ガラスおよび関連する複雑系の最先端研究
Zoom, 2021/05/11

定圧振動剪断流下における摩擦のある粉体系のスケーリング則とダイラタンシー

Scaling laws & dilatancy of frictional materials under oscillatory shear with constant pressure

Daisuke Ishima, Hisao Hayakawa

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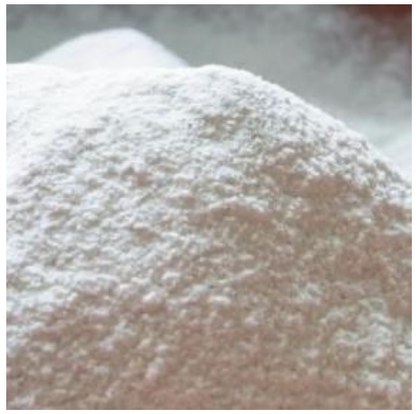
Introduction

Examples of granular materials

Sands



Powders

Balls

Pachinko balls

Characteristics

- **Athermal particles - Thermal fluctuations can be neglected.**
  - **The particles are repulsive & dissipative upon contact.**

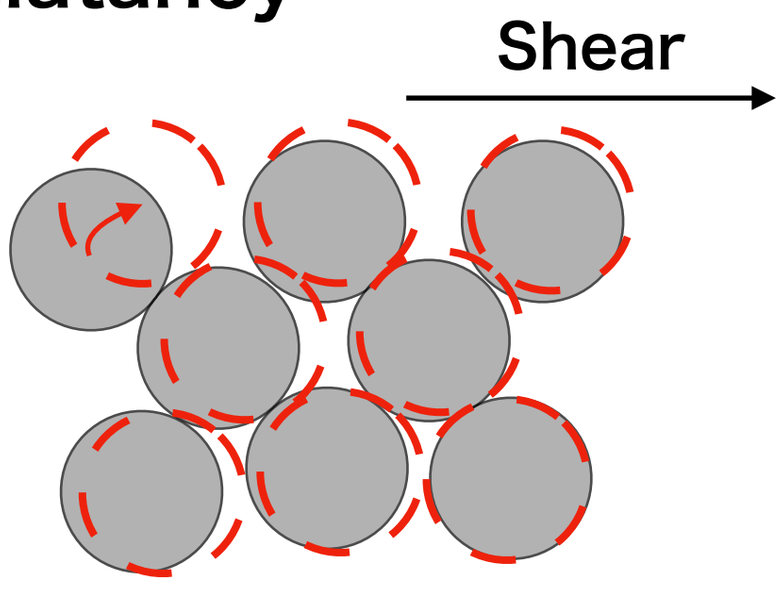
https://www.youtube.com/watch?v=29ht6SSWQMs

https://www.youtube.com/watch?v=wa9yX4xuCv4

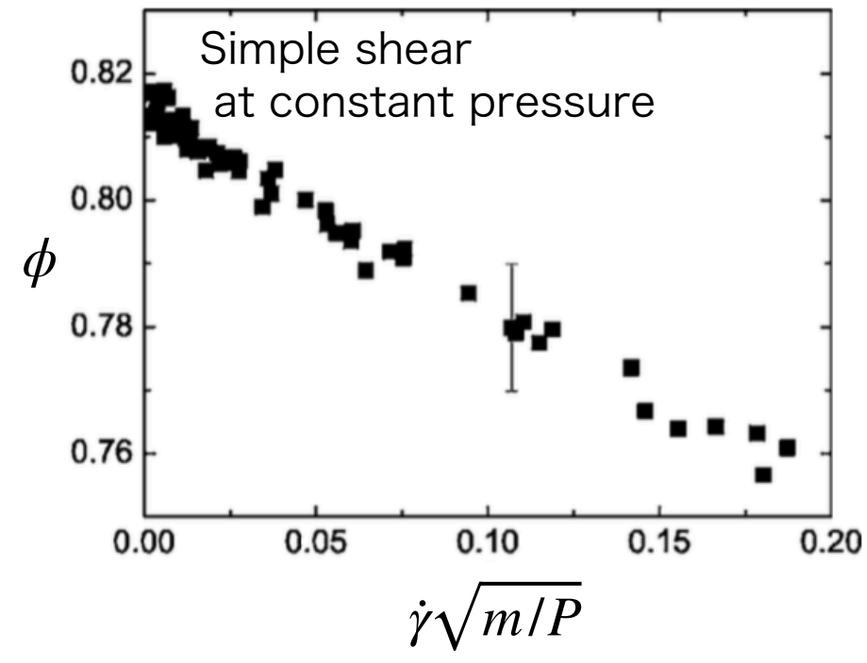
It is important to understand the behavior of granular materials.

Introduction

Dilatancy

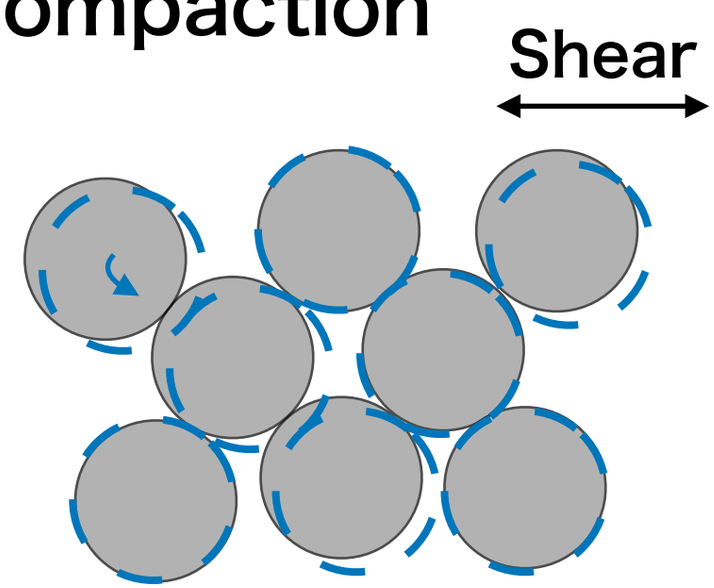


F. da Cruz et al., Phys. Rev. E **72**, *021309 (2005).*

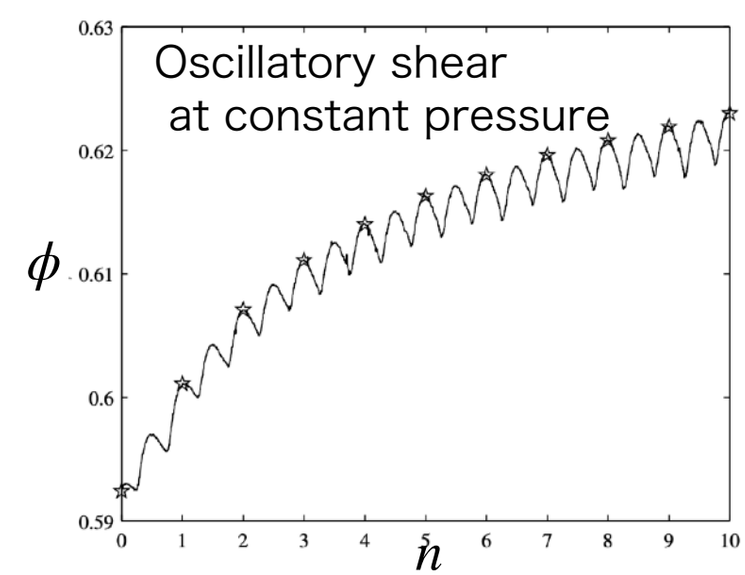


ϕ : density, $\dot{\gamma}$: shear rate, P : pressure, m : mass

Compaction



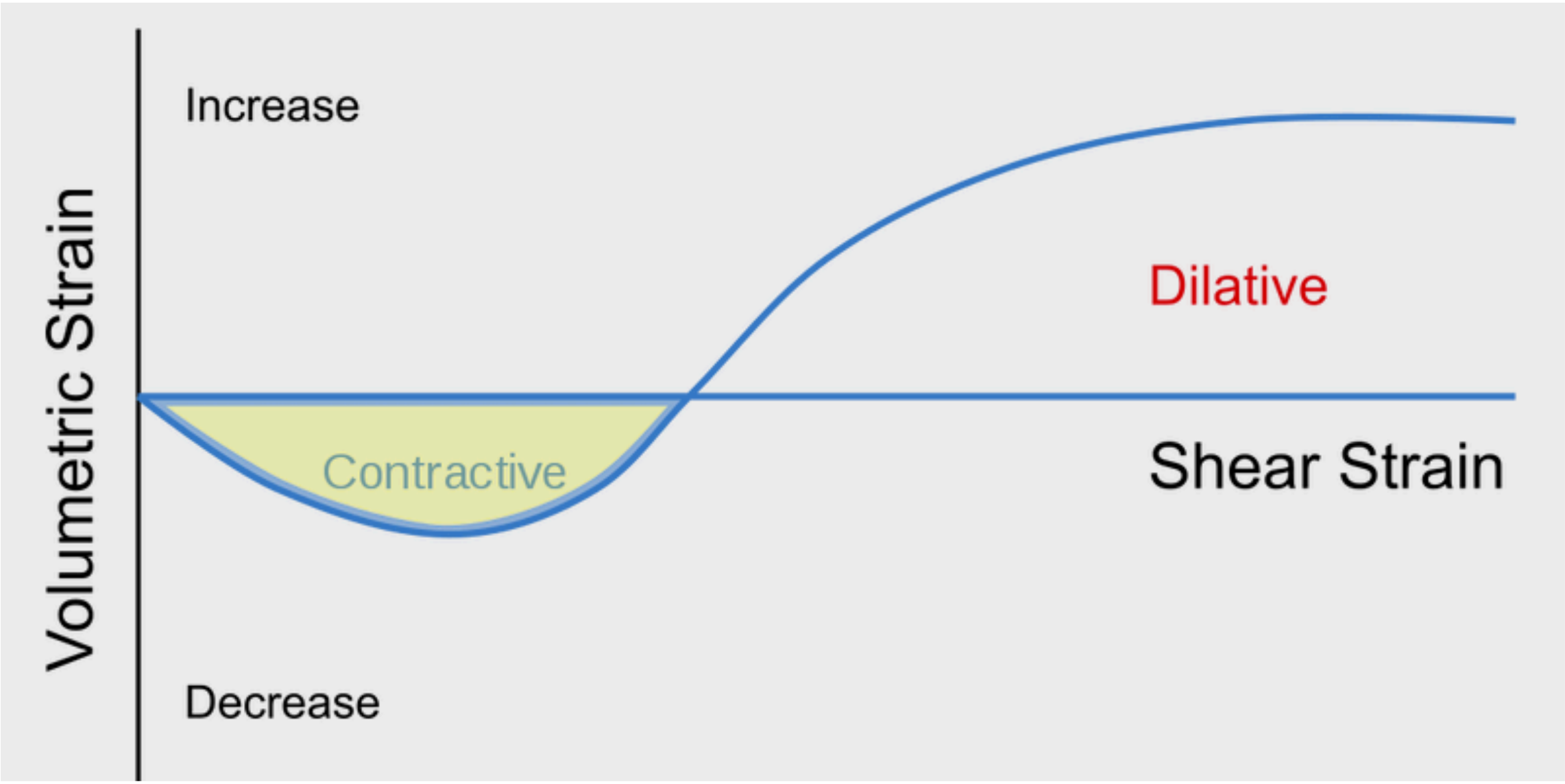
M. Nicolas et al., Eur. Phys. J. E **3**, *309 (2000).*



n : number of oscillations

Shear induced dilatancy & compaction.

Introduction



"https://en.wikipedia.org/wiki/Dilatancy_(granular_material)#cite_note-Houlsby-8"

Can we observe compaction for high pressure?

Mutual friction μ & pressure P dependence?

[3]

Rigidity & Viscosity under oscillatory shear

The shear stress σ

$$\sigma = G'\gamma(t) + \frac{G''}{\Omega}\dot{\gamma}(t) \qquad (1)$$

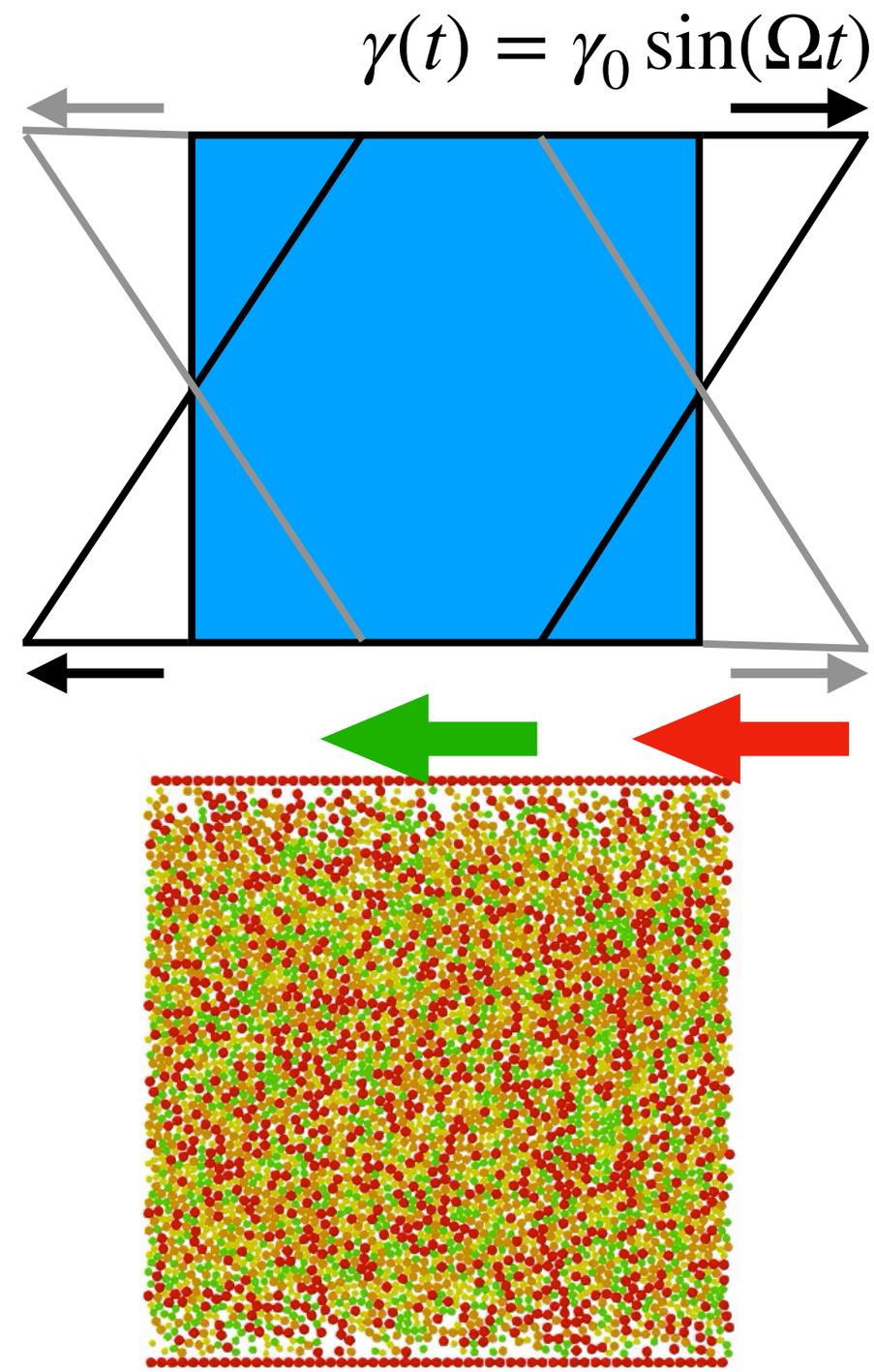
Rigidity Viscosity

Storage modulus G'

$$G' := \lim_{\gamma(t)\to\gamma_0} \frac{\sigma}{\gamma_0} \qquad (2)$$

Loss modulus G''

$$G'' := \lim_{\gamma(t)\to 0(\sigma\geq 0)} \frac{\sigma}{\gamma_0} \qquad (3)$$



We can measure G' (rigidity) & G'' (viscosity) simultaneously.

