

2分散系の稀薄慣性サスペンションの レオロジーに関する運動論

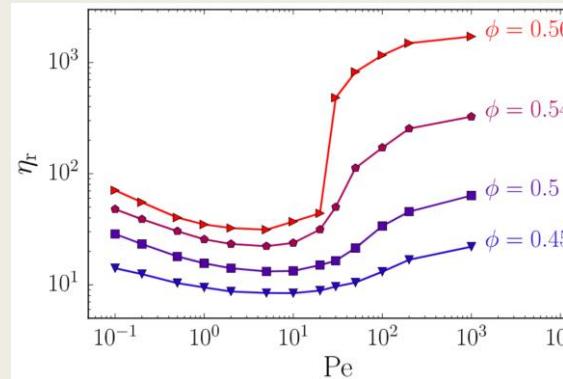


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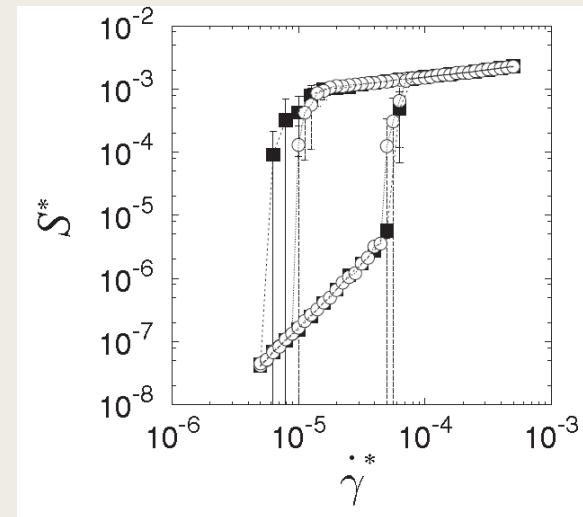
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Rheology

- Shear thickening (thinning):
viscosity: $\eta_s(\phi) = \sigma(\phi)/\dot{\gamma} \nearrow (\searrow)$
against the shear rate $\dot{\gamma}$.

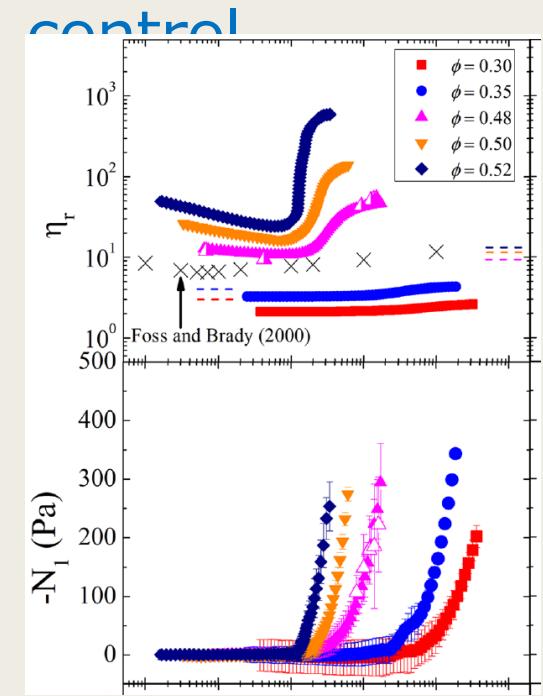


R. Mari, et al., PNAS **112**, 15326 (2015)



M.Otsuki & H. Hayakawa,
Phys. Rev. E **83**, 051301 (2011)

