

Elastic response and loop trajectories of frictional granular materials under oscillatory shear

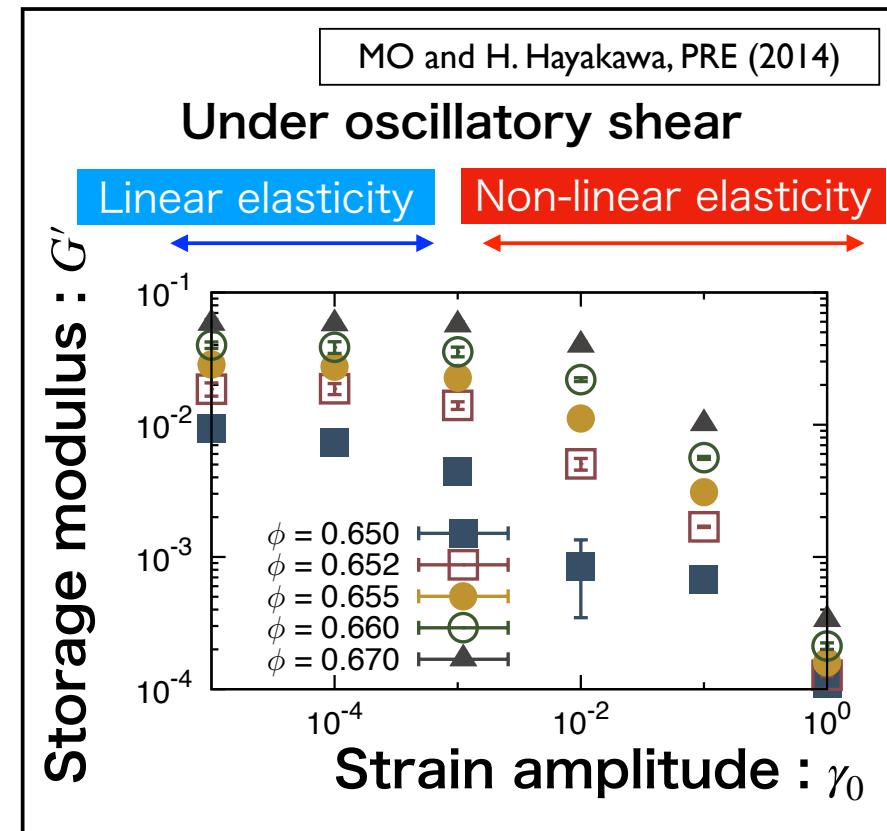
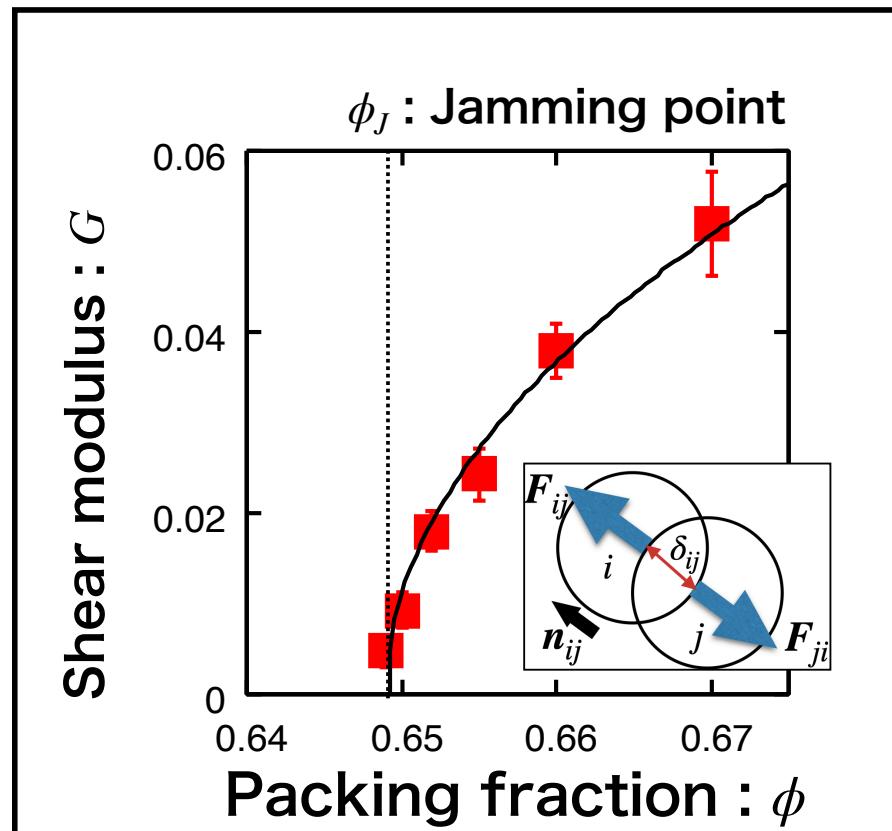
周期的剪断を受けた高密度粉体における弾性応答とループ軌道

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Hisao Hayakawa (Kyoto Univ.)

