Sound damping near jamming

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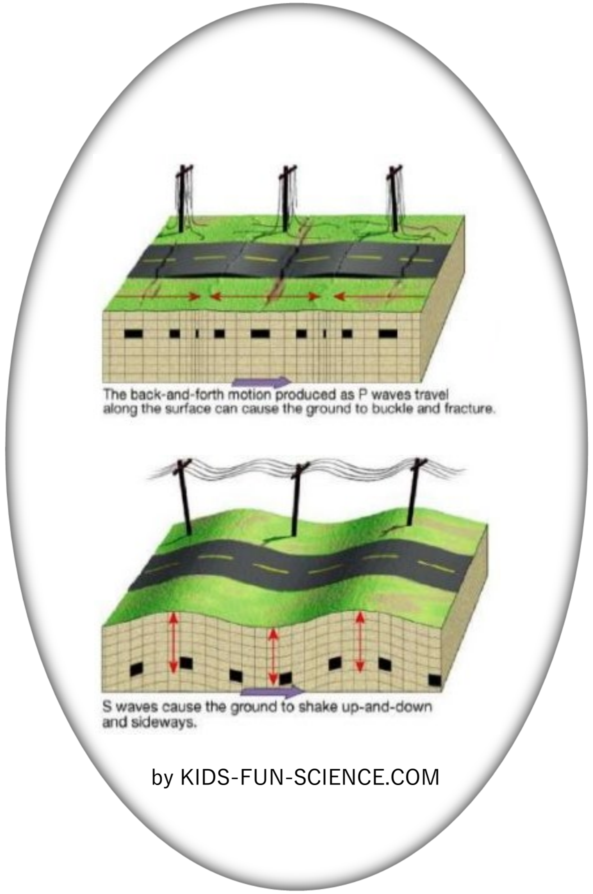
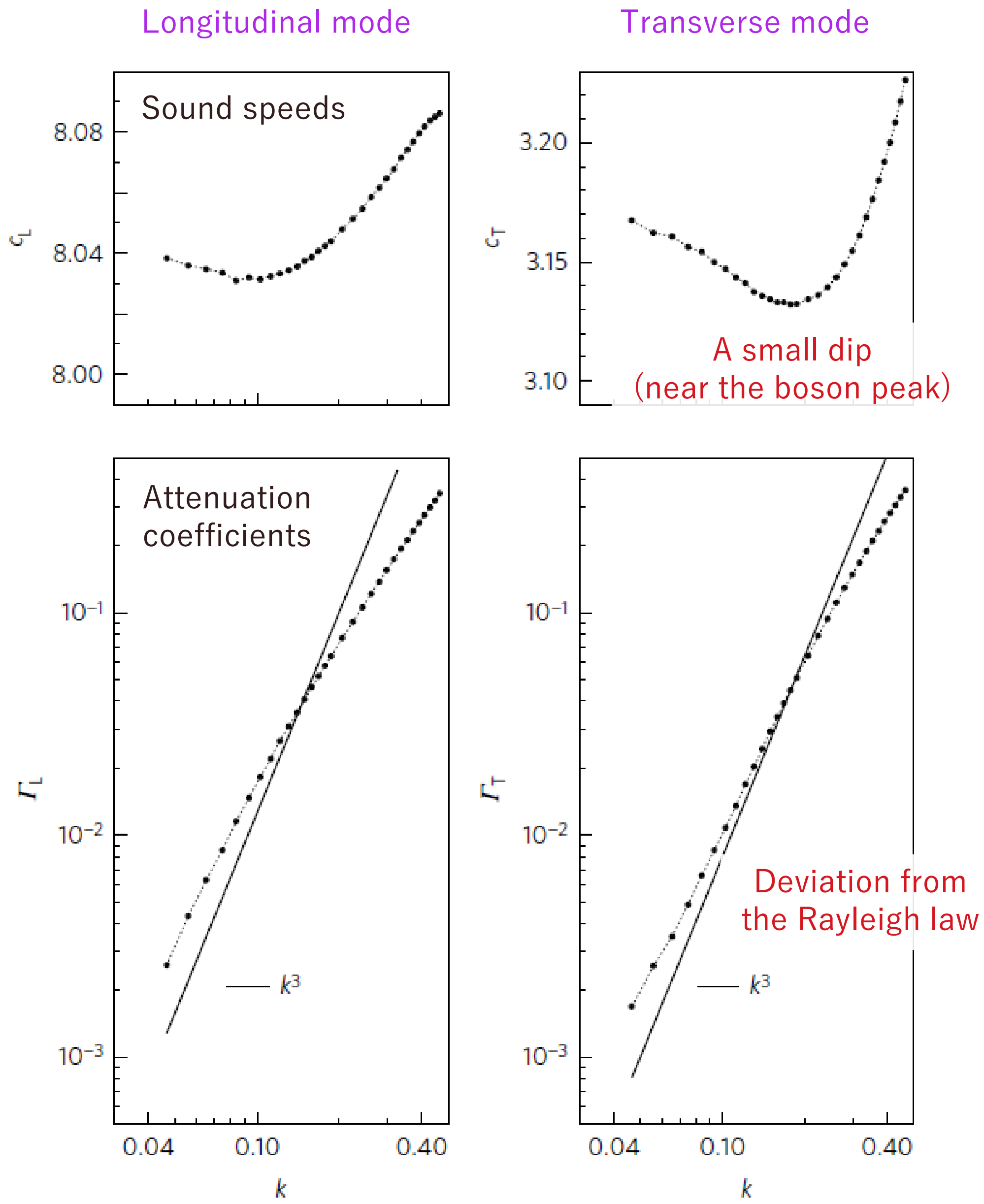
Introduction

- Sound in granular or amorphous materials, e.g. earthquakes.
- Useful for the measurements of elastic moduli, e.g. K and G .
- However, its properties are anomalous (see figures).