[4]

Previous researches

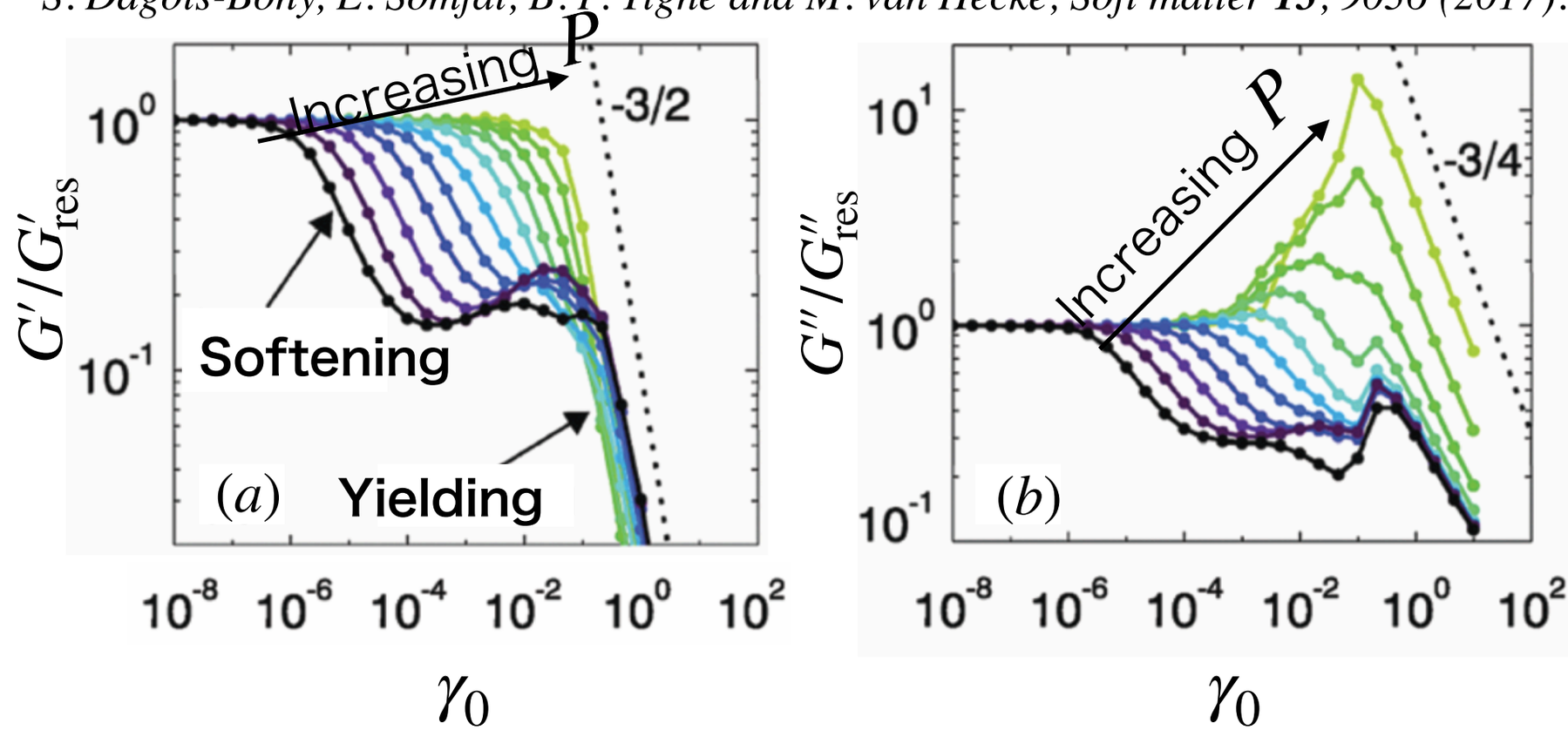
- 2D oscillatory shear
- <u>Frictionless particles</u>
- Background friction

$$(\sim -\eta \vec{\mathbf{v}})$$

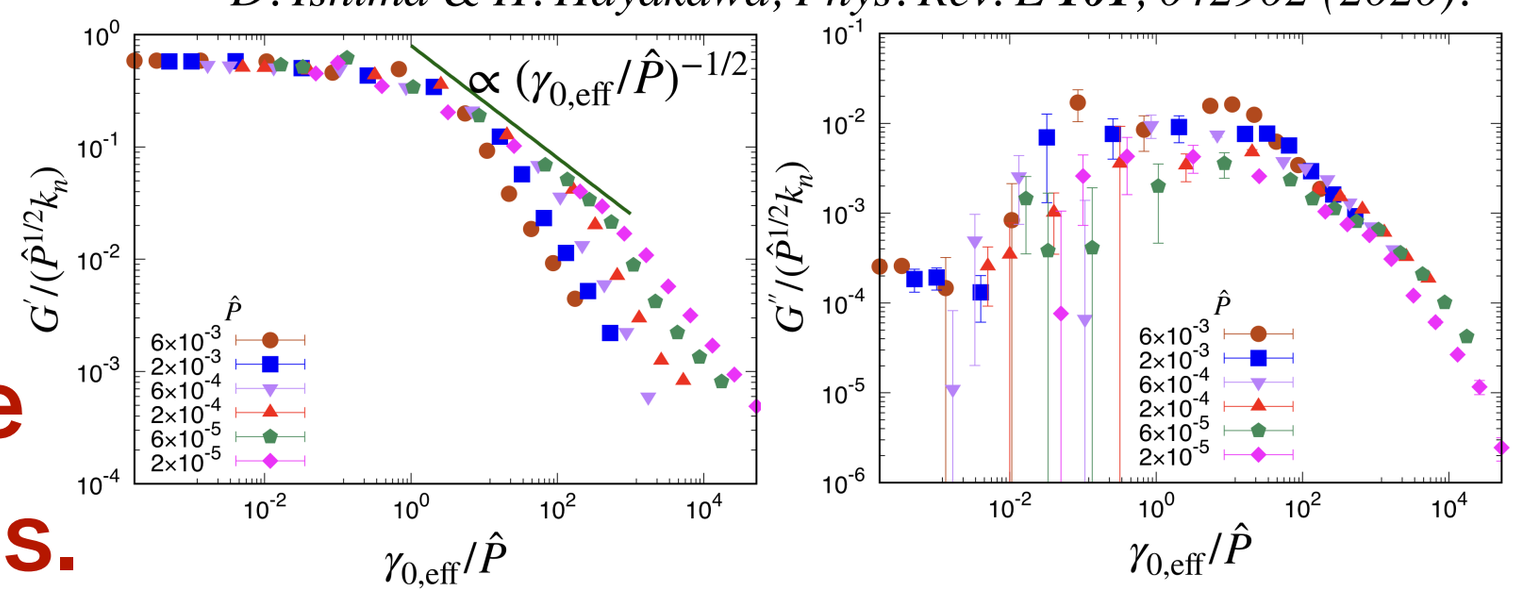
$$G'_{\mathrm{res}} := \lim_{\gamma_0 \to 0} G'$$

$$G''_{\mathrm{res}} := \lim_{\gamma_0 \to 0} G''$$

S. Dagois-Bohy, E. Somfai, B. P. Tighe and M. van Hecke, Soft matter ***13****, 9036 (2017).*



D. Ishima & H. Hayakawa, Phys. Rev. E ***101****, 042902 (2020).*



No scaling law

We cannot ignore the friction between grains.

Previous researches

- 2D oscillatory shear
- <u>Frictionless particles</u>
- Background friction

$$(\sim -\eta \vec{\mathbf{v}})$$

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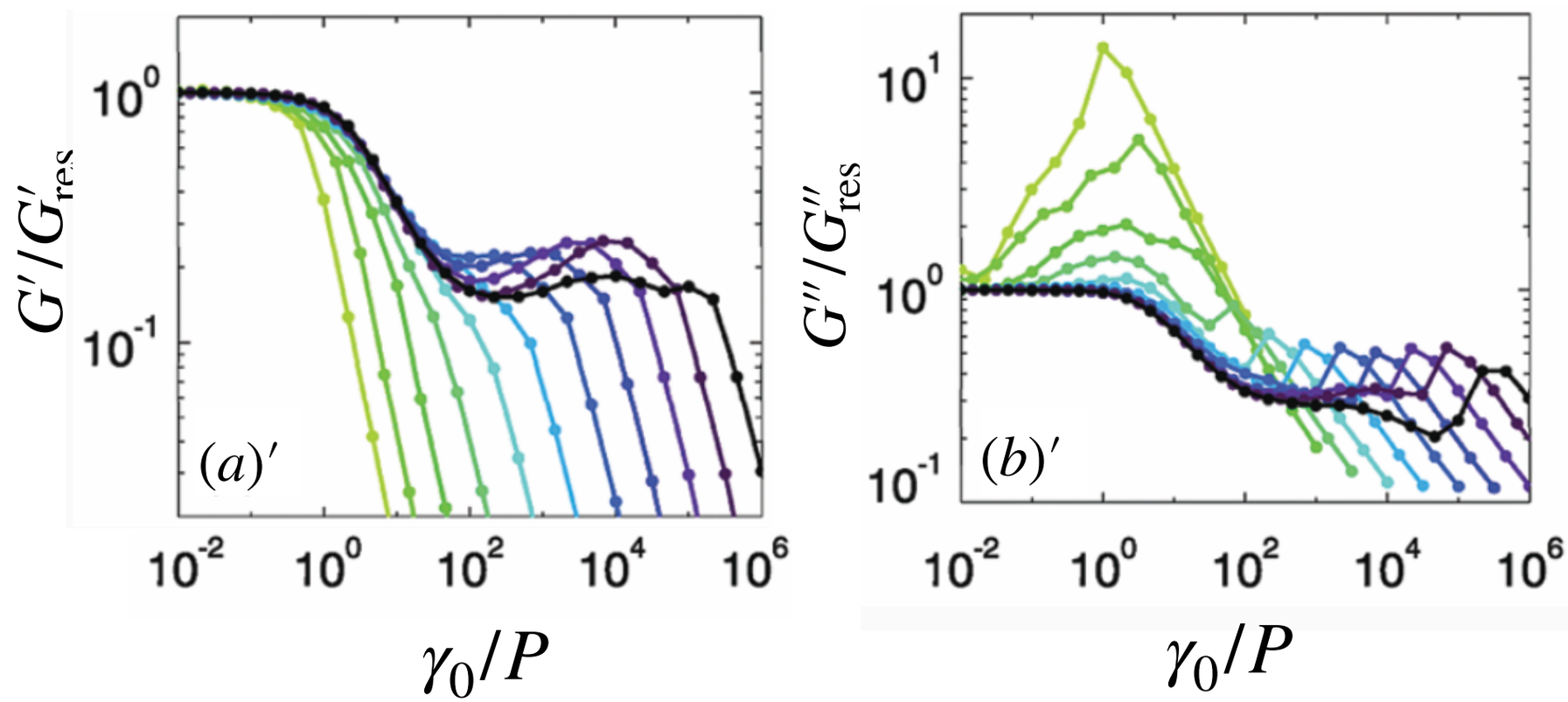

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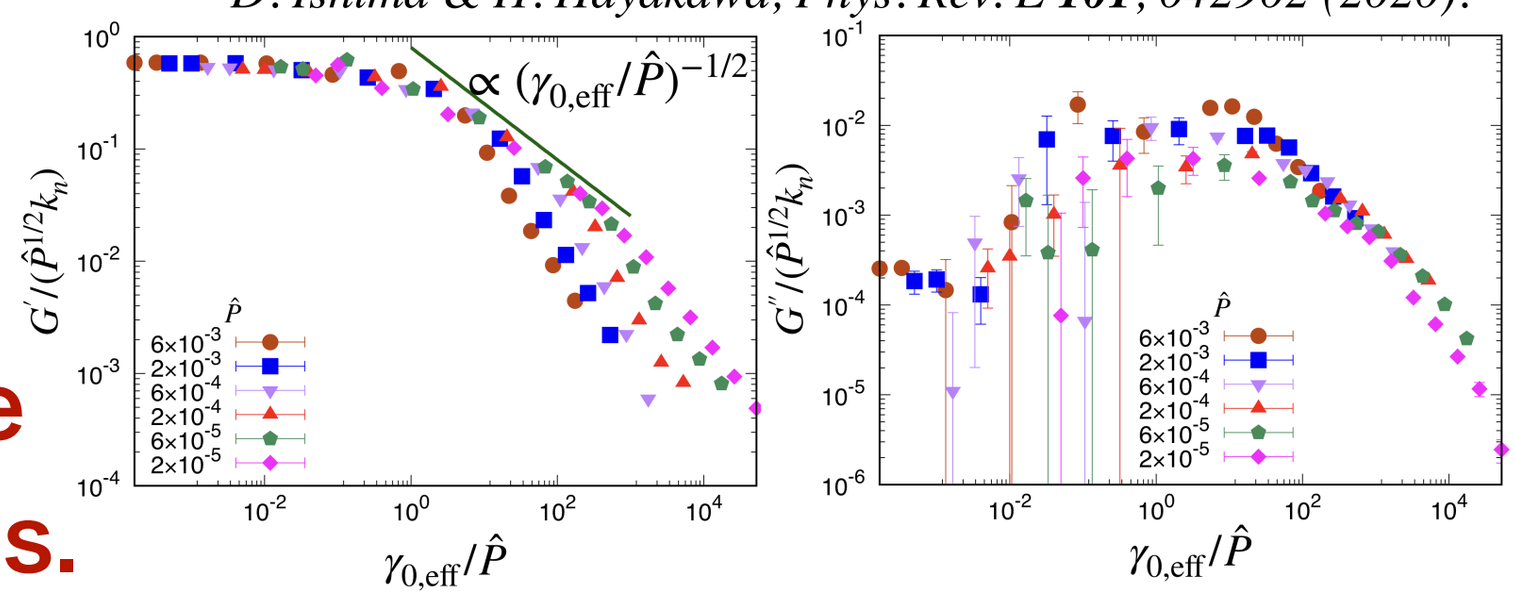