M. Otsuki and H. Hayakawa, EPJE 44, 70 (2021)

Rheology of frictionless jammed particles

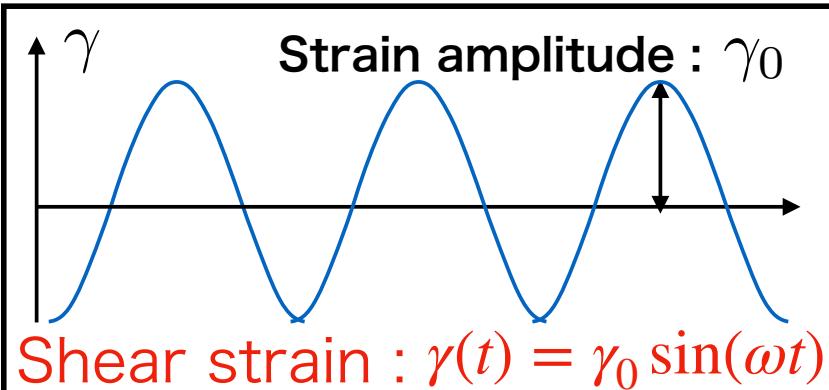
Jammed particles : collections of grains (sand, emulsion)



- Jammed particles have rigidity above ϕ_J .
- G' exhibits non-linear elasticity under finite strain.

Nonlinear elasticity in frictionless particles

MO and H. Hayakawa, arXiv:2101.07473

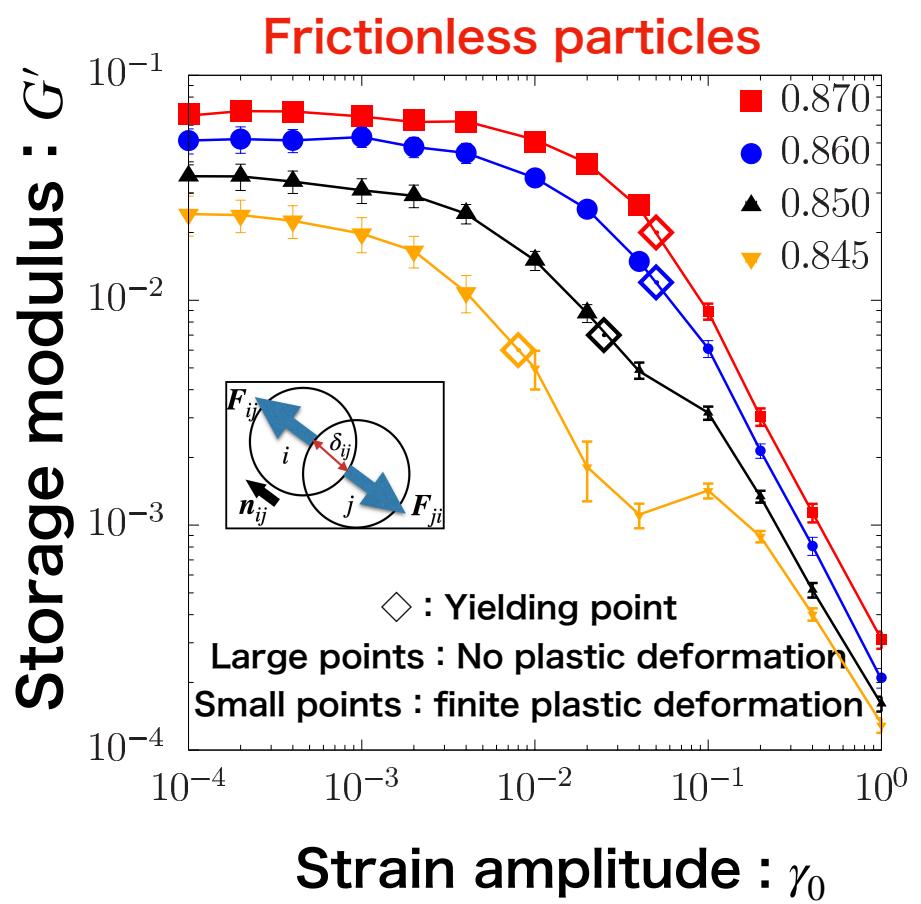


Decrease of G' before yielding

J. Boschan, et. al.,(2016),
S. Dagois-Bohy, et. al., (2017),
T. Kawasaki and K. Miyazaki, (2020)

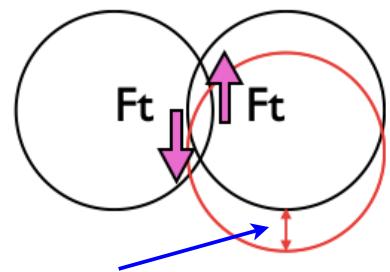
G' is related to particle trajectories.