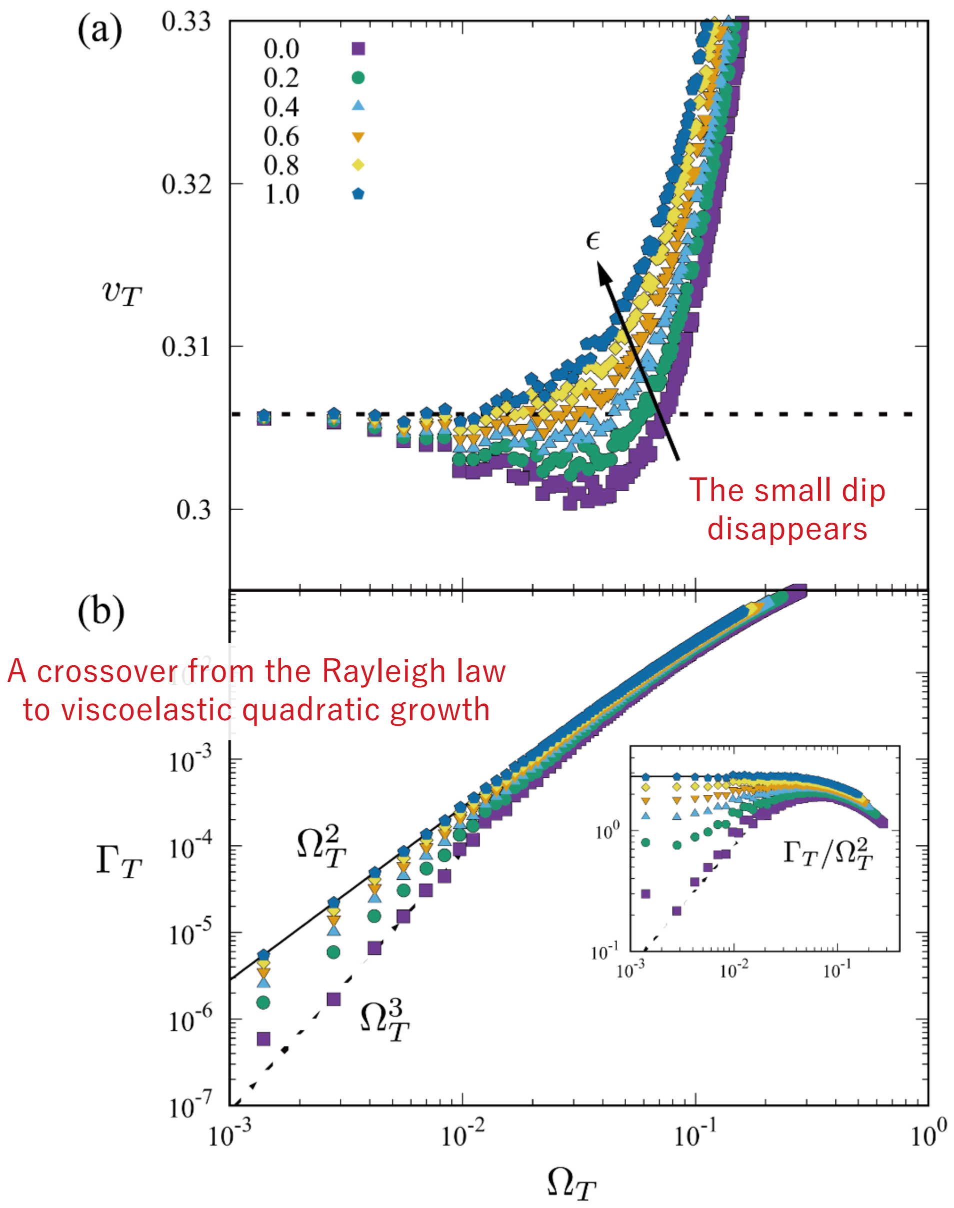
S. Gelin, H. Tanaka, and A. Lemaitre, *Nat. Materials* 15, 1177 (2016).
A. Marruzzo, W. Schirmacher, A. Fratalocchi, and G. Ruocco, *Sci. Rep.* 3, 1407 (2013).
G. Monaco and S. Mossa, *PNAS* 106, 16907 (2009).

Motivation

- The role of contact damping (see figures).

K. Saitoh and H. Mizuno, *Soft Matter* 17, 4204 (2021).

- How do the viscoelastic sound behaves near jamming, $\boldsymbol{p} \to \boldsymbol{0}$?
- What is the relation to viscoelastic properties, i.e. $\boldsymbol{G'(\omega)}$ and $\boldsymbol{G''(\omega)}$?