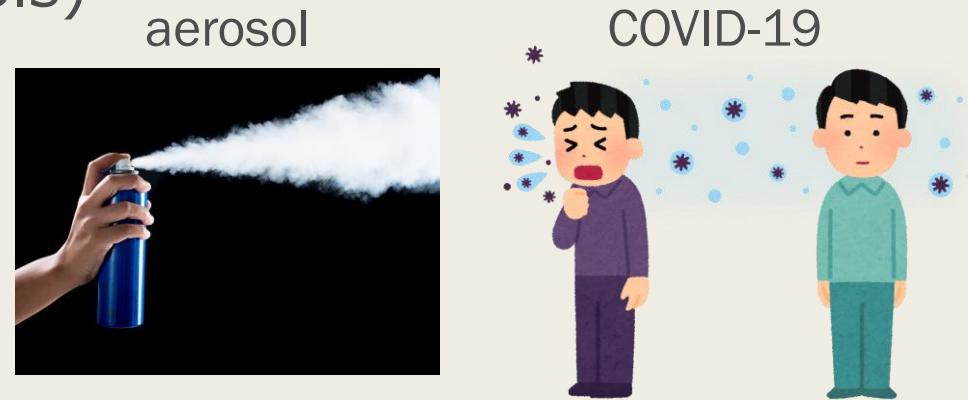
- Discontinuous shear thickening (DST):
Viscosity discontinuously increases.
- Industrial applications: protective vest and traction control
- DST is studied in many contexts and setups.
 - DST for dense systems (simulations)
Mutual friction
M.Otsuki & H. Hayakawa, Phys. Rev. E **83**, 051301 (2011)
R. Seto, et al., Phys. Rev. Lett. **111**, 218301 (2013)
 - DST for colloidal systems (experiments)
Normal stress difference
C. D. Cwalina & N.J. Wagner, J. Rheol. **58**, 949 (2014)



C. D. Cwalina & N.J. Wagner,
J. Rheol. **58**, 949 (2014)

Gas-solid suspensions

- Inertial suspensions (a model of aerosols)
- Particle size: $1 - 70 \mu\text{m}$
- Homogeneity is kept.
- Relatively dilute system
(theoretical treatment is available)

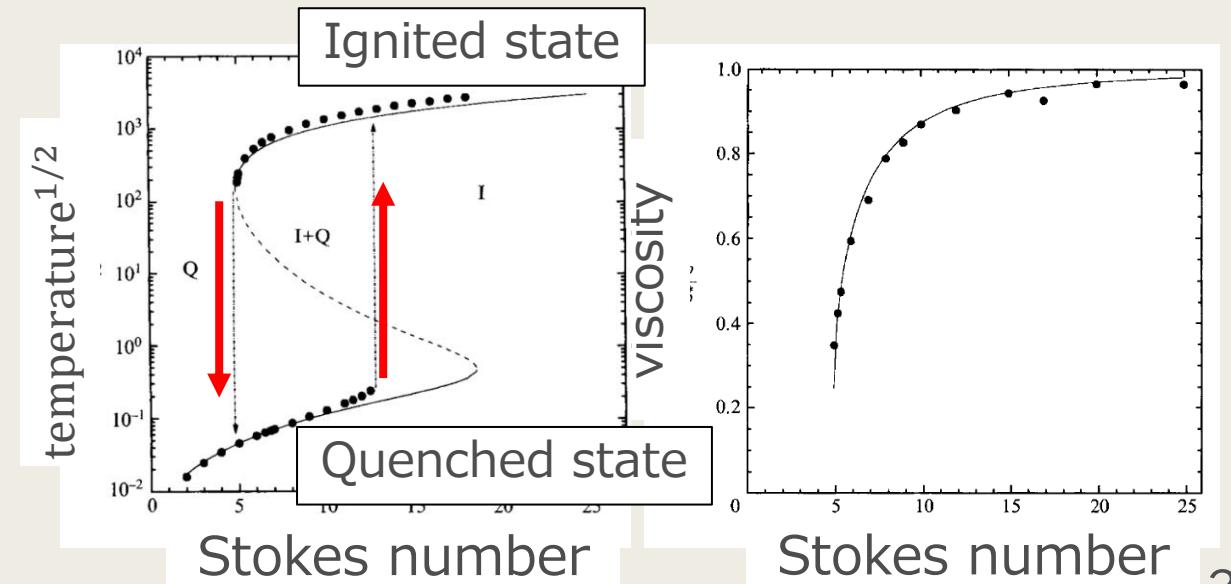


Previous (theoretical) studies for dilute systems

☞ Tsao and Koch, JFM 296, 211 (1995)

Kinetic theoretical approach
without thermal noise

⇒ DST-like (Quenched-Ignited)
transition for **temperature**,
but not for **viscosity**



Our previous works

- Kinetic theoretical approach considering **the thermal noise**
- Boltzmann-Enskog theory well describes monodisperse systems up to $\varphi \leq 0.50$.
- **DST**-like transition in the **dilute** system
 - ☞ H. Hayakawa and S. Takada, PTEP **2019**, 083J01 (2019)
- **DST**-like to **CST**-like as the density ↗
 - ☞ H. Hayakawa, S. Takada, and V. Garzó, PRE **96**, 042903 (2017)
- **Mpemba** effect in the relaxation process
 - ☞ S. Takada, H. Hayakawa, and A. Santos, Phys. Rev. E **103**, 032901 (2021)

