Impact-induced hardening in dense suspensions

Pradipto and H.Hayakawa, Phys. Rev. Fluids **6**, 033301 (2021)

Pradipto and Hisao Hayakawa
(Yukawa Institute for Theoretical Physics, Kyoto University)
Advanced research on glass and related complex systems
(ISSP, University of Tokyo)
5/11/2021

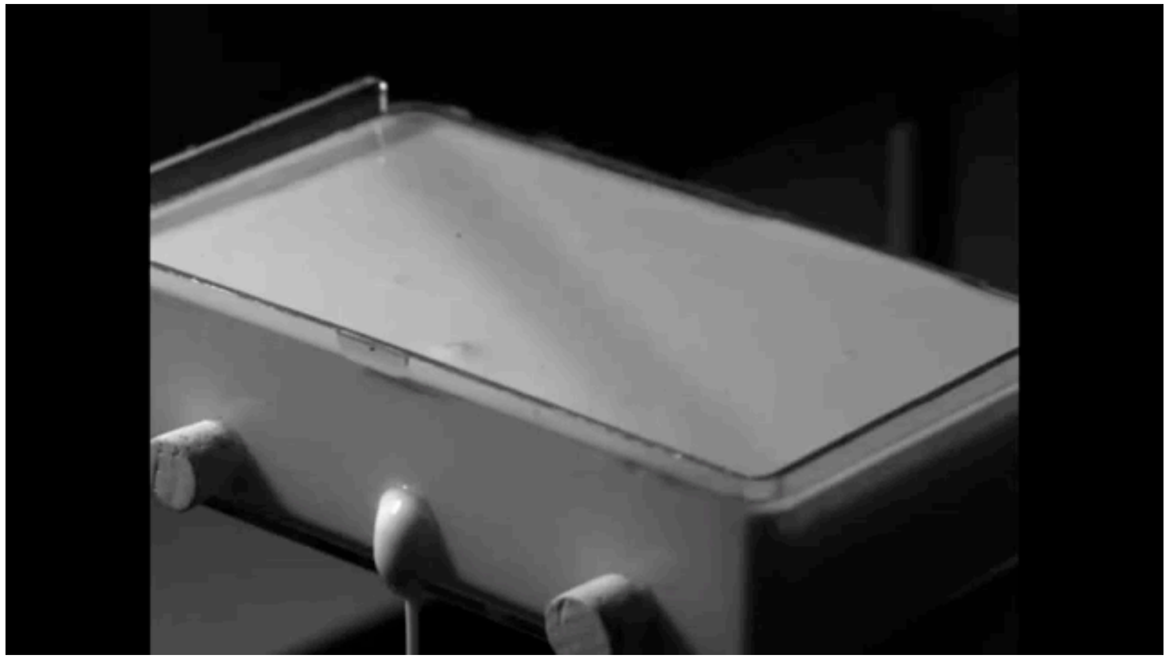
Suspensions

Mixture of macroscopic, undissolved particles in a liquid

+

We'll focus on the simplest type of suspension: hard, non-attractive particles, suspended in a Newtonian liquid.

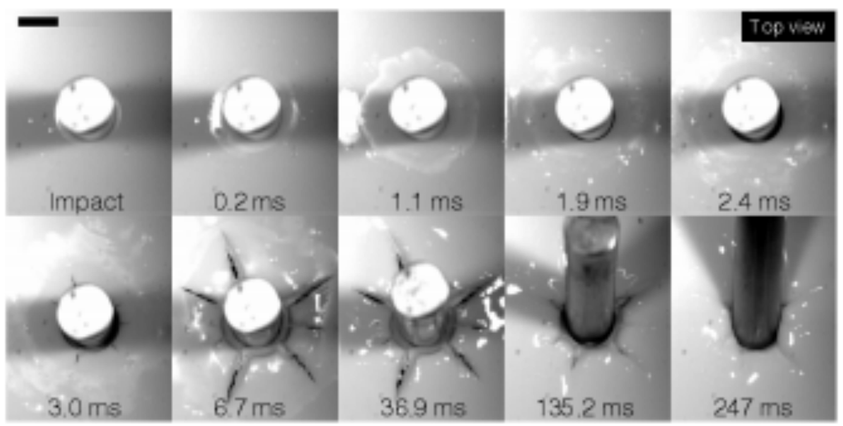
Introduction Dense suspensions under impact



source: https://www.youtube.com/watch?v=hP88C-_LgnE&list=PLVjiIPzFTOLpxiwdwrFuPYSIBpr6jFm32

Impact-induced hardening

- Occurs on the simplest type of suspensions
- Cannot be observed in liquid or particles alone
- Physical explanations remain elusive
  - Inherently far-from-equilibrium
  - Highly dissipative —> transient



Even fracture can exist!

Roche et al, PRL 2013

Run on suspension

Brown, et. al., Rep. Prog. Phys 77, 046602 (2014).

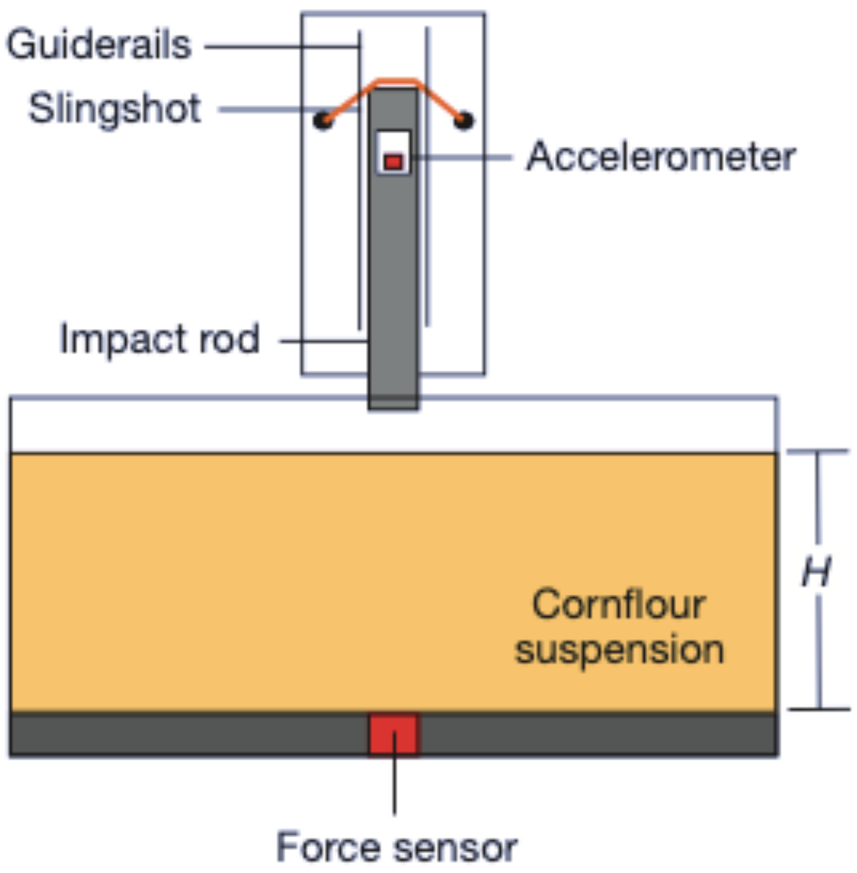
Liquid body armor

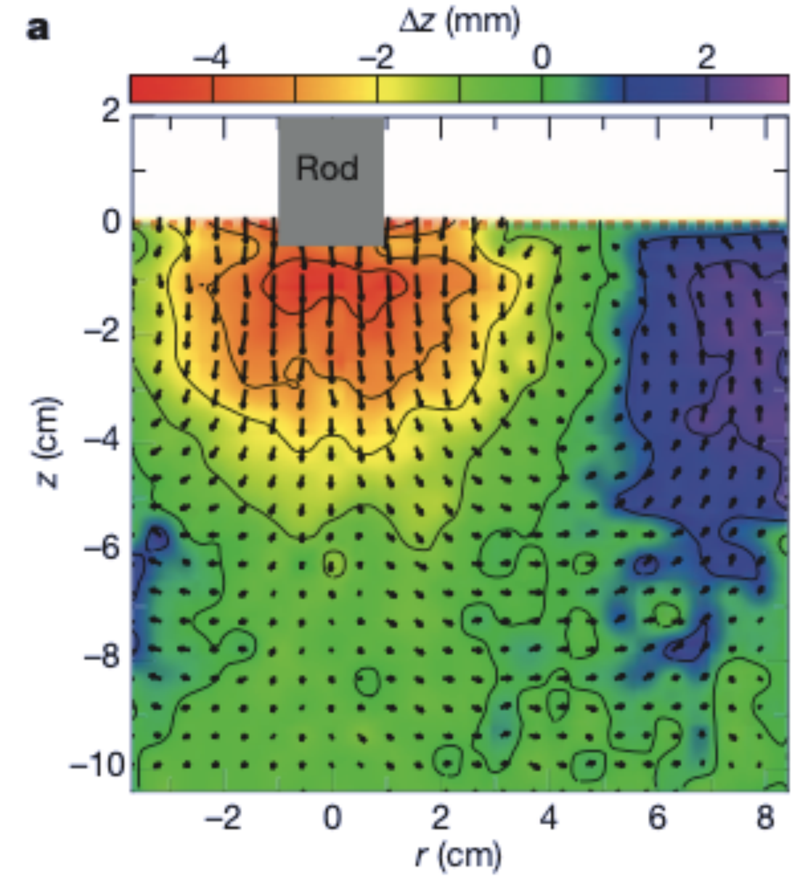
https://www.youtube.com/watch?v=L5Ts9lYZIDk

Experiments on impact-induced hardening

Impact-induced hardening is caused by the emergence of dynamically jammed region

Waitukaitis and Jaeger, Nature **487**, 205 (2012)

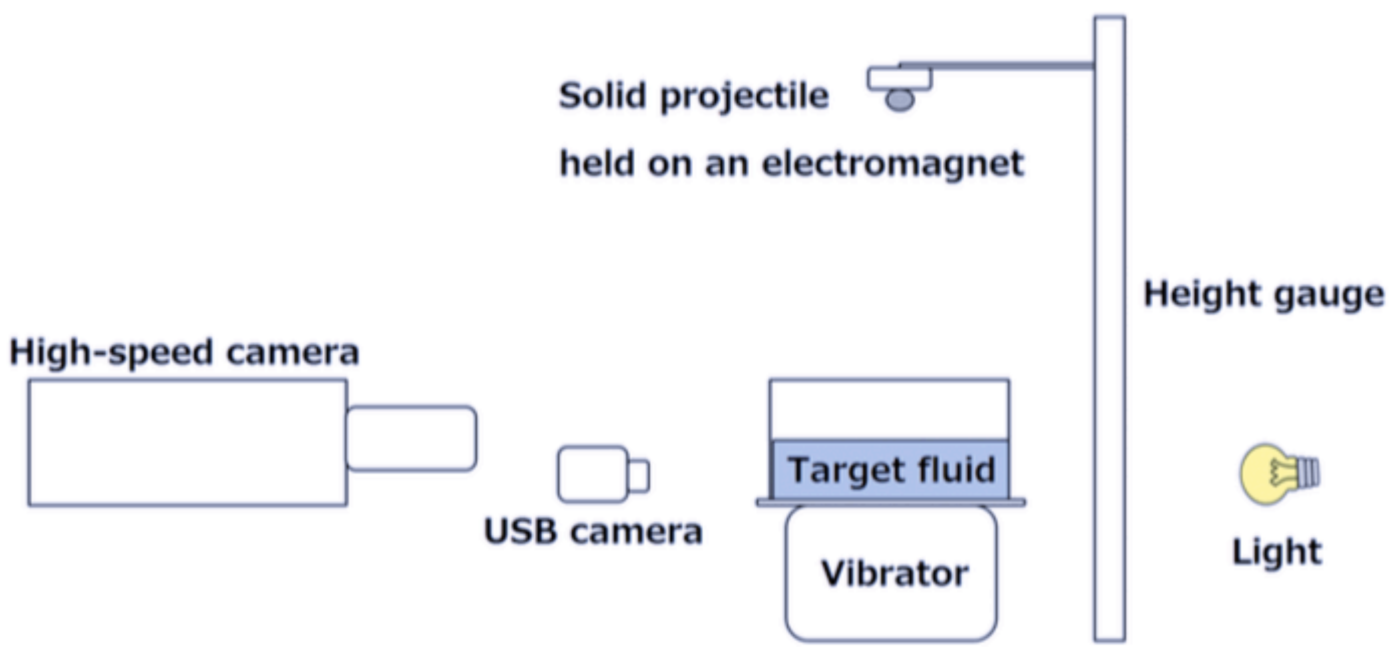


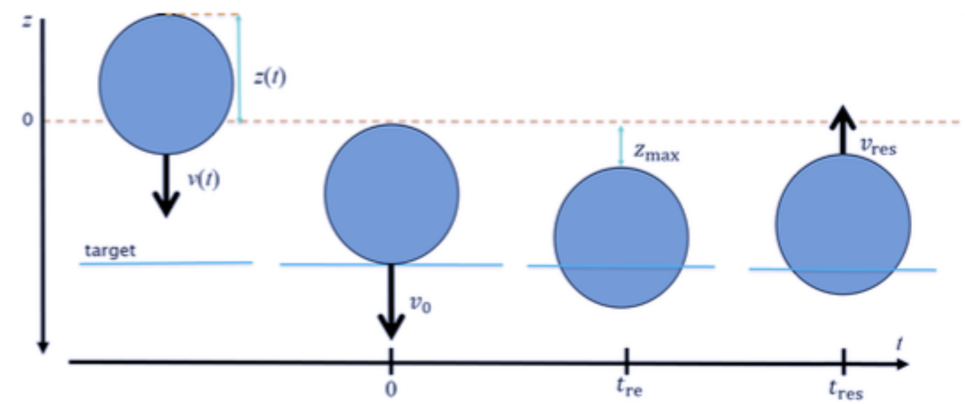


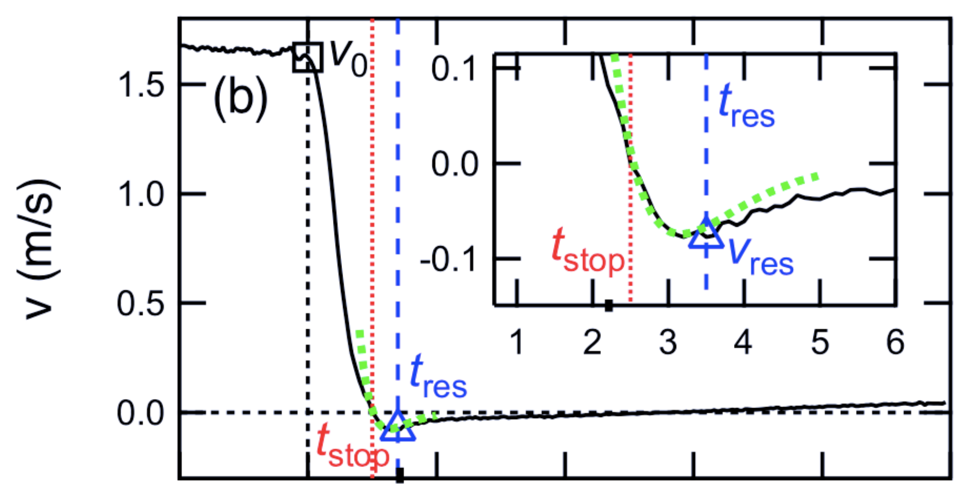
Experiments on impact-induced hardening

For shallow suspension, elastic motion (rebound) of the impactor is observed

Egawa and Katsuragi, Phys. Fluids **31**, 053304 (2019)



