OSAKA UNIVERSITY

Rheology of dense wet granular materials

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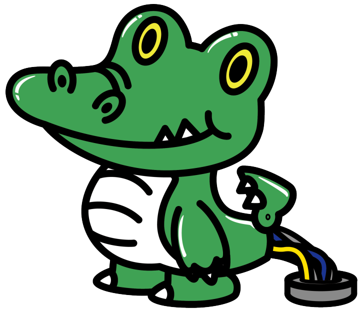

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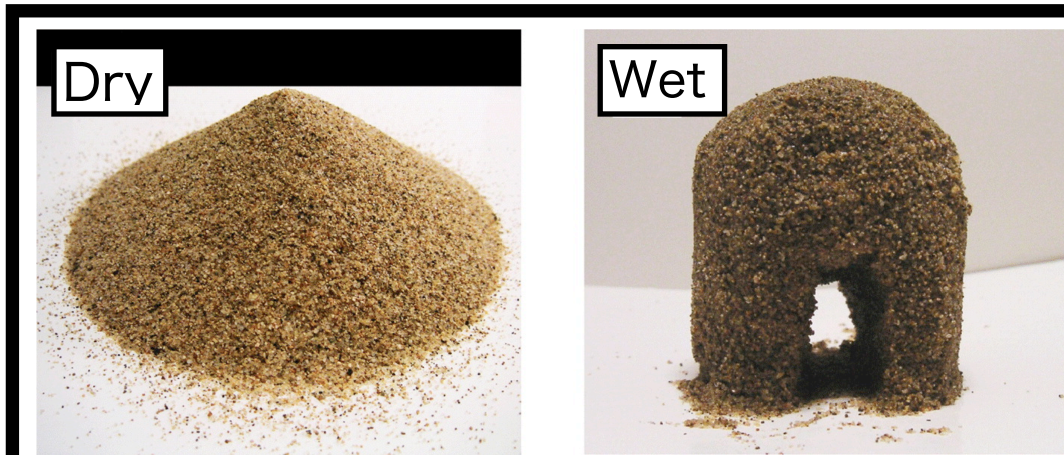
Contents

Dry & Wet granular materials

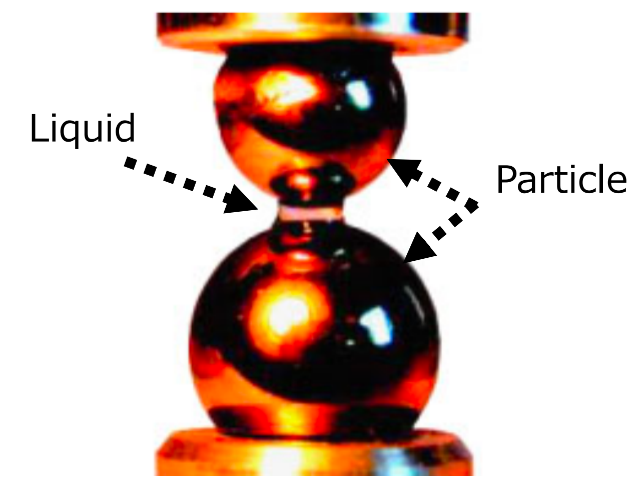
- Small amounts of liquid drastically change the rheological property of granular materials.



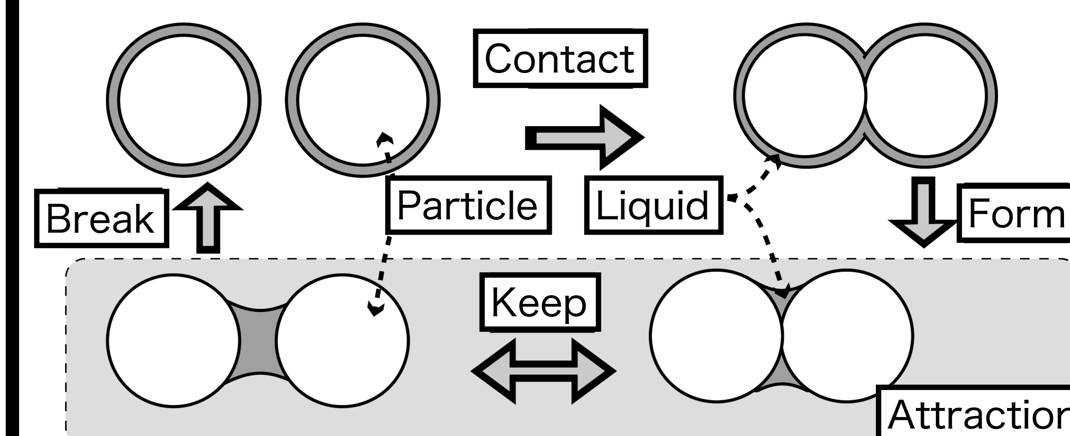
N. Mitarai, F. Nori, Adv. Phys. (2005)

- Cohesive interaction appears due to capillary bridges between grains.

- Hysteresis exists in the capillary force.



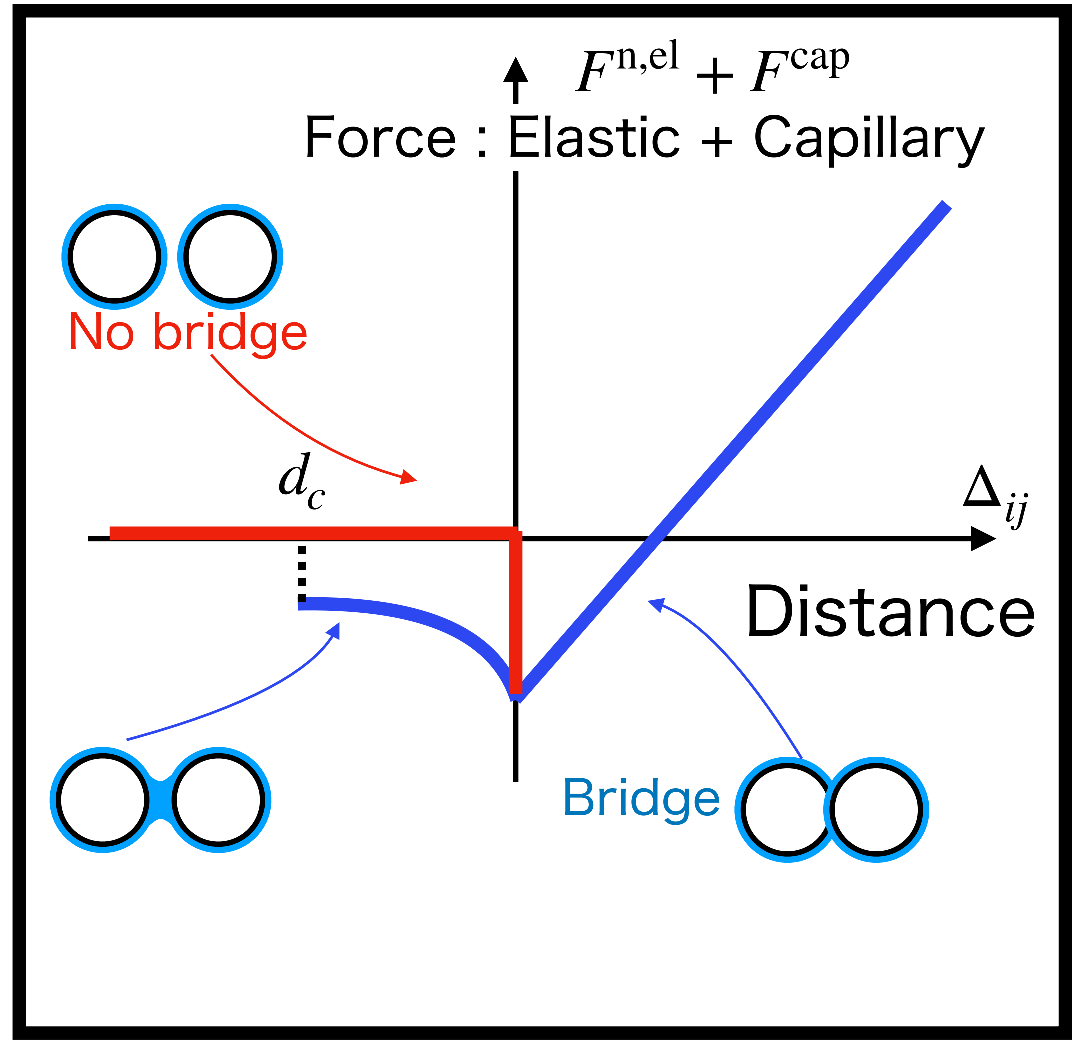
F. Soulie, et. Al, Int. J. Numer. Anal. Meth. Geomech., (2006)



Liquid bridge interaction

- Approaching region: no interaction
- Overlap region : interaction
- When $\Delta_{ij} < -d_c$: no interaction

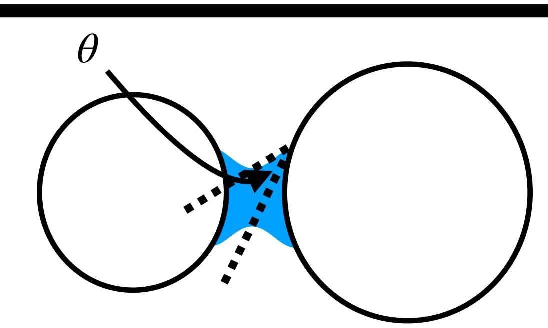
➡ Hysteresis



d_i : radius of i-th particle, $d_{ij} = d_i + d_j$, $\Delta_{ij} = d_{ij} - r_{ij}$,

$F^{\rm n,el}$: elastic interaction, $F^{\rm cap}$:capillary force

- Rupture length d_c depends on contact angle θ and liquid volume V.