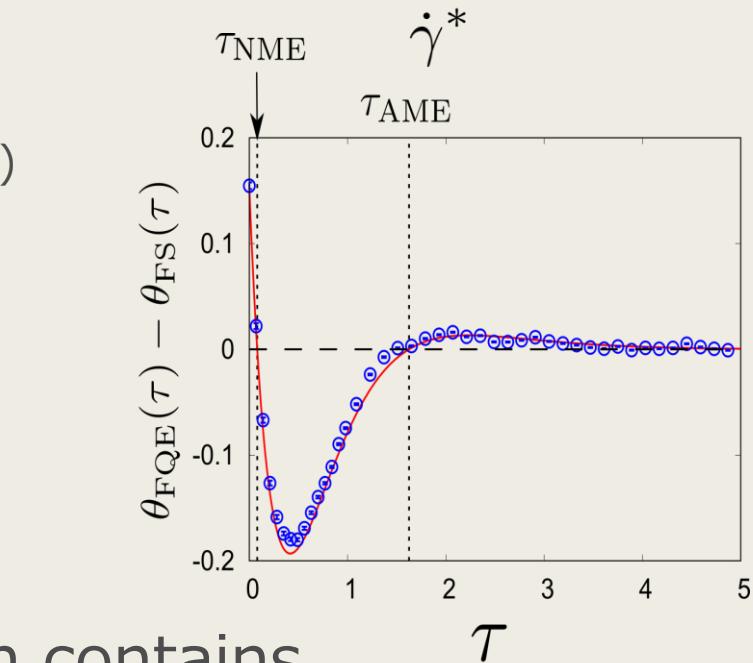
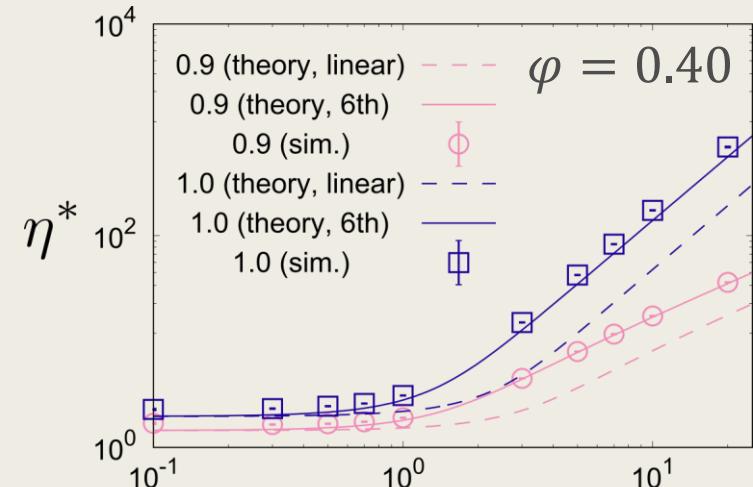
Question:

Impact of polydispersity on the rheology?



We consider a suspension which contains two kinds of spherical particles having different sizes.