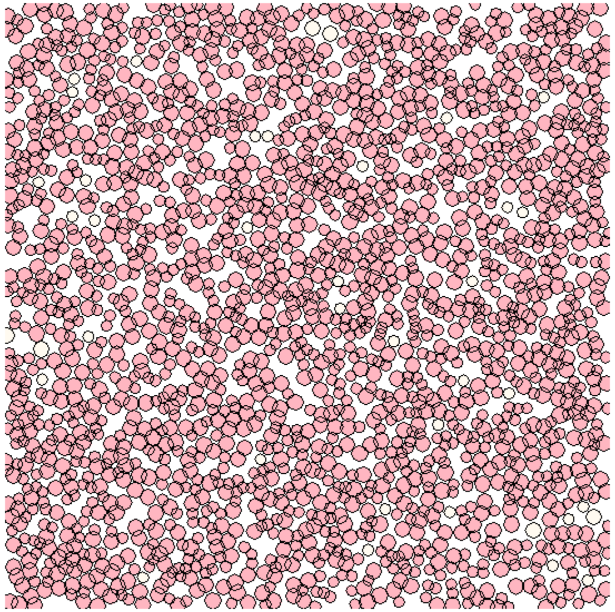


Methods

We use a binary mixture of frictionless soft particles in two dimensions, where the size ratio is $R_L/R_S = 1.4$ and the number of particles is $N = 2097152$.

- Displacements around mechanical equilibrium, $|q(t)\rangle \equiv \left(\{\boldsymbol{u}_i(t)\}_{i=1,\cdots,N}\right)^{\mathrm{T}}$
- Linear equations of motion, $m|\ddot{q}(t)\rangle = -D|q(t)\rangle - B|\dot{q}(t)\rangle$
- Dynamical matrix (Hessian), $D \equiv \left[\frac{\partial^2 E(0)}{\partial q_k \partial q_l}\right]_{k,l=1,\cdots,2N}$
- Damping matrix, $B \equiv \left[\frac{\partial^2 R(0)}{\partial \dot{q}_k \partial \dot{q}_l}\right]_{k,l=1,\cdots,2N}$
- Elastic energy, $E(t) = \frac{k_n}{2}\sum_{i<j} \xi_{ij}(t)^2$
- Dissipation function, $R(t) = \frac{\eta}{2}\sum_{i<j} \dot{\boldsymbol{u}}_{ij}(t)^2$

cf. *Durian's bubble model* or *balanced contact damping*.



Static packing made by the FIRE.

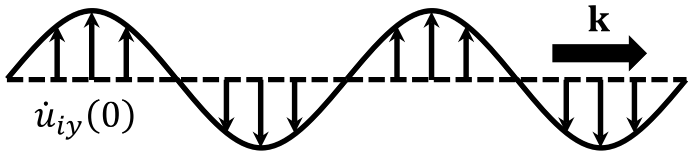
Numerical simulations

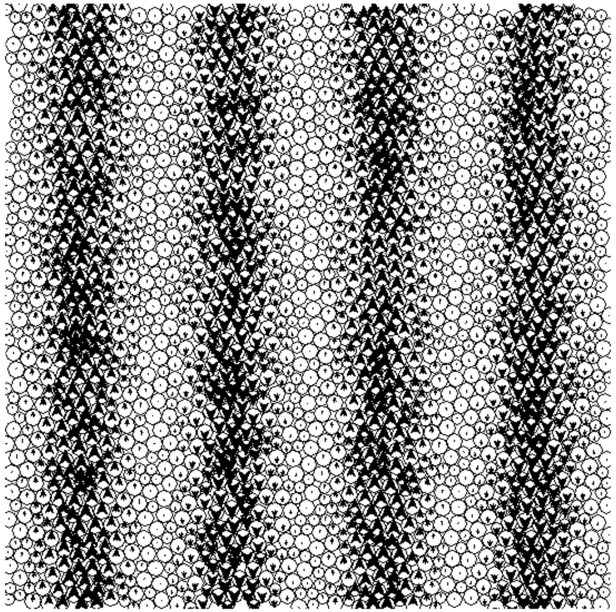
We numerically solve the linear equations of motion with initial velocities,

$$\dot{\boldsymbol{u}}_i(0) = \boldsymbol{A}\sin[\boldsymbol{k}\cdot\boldsymbol{r}_i(0)] \quad (i = 1, \cdots, N)$$

- Amplitude, $|\mathbf{A}| = 10^{-3}\, d_0/t_0$, where $d_0 \equiv R_L + R_S$ and $t_0 \equiv \sqrt{m/k_n}$.
- Wave number, $k \equiv |\boldsymbol{k}| = \frac{2\pi}{L} n \quad (n = 1,2,3,\cdots)$
- $\boldsymbol{A}\cdot\boldsymbol{k} = Ak$ for the analysis of longitudinal mode.
- $\boldsymbol{A}\cdot\boldsymbol{k} = 0$ for the analysis of transverse mode.
- Inelasticity, $\epsilon \equiv \frac{t_d}{t_0} = \frac{\eta/k_n}{\sqrt{m/k_n}} = \frac{\eta}{\sqrt{mk_n}}$, where we examine $\epsilon = 1$ and 0.1.