No scaling law

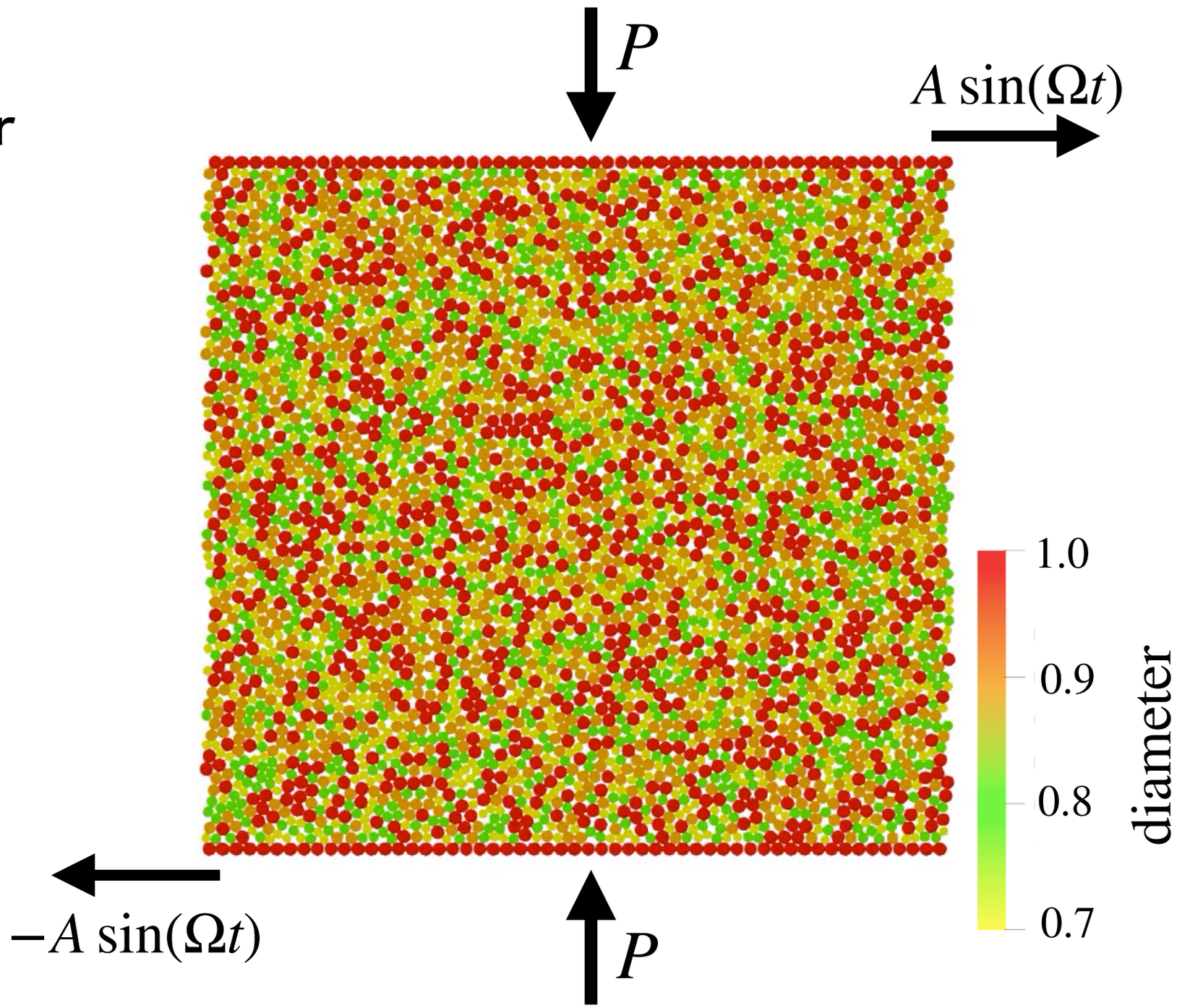
We cannot ignore the friction between grains.

How to apply shear

- 2D oscillatory shear
- Without $-\eta\vec{v}$
- <u>Frictional particles</u>
- $N = 4000$

$$\gamma_{0,\mathrm{eff}} := \frac{A}{L_x}$$

$$1.0 \times 10^{-6} \leq \gamma_{0,\mathrm{eff}} \leq 1.0$$

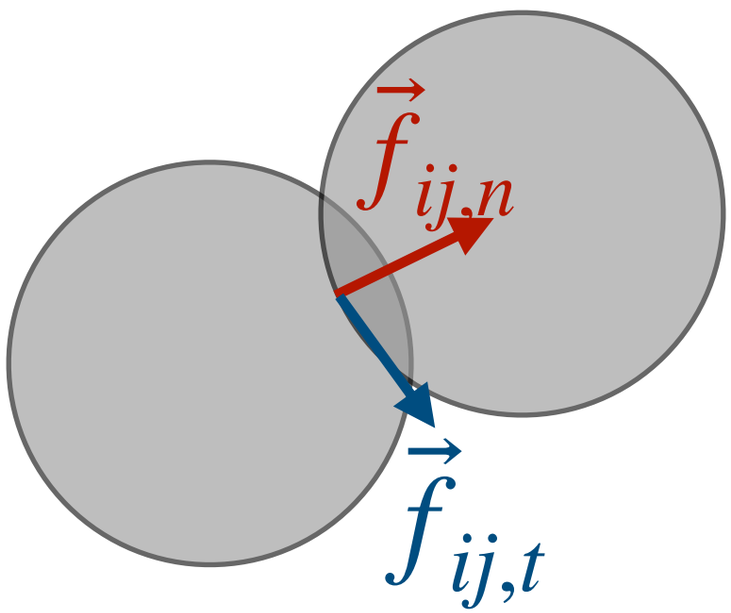


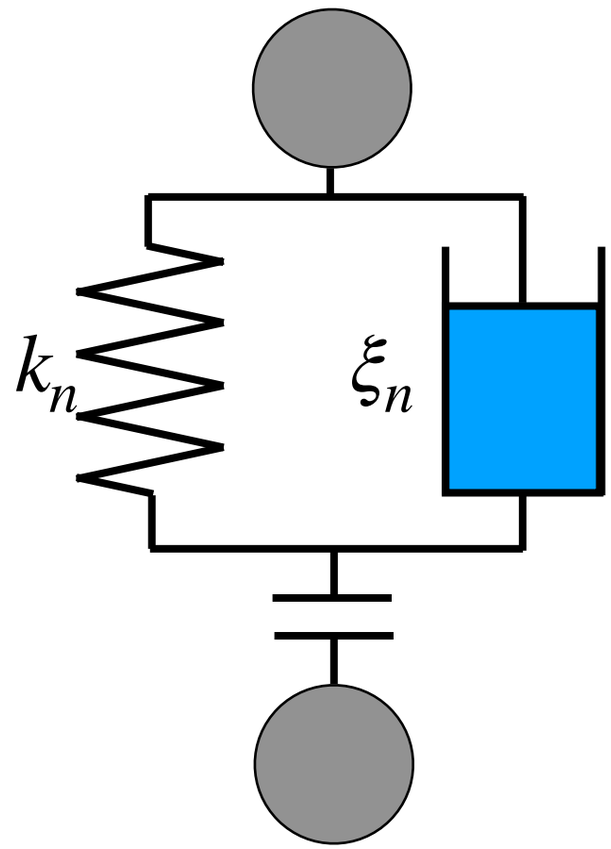
The walls are pressed with pressure P
and they move according to $\pm A\sin(\Omega t)$.

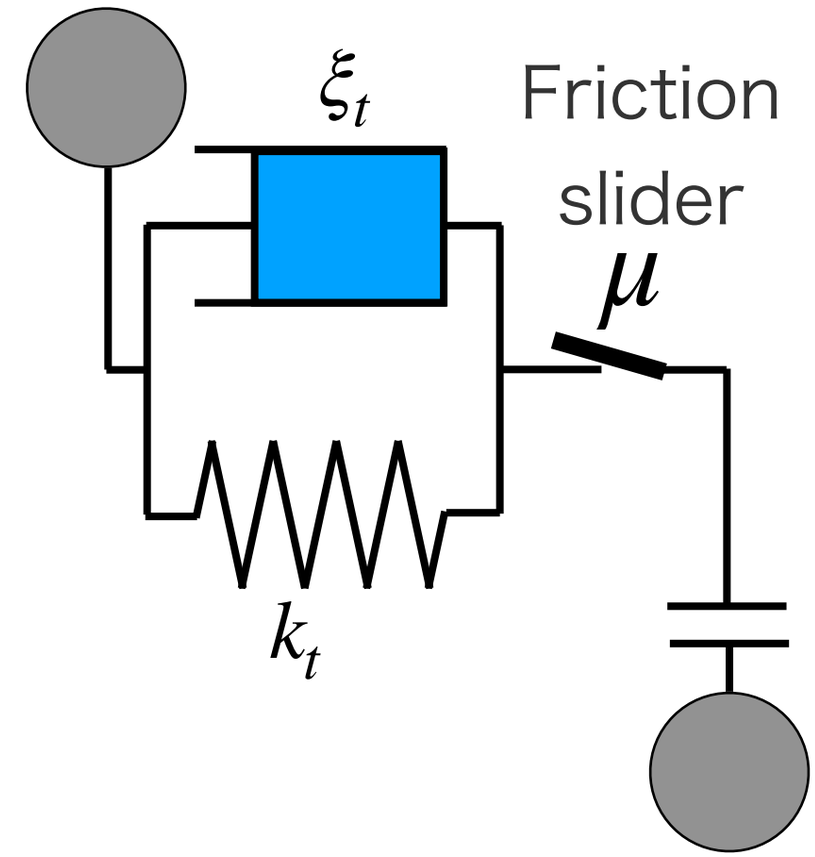
[6]

Discrete element method

Contact force: linear spring and dashpot







$\vec{f}_{ij,n}$: **Normal force:** $\vec{f}_{ij,t}$: **Tangential force**

S. Luding, Granular Matter ***10****, 235 (2008).*

Friction coefficient

$$0 \leq \mu \leq 1.0$$

Influence of preparation

We place small frictional particles at random without contact.

Increasing radius Δr after relaxation ($\Delta r = 10^{-4}$)

We obtain the state at ϕ_{ini}.

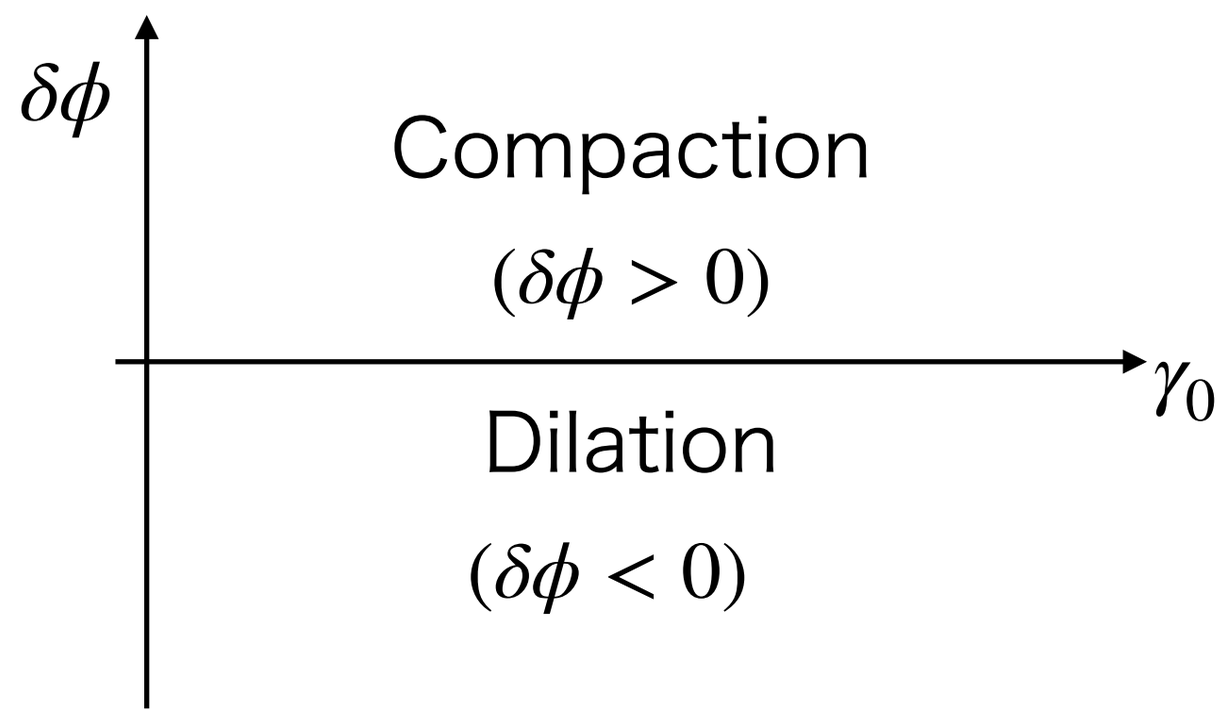
We compress both walls to achieve steady state.

We apply the system oscillatory shear.

See more details in D. Ishima & H. Hayakawa, Phys. Rev. E **101**, 042902 (2020).

$$\delta\phi := \phi(\Omega t = 2n\pi, \hat{P}, \gamma_{0,\mathrm{eff}}) - \phi_0(\hat{P})$$

$\phi_0(\hat{P})$: density without shear



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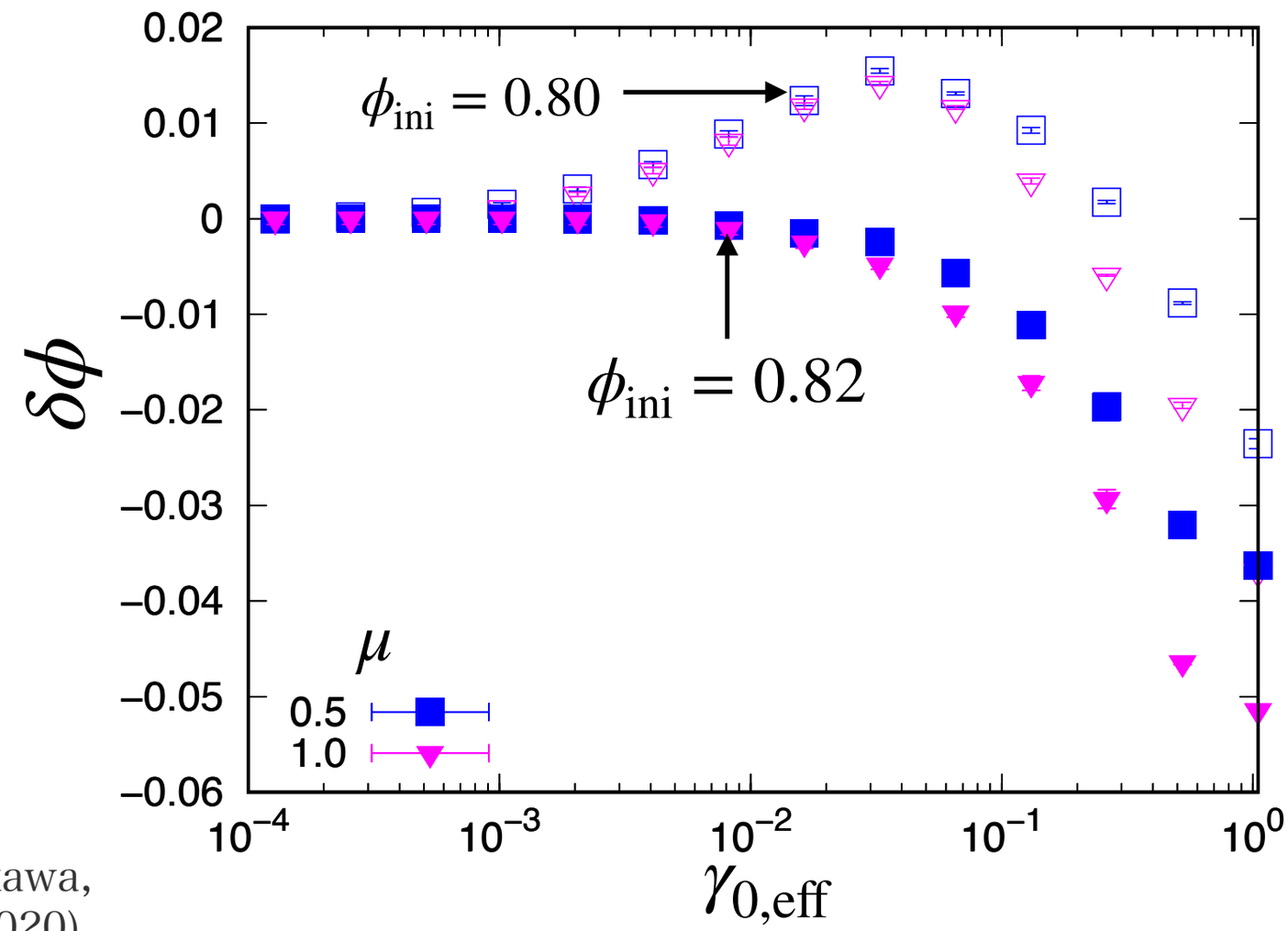
$\phi_0(\hat{P})$: density without shear

