ゲージヒッグス統合理論の現状と今後の展望

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2022/08/07 第68回原子核三者若手夏の学校

まず、
68回目の夏学が開催されること
大変喜ばしく思います!!

また、夏学講義の講師に
お招きいただき大変光栄です

世話人関係者の皆さんに
感謝しますm(_ _)m

個人的には、3度ほど夏学に参加しました

・1994 名大M1 素粒子パート準備校

現象論: 岡田 安弘さん@KEK「超対称性理論」

場の理論: 宇川 彰さん@筑波「格子ゲージ理論」

弦理論: 細谷 暁夫さん@東工大「量子重力」

講義ノート担当→素粒子論研究に投稿

これでtexを覚えた

宿泊施設は、複数の民宿に分宿

講義場所は公民館

講義をテープに録音し、書き起こした

素粒子論研究91-4
(1995)131-180

量子重力
講義ノート

述　細谷 暁夫 (東工大)
記　永谷 幸則　丸 信人 (名大)

(1995 年 5 月 17 日受理)

まえがき

この講義録は東京工業大学教授 細谷暁夫氏が１９９４年7月２５日２６日の２日間に渡り、原子核三者若手夏の学校の素粒子部会において、量子重力と題して M1 向けに行なわれた講義の記録です。この講義録は永谷と丸が共同して担当することになりました。講義

・1996 名大D1 準備校

現在の大きな会場(スキー場ホテル)で開催するスタイルを始めたのはこのときから(のはず)

準備校会計係: 参加費¥800万を郵便局から銀行へ手動で移動

青木 健一さん@金沢大「くりこみ群」講義
他は思い出せません

アトランタオリンピックの年で夏学も「マイアミの奇跡」に湧いた

・2003か2005 理研基礎科学特別研究員
研究会でトーク

講義アブスト

素粒子標準模型は、実験により精密に検証されているが、 様々な未解決問題が残され、
標準模型を超える拡張が必要とされている。
この講義では、標準模型ヒッグス場を
高次元ゲージ場の一部とみなし、 階層性問題を解決するゲージ・ヒッグス統一理論について解説する。 模型構築の基礎を詳しく説明し、
そこから予言される物理について、
標準模型との相違点を比較しながら議論する。

References

"TASI 2004 lectures: To the fifth dimension and back"
Raman Sundrum, hep-th/0508134

"New Ideas on Electroweak Symmetry Breaking"
Christophe Grojean, CERN-PH-TH/2006-172

"Holographic Methods and Gauge-Higgs Unification in Flat Extra Dimensions"
Marco Serone, 0909.5619 [hep-ph]

"Lecture on Gauge-Higgs Unification in extra dimensions"
Csaba Csa'ki, Talk slides in Ringberg Pheno. Workshop

"ゲージヒッグス統合理論 素粒子標準理論のその先へ"
細谷 裕 (サイエンス社)

"素粒子の標準模型を超えて" 林 青司 (丸善出版)

この講義の基礎的な部分は
以下の講義・セミナーをベースとしています

- 2010/10/8 セミナー@金沢・富山大合同
- 2014/3/11-12 集中講義@KMI
- 2018/9/9-12 講義@ILC夏の合宿2018
- 2018/10/26 セミナー@北大
- 2019/6/7 集中講義@名古屋
- 2020/9/10 講義@瀬戸内summer institute

スライドが英語表記ですがお許しください

PLAN

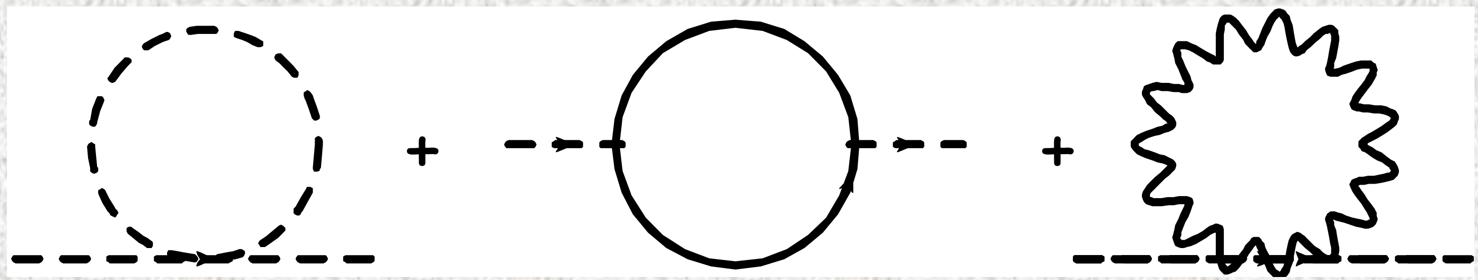
- Introduction
- Higgs mass calculation
- Gauge-Higgs sector
- Matter content &
  Yukawa coupling
- Flavor mixing & CPV
- EW symmetry breaking
- GUT extension
- Summary

Introduction

One of the problems in the Standard Model:
Hierarchy Problem

Quantum corrections to the Higgs mass
is sensitive to the cutoff scale of the theory

$\delta m_H^2 =$



…

$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}$$

Too large!!
(Natural cutoff scale is Planck scale or GUT scale)

To get Higgs mass 125 GeV,
unnatural fine tuning of parameters is required

$$m_H^2 = \underset{\substack{\uparrow \\ \text{classical}}}{m_0^2} + \underset{\substack{\uparrow \\ \text{Quantum corrections}}}{\delta m^2} \approx \mathcal{O}\left(\left(100GeV\right)^2\right)$$

classical Quantum
corrections

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unnatural fine tuning of parameters is required

$$m_H^2 = m_0^2 + \delta m^2 \approx \mathcal{O}\left(\left(100 GeV\right)^2\right)$$

classical Quantum corrections

Naively, we have $$m_0^2, \delta m^2 \approx \mathcal{O}\left(\left(10^{18} GeV\right)^2\right)$$

32 digits of fine tuning

1.00000000000000000000000000000001
- 1.00000000000000000000000000000000 !!

Problem: We have NO symmetry forbidding the scalar mass

SUSY $\qquad \phi \qquad \leftrightarrow \qquad \psi$

Mass term is forbidden
by chiral symmetry

Problem: We have NO symmetry forbidding the scalar mass

SUSY $\phi \leftrightarrow \psi$

Mass term is forbidden by chiral symmetry

Identified with Higgs in the SM

GHU $\mathcal{A}_5 \leftrightarrow \mathcal{A}_\mu$

Higher dimensional Lorentz invariance

Mass term is forbidden by the gauge symmetry

Higher dimensional gauge symmetry

Indeed, the (local) mass term A_5^2 can be forbidden by the gauge symmetry for 5th component of the gauge field

$$\because A_5 \to A_5 + \partial_5 \varepsilon(x,y) + i\left[\varepsilon(x,y), A_5\right]$$

In other words, no local counter term is allowed

$\Rightarrow$ No quadratic divergence, finite

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In other words, no local counter term is allowed

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This symmetry is very useful in the orbifold model since it is operative even on the branes $G \to H$

Gersdorff, Irges & Quiros (2002)

$$\because A_5 \to A_5 + \underbrace{\partial_5 \varepsilon_{G/H}(x,y_0)}_{Z_2 \text{ odd}} + i\left[\underbrace{\varepsilon_{G/H}(x,y_0)}_{Z_2 \text{ odd}=0}, A_5\right]$$

No quadratic divergence from brane localized Higgs mass

Explicit calculations of Higgs mass

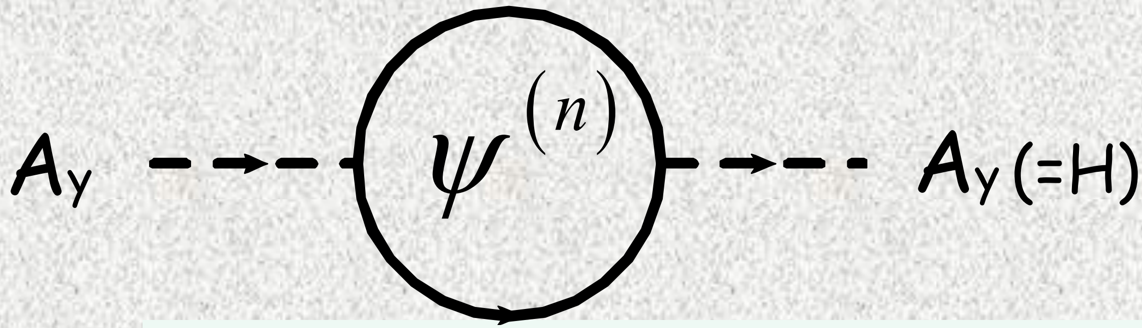
- D-dim QED on S^1@1-loop Hatanaka, Inami & Lim (1998)
- 5D Non-Abelian gauge theory on S^1/Z_2@1-loop
  Gersdorff, Irges & Quiros (2002)
- 6D Non-Abelian gauge theory on T^2@1-loop
  Antoniadis, Benakli & Quiros (2001)
- 6D Scalar QED on S^2@1-loop Lim, NM & Hasegawa (2006)
- 5D QED on S^1@2-loop
  NM & Yamashita (2006); Hosotani, NM, Takenaga & Yamashita (2007)
- 5D Gravity on S^1@1-loop (GGH) Hasegawa, Lim & NM (2004)

…

Higgs mass calculation

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



L=2πR

$$m_H^2 = i e_D^2 \int \frac{d^D k}{(2\pi)^D} \sum_{n=-\infty}^{\infty} Tr\left[\gamma_y \frac{1}{\not{k} - m} \gamma^y \frac{1}{\not{k} - m} \right]$$

$$\xrightarrow{L \to \infty} \frac{i}{D+1} e_{D+1}^2 \int \frac{d^{D+1} k}{(2\pi)^{D+1}} Tr\left[\gamma_M \frac{1}{\not{k} - m} \gamma^M \frac{1}{\not{k} - m} \right] (M = 0, 1, \cdots D)$$

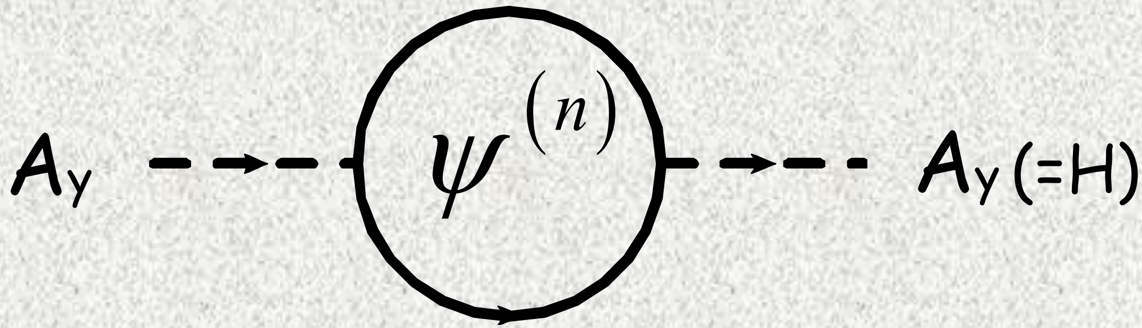
$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \int \frac{d^{D+1} k}{(2\pi)^{D+1}} \left[\frac{1-D}{k^2 - m^2} - \frac{2m^2}{(k^2 - m^2)^2} \right]$$

$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \left(1 - D + 2m^2 \frac{\partial}{\partial m^2} \right) \int \frac{d^{D+1} k}{(2\pi)^{D+1}} \frac{1}{k^2 - m^2}$$

$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \frac{-i}{(4\pi)^{(D+1)/2}} \Gamma\left(\frac{1-D}{2} \right) \left(1 - D + 2m^2 \frac{\partial}{\partial m^2} \right) (m^2)^{(D-1)/2} = 0$$

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



Boundary condition

$$\psi\left(x_\mu, y+L\right)=e^{i\alpha}\psi\left(x_\mu, y\right)$$

$$m_H^2 = ie_D^2\, 2^{[(D+1)/2]}\int\frac{d^Dk}{(2\pi)^D}\sum_{n=-\infty}^{\infty}\left[-\frac{1}{\left((2\pi n+\alpha)/L\right)^2+\rho^2}+\frac{2\rho^2}{\left[\left((2\pi n+\alpha)/L\right)^2+\rho^2\right]^2}\right]$$

$$= -ie_D^2\, 2^{[(D+1)/2]}\int\frac{d^Dk}{(2\pi)^D}\left(1+\rho\frac{\partial}{\partial\rho}\right)\left(\frac{L}{2\rho}\right)\frac{\sinh(\rho L)}{\cosh(\rho L)-\cos\alpha}$$

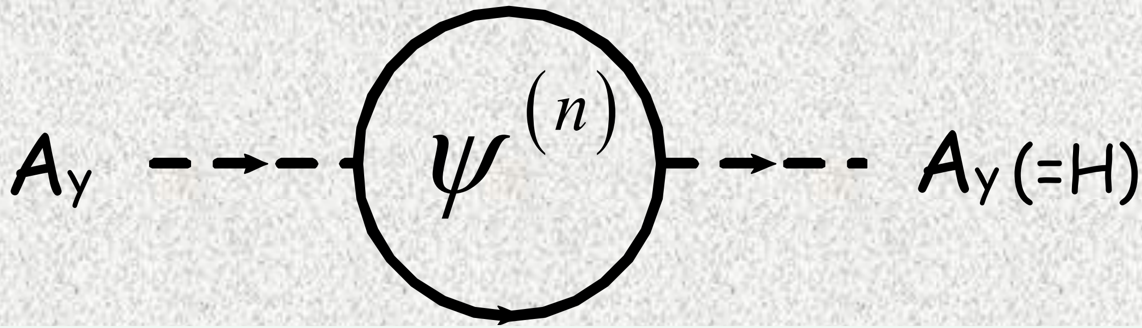
L=2πR

$$\rho^2 = -k^2+m^2$$

$$\sum_n\frac{1}{\left(\frac{2\pi n+\alpha}{L}\right)^2+\rho^2}=\left(\frac{L}{2\rho}\right)\left[\frac{\sinh(\rho L)}{\cosh(\rho L)-\cos\alpha}\right]$$

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



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$$= -i e_D^2 2^{[(D+1)/2]} \int \frac{d^D k}{(2\pi)^D} \left(1 + \rho \frac{\partial}{\partial \rho}\right) \left(\frac{L}{2\rho}\right) \frac{\sinh(\rho L)}{\cosh(\rho L) - \cos\alpha}$$

L=2πR

$$\rho^2 = -k^2 + m^2$$

$$= \frac{e_D^2 L^2}{2^{D-[(D+1)/2]} \pi^{D/2} \Gamma(D/2)} \int_0^\infty dk\ k_E^{D-1} \frac{1 - \cosh\left(\sqrt{k_E^2 + m^2}\, L\right)\cos\alpha}{\left[\cosh\left(\sqrt{k_E^2 + m^2}\, L\right) - \cos\alpha\right]^2} < \infty$$

Superconvergent!!

Ex. take D=4 (5 dimension case) & m=0,α=π

$$m_H^2 = \frac{e_D^2 L^2}{2^{D-[(D+1)/2]}\pi^{D/2}\Gamma(D/2)} \int_0^\infty dk\, k_E^{D-1} \frac{1-\cosh\left(\sqrt{k_E^2+m^2}\,L\right)\cos\alpha}{\left[\cosh\left(\sqrt{k_E^2+m^2}\,L\right)-\cos\alpha\right]^2}$$

$$= \frac{e_4^2}{4\pi^2}\frac{1}{(2\pi R)^2}\int_0^\infty ds s^3 \frac{1-\cosh s\cos\alpha}{\left[\cosh s-\cos\alpha\right]^2}\Bigg|_{\alpha=\pi}$$

$$= \frac{9e_4^2}{16\pi^4 R^2}\zeta(3) = \frac{9e_4^2}{16\pi^4}\underbrace{\zeta(3)}_{1.2} m_W^2$$

mw =π/R

Higgs mass is too small

→ generic prediction of GHU

Way out to get 125 GeV Higgs mass

1: Realizing small Higgs VEV $a \ll 1$

by choosing appropriate matter content

$$m_H \sim m_W/(4\pi a) \quad (m_W = a/R)$$

Haba, Hosotani, Kawamura & Yamashita (2004)
Adachi, NM (2018)

2: D > 5 dimensions

F_{ij}^2 contains the Higgs quartic coupling $g^2[A_i, A_j]^2$
Higgs mass is generated at leading order
$m_H = 2m_W$ is predicted in 6D on T^2/Z_3 model

Scrucca, Serone, Silvestrini & Wulzer (2003)

3: Warped dimension (ex. Randall-Sundrum model)

Higgs mass is enhanced by curvature scale $k\pi R \sim 30$

Contino, Nomura & Pomarol (2003)

Is this finite mass consistent with gauge symmetry??

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  4D gauge symmetry via compactification
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Is this finite mass consistent with gauge symmetry??

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  ⇒ No reason to exclude non-local mass term

- 5D gauge symmetry is broken to
  4D gauge symmetry via compactification
  ⇒ No reason to forbid mass of A_5

In a theory compactified on non-simply connected space, non-local Wilson-loop for a zero mode of A_5 is physical

$$W = \exp\left[ig \oint dx_5 A_5 \right]$$

Wilson-loop $W = \exp\left[ig\oint dx_5 A_5 \right]$ is

gauge invariant under gauge transformation

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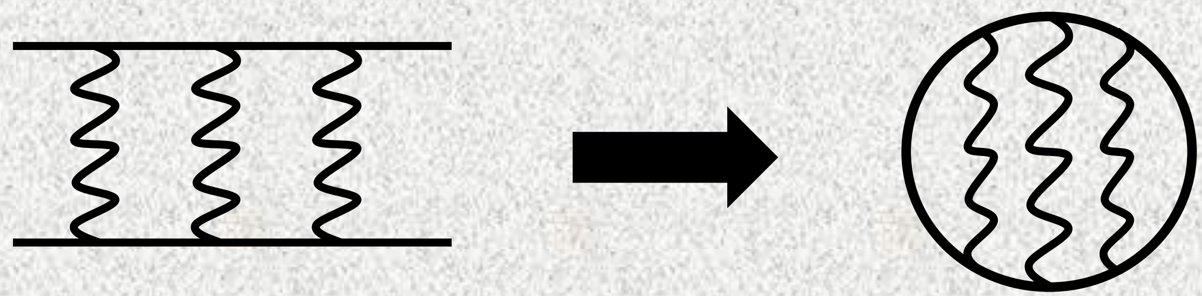
In GHU, zero mode of A_5 as SM Higgs is Wilson-line (WL) or AB phase

Higgs potential (Higgs mass) is understood as a function of non-local WL phase

$$V\left(A_5^{(0)}\right) = \frac{3}{16\pi^6 R^4} \sum_{n=1}^{\infty} \frac{\cos\left(2\pi R g A_5^{(0)} n\right)}{n^5} = \frac{3}{32\pi^6 R^4} \sum_{n=1}^{\infty} \frac{W^n + W^{\dagger n}}{n^5}$$

It was believed for the Higgs mass & potential
to be finite at any order of perturbation,
but it was shown that this is not true,
i,e, Higgs potential becomes divergent
at 4-loop through 4-Fermi interactions

Hisano, Shoji and Yamada, JHEP02 193 (2020)
Yamada, PTEP vol.9 (2021) 093B01



(log) Divergence from 4-Fermi interactions
cannot be controlled by gauge symmetry

All order finiteness might be true only for YM theory

Gauge-Higgs sector

Model building of gauge-Higgs unification

A_5 is an SU(2) adjoint originally, not SU(2) doublet

$\Rightarrow$ need to enlarge the gauge group

$G \rightarrow SU(2)_L \times U(1)_Y$

adj $\rightarrow$ doublet + other reps

$\Rightarrow$

Simplest G

SU(3)

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Simplest G
SU(3)

Consider 5D SU(3) model on S^1/Z_2 with Parity:
P = diag (-,-,+)