Initial standing wave





Numerical analyses

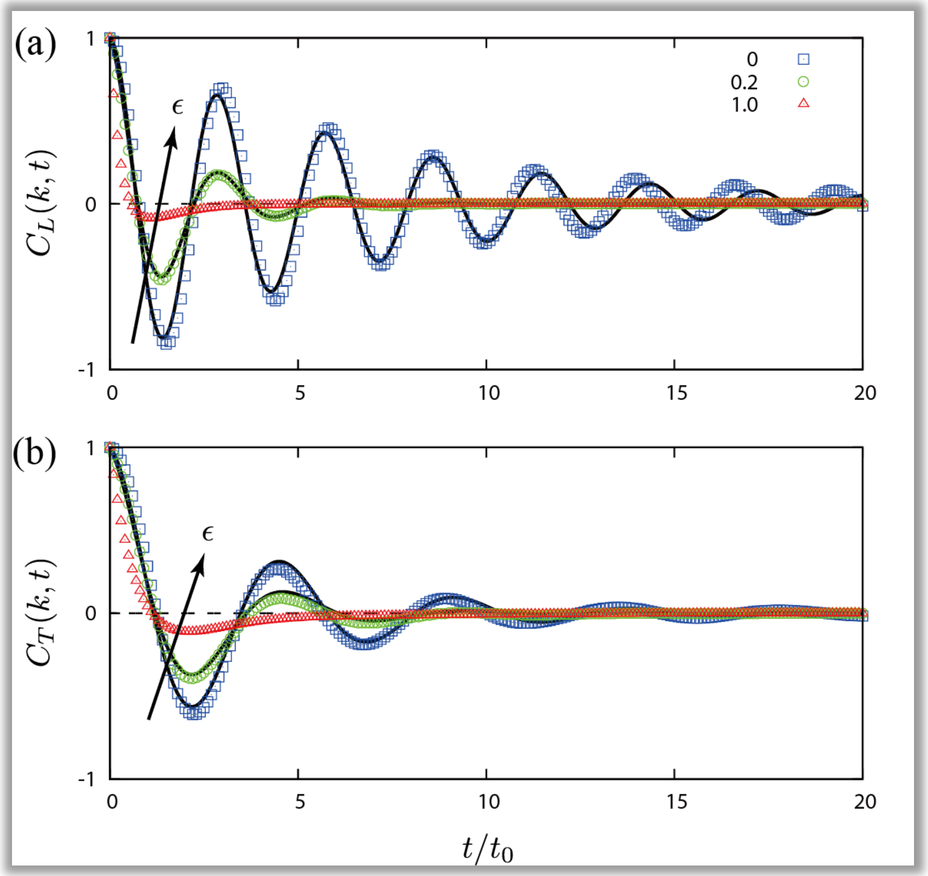
- Fourier transform, $\dot{\boldsymbol{u}}(\boldsymbol{k},t) = \sum_{i=1}^{N} \dot{\boldsymbol{u}}_i(t) e^{-I\boldsymbol{k}\cdot\boldsymbol{r}_i(t)}$

- Longitudinal mode, $\dot{\boldsymbol{u}}_L(\boldsymbol{k},t) \equiv \{\dot{\boldsymbol{u}}(\boldsymbol{k},t)\cdot\hat{\boldsymbol{k}}\}\hat{\boldsymbol{k}}$, where $\hat{\boldsymbol{k}} \equiv \boldsymbol{k}/k$.

- Transverse mode, $\dot{\boldsymbol{u}}_T(\boldsymbol{k},t) \equiv \dot{\boldsymbol{u}}(\boldsymbol{k},t) - \dot{\boldsymbol{u}}_L(\boldsymbol{k},t)$

- Autocorrelation function, $C_\alpha(k,t) = \langle \dot{\boldsymbol{u}}_\alpha(\boldsymbol{k},t)\cdot\dot{\boldsymbol{u}}_\alpha(-\boldsymbol{k},0)\rangle$, for $\alpha = L, T$.

- Fitting to a damped oscillation, $C_\alpha(k,t) \sim e^{-\Gamma_\alpha(k)t}\cos\Omega_\alpha(k)t$



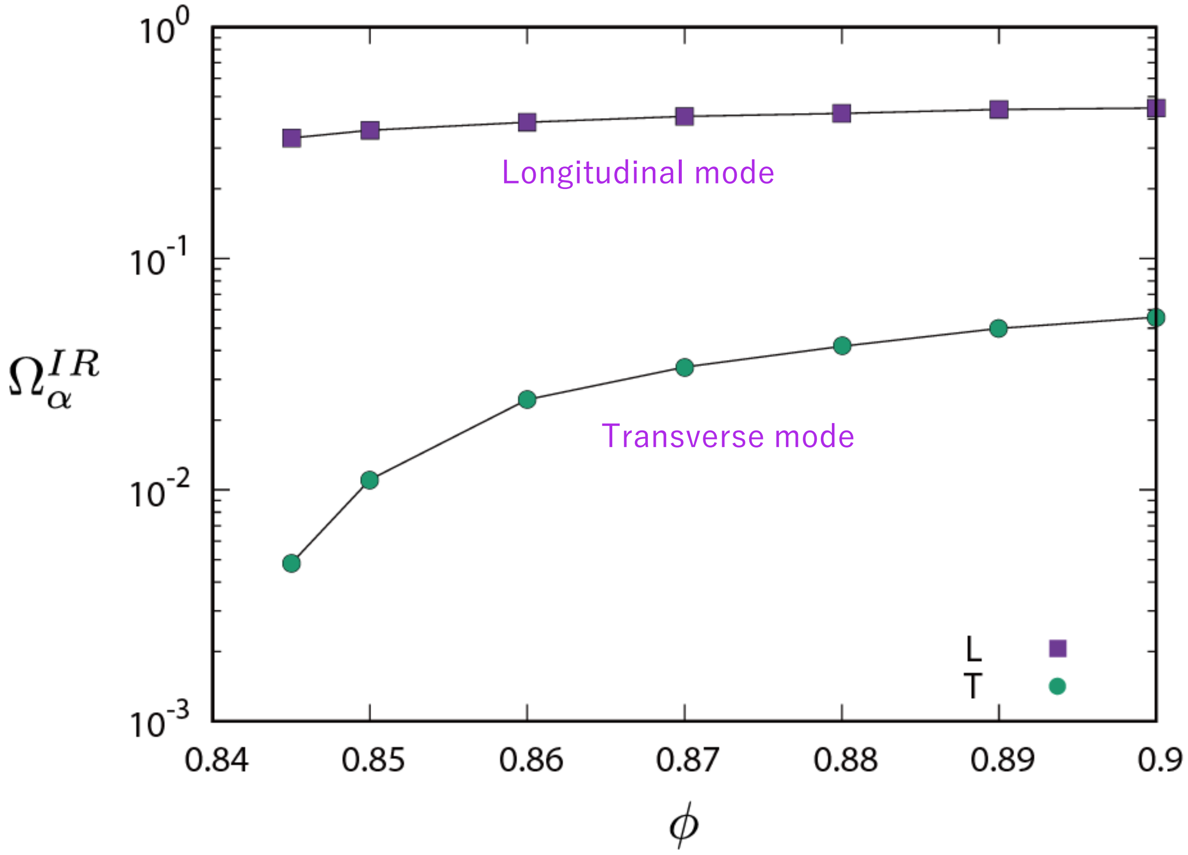
We analyze the dependence of the dispersion relation $\Omega_\alpha(k)$ and attenuation coefficient $\Gamma_\alpha(k)$ on the proximity to jamming.

