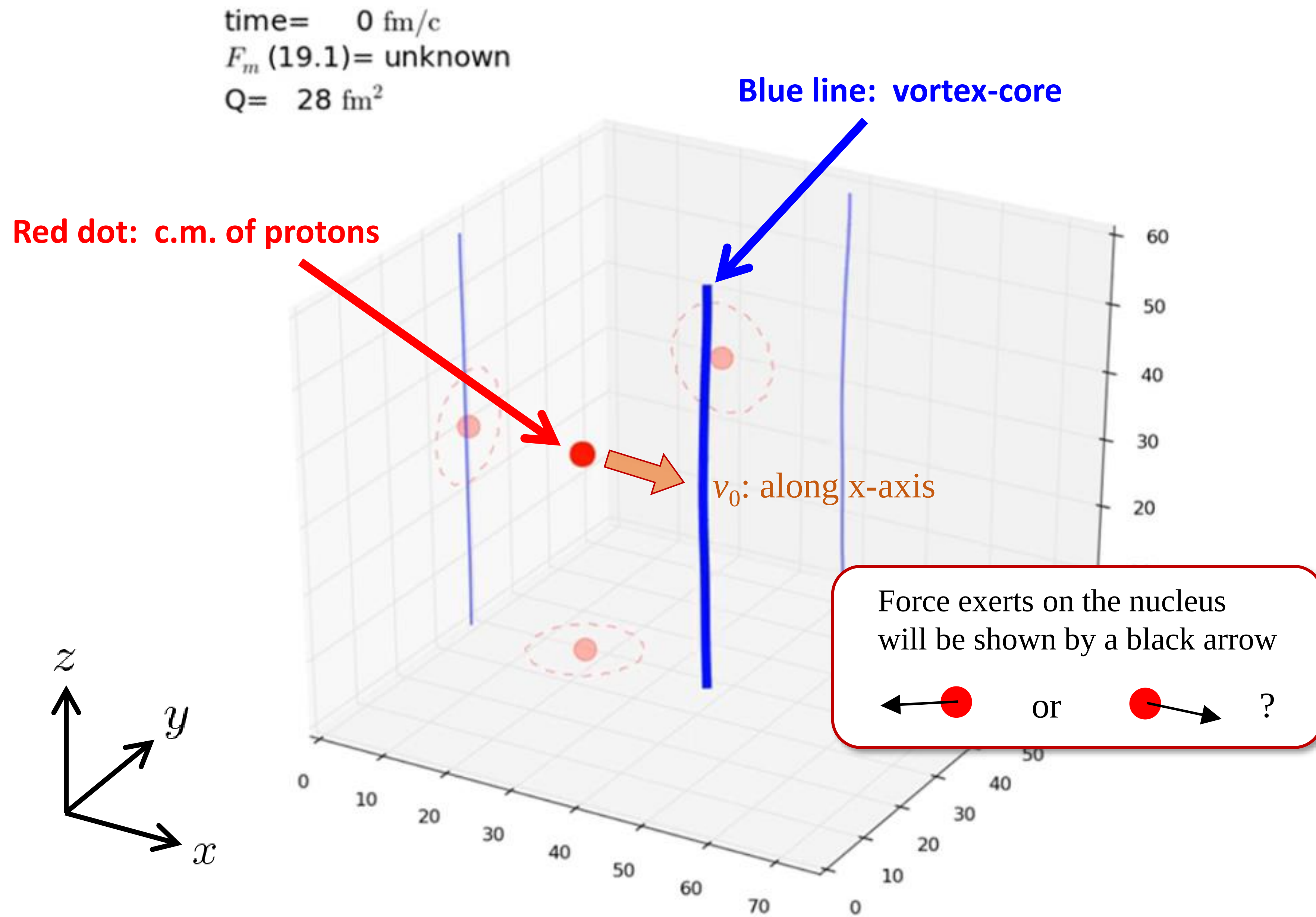
We directly measure the force $\boldsymbol{F}(R)$ in dynamical simulation