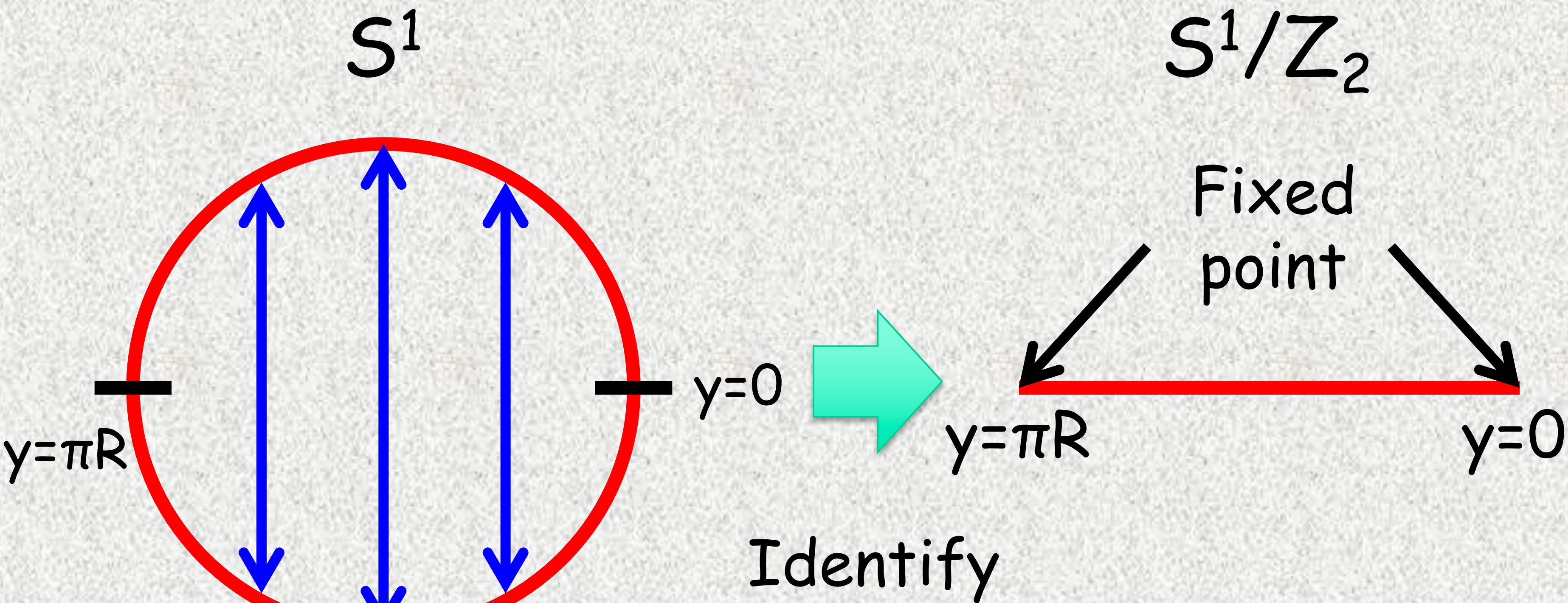
Boundary conditions

$$A_\mu(x, y+2\pi R) = A_\mu(x,y),\ A_5(x, y+2\pi R) = A_5(x,y)$$

$$PA_\mu(x,-y)P^\dagger = A_\mu(x,+y),\ PA_5(x,-y)P^\dagger = -A_5(x,+y)$$

$$PA_\mu(x,\pi R-y)P^\dagger = A_\mu(x,\pi R+y),\ PA_5(x,\pi R-y)P^\dagger = -A_5(x,\pi R+y)$$

Orbifold S^1/Z_2



$Z_2: y \to -y$

- To obtain chiral fermions
- Symmetry Breaking

Boundary conditions

$$A_\mu(x, y+2\pi R) = A_\mu(x,y),\ A_5(x, y+2\pi R) = A_5(x,y)$$

$$PA_\mu(x,-y)P^\dagger = A_\mu(x,+y),\ PA_5(x,-y)P^\dagger = -A_5(x,+y)$$

$$PA_\mu(x,\pi R - y)P^\dagger = A_\mu(x,\pi R + y),\ PA_5(x,\pi R - y)P^\dagger = -A_5(x,\pi R + y)$$

Relative parity between A_μ and A_5 should be opposite

$$F_{\mu 5} = \partial_\mu A_5 - \partial_5 A_\mu - ig\left[A_\mu, A_5\right]$$

Parities of the 1st term and the 2nd term are necessarily opposite

Boundary conditions

$$A_\mu(x,y+2\pi R)=A_\mu(x,y),\ A_5(x,y+2\pi R)=A_5(x,y)$$

$$PA_\mu(x,-y)P^\dagger=A_\mu(x,+y),\ PA_5(x,-y)P^\dagger=-A_5(x,+y)$$

$$PA_\mu(x,\pi R-y)P^\dagger=A_\mu(x,\pi R+y),\ PA_5(x,\pi R-y)P^\dagger=-A_5(x,\pi R+y)$$

$$\Rightarrow\quad A_\mu=\begin{pmatrix}(+,+) & (+,+) & (-,-)\\ (+,+) & (+,+) & (-,-)\\ (-,-) & (-,-) & (+,+)\end{pmatrix},\ A_5=\begin{pmatrix}(-,-) & (-,-) & (+,+)\\ (-,-) & (-,-) & (+,+)\\ (+,+) & (+,+) & (-,-)\end{pmatrix}$$

Mode expansions

$$A_M^{(+,+)}(x,y)=\frac{1}{\sqrt{2\pi R}}\left[A_M^{(0)}(x)+\sqrt{2}\sum_{n=1}^{\infty}A_M^{(n)}(x)\cos\left(\frac{n}{R}y\right)\right]$$

$$A_M^{(-,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}A_M^{(n)}(x)\sin\left(\frac{n}{R}y\right)$$

Only (+,+) mode has massless mode ("0 mode")

$$A_\mu^{(0)} = \frac{1}{2}\begin{pmatrix} W_\mu^3 + B_\mu^3/\sqrt{3} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + B_\mu^3/\sqrt{3} & 0 \\ 0 & 0 & -2B_\mu/\sqrt{3} \end{pmatrix}$$

$$A_5^{(0)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}$$

SU(2) x U(1) gauge fields

SU(3)→ SU(2)xU(1)

Higgs doublets

Gauge boson spectrum

$$M_{W_n} = \frac{n+a}{R}, M_{Z_n} = \frac{n+2a}{R}, M_{\gamma_n} = \frac{n}{R}, \left\langle A_5^{(0)} \right\rangle = \frac{a}{g_5 R}$$

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- W, Z, γ are identified with zero modes:

  $M_W = a/R, \ M_Z = 2a/R, \ M_\gamma = 0$

  SU(2)xU(1) → U(1) realized if a is nonzero

- $M_Z = 2M_W \rightarrow \cos\theta_W = \frac{1}{2}$ (θ_W:weak mixing angle)

  ($\sin^2\theta_W = \frac{3}{4} \gg 0.23$ (exp))

In SM, $\sin^2\theta_W = g_Y^2/(g_Y^2 + g_2^2)$ is NOT be predictable since g_Y & g_2 are independent

In GHU, predictable since g_Y & g_2 are related

$$M_{W_n} = \frac{n+a}{R}, M_{Z_n} = \frac{n+2a}{R}, M_{\gamma_n} = \frac{n}{R}, \left\langle A_5^{(0)} \right\rangle = \frac{a}{g_5 R}$$

- W, Z, γ are identified with zero modes:

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  ($\sin^2\theta_W = \frac{3}{4} \gg 0.23$ (exp))

- Non-zero KK modes of A_5 are eaten by non-zero KK modes of A_μ

  ("Higgs mechanism")

Wrong prediction of θ_W

Check the hypercharge of Higgs doublet

$$\delta_{U(1)} A_5^{(0)} = g\left[T^8, A_5^{(0)}\right] = \frac{g}{2\sqrt{3}}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}\right]$$

$$= \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -2H^- & -2H^{0*} & 0 \end{pmatrix} - \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & -2H^+ \\ 0 & 0 & -2H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} = \frac{g\sqrt{3}}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -H^- & -H^{0*} & 0 \end{pmatrix}$$

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$$= \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -2H^- & -2H^{0*} & 0 \end{pmatrix} - \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & -2H^+ \\ 0 & 0 & -2H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} = \frac{g\sqrt{3}}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -H^- & -H^{0*} & 0 \end{pmatrix}$$

$$\Rightarrow \quad \sin^2\theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{\left(\sqrt{3}g\right)^2}{g^2 + \left(\sqrt{3}g\right)^2} = \frac{3}{4} \neq 0.23(\text{Exp})$$

Too Big!!

Wrong prediction of θ_W

Check the hypercharge of Higgs doublet

$$\delta_{U(1)} A_5^{(0)} = g\left[T^8, A_5^{(0)}\right] = \frac{g}{2\sqrt{3}}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}\right]$$

$$= \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -2H^- & -2H^{0*} & 0 \end{pmatrix} - \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & -2H^+ \\ 0 & 0 & -2H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} = \frac{g\sqrt{3}}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -H^- & -H^{0*} & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{\left(\sqrt{3}g\right)^2}{g^2 + \left(\sqrt{3}g\right)^2} = \frac{3}{4} \neq 0.23(\text{Exp})$$

Too Big!!

Well-known by Fairlie, Manton (6D on S^2 w/ monopole bkgd)

	G_2	$SO(5)$	$SU(3)$
$\sin^2\theta_W$	1/4	1/2	3/4

Way out to get a correct θ_W

1: Additional U(1) $SU(3) \times U(1)' \rightarrow SU(2)_L \times U(1)_Y \times U(1)_X$

Scrucca, Serone & Silvestrini (2003)

$$A_Y = \frac{g' A_8 + \sqrt{3} g A'}{\sqrt{3g^2 + g'^2}}, A_X = \frac{\sqrt{3} g A_8 - g' A'}{\sqrt{3g^2 + g'^2}} \Rightarrow g_Y = \frac{\sqrt{3} g g'}{\sqrt{3g^2 + g'^2}}$$

$$\because A_8 = \frac{g' A_Y + \sqrt{3} g A_X}{\sqrt{3g^2 + g'^2}} \Rightarrow g A_8 \supset \frac{g'}{\sqrt{3g^2 + g'^2}} g A_Y$$

Gell-Mann matrices

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \qquad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

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$$\Rightarrow \quad \sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{3}{4 + 3 g^2 / g'^2}$$

adjustable of θ_W by tuning g'

Way out to get a correct Θ_W

2: Localized gauge kinetic terms

$$\mathcal{L} = -\frac{1}{2g_5^2} TrF_{MN}F^{MN} - \underbrace{\left[\frac{1}{2g_4^2}\delta(y) + \frac{1}{2g_4'^2}\delta(y-\pi R)\right] TrF_{\mu\nu}F^{\mu\nu}}_{\text{SU(2) x U(1) invariant}}$$

SU(3) invariant

SU(2) x U(1) invariant

4D effective Gauge coupling

$$\frac{1}{g_{eff}^2} = \frac{1}{g_5^2}\int_0^{\pi R} dy \left(f_{A_\mu}^{(0)}(y)\right)^2 + \frac{1}{g_4^2} + \frac{1}{g_4'^2}$$

By tuning g_4, g'_4, $\sin\Theta_W$ is adjustable

Matter Content
&
Yukawa Coupling

Quark & Lepton embedding

Consider a fundamental rep of SU(3)

$$\mathbf{3} = (q, q-1, 1-2q)^T \ (q\text{: electric charge})$$

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$$\mathbf{3} = \mathbf{2}_{1/6} + \mathbf{1}_{-1/3} = (2/3, -1/3, -1/3)^T = (u_L, d_L, d_R)^T$$

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This can be obtained by Z_2 parity as

$$\psi(-y) = P\gamma_5\psi(y), \quad P = diag(-,-,+)$$

Relative parity between LH and RH should be opposite

$$\mathcal{L}_{fermion} \supset \bar{\psi}_R \partial_5 \psi_L$$

$\Rightarrow \gamma_5$ insertion

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Only fundamental reps cannot incorporate right-handed up-type quarks as well as leptons

Quark & Lepton embedding

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As one of the embeddings, tensor product is useful

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$$\psi(-y) = (P \otimes P)\gamma_5 \psi(y), \;\; \psi(-y) = (P \otimes P \otimes P)\gamma_5 \psi(y)$$

2-rank sym: $\mathbf{6}^* = \begin{cases} \mathbf{3}_{L-1/3} + \mathbf{2}_{L1/6}\,(Q) + \mathbf{1}_{L2/3} \\ \mathbf{3}_{R-1/3} + \mathbf{2}_{R1/6} + \mathbf{1}_{R2/3}\,(u_R) \end{cases}$

$\because\; \mathbf{3}^* \times \mathbf{3}^* = (\mathbf{2}_{-1/6} + \mathbf{1}_{1/3}) \times (\mathbf{2}_{-1/6} + \mathbf{1}_{1/3})$

Many massless exotics $\Rightarrow$ brane localized mass term

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$\because\ 3^* \times 3^* = (2_{-1/6} + 1_{1/3}) \times (2_{-1/6} + 1_{1/3})$

P x P = diag(-,-,+) x diag(-,-,+)

Many massless exotics ⇒ brane localized mass term

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RH neutrino can be embedded into SU(3) singlet

Fermion mass

In SM, quarks & leptons obtain masses through Yukawa coupling

$$y\bar{\psi}_R H \psi_L \overset{H=\langle H\rangle}{\Rightarrow} m_{q,l}\bar{\psi}_R\psi_L$$

In SM, fermion mass (Yukawa) hierarchy cannot be explained

Big Hurdle

In gauge-Higgs unification,

Yukawa coupling = gauge coupling

How can we get fermion mass hierarchy???

As will be shown below,
fermion masses except for top quark are relatively easy

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As will be shown below,
fermion masses except for top quark are relatively easy

1: Localizing fermions@different point in 5th direction

Yukawa ~ exponentially suppressed overlap integral

Arkani-Hamed & Schmaltz (1999)

2: Bulk fermions mixed with localized fermions @the fixed points

Non-local Yukawa coupling

Csaki, Grojean & Murayama (2002)

1: Yukawa coupling from localizing fermions @different points

1: To localize fermions at different points along the 5^{th} direction, bulk masses are introduced

2: To be consistent with Z_2 orbifold, Z_2 parity of bulk mass must be odd ⇒ kink mass

1: Yukawa coupling from localizing fermions @different points

1: To localize fermions at different points along the 5th direction, bulk masses are introduced

2: To be consistent with Z_2 orbifold, Z_2 parity of bulk mass must be odd ⇒ kink mass

Consider a 5D fermion satisfying the following Dirac equation

$$0 = \left[i\Gamma^M D_M - M\varepsilon(y) \right] \psi(x,y)$$

$$D_M = \partial_M - igA_M, \Gamma^M = (\gamma^\mu, i\gamma^5), (M = 0,1,2,3,5), \varepsilon(y) = \begin{cases} 1 \ (y > 0) \\ -1 \ (y < 0) \end{cases}$$

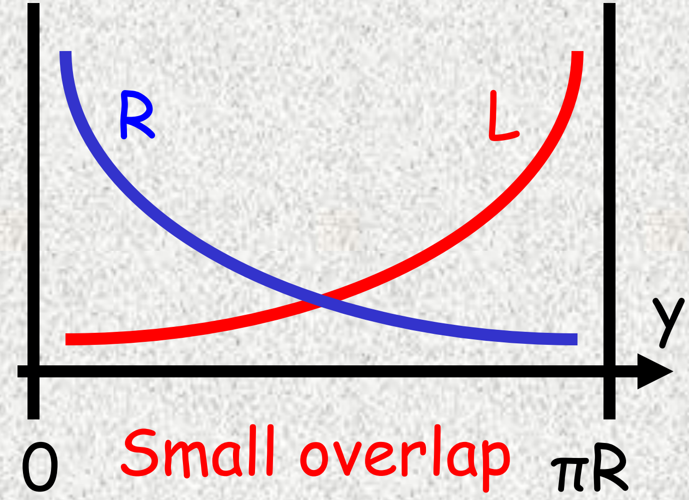
Focusing zero modes

$$\psi(x,y) \sim \psi^{(0)}_{L(R)}(x) f^{(0)}_{L(R)}(y), \gamma^5 \psi_{L(R)} = (-)\psi_{L(R)}$$

Zero mode wave functions

$$0=\left[\partial_y - M\varepsilon(y)\right] f_L^{(0)}(y) \rightarrow f_L^{(0)}(y)=\sqrt{\frac{2M}{e^{2\pi MR}-1}}\,e^{M|y|}$$

$$0=\left[\partial_y + M\varepsilon(y)\right] f_R^{(0)}(y) \rightarrow f_R^{(0)}(y)=\sqrt{\frac{2M}{1-e^{-2\pi MR}}}\,e^{-M|y|}$$



4D effective Yukawa coupling

$$Y=g_4\int_0^{\pi R} dy f_L^{(0)}(y) f_R^{(0)}(y) = g_4 \int_0^{\pi R} dy \sqrt{\frac{4M^2}{\left(1-e^{-2\pi MR}\right)\left(e^{2\pi MR}-1\right)}}$$

$$\approx 2\pi MR g_4 e^{-\pi MR} \le g_4 \Leftrightarrow m_f \le m_W$$

$\pi MR \gg 1$

Fermion masses except top is easy, but top is hard

No need of unnatural fine-tuning for 5D parameters M,R

Top mass generation

Cacciapaglia, Csaki & Park (2005)

Consider large dimensional reps,
then an upper bound on fermion mass is modified as follows

$$m_t \leq \sqrt{n}\, m_W$$

(n: # of indices of rep)

For $m_t = 2m_W \Rightarrow$ need a 4-index rep top is embedded
To saturate this bound, bulk mass should be zero

Simplest example: 

$(\mathbf{15^*})_{-2/3} \longrightarrow (\mathbf{1}, 2/3)(t_R) + (\mathbf{2}, 1/6)(t_L)$
$+ (\mathbf{3}, -1/3) + (\mathbf{4}, -5/6) + (\mathbf{5}, -4/3)$

$\sqrt{N}$ enhancement

Consider a rank N symmetric tensor of SU(3)

$\boxed{}\boxed{}\boxed{}\boxed{\;\cdots\;}\boxed{}$ N boxes

Decompose it into SU(2) reps as $3 = 2 + 1$
and make a singlet & a doublet

singlet $\boxed{1}\boxed{1}\boxed{1}\boxed{\;\cdots\;}\boxed{1}$ unique

doublet $\boxed{1}\boxed{1}\boxed{2}\boxed{\;\cdots\;}\boxed{1}$ etc N patterns

Canonical kinetic term $\Rightarrow 1/\sqrt{N}$

$$\text{Yukawa} = 1_R\, 2_L\, 2_H \Rightarrow N \times 1/\sqrt{N} = \sqrt{N}$$

Fermion matter content

$3 = 2_{L1/6}(Q) + 1_{L-1/3}$
$\quad 2_{R1/6} + 1_{R-1/3}(d_R)$

Down quark sector

$6^* = 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3}$
$\quad 3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(u_R)$

Up quark sector (except for top)

$10 = 4_{L1/2} + 3_{L0} + 2_{L-1/2}(L) + 1_{L-1}$
$\quad 4_{R1/2} + 3_{R0} + 2_{R-1/2} + 1_{R-1}(e_R)$

Charged lepton sector

$15^* = 5_{L-4/3} + 4_{L-5/6} + 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3}$
$\quad 5_{R-4/3} + 4_{R-5/6} + 3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(t_R)$

Top quark

Unwanted massless exotics (blue reps) & two extra Qs must be massive by brane localized mass terms

2: Mixing between bulk and boundary localized fermions

Csaki, Grojean & Murayama (2002)