The Ioffe-Regel limits

- Note that $C_\alpha(k,t)$ oscillates only if
$$\frac{\pi\Gamma_\alpha(k)}{\Omega_\alpha(k)} < 1$$
- Otherwise, $C_\alpha(k,t)$ is overdamped.

- The ratio is an increasing function of k and less than unity if $k < k_\alpha^{IR}$.

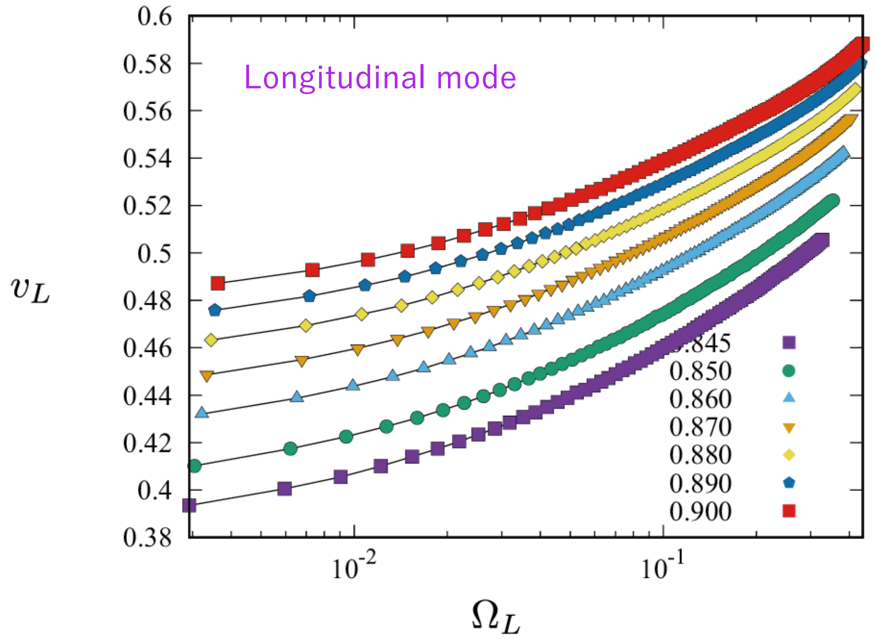
- The Ioffe-Regel limit is defined as
$$\Omega_\alpha^{IR}(k_\alpha^{IR}) \equiv \pi\Gamma_\alpha(k_\alpha^{IR})$$

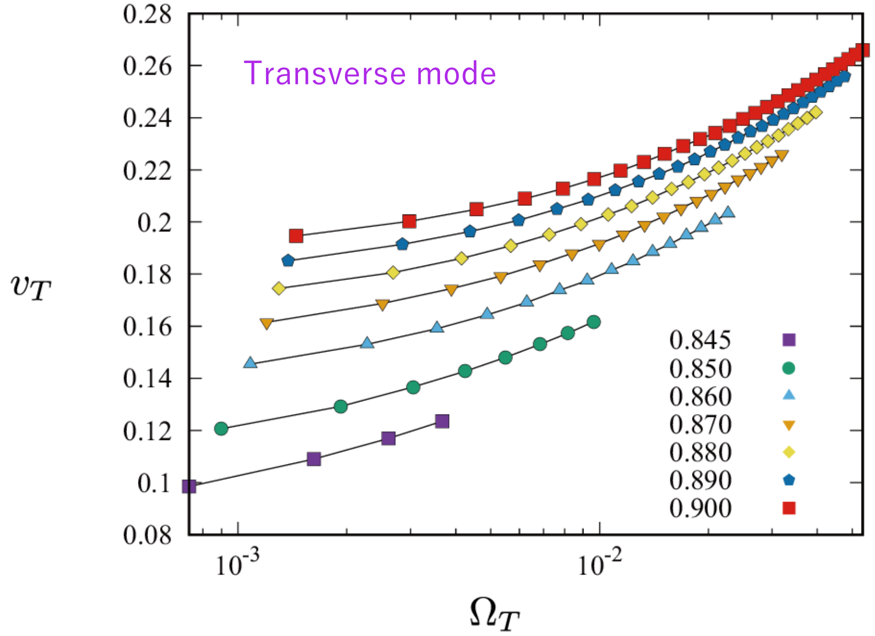


H. Mizuno, S. Mossa, and J.-L. Barrat, *PNAS* 111, 11949 (2014).

Sound speeds

- Defined as $v_\alpha(k) \equiv \Omega_\alpha(k)/k$ with $\alpha = L, T$.
- Parametric plots of $v_\alpha(k)$ and $\Omega_\alpha(k)$.
- Dependence on the packing fraction ϕ (as listed).
- Inelasticity is $\epsilon = 1$.