$$d_c = \left(1 + \frac{\theta}{2}\right) V^{1/3}$$

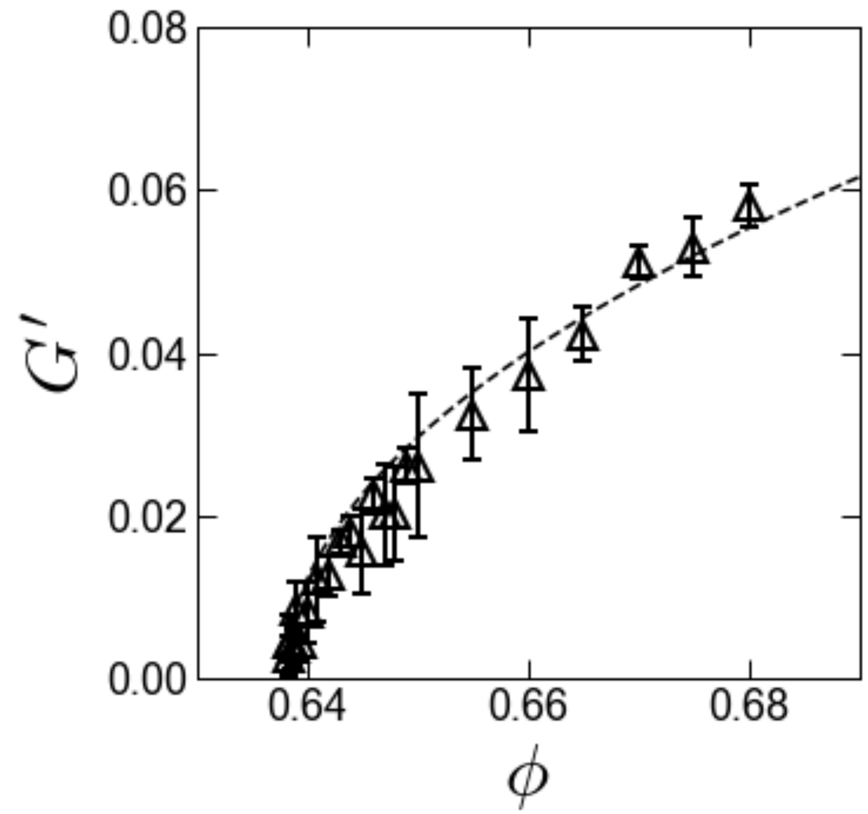
C. D. Willett, et.al, Langmuir (2000)

Shear modulus in dry system

Dry frictionless particles with repulsive interaction

$$\begin{cases} G \propto (\phi - \phi_J)^{1/2} \\ P \propto (\phi - \phi_J) \\ Z - Z_c \propto (\phi - \phi_J)^{1/2} \end{cases}$$

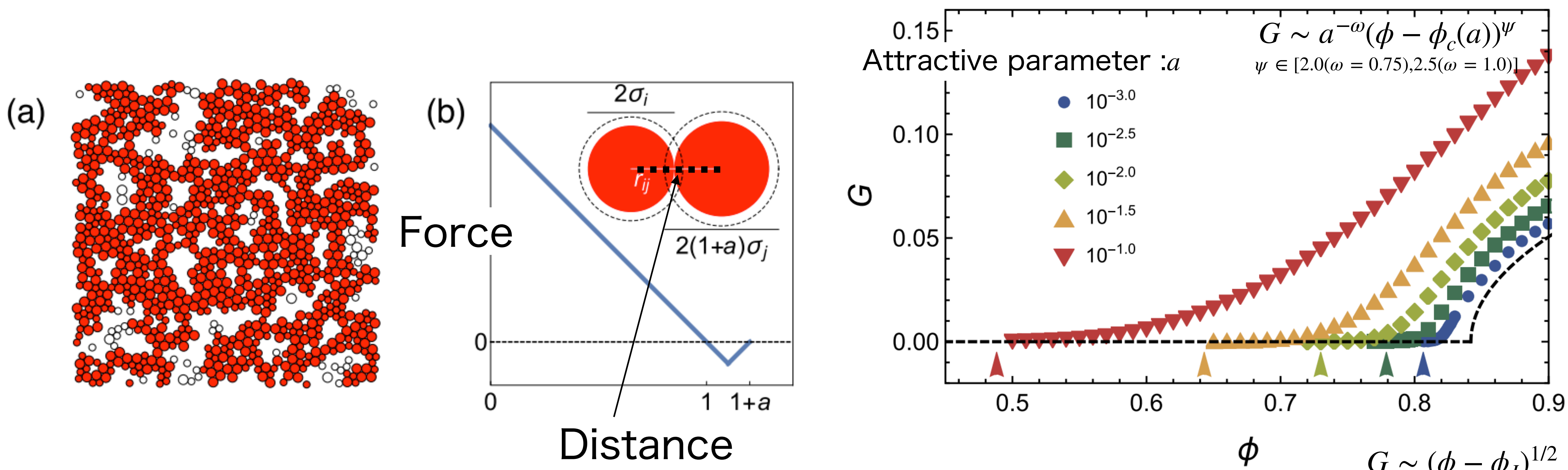
C. S. O'Herm et.al., Phys. Rev. E (2003)



- Dry granular materials exhibit critical scaling laws.
- Cohesive interaction between particles exists in many realistic situations.
  e.g.) van der Waals force, capillary force, electromagnetic force

Shear modulus in sticky system

2D frictionless particles with simple attractive interaction



- Shear modulus G appears below ϕ_J.
- Effective transition point $\phi_c(a)$ depend on a.

ϕ_J : Jamming dencity of dry granular materials

D. J. Koeze et.al., Phys. Rev. Research (2020)

Purpose：
To reveal elastic response of 3D wet granular materials with hysteresis

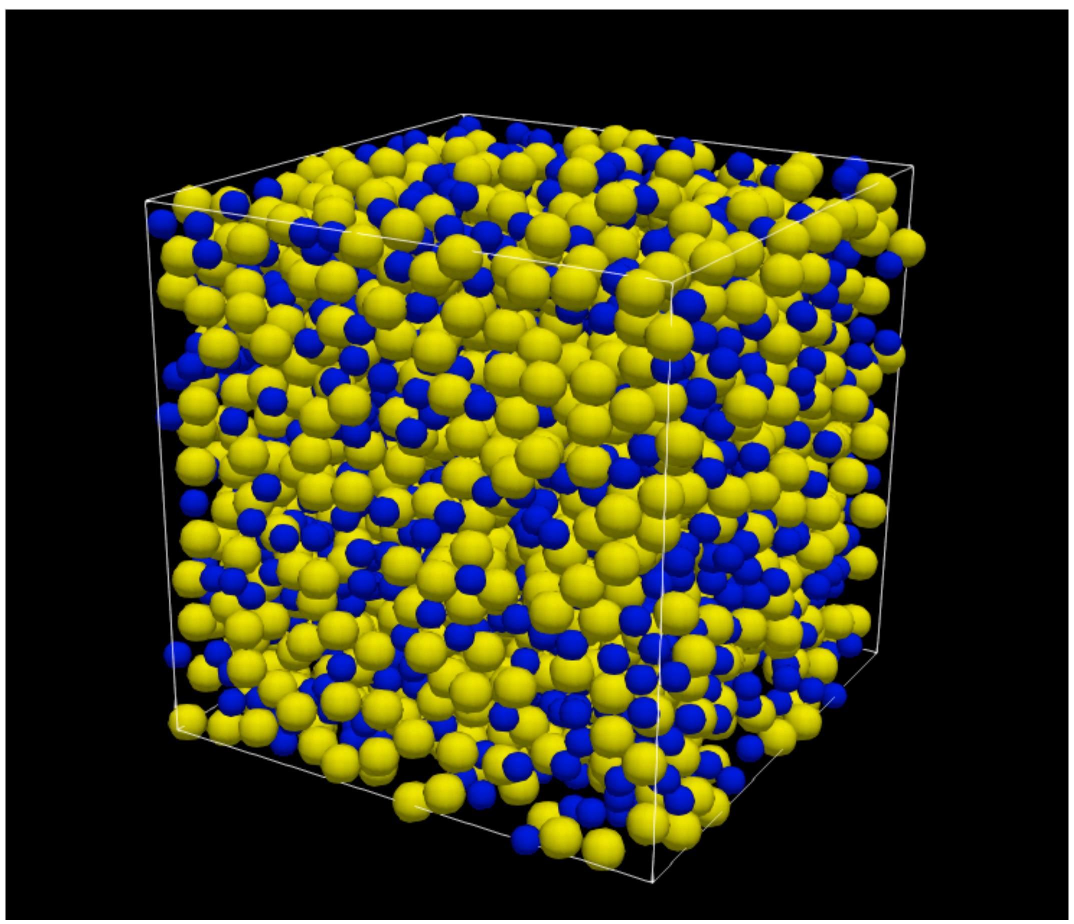
Model : wet granular particles

Equation of motion

$$m_i \frac{d^2 \boldsymbol{r}_i}{dt^2} = \sum_{j \neq i} (F_{ij}^{\mathrm{con}} + F_{ij}^{\mathrm{cap}}) \boldsymbol{n}_{ij}$$

m_i : mass $\boldsymbol{n}_{ij}$: normal vector

Constant volume

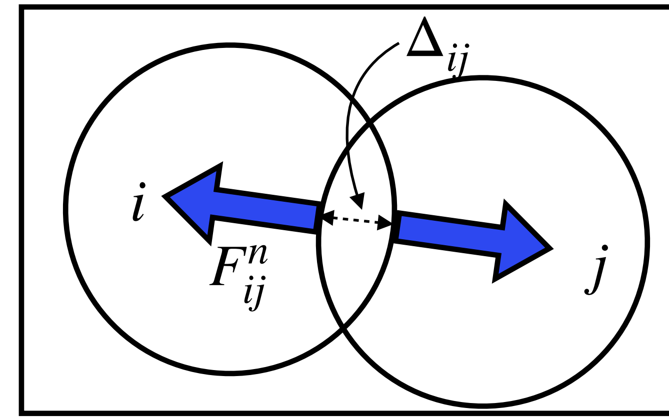


N=3000, monodisperse

Contact force : F_{ij}^{con}

$$\boldsymbol{F}_{ij}^{\mathrm{con}} = k\Delta_{ij}\boldsymbol{n}_{ij} - \eta v_{ij}\boldsymbol{n}$$

- Δ_{ij} : overlap
- v_{ij} : relative velocity
- k : spring constant
- η : viscosity coefficient