Consider the massive bulk fermion
coupling to SM fermions on the branes

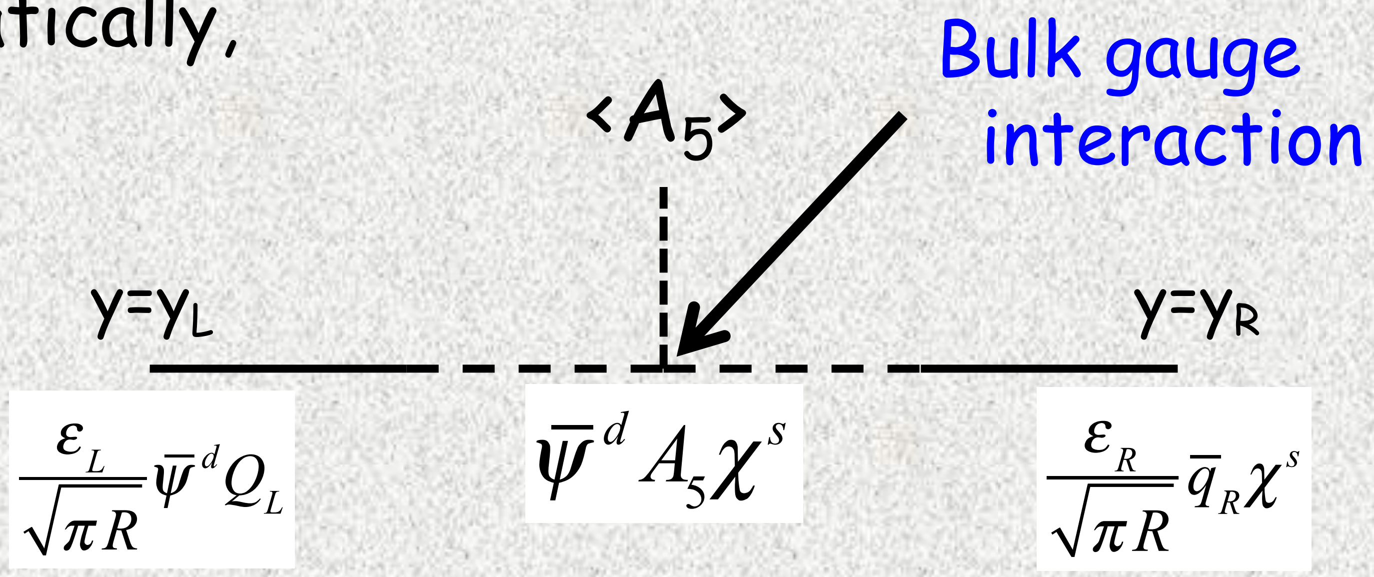
$$\mathcal{L}_{Bulk} = \bar{\Psi} i \not{D} \Psi + \bar{\tilde{\Psi}} i \not{D} \tilde{\Psi} - M\left(\bar{\Psi}\tilde{\Psi} + \bar{\tilde{\Psi}}\Psi\right)$$

$\Psi \supset \psi^d, \chi^s$ $\tilde{\Psi}$ opposite parity to Ψ

$$\mathcal{L}_{Brane} = \delta(y - y_L)\left[i\bar{Q}_L \bar{\sigma}^\mu \partial_\mu Q_L + \frac{\varepsilon_L}{\sqrt{\pi R}} \bar{\psi}^d Q_L + h.c. \right] + \delta(y - y_R)\left[i\bar{q}_L \bar{\sigma}^\mu \partial_\mu q_L + \frac{\varepsilon_R}{\sqrt{\pi R}} \bar{q}_R \chi^s + h.c. \right]$$

Mixing mass term between bulk & brane fermions

Schematically,



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Mixing mass term between bulk & brane fermions

Integrating out massive fermion generates mass term as

$$\varepsilon_L \varepsilon_R \pi M R e^{-\pi M R} \bar{q}_R e^{ig\int_0^{\pi R} dy A_y} Q_L \Rightarrow m_f \propto \varepsilon_L \varepsilon_R \pi M R e^{-\pi M R} M_W$$

Exponentially suppressed coupling

⇒ easy to generate fermion masses except for top

How do we obtain top mass???

2: Mixing between bulk and boundary localized fermions

Csaki, Grojean & Murayama (2002)

Consider the massive bulk fermion
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$$\varepsilon_L \varepsilon_R \pi M R e^{-\pi M R} \bar{q}_R e^{ig\int_0^{\pi R} dy A_y} Q_L \Rightarrow m_f \propto \varepsilon_L \varepsilon_R \pi M R e^{-\pi M R} M_W$$

How do we obtain top mass???

By mixing effects ε

Flavor Mixing $ CPV

Flavor mixing in SM

Yukawa coupling can be diagonalized
by bi-unitary transformations

$$\mathcal{L}_{yukawa} = -y_d^{ij}\bar{Q}_L^i H d_R^j - y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j + h.c.$$

$$\rightarrow -\bar{Q}_L^l U_d^{il\dagger} y_d^{ij} H V_d^{jk} d_R^k - y_u^{ij}\bar{Q}_L^l U_u^{il\dagger} y_u^{ij} \tilde{H} V_u^{jk} u_R^k + h.c.$$

$$= -\bar{Q}_L^l \tilde{y}_d^{ii} H d_R^k - y_u^{ij}\bar{Q}_L^l \tilde{y}_u^{ii} \tilde{H} u_R^k + h.c$$

In mass eigenstates, flavor mixings appear
in charged current

$$\mathcal{L}_W \sim \bar{u}_L^i W_\mu^+ \gamma^\mu \underbrace{\left(U_u^\dagger U_d\right)^{ij}}_{V_{CKM}} d_L^j + \bar{d}_L^i W_\mu^- \gamma^\mu \underbrace{\left(U_d^\dagger U_u\right)^{ij}}_{V_{CKM}^\dagger} u_L^j$$

Flavor mixing in GHU

In GHU, yukawa coupling is gauge coupling,
which seems to be no flavor mixing

If the bulk mass are flavor non-diagonal,
flavor mixing seems to be generated
but it is NOT true

$$M_{ij}\bar{\psi}^i\psi^j$$

can be diagonalized
by a suitable unitary transformation,
leaving the kinetic term invariant

We are led to introduce

brane localized mass terms,

which are necessary to make exotics heavy

& are the sources of flavor mixing

as will be seen below

$$\mathcal{L} = -\frac{1}{4}F^{MN}F_{MN} + \bar{\psi}_3^i\left(i\not{D} - M^i\varepsilon(y)\right)\psi_3^i + \bar{\psi}_{\bar{6}}^i\left(i\not{D} - M^i\varepsilon(y)\right)\psi_{\bar{6}}^i$$

$$+\delta(y)\sqrt{2\pi R}\bar{Q}_R^i(x)\left[\eta_{ij}Q_{3L}^j(x,y) + \lambda_{ij}Q_{\bar{6}L}^j(x,y)\right] + \cdots$$

Brane localized fields

Brane mass matrices (off-diagonal elements are generically allowed)

⇒ "Flavor mixing"

$$\mathcal{L}_{\mathrm{BM}}^{\mathrm{Q}} \sim \delta(y)\bar{Q}_R\begin{bmatrix}\eta & \lambda\end{bmatrix}\begin{bmatrix}Q_3\\ Q_6\end{bmatrix}_L = \delta(y)\bar{Q}'_R\begin{bmatrix}m_{diag} & 0\end{bmatrix}\begin{bmatrix}Q_H\\ Q_{SM}\end{bmatrix}_L$$

$$\begin{bmatrix}Q_3\\ Q_6\end{bmatrix}_L = \begin{bmatrix}U_1 & U_3\\ U_2 & U_4\end{bmatrix}\begin{bmatrix}Q_H\\ Q_{SM}\end{bmatrix}_L, \quad U^{\bar{Q}}Q_R = Q'_R$$

"2Nx2N unitary matrix"

Yukawa coupling

$$\begin{aligned}\mathcal{L}_{Yukawa} &= g_5 A_y^6 \bar{d}^i Q_3^i + g_5 A_y^6 \bar{u}^i Q_6^i \\ &\supset g_5 A_y^6 \bar{d}^i U_3^{ij} Q_{SM}^j + g_5 A_y^6 \bar{u}^i U_4^{ij} Q_{SM}^j \\ &\to g_5 \left\langle A_y^6 \right\rangle \left(\bar{d}_R^{i(0)} Y_d^{ii} U_3^{ij} Q_{SM}^{j(0)} + \bar{u}_R^{i(0)} Y_u^{ii} U_4^{ij} Q_{SM}^{j(0)} \right)\end{aligned}$$

Yukawa coupling with flavor mixing

$$Y^{ii} = \int_{-\pi R}^{\pi R} dy f_L^{i(0)} f_R^{i(0)} \approx 2\pi R M^i e^{-\pi R M^i}$$

Diagonalization

$$\begin{cases} \hat{Y}_d = V_{dR}^\dagger Y_d U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^\dagger Y_u U_4 V_{uL} \end{cases} \qquad V_{CKM} = V_{uL}^\dagger V_{dL} \quad \left(U_3^\dagger U_3 + U_4^\dagger U_4 = 1_{N\times N} \right)$$

$M_{3,6} \propto 1$ ($Y_{u,d} \propto 1$) case (flavor symmetry restored)

$$\begin{cases} \hat{Y}_d = V_{dR}^\dagger U_3 V_{dL} \rightarrow \hat{Y}_d^2 = V_{dL}^\dagger U_3^\dagger U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^\dagger U_4 V_{uL} \rightarrow \hat{Y}_u^2 = V_{uL}^\dagger U_4^\dagger U_4 V_{uL} \end{cases} \xrightarrow{U_3^\dagger U_3 + U_4^\dagger U_4 = 1} V_{uL} \propto V_{dL}$$

$$\Rightarrow V_{CKM} = V_{uL}^\dagger V_{dL} \propto V_{dL}^\dagger V_{dL} = 1 \text{(No mixing)}$$

Lesson

To get flavor mixing,
we need non-degenerate bulk masses
as well as the off-diagonal brane masses
(specific to gauge-Higgs unification)

FCNC in SM

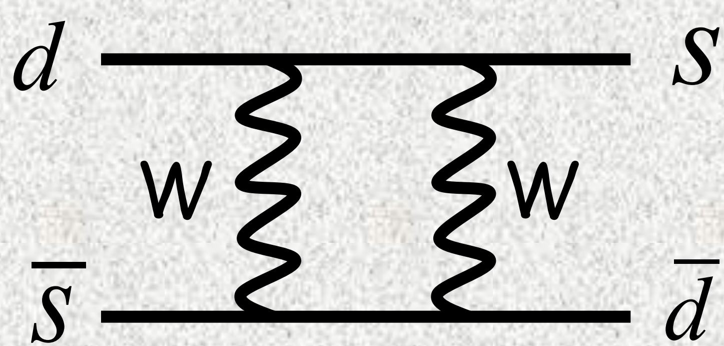
Flavor Changing Neutral Current processes are severely constrained

$K^0-\bar{K}^0$ mixing

$$\frac{c}{\Lambda^2}\bar{s}d\bar{d}s \Rightarrow \Lambda \geq 100TeV, c \approx \mathcal{O}(1)(\exp)$$

- No tree level process by Z boson exchange
- 1-loop process by charged current suppressed

$\Rightarrow$ GIM mechanism



$$\approx \frac{g^2}{16\pi^2}\frac{\left(m_c^2-m_u^2\right)^2}{M_W^2 m_c^2}\left(\sin\theta_c\cos\theta_c\right)^2\frac{g^2}{M_W^2} \approx 10^{-8\sim-7}\frac{g^2}{M_W^2}$$

FCNC in GHU

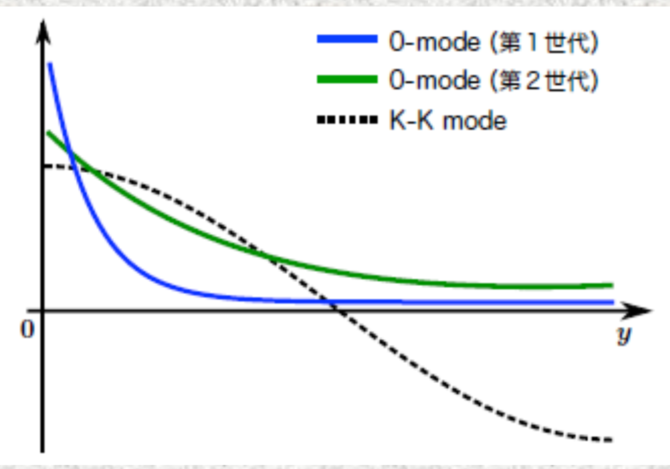
FCNC@tree level even in QCD sector

$$\mathcal{L}_{strong} \supset \frac{g_s}{\sqrt{2\pi R}} G_\mu^{(0)} \left(\bar{\psi}_R^{i(0)} \gamma^\mu \psi_R^{i(0)} + \bar{\psi}_L^{i(0)} \gamma^\mu \psi_L^{i(0)} \right)$$
$$+ g_s G_\mu^{(n)} \bar{\psi}_R^{i(0)} \gamma^\mu \psi_R^{j(0)} \left(V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{ij}$$
$$+ g_s G_\mu^{(n)} \bar{\psi}_L^{i(0)} \gamma^\mu \psi_L^{j(0)} \left[V_{dL}^\dagger \left(U_3^\dagger I_{LL}^{(0n0)} U_3 + U_4^\dagger I_{LL}^{(0n0)} U_4 \right) V_{dL} \right]_{ij}$$

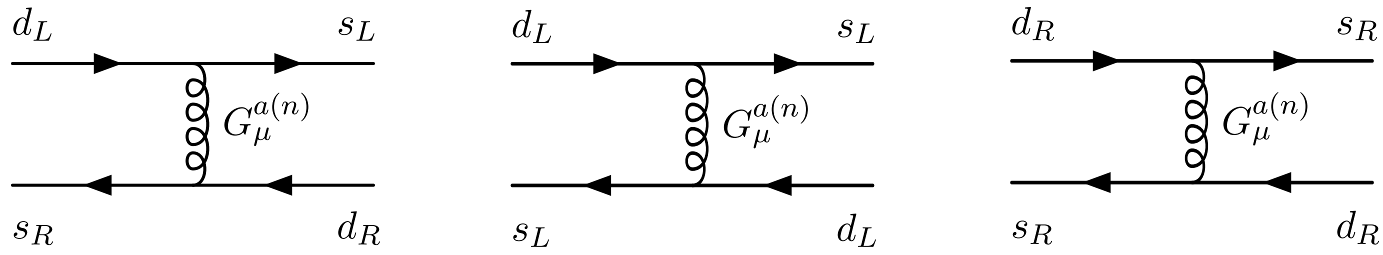
0 mode sector (flat mode function):
No mixing O.K.

Nonzero KK gluon couplings induce nontrivial flavor mixing

$\Rightarrow$ flavor mixing@tree level

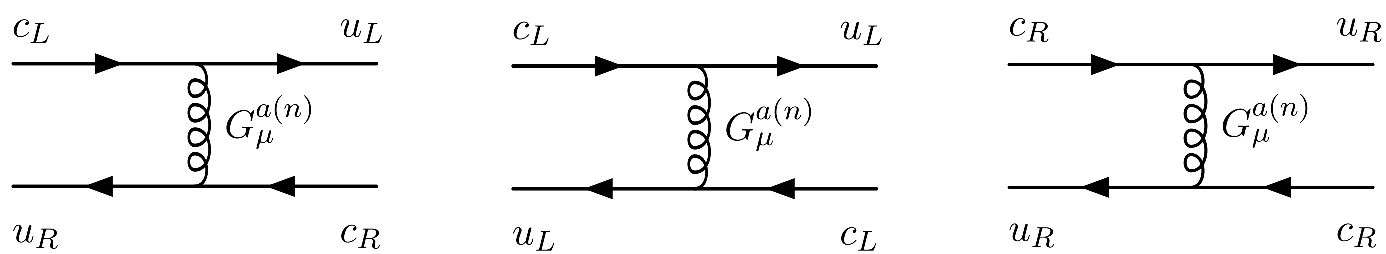


$K^0 - \bar{K}^0$



(i) LR type (ii) LL type (iii) RR type

$D^0 - \bar{D}^0$

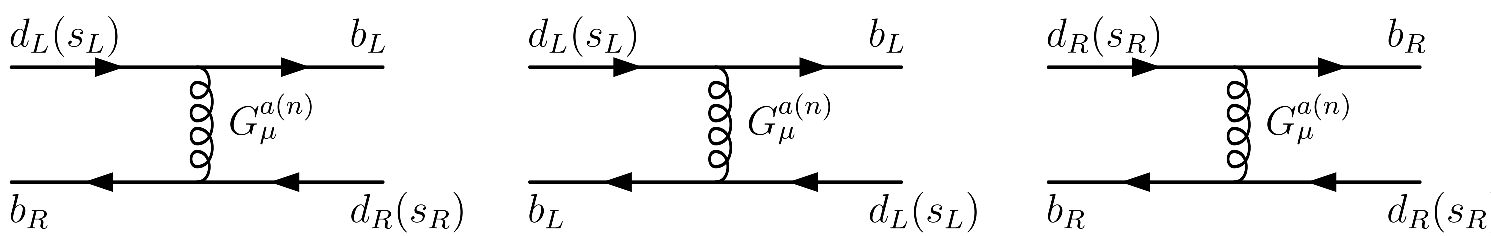


(i) LR type (ii) LL type (iii) RR type

$B_d^0 - \bar{B}_d^0$

&

$B_s^0 - \bar{B}_s^0$



(i) LR type (ii) LL type (iii) RR type

Lower bounds of compactification scale in GHU

$$K^0 - \bar{K}^0 : \mathcal{O}(10)\,TeV$$

$$D^0 - \bar{D}^0 : \mathcal{O}(1)\,TeV$$

$$B_d^0 - \bar{B}_d^0, B_s^0 - \bar{B}_s^0 : \mathcal{O}(1)\,TeV$$

"K^0-K^0bar", Adachi, Kurahashi, Lim and NM, JHEP1011 (2010) 015
"D^0-D^0bar", Adachi, Kurahashi, Lim and NM, JHEP1201 (2012) 047
"B^0-B^0bar", Adachi, Kurahashi, NM and Tanabe, PRD85 (2012) 096001

"GIM-like" mechanism

The above results are smaller than that from naïve order estimate

$$\frac{1}{M_{KK}^2}\bar{\psi}\psi\bar{\psi}\psi \Rightarrow \begin{cases} M_{KK} \geq 1000\,TeV\left(K^0 - \bar{K}^0, D^0 - \bar{D}^0\right) \\ M_{KK} \geq 400\,TeV\left(B_d^0 - \bar{B}_d^0\right) \\ M_{KK} \geq 70\,TeV\left(B_s^0 - \bar{B}_s^0\right) \end{cases}$$

This apparent discrepancy can be understood since the "GIM-like" mechanism works in GHU

i.e. FCNC processes are automatically suppressed for 1st & 2nd generation of quarks

In the large bulk mass limit, the KK mode sum can be approximated as follows

$$S_{KK}^{LR} = \pi R \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2$$

$$\simeq -\frac{\pi^2}{2} \left(e^{-2\pi RM^1} + e^{-2\pi RM^2} \right)$$

$$-\frac{\pi}{2R} \frac{\left(M^1\right)^2 - M^1 M^2 + \left(M^2\right)^2}{M^1 M^2 \left(M^1 - M^2\right)} \left(e^{-2\pi RM^1} - e^{-2\pi RM^2} \right) \left(\pi RM^i \gg 1 \right)$$

exponential suppression!!

$$\boxed{e^{-2\pi RM^i} \Leftrightarrow \frac{m_{q^i}^2}{m_W^2}}$$

similar to GIM suppression

$$\boxed{\frac{m_c^2 - m_u^2}{m_W^2}}$$

$$S_{KK}^{LL(RR)} = \pi R \sum_{n=1}^{\infty} \frac{1}{n^2} \left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2 \simeq \frac{\pi}{8R} \frac{\left(M^1 - M^2\right)^2}{M^1 M^2 \left(M^1 + M^2\right)}$$

Power suppression

CP violation

In SM, CP symmetry is broken by the CKM phase
$\Rightarrow$ Nobel prize of Kobayashi & Maskawa