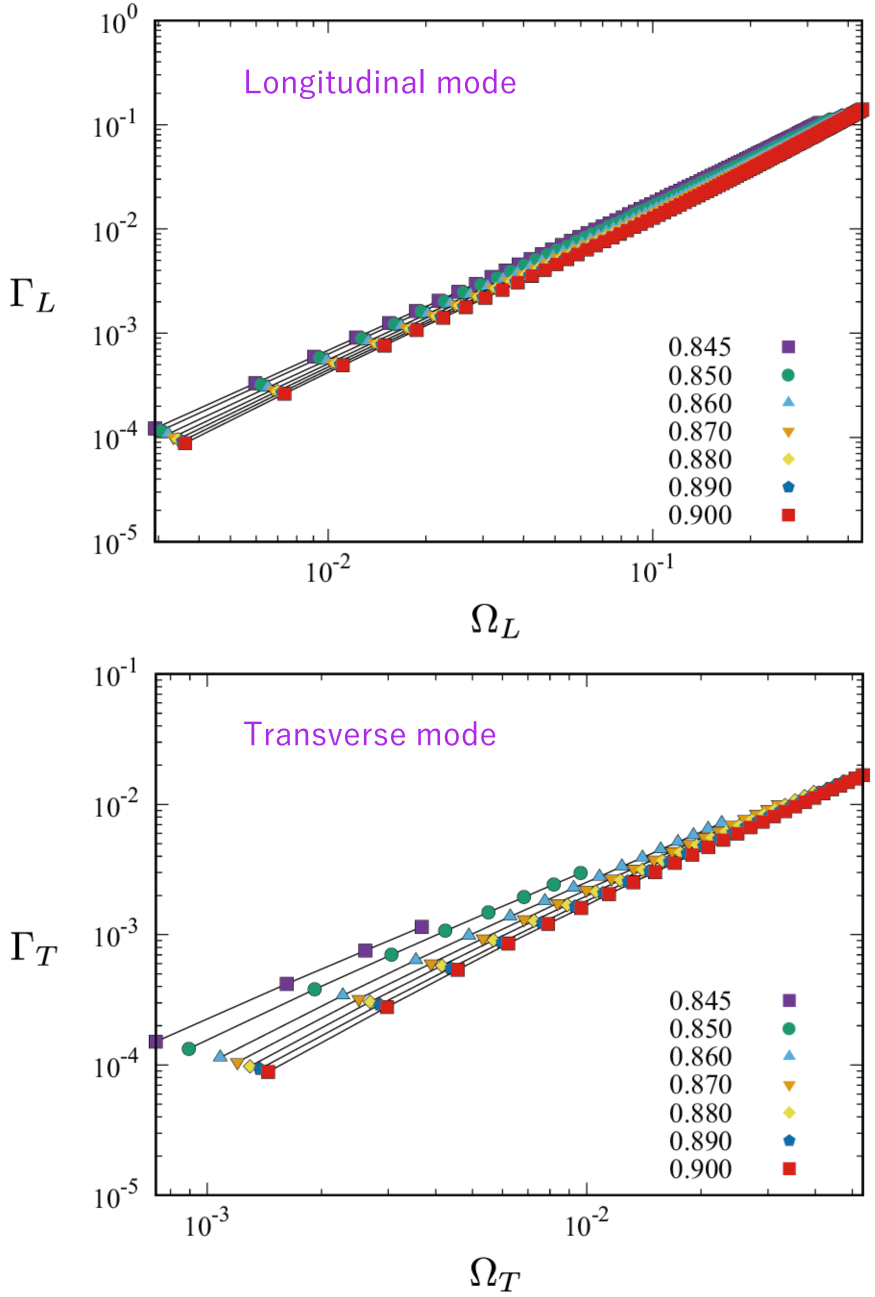




Attenuation coefficients

- Dependence on ϕ (as listed).
- Inelasticity is $\epsilon = 1$.
- We show the data below the Ioffe-Regel limit,

$$\Omega_\alpha < \Omega_\alpha^{IR}$$



Relations to G' and G''

- Storage and loss moduli, $G'(\omega)$ and $G''(\omega)$.
- In long wave lengths,

$$\Omega_T(k) \propto \sqrt{G'(\omega)}k$$

$$\Gamma_T(k) \propto \frac{G''(\omega)}{\omega}k^2$$

H. Mizuno and R. Yamamoto, *Phys. Rev. Lett.* 110, 095901 (2013)

In long wave lengths, transverse sound speed is

$$v_T(k) = \frac{\Omega_T(k)}{k} \propto \sqrt{G'(\omega)}$$

and the attenuation coefficient is

$$\therefore \Gamma_T(k) \propto \frac{G''(\omega)}{\omega}\left(\frac{\Omega_T(k)}{v_T(k)}\right)^2 \sim \frac{G''(\omega)}{\omega G'(\omega)}\Omega_T(k)^2$$

Critical scaling of G' and G''

- Near jamming, pressure, $p \to 0$
- Critical scaling,

$$G'(\omega) \sim \begin{cases} p^{1/2} \\ \omega^{1/2} \end{cases}$$

$$G''(\omega) \sim \begin{cases} \omega/p^{1/2} & (\omega < p) \\ \omega^{1/2} & (\omega > p) \end{cases}$$