- Newton's law

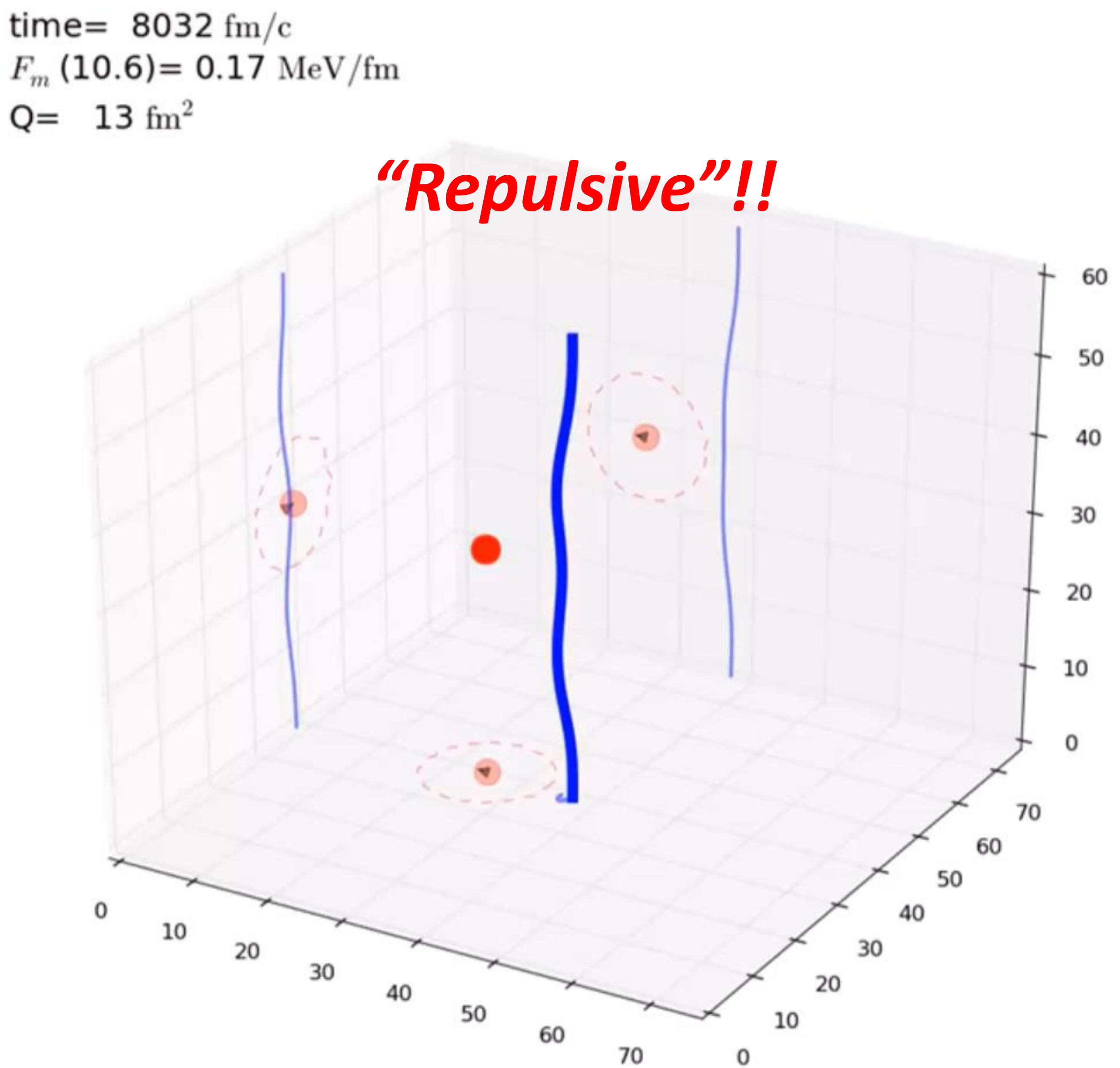
$$\boldsymbol{F} = M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = 0 \quad \text{if} \quad \boldsymbol{F} = 0$$

- We keep a nuclear motion in a constant velocity $v_0 \ (\ll v_{\mathrm{crit}})$