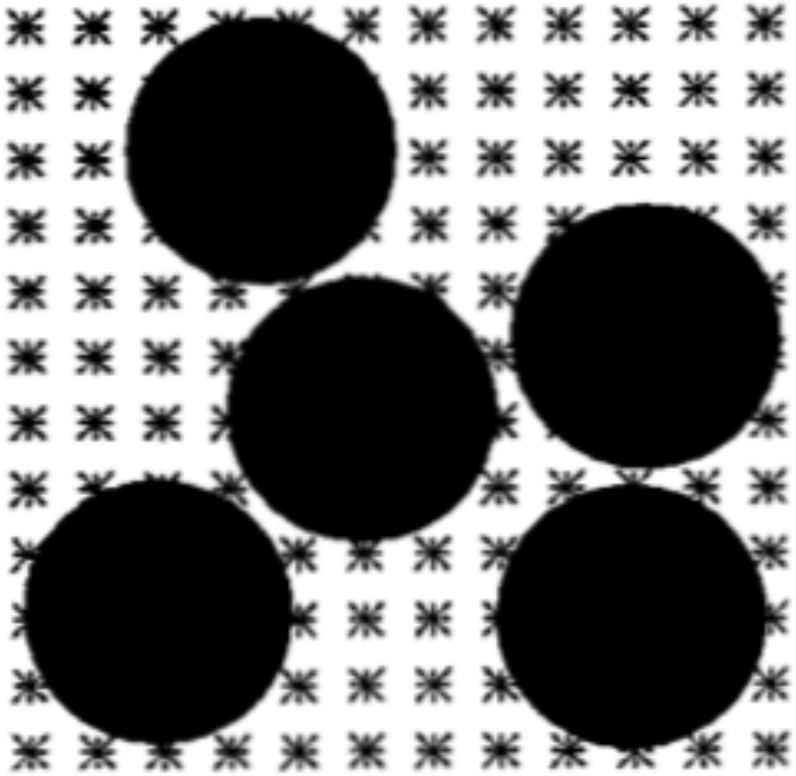




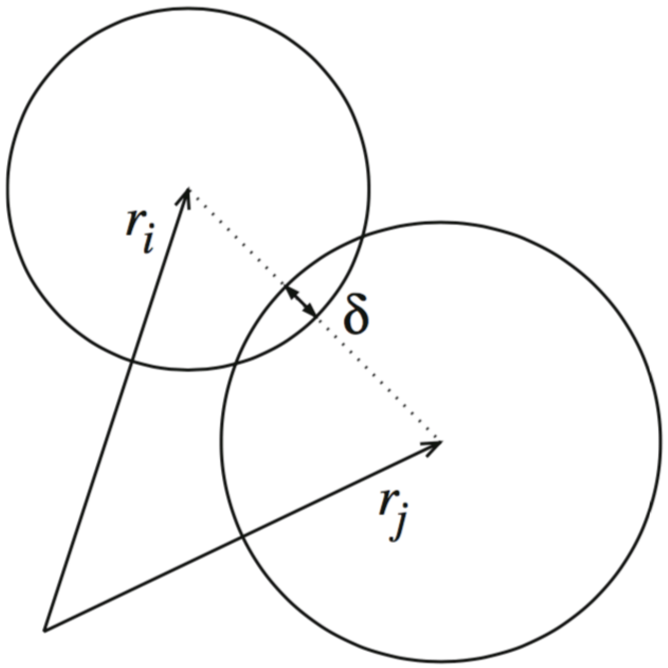
Part 1: Numerical simulations of impact-induced hardening

Ingredients of our simulation

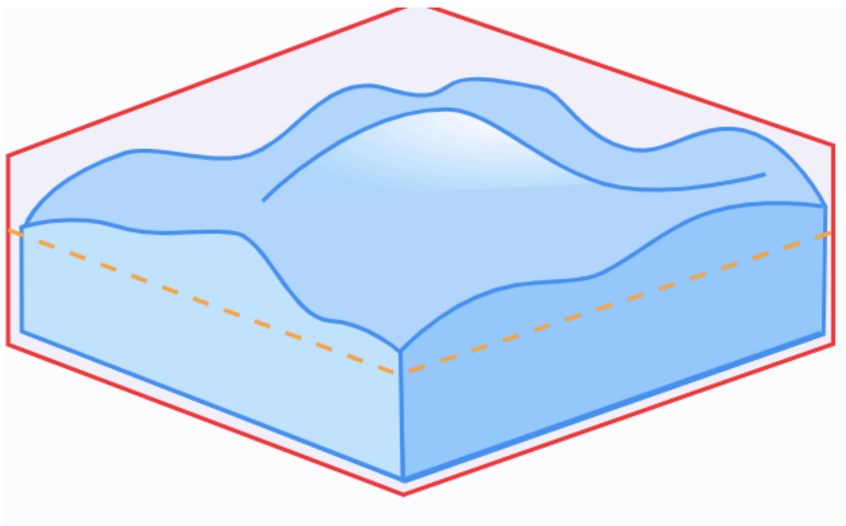
Hydrodynamic Interaction



Contact between particles



Free surface of the liquid

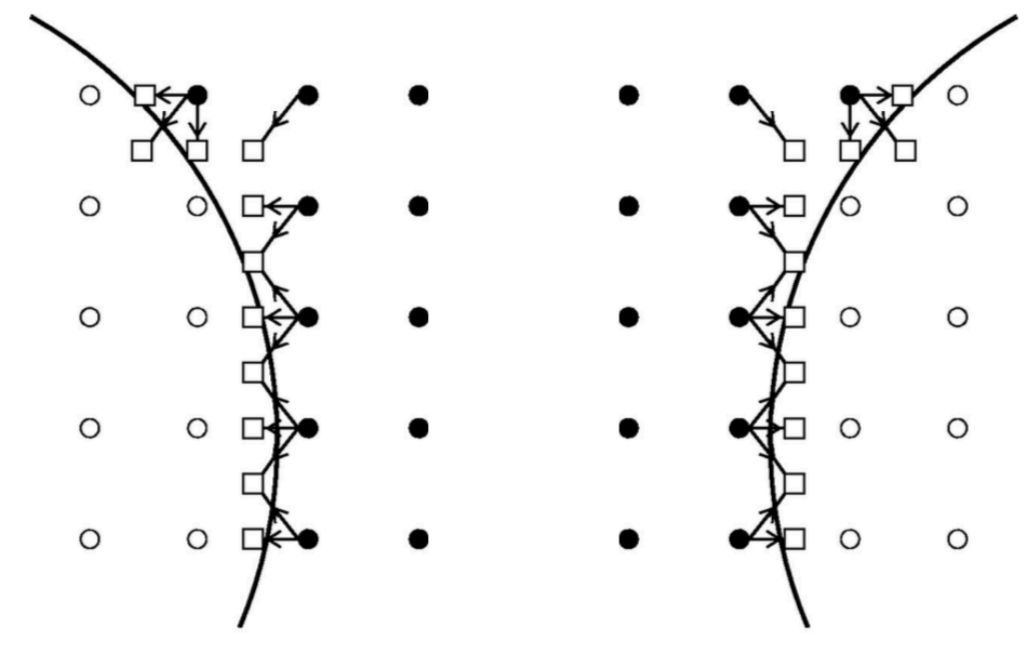


Long range hydrodynamic: lattice Boltzmann method

Ladd, J. Fluid. Mech, **271,** 285 (1994)

Svec, et al., J. Non-Newton. Fluid, **179,** 32-42 (2012)

- Calculate hydrodynamic field on lattices
- Calculate force on boundary nodes

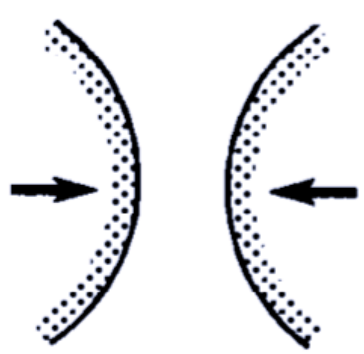


Lubrication force

- Pairwise resistance matrix

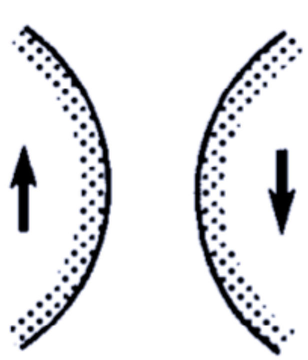
Kim and Karilla, *Microhydrodynamics* (1991)

$1/\varepsilon$



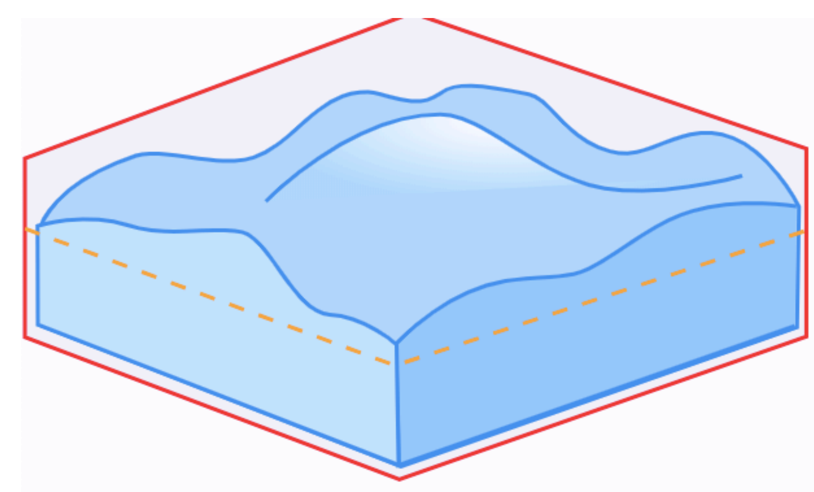
Squeezing

$\ln(1/\varepsilon)$



Shearing

Free surface



Mass-tracking algorithm

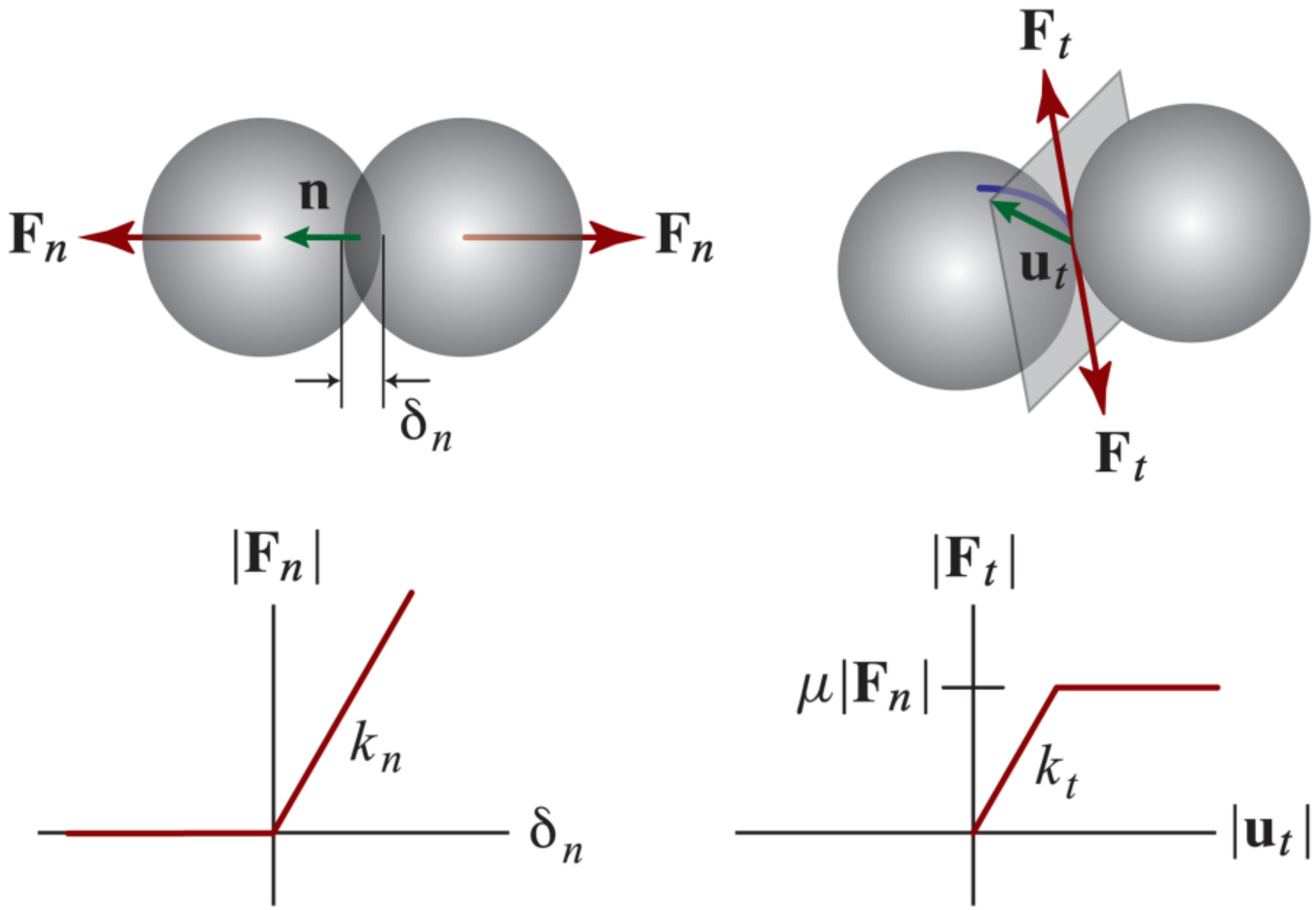
Leonardi, et al., Phys. Rev. E **92,** 052204 (2015)