K. Baumgarten and B.P. Tighe, *Soft Matter* 13, 8368 (2017)

In long wave lengths ($\omega < p$),

$$G'(\omega) \sim p^{1/2} \qquad \frac{G''(\omega)}{\omega} \sim \frac{1}{p^{1/2}}$$

Therefore,

$$v_T(k) \propto \sqrt{G'(\omega)} \sim p^{1/4}$$

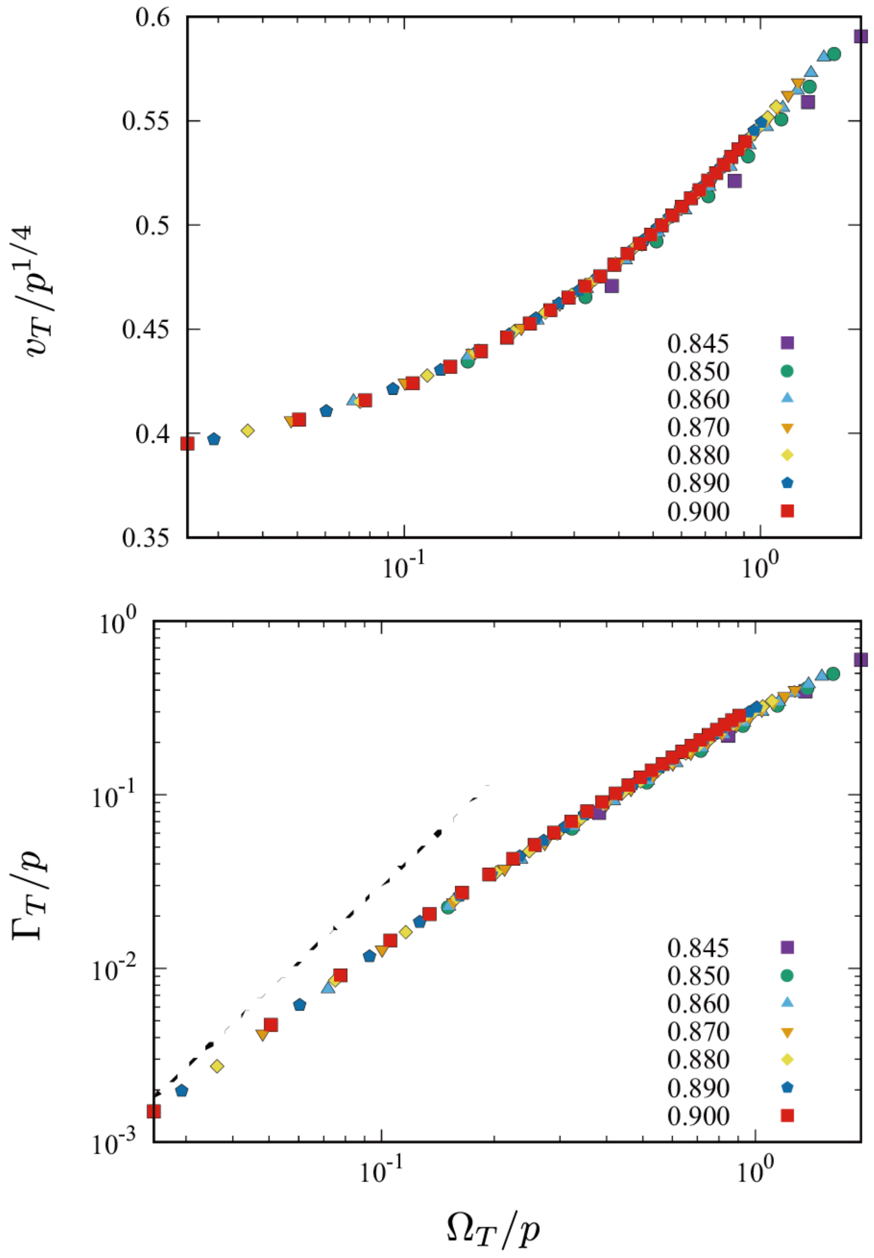
$$\Gamma_T(k) \propto \frac{G''(\omega)}{\omega G'(\omega)} \Omega_T(k)^2 \sim \frac{\Omega_T(k)^2}{p}$$