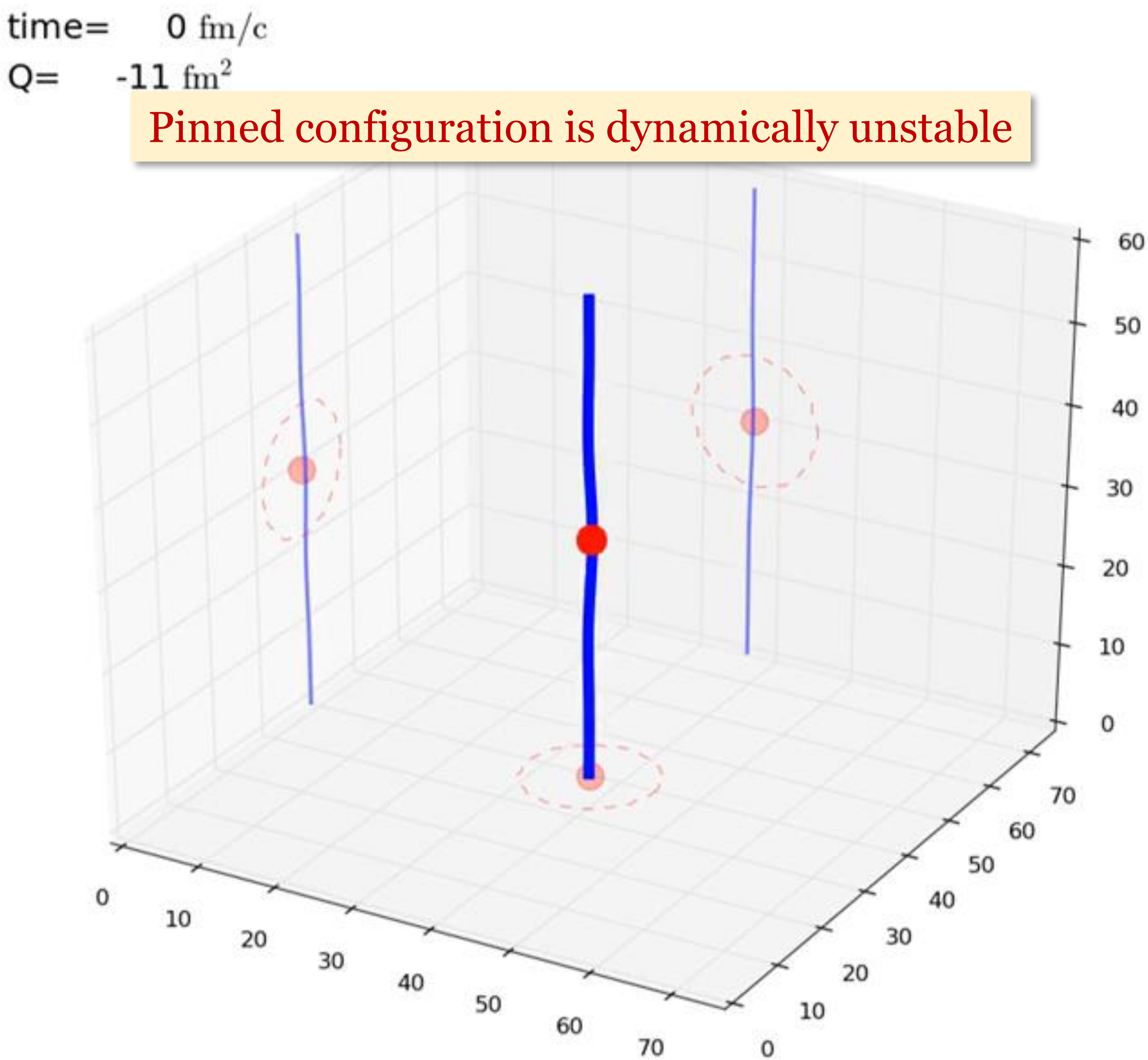


time= 0 fm/c
F_m (19.1)= unknown
Q= 28 fm^2
Blue line: vortex-core
Red dot: c.m. of protons
v_0: along x-axis
Force exerts on the nucleus will be shown by a black arrow
or
?
z
y
x

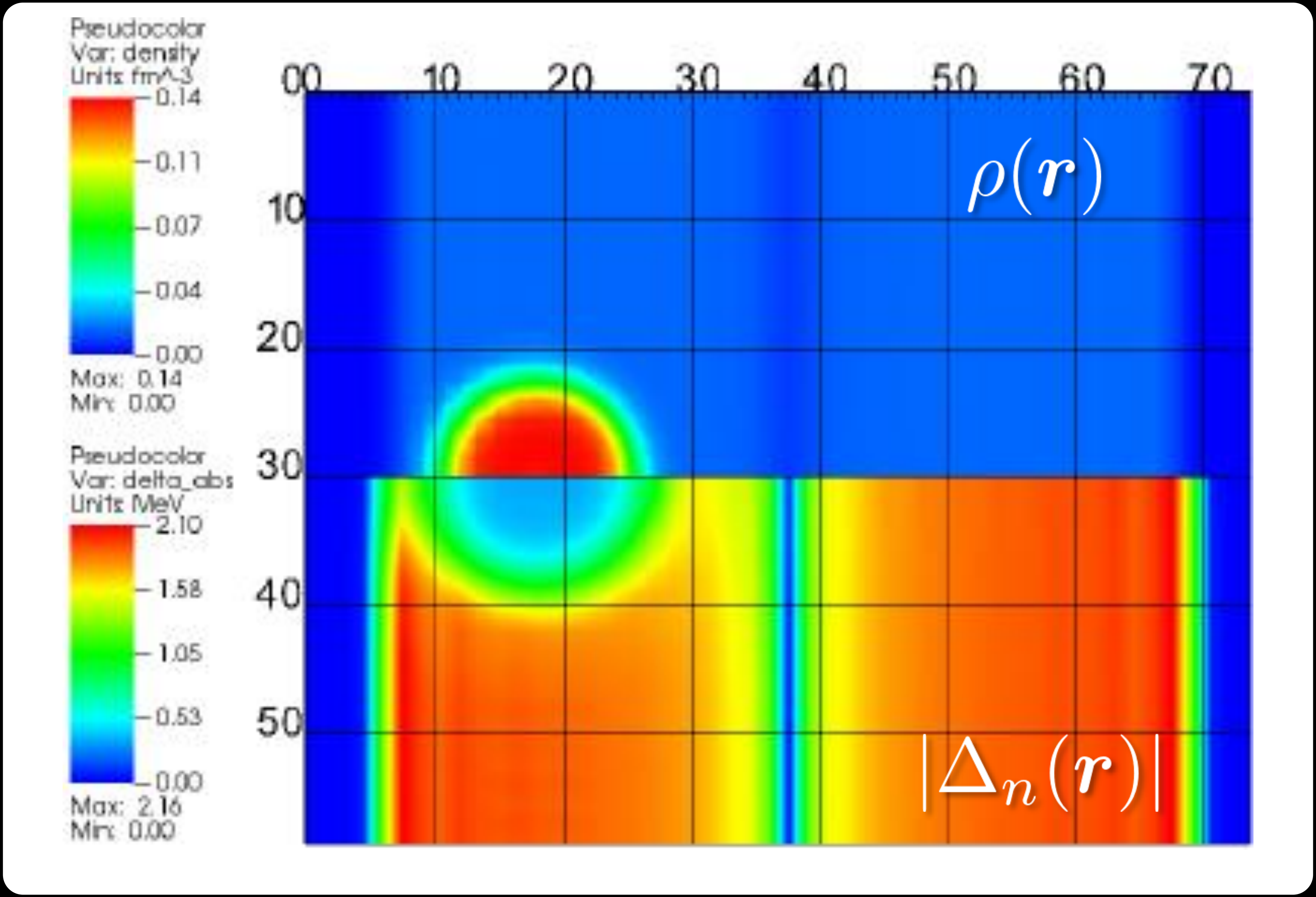


time= 8032 fm/c
F_m (10.6)= 0.17 MeV/fm
Q= 13 fm^2
"Repulsive"!!
60
50
40
30
20
10
0
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70