Contact between suspended particles

Luding. S, Granular Matter, 2008, **10**, 235

Normal part

Tangential part



Coulomb friction

$N = 2000$ Particles

Bidisperse $a_{\mathrm{max}} = 1.2a_{\mathrm{min}}$

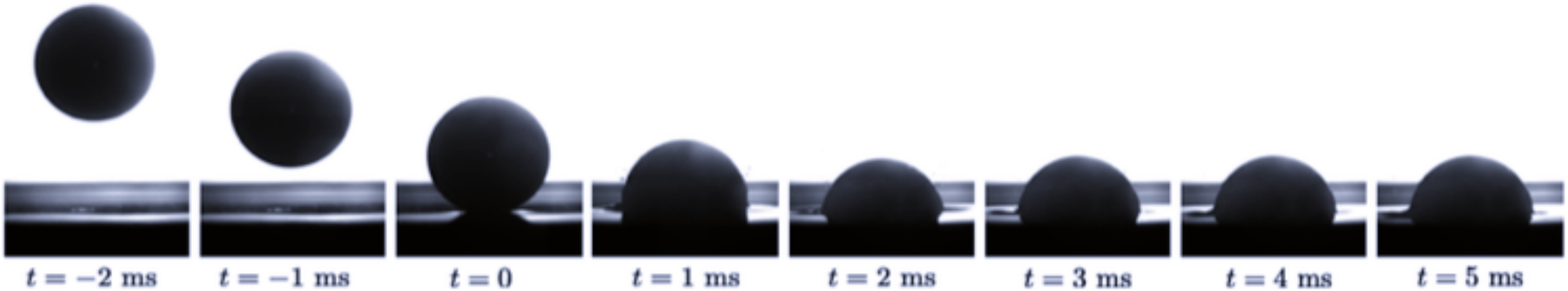
Confined in rectangular box $W \times D \times H$

$W/a_I = 8$, $D/a_I = 8$, and $H/a_I = 4$

Impactor $a_I = 4.5a_{a_{\mathrm{min}}}$

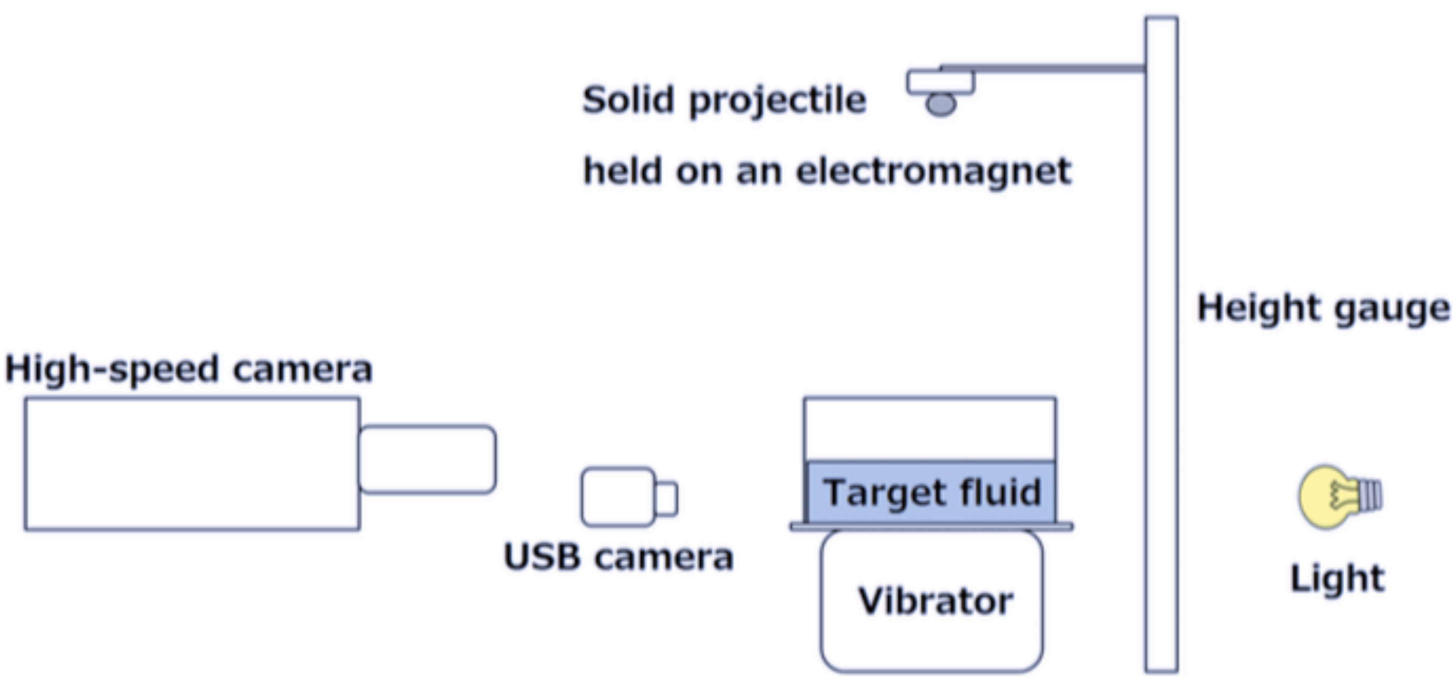
Impactor density $\rho_I/\rho_f = 4$

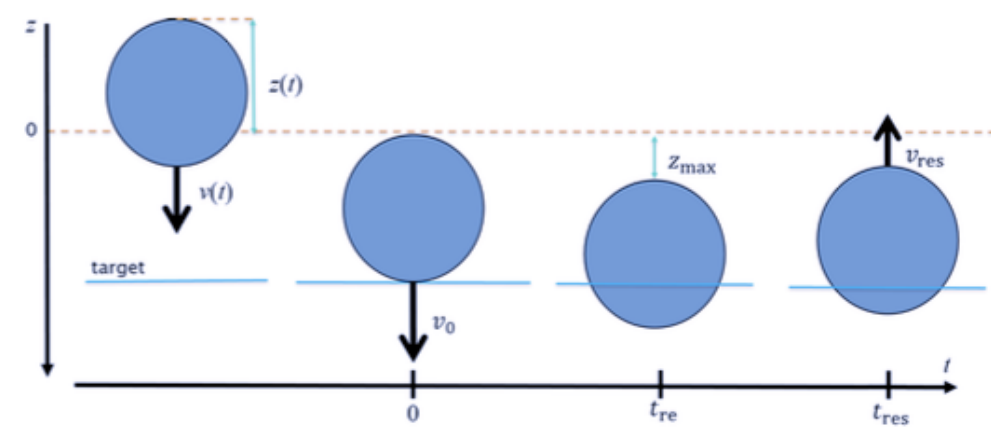
Rebound of free-falling impactor

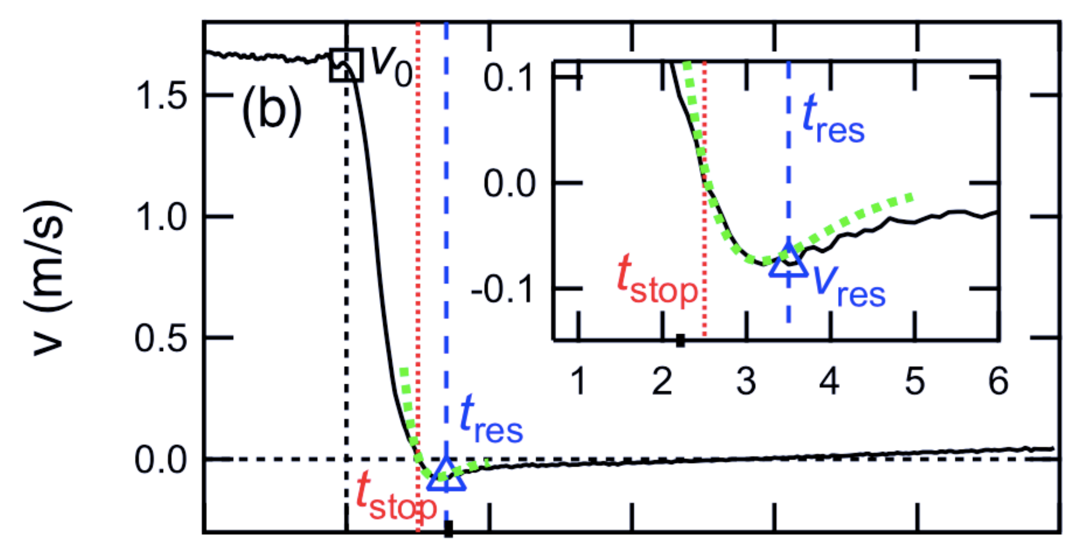


Setup

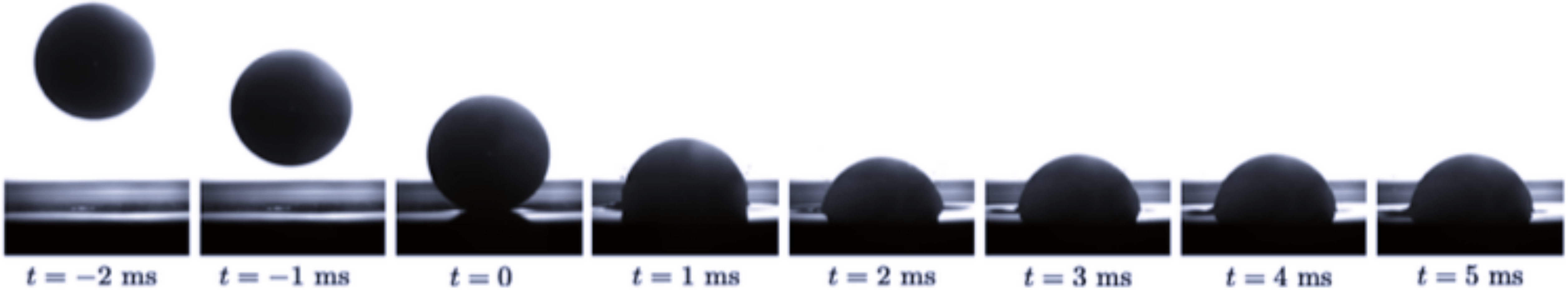
Egawa and Katsuragi, Phys. Fluids **31**, 053304 (2019)