DST = discontinuous shear thickening
CST = continuous shear thickening



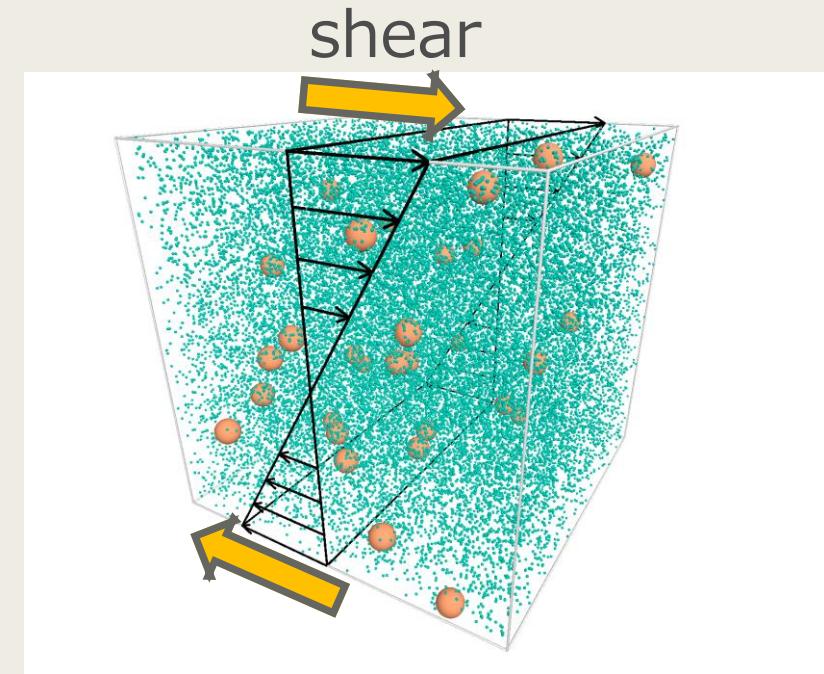
Setup and model

Binary particles are distributed.

	Larger particles species “1”	Smaller particles species “2”
Diameter	σ_1	σ_2
Mass	$m_1 (\propto \sigma_1^3)$	$m_2 (\propto \sigma_2^3)$
Num. particles	N_1	$N_2 (= N - N_1)$

Control parameters of the system:

- Packing fraction: $\varphi (\ll 1)$
- Restitution coeff.: e
- Shear rate: $\dot{\gamma}$
- Environmental temperature: T_{env}



Setup and model

Equation of motion for k -th particle of species “ i ”:

Langevin equation:
$$\frac{d\mathbf{p}_k^{(i)}}{dt} = -\zeta_i \mathbf{p}_k^{(i)} + \mathbf{F}_k^{\text{(imp)}} + m_i \boldsymbol{\xi}_k^{(i)}$$

$\mathbf{p}_k^{(i)} \equiv m_i (\mathbf{v}_k^{(i)} - \dot{\gamma} y_k \mathbf{e}_x) = m_i \mathbf{V}_k^{(i)}$; peculiar momentum

☞ D. J. Evans & G. Morriss, “Statistical Mechanics of Nonequilibrium Liquids”

① drag term $\zeta_i \propto \sigma_i$ (Stokes' drag)

② impulsive force due to collisions

$$\mathbf{v}'_1 = \mathbf{v}_1 - \frac{m_{ij}}{m_i} (1 + e_{ij}) (\mathbf{v}_{12} \cdot \hat{\boldsymbol{\sigma}}) \hat{\boldsymbol{\sigma}}$$

$$\mathbf{v}'_2 = \mathbf{v}_2 - \frac{m_{ij}}{m_j} (1 + e_{ij}) (\mathbf{v}_{12} \cdot \hat{\boldsymbol{\sigma}}) \hat{\boldsymbol{\sigma}}$$

$$m_{ij} \equiv \frac{m_i m_j}{m_i + m_j}$$
: reduced mass

③ thermal noise term satisfies

$$\langle \boldsymbol{\xi}_k^{(i)}(t) \rangle = 0,$$

$$\langle \boldsymbol{\xi}_{k,\alpha}^{(i)}(t) \boldsymbol{\xi}_{\ell,\beta}^{(j)}(t') \rangle = \frac{2\zeta_i T_{\text{env}}}{m_i} \delta_{ij} \delta_{k\ell} \delta_{\alpha\beta} \delta(t - t')$$

Simulations are also performed by solving the **Langevin eq.**

Kinetic theory

Boltzmann kinetic equation for the inertial suspension

Langevin
Equation



$$\left(\frac{\partial}{\partial t} - \dot{\gamma} V_y \frac{\partial}{\partial V_x} \right) f_i(\mathbf{V}, t) = \zeta_i \frac{\partial}{\partial \mathbf{V}} \cdot \left(\left[\mathbf{V} + \frac{T_{\text{env}}}{m_i} \frac{\partial}{\partial \mathbf{V}} \right] f_i(\mathbf{V}, t) \right) + \sum_j J_{ij}(\mathbf{V} | f_i, f_j)$$

shear

drag from the solvent

collision

※ $f_i(\mathbf{V}, t)$: velocity dist. for species "i"

Collision integral: $J_{ij}(\mathbf{V}_1 | f_i, f_j) = \sigma_{ij}^2 \int d\mathbf{V}_2 \int d\hat{\boldsymbol{\sigma}} \Theta(\hat{\boldsymbol{\sigma}} \cdot \mathbf{V}_{12}) (\hat{\boldsymbol{\sigma}} \cdot \mathbf{V}_{12}) \left[\frac{f_i(\mathbf{V}'_1, t) f_j(\mathbf{V}'_2, t)}{e_{ij}^2} - f_i(\mathbf{V}_1, t) f_j(\mathbf{V}_2, t) \right]$