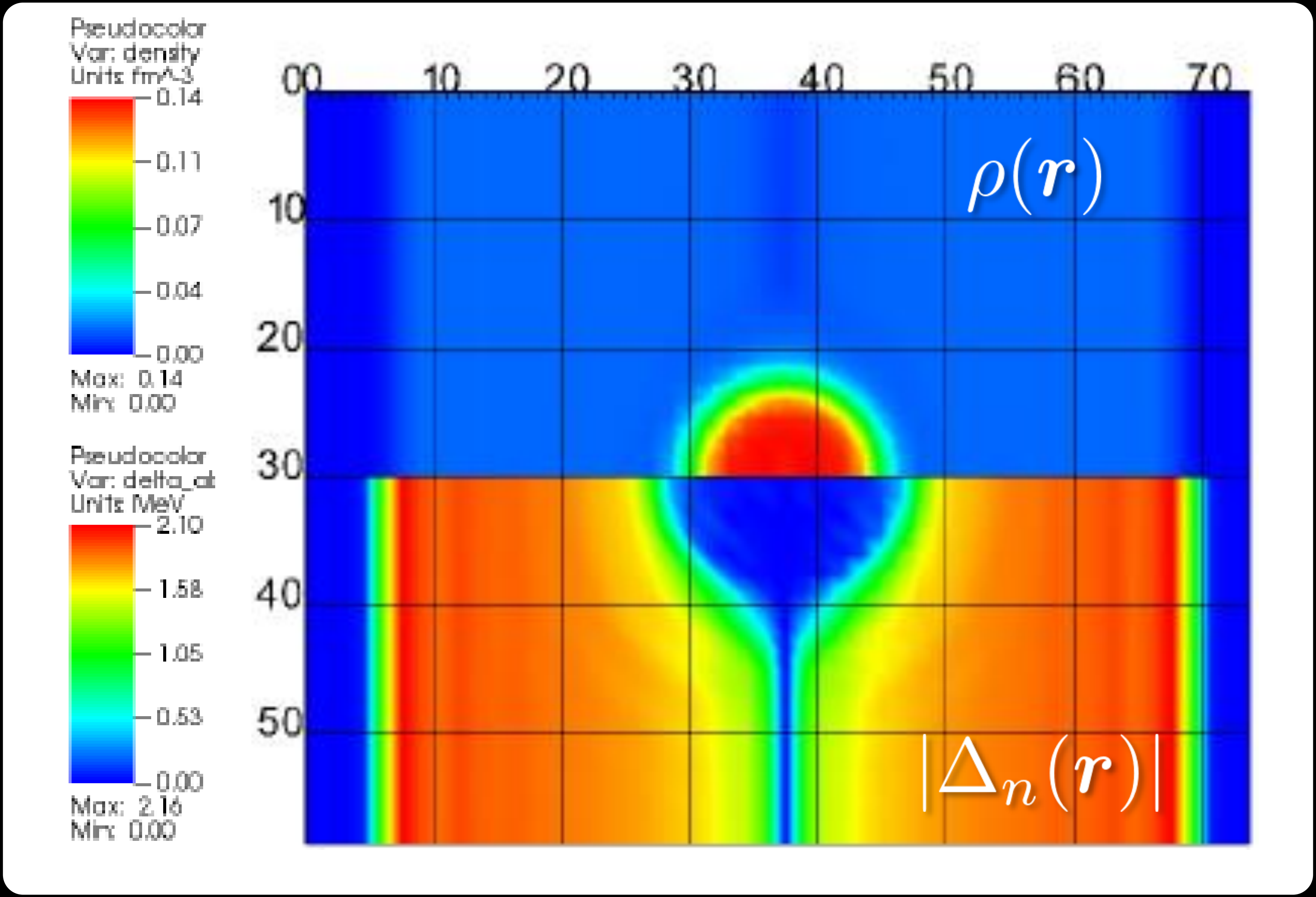
Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



"Unpinned configuration"

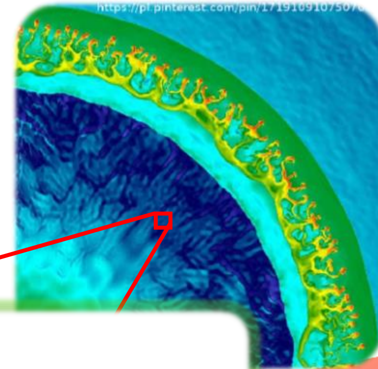


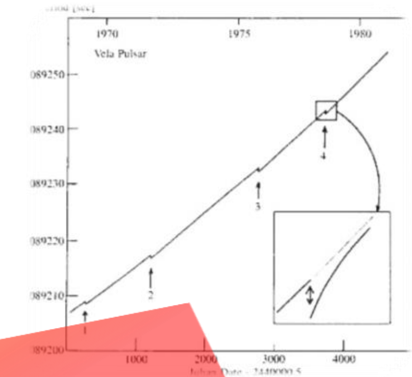
"Pinned configuration"