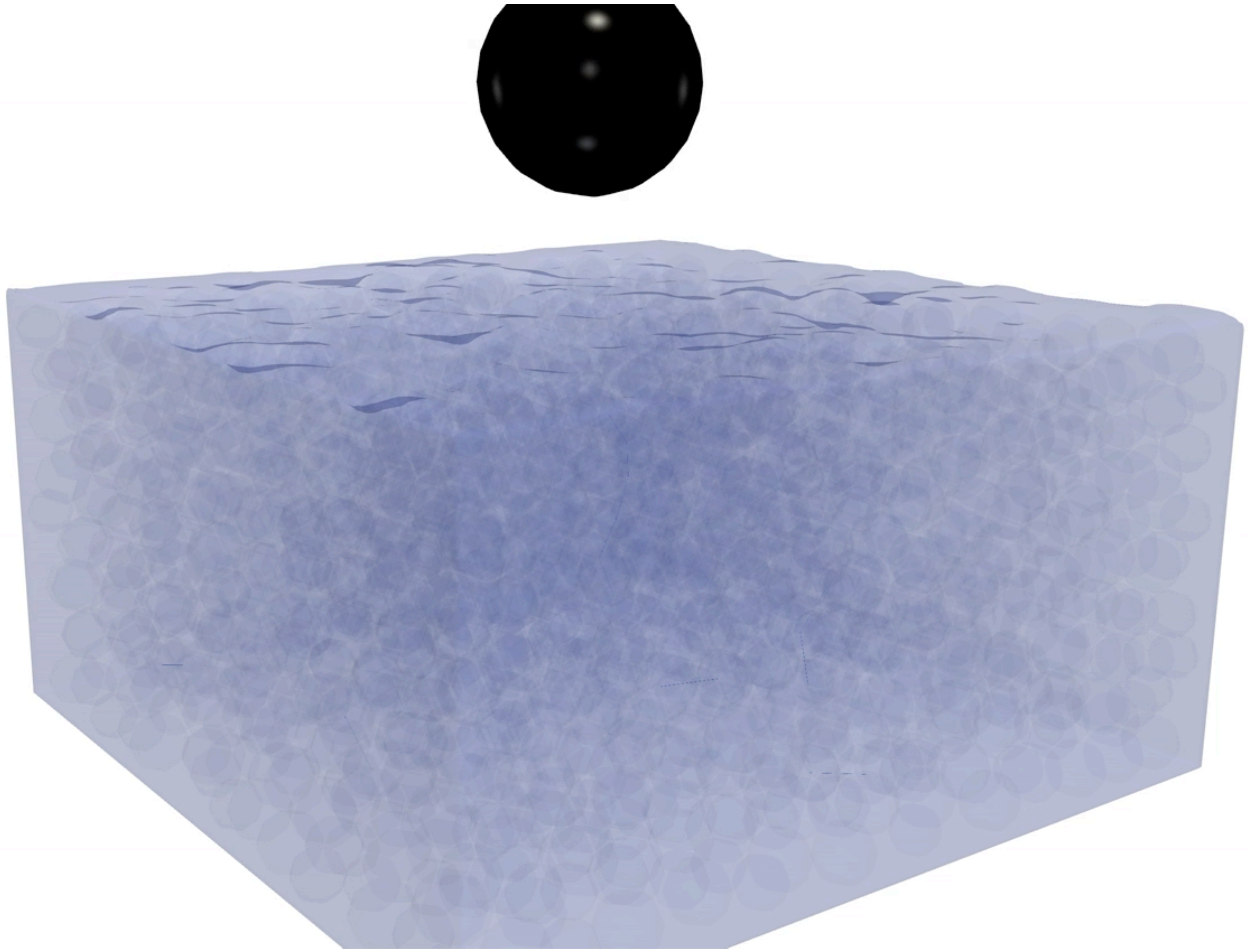






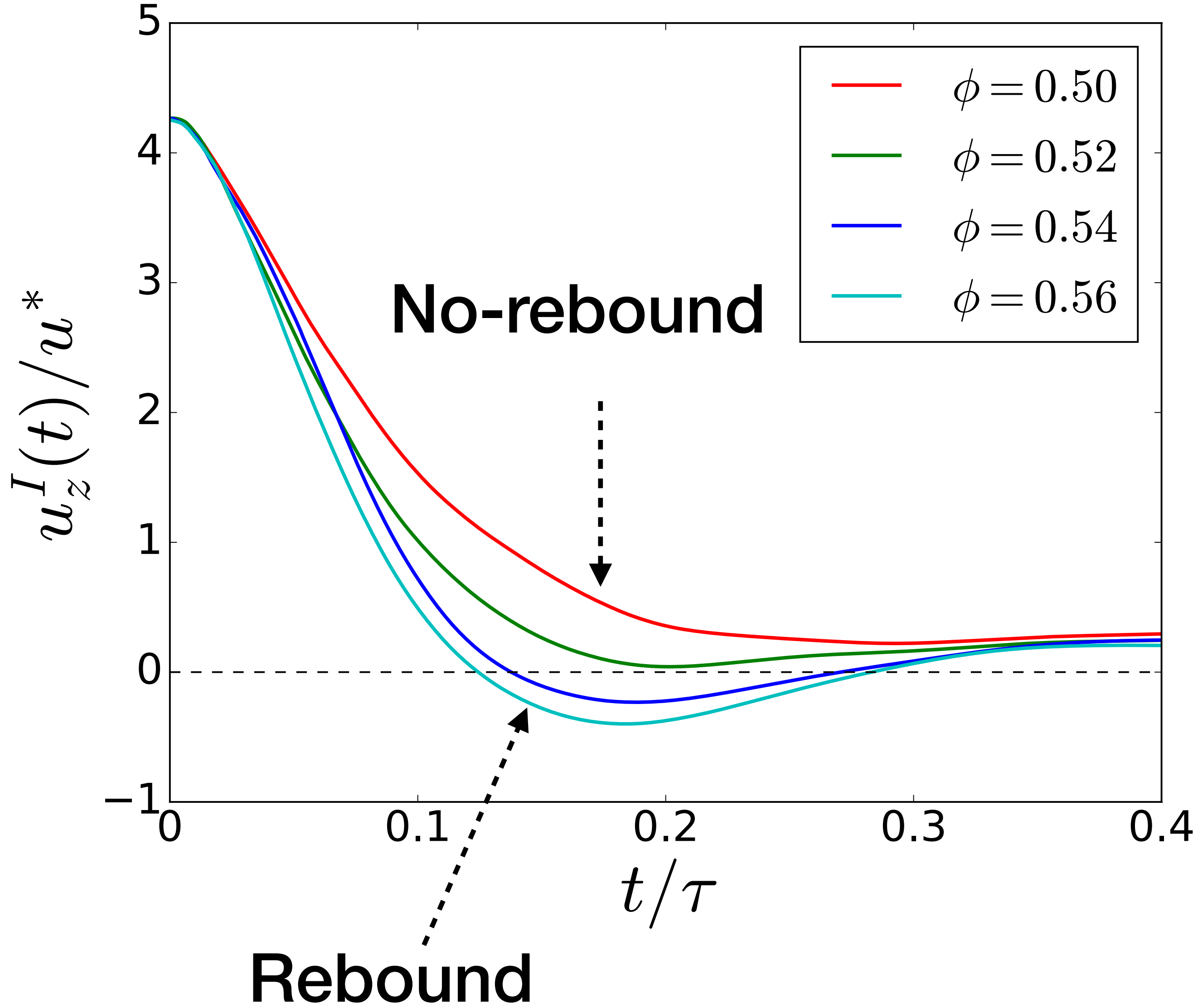
Rebound of free-falling impactor





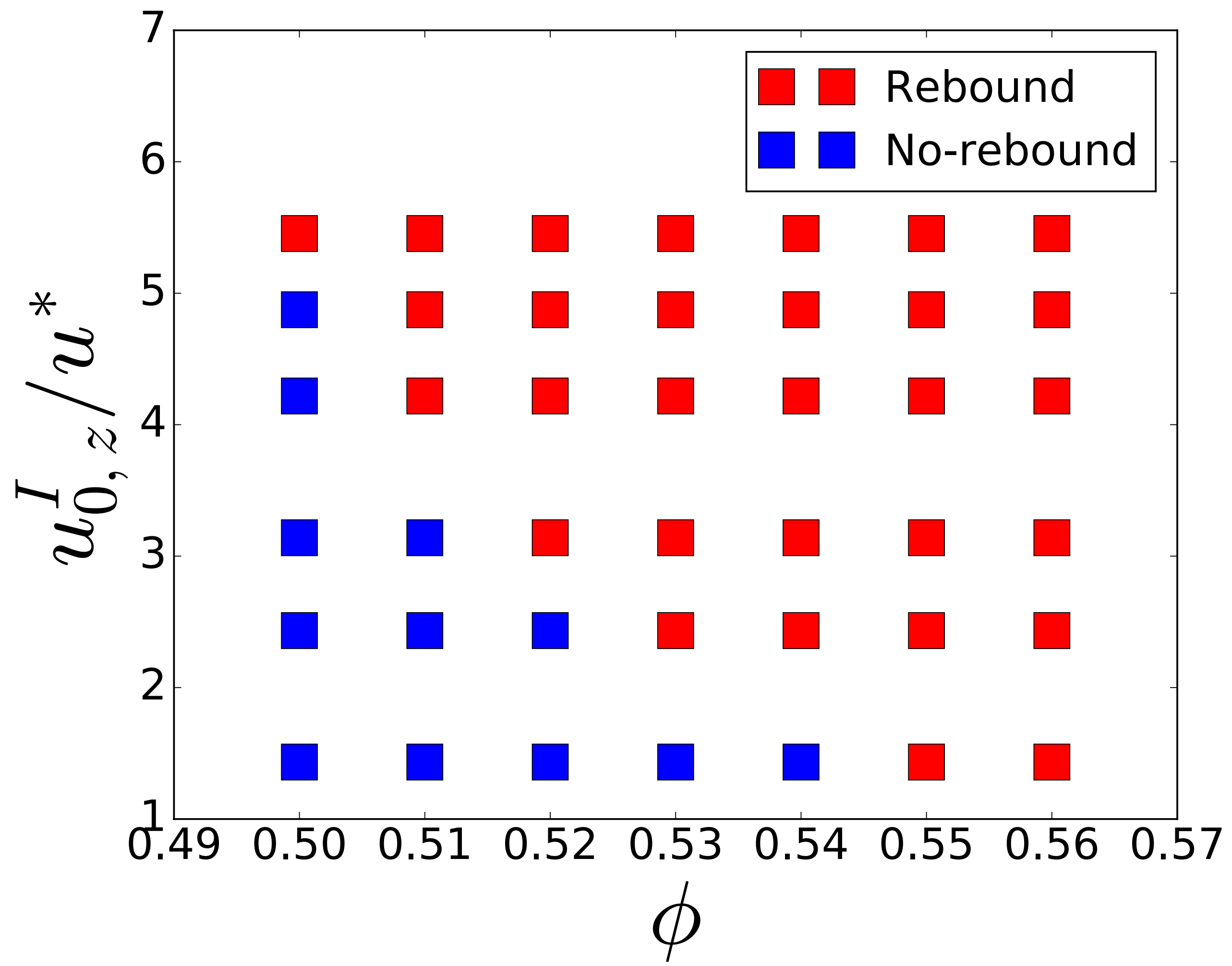
Results: Impactor motion around impact moment

$$\phi = \frac{\text{Total solid particles volume}}{\text{Volume of the container}}$$



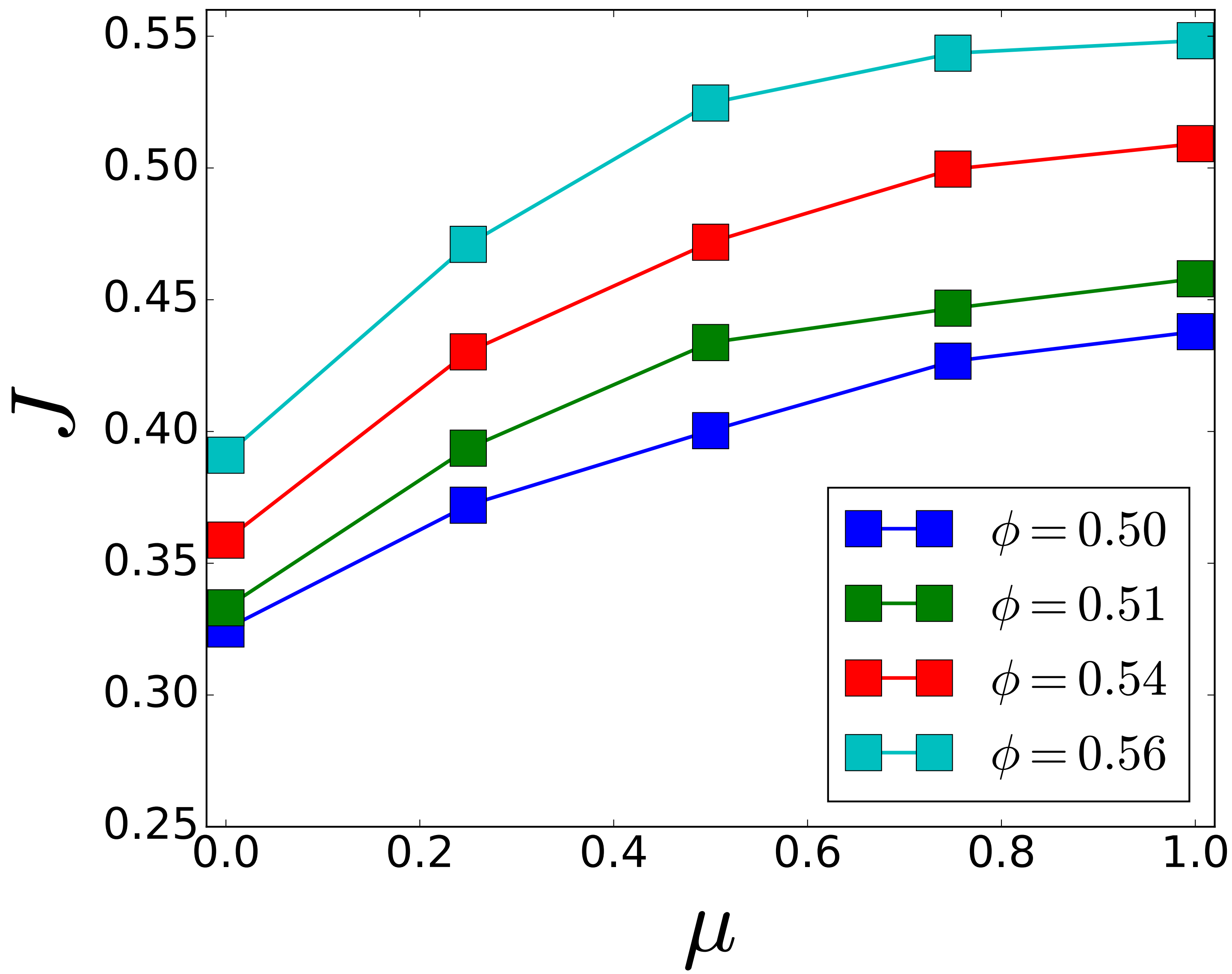
$$u^* = \sqrt{2ga_{min}}$$

Rebounds also depend on the impact velocity



We can run on top suspension but we'll sink if we walk!