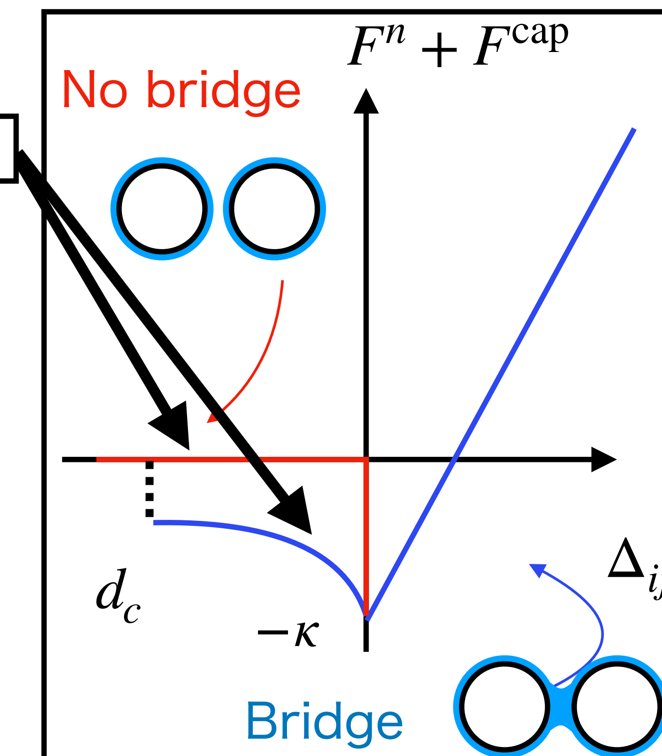
Frictionless particles

Capillary force : F_{ij}^{cap}

$$F_{ij}^{\mathrm{cap}} = \begin{cases} -\kappa & (\Delta_{ij} \geq 0) \\ -\kappa \exp\left(\frac{\Delta_{ij}}{\lambda}\right) & (-d_c < \Delta_{ij} < 0) \\ 0 & (\text{otherwise}) \end{cases}$$

$(\kappa = 2\pi R\gamma_s \cos\theta,\ R = \sqrt{r_i r_j},\ \gamma_s = 0.03,\ \theta = \frac{\pi}{9},\ \lambda = 1)$

Hysteresis



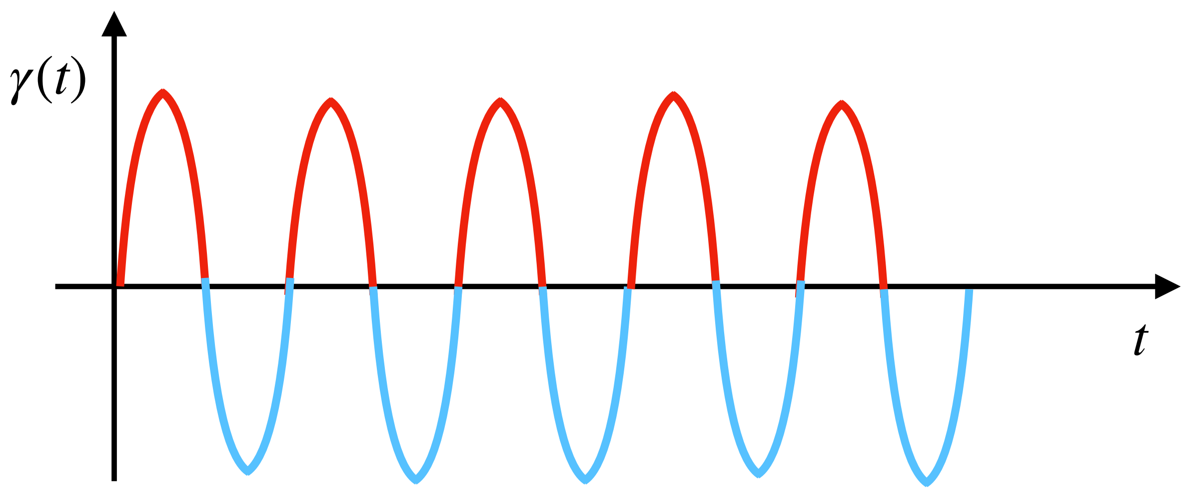
V. Thanh Trung, et al. Nat. comm. (2020)

Protocol

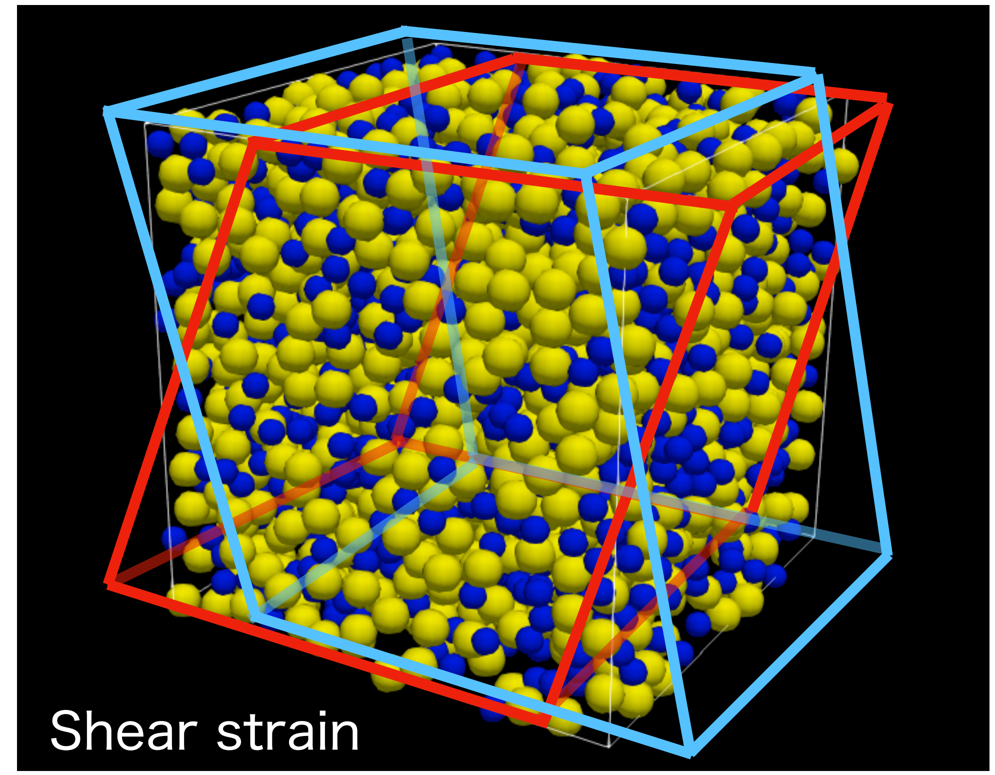
- Initial state is obtained by compression.
- Oscillatory shear : $\gamma(t) = \gamma_0 \sin \omega t$

Frequency : $\omega = 1.0 \times 10^{-4}\sqrt{m/k}$

Amplitude : $\gamma_0 = 1.0 \times 10^{-5}$



Oscillatory shear strain



Lees-Edwards B.C.

SLLOD equation

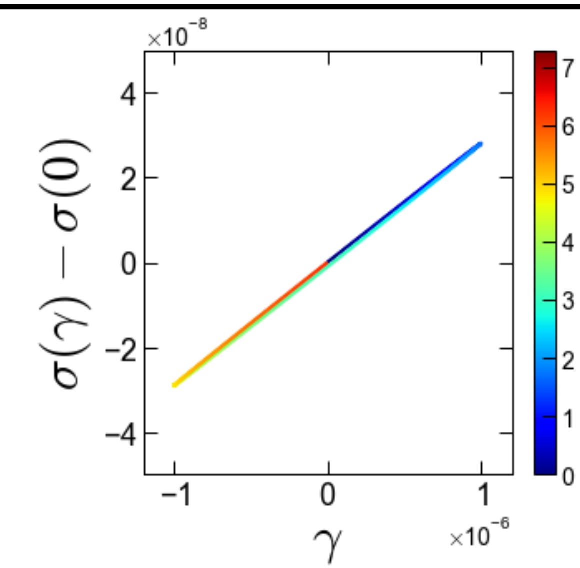
$$\frac{d\boldsymbol{r}_i}{dt} = \frac{\boldsymbol{p}_i}{m_i} + \dot{\gamma}(t) r_{i,y} \boldsymbol{e}_x$$

$$\frac{d\boldsymbol{p}_i}{dt} = \sum_{j \neq i} (F_{ij}^{\mathrm{con}} + F_{ij}^{\mathrm{cap}}) \boldsymbol{n}_{ij} - \dot{\gamma}(t) p_{i,y} \boldsymbol{e}_x$$

- In the last cycle, we measured the shear modulus.