$$\hat{P} = 2.0 \times 10^{-5}$$



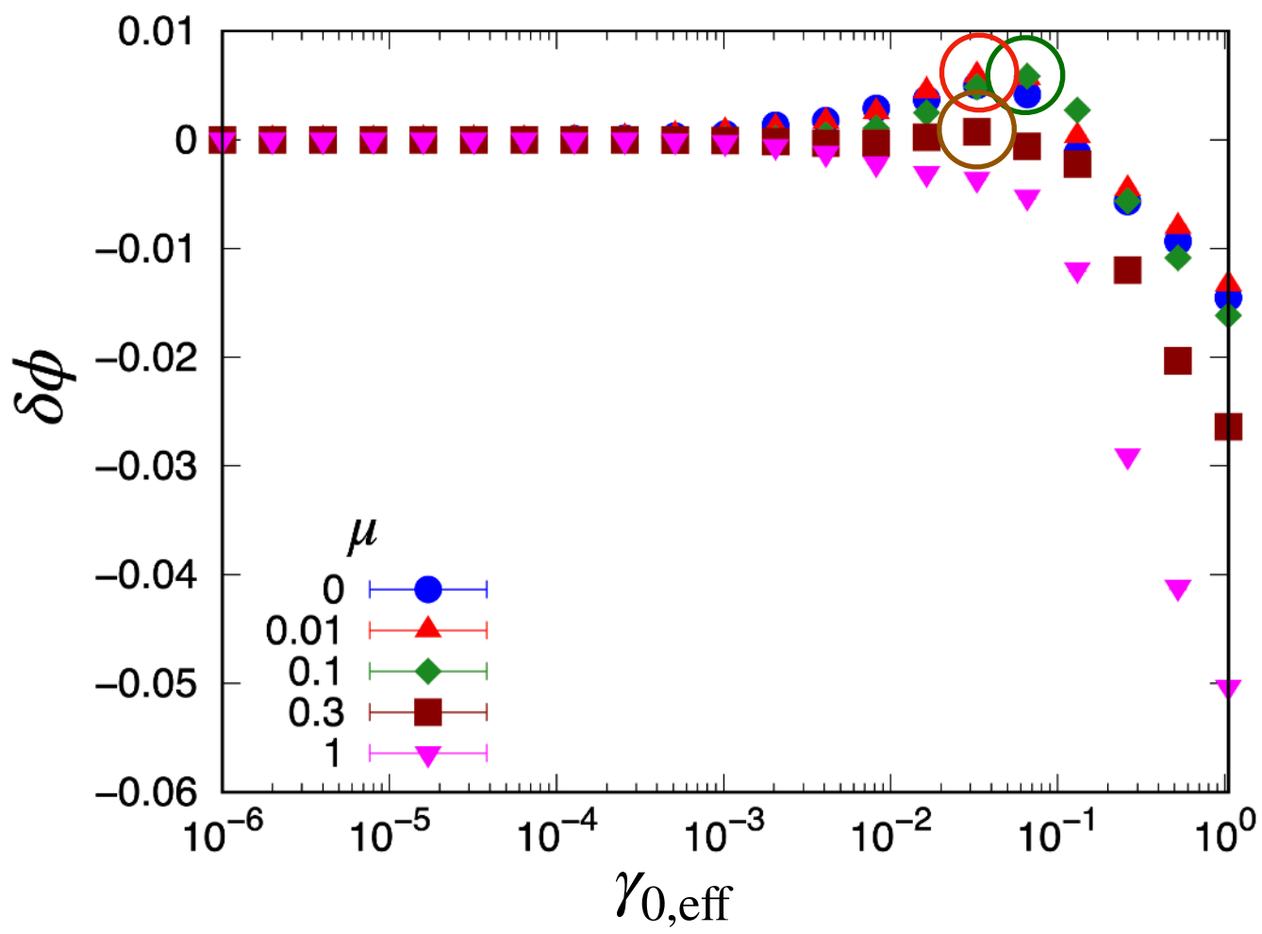
$\delta\phi$ depends on how to prepare initial configuration.

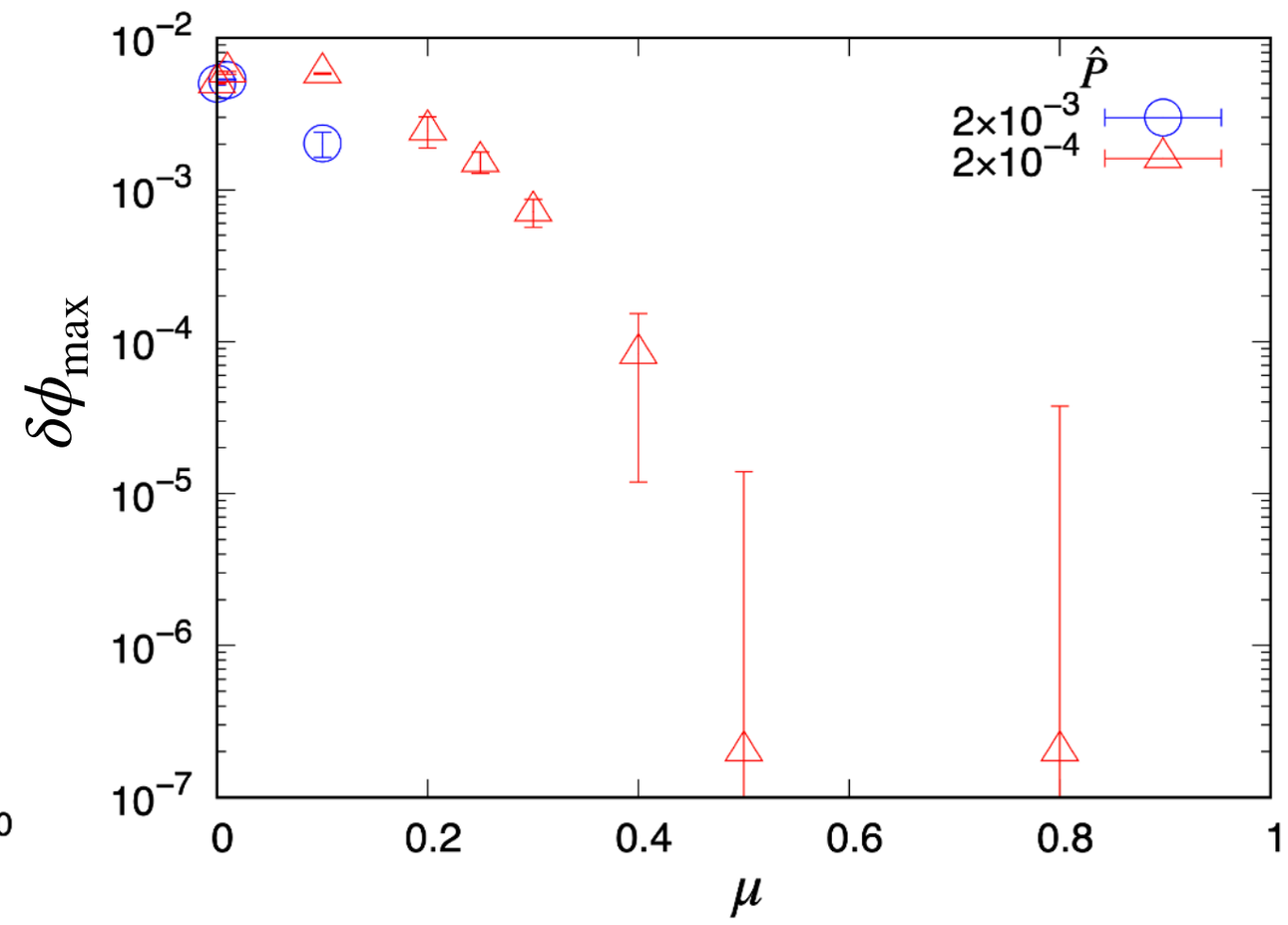
Influence of μ for dilation & compaction

$$\delta\phi := \phi(\Omega t = 2n\pi, \hat{P}, \gamma_{0,\mathrm{eff}}) - \phi_0(\hat{P})$$

$\phi_0(\hat{P})$: density without shear

$$\hat{P} = 2.0 \times 10^{-4} \quad (\hat{P} := P/k_n)$$

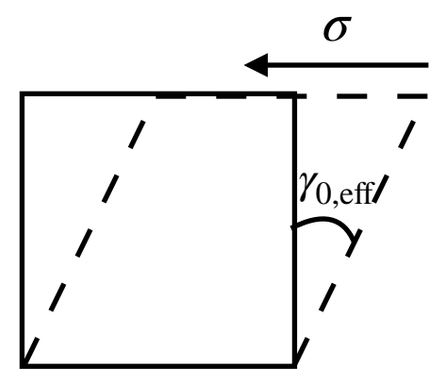




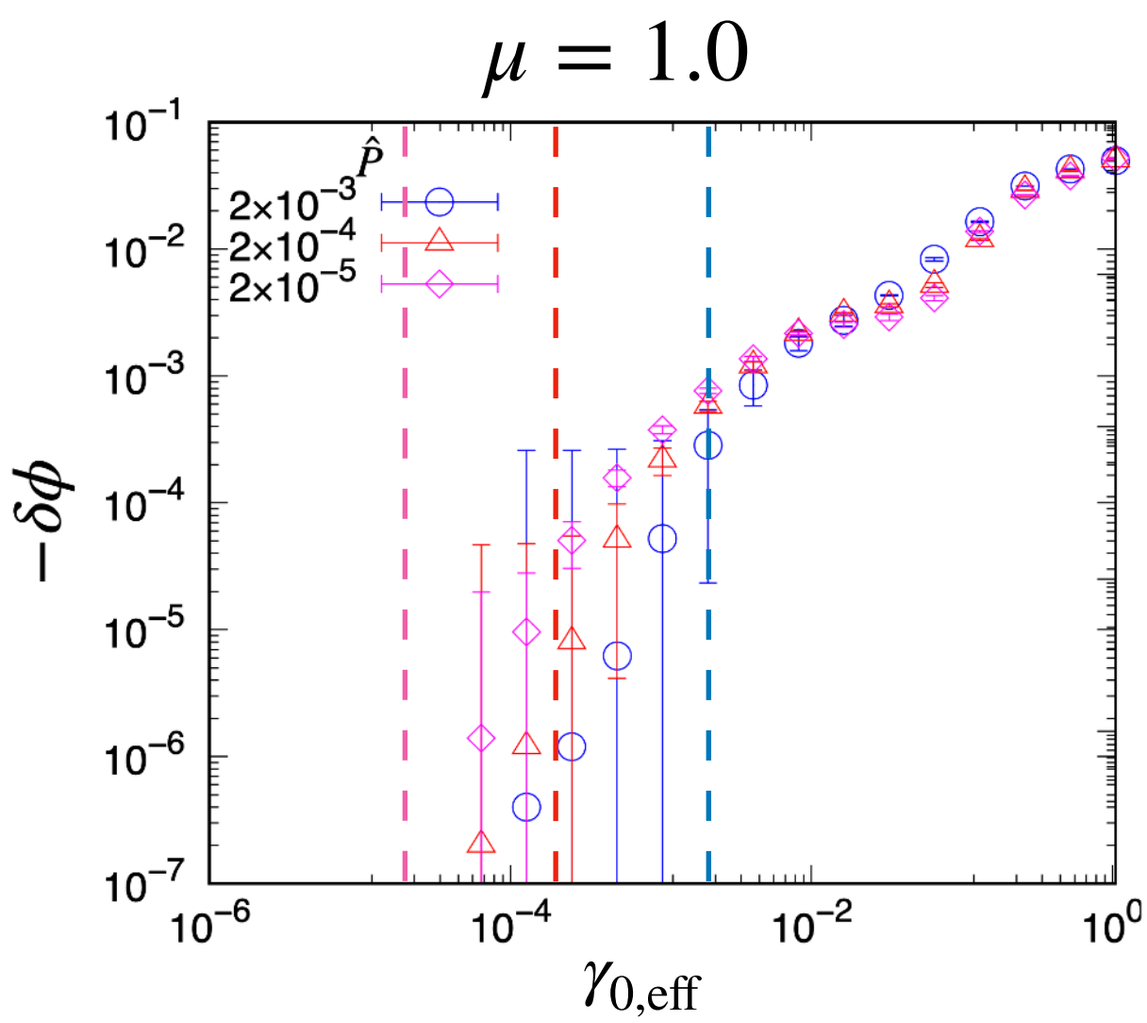
Dilatancy always takes place at large $\gamma_{0,\mathrm{eff}}$.