MO and H. Hayakawa, arXiv:2101.07473



Effect of friction for granular materials

Tangential friction between particles



F_n : Normal repulsion

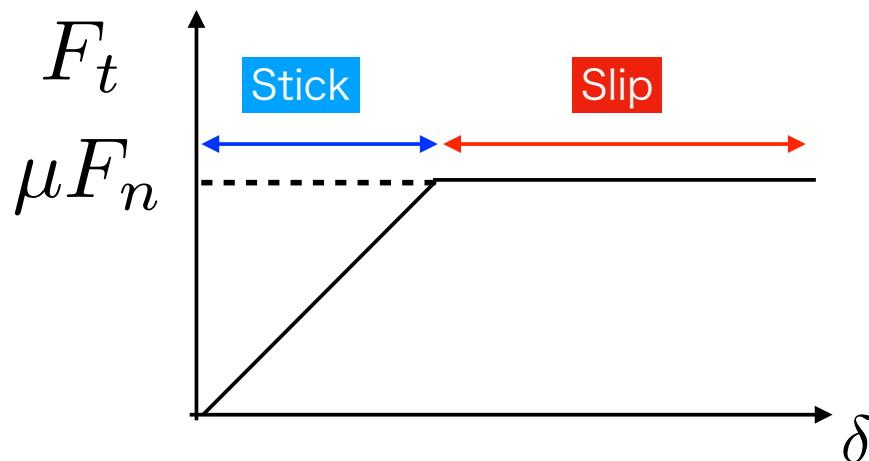
F_t : Tangential friction

μ : Friction coefficient

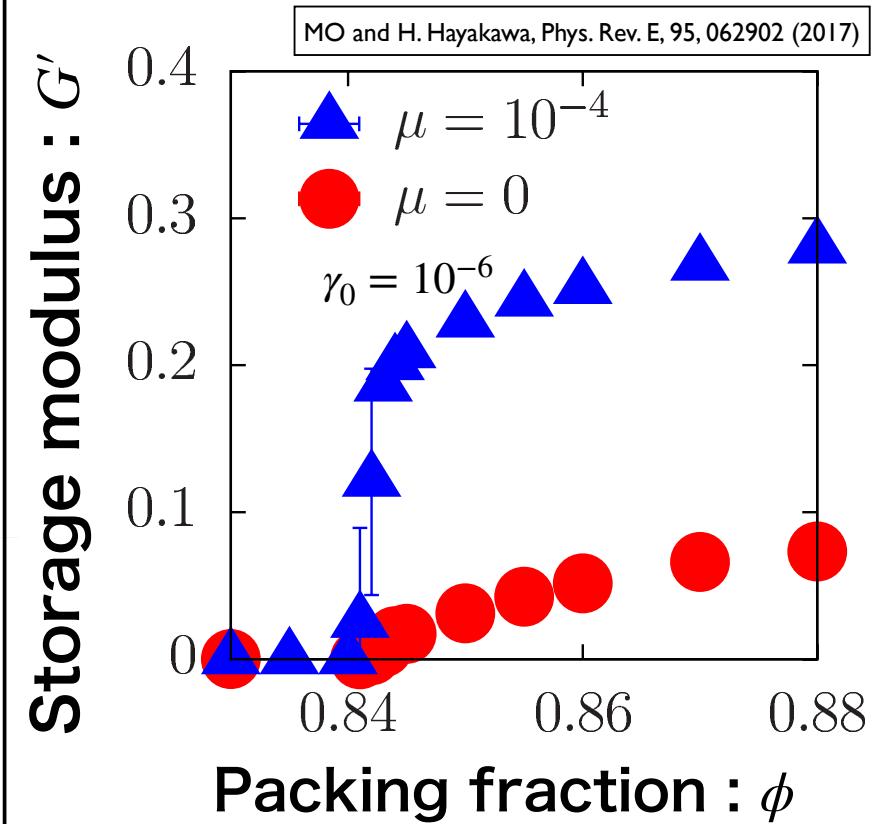
$$F_t \leq \mu F_n$$

δ : Tangential displacement

Coulomb's law of friction



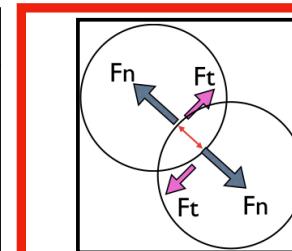
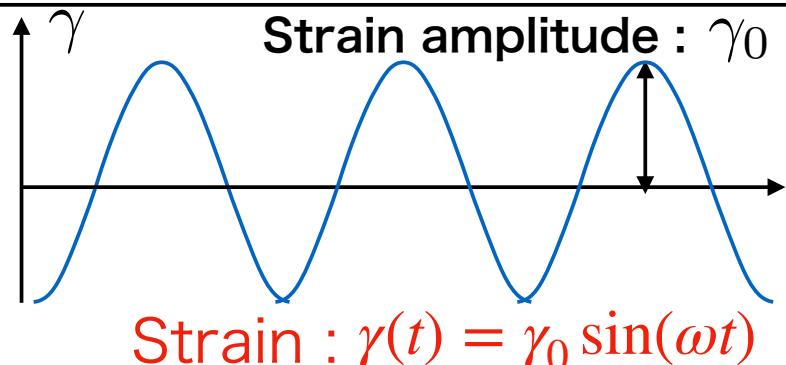
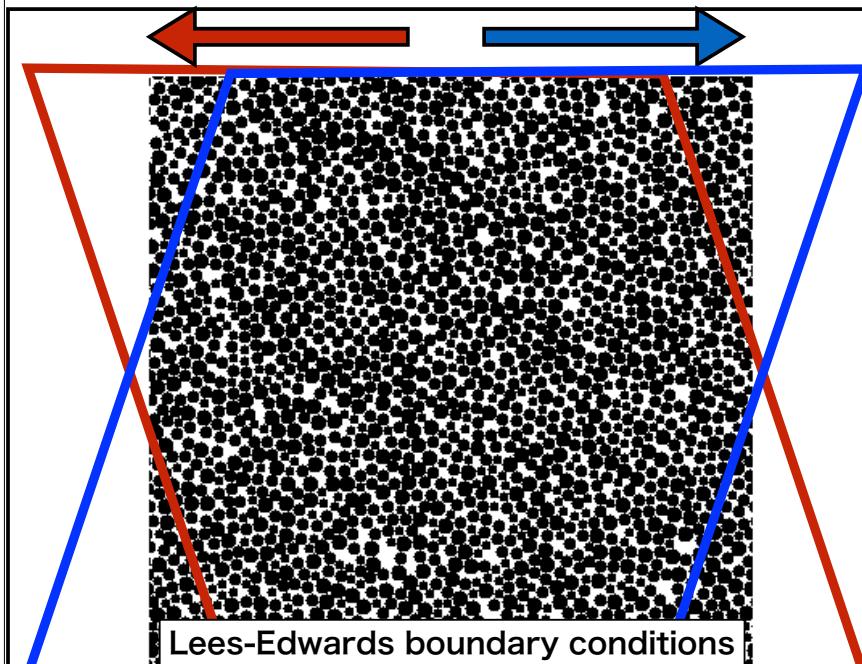
Elasticity in linear response regime