Shear modulus

$$G' = \frac{\omega}{\pi} \int_0^{2\pi/\omega} dt \sigma(t) \sin(\omega t)/\gamma_0$$

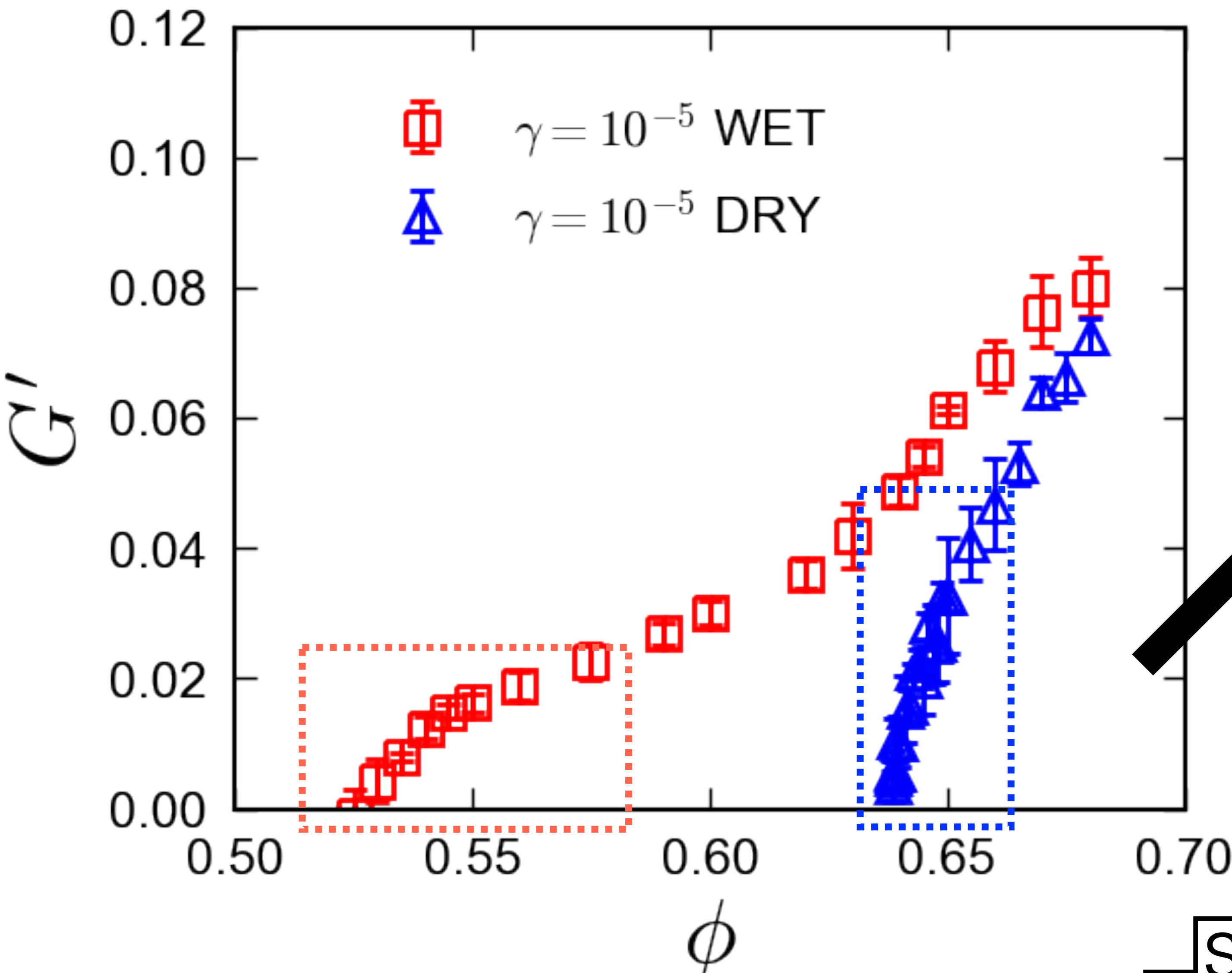


Contents

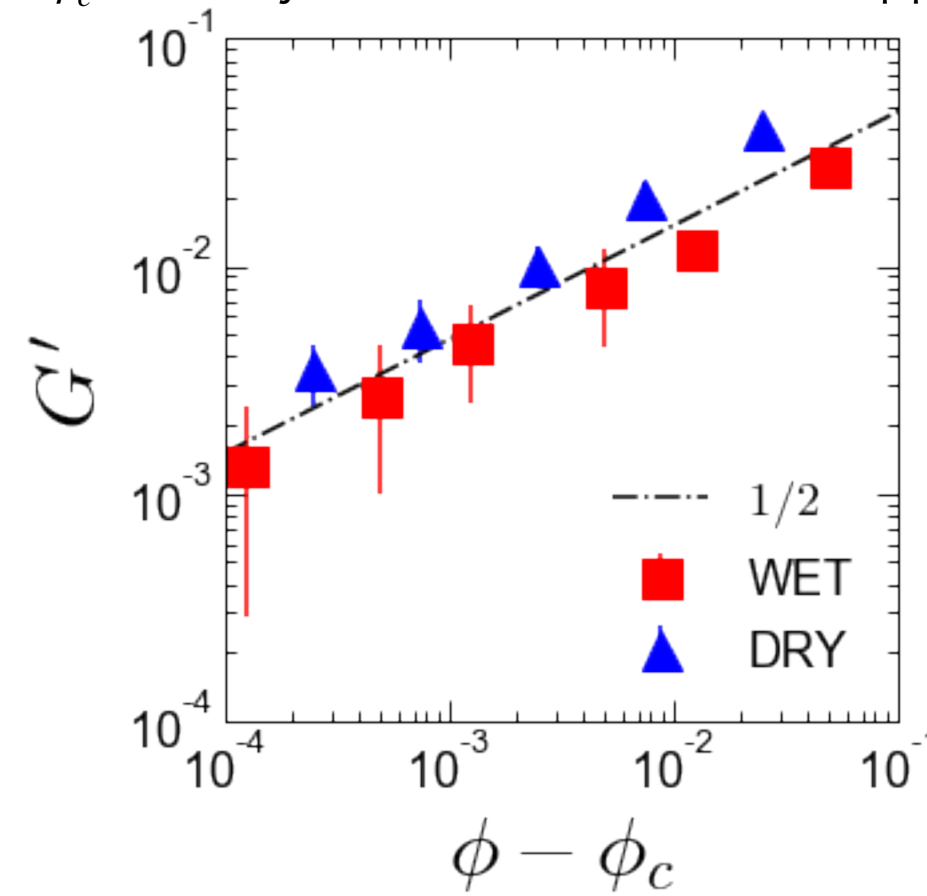
Shear modulus

Dry : Contact force(F_{ij}^{con})

Wet : Contact force(F_{ij}^{con}) + Capillary force (F_{ij}^{cap})