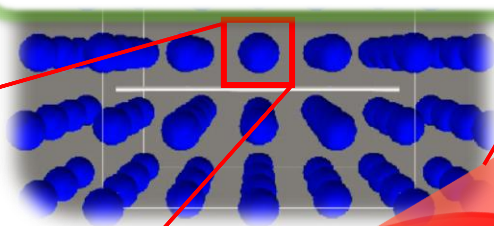
Our goal and strategy

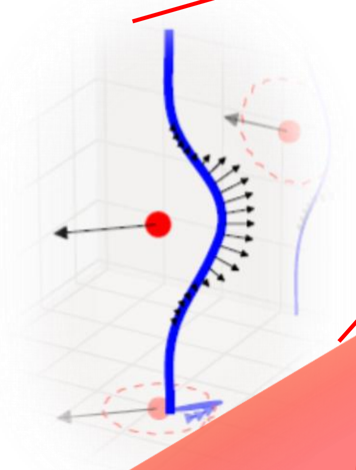
Goal: Unveil the mechanism of glitches





New collaboration started:
Nicolaus Copernicus Astronomical Centre
B. Haskell et al.





10^{4}m

Macroscopic

- observations
- hydrodynamics

$\sim 10^{-10}$m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

Microscopic

Nuclear Physics!!

- vortex-nucleus dynamics from *neutrons and protons*

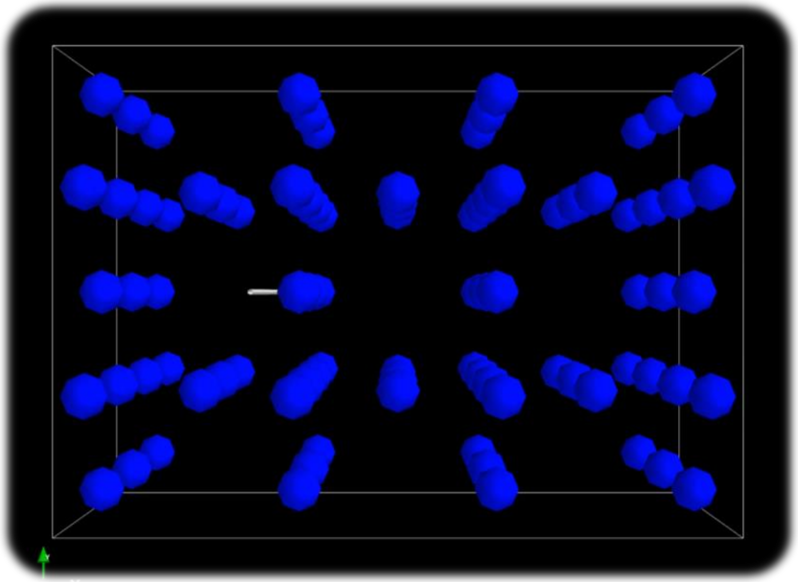
Ongoing project - I

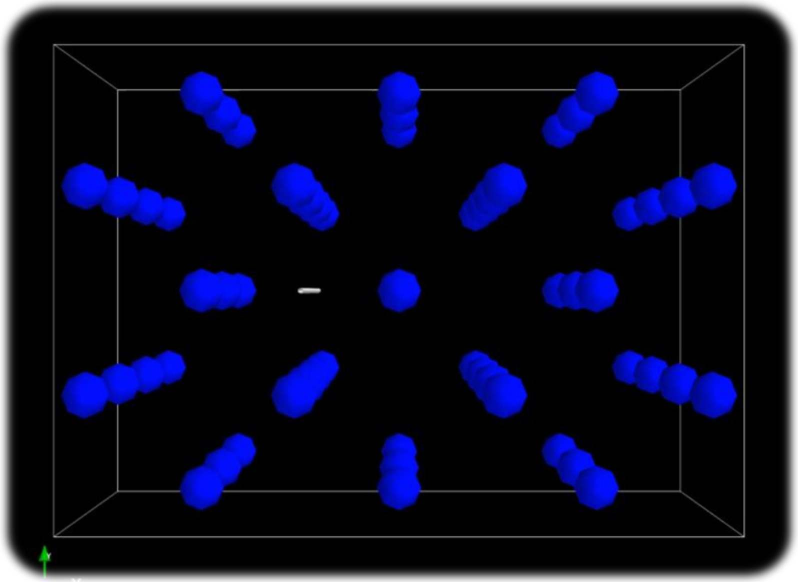
Mesoscopic simulation of pinning force with the Vortex Filament Model (VFM)