Discontinuous transition

Effect of friction on non-linear elasticity?

Model : frictional granular materials



F_n : Normal repulsive force

F_t : Tangential friction

Coulomb's law $F_t \leq \mu F_n$

μ : Friction coefficient

SLLOD eq. :

$$\frac{d\mathbf{r}_i}{dt} = \frac{\mathbf{p}_i}{m_i} + \dot{\gamma}(t)r_{i,y}\hat{\mathbf{x}},$$

$$\frac{d\mathbf{p}_i}{dt} = \sum_{j \neq i} \mathbf{F}_{ij} - \dot{\gamma}(t)p_{i,y}\hat{\mathbf{x}},$$

Shear stress: $\sigma(t)$

Storage modulus:

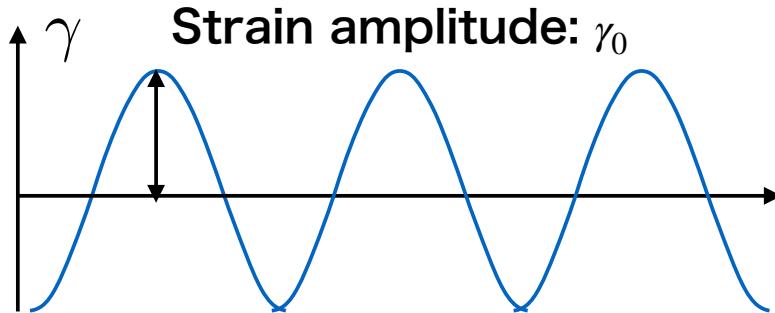
$$G' = \frac{\omega}{\pi} \int_0^{2\pi/\omega} d\theta \frac{\sigma(t)\sin(\omega t)}{\gamma_0}$$

Loss modulus:

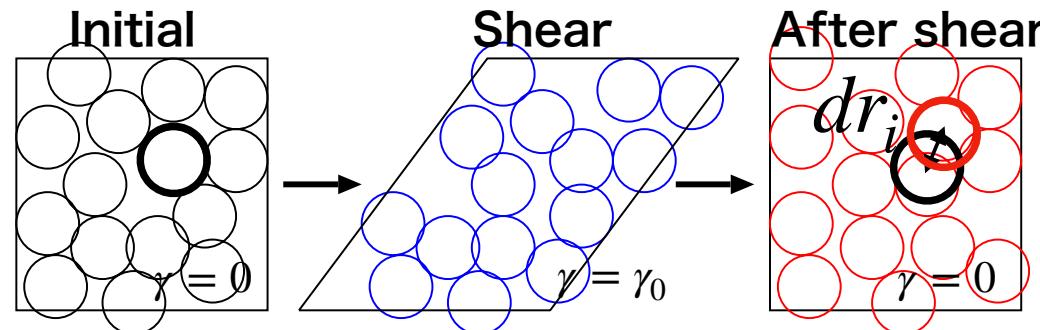
$$G'' = \frac{\omega}{\pi} \int_0^{2\pi/\omega} d\theta \frac{\sigma(t)\cos(\omega t)}{\gamma_0}$$

Microscopic plasticity

c.f. T. Kawasaki and L. Breathier, Phys. Rev. E 94, 022615 (2016)