ϕ_c : density whose shear modulus appears

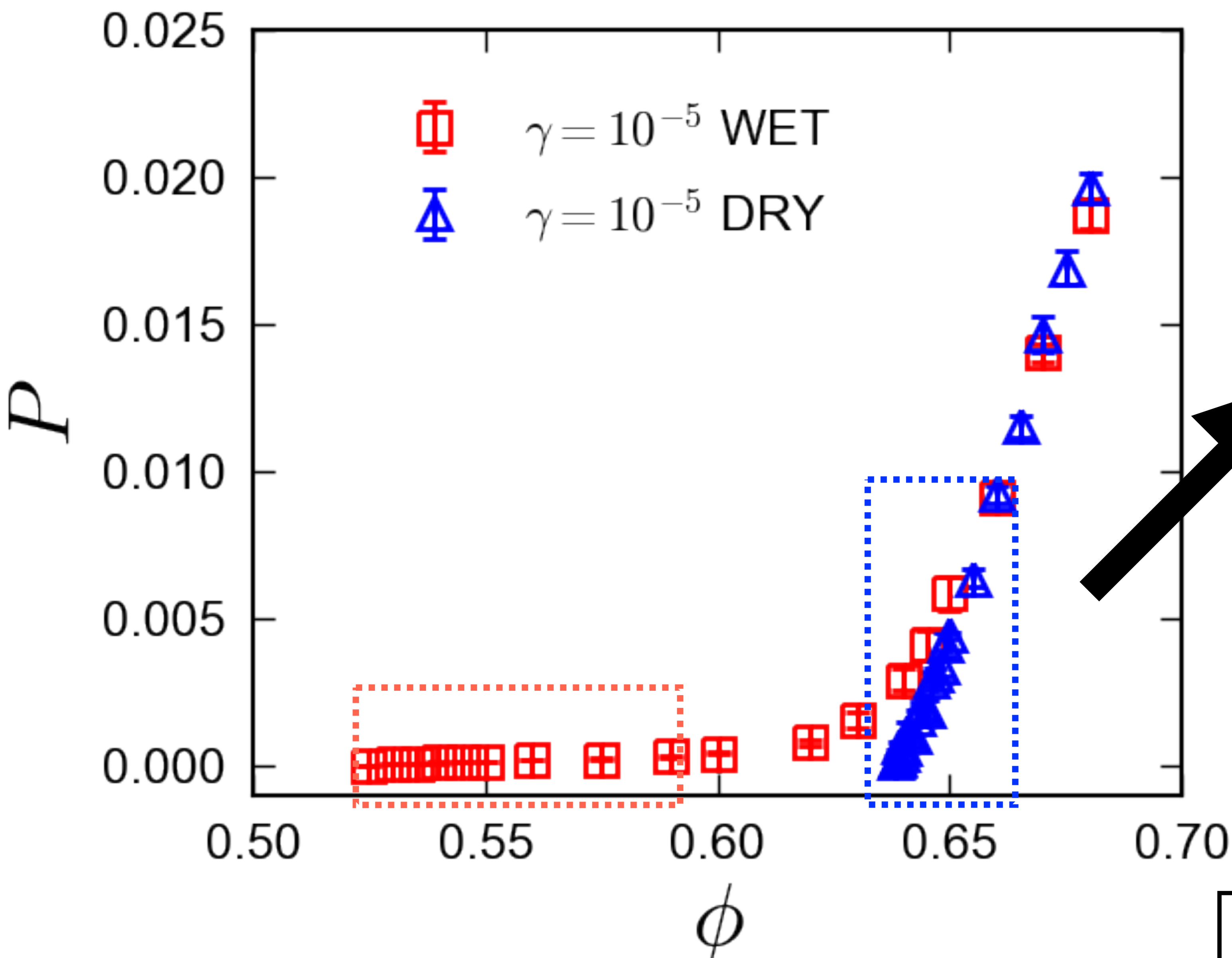


Scaling laws

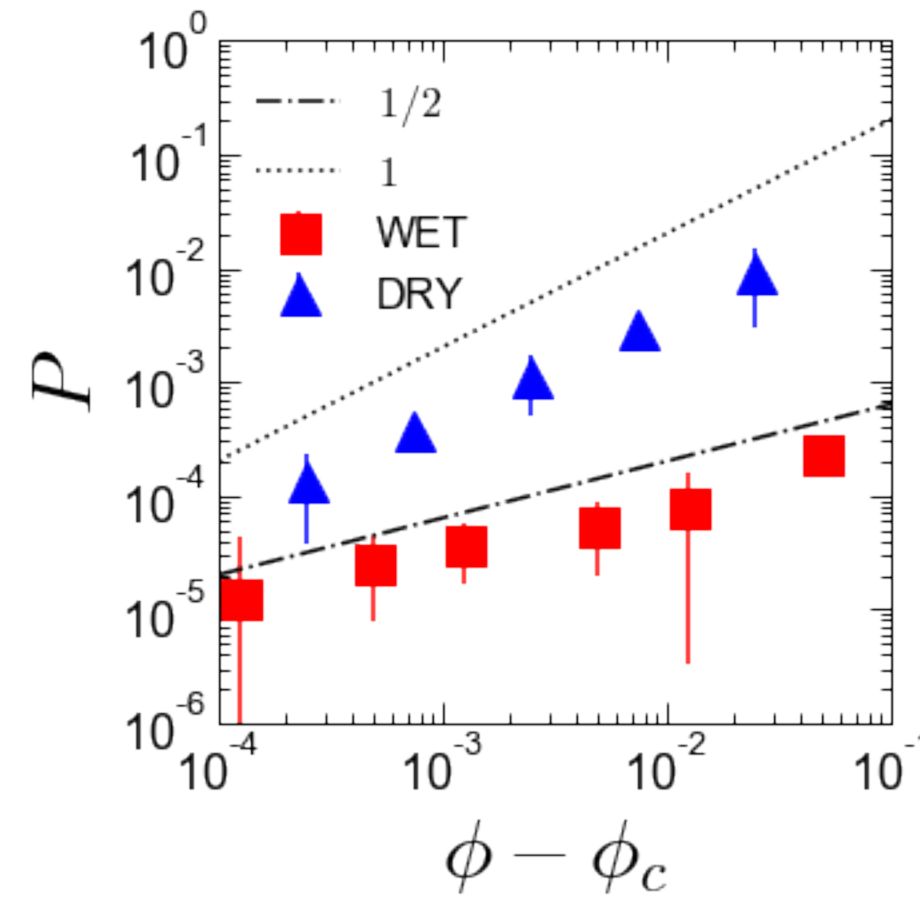
Dry & Wet particles : $G \propto (\phi - \phi_c)^{1/2}$

- Dry and wet systems satisfy the same scaling relation.

Pressure



ϕ_c : density whose shear modulus appears



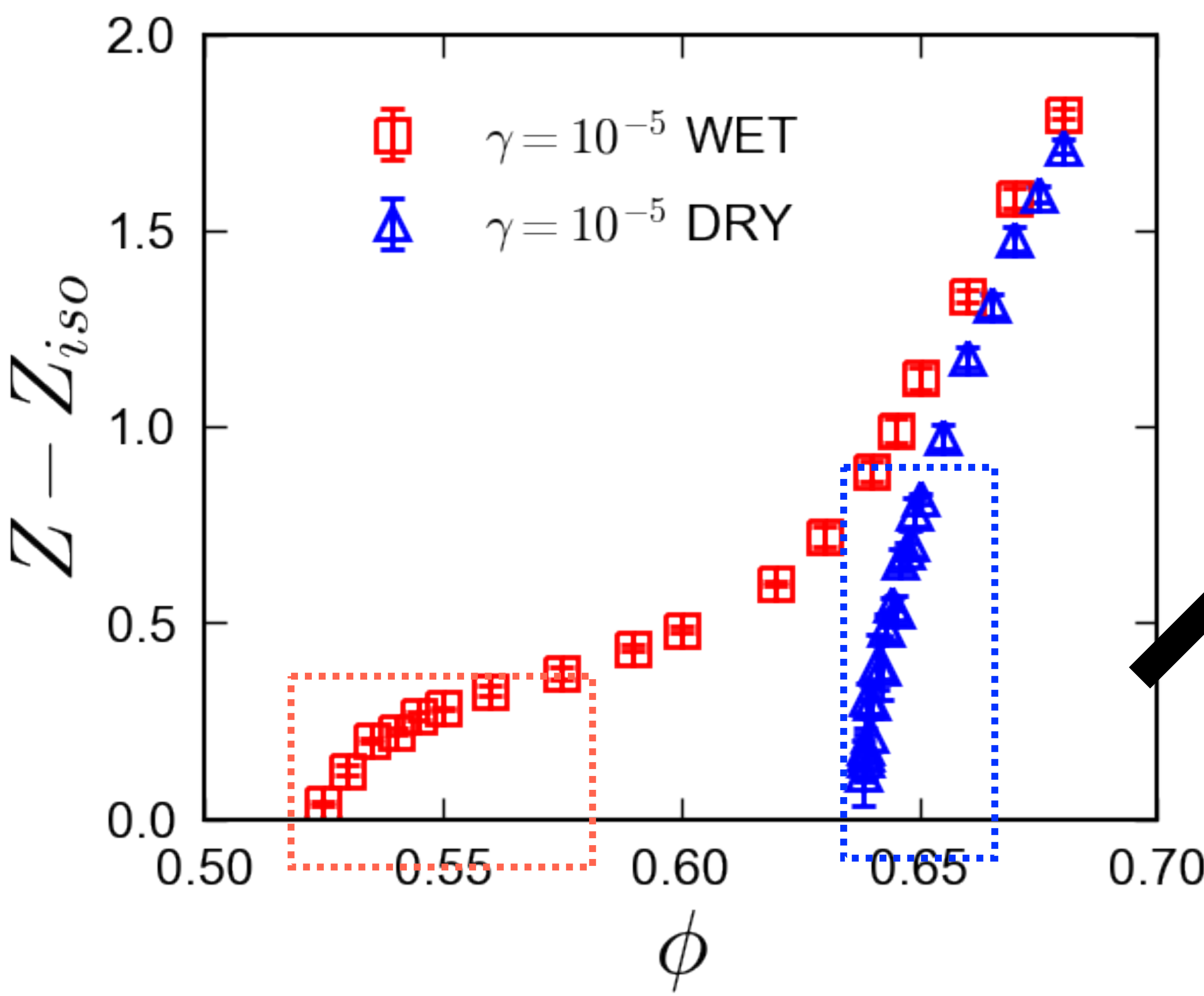
Scaling laws

Dry particles : $P \propto (\phi - \phi_c)$

Wet particles : $P \propto (\phi - \phi_c)^{1/2}$

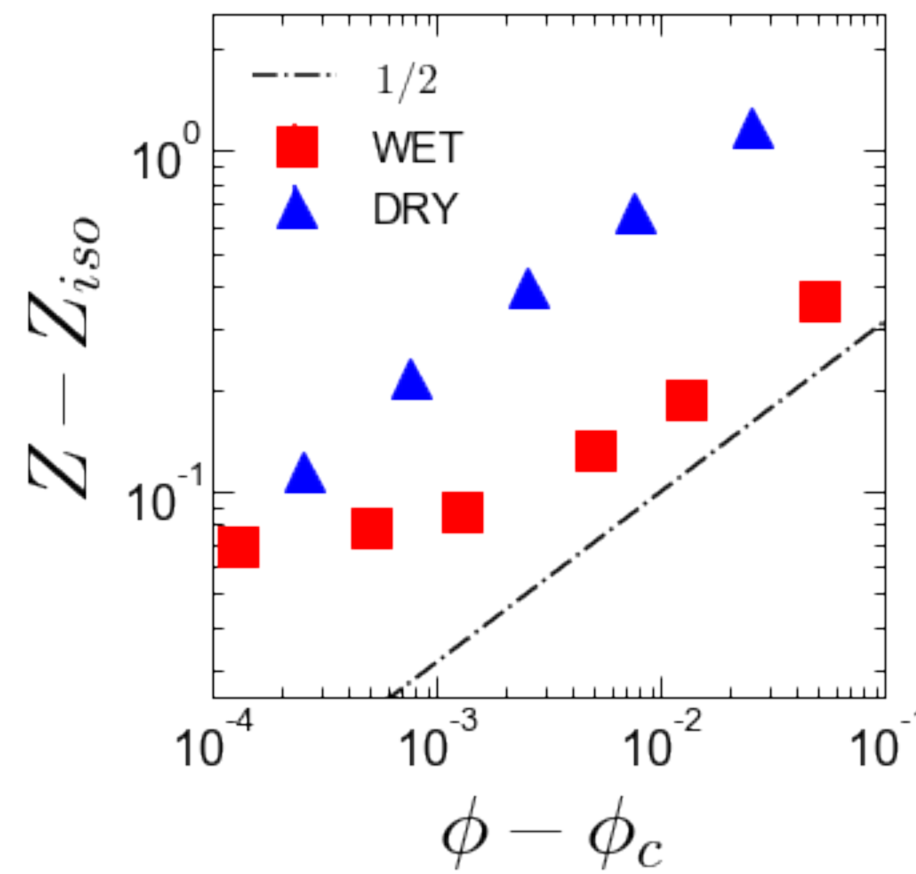
- The exponent for P is different between dry and wet systems.