- Even for a repulsive interaction, vortices could be immobilized by the lattice

Simulations by K. Kobczewski (a PhD student at WUT)

$$n_s \boldsymbol{\kappa} \times (\boldsymbol{v}_{vor} - \boldsymbol{v}_{ext}) = \boldsymbol{f}_{VN} + \boldsymbol{f}_{tension} + \boldsymbol{f}_{dissipation}$$





Talk by K. Kobczewski at POLNS18, March 26-28, 2018: https://indico.camk.edu.pl/event/10/contribution/8

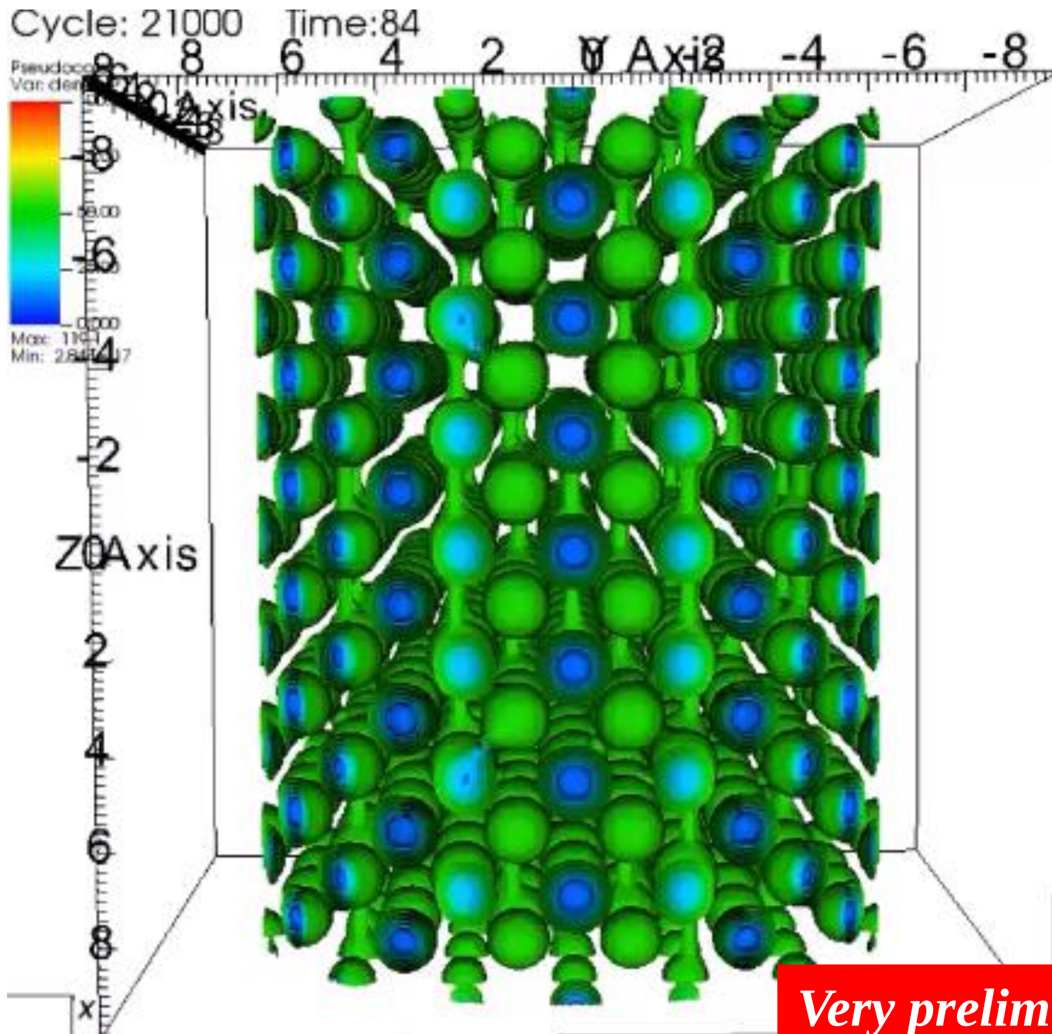
Very preliminary

Ongoing project - II

Microscopic simulation of vortex dynamics with Gross-Pitaevskii Equation (GPE)

- Vortices could be bent, and dynamics might be more complicated.

Simulation by T. Sasaki (a Master's course student at Niigata University)