Results: Dependence on friction



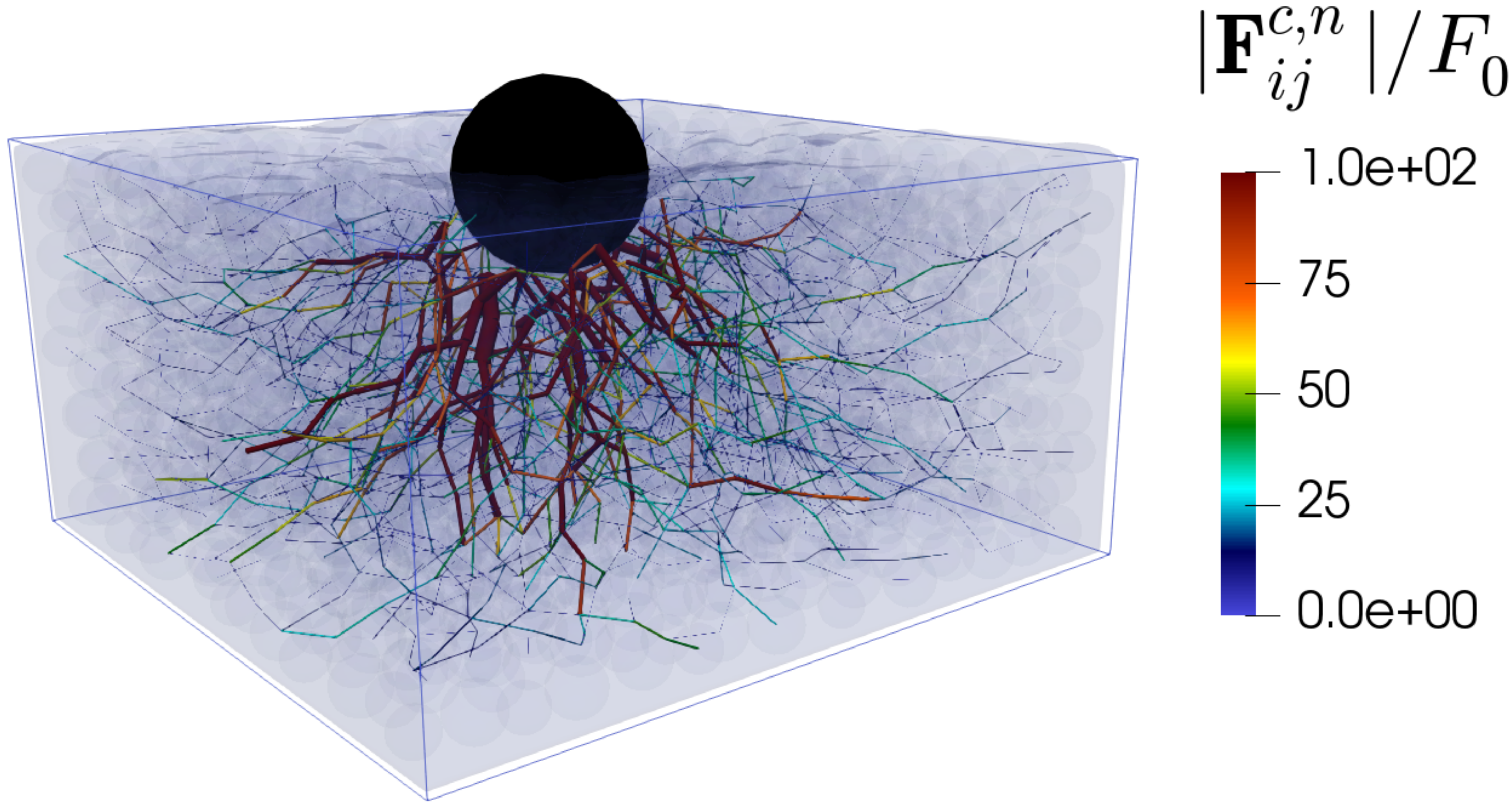
Impulse

$$J = \int_{t=0}^{t=0.1} F_z^I(t)dt$$

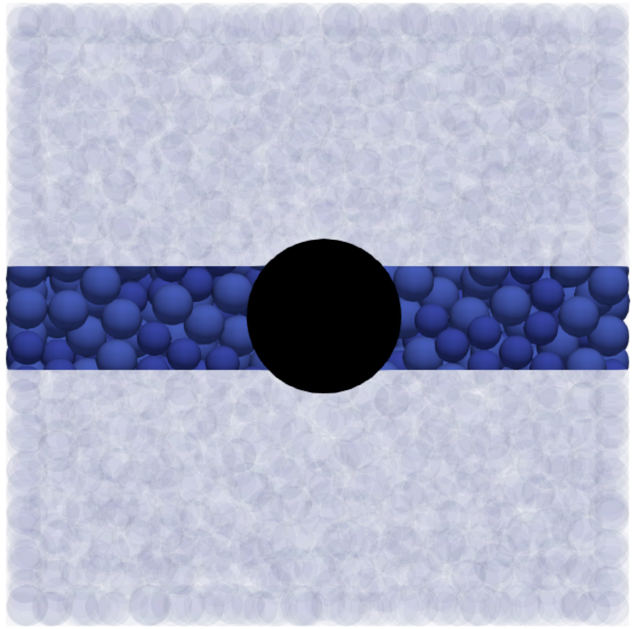
$$\frac{u_{0,z}}{u^*} = 4.2$$

Frictional interaction increases the contact duration between particles that leads to a stronger hardening

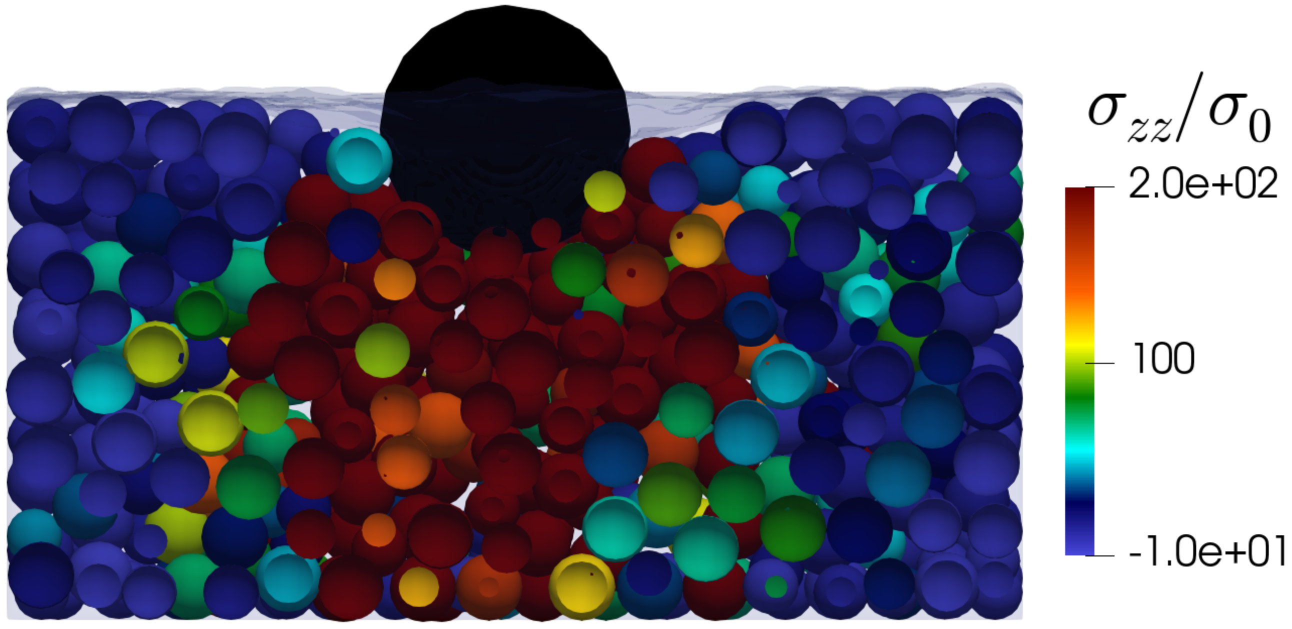
Force chains



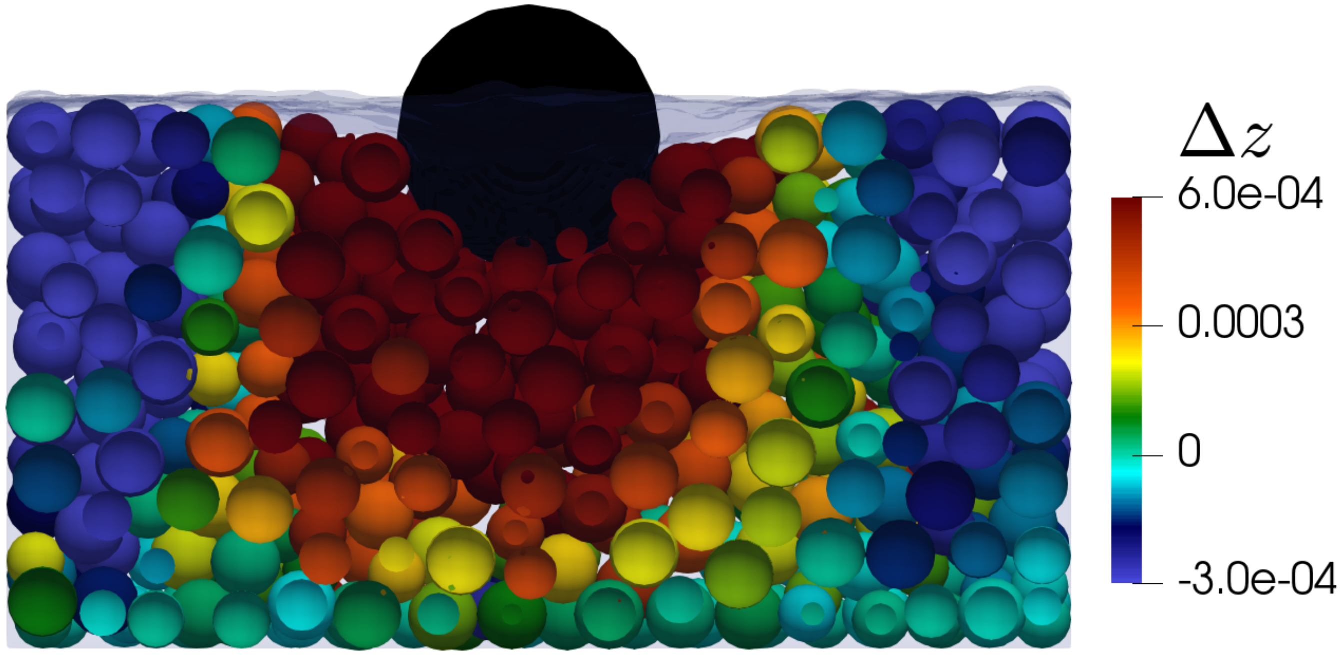
Slice on the middle of the box



Normal stress



Displacement



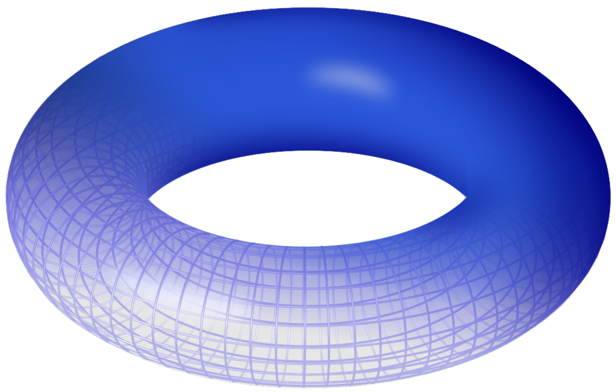
"Dynamically jammed region"

Persistent Homology

Kramar, et al., Phys. Rev. E 90, 052203 (2014)
Gameiro, et al., Phys. Rev. Fluids 5, 034307 (2020)
Hiraoka, et. al., PNAS 113, 26 (2019)
Takahashi, et al., Phys. Rev. E 97, 012906 (2018)

Homology

Shape



→ **Homology** →

Vector spaces

$$\begin{bmatrix} \mathbb{R}^{\beta_0} \\ \mathbb{R}^{\beta_1} \\ \mathbb{R}^{\beta_2} \\ \vdots \end{bmatrix} \qquad \begin{aligned} \beta_0 &= 1 \\ \beta_1 &= 2 \\ \beta_2 &= 1 \end{aligned}$$

Betti numbers

β_n ⟶ Number of each topological structure

$n = 0$ Connected components

$n = 1$ Loops

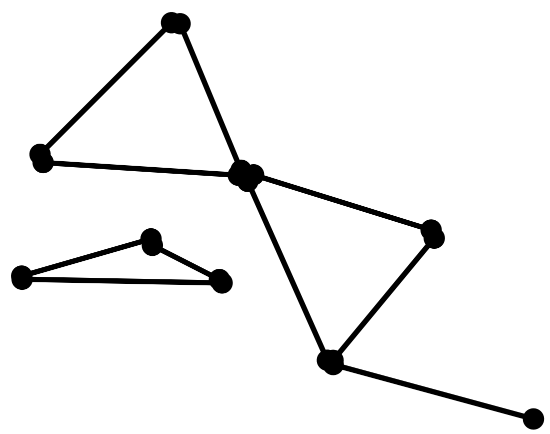
$n = 2$ Enclosed volume (Cavity)

Persistent Homology

Kramar, et al., Phys. Rev. E 90, 052203 (2014)
Gameiro, et al., Phys. Rev. Fluids 5, 034307 (2020)
Hiraoka, et. al., PNAS 113, 26 (2019)
Takahashi, et al., Phys. Rev. E 97, 012906 (2018)

Homology

Shape



Vector spaces

$$\begin{bmatrix} \mathbb{R}^{\beta_0} \\ \mathbb{R}^{\beta_1} \\ \mathbb{R}^{\beta_2} \\ \vdots \end{bmatrix} \qquad \begin{aligned} \beta_0 &= 2 \\ \beta_1 &= 3 \\ \beta_2 &= 0 \end{aligned}$$

Betti numbers

$\beta_n \longrightarrow$ Number of each topological structure

$n = 0$ Connected components

$n = 1$ Loops

$n = 2$ Enclosed volume (Cavity)

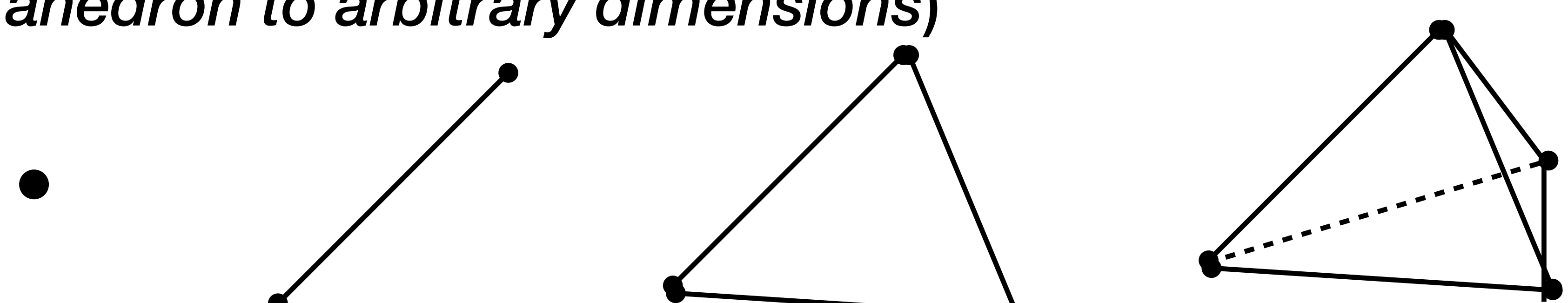
Homology of a simplicial complex is computable via linear algebra