Evol. Eq. for the kinetic stress:

$$\frac{\partial}{\partial t} P_{\alpha\beta}^{(i)} + \dot{\gamma} \left(\delta_{\alpha x} P_{y\beta}^{(i)} + \delta_{\beta x} P_{y\alpha}^{(i)} \right) = -2\zeta_i \left(P_{\alpha\beta}^{(i)} - n_i T_{\text{env}} \delta_{\alpha\beta} \right) - \sum_j \Lambda_{\alpha\beta}^{(ij)}$$

Kinetic stress of species "i":

$$P_{\alpha\beta}^{(i)} = m_i \int d\mathbf{V} V_\alpha V_\beta f_i(\mathbf{V}, t)$$

Moment of the collision integral: $\Lambda_{\alpha\beta}^{(ij)} = -m_i \int d\mathbf{V} V_\alpha V_\beta J_{ij}(\mathbf{V} | f_i, f_j)$

**Equations are
NOT closed!!**

Kinetic theory

Maxwellian dist.: $f_{i,M}(V) = n_i \left(\frac{m_i}{2\pi T_i} \right)^{\frac{3}{2}} \exp \left(-\frac{m_i V^2}{2T_i} \right)$

Boltzmann
kinetic equation



Closure: Grad's approximation

$$f_i(V) = f_{i,M}(V) \left[1 + \frac{m_i}{2T_i} \left(\frac{P_{\alpha\beta}^{(i)}}{n_i T_i} - \delta_{\alpha\beta} \right) V_\alpha V_\beta \right]$$

A set of dynamic equations:

Temperature: $\partial_\tau \theta_i = -\frac{2}{3} \dot{\gamma}^* \Pi_{xy}^* + 2\zeta_i^* (1 - \theta_i) - \frac{1}{3} \sum_j \Lambda_{\alpha\alpha}^{(ij)*}$

Anisotropic temp: $\partial_\tau \Delta\theta_i = -2\dot{\gamma}^* \Pi_{xy}^* - 2\zeta_i^* \Delta\theta_i - \sum_j (\Lambda_{xx}^{(ij)*} - \Lambda_{yy}^{(ij)*})$

Shear stress: $\partial_\tau \Pi_{xy}^{(i)*} = -\dot{\gamma}^* \left(\theta_i - \frac{1}{3} \Delta\theta_i \right) - 2\zeta_i^* \Pi_{xy}^{(i)*} - \sum_j \Lambda_{xy}^{(i)*}$

Nondim. quantities: $\theta_i \equiv \frac{T_i}{T_{\text{env}}}, \Delta\theta_i \equiv \frac{P_{xx}^{(i)} - P_{yy}^{(i)}}{n_i T_{\text{env}}}, \Pi_{xy}^{(i)*} \equiv \frac{P_{xy}^{(i)}}{n_i T_{\text{env}}} - \theta_i \delta_{\alpha\beta}$.

$\Lambda_{\alpha\alpha}^{(ij)*}, \Lambda_{xx}^{(ij)*} - \Lambda_{yy}^{(ij)*}, \Lambda_{xy}^{(i)*}$: functions of $\theta_i, \Delta\theta_i, \Pi_{xy}^{(i)*}$.



We evaluate the steady solution of this set of equations.

Rheology ($N_1\sigma_1^3 = N_2\sigma_2^3$)

The theoretical flow curves work well.

- Mean viscosity: $\eta = -\sum_i P_{xy}^{(i)} / \dot{\gamma}$

Bidispersity is irrelevant.

DST for $\sigma_1 \gtrsim \sigma_2$ (known result)

- Viscosity ratio: $(\eta_1/\eta_2, \eta_i = -\sum_i P_{xy}^{(i)} / \dot{\gamma})$