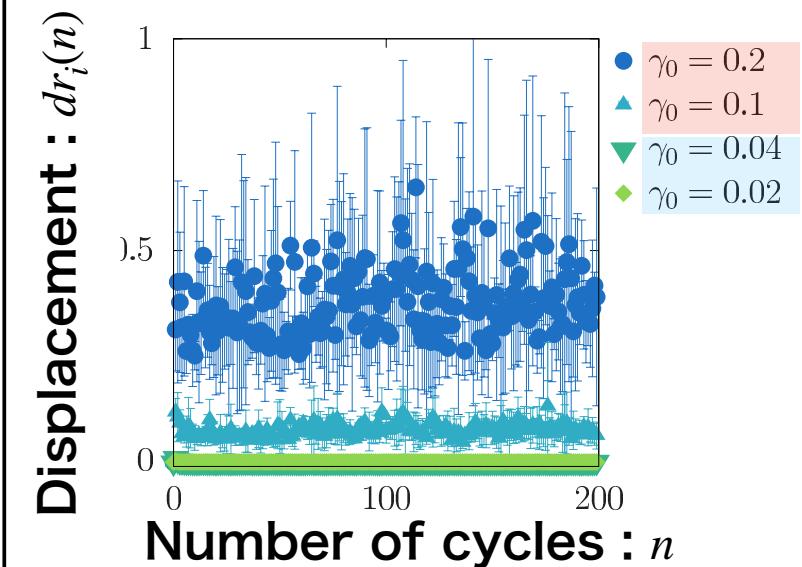


Large strain amplitude

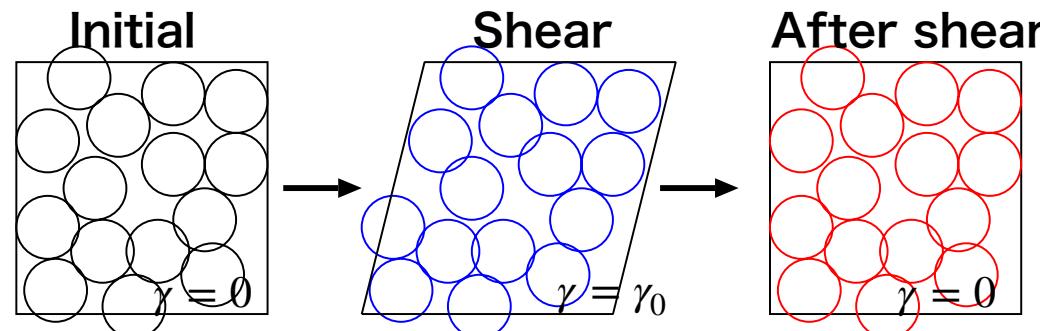


Position after n cycles : $r_i(n)$

Displacement : $dr_i(n) = |r_i(n) - r_i(n - 1)|$



Small strain amplitude



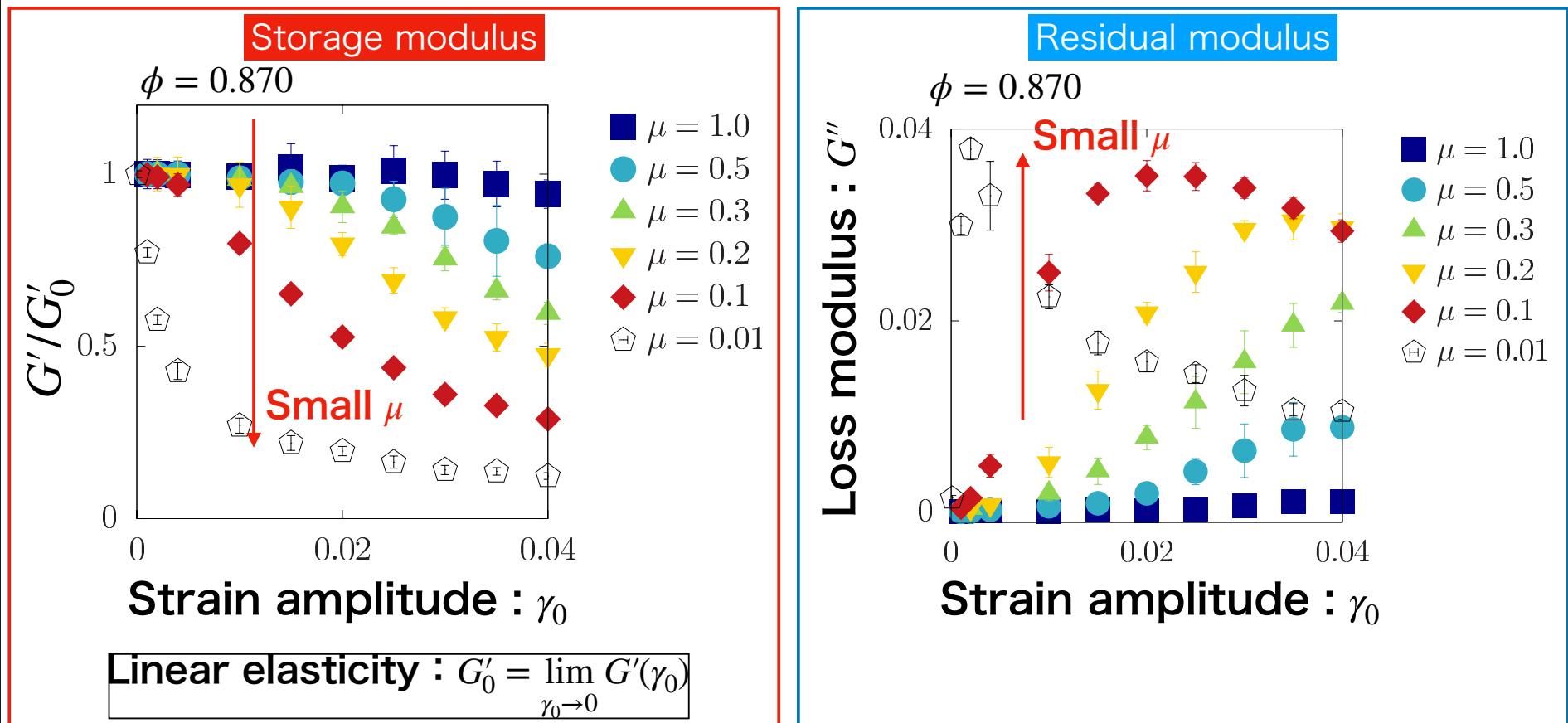
Plastic state

$dr(n) > 0$: Irreversible deformation

Absorbing state

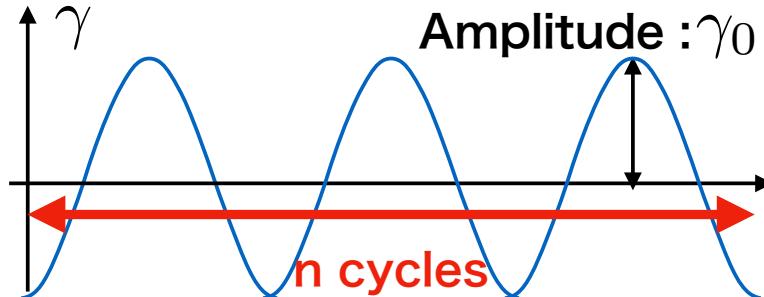