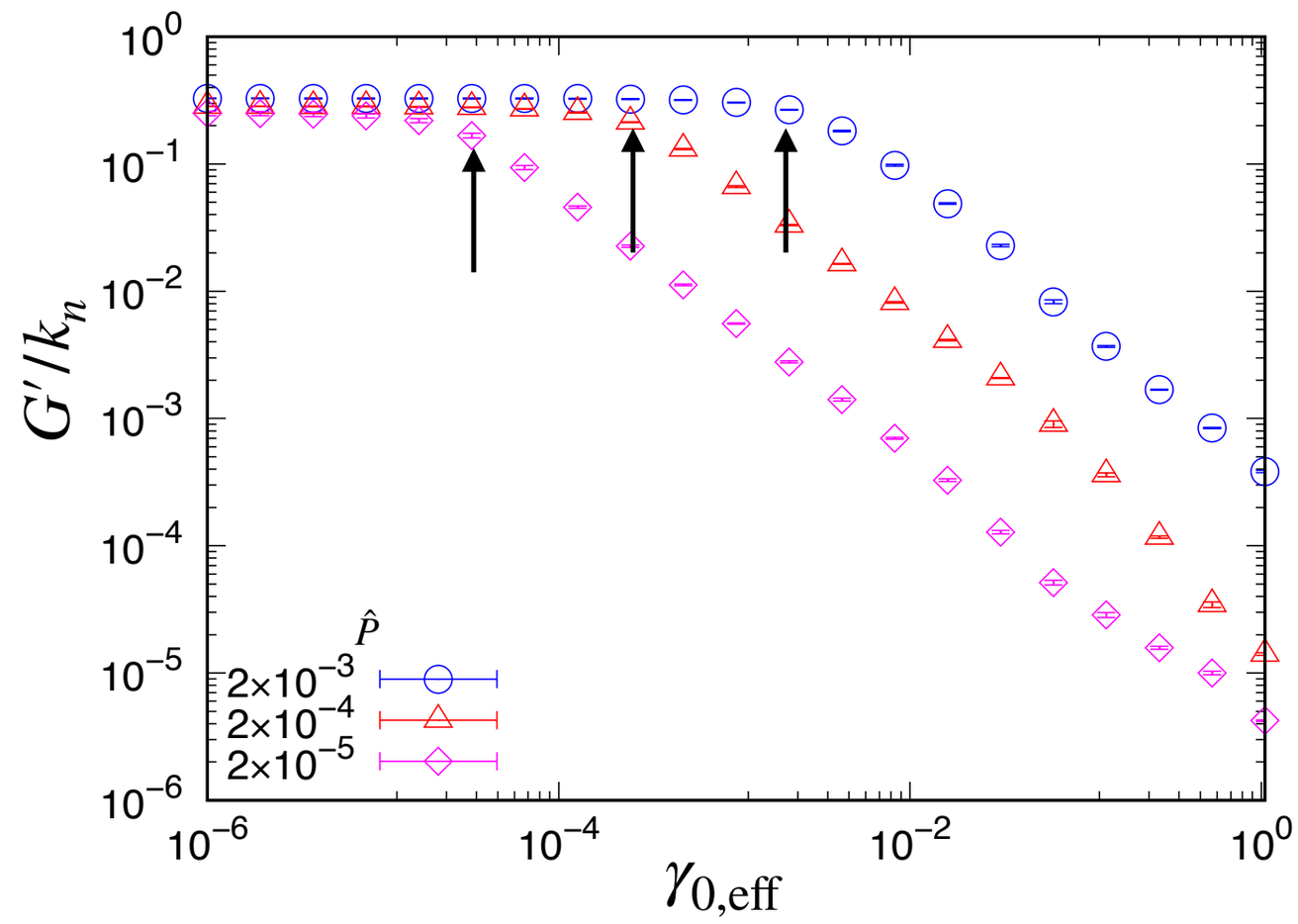
Compaction can be observed for $\mu < 0.4$ at intermediate $\gamma_{0,\mathrm{eff}}$.

Influence of $\hat{P}$ for dilatancy



$$G' := \frac{\sigma}{\gamma_{0,\mathrm{eff}}} \text{ : Rigidity,}$$





D. Ishima & H. Hayakawa, Phys. Rev. E ***101****, 042902 (2020).*

Dilatancy is almost independent of $\hat{P}$.

Bending point of G' depends on $\hat{P}$.

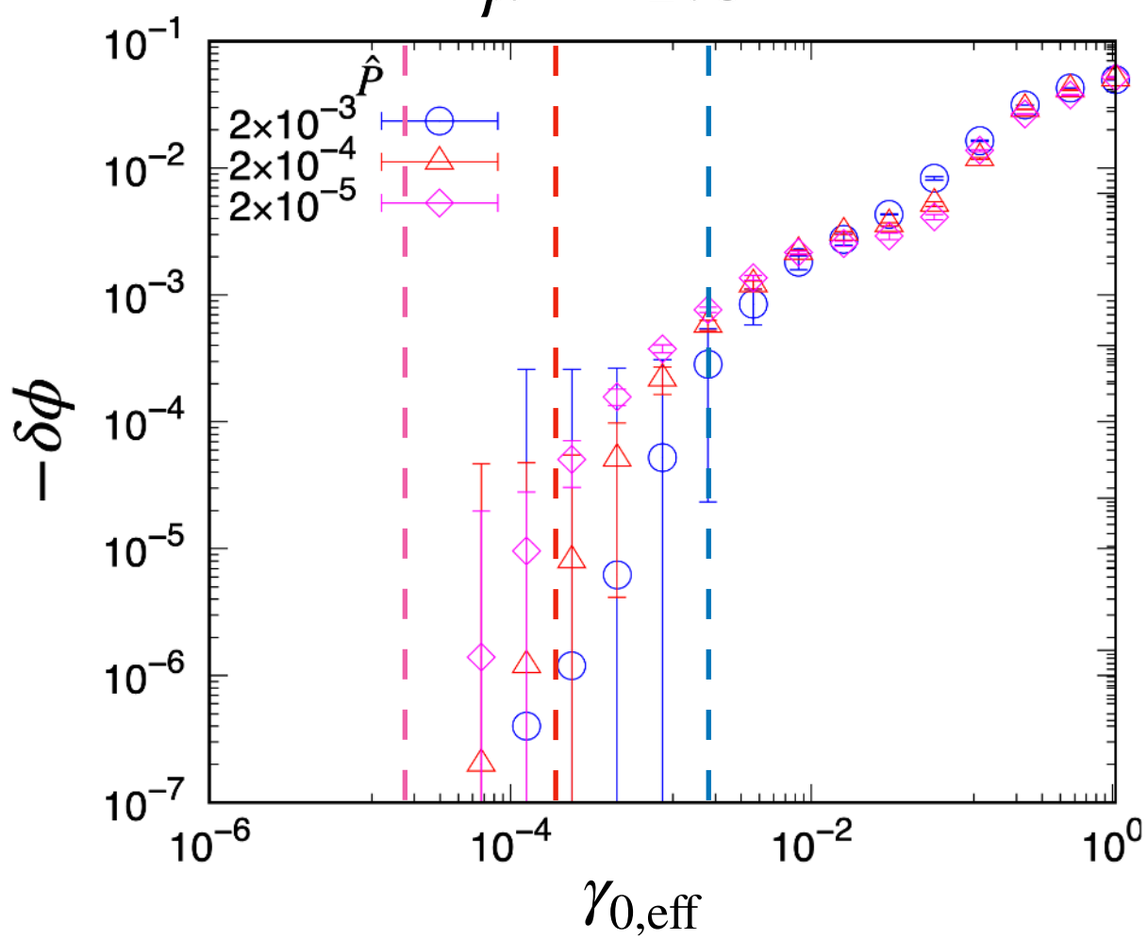
Relation - Mean square displacement

$$\Delta x^2(m) := \frac{1}{N}\sum_i |\vec{x}_i(m,\psi=0) - \vec{x}_i(0,\psi=0)|^2,$$

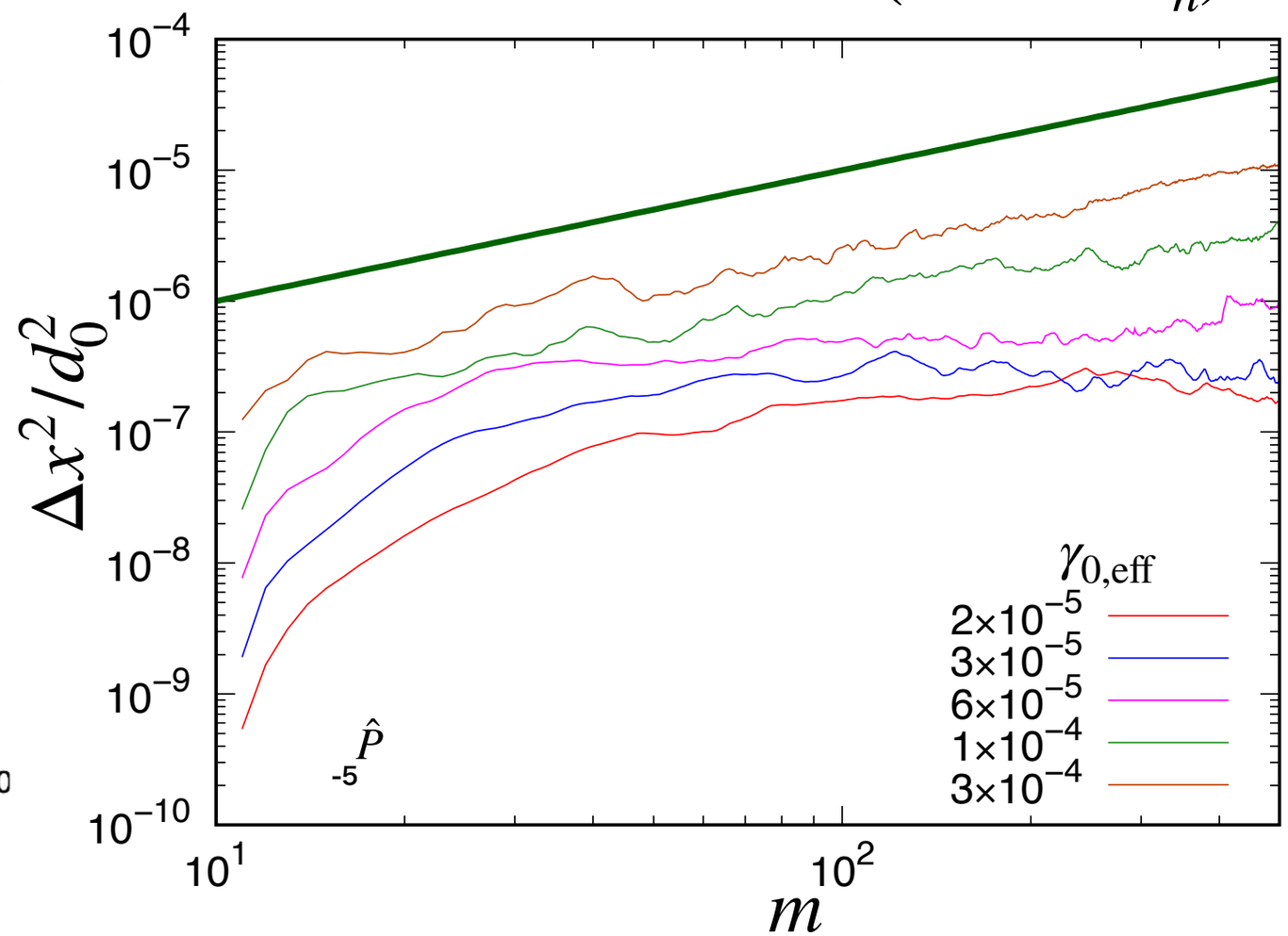
$$t = \frac{2\pi}{\Omega}(m+\psi)$$

m : number of oscillation
ψ : oscillatory phase

$\mu = 1.0$



$\hat{P} = 2.0\times10^{-5} \quad (\hat{P} := P/k_n)$



Reversible motion above bending point of G' at $\hat{P} = 2.0\times10^{-5}$

Relation between MSD & dilatancy

Relation - Mean square displacement

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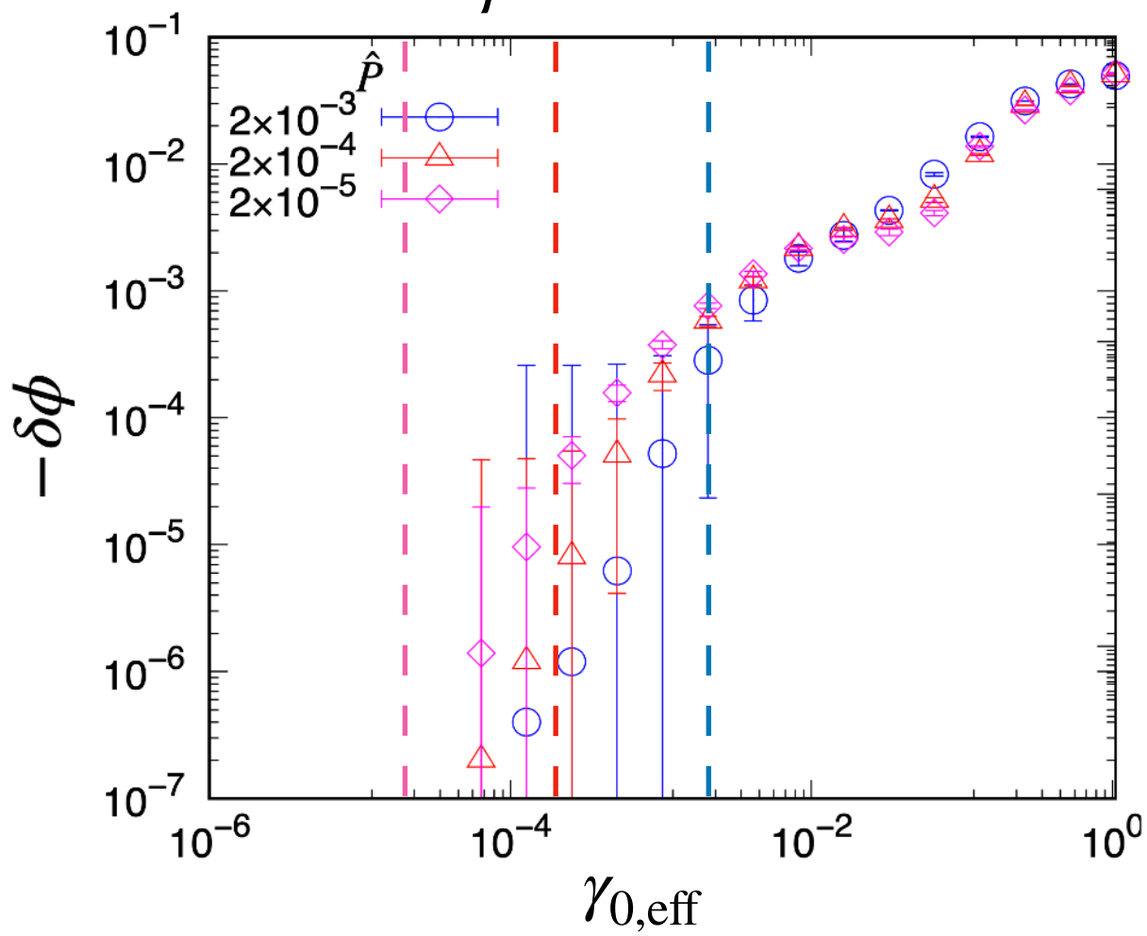
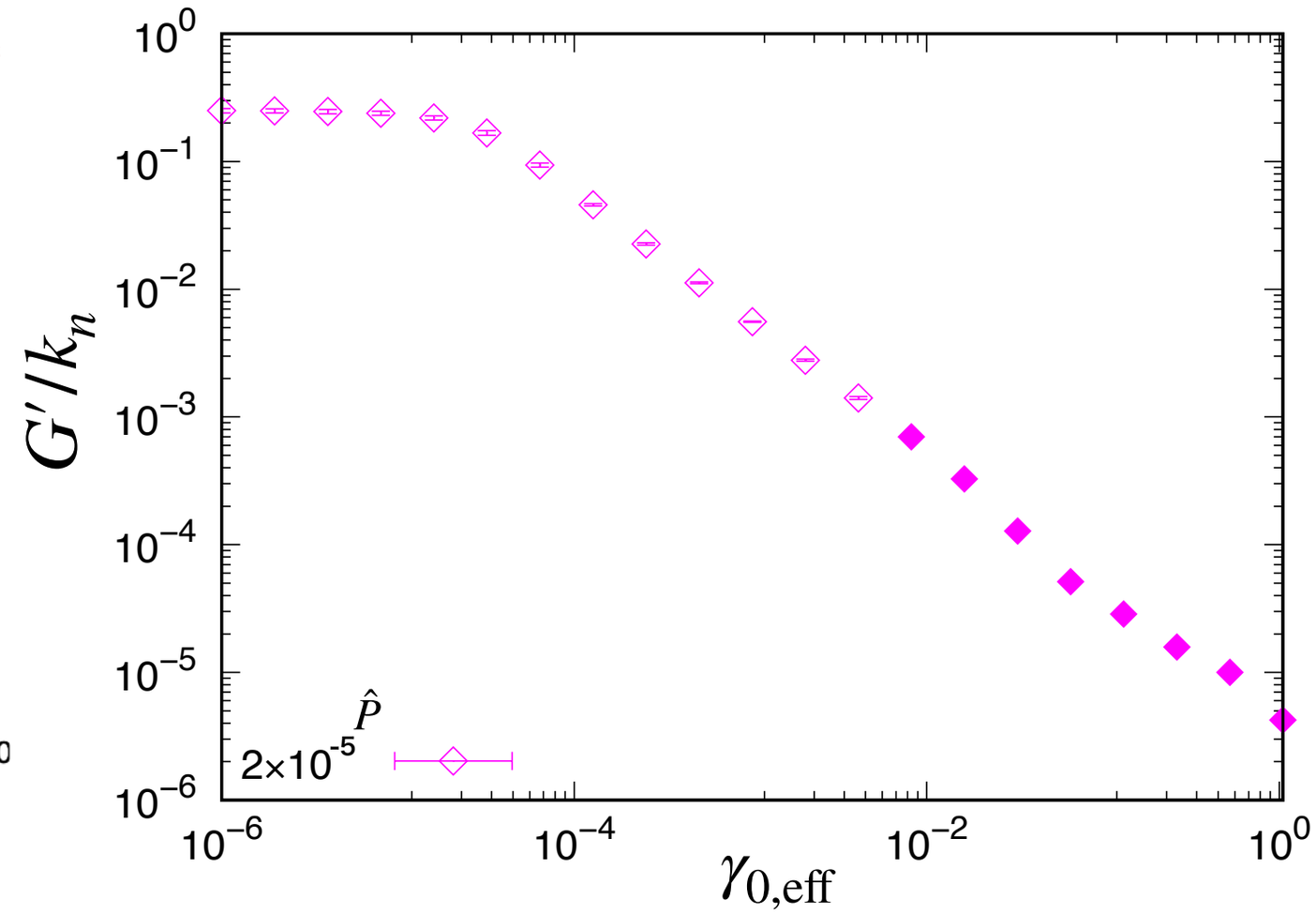