◆ Discontinuous change at $\dot{\gamma}^* \simeq 30.0$

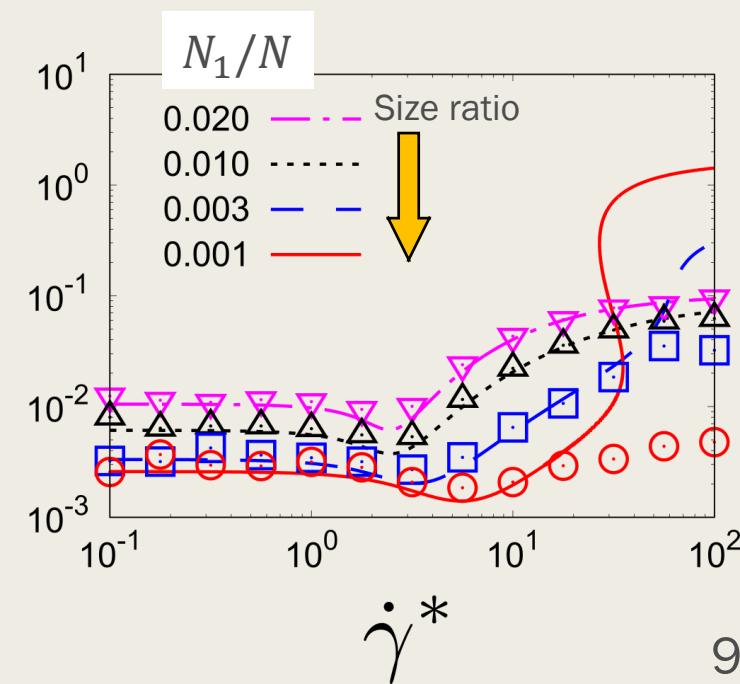
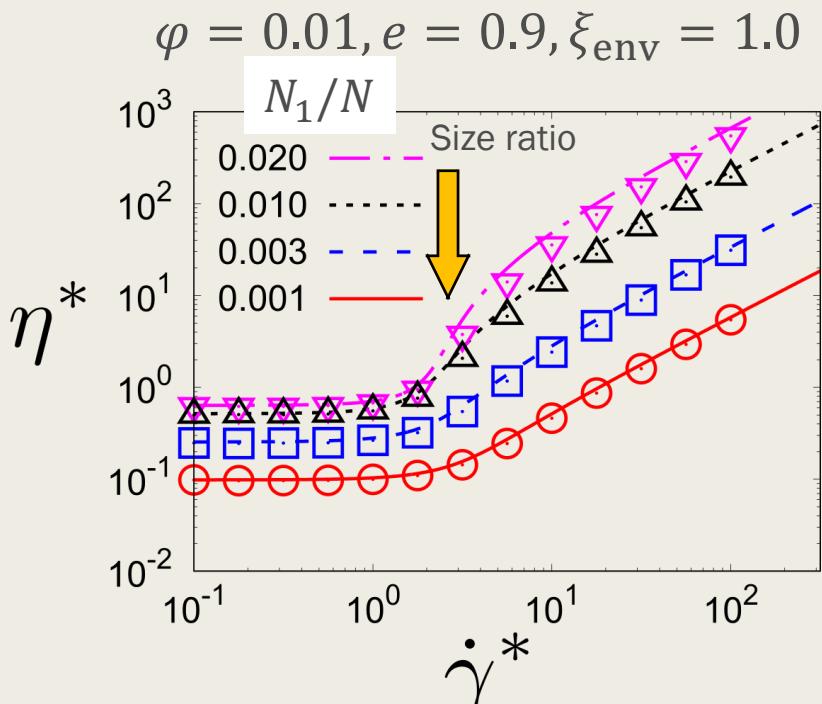
◆ Discrepancy exists for

$\sigma_1/\sigma_2 \gg 1$ & $\dot{\gamma}^* \geq 30.0$ when $N = 10^3$ $\frac{\eta_1}{\eta_2}$

(☞ see the next slide)

◆ " $\dot{\gamma}^* \rightarrow \infty$ " = granular gas limit

☞ Montanero & Garzó, Physica A **310**, 17 (2002).



Rheology ($N_1\sigma_1^3 = N_2\sigma_2^3$)