$$\therefore \frac{\Gamma_T(k)}{p} \sim \left(\frac{\Omega_T(k)}{p}\right)^2$$

⋯ quadratic

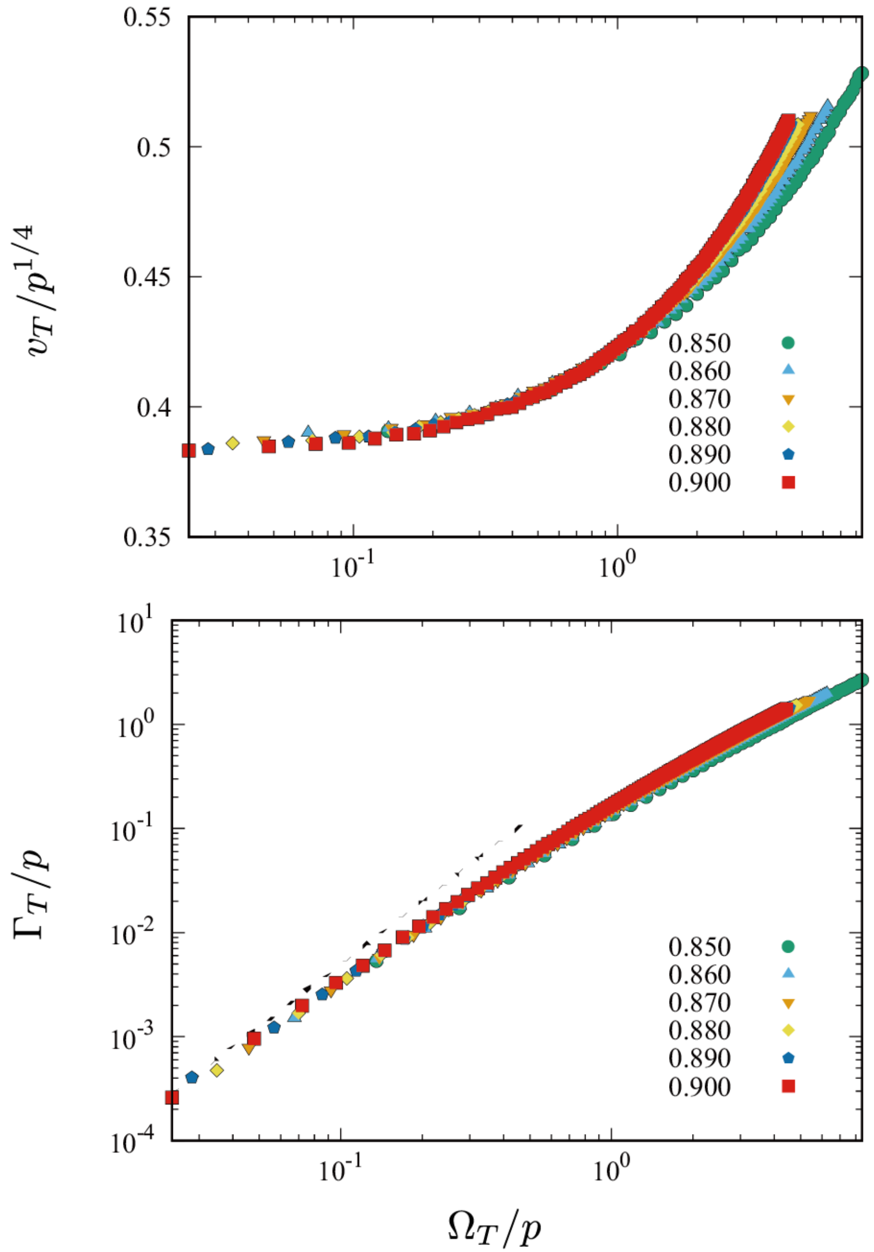
Data collapses

- Sound characteristics near jamming.
- Inelasticity is $\epsilon = 1$.
- The dashed line is quadratic.



Effects of ϵ

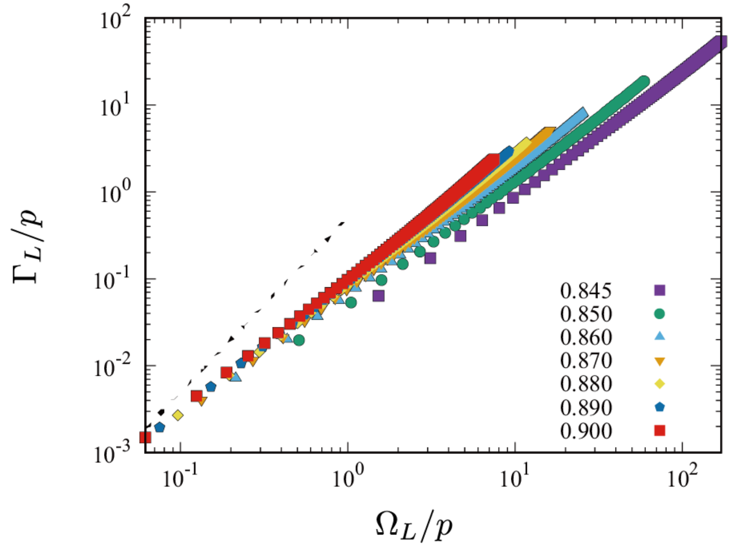
- Data for $\epsilon = 0.1$.
- The same scaling seems to work.



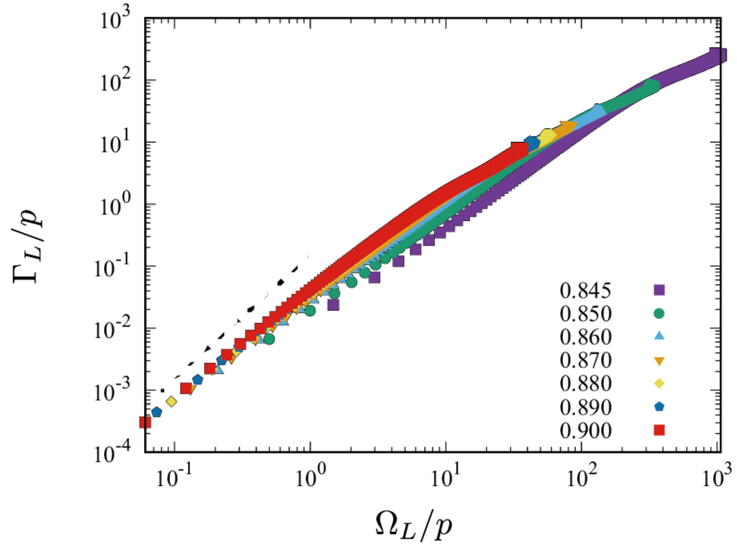
Summary & discussion

- We numerically investigated sound properties of soft frictionless particles in two dimensions.
- Critical scaling of $\mathbf{\Gamma_\alpha}$ is NEW.
- Longitudinal modes are future work.

Counterparts for isotropic compression, $K'(\omega)$ and $K''(\omega)$.



$\epsilon = 1.0$



$\epsilon = 0.1$