$\hat{P} = 2.0\times10^{-5} \quad (\hat{P} := P/k_n)$



Reversible motion above bending point of G' at $\hat{P} = 2.0\times10^{-5}$

Relation between MSD & dilatancy

Relation - Mean square displacement

$$\Delta x^2(m) := \frac{1}{N}\sum_i |\vec{x}_i(m,\psi=0) - \vec{x}_i(0,\psi=0)|^2,$$

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ψ : oscillatory phase

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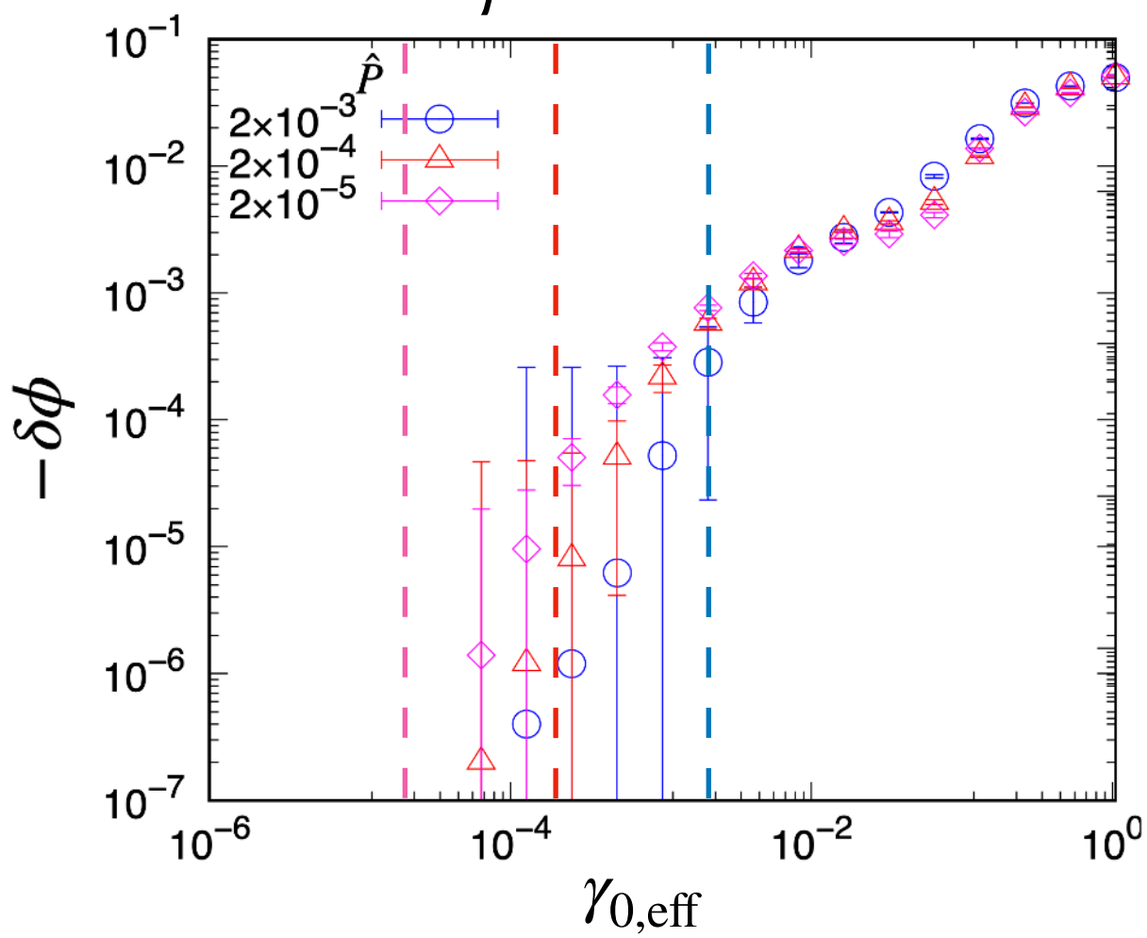
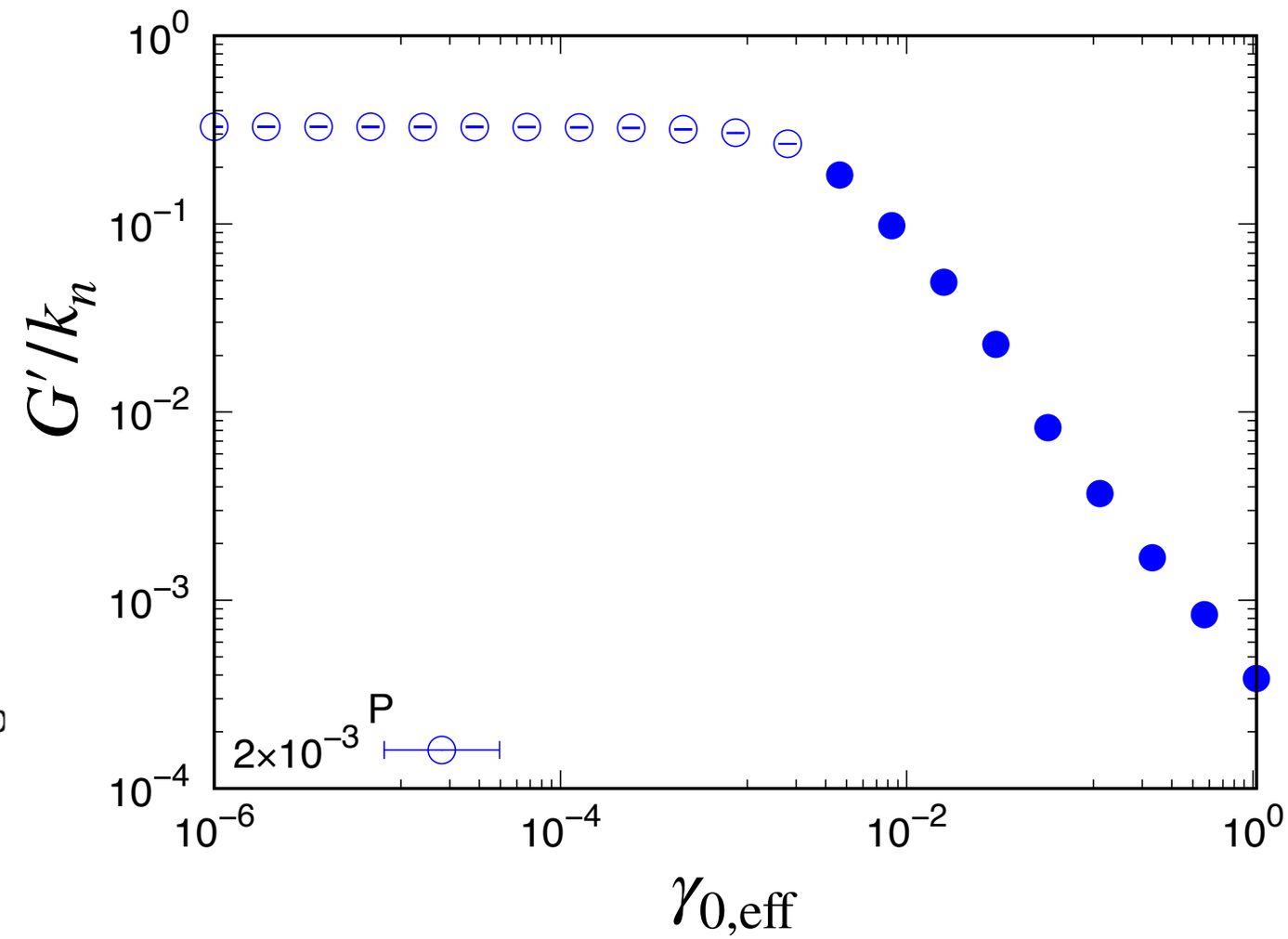


$\hat{P} = 2.0 \times 10^{-3} \quad (\hat{P} := P/k_n)$

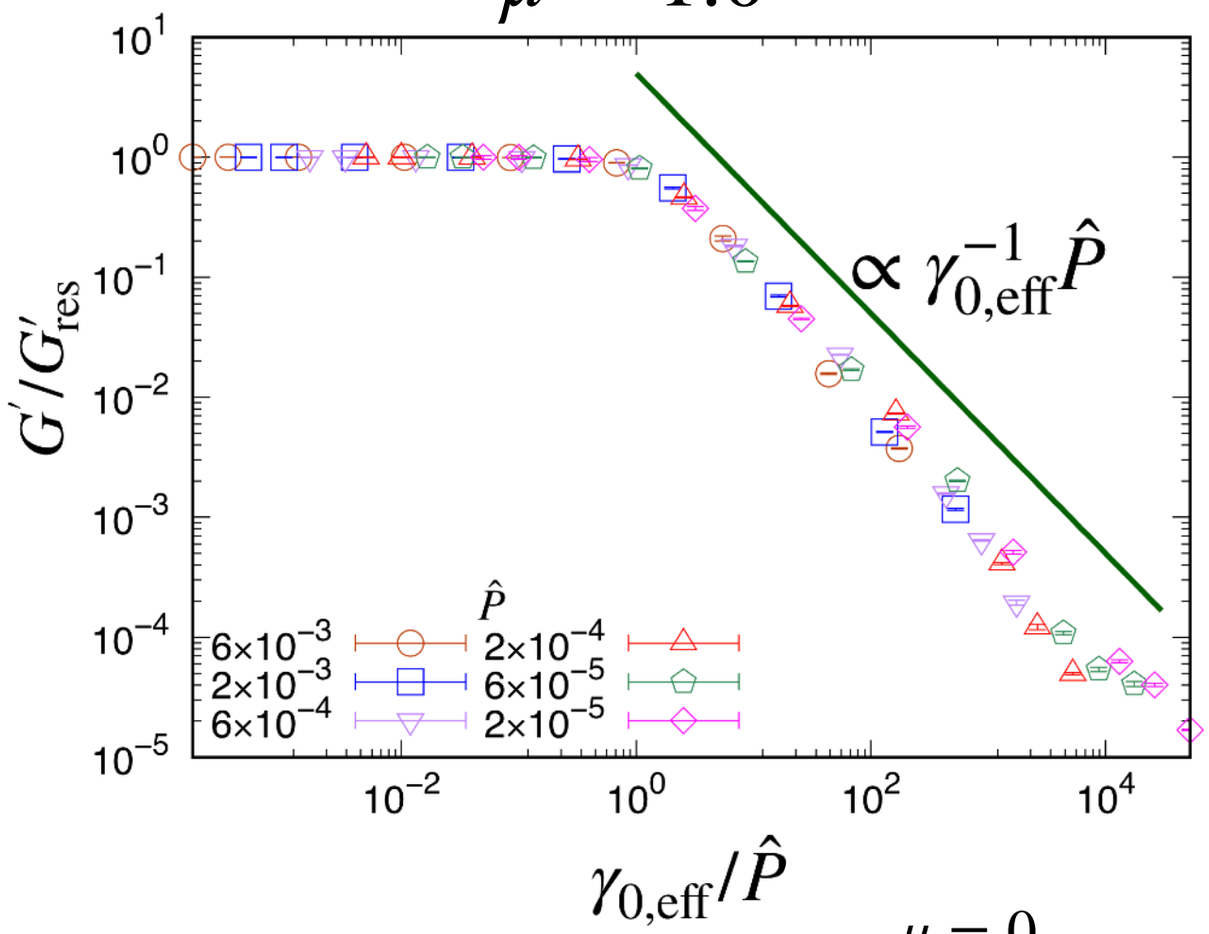


Reversible motion above bending point of G' at $\hat{P} = 2.0 \times 10^{-5}$

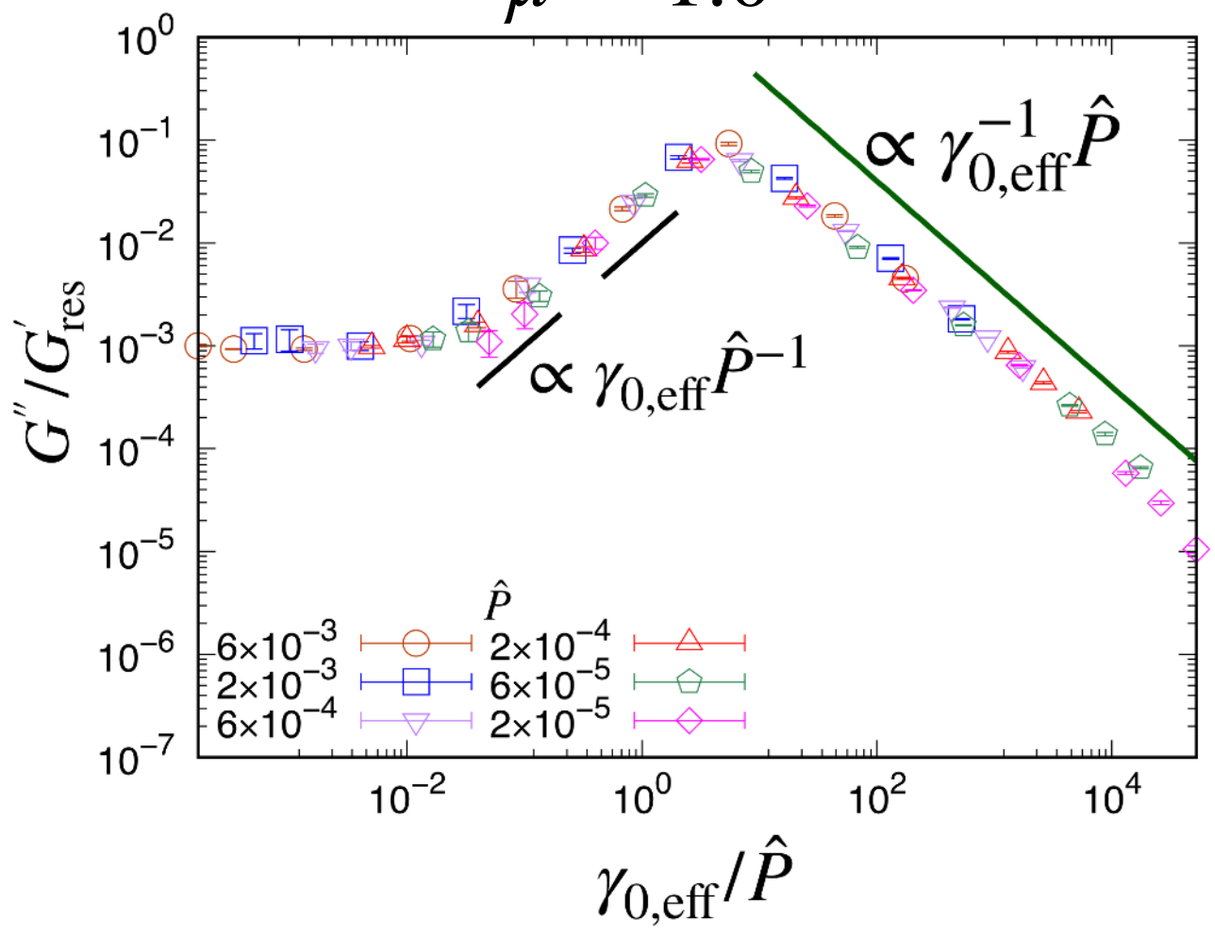
Relation between MSD & dilatancy

Scaling laws for G' & G''