↓

Perseus http://people.maths.ox.ac.uk/nanda/perseus/

(Open source program based on discrete Morse theory)

Simplicial complex (*set of generalization of the notion of a triangle or tetrahedron to arbitrary dimensions*)



0-simplex 1-simplex 2-simplex 3-simplex

Persistent Homology

Kramar, et al., Phys. Rev. E 90, 052203 (2014)

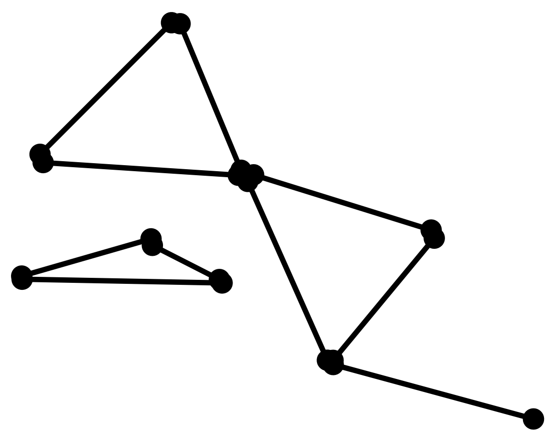
Gameiro, et al., Phys. Rev. Fluids 5, 034307 (2020)

Hiraoka, et. al., PNAS 113, 26 (2019)

Takahashi, et al., Phys. Rev. E 97, 012906 (2018)

Homology

Shape



→ **Homology** →

Vector spaces

$$\begin{bmatrix} \mathbb{R}^{\beta_0} \\ \mathbb{R}^{\beta_1} \\ \mathbb{R}^{\beta_2} \\ \vdots \end{bmatrix} \qquad \begin{aligned} \beta_0 &= 2 \\ \beta_1 &= 3 \\ \beta_2 &= 0 \end{aligned}$$

Betti numbers

β_n → Number of each topological structure

$n = 0$ Connected components

$n = 1$ Loops

$n = 2$ Enclosed volume (Cavity)

Homology of a simplicial complex is computable via linear algebra

↓

Perseus http://people.maths.ox.ac.uk/nanda/perseus/

(Open source program based on discrete Morse theory)

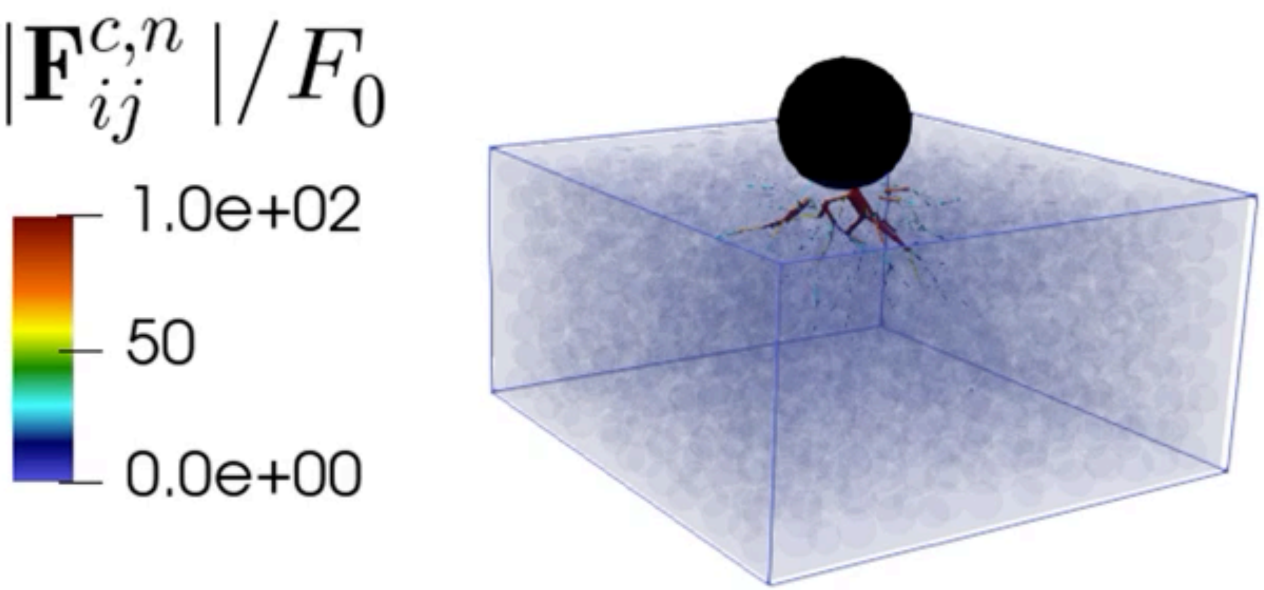
Persistent homology applies **filtration** to the data, then record the **appearance** and **merging** of each simplicial complex

Appearance $\theta_{f,b}$

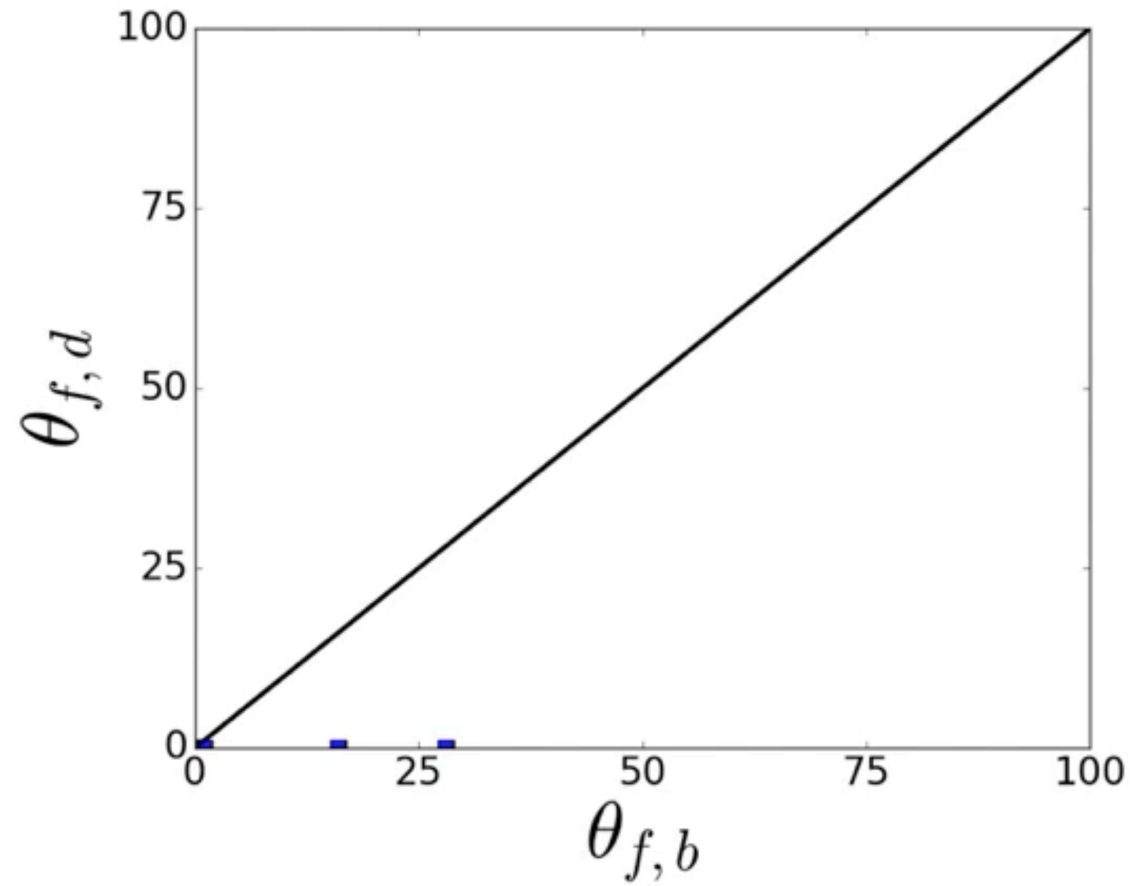
Merge $\theta_{f,d}$

Results: Persistent homology

Force chains



Persistence diagram for β_0



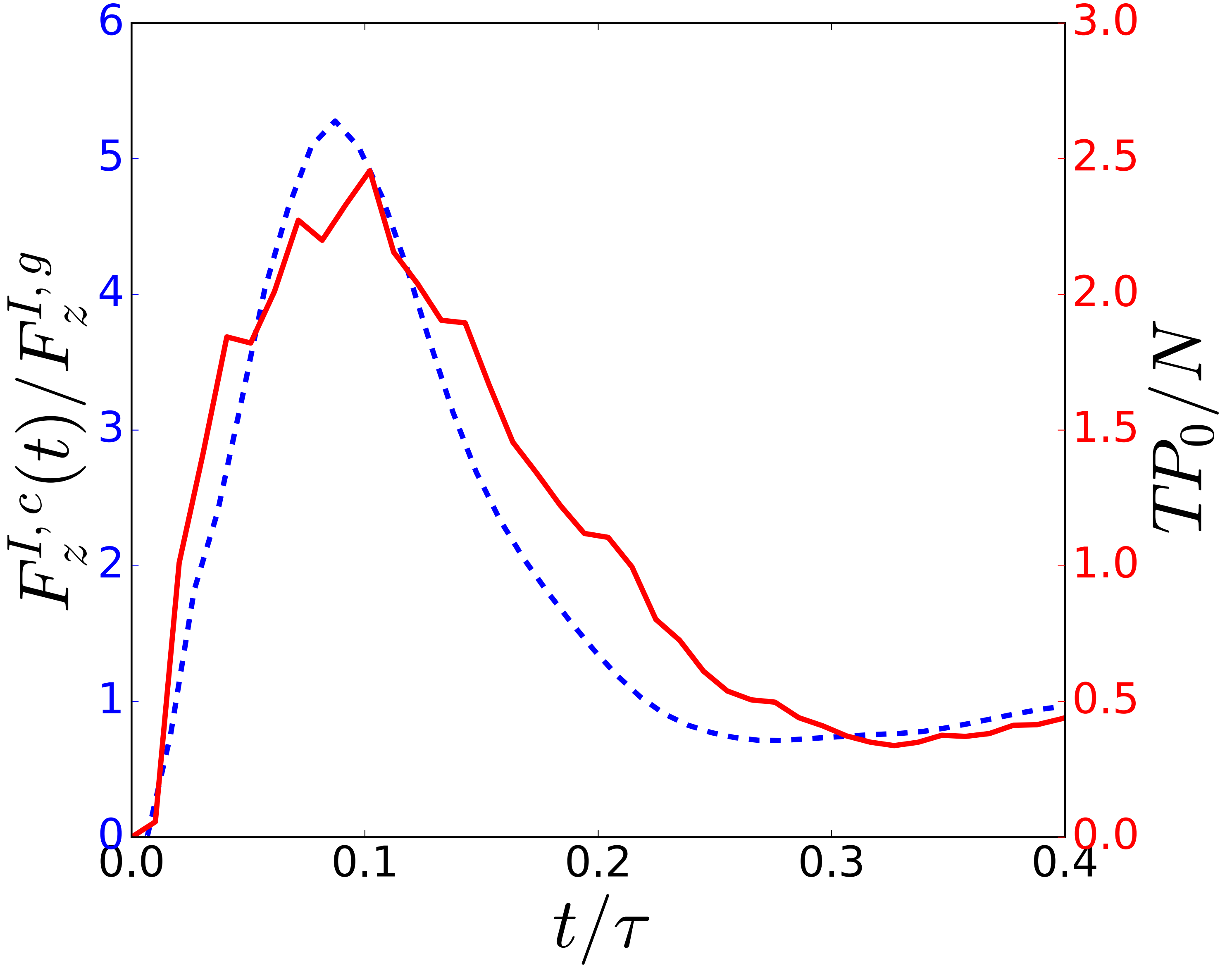
$$\phi = 0.54 \qquad \frac{u_{0,z}^I}{u^*} = 4.26$$

Points far from diagonal —> long chains

Persistent homology emphasize the length of each chain, instead of its magnitude

Total Persistence

$$TP_0 = \sum_{(\theta_{f,d},\theta_{f,b})} (\theta_{f,d} - \theta_{f,b})$$



Persistent homology elucidates the importance of the topological properties of the force chains

Conclusions of part 1

The impact-induced hardening is still not well understood despite it occurs on the simplest type of suspension