One of the approaches to explain
baryon asymmetry is EW baryogenesis

CP violation is required to work this mechanism
(one of the Sakharov's conditions)

CP violation from CKM phase is NOT enough and
additional CP violation is necessary

Now, higher dimensional origin
of CP violation is discussed

CP violation

Adachi, Lim & NM, PRD(2009)

Parity

$$\begin{cases} \mathcal{P}:\left(x^{\mu},y\right)=\left(x_{\mu},-y\right) \\ \mathcal{P}:\psi\left(x^{\mu},y\right)=\gamma^{0}\psi\left(x_{\mu},-y\right) \\ \mathcal{P}:\left(A^{\mu},A^{y}\right)\left(x^{\mu},y\right)=\left(A_{\mu},-A^{y}\right)\left(x_{\mu},-y\right) \end{cases}$$

CP violation

Adachi, Lim & NM, PRD(2009)

Parity

$$\begin{cases} \mathcal{P}:\left(x^\mu, y\right)=\left(x_\mu,-y\right) \\ \mathcal{P}:\psi\left(x^\mu, y\right)=\gamma^0\psi\left(x_\mu,-y\right) \\ \mathcal{P}:\left(A^\mu, A^y\right)\left(x^\mu, y\right)=\left(A_\mu,-A^y\right)\left(x_\mu,-y\right) \end{cases}$$

Charge Conjugation

$$\begin{cases} \mathcal{C}:\left(x^\mu, y\right)=\left(x^\mu,-y\right) \\ \mathcal{C}:\psi\left(x^\mu, y\right)=i\gamma^2\psi^*\left(x^\mu,-y\right) \\ \mathcal{C}:\left(A^\mu, A^y\right)\left(x^\mu, y\right)=\left(-A^\mu, A^y\right)^T\left(x^\mu,-y\right) \end{cases}$$

Origin of $-y$ from $\mathcal{C}^\dagger\gamma^\mu\mathcal{C}=-\left(\gamma^\mu\right)^T, \mathcal{C}^\dagger\gamma^5\mathcal{C}=\left(\gamma^5\right)^T$

CP violation

CP

$$\begin{cases} \mathcal{CP}: \left(x^{\mu}, y\right) = \left(x_{\mu}, y\right) \\ \mathcal{CP}: \psi = i\gamma^{0}\gamma^{2}\psi^{*} \\ \mathcal{CP}: \left(A^{\mu}, A^{y}\right) = \left(-A_{\mu}, -A^{y}\right)^{T} \end{cases}$$

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} - \gamma^{5}\partial_{y} - M\varepsilon\left(y\right) + gA^{y}\gamma^{5}\right]\psi$$

is CP invariant

$\left\langle A^{y} \right\rangle \neq 0$ $\Rightarrow$ CP is spontaneously broken

because A^y is CP odd

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} - \gamma^{5}\partial_{y} - M\varepsilon\left(y\right) + gA^{y}\gamma^{5} \right]\psi$$

Even if M=0, CP seems to be broken ⇒ Not true

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} - \gamma^{5}\partial_{y} - M\varepsilon(y) + gA^{y}\gamma^{5} \right]\psi$$

Even if M=0, CP seems to be broken $\Rightarrow$ Not true

In M=0 case, chiral rotation $\psi \to e^{i\frac{\pi}{4}\gamma^{5}}\psi$ can remove $i\gamma^5$ from the last term in Lagrangian keeping other terms invariant

$\Rightarrow$ A^y is CP even in this case

To break CP, an interplay A^y and M is important

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} - \gamma^{5}\partial_{y} - M\varepsilon(y) + gA^{y}\gamma^{5} \right]\psi$$

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⇒ A^y is CP even in this case

To break CP, an interplay A^y and M is important

Neutron EDM (P, CP violating)

5D GHU@1-loop 1/R > 2.6TeV (SM@3-loop)

Y. Adachi, C.S. Lim, NM, PRD80 (2009) 055025

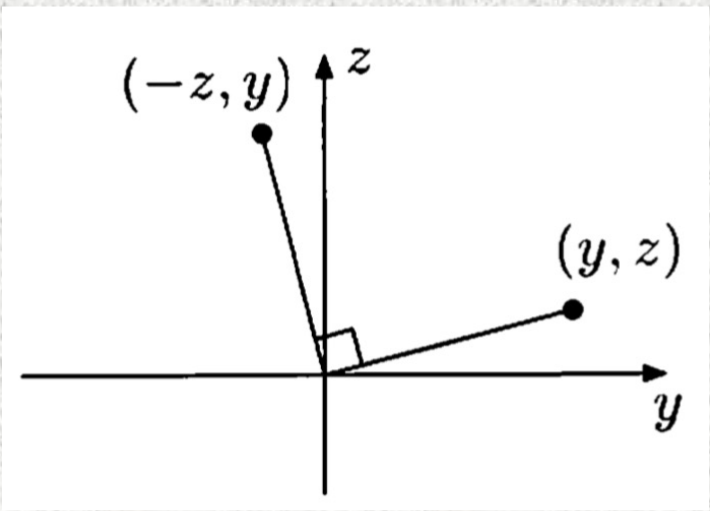
CP violation in even extra dimensions

C.S. Lim, NM, K. Nishiwaki, PRD81(2010) 076006

CP can be broken by the geometry
of compactified spaces

Consider 6D theory compactified on T^2/Z_4

T^2/Z_4: points on 2-dim torus
by Z_4 transformation
are identified



Z_4 transformation

C, P transformations preserving 4D C, P acting on 6D Dirac fermions

$$P:\Psi_6 \to \left(\gamma^0 \otimes \sigma_3\right)\Psi_6, \quad C:\Psi_6 \to \left(C \otimes \sigma_3\right)\bar{\Psi}_6^T$$

C, P transformations preserving 4D C, P acting on 6D Dirac fermions

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Invariance of $\bar{\Psi}_6 i\Gamma^M \partial_M \Psi_6$ leads to

$$P:\left(y,z\right)\to\left(y,z\right), \quad C,CP:\left(y,z\right)\to\left(y,-z\right)$$

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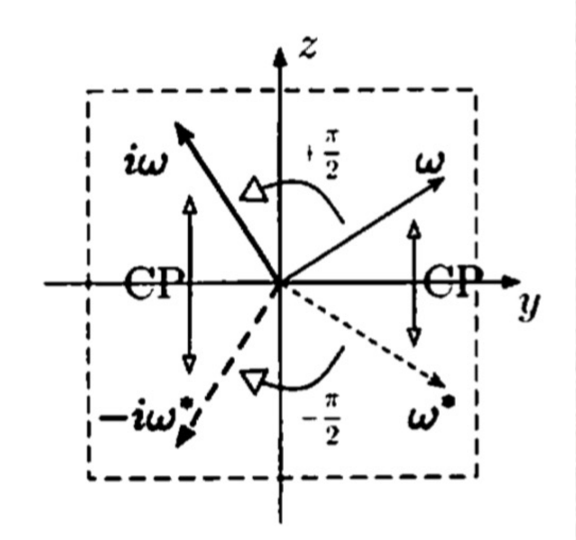
CP is a complex conjugate of ω = (y+iz)/√2

Z_4 after CP is −90° rotation NOT +90°

⇒ CP & T^2/Z_4 is incompatible



C, P transformations preserving 4D C, P acting on 6D Dirac fermions

$$P:\Psi_6 \to \left(\gamma^0 \otimes \sigma_3\right)\Psi_6, \quad C:\Psi_6 \to \left(C \otimes \sigma_3\right)\bar{\Psi}_6^T$$

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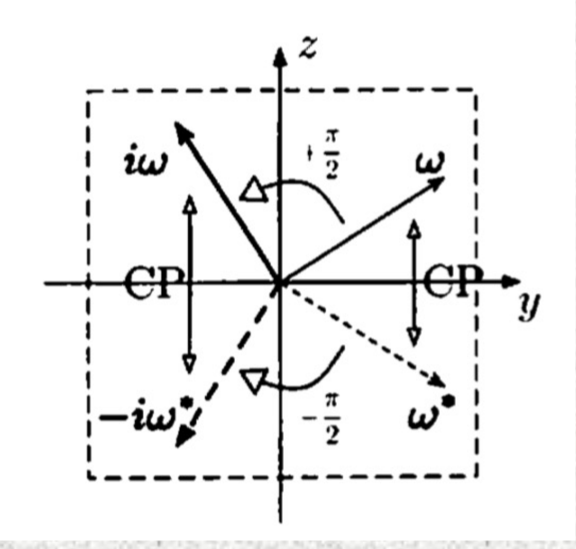


Z_2 after CP is −180° rotation equivalent to +180°

⇒ CP & T^2/Z_2 is compatible

For those interested in these issues

最近の研究から

高次元時空から見たCP対称性の破れの起源

丸　信人　〈中央大学理工学部物理学科　112-8551 東京都文京区春日 1-13-27　e-mail: maru@phys.chuo-u.ac.jp〉

小林・益川両氏によって素粒子の標準模型における「CP対称性の破れ」の機構が提案され実験でも検証されたが，現在の物質・反物質の非対称性を十分に説明できない．本稿では，高次元理論に基づいた新しいCP対称性の破れの機構について最近の研究成果を解説する．

1. はじめに

2008年ノーベル物理学賞は，南部陽一郎，益川敏英，小林誠各氏による「対称性の破れ」の業績に対して贈られ，日本物理学会は大いに盛り上がった．その中でも小林・益川両氏による研究は，素粒子の標準模型における「CP対称性の破れ」の機構を提案し，実験でも検証された．しかし，小林・益川理論によるCP対称性の破れでは，現在の宇宙の物質と反物質の非対称性を生成するには十分でないことが知られている．従って，標準模型にはないCP対称

"右巻き"（"左巻き"）のクォークを表し，$\langle H \rangle$はヒッグス場の真空期待値（実数）である．湯川結合定数が複素共役Y^*に変換することに注目すると，Yが実数でないかぎりCP対称性が破れ得ることがわかる．ただし，クォークには物理を変えない位相変換の自由度があり，この自由度を使っても物理的位相が残るためには少なくとも6種類のクォークが必要である．これがまさに小林・益川理論の予言であった．[1),*2]

CP対称性の破れは，宇宙における粒子・反粒子の非対

EW symmetry breaking

SM Higgs potential

SM Higgs potential is fixed
by the following requirements

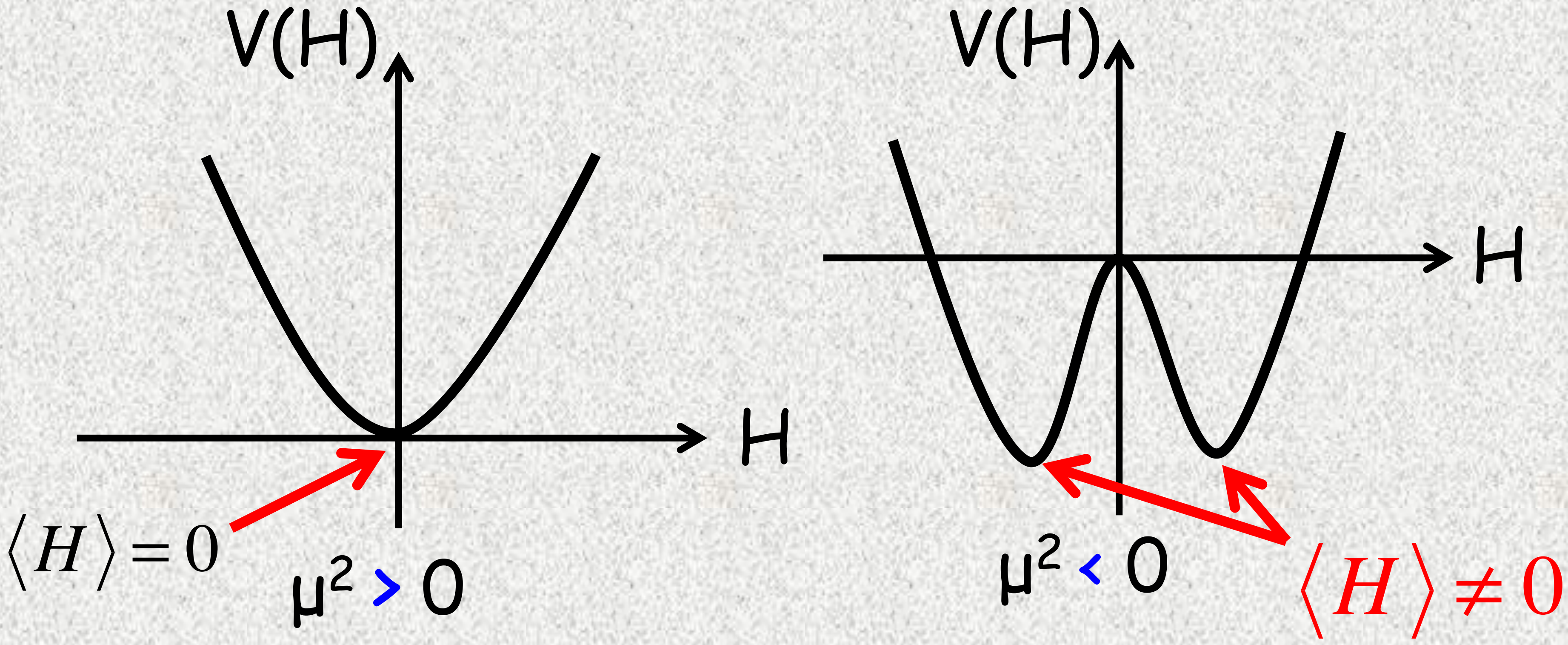
- $SU(2)_L \times U(1)_Y$ invariance
- Renormalizability

$$\Rightarrow \quad V(H) = \mu^2 H^\dagger H + \lambda \left(H^\dagger H \right)^2$$

Vacuum stability $\lambda > 0$ assumed
$\rightarrow$ cannot be predicted in SM

SM Higgs potential

Vacuum depends on a sign of μ^2



$\mu^2 < 0 \Rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

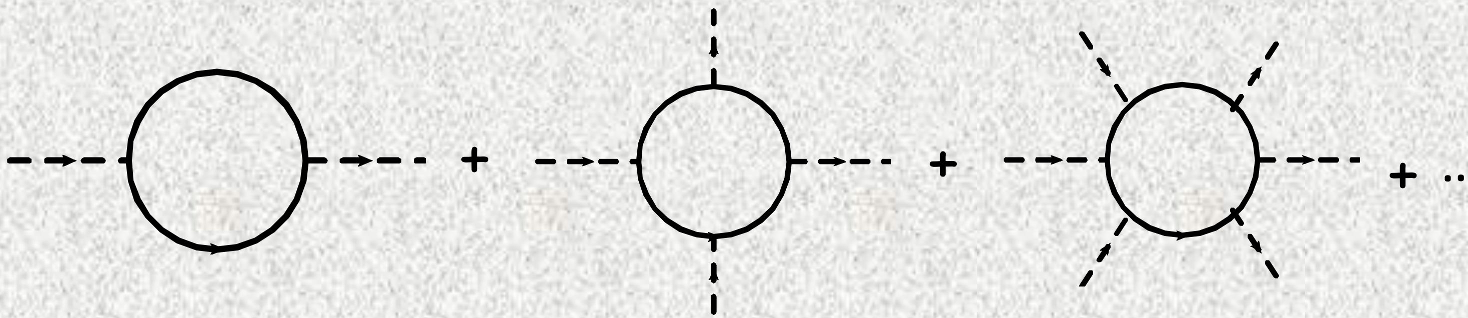
(sign of μ^2 cannot be predicted in SM)

Electroweak symmetry breaking

In GHU, EW symmetry is dynamically broken
by the Hosotani mechanism Hosotani (1983,1989)

Higgs potential is radiatively generated
since the tree level potential is forbidden
by gauge invariance (Coleman-Weinberg potential)

$V(A_5) =$



$$V(a) = (-1)^F \frac{\text{(DOF)}}{2} \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{2\pi R} \sum_n \log\left(p_E^2 + m_n^2\right)$$

KK mass

Calculation of the effective potential (Adj rep)

$$I(a) \equiv \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \log\left[p^2 + \left(\frac{n+a}{R}\right)^2 \right]$$

$$\frac{dI(a)}{da} = \frac{2}{R} \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{\left(\frac{n+a}{R}\right)}{p^2 + \left(\frac{n+a}{R}\right)^2} = \frac{2}{R} \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \left(\frac{n+a}{R}\right) \int_0^\infty dt \exp\left[-\left\{ p^2 + \left(\frac{n+a}{R}\right)^2 \right\} t \right]$$

$$= \frac{2}{R} \sum_{n=-\infty}^{\infty} \frac{n+a}{R} \int_0^\infty dt \frac{1}{(4\pi t)^2} \exp\left[-\left(\frac{n+a}{R}\right)^2 t \right]$$

$$= \frac{2}{R} \frac{1}{(4\pi)^2} \int_0^\infty dt \frac{1}{t^2} \sum_{n=-\infty}^{\infty} R^2 \sqrt{\frac{\pi}{t^3}} i\pi n \exp\left[-\frac{(\pi R n)^2}{t} - 2\pi i n a \right] = \frac{3R}{16(\pi R)^5} \sum_{n=1}^{\infty} \frac{1}{n^4} \sin(2\pi n a)$$

$$\Rightarrow I(a) = -\frac{3R}{32\pi^6 R^5} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(2\pi n a) + (\text{a-independent})$$

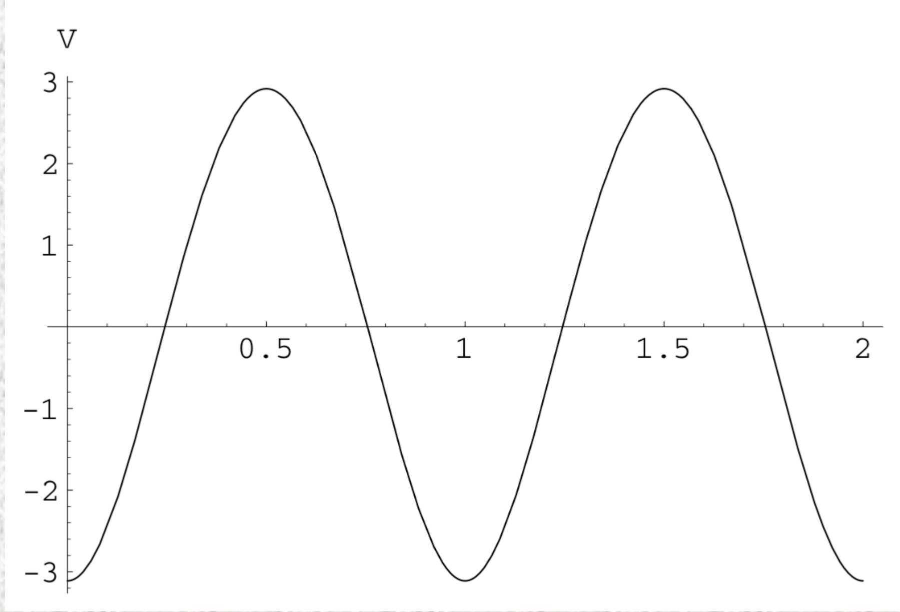
Poisson resummation

$$\sum_{n=-\infty}^{\infty} \left(\frac{n+a}{R}\right) \exp\left[-\left(\frac{n+a}{R}\right)^2 t \right] = \sum_{m=-\infty}^{\infty} R^2 \sqrt{\frac{\pi}{t^3}} (i\pi m) \exp\left[-\frac{(\pi R m)^2}{t} - 2\pi i m a \right]$$