- Finite size effect of the simulation

As N (num. particles) increases,

**Simulations
results**

converge



**Theoretical
values**

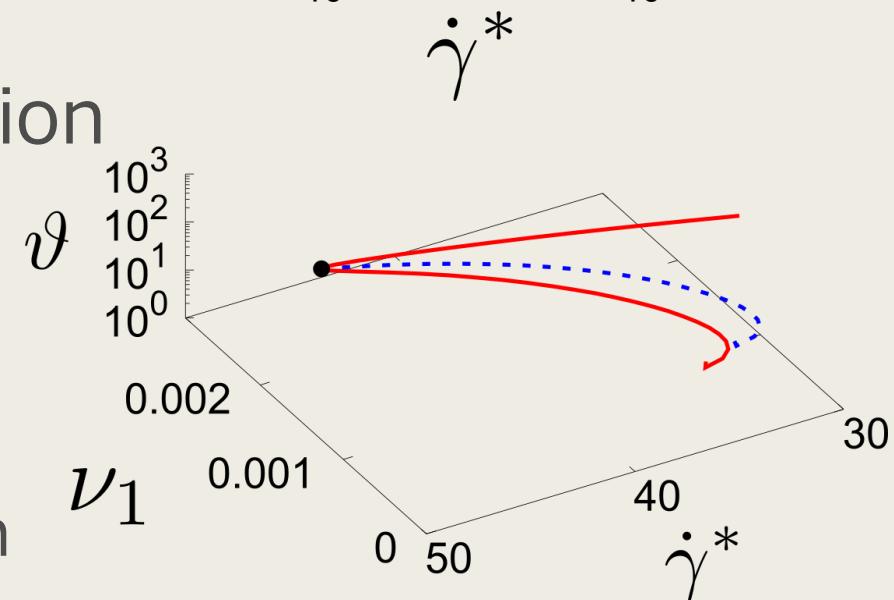
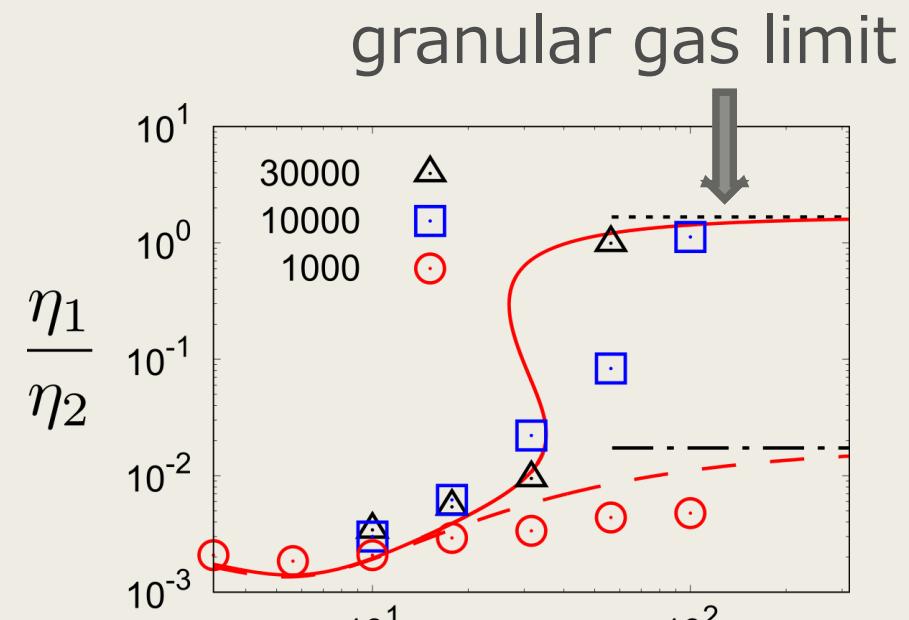
- Viscosity ratio shows DST-like transition as the partial density decreases.

$$(\nu_{1,c} \equiv \frac{N_1}{N} \simeq 0.0228)$$

Red lines: $\frac{\partial \dot{\gamma}^*}{\partial \vartheta} = 0$, Blue line: $\frac{\partial^2 \dot{\gamma}^*}{\partial \vartheta^2} = 0$