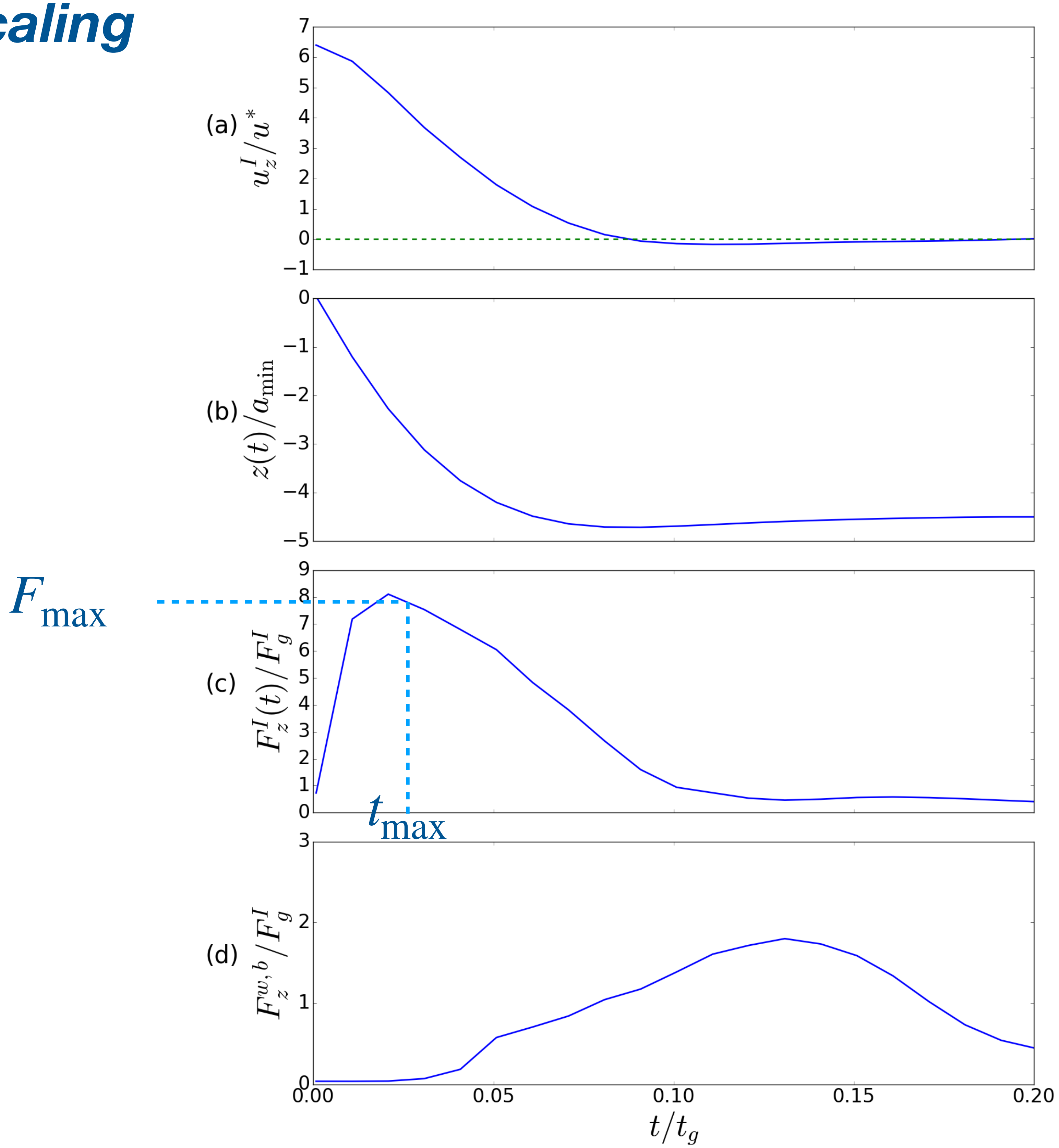
We have performed a simulation of free-falling impactor onto suspensions

The impact-induced hardening takes place due to dynamically jammed region formed by percolating force chains of contacting frictional particles

Persistent homology elucidates the significance of the topological structure of the force chains

Part 2: Scaling law and viscoelastic response

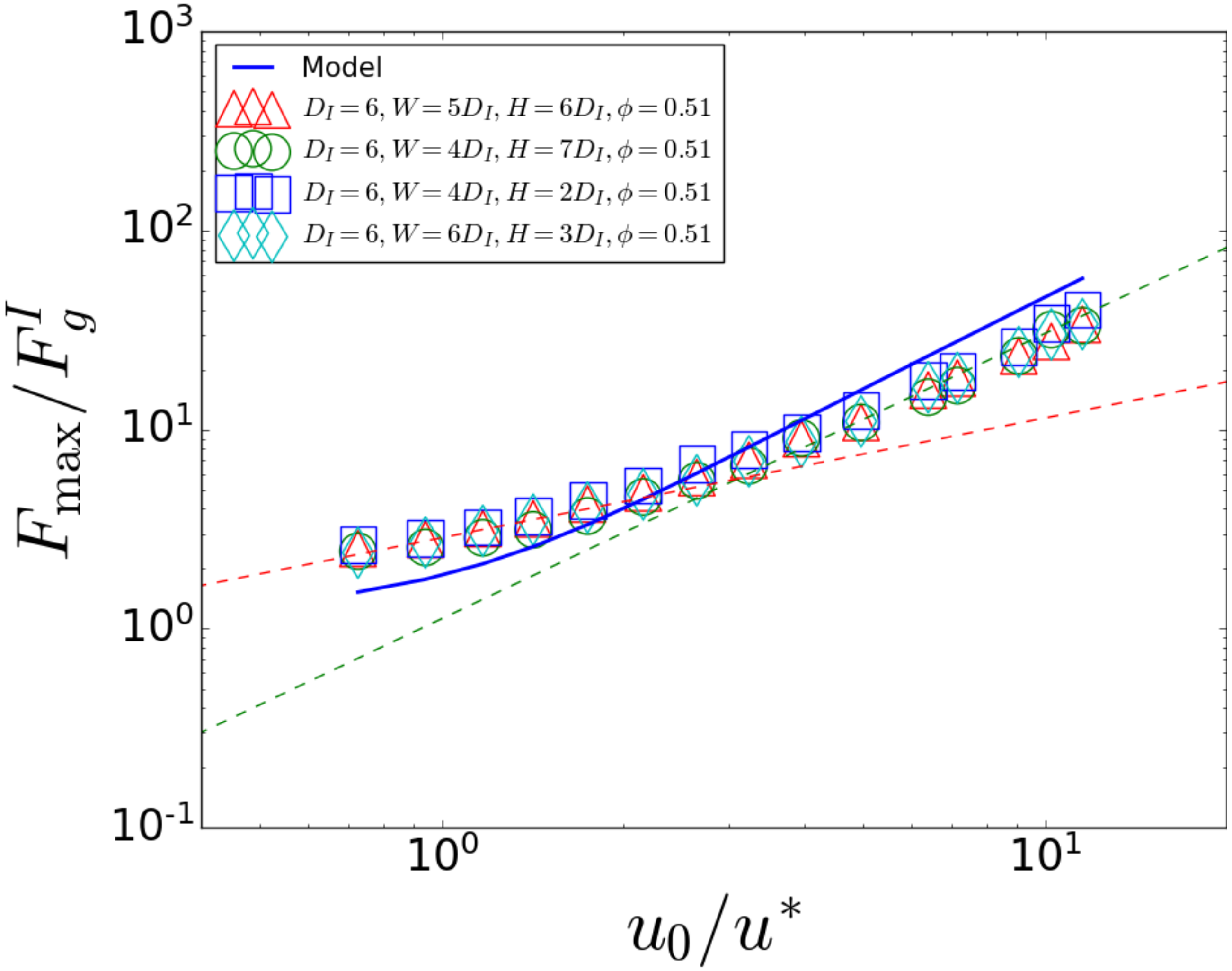
$F_{\max}$ and $t_{\max}$ scaling

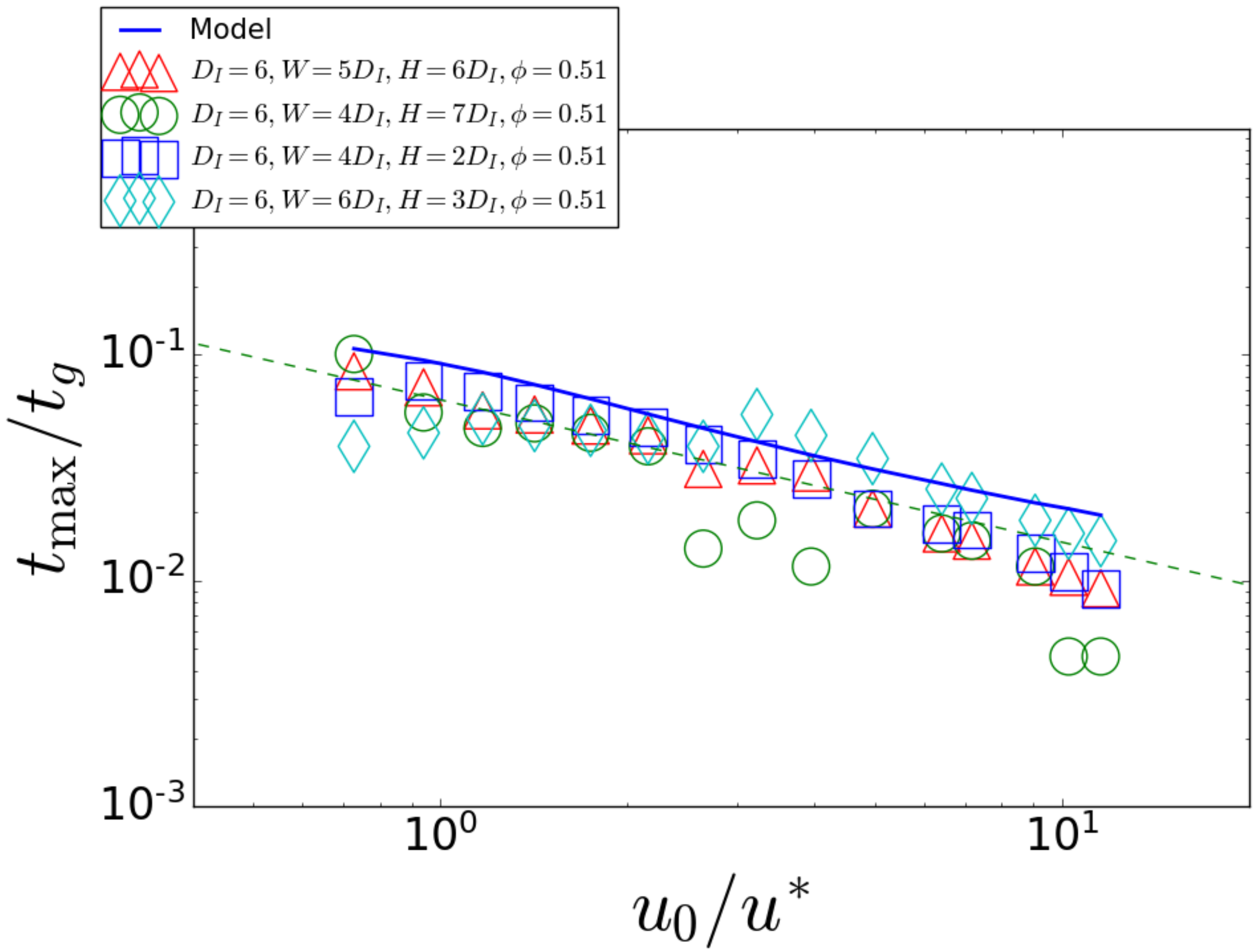


$F_{\max}$ emerge at very early stage after the impact

$F_{\max}$ and $t_{\max}$ might not be related to the elastic response (rebound)