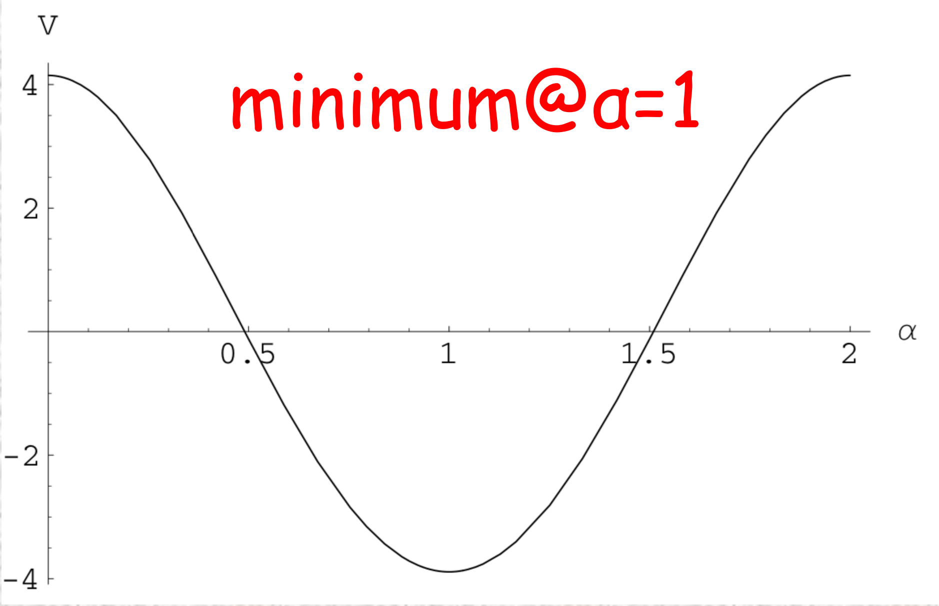
Ex1. 5D SU(2) model on S^1/Z_2 with N_f fundamental fermions

Kubo, Lim & Yamashita (2002)

$$V(a)=\frac{3}{128\pi^7R^5}\sum_{n=1}^{\infty}\frac{1}{n^5}\left[\underset{\text{Gauge + ghost}}{-3\cos[2\pi na]}+4N_f\underbrace{\cos[\pi na]}_{\text{fund}}\right]$$



YM: SU(2) unbroken



Fermion in fundamental rep

Gauge symmetry breaking

$$SU(2) \to U(1) @ S^1/Z_2 \to ?? @ a=1$$

Wilson line phase

$$\langle W \rangle = \mathcal{P}\exp\left(ig \oint_{S^1} dy \langle A_5^1 \rangle \frac{\sigma_1}{2} \right) = \exp\left(ig \frac{a}{gR} \frac{\sigma_1}{2} 2\pi R \right) = \exp\left(i\pi a \sigma_1 \right)$$

$$\Rightarrow \exp\left(i\pi\sigma_1 \right) = -I \to \left[\langle W \rangle, T^3 \right] = 0$$

$$\Rightarrow \quad SU(2) \to U(1) \to U(1)$$

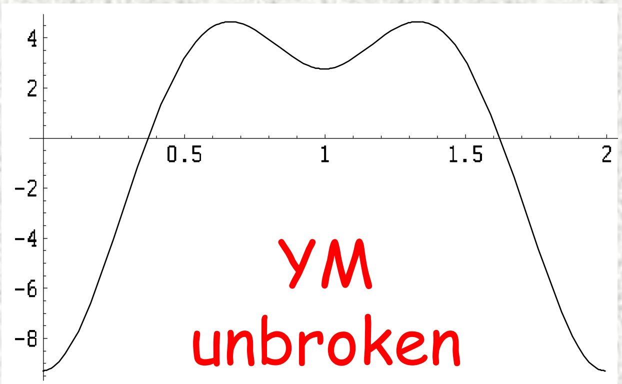
U(1) is unbroken

Ex. 5D SU(3) model on S^1/Z_2 with N_f fundamental & N_a adjoint fermions

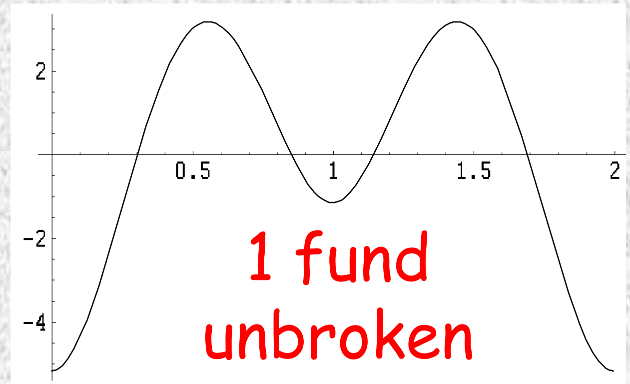
Kubo, Lim & Yamashita (2002)

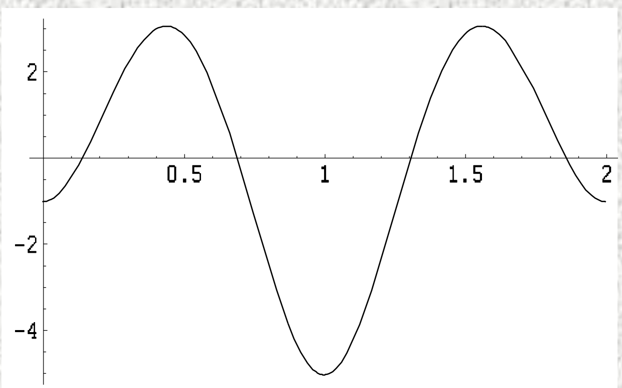
$$V(a)=\frac{3}{128\pi^7R^5}\sum_{n=1}^{\infty}\frac{1}{n^5}\left[\underset{\text{Gauge + ghost}}{(4N_a-3)}\underbrace{\left(\cos[2\pi na]+2\cos[\pi na]\right)}_{\text{adjoint}}+4N_f\underbrace{\cos[\pi na]}_{\text{fund}}\right]$$

V(a)



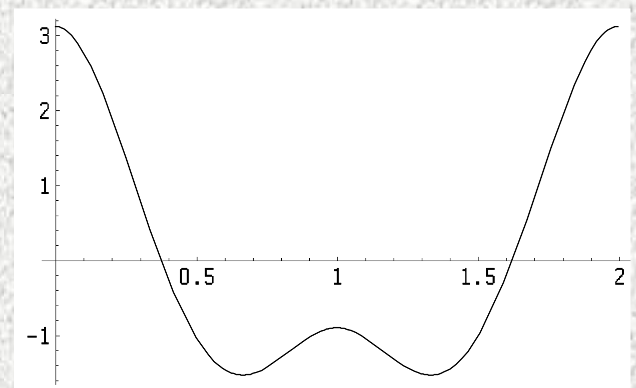
a





2 fund

SU(2)xU(1) -> U(1)xU(1)



1 adj

SU(2)xU(1) -> U(1)

Wilson line phase

$$\langle W\rangle = \mathcal{P}\exp\left(ig\oint_{S^1} dy\langle A_5\rangle\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i\sin(\pi a) \\ 0 & i\sin(\pi a) & \cos(\pi a) \end{pmatrix} \quad (a \bmod 2) = \begin{cases} SU(2)\times U(1) \text{ for a = 0} \\ U(1)'\times U(1) \text{ for a = 1} \\ U(1)_{em} \text{ for other cases} \end{cases}$$

$$\langle A_5\rangle = \frac{a}{gR}\frac{T^6}{2} \equiv A_5^{6(0)}\frac{T^6}{2}$$

$$T^3 = diag(1,-1,0)$$

$$T^8 = diag(1,1,-2)/\sqrt{3}$$

$$a = 1 : \langle W\rangle = diag(1,-1,-1) \Rightarrow \left[\langle W\rangle, T^3\right] = \left[\langle W\rangle, T^8\right] = 0$$

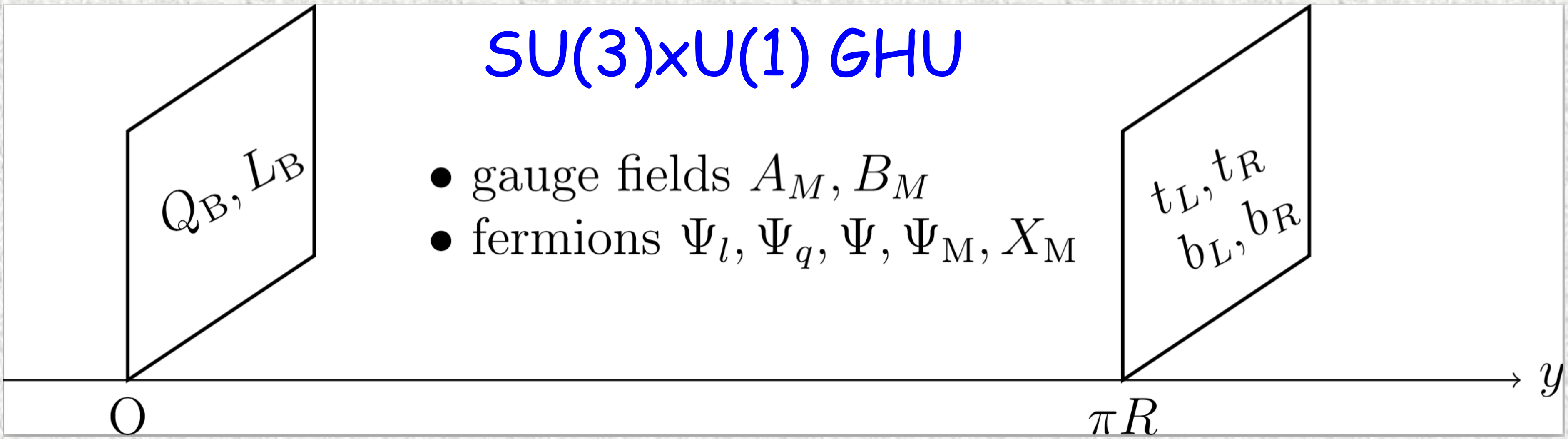
U(1) x U(1)' unbroken

$$0 < a < 1 : \left[\langle W\rangle, \sqrt{3}T^3 + T^8\right] = 2\left[\langle W\rangle, \sin\theta_W \lambda^3 + \cos\theta_W \lambda^8\right] = 0$$

$U(1)_{em}$ unbroken

Simplified model

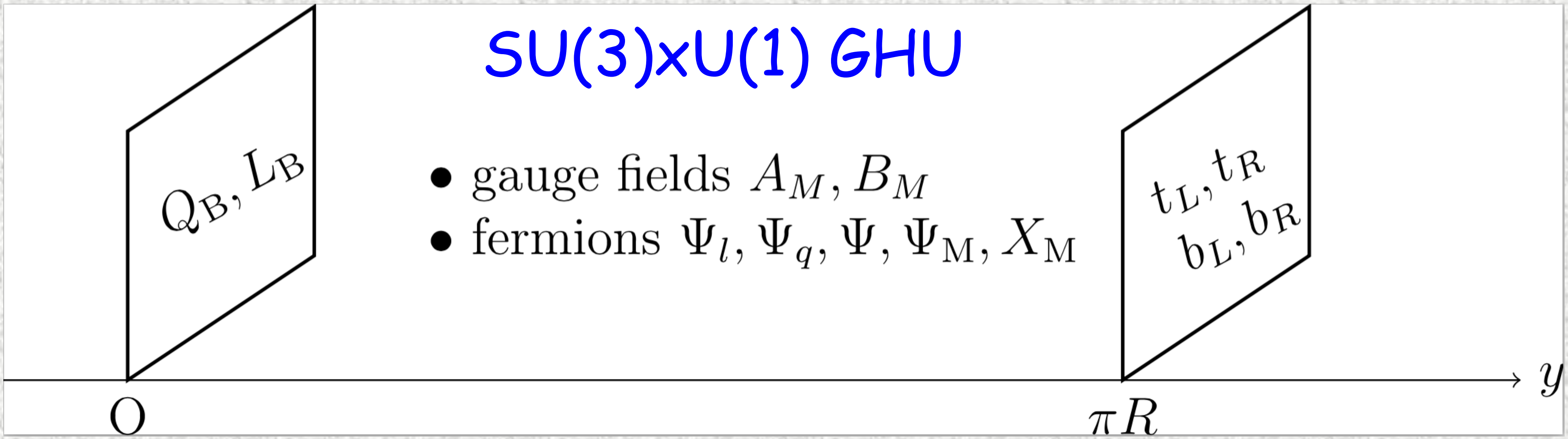
Adachi & NM, PRD98 (2018) 015022



- 3rd generation quarks: $(t_L, b_L)^T$, t_R, b_R
  brane localized fermions@$y=\pi R$
- Messenger fermions: Ψ(3(b), 15*(t))
  linear combination of Q_{3R} & Q_{15^*R} couple to (t_L, b_L)
  B_{3L} & T_{15^*L} couple to b_R, & t_R

Simplified model

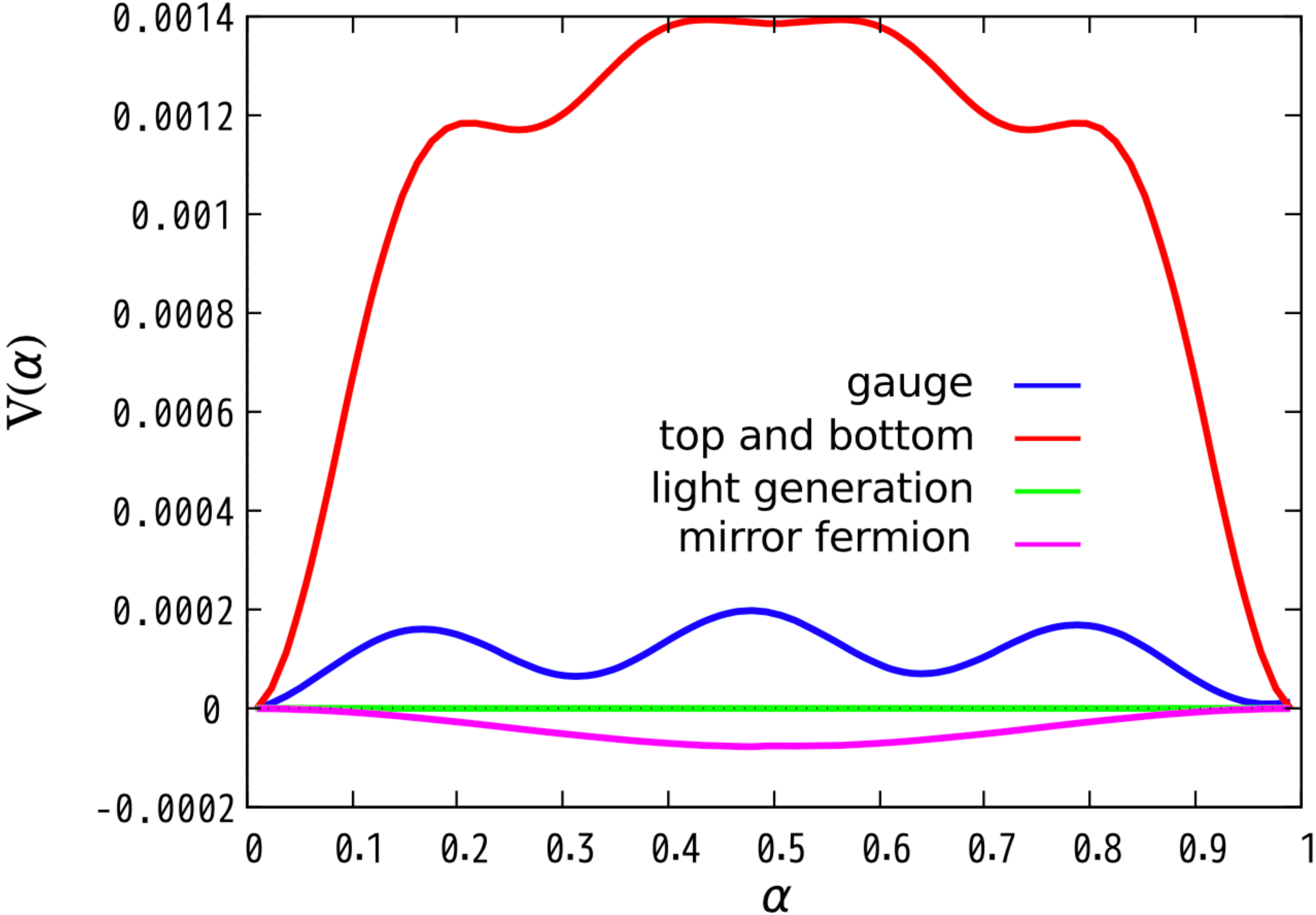
Adachi &NM, PRD98 (2018) 015022



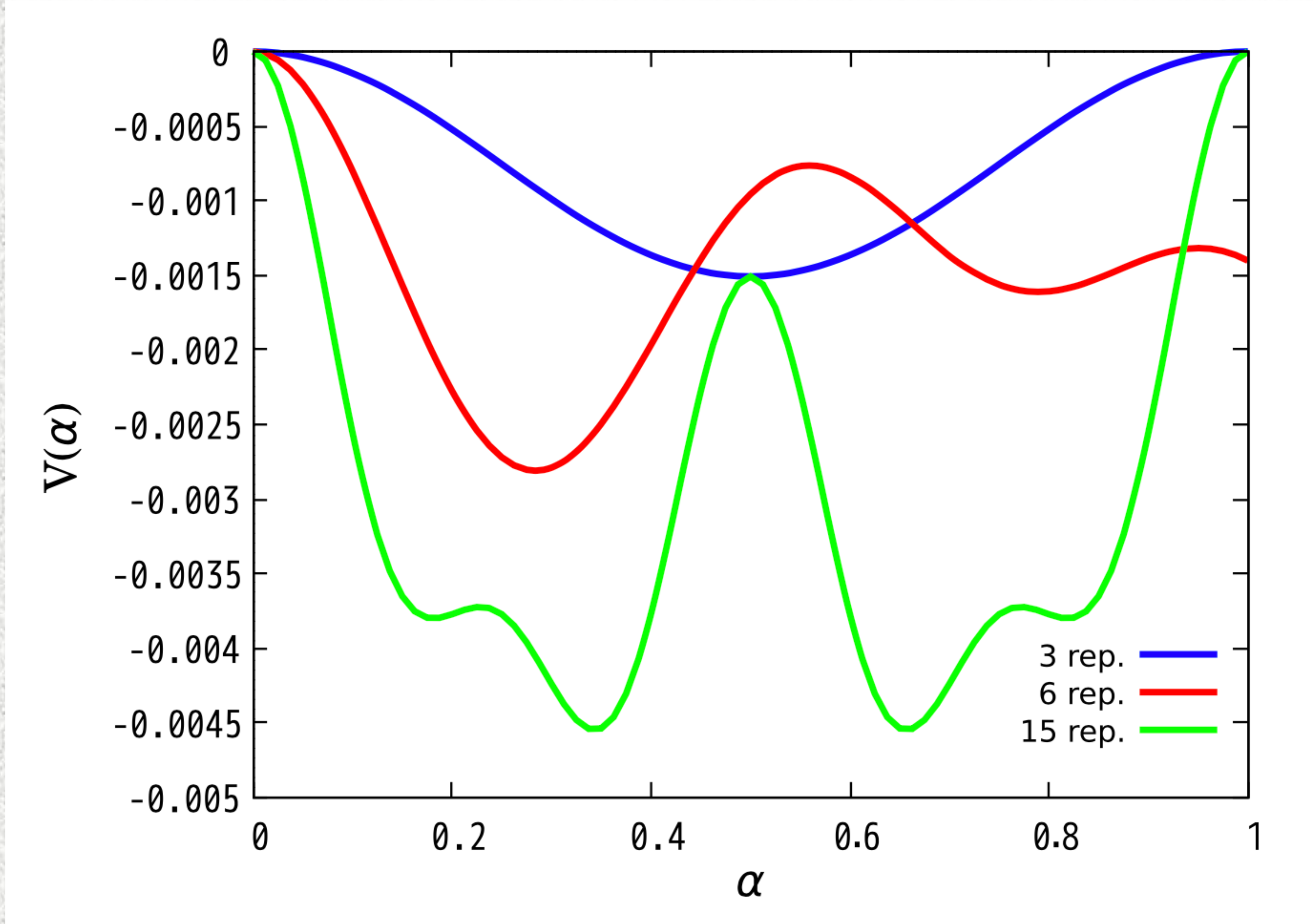
- 1^{st} & 2^{nd} generations of q & l: bulk fields (3, 3*)