Very preliminary

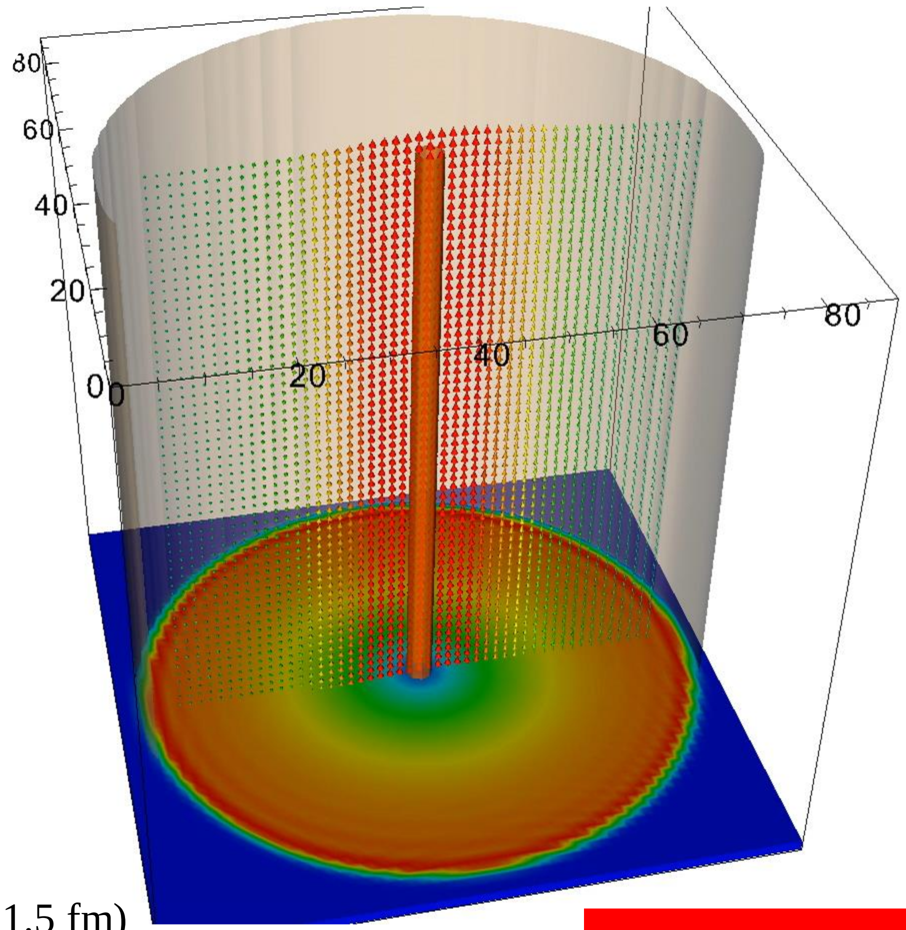
Ongoing project - III

Microscopic simulation of Donnelly-Glaberson instability within TDSLDA

- Kelvin waves are amplified, which may cause a turbulent state

Simulation by D. Pecak (a postdoc at WUT)

Normal flow



$n_n = 0.019\ \mathrm{fm}^{-3}$

Box: 90^3 fm^3 (Δx=1.5 fm)

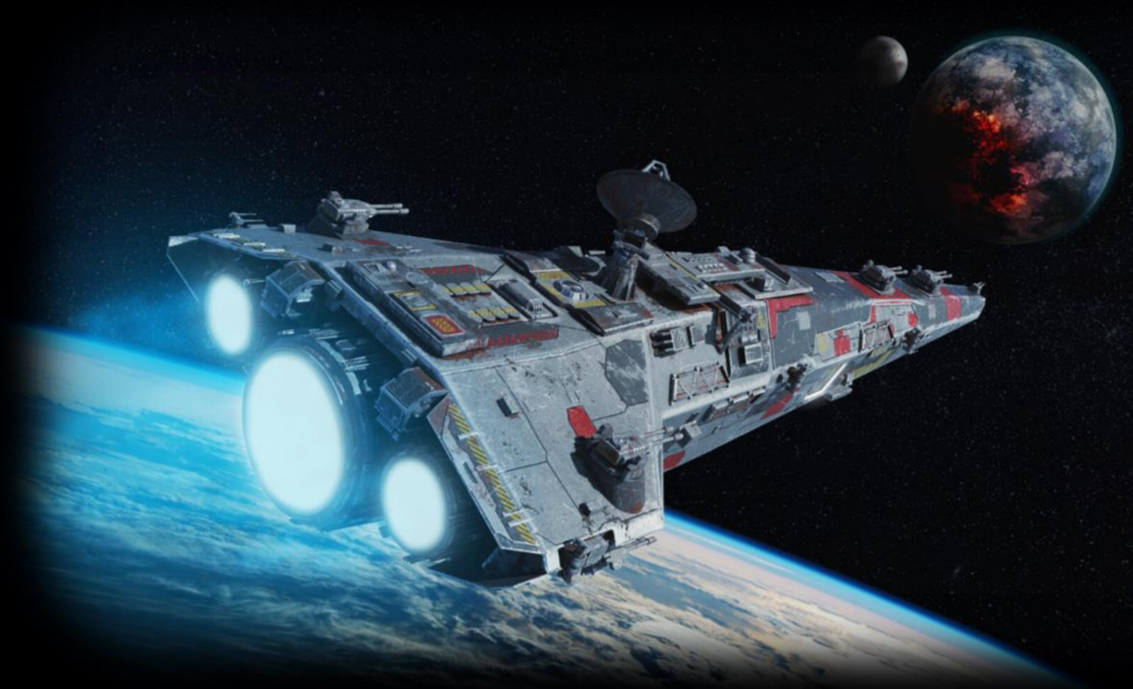
EDF: BSk31

Very preliminary

Summary

I hope you enjoyed our exciting adventure!

with a magic of DFT and TDDFT..