Contact number



ϕ_c : density whose shear modulus appears

Z_{iso} : contact number at isostatic state



Dry particles : $Z - Z_{iso} \propto (\phi - \phi_c)^{1/2}$

Wet particles : $\lim_{\phi \to \phi_c} Z(\phi) - Z_{iso} > 0$

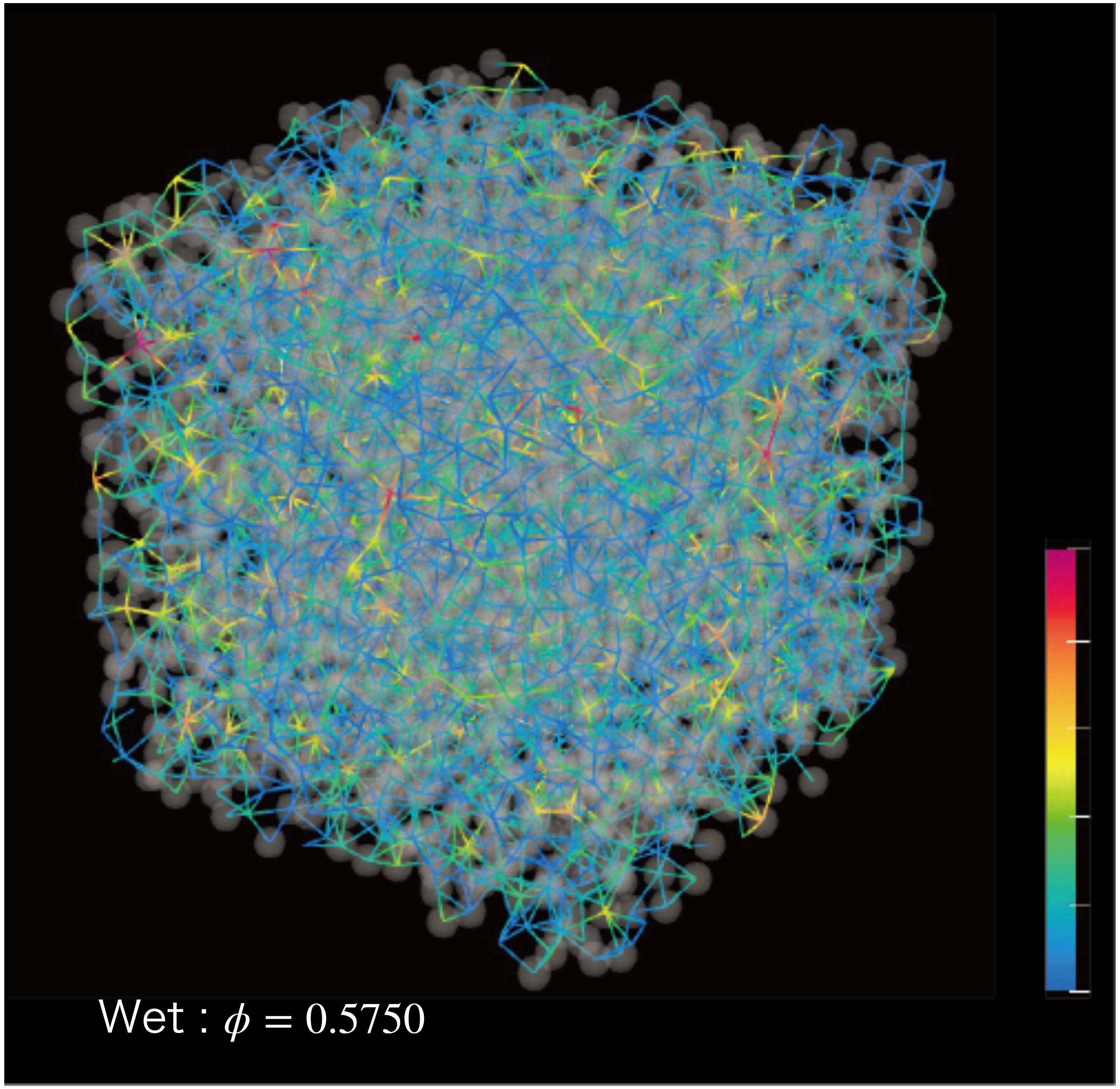
- Wet : $Z - Z_{iso} > 0$ for $\phi \to \phi_c$

This behave is consistent with 2D attractive system.

D. J. Koeze et.al., Phys. Rev. Research (2020)

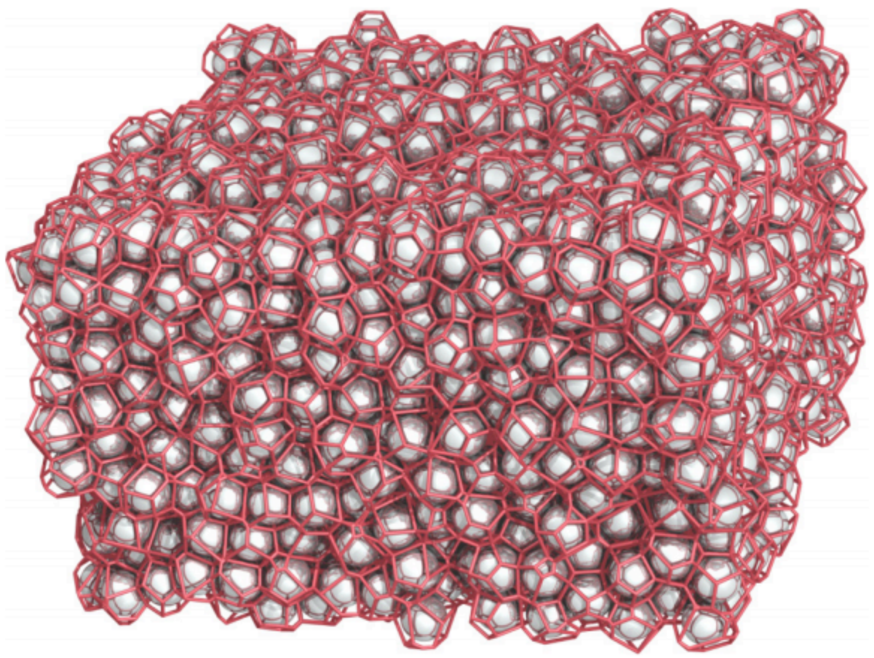
Contents

Force chain



Local packing fraction

Voronoi tessellation



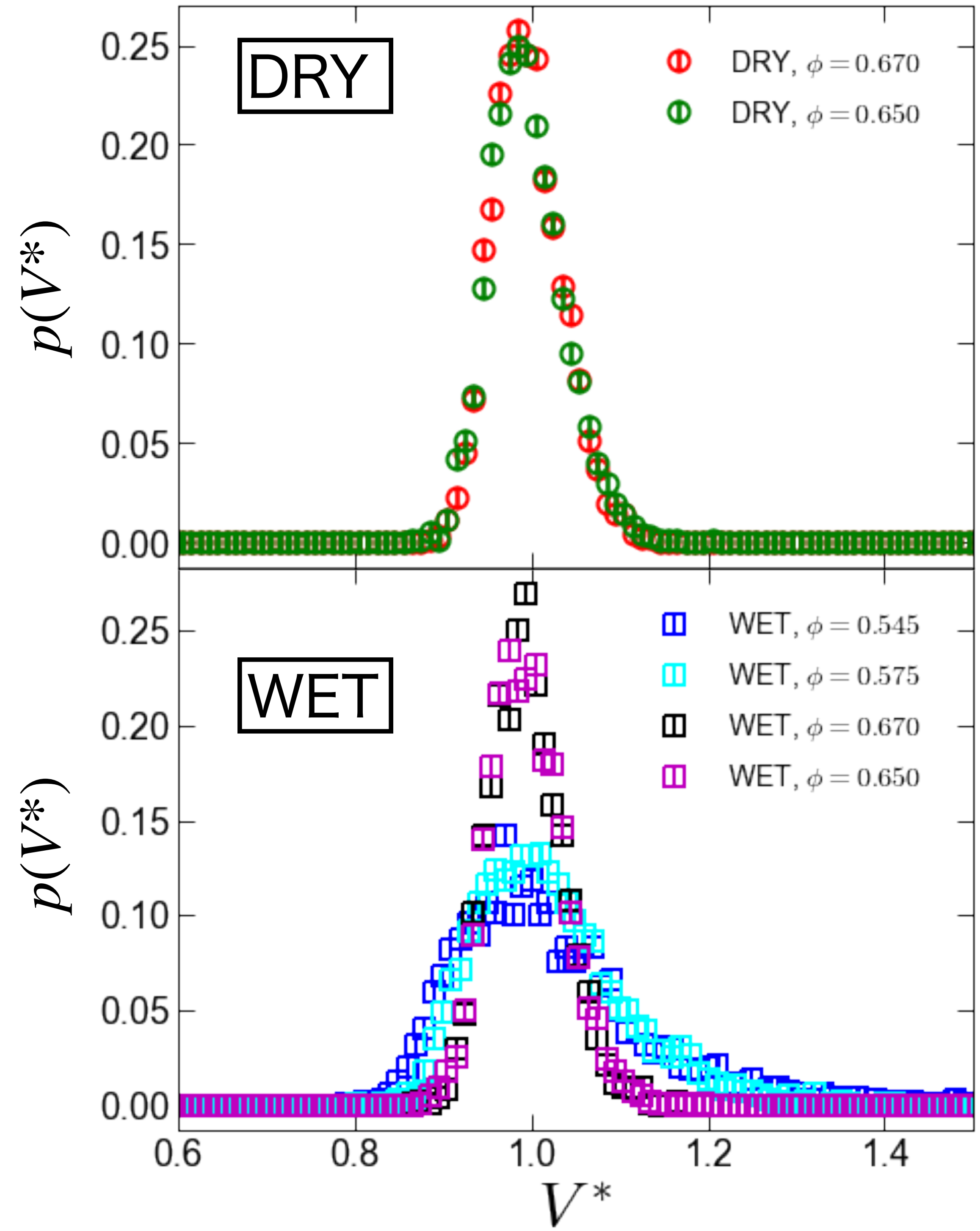
M. A. Klatt & S. Torquato. Phys. Rev. E (2014)