$dr(n) = 0$: No change in configuration
No plastic deformation

Mechanical response in absorbing state



- Softening in absorbing state (SAS): G' decreases without plastic deformation.
- SAS becomes significant for small μ and large γ_0 .
- G'' increases for small μ and large γ_0 .

Particle trajectories in absorbing state

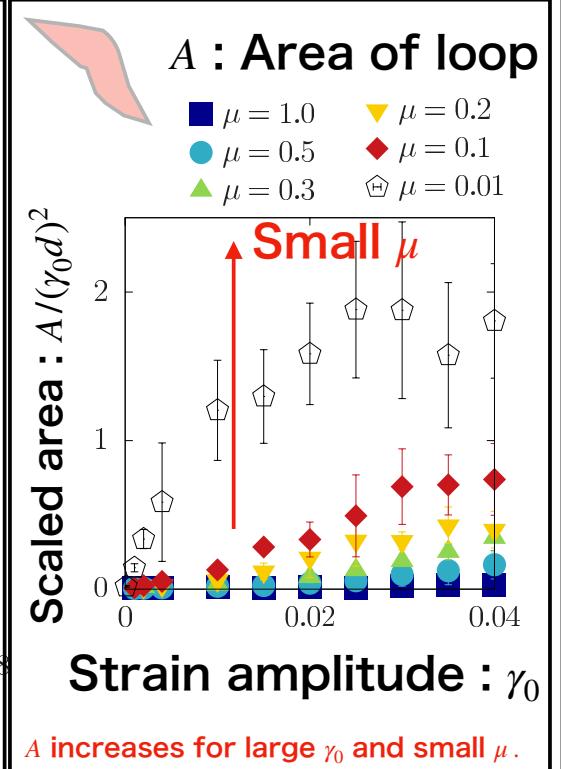
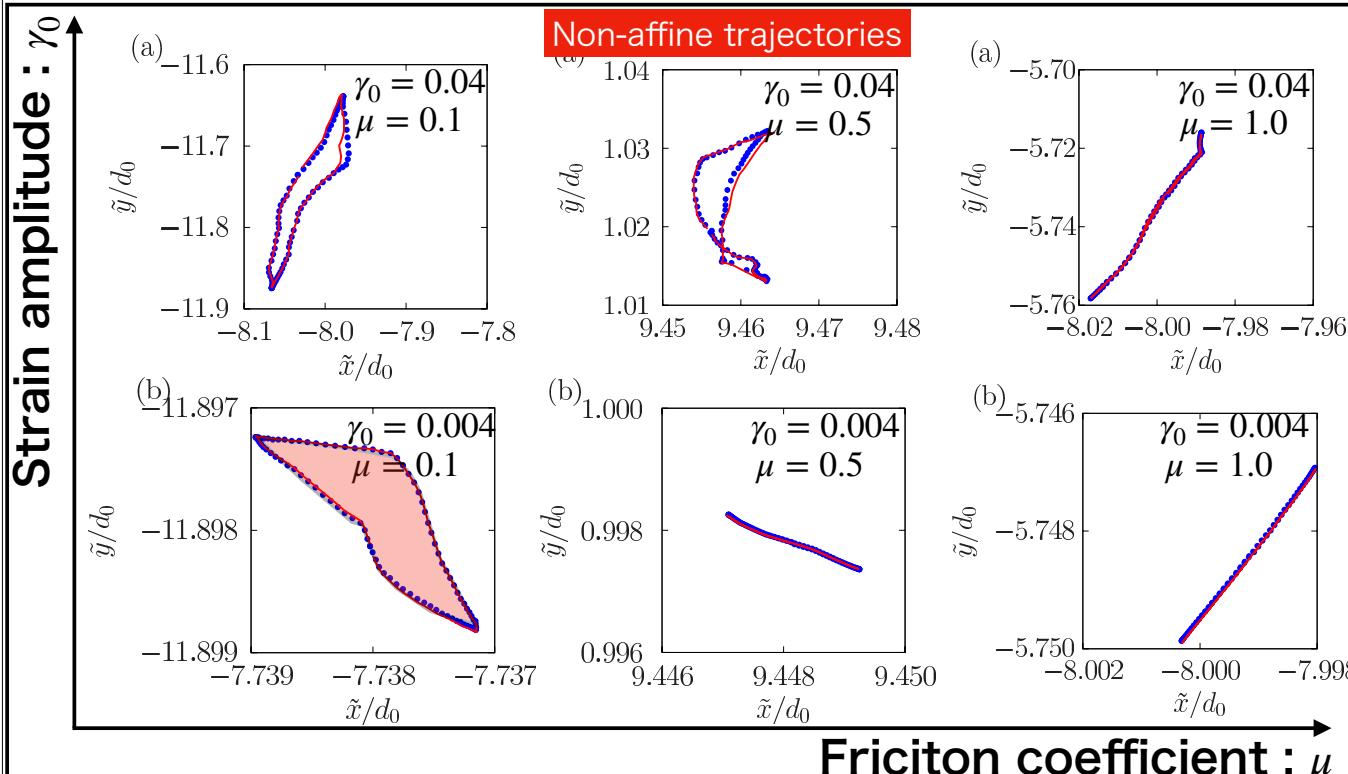