  3(Q, d_R), 3*(Q, u_R), 3(L, e_R), 3*(L, ν_R)
- Q_B, L_B: brane localized fermions@y=0

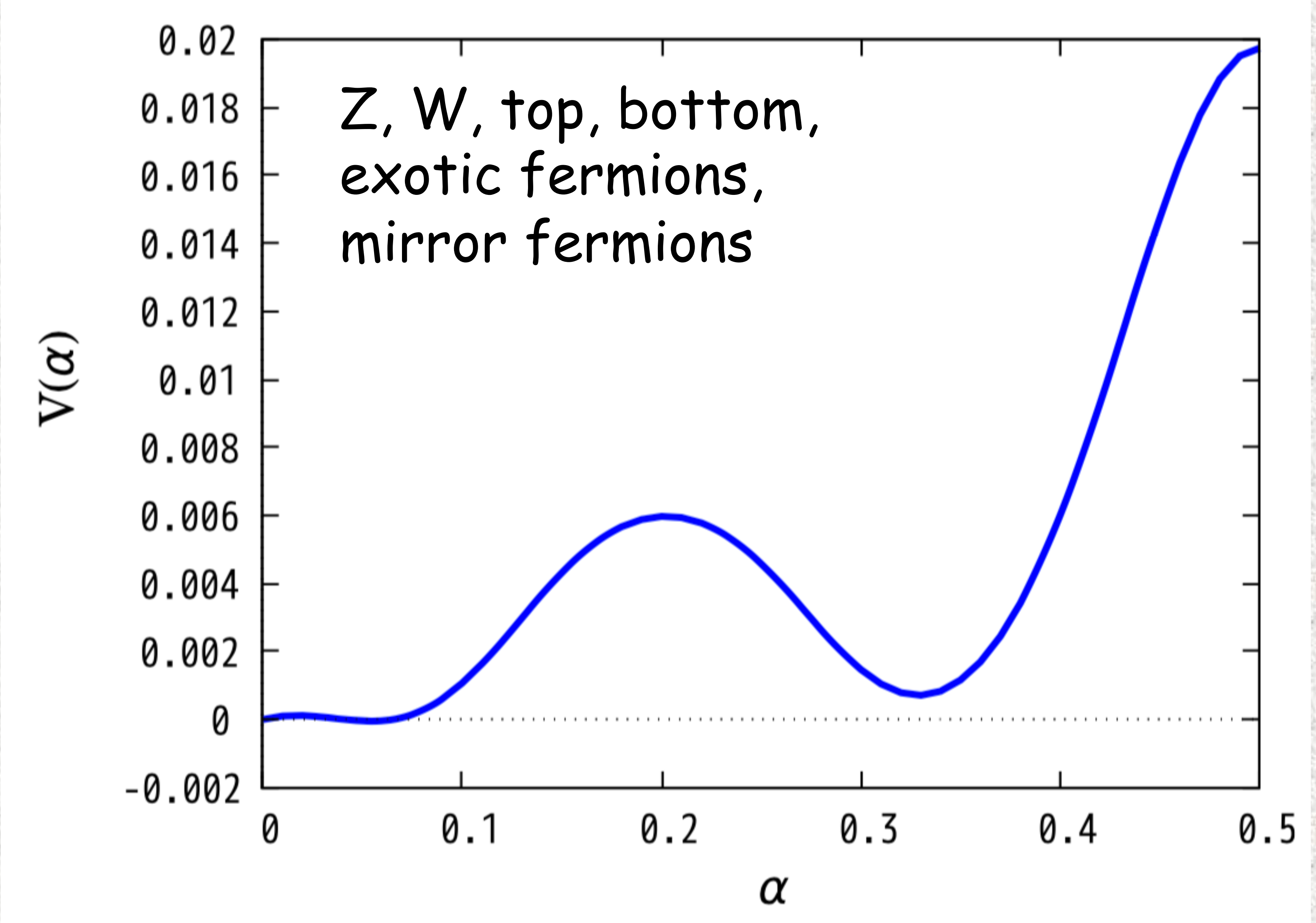
  to remove exotic SU(2) doublets
- Mirror fermions: Ψ_M, X_M (15*, 15*) for EWSB


V(α)
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
-0.0002
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
α
gauge
top and bottom
light generation
mirror fermion

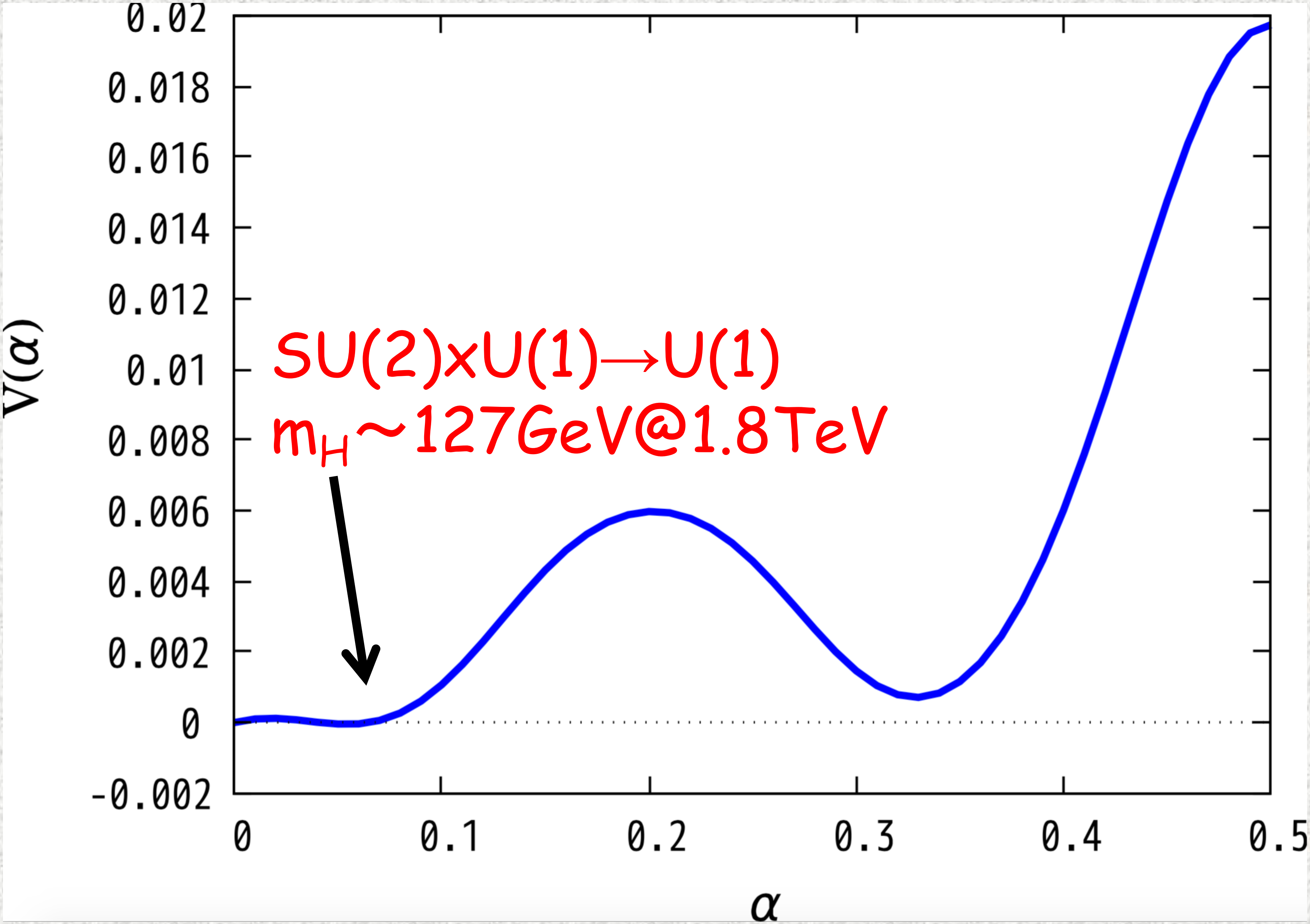
Higgs potential from mirror fermions



Higgs potential



EW symmetry breaking & Higgs mass



GUT Extension 1 of GHU

"Towards a Realistic Grand Gauge-Higgs Unification"

C.S. Lim and N.M., PLB653 (2007) 320

Motivations of GUT

In SM,

- Interactions are NOT unified
- Matter fields are NOT unified
- Quatization of EM charge cannot be explained → prediction of θ_W
- etc…

⇒ Extension of SM

⇒ Minimal model is SU(5)

Quick review of SU(5) GUT

- Choice of gauge group G_{GUT}

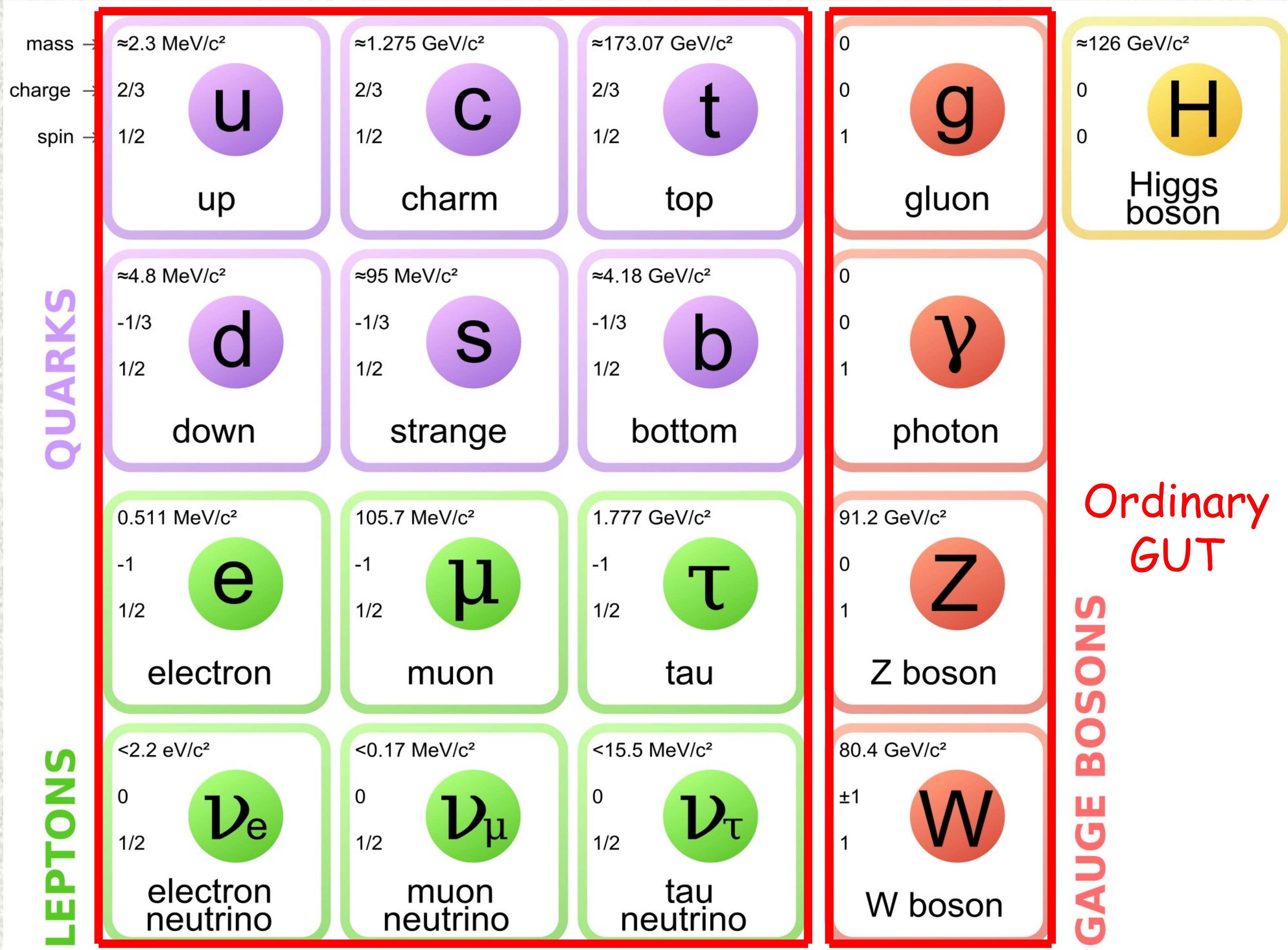
For G_{GUT} to include SM group SU(3) x SU(2) x U(1)
Rank of G_{GUT} must be more than 4

Rank 4 simple group: SU(5), SO(8), SO(9), Sp(8), F_4

To obtain chiral fermions,
fermions must belong to complex representations
(real representation can have mass term)

$\Rightarrow$ SU(5)

Georgi-Glashow (1974)



https://www.quantumdiaries.org/

Unification of gauge fields

Adjoint representation of SU(5): 24-dim

$$A_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} & & X_{1\mu} & Y_{1\mu} \\ G_\mu - \frac{2}{\sqrt{30}} B_\mu & & X_{2\mu} & Y_{2\mu} \\ & & X_{3\mu} & Y_{3\mu} \\ X^1_\mu \quad X^2_\mu \quad X^3_\mu & & \frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} & W^+_\mu \\ Y^1_\mu \quad Y^2_\mu \quad Y^3_\mu & & W^-_\mu & -\frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} \end{pmatrix}$$

Unification of fermions

Traceless = sum of $U(1)_{em}$ charge must be zero

ex. $(\nu_e, e)_L$ $Q_{\nu e} + Q_e = -1 \Rightarrow 1/3\,(Q_{d^c_L}) \times 3 = 1$ canceled

$$5^* = (3^*, 1)(d_R) + (1, 2^*)(\nu_e, e)_L$$

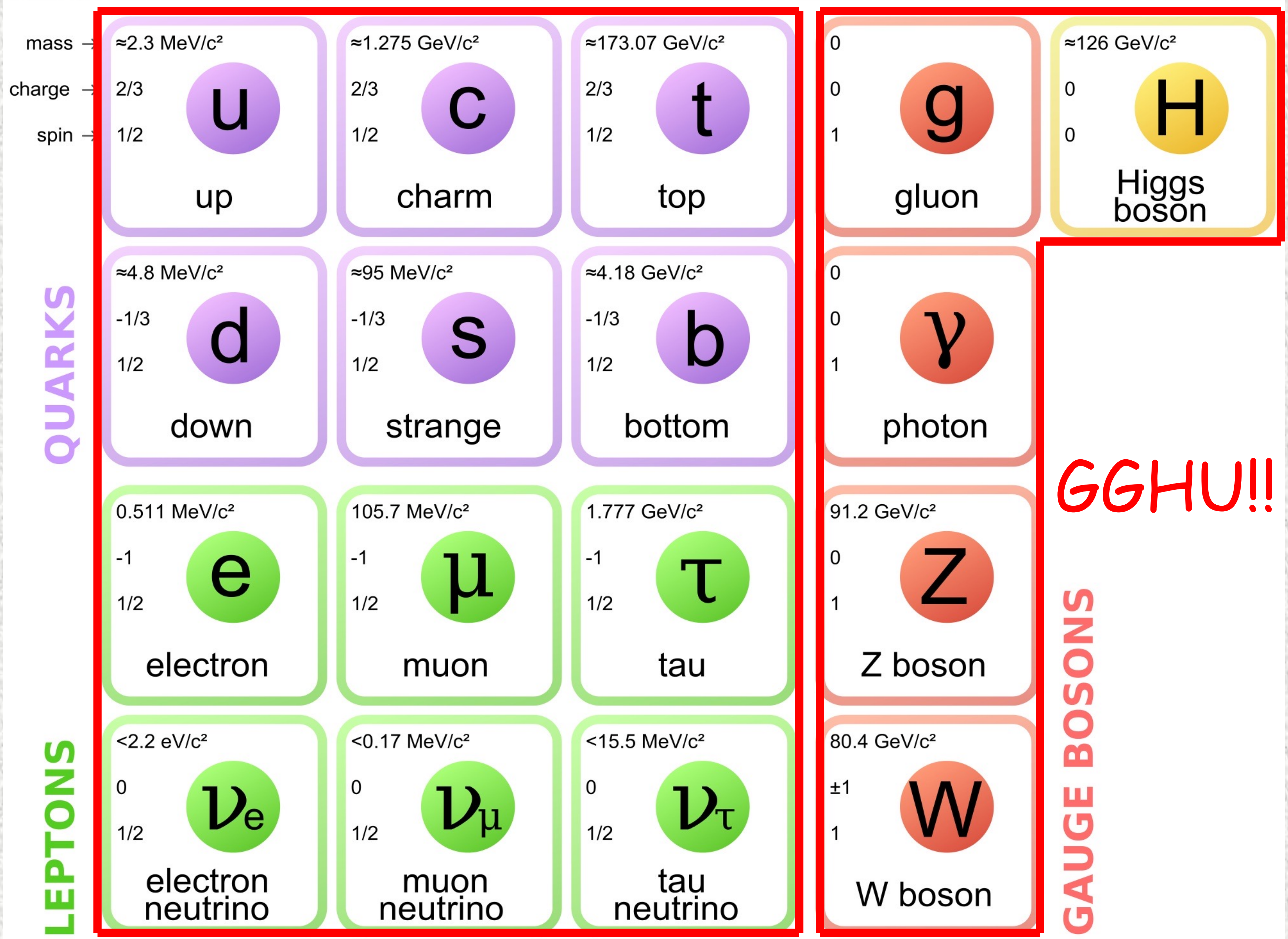
$$\psi^i\left(5^*\right) = \begin{pmatrix} d^{1c} \\ d^{2c} \\ d^{3c} \\ e \\ -\nu_e \end{pmatrix}_L, \quad \psi_{ij}\left(10\right) = \begin{pmatrix} 0 & u^{3c} & -u^{2c} & -u_1 & -d_1 \\ -u^{3c} & 0 & -u^{1c} & -u_2 & -d_2 \\ u^{2c} & -u^{1c} & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L$$

Unifications of fermions

Remaining fermions (quark doublet, RH up quarks, RH ν) are embedded in tensor product of 5 and/or 5*

$10 = 5\times 5_{asym}$
$= (3, 2)(u, d)_L + (3, 1)(u_R) + (1, 1)(e_R)$

$$\psi^i\left(5^*\right)=\begin{pmatrix} d^{1c} \\ d^{2c} \\ d^{3c} \\ e \\ -\nu_e \end{pmatrix}_L , \quad \psi_{ij}\left(10\right)=\begin{pmatrix} 0 & u^{3c} & -u^{2c} & -u_1 & -d_1 \\ -u^{3c} & 0 & -u^{1c} & -u_2 & -d_2 \\ u^{2c} & -u^{1c} & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L$$



https://www.quantumdiaries.org/

It is meaningful to consider the grand unified version of gauge-Higgs unification scenario ("Grand Gauge-Higgs unification") since the hierarchy problem was originally addressed in the GUT framework

In this lecture,
we discuss some attempts towards a realistic grand gauge-Higgs unification

$\Rightarrow$ 5D SU(6) model on S^1/Z_2

Parity assignments

$$P_0 = diag(+,+,+,+,+,-) @ y = 0$$

$$\Rightarrow SU(6) \to SU(5) \times U(1) @ y = 0$$

$$P_1 = diag(+,+,-,-,-,-) @ y = \pi R$$

$$\Rightarrow SU(6) \to SU(2) \times SU(4) \times U(1) @ y = \pi R$$

For gauge field

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

KK mode expansions Only (+,+) mode has a massless mode

$$\Phi_{(+,+)}(x,y)=\frac{1}{\sqrt{2\pi R}}\left[\phi^{(0)}_{(+,+)}(x)+\sqrt{2}\sum_{n=1}^{\infty}\phi^{(n)}_{(+,+)}(x)\cos\left(\frac{n}{R}y\right)\right]$$

$$\Phi_{(+,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi^{(n)}_{(+,-)}(x)\cos\left(\frac{n+\frac{1}{2}}{R}y\right)$$

$$\Phi_{(-,+)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi^{(n)}_{(-,+)}(x)\sin\left(\frac{n+\frac{1}{2}}{R}y\right)$$

$$\Phi_{(-,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\phi^{(n)}_{(-,-)}(x)\sin\left(\frac{n}{R}y\right)$$

Focus on 0 modes,

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

Gauge symmetry breaking

$SU(6) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$

Weinberg angle $\sin^2\theta_W = \frac{3}{8}$ Same as the Georgi-Glashow SU(5) GUT

Prediction of θ_W

Hypercharge $U(1)_Y$ generator is the same as SU(5) GUT

$$T(24) = \frac{1}{2\sqrt{15}} \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} = \sqrt{\frac{3}{5}} Y = \sqrt{\frac{3}{5}} diag\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right)$$

$$\Rightarrow \quad \sin^2\theta_W = \frac{g_{U(1)}^2}{g_{SU(2)}^2 + g_{U(1)}^2} = \frac{\left(\sqrt{3/5}\, g_{SU(5)}\right)^2}{g_{SU(5)}^2 + \left(\sqrt{3/5}\, g_{SU(5)}\right)^2} = \frac{3}{8}$$

This gauge symmetry breaking pattern is well-known

SUSY gauge-Higgs ⇒ Burdman & Nomura, NPB656 (2003) 3
Non SUSY gauge-Higgs
⇒Haba, Hosotani, Kawamura & Yamashita, PRD70 (2004) 015010

This symmetry breaking structure was also considered in the pseudo NG boson scenario of Higgs boson as global symmetry breaking

Inoue, Kakuto & Komatsu, PTP75(1986) 664 etc...