$V^* = V/\bar{V}$: scaled volume

V : volume of voronoi cells,

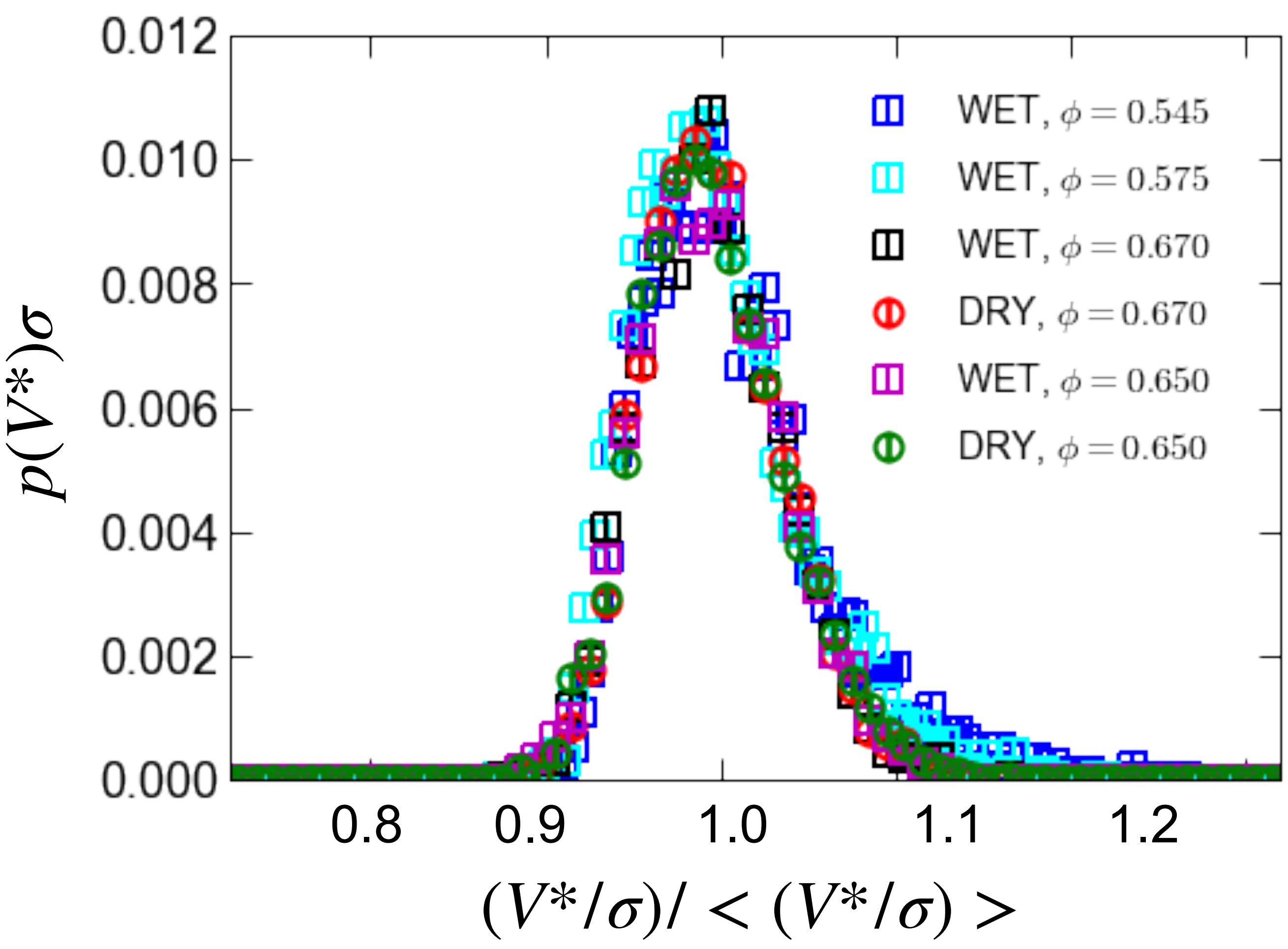
$\bar{V} = \frac{\sum V}{N}$: average volume

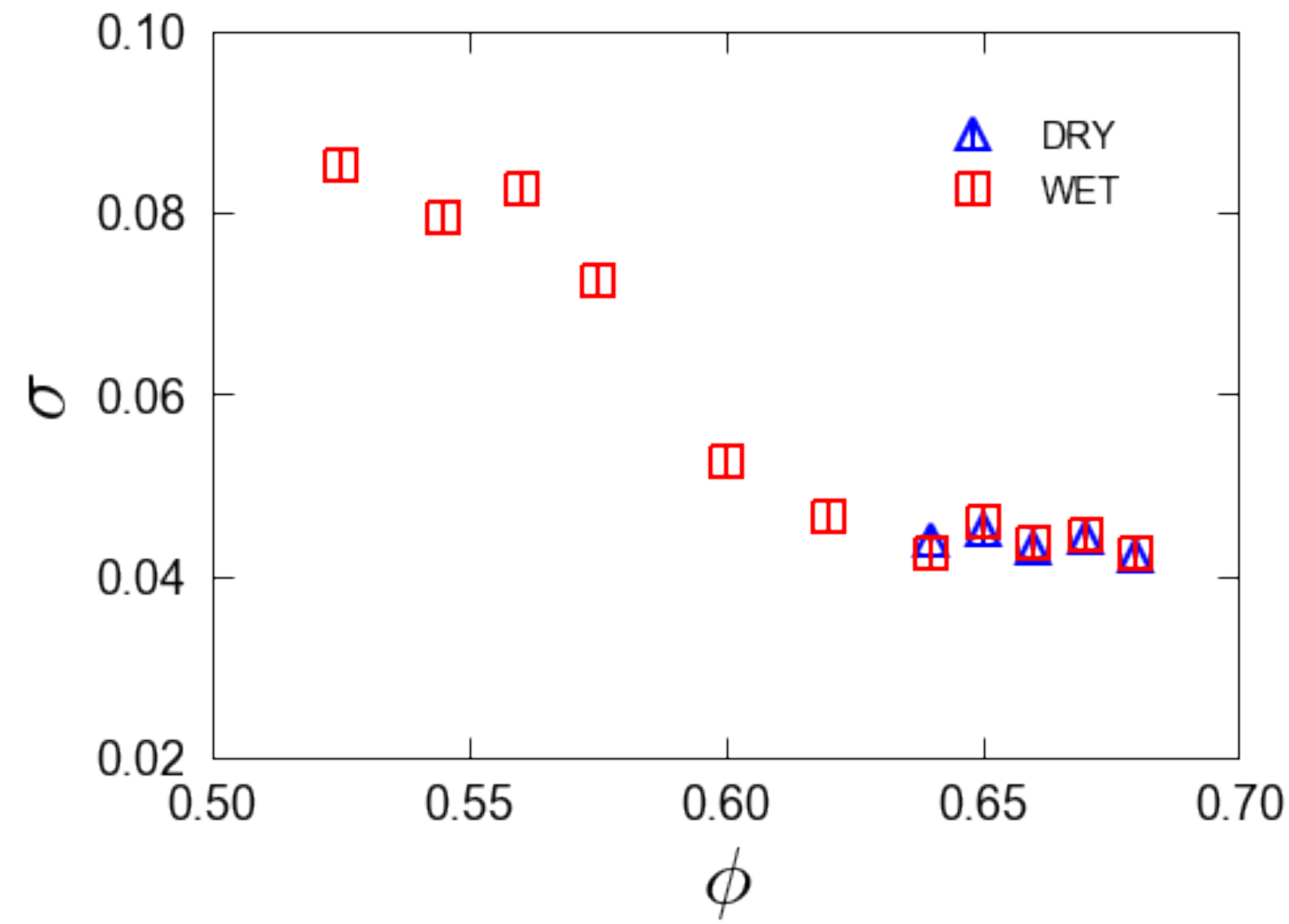
$p(V^*)$: distribution function of V^*



- Dry : The distribution does not depend on ϕ.
- Wet : The distribution becomes broad for low ϕ.