$\mu = 1.0$

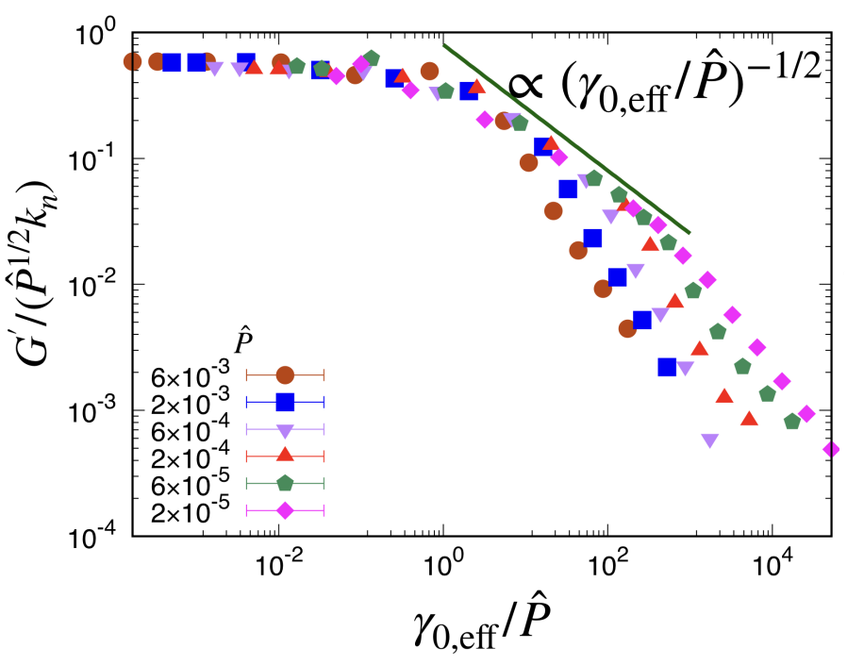


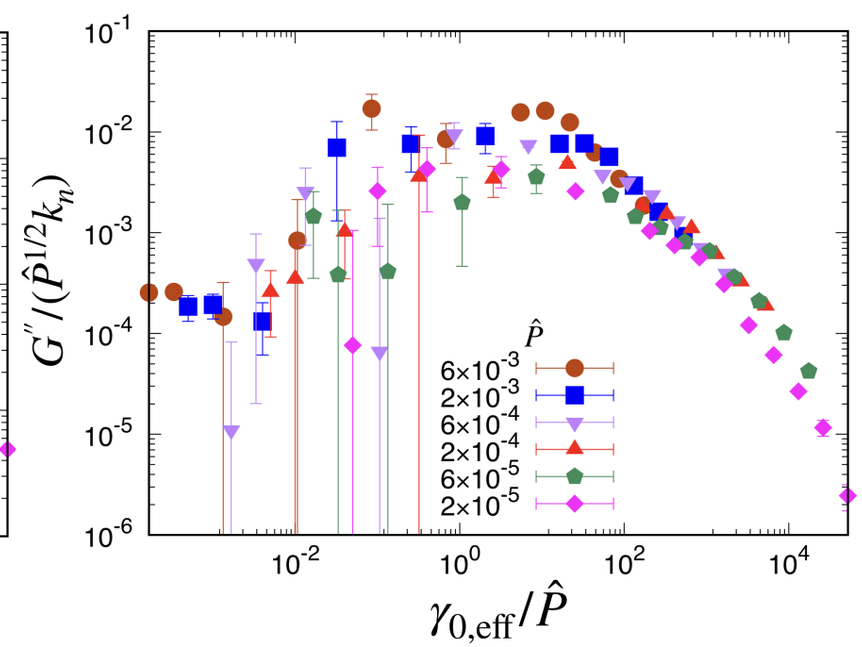
$\mu = 1.0$



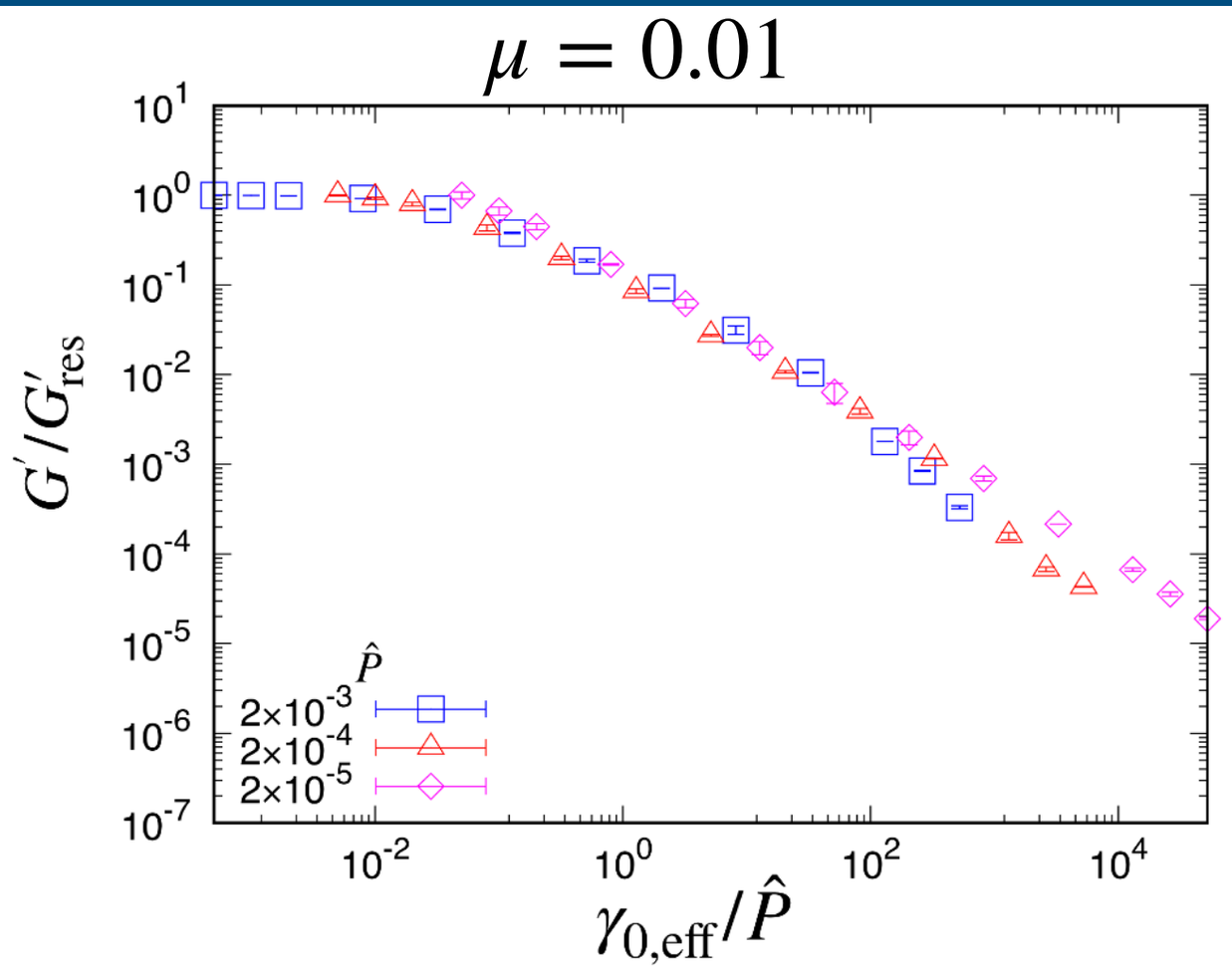
$\mu = 0$

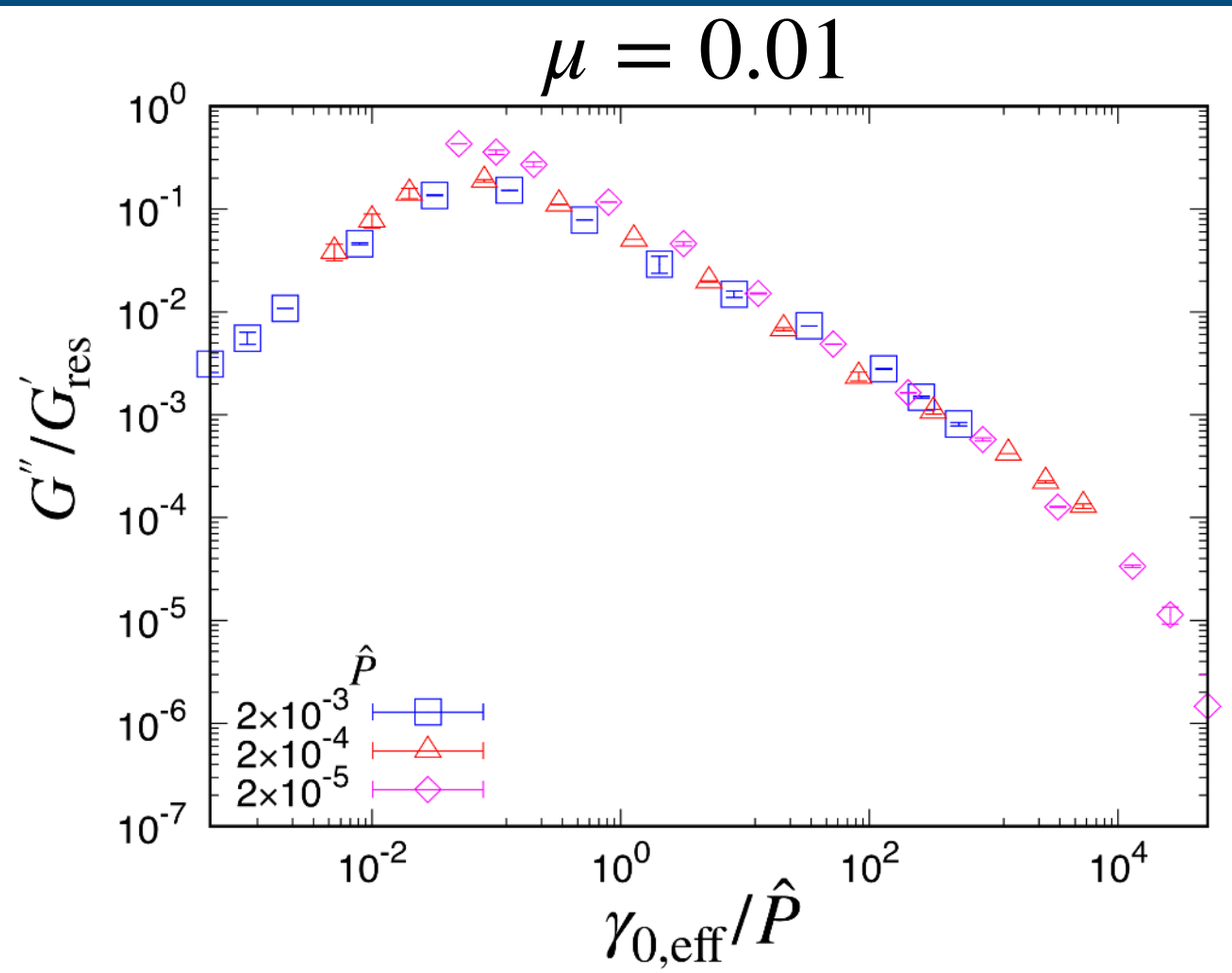
G' & G'' satisfy the scaling laws





Scaling laws for G' & G''





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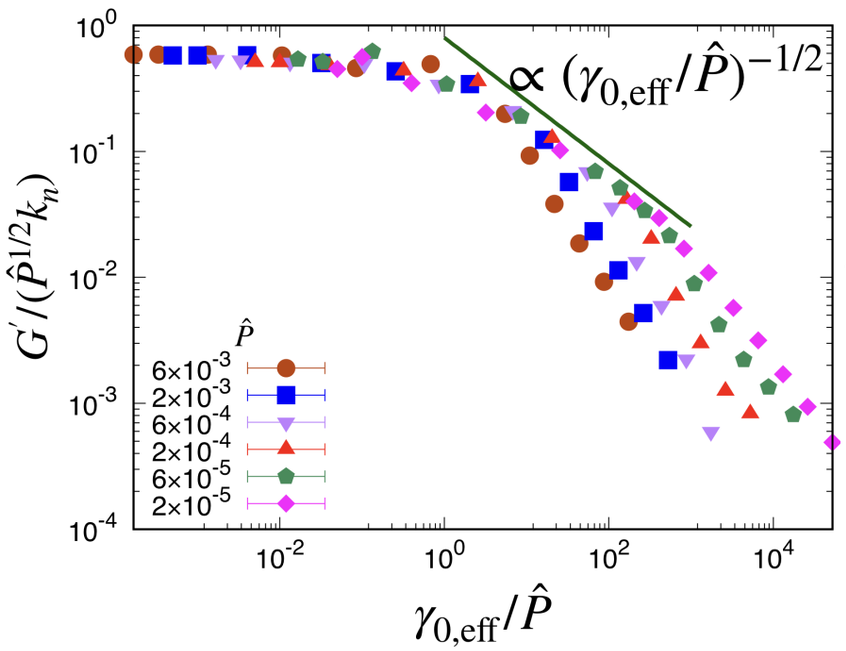
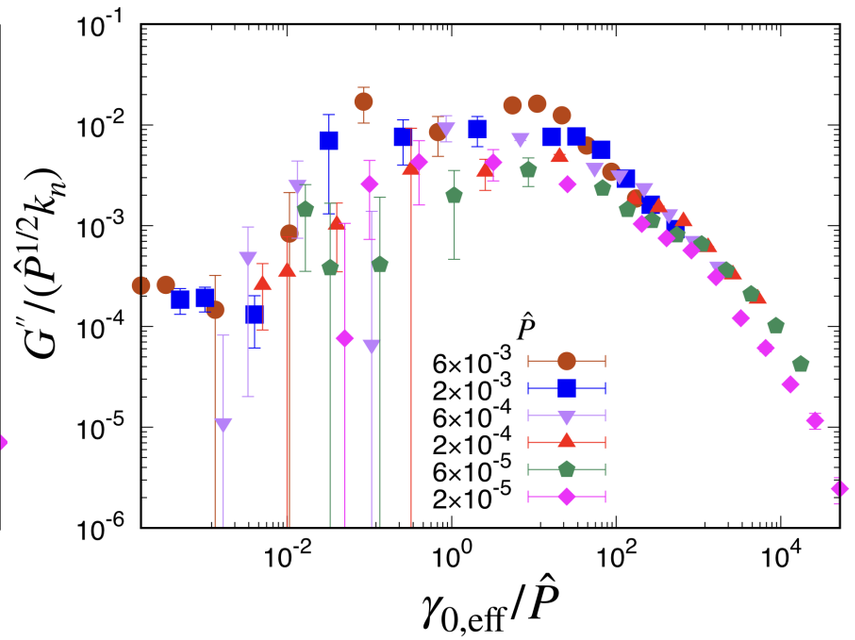