↓

Very natural from the viewpoint of AdS/CFT

$\because$ global sym of 4D ⇔ gauge sym in 5D

Focus on 0 modes,

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

A_5 has a zero mode transforming as an $SU(2)_L$ doublet

Standard Model Higgs!!

Furthermore,

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

Colored Higgs has NO zero mode

Doublet-Triplet mass splitting works
(doublet Higgs: massless, Triplet Higgs: 1/2R)
Kawamura, PTP105 (2001) 691, 999

Doublet-Triplet mass splitting problem

Fine-tuning problem between SM Higgs mass
and colored Higgs mass

Higgs potential in SU(5) GUT

$$V(\Sigma, H) = V(\Sigma) + V(H) + \lambda_4 \left(tr\Sigma^2 \right) H^\dagger H + \lambda_5 H^\dagger \Sigma^2 H$$

$$V(\Sigma(24)) = -M^2 tr\Sigma^2 + \lambda_1 \left(tr\Sigma^2 \right)^2 + \lambda_2 tr\Sigma^4$$

$$V(H(5)) = -m^2 H^\dagger H + \lambda_3 \left(H^\dagger H \right)^2, \quad H = \begin{pmatrix} H_3 \\ H_{SM} \end{pmatrix}$$

Reormalizability, Σ ↔ -Σ, H ↔ -H assumed

$$SU(5) \xrightarrow{\langle\Sigma\rangle = V diag(2,2,2,-3,-3)} SU(3)\times SU(2)\times U(1)$$ @O(M_{GUT})

$$V\left(\langle\Sigma\rangle, H\right) \supset \lambda_4\left(tr\langle\Sigma\rangle^2\right)H^\dagger H + \lambda_5 H^\dagger\langle\Sigma\rangle^2 H - m^2 H^\dagger H + \lambda_3\left(H^\dagger H\right)^2$$

$$= m_3^2 H_3^\dagger H_3 + m_2^2 H_{SM}^2$$

$$\Rightarrow \begin{cases} m_3^2 = -m^2 + \left(30\lambda_4 + 4\lambda_5\right)V^2 \sim \mathcal{O}\left(M_{GUT}\right) \\ m_2^2 = -m^2 + \left(30\lambda_4 + 9\lambda_5\right)V^2 \approx 0 \end{cases}$$ $\leftarrow$ Fine-tuning!!

m~O(M_{GUT}), V~O(M_{GUT})

1 generation of SM q & l are elegantly embedded into the following representations (w/ RH ν)

$$6^* = \begin{cases} 6^*_L = \underbrace{(3^*,1)^{(+,-)}_{(1/3,-1)} \oplus l_L(1,2)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{(3^*,1)^{(-,+)}_{(1/3,-1)} \oplus (1,2)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R(1,1)^{(+,+)}_{(0,5)}}_{1} \end{cases}$$

$$6^* = \begin{cases} 6^*_L = \underbrace{(3^*,1)^{(-,-)}_{(1/3,-1)} \oplus (1,2)^{(-,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(+,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{d_R(3^*,1)^{(+,+)}_{(1/3,-1)} \oplus (1,2)^{(+,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,+)}_{(0,5)}}_{1} \end{cases}$$

$$20 = \begin{cases} 20_L = \underbrace{q_L(3,2)^{(+,+)}_{(1/6,-3)} \oplus (3^*,1)^{(+,-)}_{(-2/3,-3)} \oplus (1,1)^{(+,-)}_{(1,-3)}}_{10} \oplus \underbrace{(3^*,2)^{(-,+)}_{(-1/6,3)} \oplus (3,1)^{(-,-)}_{(2/3,3)} \oplus (1,1)^{(-,-)}_{(-1,3)}}_{10^*} \\ 20_R = \underbrace{(3,2)^{(-,-)}_{(1/6,-3)} \oplus (3^*,1)^{(-,+)}_{(-2/3,-3)} \oplus (1,1)^{(-,+)}_{(1,-3)}}_{10} \oplus \underbrace{(3^*,2)^{(+,-)}_{(-1/6,3)} \oplus u_R(3,1)^{(+,+)}_{(2/3,3)} \oplus e_R(1,1)^{(+,+)}_{(-1,3)}}_{10^*} \end{cases}$$

The difference between L & R components are only a relative parity sign

In 2nd 6* rep, the signs of parity @y=0 are flipped

1 generation of SM q & l are elegantly embedded into
the following representations (w/ RH ν)

$$6^* = \begin{cases} 6^*_L = \underbrace{(3^*,1)^{(+,-)}_{(1/3,-1)} \oplus l_L(1,2)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{(3^*,1)^{(-,+)}_{(1/3,-1)} \oplus (1,2)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R(1,1)^{(+,+)}_{(0,5)}}_{1} \end{cases}$$

$$6^* = \begin{cases} 6^*_L = \underbrace{(3^*,1)^{(-,-)}_{(1/3,-1)} \oplus (1,2)^{(-,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(+,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{d_R(3^*,1)^{(+,+)}_{(1/3,-1)} \oplus (1,2)^{(+,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,+)}_{(0,5)}}_{1} \end{cases}$$

$$20 = \begin{cases} 20_L = \underbrace{q_L(3,2)^{(+,+)}_{(1/6,-3)} \oplus (3^*,1)^{(+,-)}_{(-2/3,-3)} \oplus (1,1)^{(+,-)}_{(1,-3)}}_{10} \oplus \underbrace{(3^*,2)^{(-,+)}_{(-1/6,3)} \oplus (3,1)^{(-,-)}_{(2/3,3)} \oplus (1,1)^{(-,-)}_{(-1,3)}}_{10^*} \\ 20_R = \underbrace{(3,2)^{(-,-)}_{(1/6,-3)} \oplus (3^*,1)^{(-,+)}_{(-2/3,-3)} \oplus (1,1)^{(-,+)}_{(1,-3)}}_{10} \oplus \underbrace{(3^*,2)^{(+,-)}_{(-1/6,3)} \oplus u_R(3,1)^{(+,+)}_{(2/3,3)} \oplus e_R(1,1)^{(+,+)}_{(-1,3)}}_{10^*} \end{cases}$$

No Massless Exotics!!
(zero modes are just SM)

No SM Anomalies!!
(U(1)x broken by anomalies)

Difference from the conventional GUT

⇒ These fields belong to the same multiplets in the usual GUT

$$6^* = \begin{cases} 6^*_L = \underbrace{\left(3^*,1\right)^{(+,-)}_{(1/3,-1)} \oplus l_L\left(1,2\right)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\left(1,1\right)^{(-,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{\left(3^*,1\right)^{(-,+)}_{(1/3,-1)} \oplus \left(1,2\right)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R\left(1,1\right)^{(-,-)}_{(0,5)}}_{1} \end{cases},\ 6^* = \begin{cases} 6^*_L = \underbrace{\left(3^*,1\right)^{(-,-)}_{(1/3,-1)} \oplus \left(1,2\right)^{(-,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\left(1,1\right)^{(+,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{d_R\left(3^*,1\right)^{(+,+)}_{(1/3,-1)} \oplus \left(1,2\right)^{(+,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\left(1,1\right)^{(-,+)}_{(0,5)}}_{1} \end{cases}$$

$$20 = \begin{cases} 20_L = \underbrace{q_L\left(3,2\right)^{(+,+)}_{(1/6,-3)} \oplus \left(3^*,1\right)^{(+,-)}_{(-2/3,-3)} \oplus \left(1,1\right)^{(+,-)}_{(1,-3)}}_{10} \oplus \underbrace{\left(3^*,2\right)^{(-,+)}_{(-1/6,3)} \oplus \left(3,1\right)^{(-,-)}_{(2/3,3)} \oplus \left(1,1\right)^{(-,-)}_{(-1,3)}}_{10^*} \\ 20_R = \underbrace{\left(3,2\right)^{(-,-)}_{(1/6,-3)} \oplus \left(3^*,1\right)^{(-,+)}_{(-2/3,-3)} \oplus \left(1,1\right)^{(-,+)}_{(1,-3)}}_{10} \oplus \underbrace{\left(3^*,2\right)^{(+,-)}_{(-1/6,3)} \oplus u_R\left(3,1\right)^{(+,+)}_{(2/3,3)} \oplus e_R\left(1,1\right)^{(+,+)}_{(-1,3)}}_{10^*} \end{cases}$$

Down-type quark Yukawa & charged lepton Yukawa cannot be generated by the gauge interaction…

GUT Extension 2 of GHU

"Fermion Mass Hierarchy in Grand Gauge-Higgs Unification"
N.M. and Yoshiki Yatagai, PTEP (2019) 8, 083B03

"Improving Fermion Mass Hierarchy
in Grand Gauge-Higgs Unification
with Localized Gauge Kinetic Terms"
N.M. and Yoshiki Yatagai, EPJC80 (2020), 10, 933

"Fermion Mass Hierarchy and Mixing
in Simplified Grand Gauge-Higgs Unification"
N.M., Haruki Takahashi and Yoshiki Yatagai, 2205.05824 [hep-ph]

"Gauge Coupling Unification
in Simplified Grand Gauge-Higgs Unification"
N.M., Haruki Takahashi and Yoshiki Yatagai, 2207. [hep-ph]

"Fermion Mass Hierarchy
in Grand Gauge-Higgs Unification"

N.M. and Yoshiki Yatagai, PTEP (2019) 8, 083B03

- SM quarks & leptons are localized on the boundary
- Yukawa couplings are generated by bulk & boundary couplings
- Quark & lepton masses except for top quark are reproduced
- 125 GeV Higgs mass is obtained by introducing extra bulk fermions

"Improving Fermion Mass Hierarchy in Grand Gauge-Higgs Unification with Localized Gauge Kinetic Terms"

N.M. and Yoshiki Yatagai, EPJC80 (2020), 10, 933

- Localized gauge kinetic terms are introduced to reproduce top quark mass

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^{MN}F_{MN} - 2\pi R c_1 \delta(y)\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - 2\pi R c_2 \delta(y-\pi R)\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\Rightarrow g_4^2 \to (1+c_1+c_2)g_4^2$$

$$\Rightarrow m_{fermion} \to \sqrt{1+c_1+c_2}\, m_{fermion}$$

- Fermion mass hierarchy except for ν is reproduced (bulk fermions: 20, 15, 6 reps/gen)

"Improving Fermion Mass Hierarchy in Grand Gauge-Higgs Unification with Localized Gauge Kinetic Terms"

N.M. and Yoshiki Yatagai, EPJC80 (2020), 10, 933

- Electroweak symmetry breaking and 125 GeV Higgs mass are obtained by introducing extra massive bulk fermions (simplified than 120 rep in previous paper)

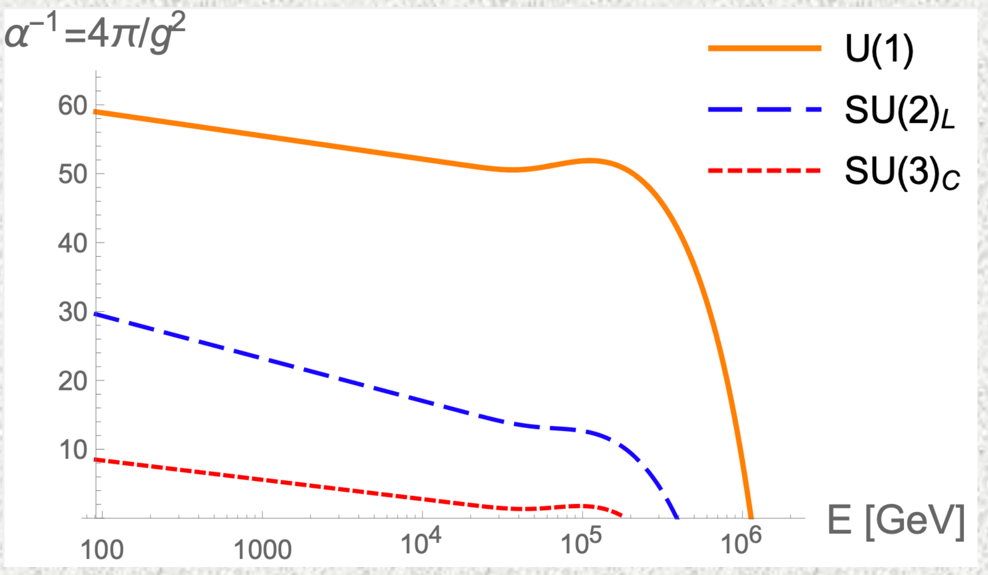
1: 15 rep x 3 $\rightarrow$ $1/R \sim 8$TeV, $m_{15} \sim 1.6$TeV

2: 6 rep x 5 $\rightarrow$ $1/R = 16.2$TeV, $m_6 \sim 3$TeV

"Fermion Mass Hierarchy and Mixing
in Simplified Grand Gauge-Higgs Unification"
N.M., Haruki Takahashi and Yoshiki Yatagai, 2205.05824 [hep-ph]
"Gauge Coupling Unification
in Simplified Grand Gauge-Higgs Unification"
N.M., Haruki Takahashi and Yoshiki Yatagai, 2207. [hep-ph]

Unsatisfying points in the previous model

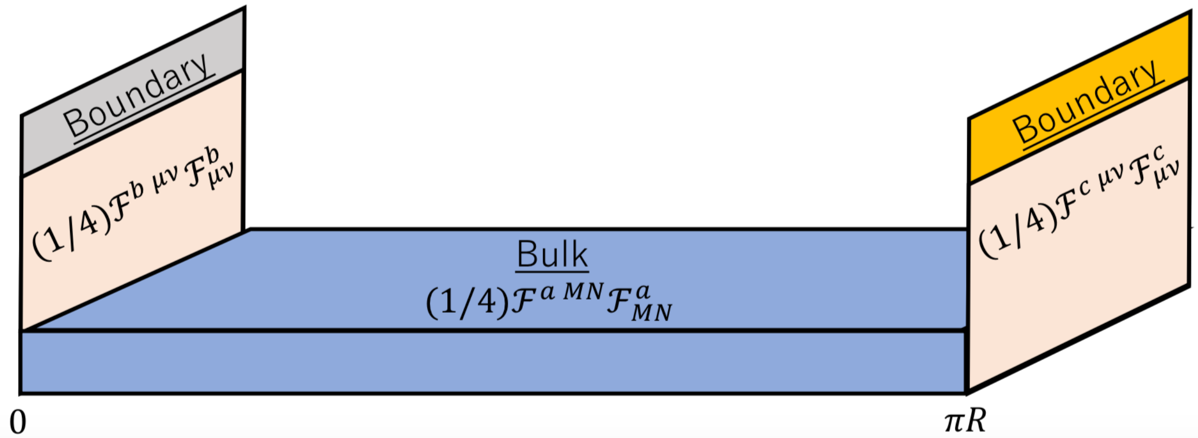
- generation mixings and CP phase not reproduced
- too many bulk fermions
  ⇒ Landau pole



Gauge sector with localized gauge kinetic terms

	Boundary at $y=0$	Boundary at $y=\pi R$
$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}\mathcal{F}^{a\,MN}\mathcal{F}^{a}_{MN}$	$-2\pi R\,\boxed{c_1}\,\delta(y)\frac{1}{4}\mathcal{F}^{b\,\mu\nu}\mathcal{F}^{b}_{\mu\nu}$	$-2\pi R\,\boxed{c_2}\,\delta(y-\pi R)\frac{1}{4}\mathcal{F}^{c\,\mu\nu}\mathcal{F}^{c}_{\mu\nu}$
$a : SU(6)$	$b : SU(5)\times U(1)_X \to SU(5)$	$c : SU(2)\times SU(4)\times U(1)' \to SU(3)_c\times SU(2)_L\times U(1)_Y$

c_i : dimensionless free parameters
$c = c_1 + c_2$



Lagrangian for the bulk and mirror fermions

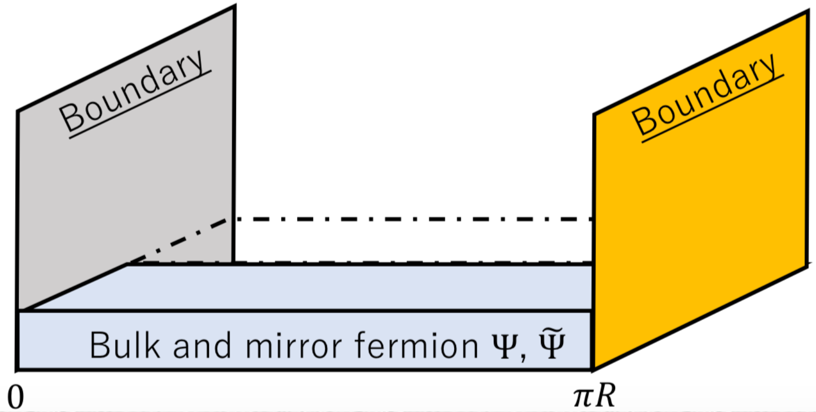
$$\mathcal{L}_{\text{bulk+mirror}} = \underbrace{\bar{\Psi} i\Gamma^M D_M \Psi}_{\text{Bulk fermion } \Psi} + \underbrace{\bar{\tilde{\Psi}} i\Gamma^M D_M \tilde{\Psi}}_{\text{Mirror fermion } \tilde{\Psi}} + \left(\boxed{\boxed{M}\bar{\Psi}\tilde{\Psi} + \text{h.c.}}\right)$$

with opposite Z_2 parities each other

$M = \dfrac{\lambda}{\pi R}$

dimensionless parameters

Mass term in the bulk
to avoid exotic massless fermions



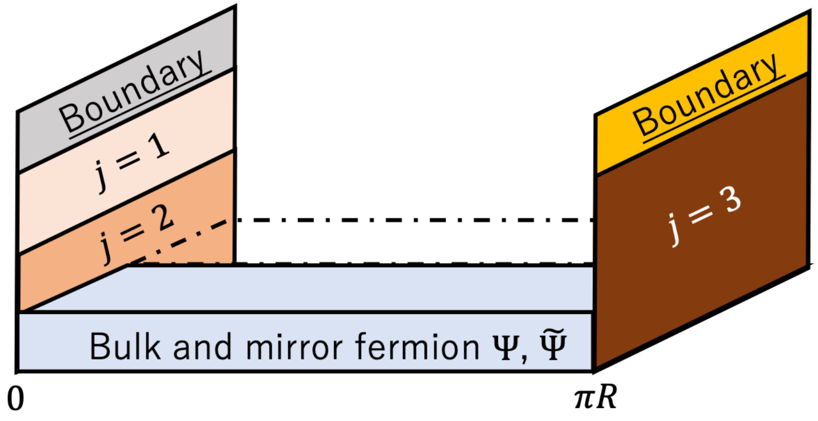
Ψ: 20, 15, 15', 6, 6' reps greatly reduced