“Flat Earth” by [iStock](); “Fanciful view of ship sailing over edge of Earth” by [Georgia Studies Images](); [“Star Wars Spaceship” by Valérian Pierret @Artstation]()

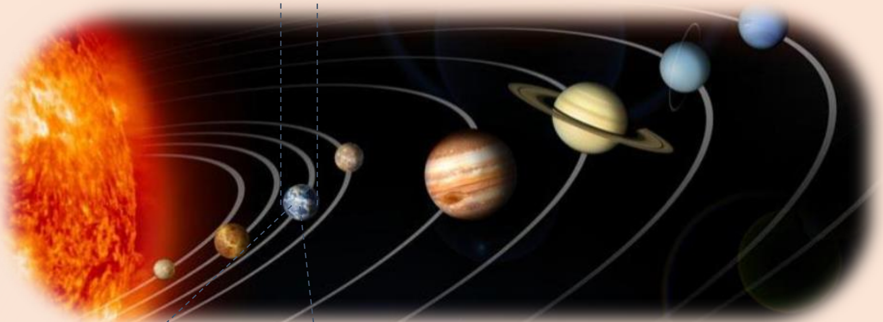
The transdisciplinary character is
one of the fascinating points of Nuclear Physics

What I showed in my lectures are only a tiny part of the huge field! :)

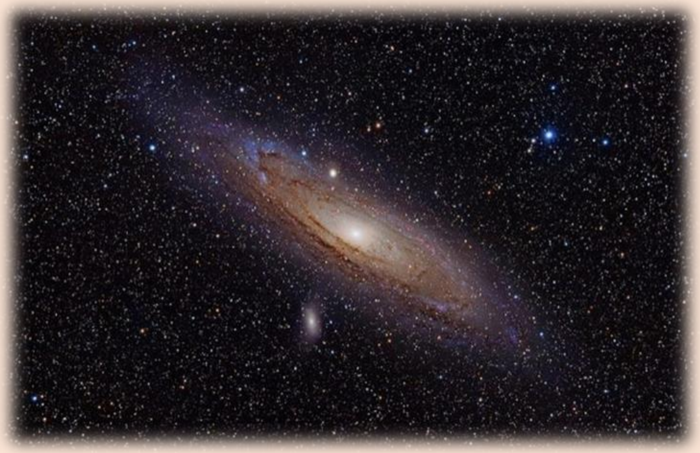
Takeaway message

Sun
(~10^{9}m)

Earth
(~10^{7}m)

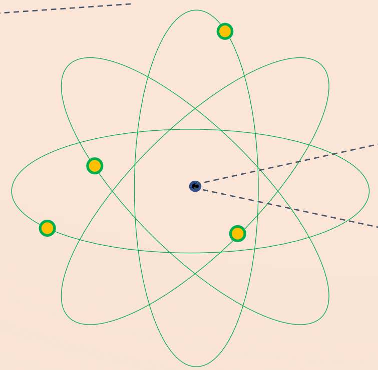


Solar system
(~10^{21}m)

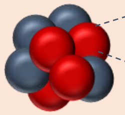


Universe

Daily life
(~1m)

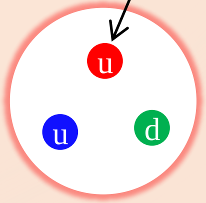


Atom (~10^{-10}m)



Nucleus
(~10^{-14}m)

Quark
(<10^{-19}m)

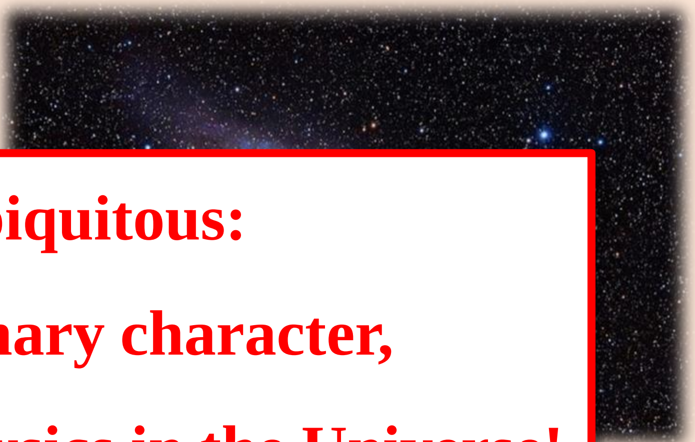


Proton
(~10^{-15}m)

Takeaway message

Sun
(~10^9m)

Earth
(~10^7m)

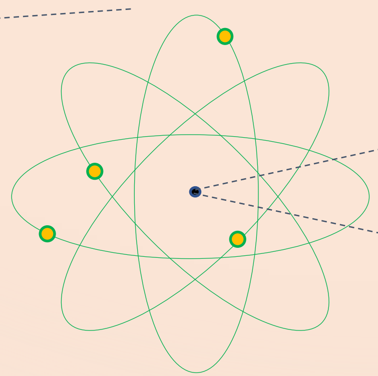


Nuclear physics is ubiquitous:

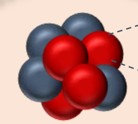
It has highly transdisciplinary character,

connecting different scales of physics in the Universe!

Daily life
(~1m)

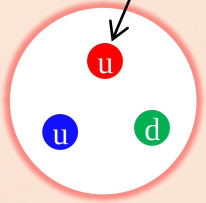


Atom (~10^{-10}m)



Nucleus
(~10^{-14}m)

Quark
(<10^{-19}m)



Proton
(~10^{-15}m)

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