Scaling law for G'

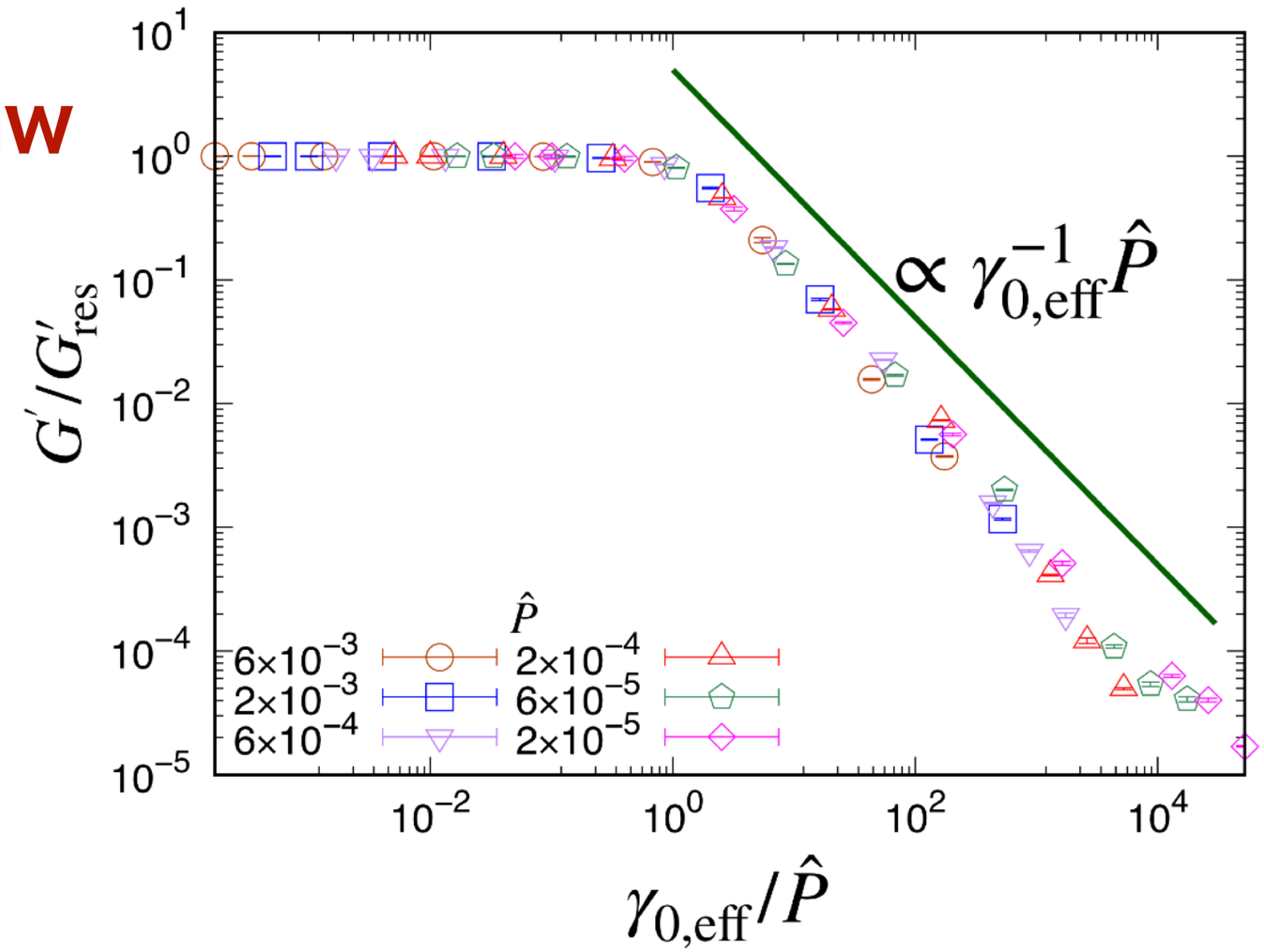
G' satisfies the scaling law

The scaling function of G':

$$G'(\gamma_{0,\mathrm{eff}}, \hat{P}) := G'_{\mathrm{res}}(\hat{P})\mathscr{G}'\left(\frac{\gamma_{0,\mathrm{eff}}}{\hat{P}^{\beta'_\mu}}\right)$$

$$G'_{\mathrm{res}}(\hat{P}) := \lim_{\gamma_{0,\mathrm{eff}}\to 0} G'(\gamma_{0,\mathrm{eff}}, \hat{P}),$$

$$\lim_{x\to 0}\mathscr{G}'(x) = 1, \quad \lim_{x\to\infty}\mathscr{G}' \sim x^{-1}.$$



Phenomenological discussion for G'

- $\sigma \propto \gamma_{0,\mathrm{eff}}$ for Hookean regime,

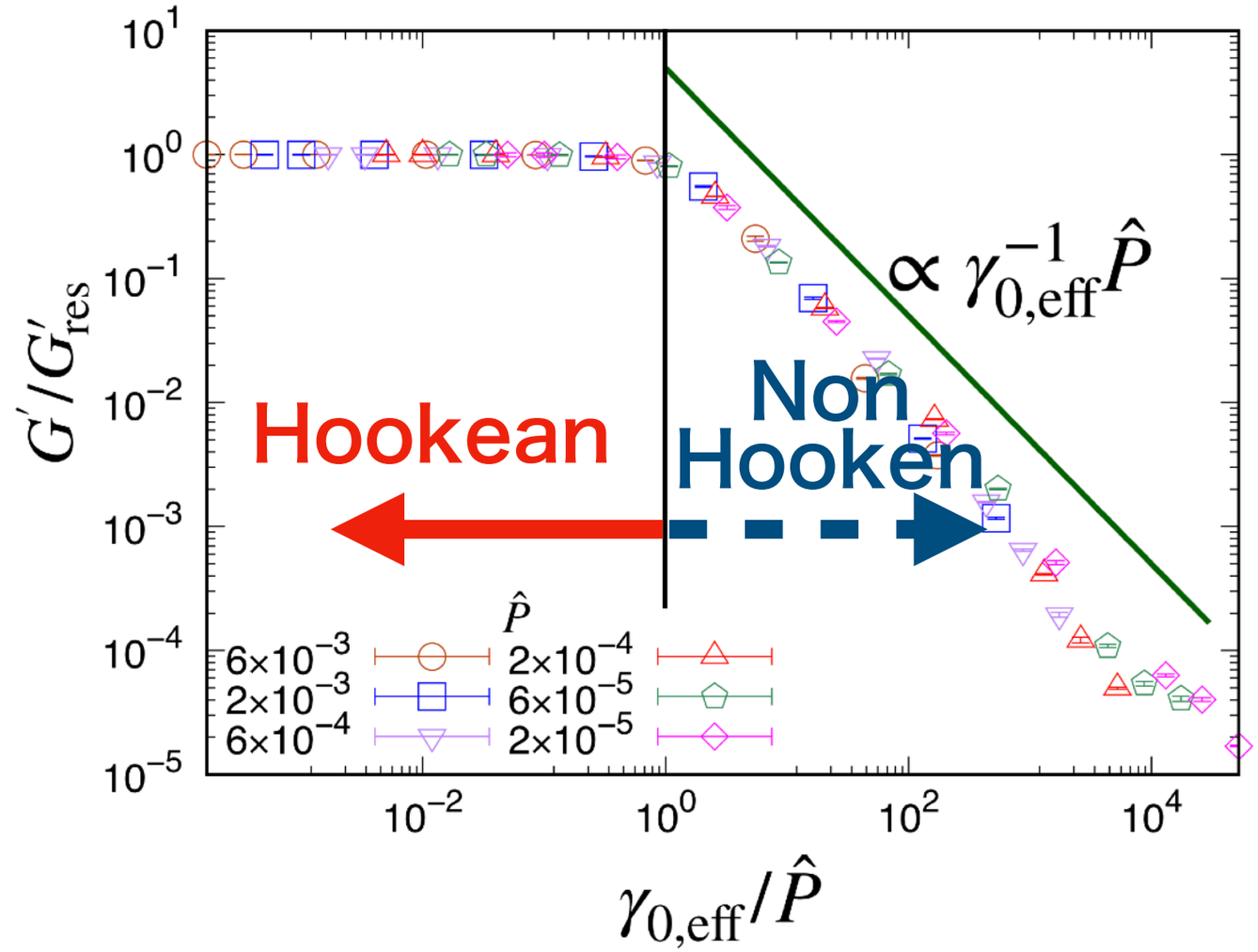
  $\therefore$ For Hookean regime

  G' is independent of $\gamma_{0,\mathrm{eff}}$.

- Because $\mu_{\mathrm{m}} := \sigma/P$ is constant for plastic regime, $\sigma \propto P$.

  $\therefore$ For plastic regime

  $G' \simeq \sigma/\gamma_{0,\mathrm{eff}} \propto P/\gamma_{0,\mathrm{eff}}$,



We can estimate that the turning point between

$G' \simeq G'_{\mathrm{res}} \simeq k_t$ & $G' \simeq P/\gamma_{0,\mathrm{eff}} \sim O(1)$ is $\gamma_{0,\mathrm{eff}}/\hat{P} \sim O(1)$.

$(\hat{P} := P/k_n, \quad k_t/k_n = 0.5)$

Scaling law for G''

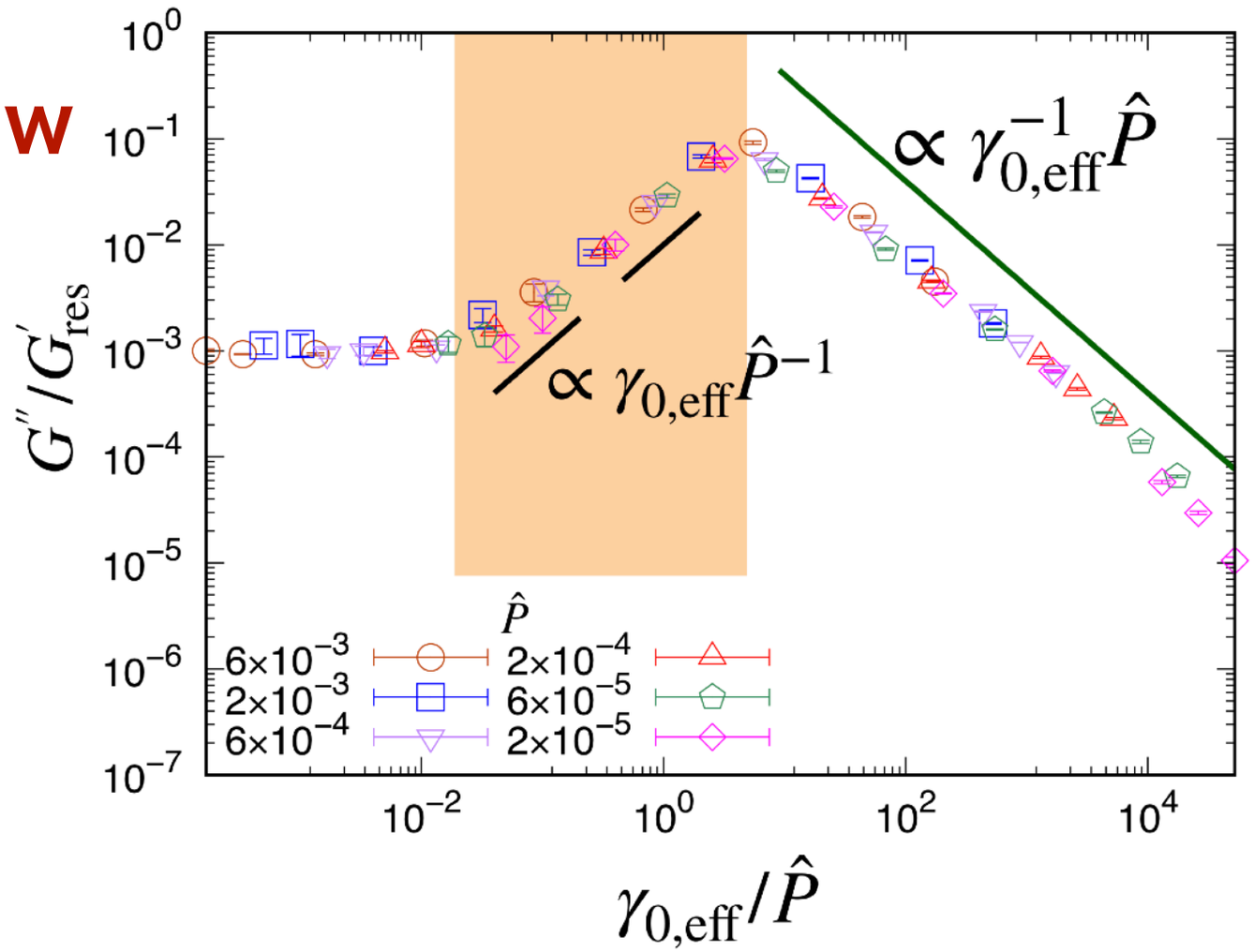
G'' satisfies the scaling law

The scaling function of G'':

$$G''(\gamma_{0,\mathrm{eff}}, \hat{P}) := G'_{\mathrm{res}}(\hat{P})\mathscr{G}''\left(\frac{\gamma_{0,\mathrm{eff}}}{\hat{P}^{\beta''_{\mu}}}\right)$$

$$\lim_{x\to 0} \mathscr{G}''(x) = \mathrm{const.},$$

$$\lim_{x\to\infty} \mathscr{G}'' \sim x^{-1}$$



Because of the limitation of my talk time,

I would like to skip the explanation of $G'' \propto \gamma_{0,\mathrm{eff}}/\hat{P}$.

For details see D.Ishima & H. Hayakawa, Phys. Rev. E, 101, 042902 (2020).

Summary

We have investigated the oscillatory sheared granular materials under constant pressure.

- **Compaction ⋯ It depends on μ & how to prepare initial configuration.**
- **Dilatancy is caused by irreversible motion of particles. However, it is not directly related to bending point of G'.**
- **Scaling laws ⋯ G' & G'' satisfy scaling laws for frictional grains.**

See more details in

D.Ishima & H. Hayakawa, Phys. Rev. E, **101**, 042902 (2020),

D.Ishima & H. Hayakawa, arXiv: 2011.06891.