Lagrangian for the SM fermions

$$\mathcal{L}_{\mathrm{SM}}^{j=1,2} = \delta(y)\left[\bar{\chi}_{10}^{j} i\Gamma^{\mu} D_{\mu} \chi_{10}^{j} + \bar{\chi}_{5^*}^{j} i\Gamma^{\mu} D_{\mu} \chi_{5^*}^{j} + \bar{\chi}_{1}^{j} i\Gamma^{\mu} D_{\mu} \chi_{1}^{j}\right]$$

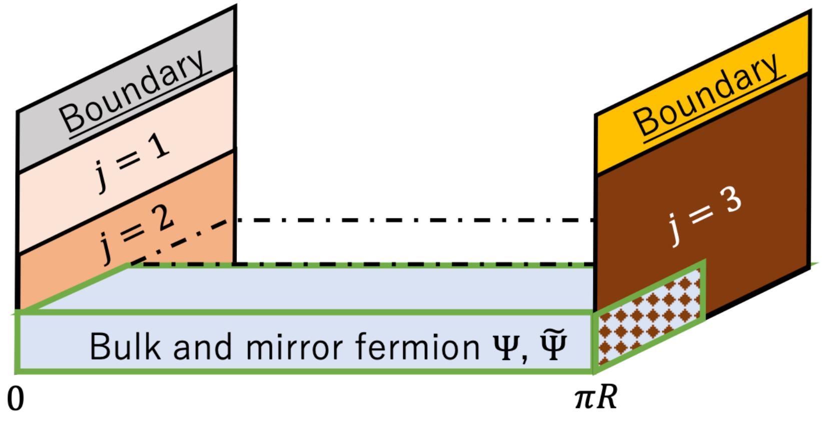
$$\mathcal{L}_{\mathrm{SM}}^{j=3} = \delta(y-\pi R)\left[\overline{q_L^3} i\Gamma^{\mu} D_{\mu} q_L^3 + \overline{u_R^3} i\Gamma^{\mu} D_{\mu} u_R^3 + \overline{d_R^3} i\Gamma^{\mu} D_{\mu} d_R^3 + \overline{l_L^3} i\Gamma^{\mu} D_{\mu} l_L^3 + \overline{e_R^3} i\Gamma^{\mu} D_{\mu} e_R^3 + \overline{\nu_R^3} i\Gamma^{\mu} D_{\mu} \nu_R^3\right]$$

j : "Generation" of the SM fermions



Mixing mass terms between the bulk fermions and the SM fermions

Boundary at $y = 0$	Bulk	Boundary at $y = \pi R$
$\epsilon_{20}^{j}\left(\bar{\chi}_{10}^{j}\Psi_{10\subset 20} + \bar{\chi}_{10}^{j,c}\Psi_{10^{*}\subset 20}\right)$	Ψ_{20} Ψ_{15} $\Psi_{15'}$ Ψ_{6} $\Psi_{6'}$	$\epsilon_{20e}\left(\overline{e_R}^3 E_{20} + \overline{u_R}^3 U_{20}\right) + \epsilon_{20q}\overline{q_L}^3 Q_{20}$



Mixing mass terms between the bulk fermions and the SM fermions

$$\mathcal{L}_{\mathrm{SM}} = \delta(y)\left[\bar{\psi}_{\mathrm{SM}\,L} i\Gamma^{\mu} D_{\mu} \psi_{\mathrm{SM}\,L} + \cdots\right]$$

$$\mathcal{L}_{\mathrm{SM+bulk}} = \delta(y)\{\epsilon \bar{\psi}_{\mathrm{SM}} \Psi + \cdots\}$$

$$\mathcal{L}_{\mathrm{SM}} = \delta(y)\left[\bar{\psi}_{\mathrm{SM}\,R} i\Gamma^{\mu} D_{\mu} \psi_{\mathrm{SM}\,R} + \cdots\right]$$

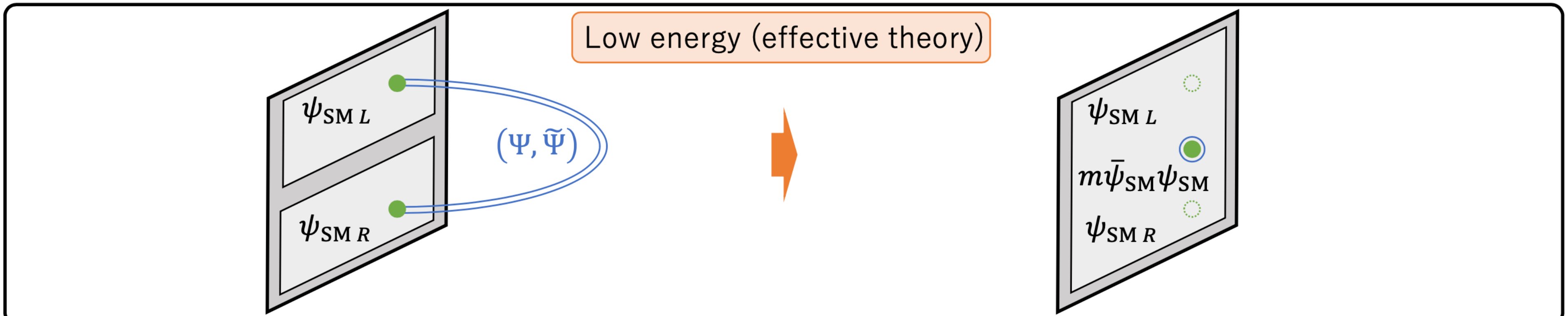
$$\mathcal{L}_{\mathrm{bulk+mirror}} = \bar{\Psi} i\Gamma^{M} D_{M} \Psi + \bar{\tilde{\Psi}} i\Gamma^{M} D_{M} \tilde{\Psi} + \left(M \bar{\Psi} \tilde{\Psi} + \mathrm{h.c.}\right)$$

$$\mathcal{L}_{\mathrm{SM}} = \delta(y)\left[\bar{\psi}_{\mathrm{SM}\,L} i\Gamma^{\mu} D_{\mu} \psi_{\mathrm{SM}\,L} + \cdots\right]$$

$$m \bar{\psi}_{\mathrm{SM}} \psi_{\mathrm{SM}}$$

$$\mathcal{L}_{\mathrm{SM}} = \delta(y)\left[\bar{\psi}_{\mathrm{SM}\,R} i\Gamma^{\mu} D_{\mu} \psi_{\mathrm{SM}\,R} + \cdots\right]$$

Mixing mass terms between the SM fermions and the bulk fermions produce the SM fermion masses.



Quark masses, mixings and CP phase

https://pdg.lbl.g

c	m_u	m_c	m_t
70	1.724 MeV	1.291 GeV	181.918 GeV
75	2.413 MeV	1.271 GeV	177.497 GeV
80	2.223 MeV	1.290 GeV	178.684 GeV
Data	$2.16^{+0.49}_{-0.26}$ MeV	1.27 ± 0.02 GeV	172 ± 0.30 GeV

c	m_d	m_s	m_b
70	5.119 MeV	94.0 MeV	4.928 GeV
75	4.727 MeV	85.2 MeV	5.090 GeV
80	4.856 MeV	84.5 MeV	5.150 GeV
Data	$4.67^{+0.48}_{-0.17}$ MeV	93^{+11}_{-5} MeV	$4.18^{+0.13}_{-0.02}$ GeV

c	$\sin\theta_{12}$	$\sin\theta_{13}$	$\sin\theta_{23}$	δ
70	0.157976	0.003336	0.041942	0.9834
75	0.165093	0.003767	0.048009	1.3759
80	0.168864	0.003985	0.044065	1.3053
Data	0.22650 ± 0.00048	$0.00361^{+0.00011}_{-0.00009}$	$0.04053^{+0.00083}_{-0.00061}$	$1.196^{+0.045}_{-0.043}$

Lepton masses, mixings and CP phase

https://pdg.lbl.g

c	m_e	m_μ	m_τ
70	0.5093 MeV	106.358 MeV	1912.20 MeV
75	0.5125 MeV	103.804 MeV	1856.99 MeV
80	0.5100 MeV	105.381 MeV	1899.96 MeV
Data	0.5109989461(31) MeV	105.6583745(24) MeV	1776.86(12) MeV

c	Δm_{21}^2	Δm_{32}^2 (Normal)
70	7.7514×10^{-5} eV^2	2.4777×10^{-3} eV^2
75	7.6760×10^{-5} eV^2	2.4367×10^{-3} eV^2
80	7.7279×10^{-5} eV^2	2.4670×10^{-3} eV^2
Data	$(7.53\pm0.18)\times10^{-5}$ eV^2	$(2.453\pm0.033)\times10^{-3}$ eV^2

c	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$ (Normal)	δ
70	0.4421	2.234×10^{-2}	0.5200	1.729π rad
75	0.4567	2.127×10^{-2}	0.5197	1.626π rad
80	0.3855	2.225×10^{-2}	0.4108	1.916π rad
Data	0.307 ± 0.013	$(2.20\pm0.07)\times10^{-2}$	0.546 ± 0.021	$1.36^{+0.20}_{-0.16}\,\pi$ rad

RGE of SM gauge couplings

In higher dimensional theory, gauge coupling has a power-law dependence on energy scale since gauge coupling is dimensionful

Dienes, Dudas & Gherghetta (1998, 1999)

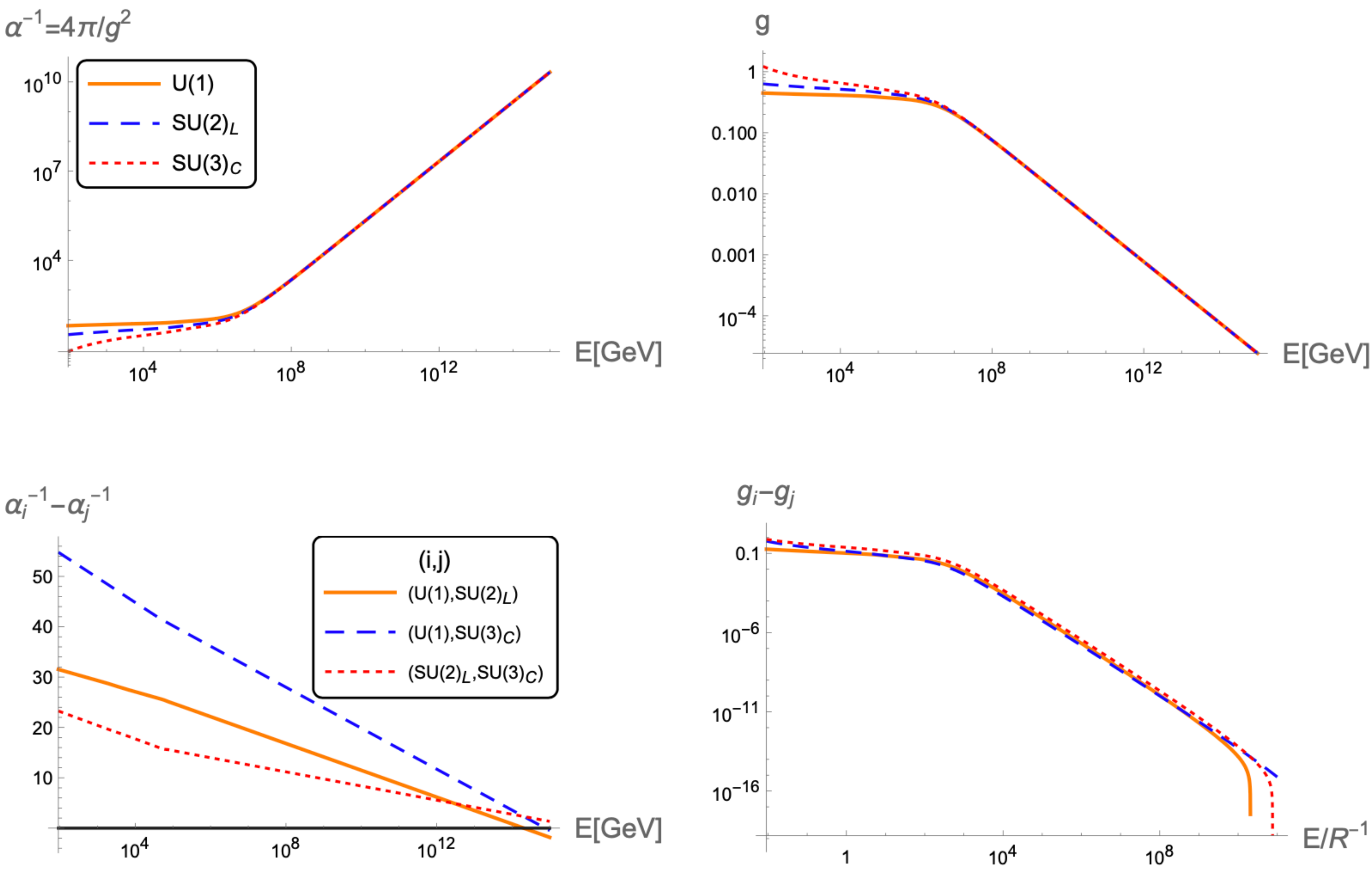
⇒ unification scale is likely to be naively lower than 4D GUT scale

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu) - \frac{b_i - \tilde{b}_i^{(+)}}{4\pi} \ln \frac{\Lambda}{\mu} - \frac{\tilde{b}_i^{(+)} + \tilde{b}_i^{(-)}}{\pi} R(\Lambda - \mu).$$

b_i: β-function of SM fields
$\tilde{b}_i^{(\pm)}$: β-function of bulk field with (anti-)periodic BC

The last term depends linearly on energy scale

Gauge coupling unification



Unification scale is relatively high

$\sim O(10^{14})$ GeV!!

c	r	R^{-1}	M_G	α_G^{-1}	$(\alpha_G^{-1} - \alpha_3^{-1})/\alpha_G^{-1}$	$\alpha_3^{-1}(M_Z)$
80	0	10 TeV	2.1×10^{14} GeV	4.4×10^{9}	5.26×10^{-10}	10.7
80	0	15 TeV	2.2×10^{14} GeV	3.2×10^{10}	6.12×10^{-10}	10.4
90	0	10 TeV	2.1×10^{14} GeV	4.3×10^{9}	5.25×10^{-10}	10.7
90	0	15 TeV	2.3×10^{14} GeV	3.2×10^{9}	6.1×10^{-10}	10.4

Bulk fermions are embedded in SU(6) reps

$\Rightarrow \tilde{b}_i^{(+)} + \tilde{b}_i^{(-)} = -2/3$

$\Rightarrow$ the difference of gauge couplings is dominated by log contributions

Summary

- Gage-Higgs unification is a very attractive scenario beyond the SM alternative to SUSY and an effective field theory of string theory
- Controlled by gauge principle & very predictive
  Higgs mass, potential → finite
- Fermion masses except for top are easy,
  but nontrivial for top Yukawa
- Flavor mixings and CP phase are harder
  to be realized, but possible

Summary

- EWSB @loop level

  Once the matter content is fixed,
  1/R is a unique free parameter in Higgs potential
  ⇒ very predictive contrary to SM case

- GUT extension is interesting
  ⇒ relatively high unification scale

Outlook

In addition to building more realistic models of EW symmetry breaking, many issues & problems to be explored

- **DM**
  - Fermion DM in GHU

    NM, T. Miyaji, N. Okada, S. Okada, JHEP07 (2017) 048

    NM, N. Okada, S. Okada, PRD96 (2017) 115023
  - Vector DM in GHU

    NM, N. Okada, S. Okada, PRD98 (2018) 075021
- **Strong CP**

  Y. Adachi, C.S. Lim, NM, PTEP2022 (2022) 5,053B06; arXiv: 2205.00161 (accepted in PTEP)

Outlook

In addition to building more realistic models of EW symmetry breaking,
many issues & problems to be explored

- **Baryon asymmetry**
  - EW baryogenesis in GHU

    NM, K. Takenaga, PRD72 (2005) 046003

    Y. Adachi, NM, PRD101 (2020) 036013
- **Inflation**

  N.Arkani-Hamed, H-C. Cheng, P. Creminelli, L. Randall, PRL90 (2003) 221302
- **More...**

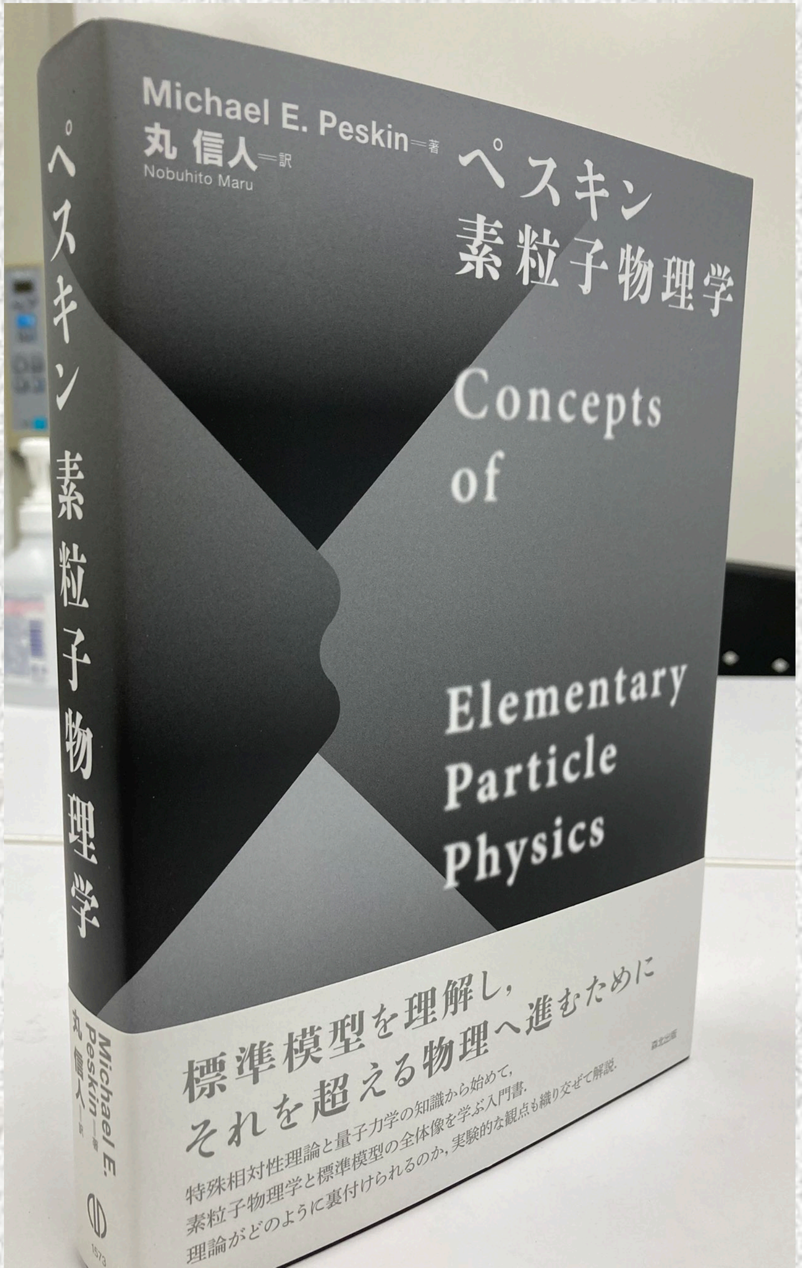
Outlook

In addition to building more realistic models of EW symmetry breaking, many issues & problems to be explored

Personally, recent interest is focused on GHU in magnetic flux compactification, too

T. Hirose, NM, JHEP1908 (2019) 054; J. Phys. G48 055005
JHEP2106 (2021) 159
K. Akamatsu, T. Hirose, NM, arXiv: 2205.09320

ペスキンの素粒子物理本、翻訳しました!!



Thank you!!