Position : $r_i(t) = R_i + \underline{\gamma(t)Y_i e_x} + \tilde{r}_i(t)$

Refference position : $R_i = (X_i, Y_i)$

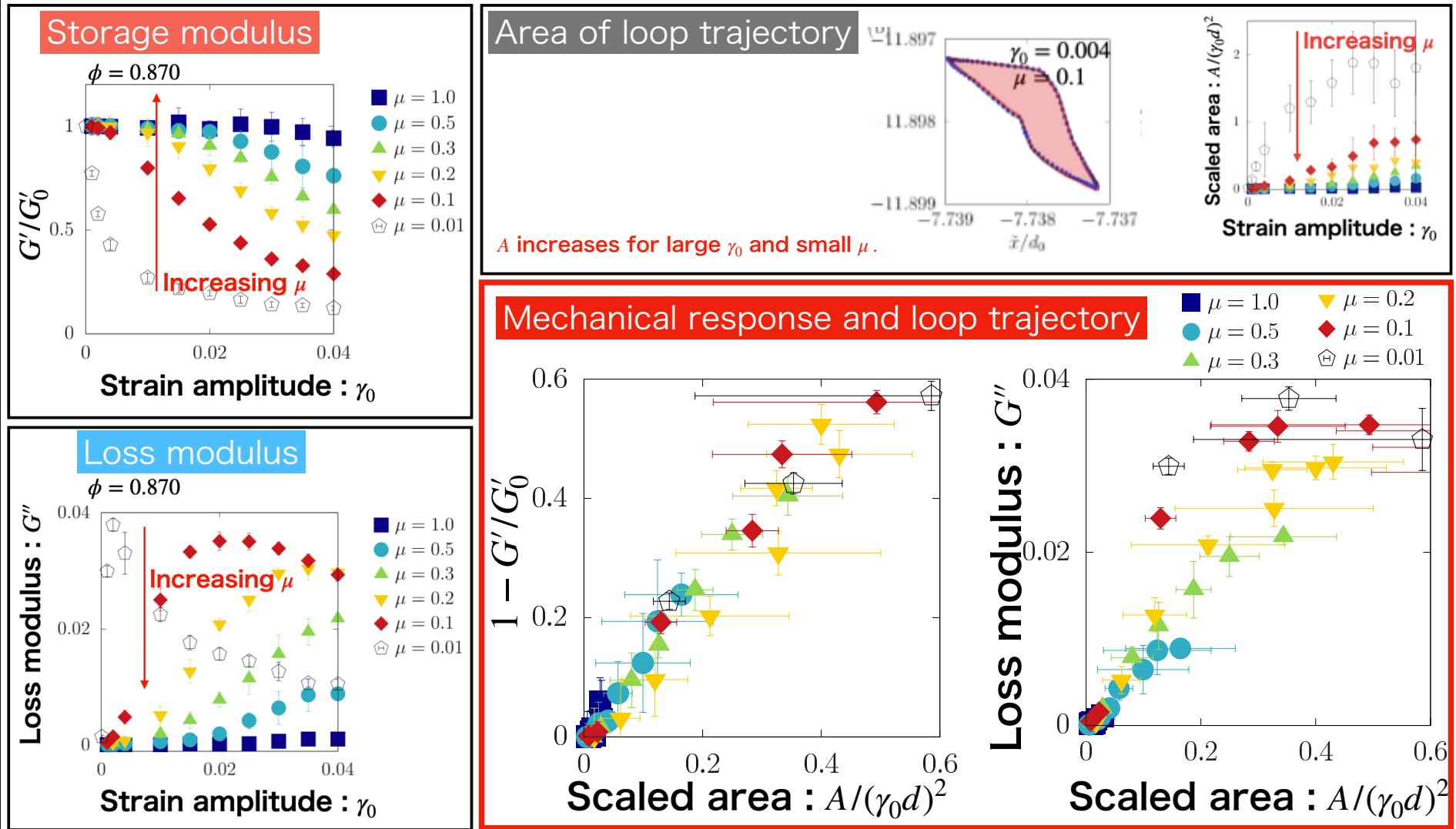
Affine motion : $\gamma(t)Y_i e_x$ (Macroscopic shear)

Non-affine motion : $\tilde{r}_i(t)$



c.f. $A \sim (\gamma_0 d)^2$ if the trajectory is scaled by $\gamma_0 d$ with the diameter of grain d .