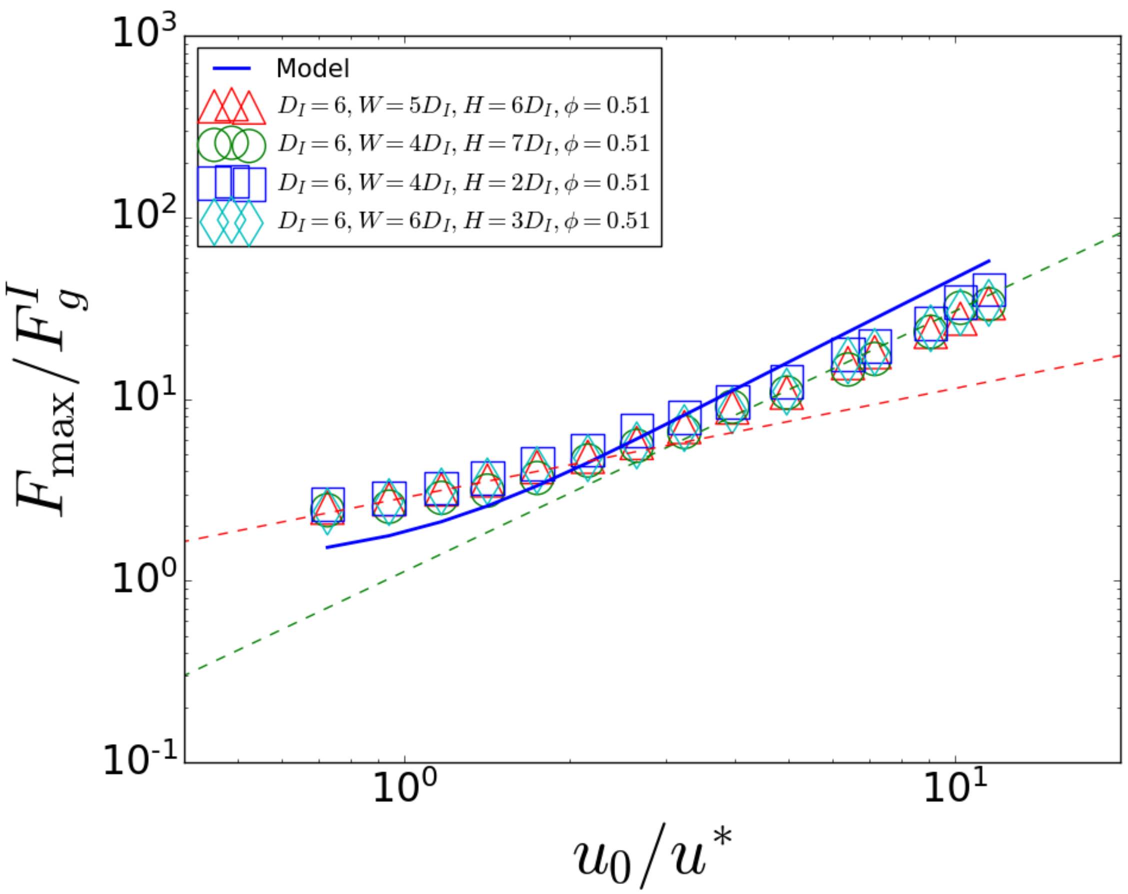
$F_{\max}$ and $t_{\max}$ scaling

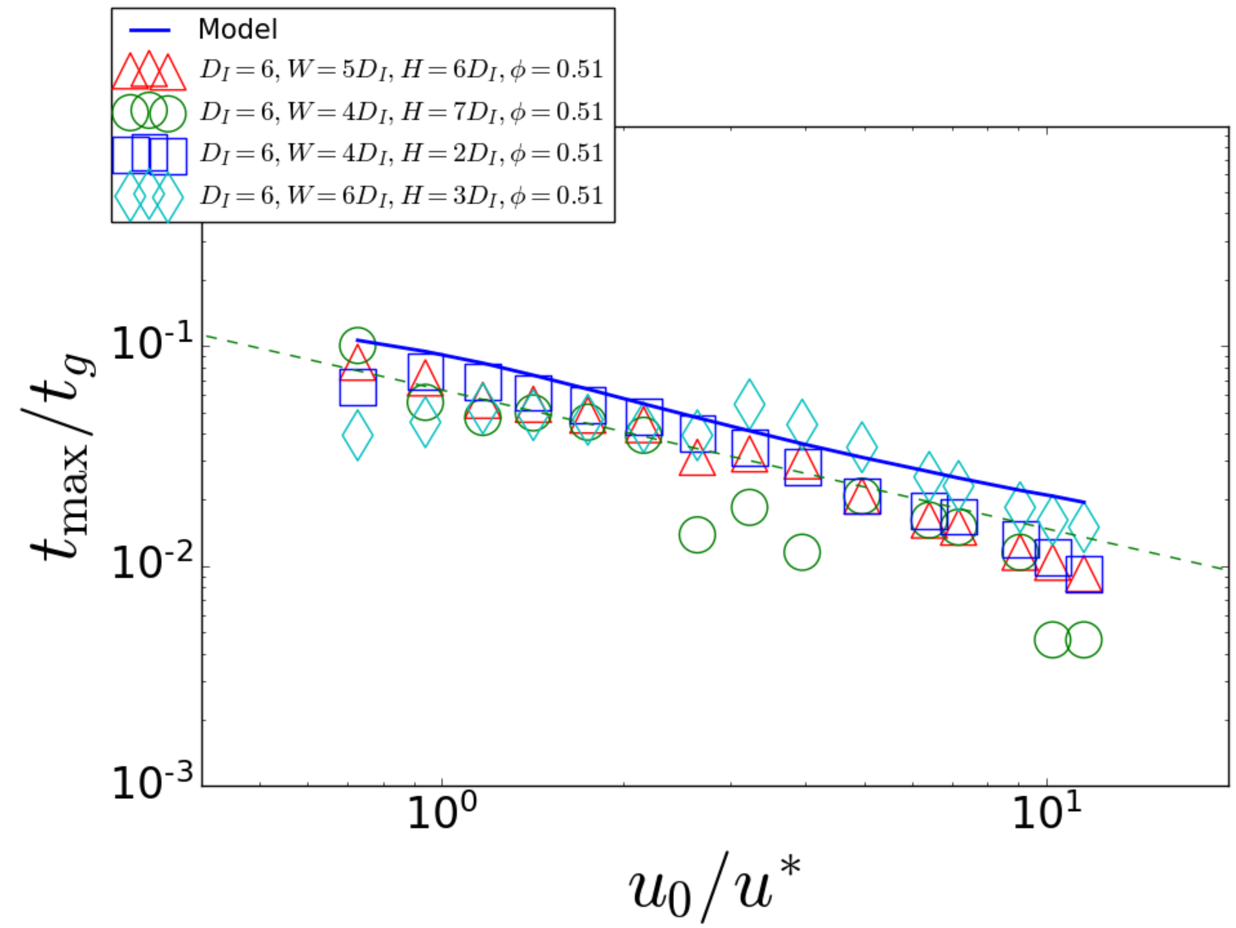




- Universal scaling with **crossover** from low u_0 to high u_0 regime
- $F_{\max} \propto u_0^{1.432}$ and $t_{\max} \propto u_0^{-0.523}$

$F_{\max}$ and $t_{\max}$ scaling: phenomenology





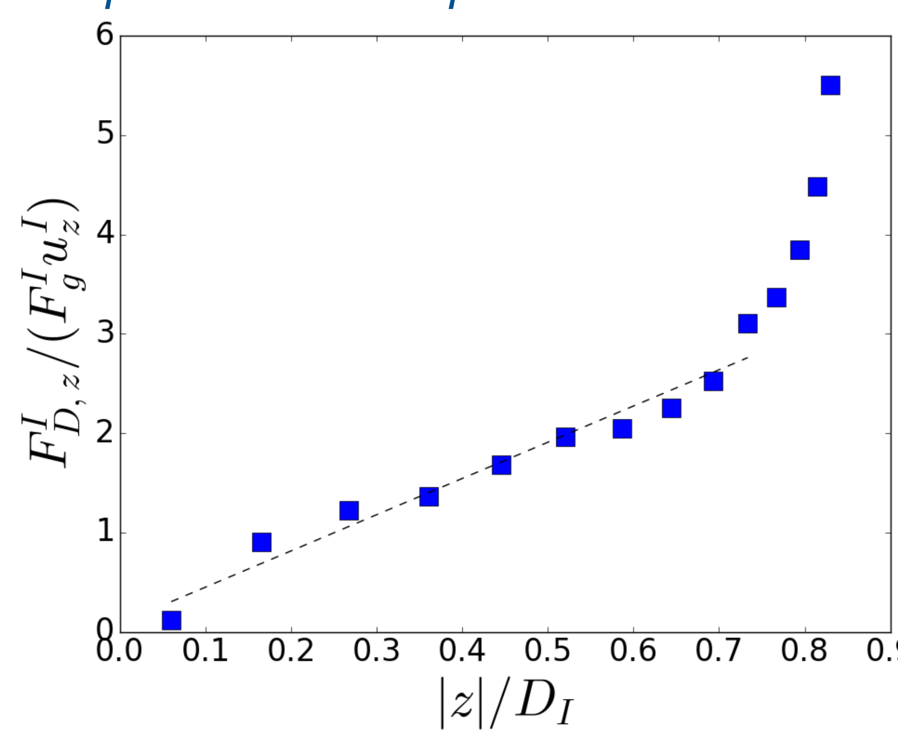
Floating model

$$m_I \ddot{z}_I = -m_I \tilde{g} - 3\pi\eta \dot{z}_I |z|$$

Brassard, et al, arXiv:2011.11824

$$\tilde{g} = \frac{\rho_I - \rho_f}{\rho_f}$$

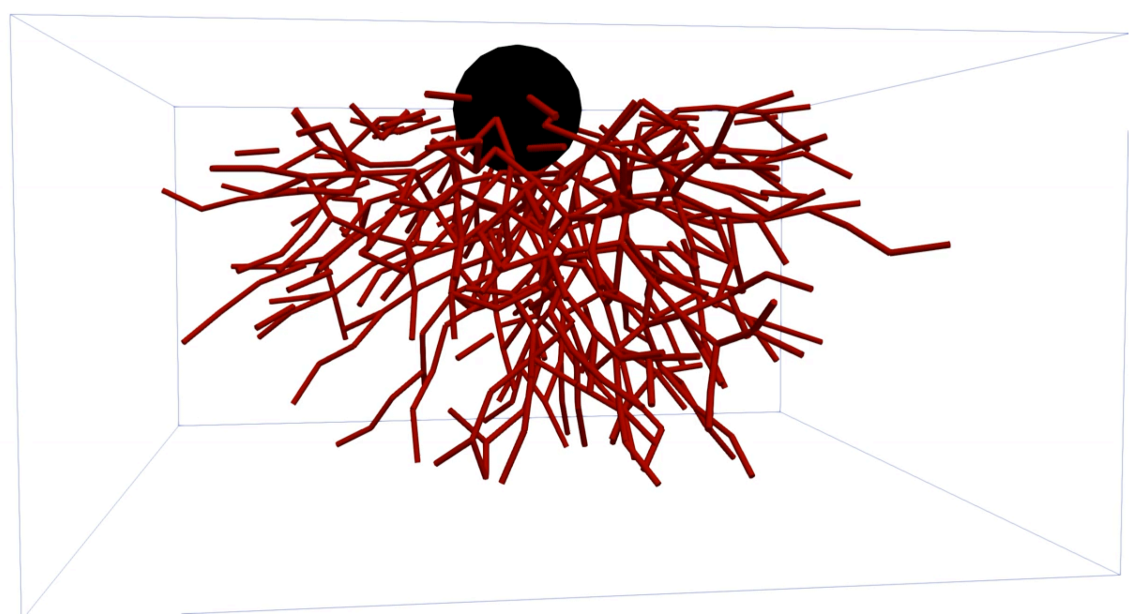
Drag vs position of the deepest point of the impactor

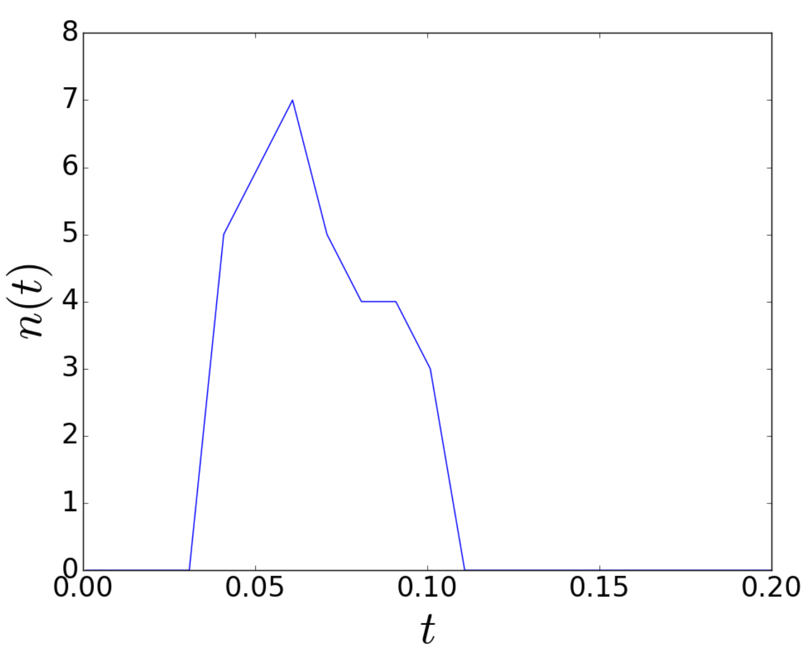


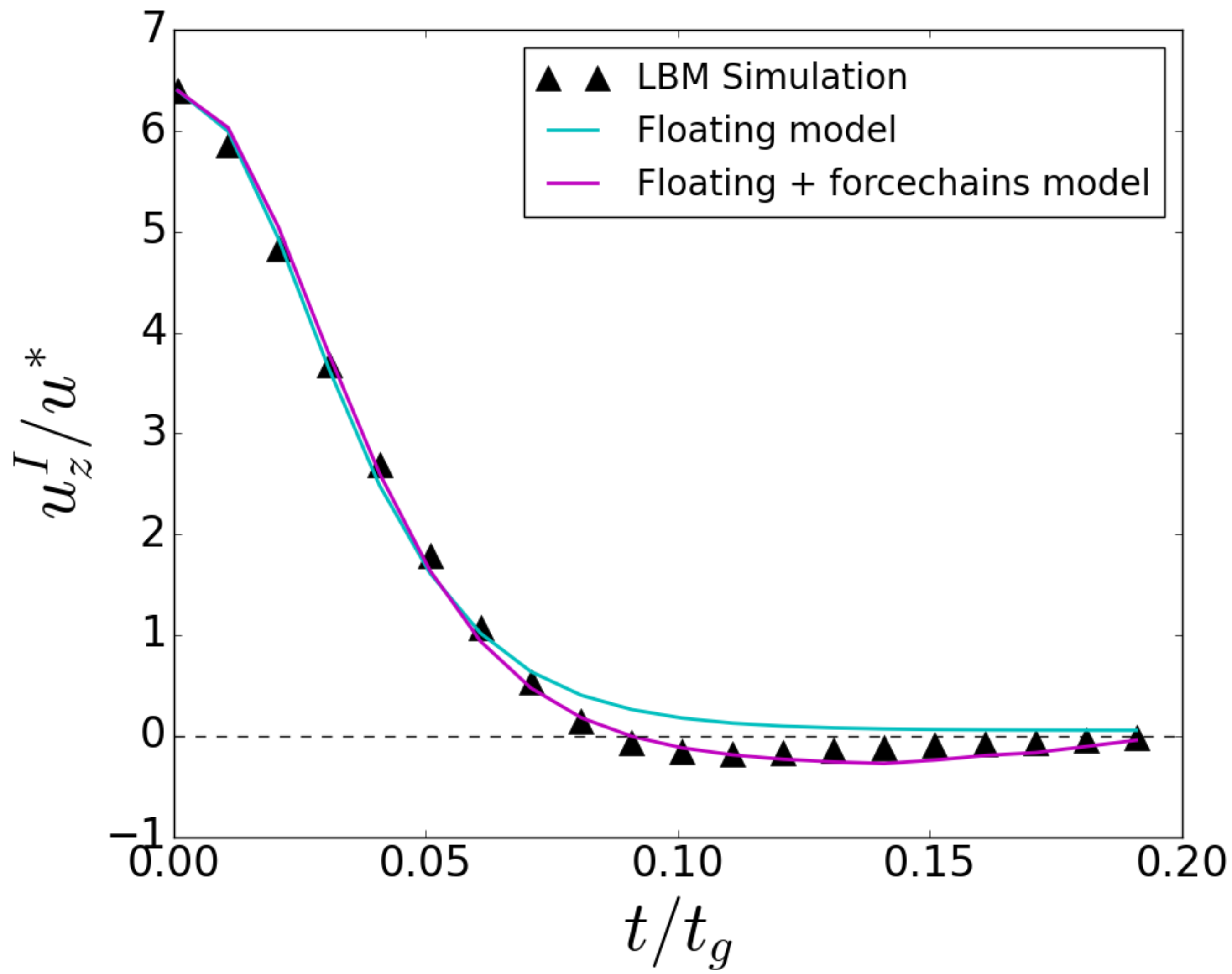
Floating + force chains model

$$m_I \ddot{z}_I = -m_I \tilde{g} - 3\pi\eta \dot{z}_I |z| - n k_n z_I,$$

$n(t) \longrightarrow$ Number of percolated force chains from the impactor to the bottom boundary







Rebound of the impactor recovered!

Conclusions of part 2

Universal scalings of $F_{\max}$ and $t_{\max}$ are observed in dense suspension under impact with crossover from low u_0 to high u_0 regime.

Floating model with viscous drag that depends linearly on the position of the deepest point of the impactor can recover the crossover and the scaling exponents

The rebound motion is accurately recovered by introducing an elastic term to the model based on the number of the connected force chains from the impactor to the bottom plate.