Volume of voronoi : distribution

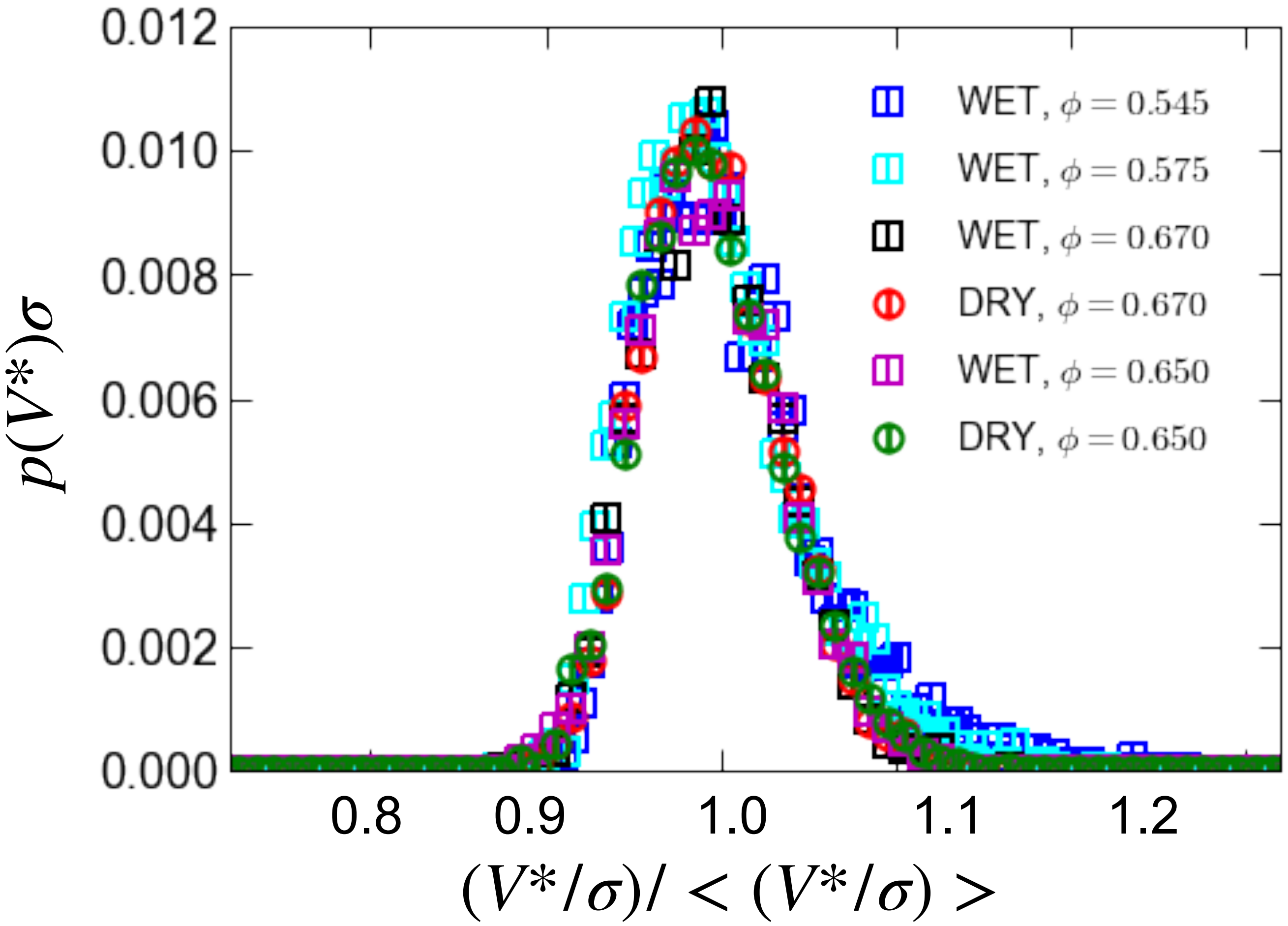


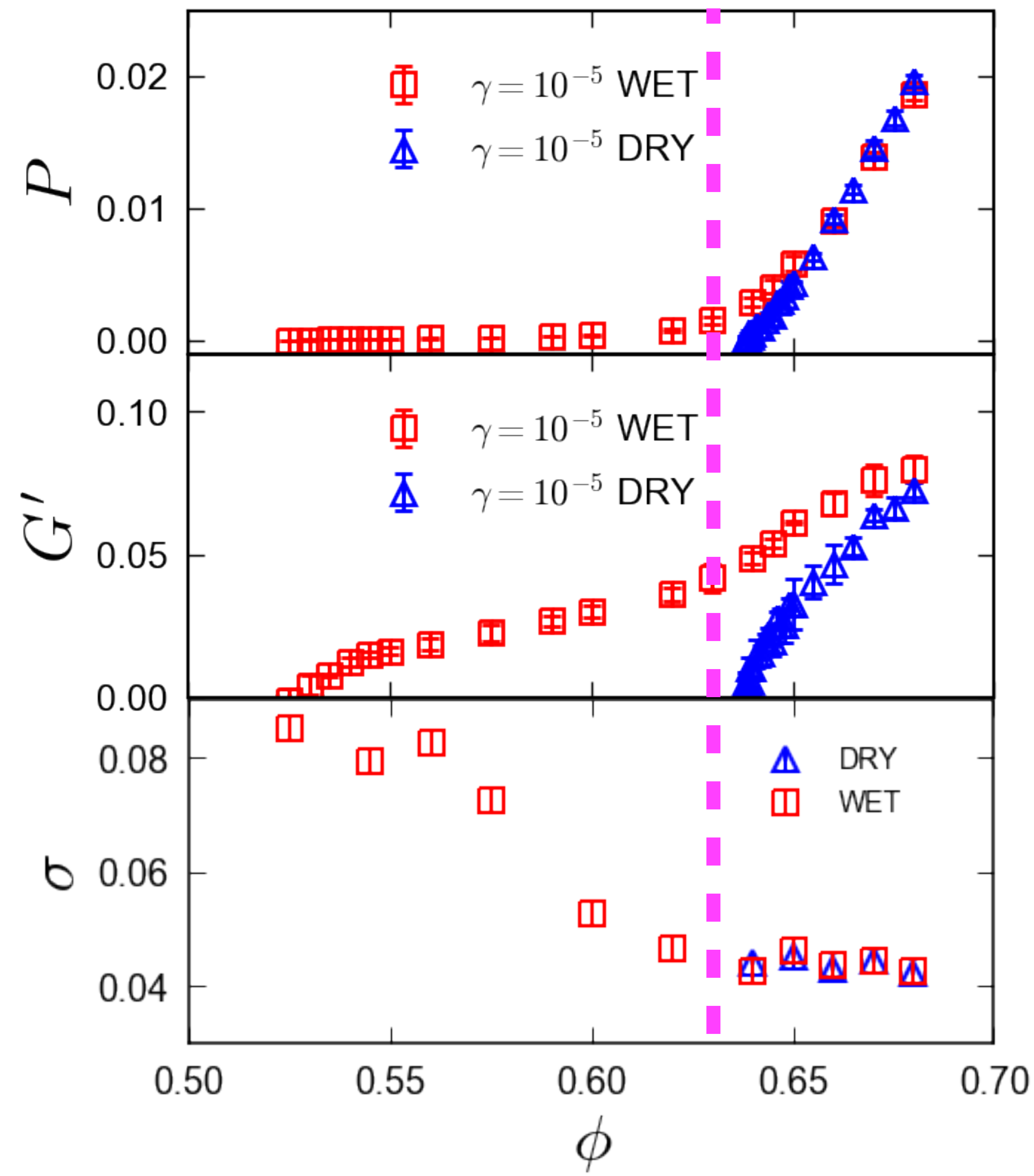


$p(V^*, \sigma(\phi)) = \sigma^{-1} p(V^*/\sigma)$: scaled frequency distribution of V^*
σ : variance of $p(V^*)$

- Scaled $p(V^*)$ has the same form for different parameters.
- Dry: σ is constant.
- Wet : As ϕ decreases, σ increases.

Volume of voronoi : distribution





■ Changes of shear modulus, and pressure correspond to change of variance.

Future Work : Exponent of P

Why are the exponents of P differs?

DRY : $P \propto (\phi - \phi_c)$ / WET : $P \propto (\phi - \phi_c)^{1/2}$

Theory : Dry grains

$$P \simeq \frac{S_D n^2}{2} \int_0^{d_0} dr r^D F^{\mathrm{el}}(r) g(r, \phi - \phi_c) \longrightarrow P \propto (\phi - \phi_c)$$

D : dimension, S_D : surface area of D-sphere, F^{el} : elastic interaction, $g(r)$: pair correlation function

M. Otsuki and H. Hayakawa, Phys. Rev. E (2009)

Can we determine the exponent of P in wet system?

- The interaction force changes. ($F^{\mathrm{el}}(r) \to F^{\mathrm{el}}(r) + F^{\mathrm{cap}}(r)$)
- The scaling $g(r, \phi - \phi_c)$ might change.

Work in progress…

Contents

Summary

- ☑ Scaling laws

DRY

$$\begin{cases} G \propto (\phi - \phi_c)^{1/2} \\ P \propto (\phi - \phi_c) \\ \lim_{\phi \to \phi_c} Z(\phi) - Z_{iso} = 0 \end{cases}$$

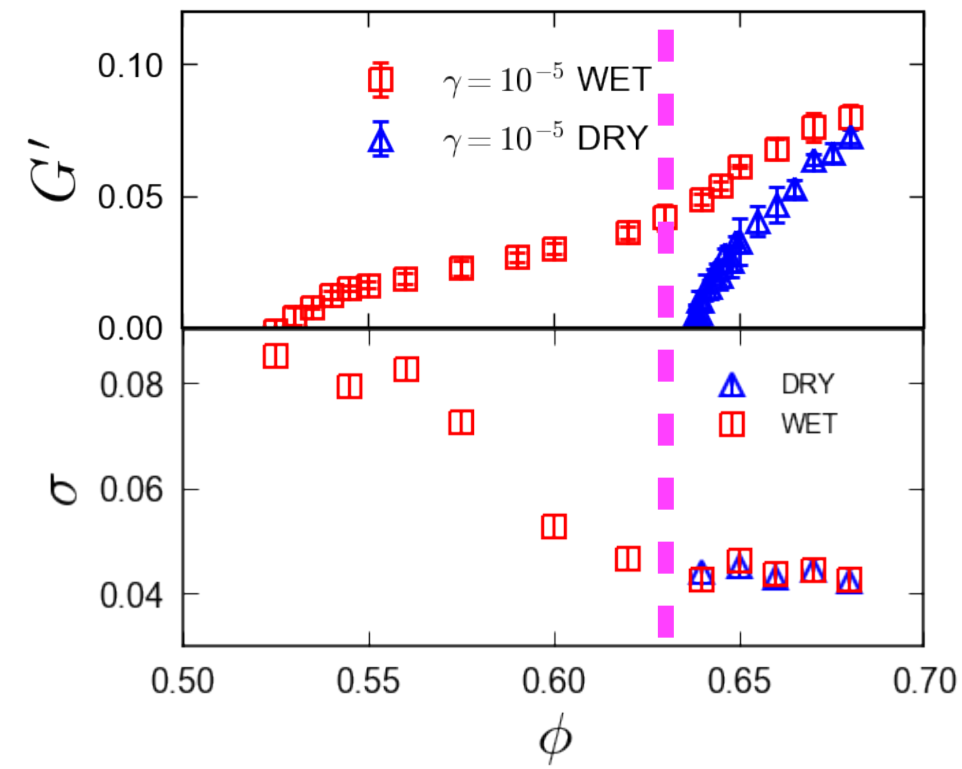
WET

$$\begin{cases} G \propto (\phi - \phi_c)^{1/2} \\ P \propto (\phi - \phi_c)^{1/2} \\ \lim_{\phi \to \phi_c} Z(\phi) - Z_{iso} > 0 \end{cases}$$

ϕ_c : density whose shear modulus appears

- ☑ Structure

  Change of shear modulus, pressure and contact number corresponds to change of variance.



- ☐ Future work : Exponents of P

PHYSICAL REVIEW E **90**, 032206 (2014)

Rheology of cohesive granular materials across multiple dense-flow regimes

$$p = \begin{cases} p_{\mathrm{QS}} + (p_{\mathrm{int}} + p_{\mathrm{coh},2}) & \text{for} \quad \phi \geqslant \phi_c \\ [(p_{\mathrm{inert}} + p_{\mathrm{coh},1})^{-1} + (p_{\mathrm{int}} + p_{\mathrm{coh},2})^{-1}]^{-1} & \text{for} \quad \phi < \phi_c, \end{cases}$$

$$p_{\mathrm{QS}} d/k = \alpha_{\mathrm{QS}} |\phi - \phi_c|^{\epsilon},$$

$$p_{\mathrm{int}} d/k = \alpha_{\mathrm{int}} \hat{\dot{\gamma}}^{2\epsilon/(\epsilon+\chi)},$$

$$p_{\mathrm{inert}} d/k = \frac{\alpha_{\mathrm{inert}} \hat{\dot{\gamma}}^2}{|\phi_c - \phi|^{\chi}}.$$

$$p_{\mathrm{coh},1} d/k = \alpha_{\mathrm{coh},1} \mathrm{Bo}^* \frac{|\phi - \phi_a|}{|\phi_c - \phi|},$$

$$p_{\mathrm{coh},2} d/k = \alpha_{\mathrm{coh},2} (\mathrm{Bo}^*)^{\epsilon/(\epsilon+\chi)}.$$