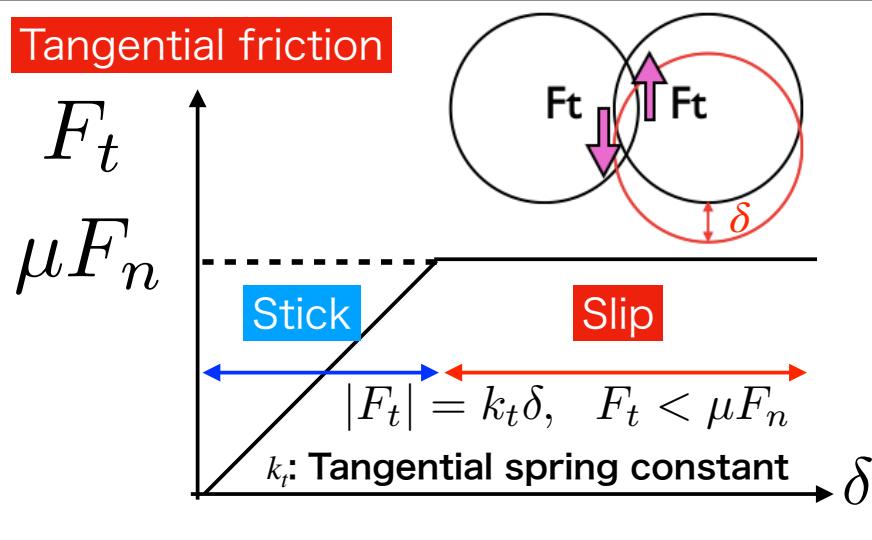
Mechanical response and loop



Mechanical response is scaled by the area of loops.

Scaling based on slip between particles

Non-linear elasticity might be related to the slip between particles.



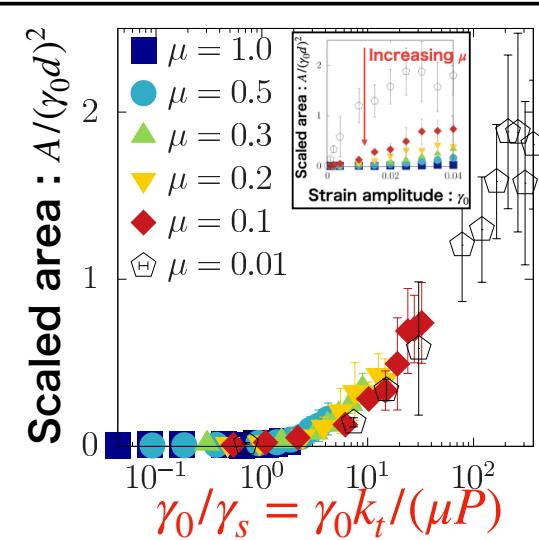
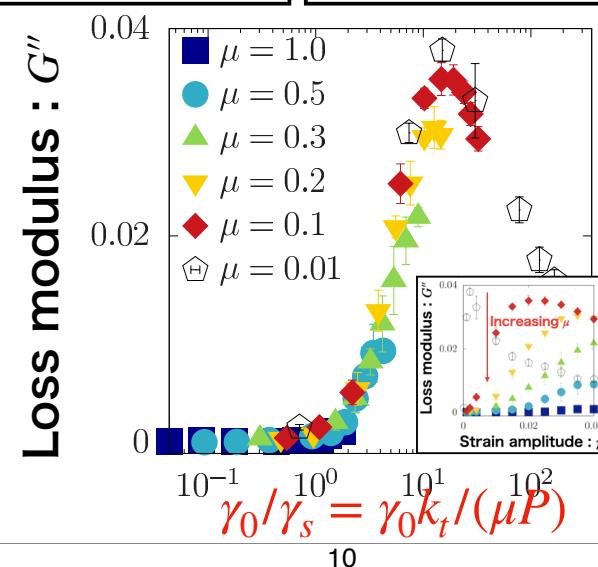
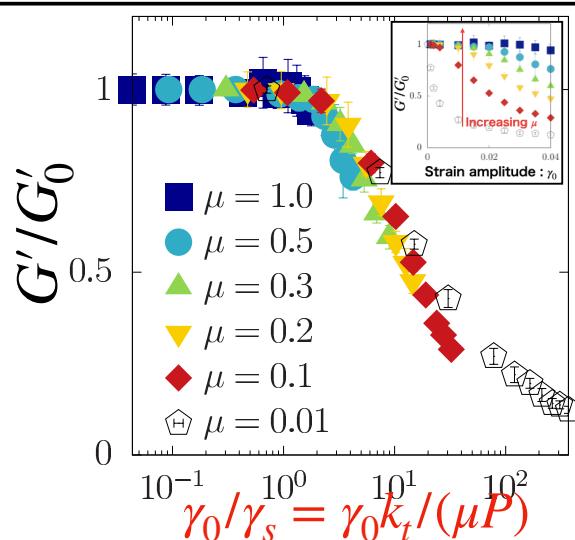
Strain for slip

Displacement for slip : $\delta_s = \mu F_n / k_t$

Pressure : $P \sim F_n/d$

Normal force: F_n , Diameter of grain: d

Strain for slip : $\gamma_s = \delta_s / d \sim \mu P / k_t$



Summary

M. Otsuki and H. Hayakawa, EPJE 44, 70 (2021)

- Topic : Non-linear elastic response in frictional grains
- G' decreases without plastic deformation.
- Loss modulus increases for large γ_0 and small μ .
- Non-linear response is related to the area of trajectories.
- Shear modulus is scaled by the strain amplitude for slip.
- Open question: origin of loss modulus without dissipation.