\Rightarrow analogous to the first order transition

☞ van der Waals fluid



Discussion

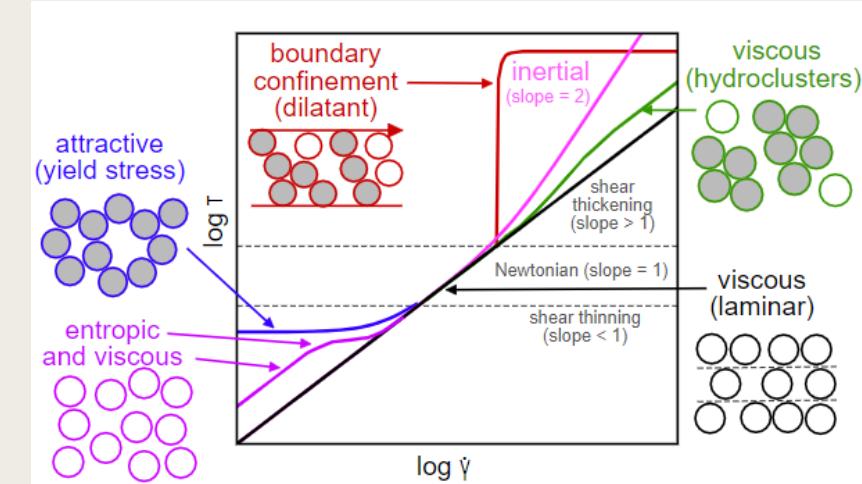
■ More realistic system

Resistance matrix

instead of the drag coeff. ζ_i (scalar)

(This depends on the config. of particles.)

$$\mathbf{F}_{\text{drag}} = -\mathbf{R}(\mathbf{r}) \cdot (\mathbf{v} - \dot{\gamma} y \mathbf{e}_x) \quad \rightleftharpoons \text{Kim \& Karrila, } \\ \text{"Microhydrodynamics"} \\ (\text{Dover, 1991})$$

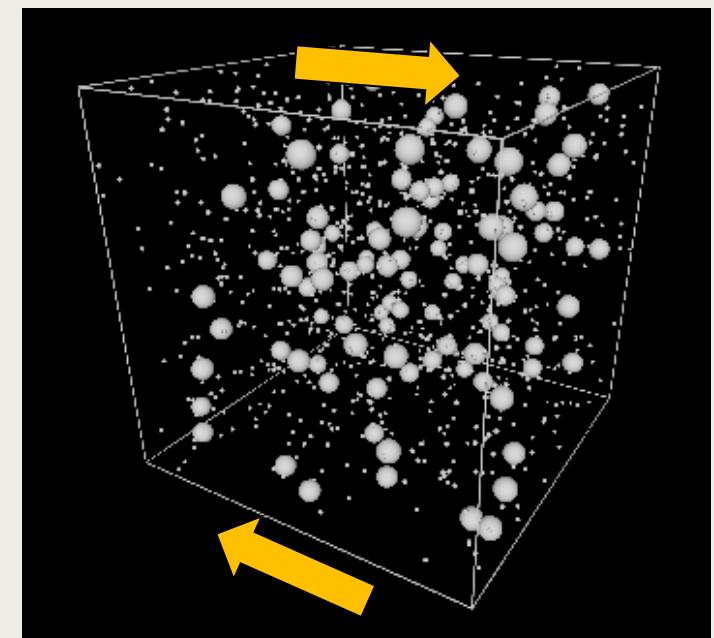
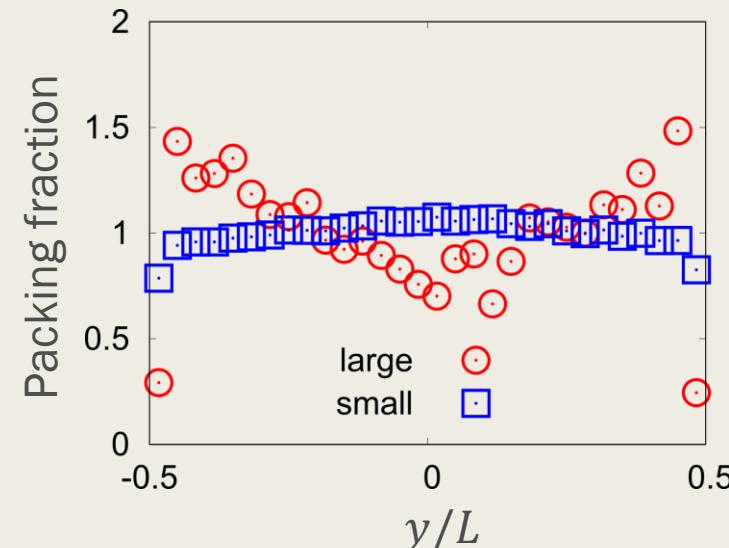


Brown and Jaeger,
Science 333, 1230 (2011)

■ Inhomogeneous situations

diffusion,
segregation,

...



Summary

We have examined the kinetic theory of the rheology for a dilute binary inertial suspension.

⇒ Kinetic theory with Grad's approx. works well.

- known result

DST for the mean viscosity.

(similar to monodisperse systems)

- new result

Discontinuous change of the viscosity ratio for larger size ratio.